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Desktop Chaotic Systems: Intuition and Visualization

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Abstract

This paper presents a dynamic study of the Wildwood® Pendulum, a commercially available desktop system which exhibits a strange attractor. The purpose of studying this chaotic pendulum is two-fold: to gain insight in the paradigmatic approach of modeling, simulating, and determining chaos in nonlinear systems, and to provide a desktop model of chaos as a visual tool. For this study the nonlinear behavior of this chaotic pendulum is modeled, a computer simulation is performed, and an experimental performance is measured. An assessment of the pendulum in the phase plane shows the strange attractor. Through the use of a box-assisted correlation dimension methodology, the attractor dimension is determined for both the model and the experimental pendulum systems. Correlation dimension results indicate that the pendulum and the model are chaotic and their fractal dimensions are similar.

2. Motivation

When studying nonlinear dynamics and chaotic behavior, there is a need to clearly present the concepts of chaos, strange attractors, and correlation dimension to a non specialist. Many software tools exist to assist in the graphical display and visualization of chaotic phenomena. However, there is a definite need for a variety of desktop chaotic systems, that is, small scale devices that clearly demonstrate chaotic behavior to a variety of audiences, yet are easily modeled, and can be used as teaching tools.

Some of these desktop chaotic devices have been identified in Moon [1], for example, the coupled neon bulb chaotic relaxation oscillator and Rott's coupled pendula. "The neon bulb system consists of two neon bulb circuits coupled together and the two circuits exhibit chaotic dynamics in the form of the flashing bulbs." The other example, Rott's coupled pendula, "allows for transient chaotic behavior and is designed like a three-armed pendulum puppet". Both these systems are widely exercised to gain intuition into their dynamics and for a means to visualize chaos. Other constructable desktop systems can be found in [1]. In this paper another chaotic desktop system is presented, the Wildwood Pendulum®. This system is chosen because it exhibits chaotic motion, is commercially available, is inexpensive, and its unpredictable behavior is intuitive and easily observed.

3. System Description

The Wildwood Pendulum® is a ferromagnetic end mass on a flexible shaft oscillating above an electromagnet and five permanent magnets. The pivot point is a magnetic hinge with very low damping. Figure 1a shows an isometric drawing of the pendulum. Figures 1b and 1c provide top and side views which show the arrangement of the pendulum system and define key parameters. Table A provides a list of measured parameter values used in the pendulum model.

PARAMETER	DESCRIPTION	VALUE	UNITS
G	gravitational constant	9.8	m/s
w	width of base	93	mm
l	length of base	164	mm
M	mass of pendulum (rod and bob)	6.935	grams
ζ	pendulum damping factor	0.015	
k_p	field strength of bob magnet	300	gauss
R_p	length of pendulum bob	145	mm
R_b	distance from hinge to base plane	150	mm
N_m	number of permanent magnets in base	5	
R_m	radius of permanent magnets in base	26.77	mm
r_m	radius of each permanent magnet	6.35	mm
k_m	field strength of permanent magnet	100	gauss
R_e	radius of electromagnet	8.5	mm

Table A - Parameter Values for the Wildwood Pendulum®

The pendulum's chaotic behavior is observed when the bob is perturbed about the unforced equilibrium position, θ_o, which is shown in Figure 1c. As the bob moves toward θ_o, a sense coil engages the electromagnet which exerts an attractive force on the bob. As the bob moves away from θ_o, the sense coil disengages the electromagnet. This has the effect of adding energy to the system by increasing the acceleration of the pendulum bob. A diagram of the patented circuit for the electromagnet and sense coil is shown in Figure 2. While the electromagnet exerts an attractive force on the pendulum, the ferromagnets exert a constant repulsive force.

4. Equations of Motion

To better understand the dynamics of this pendulum and to improve our knowledge of chaotic systems, a mathematical analysis of this system is performed. To model the pendulum, two different coordinate systems are used for ease of implementation, and to relate the magnetic forces of the pendulum bob to the magnetic forces located in the pendulum base. The coordinate system for the base is shown in Figure 3a. The base plane coordinates have unit vectors $\hat{e}_x$, $\hat{e}_y$, and $\hat{e}_z$, in the directions of the x, y, and z axes respectively. Here, the origin is located in a horizontal plane parallel to the top of the permanent magnets and directly beneath the pendulum hinge point. Spherical coordinates for the pendulum bob are shown in Figure 3b. This coordinate system has unit vectors $\hat{e}_\theta$, $\hat{e}_\phi$, and $\hat{e}_r$ in the theta, phi and radial directions. The origin of the spherical coordinate system is at the pendulum hinge point, θ_o.

