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THE EFFICIENCY OF A WIND TUNNEL.

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Obviously the most satisfactory definition of the efficiency of a wind tunnel should give as extreme limits of numerical evaluation, zero and unity. It is highly desirable that the mathematical expression resulting from such a definition should show how such factors as the motor and fan efficiencies affect the overall efficiency; and, furthermore, one should be able to develop from such an expression the efficiency of the "tube" alone.

If, by some means, a steady state of motion of a perfect fluid were established in an ideal wind tunnel, there would be no losses, and the motion would persist indefinitely. In the actual tunnel, the function of the motor-fan group is to overcome the total loss of head in the tube due to friction and eddies. The <u>useful</u> energy should therefore be taken as the kinetic energy E_c in the experimental chamber, that is

unit time

not as used ordinarily

$$E_c = \frac{\rho_1}{2g} A_1 V_1^3 \qquad (1)$$

where ρ_1 is the specific weight of the air in the experimental chamber A_1 the cross-section area of the chamber, and V_1 the mean velocity.

density

The total amount of energy transformed in unit time by the complete combination of motor, fan and tube, is

$$E_c + E_s$$

where E_s is the energy supplied to the motor, and going ultimately to heat the air. The overall efficiency of the complete unit may then be defined as

$$\eta_o = \frac{E_c}{E_c + E_s} \tag{2}$$

If the brake power and efficiency of the motor be respectively designated by P_m and η_m, it is evident that we can also write

$$\eta_o = \frac{E_c}{E_c + \frac{P_m}{\eta_m}} \tag{3}$$

In defining the efficiency of the tube and fan as a separate unit, we need only charge to the unit the amount of energy given by

$$E_c + E_s - (E_s - P_m) = E_c + P_m = E_c + \eta_m E_s$$

since P_m is the brake power absorbed by the fan. For this unit with efficiency η, we have

$$\eta = \frac{E_c}{E_c + P_m} = \frac{E_c}{E_c + \eta_m E_s} \tag{4}$$

It is now easy to see that in order to obtain a definition of the tube efficiency, we need only charge to the tube the kinetic energy in the experimental chamber plus the work done by the propeller on the air. That is, we subtract from the denominator of the expression for overall efficiency the motor and propeller losses; thus

$$E_c + E_s - (E_s - P_m) - (P_m - P_u) = E_c + P_u$$

where P_u is the useful power delivered by the fan, and must be equal to the total loss of head h_f due to friction, multiplied by the weight of air handled in unit time (since we are using ? engineering units). That is

$$P_u = \rho h_f A V$$

where ρ, A, and V are, respectively, the specific weight, cross section area and mean velocity at any section of the tube. The tube efficiency η_t is, then

$$\eta_t = \frac{E_c}{E_c + P_u}$$

But also

$$E_c = \frac{\rho_1}{2g} A_1 V_1^3 = \left(\frac{\rho_1}{2g} V_1^2\right) A_1 V_1$$

And since

$$\Delta p_1 = \frac{\rho_1}{2g} V_1^2$$

Then

$$E_c = \Delta p_1 A_1 V_1 = \rho_1 h_c A_1 V_1$$

where Δp_1 and h_c are, respectively, the theoretical pressure drop with no friction and the velocity head in the experimental chamber. On account of continuity

$$\rho_1 A_1 V_1 = \rho A V$$

Hence

$$P_u = \rho_1 h_f A_1 V_1 \tag{5}$$

Therefore the tube efficiency may be written

$$\eta_t = \frac{1}{1 + \frac{h_f}{h_c}} \tag{6}$$

which shows that the efficiency of the tube is a function only of