In order to simplify the model, the pendulum shaft is assumed to be rigid and of fixed length. Therefore, $\mathbf{r}$ is a constant and velocity, $\dot{\mathbf{r}}$, and acceleration, $\ddot{\mathbf{r}}$, in the radial direction are zero. In this case the spherical equations of motion [2] reduce to:

position: $\mathbf{r} = r\hat{e}_r$

velocity: $\mathbf{v} = r\dot{\theta}\hat{e}_\theta + r\dot{\phi}\sin\theta\,\hat{e}_\phi$

acceleration:
$$\mathbf{a} = (r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{e}_\theta + (r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\hat{e}_\phi$$
$$= \frac{F_\theta}{m}\hat{e}_\theta + \frac{F_\phi}{m}\hat{e}_\phi$$

Solving the acceleration equation for the acceleration terms $\ddot{\theta}$ and $\ddot{\phi}$, yields the following differential equations which may be integrated to obtain the pendulum's velocity and position.

$$\hat{e}_\theta: \quad \ddot{\theta} = \frac{1}{r}\left[\frac{F_\theta}{m} + r\dot{\phi}^2\sin\theta\cos\theta\right]$$

$$\hat{e}_\phi: \quad \ddot{\phi} = \frac{1}{r\sin\theta}\left[\frac{F_\phi}{m} - 2r\dot{\theta}\dot{\phi}\cos\theta\right]$$

To solve for the required derivatives, the forces exerted on the pendulum bob by the various magnets must be computed. For simplicity, it is assumed that the force of a particular magnet on the bob is proportional to the inverse of the distance squared between the pendulum and the magnet. The force equation is then:

$$\mathbf{F} = \frac{K_p K_m}{\Delta r^2} = \frac{K_p K_m}{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

Here, K_p is the force constant for the permanent magnet in the pendulum bob, and K_m is the force constant for permanent magnets in the base. The distance between the two magnets in question, Δr, is computed using base plane coordinates. The sign of the magnetic force constants determines whether the force is attractive or repulsive in nature. If signs are opposite, the magnets attract. If signs are the same, they repel.

This same method is used to compute the force of the electromagnet on the pendulum bob. As the magnetic pendulum bob accelerates towards the electromagnet, the above force equation is assumed; however, when the pendulum bob moves away from the electromagnet, the force is assumed to be zero because the electromagnet is switched off when $\dot{\theta} < 0$.

When all of the forces have been computed, forces in the same coordinate direction are summed. the resulting base plane force vector is then transformed to the spherical coordinate system of the pendulum bob. These forces are then used to compute the acceleration terms $\ddot{\theta}$ and $\ddot{\phi}$.

At this point, some discussion concerning the peculiarities of the model is in order. All of the parameters were measured, except for the magnetic strength of the electromagnet. This parameter is given a relative magnitude with respect to the measured values of the other magnets and observed behavior of the pendulum. Initially, an attempt was made to measure this value, however, due to the gaussmeter used, and the interacting field strengths of the base magnets, an accurate measurement of the electromagnet was not made.

Additionally, a single point source was used initially to represent each magnet. This was insufficient due to the interacting field strengths of the magnets, because the size of the base magnets is the same order of magnitude as the distance between the magnets. Representing each magnet in the base as a cluster of point sources produced behavior more accurately representative of the physical system. This clustering of point sources is shown in Figure 4.

The simulation of the Wildwood Pendulum© was performed using the MATLAB® 4.0 matrix math package. A copy of the simulation code is given at the end of this paper. Figure 4 gives a typical response of the simulation in the x-y plane. This time response of the system was obtained by integrating values of the differential equations using a Runge-Kutta-Fehlberg method. Figure 5 shows a phase plane representation of this simulation data for $\dot{\theta}$ vs. θ. This phase plane representation gives evidence of the chaotic nature of this pendulum simulation. This simulation data is used in the dimension determination section to calculate an estimate of the correlation dimension.

5. Experimental Data

To analyze the chaotic behavior of the actual pendulum system, data was gathered from the Wildwood Pendulum© using the photodiode circuit shown in Figure 6. As the pendulum passes through the beam of light, the voltage across the photodiode decreases in proportion to the amount of light observed. This is equivalent to capturing the dynamics of the pendulum in the plane of the light beam. The time series plot of the voltage over time for the pendulum is shown in Figure 7. The time between zero crossings of the voltage signal represent the chaotic nature of the pendulum system.

To compare this voltage data with the simulation data, a phase plane portrait of the voltage data is reconstructed using 10000 data points. To reconstruct the attractor, an independent time series of data was generated using the following time delay embedding method[3]:

$$v(t) = (x(t), x(t+T), ..., x(t+(m-1)T).$$

where v(t) is the independent time series, x(t) is the voltage signal, m is the embedding dimension, and T is the reconstruction time delay. For this voltage data, the values of m=4 and T=1 were used for the reconstruction. This reconstruction is used in the next section for determining the dimension of the experimental system.

6. Dimension Determination

Verification of the fractal behavior of this pendulum is provided using a box-assisted correlation dimension algorithm [4,5] to determine the dimension of both the simulation data and the measured pendulum data. Two box-counting correlation dimension algorithms are used to ensure the dimension is accurate.

For the correlation dimension method, 4000 data points are used and the first 1000 points are thrown out as transient. Using 3000 data points for both the experimental data and the simulation data allows for accurate prediction of correlation dimension up to approximately 7, according to the following equation from Eckmann and Ruelle[6] which states:

Maximum Accurate Dimension < 2 log(number of data points).

With this limit in mind, the correlation dimension, (or correlation exponent), was calculated for both pendulum systems using a Grassberger-Procaccia correlation integral.[7] This correlation exponent is closely related to the fractal dimension and gives a lower bound to the fractal dimension. The Grassberger-Procaccia algorithm using Theiler's method[4,5] considers the spatial correlation between pairs of points on a reconstructed attractor, and is measured with the following correlation integral:

$$C(N,r) = \frac{2}{N(N-1)} \sum H(r - \| x_i - x_j \|),$$

where H(x) is the Heaviside step function. The summation counts the number of pairs (x_i, x_j) for which this distance is less than r, where r is the box size. Then C(N,r) scales like a power of v so that:

$$C(N,r) = r^v$$

where v is the correlation dimension which is the slope of the log-log plot of C(N,r) vs. r.

The correlation dimension is calculated first for the experimental pendulum data with Theiler's box-assisted method, using embedding dimensions from m=4 to m=7. The correlation dimension from this method is approximately 3.47. The same correlation dimension method is used to calculate the dimension for the simulation data. The log-log plots of C(N,r) vs. r for the simulation data only converged for embedding dimensions of m=4 and m=5, with a resulting correlation dimension of 3.2.

To corroborate the dimension results for the simulation data, another box-counting method using the Grassberger-Procaccia algorithm was implemented. This method was used only on the simulation data. From this analysis the simulation equations exhibit a fractal dimension of approximately 3.3.

From these dimension results it is concluded that the modeled pendulum equations closely match the behavior exhibited by the actual Wildwood® chaotic pendulum. Both the simulated data and the measured data resulted in a similar correlation dimension of approximately 3.4.

7. Conclusions

From the study of desktop chaotic systems insight is gained in modeling, simulating, and analyzing chaos in nonlinear systems. The Wildwood® pendulum is chosen for analysis because its chaotic behavior is easily observed and its fractal dimension has not been determined previously. For this pendulum, the model equations are given and analyzed using both correlation dimension techniques and Lyapunov characteristic exponent methods. Experimental data is gathered for the pendulum, and the attractor dimension of the measured data is calculated. Both the modeled pendulum and the measured pendulum data represent a chaotic system with similar attractor dimension. This process was made easier with the use of desktop systems like the chaotic pendulum, which provided a system to easily visualize chaotic behavior.

8. Acknowledgments

The authors would like to thank Walter Merrill for his advice on this paper, Donald Simon for help in capturing experimental data, and the manufacturers of the Wildwood° pendulum, Alan and Roger Andrews. With the assistance of the Andrews Manufacturing Co., all the pendulum components for this experiment, as well as patented circuit diagrams, were made available to NASA. Thank you all for your assistance.

9. References

[1] Francis C. Moon, Chaotic and Fractal Dynamics: An Introduction for Applied Scientists and Engineers, John Wiley and Sons, Inc., 1992.

[2] Donald T. Greenwood, Principles of Dynamics, Prentice-Hall, Inc., 1965.

[3] N.H. Packard, et. al., "Geometry from a Time Series", Physical Review Letters, Volume 45, pp. 712-716, 1980.

[4] James Theiler, "Efficient Algorithm for Estimating the Correlation Dimension from a Set of Discrete Points", Physical Review A, Volume 36, No. 9, p. 4456-4462.

[5] James Theiler, "Spurious Dimension from Correlation Algorithms Applied to Limited Time-Series Data", Physical Review A, Volume 34, No. 3, p.2427-2432.

[6] J.-P. Eckmann and D. Ruelle, "Fundamental Limitations for Estimating Dimensions and Lyapunov Exponents in Dynamical Systems", Physica D, Volume 56, 1992, pp. 185-7.

[7] Peter Grassberger and Itamar Procaccia, "Characterization of Strange Attractors", Physical Review Letters, Volume 50, No. 5, p. 346-349.

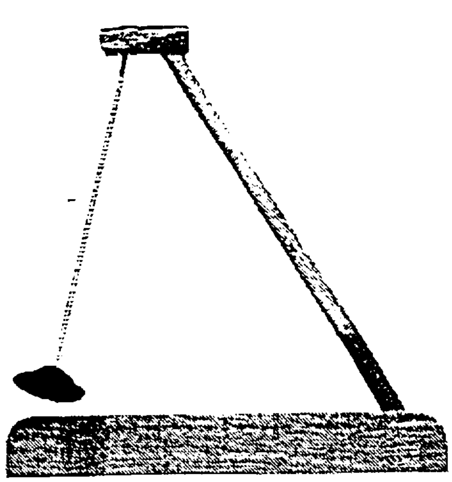

Figure 1a. The Wildwood Chaotic Pendulum

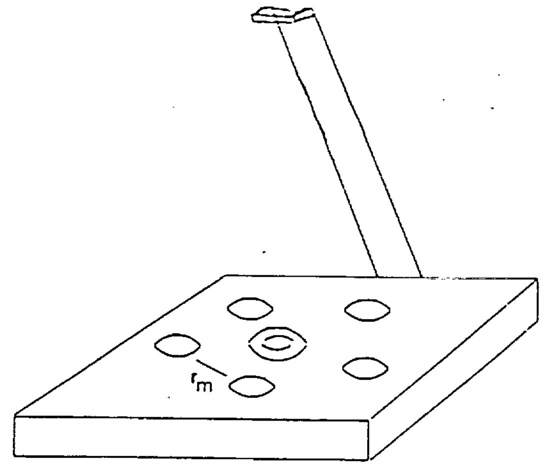

Figure 1b. Top View of the Chaotic Pendulum

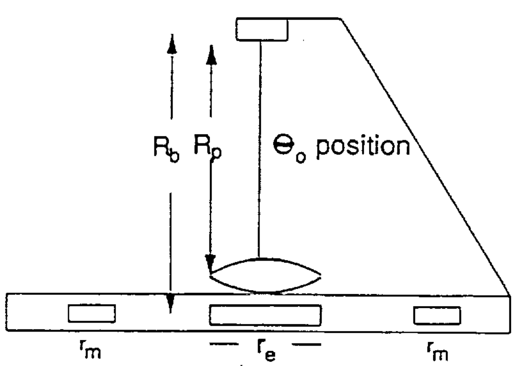

Figure 1c. Side View of the Chaotic Pendulum

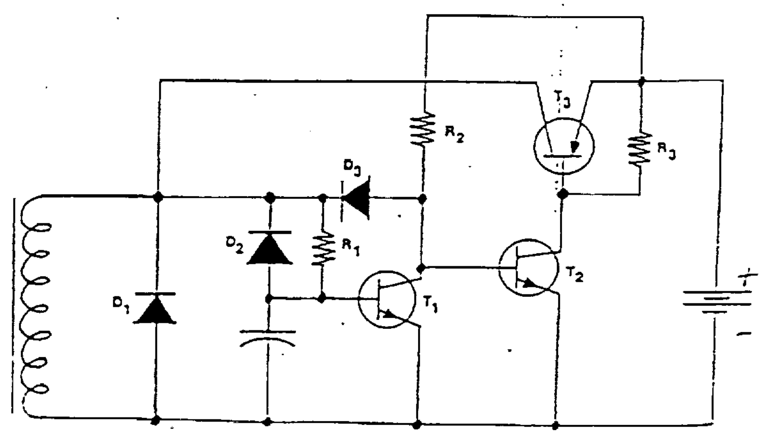

Figure 2. The Electromagnet and Sense Coil Circuit

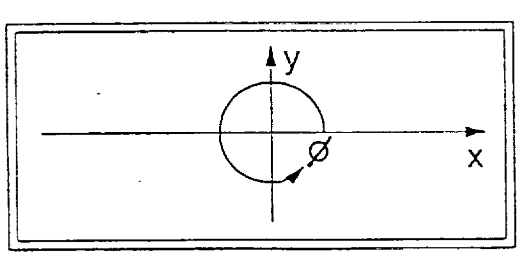

Figure 3a. Base Plane Coordinate System

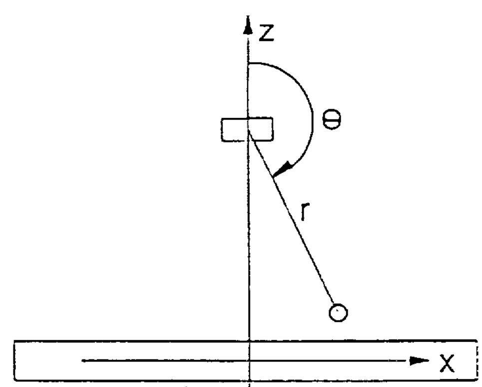

Figure 3b. Spherical Coordinate System

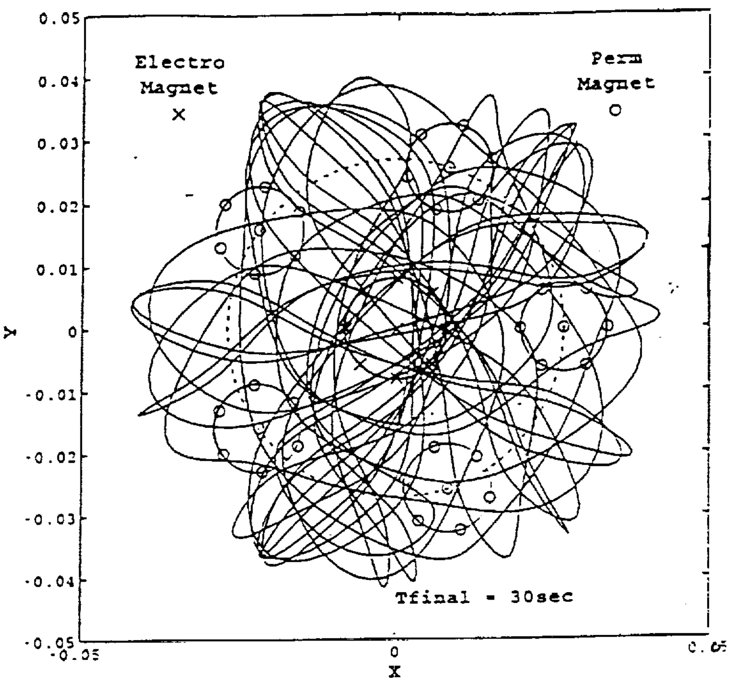

Figure 4. X-Y Plane Plot of Pendulum Simulation

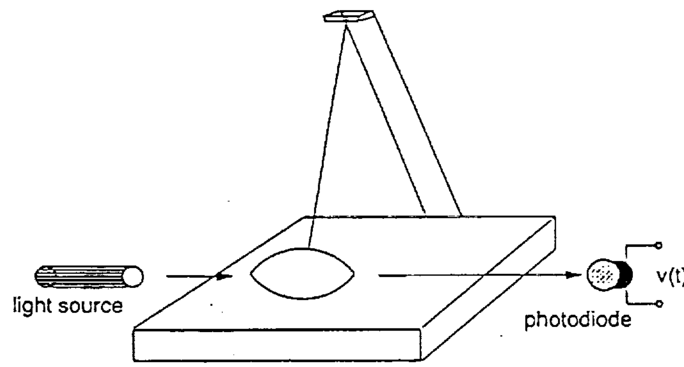

Figure 6. Photodiode Circuit for Experiment

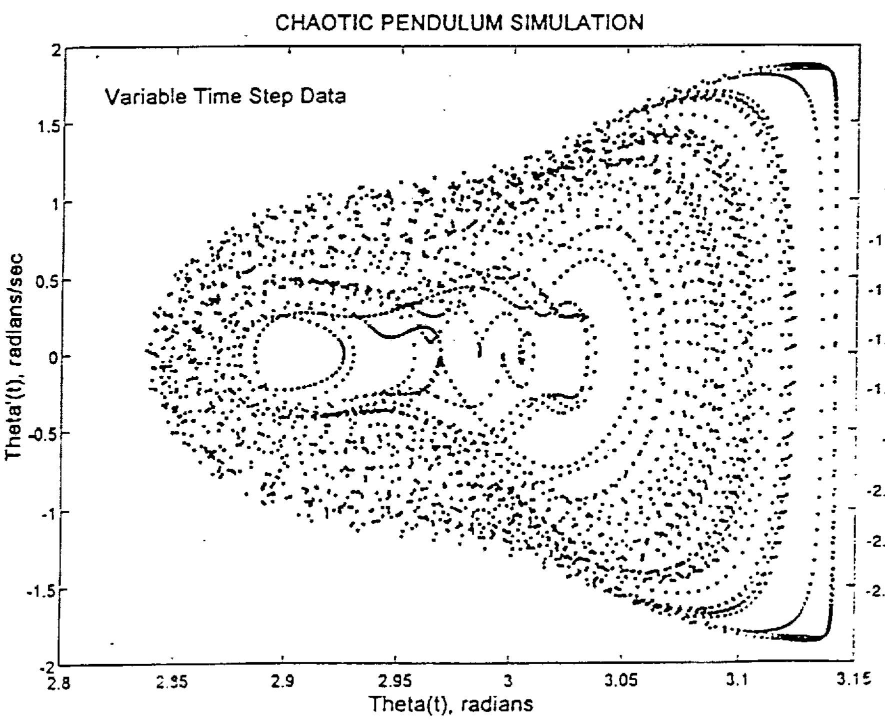

Figure 5. Phase Plane Plot of the Pendulum Simulation

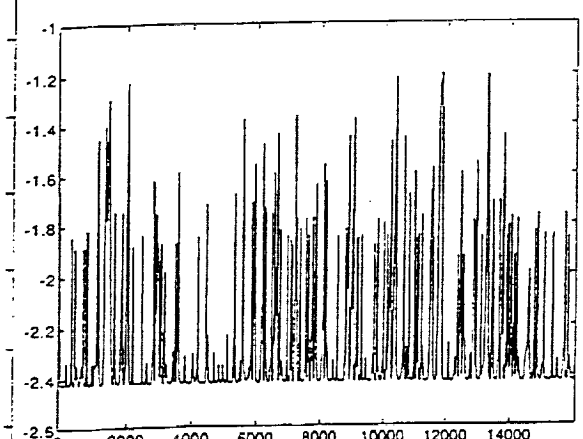

Figure 7. Voltage Across Photodiode Versus Time

```
%PEND3RUN.M ... Matlab M-file used to Run Wildwood Pendulum Simulation

 %Global Variables ...
 global G;                  %Gravitational Cosntant
% global W;                 %Weight of Pendulum
 global M;                  %Mass of Pendulum
 global Kd;                 %Pendulum Damping Constant
 global Kp;                 %Pendulum Magnetic field Strength Const (Perm Magnet)
 global Rp;        -        %Pendulum Radius
 global Rb;                 %Distance from Pendulum Support to Base Plane

% global Nm;                %Number of Permanent Magnets
% global Mm;                %Number of Point Sources/Perm Magnet
% global NMm;               %Total Number of Point Sources for Perm Magnets
% global Rm;                %Radius of Permanent Magnets in Base Plane
% global rm;           %Radius of Individual Permanent Magnet
 global Xm;                 %X Loc of Permanet Magnet Sources in Base Plane Coords
 global Ym;                 %Y Loc of Permanet Magnet Sources in Base Plane Coords
 global Km;                 %Field Strength Const of Individual Perm Magnets)

% global NMe;               %Total Number of Point Sources for Electro Magnet
% global re;           %Radius of the Electro Magnet
 global Xe;               . %X Loc of Electromagnet Sources in Base Plane Coords
 global Ye;                 %Y Loc of Electromagnet Sources in Base Plane Coords
 global Ke;                 %Field Strength Const of Electromagnet

%If "CONTINUE" is set then start simulation from where it left off ...
if exist('CONTINUE')~=1;

 %K0 is used to Vary different parameters in the simulation
 if exist('K0')~=1; K0=1, end;

 %Define Parameters for Pendulum Bob
 G  = 9.8;                  %meters/sec/sec
 M  = 0.007;                %kg
 W  = M*G;                  %newtons
 Kp = 3.0e-3;
 Rp = 0.145;                %meters
 Rb = 0.150;                %meters

 %Calculate some reasonable numbers for damping
 % Damping = -Kd*THd
 zeta = 0.015;              %damping constant for pendulum
 wn   = sqrt(G/Rp);         %undamped natural frequency of simple pendulum
 Kd   = 2*zeta*wn;          %kg/sec

 %Define Parameters for Permanent Magnets
 Rm = 0.027;                %meters
 rm = 0.007;                %meters
 %Use 5 Evenly Spaced Magnets.  Each Magnet has 7 Point Sources.
  Nm  = 5;                  %Number of Permanent Magnets (best odd)
  Mm  = 7;                  %Number of Point Sources/Magnet (even perimeter best)
  NMm = Nm*Mm;                       %Total Number of Magnets
  %Calculate Position of Magnets in Base Plane Coords
   THc = 2*pi*[0:Nm-1]/Nm;          %Theta Position Each Magnet
   %Convert Magnet Position From Polar to Cart Coords
   [Xc,Yc]=pol2cart(THc,Rm*ones(size(THc)));
  %Calculate Position of Point Sources relative to center of each magnet
  % Save one point source for the center
   THm=2*pi*[0:Mm-2]'/(Mm-1)+pi;           %Theta Position
   %Convert Point Source Position to Cart Coords
   [Xt,Yt]=pol2cart(THm,rm*ones(size(THm)));
  %Source Absolute Position = Source Relative Position + Magenet Mid Point
  % Set Source at Mid Point of each magnet
```

```
  Xm=[Xc; ones(size(Xt))*Xc+Xt*cos(THc)-Yt*sin(THc)];  Xm=Xm(:)';
  Ym=[Yc; ones(size(Yt))*Yc+Xt*sin(THc)+Yt*cos(THc)];  Ym=Ym(:)';
 %Force Const for Each Magnet
 % Need same sign as Kp for repulsive force
  Km = (Kp/3)*ones(size(Xm))/Mm;

%Define Parameters for Electro Magnets
re = 0.008;                    %meters
%Use a Single Magnet with 9 Point Sources.
NMe = 9;                       %Total Number of Magnets
 %Calculate Position of Point Sources relative to center of magnet
 % Save one Point Source for Center of each magnet
  THe=2*pi*[0:NMe-2]'/(NMe-1)+pi;          %Theta Position
  %Convert Point Source Position to Cart Coords
  [Xe,Ye]=pol2cart(THe,re*ones(size(THe)));
 %Set Point Source at center of each magnet
  Xe=[0 Xe(:)'];
  Ye=[0 Ye(:)'];
 %Force Const for the Magnet
 % Need sign opposite of Kp for attraction
  Ke = -Kp/40*ones(size(Xe))/NMe;

%Define Start (T) and Stop (Tfinal) Times for Integration Routine
T=0; Tfinal=10; Tc2Ts=0;
%Define Initial Conditions of the Pendulum Relative to Hinge Point
%... [Theta ThetaDOT Phi PhiDOT] (angles in radians)
%     0 >= Theta <= pi
%    -pi >=  Phi  <= pi
 STATES=[pi-pi/11 0 pi/4 0];
end;

%Break out of Integration Routine to save data and report to user
% every DTR seconds
DTR=(Tfinal-max(T))/20;

%Run System Simulation
pend2plt; set(gca,'DrawMode','fast'); drawnow
if ( max(T)<Tfinal-eps )
  Tc0 = clock; Ts0=max(T);
  while (max(T)+eps<Tfinal)
   %Calculate Time Limits for Current Segment of Simulation
    n=length(T); t0=T(n); t1=min(t0+DTR,Tfinal);
   %Report Status to the User
    note=['Running the Simulation from T=',n2s(t0),' to T=',n2s(t1)];
    if Tc2Ts>0; note=[note '; Clock/Sim Time=',n2s(Tc2Ts)]; end
    disp(note)
   %Run Simulation for Specified Segment
    [t,states] = ode23('pend3ode',t0,t1,STATES(n,:),2.0e-4);
   %Append Results to Existing States
    [n,m]=size(states); T=[T;t(2:n)]; STATES=[STATES;states(2:n,:)];
   %Plot Current Data
    hold on;
    plot(Rp*sin(states(:,1)).*cos(states(:,3)),  ...
         Rp*sin(states(:,1)).*sin(states(:,3)),'m.',  ...
         'EraseMode','none');
    hold off;
    if exist('hTf')==1;
      set(hTf,'String',['Tfinal = ',num2str(max(T)),'sec']);
    end;
    drawnow
   %Give User a Measure  of the System Performance
    Tc2Ts = fix(etime(clock,Tc0)/(max(t)-Ts0));
  end
end;
```

```
function [derivs]=pend2ode(t,states)
%
% FUNCTION: pend2ode
%
% PURPOSE: Compute derivatives for a conical pendulum in a chaotic system.
%
% SYNTAX:  [derivs]=pend2ode(t,states)
%

% Global Variables ...
 global G;             %Gravitational Cosntant
% global W;            %Weight of Pendulum
 global M;             %Mass of Pendulum
 global Kd;            %Pendulum Damping Constant
 global Kp;            %Pendulum Magnetic field Strength Const (Perm Magnet)
 global Rp;            %Pendulum Radius
 global Rb;            %Distance from Pendulum Support to Base Plane

% global Nm;           %Number of Permanent Magnets
% global Mm;           %Number of Point Sources/Perm Magnet
% global NMm;          %Total Number of Point Sources for Perm Magnets
% global Rm;           %Radius of Permanent Magnets in Base Plane
% global rm;        %Radius of Individual Permanent Magnet
 global Xm;            %X Location of Permanet Magnet Sources in Base Plane Coords
 global Ym;            %Y Location of Permanet Magnet Sources in Base Plane Coords
 global Km;            %Field Strength Const of Individual Perm Magnets)

% global NMe;          %Total Number of Point Sources for Electro Magnet
% global re;        %Radius of the Electro Magnet
 global Xe;            %X Location of Electromagnet Sources in Base Plane Coords
 global Ye;            %Y Location of Electromagnet Sources in Base Plane Coords
 global Ke;            %Field Strength Const of Electromagnet

%Extract States from "state" vector
 %THETA: measured counter-clockwise from the x-axis in the x-y plane)
  TH  = states(1);  %Theta
  THd = states(2); %Theta dot
 %PHI: measured clockwise from the z-axis in the y-z plane)
  PH  = states(3);  %Phi
  PHd = states(4);  %Phi dot

%Calculate Location of Pendulum in Cart Base Plane Coords
 Xp = Rp*sin(TH)*cos(PH);
 Yp = Rp*sin(TH)*sin(PH);
 Zp = Rb+Rp*cos(TH);

%Calculate the Distance Between the Permanent Magnets and the Pendulum
 dXm = Xp*ones(size(Xm))-Xm;%     %x dist pend to mag
 dYm = Yp*ones(size(Ym))-Ym;%     %y dist pend to mag
 rpsq = dXm.^2 + dYm.^2;%         %dist in plane squared
 rp   = sqrt(rpsq);%              %dist in plane
 rmsq = rpsq + Zp*Zp;%            %dist pt to pt squared
 rm   = sqrt(rmsq);%              %dist pt to pt

%Calculate Forces of Permanent Magnets on Pendulum
% Assume that the Force is proportional to K1*K2/(r*r),
% where 'r' is the distance between the pendulum and a magnet
% If Kp*Km < 0 force is attracting,
% If Kp*Km = 0 No force
% If Kp*Km > 0 force repelling
 Fm  =  Kp*Km./rmsq;%    %Total force
%Reduce the Forces to Their Cart Coord Components
 Fmx = Fm.*dXm./rm;%    %x force in base plane coords
 Fmy = Fm.*dYm./rm;%    %y force in base plane coords
```

```
Fmz = Fm.*Zp./rm;%        %z force in base plane coords

%Caluculate Force of Electromagnet cluster (at 0;0) on pendulum
% Assume that the Force is proportional to Kp*Ke/(r*r),
% where 'r' is the distance between the pendulum and a electromagnet
% If Kp*Ke < 0 force is attracting,
% If Kp*Ke = 0 No force
% If Kp*Ke > 0 force repelling
dXe = Xp*ones(size(Xe))-Xe;         %x dist pend to mag
dYe = Yp*ones(size(Ye))-Ye;         %y dist pend to mag
rpsq = dXe.^2 + dYe.^2;             %dist in plane squared
rp   = sqrt(rpsq);            %dist in plane
resq = rpsq + Zp*Zp;                %dist pt to pt squared
re   = sqrt(resq);            %dist pt to pt
%The Electro Magnet is on only if the Pendulum Bob is moving
% toward it.
if ( THd > 0 )
  Fe =  Kp.*Ke./resq;         %Total force
else
  Fe = 0;
end
%Reduce the Forces to Their Cart Coord Components
Fex = Fe.*dXe./re;%        %x force in base plane coords
Fey = Fe.*dYe./re;%        %y force in base plane coords
Fez = Fe.*Zp./re;%         %z force in base plane coords

%Calculate Gravitational Force on Pendulum
Fgz = -M*G;%    %Force Due to Gravity in base plane coords

%Calculate Total Forces in Base Plane
Ftx = sum( [ Fmx, Fex ] );%                %x Force in base plane coords
Fty = sum( [ Fmy, Fey ] );%                %y Force in base plane coords
Ftz = sum( [ Fmz, Fez, Fgz ] );%     %z Force in base  plane coords

%Transform Base Plane Forces to Spherical Coords
% Define Transformation Matrix
 TM=[ cos(PH)*cos(TH)  sin(PH)*cos(TH)  -sin(TH)
        -sin(PH)         cos(PH)         0
     cos(PH)*sin(TH)  sin(PH)*sin(TH)   cos(TH)
   ];
%Calculate the Forces in the Pendulums Coord
FTH = TM(1,:)*[Ftx;Fty;Ftz];
FPH = TM(2,:)*[Ftx;Fty;Ftz];
FR  = TM(3,:)*[Ftx;Fty;Ftz];

%Calculate the Derivatives
RpSq = Rp*Rp; THdc = THd^3;
THdd = (FTH/M + Rp*PHd*PHd*sin(TH)*cos(TH) -Kd*THd*Rp)/Rp;
if ( TH ~= 0 )
  PHdd = (FPH/M - 2*Rp*THd*PHd*cos(TH))/(Rp*sin(TH));
end
derivs = [ THd THdd PHd PHdd];
```

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13. ABSTRACT ***(Maximum 200 words)***

This paper presents a dynamic study of the Wildwood° Pendulum, a commercially available desktop system which exhibits a strange attractor. The purpose of studying this chaotic pendulum is twofold: to gain insight in the paradigmatic approach of modeling, simulating, and determining chaos in nonlinear systems, and to provide a desktop model of chaos as a visual tool. For this study the nonlinear behavior of this chaotic pendulum is modeled, a computer simulation is performed, and an experimental performance is measured. An assessment of the pendulum in the phase plane shows the strange attractor. Through the use of a box-assisted correlation dimension methodology, the attractor dimension is determined for both the model and the experimental pendulum systems. Correlation dimension results indicate that the pendulum and the model are chaotic and their fractal dimensions are similar.

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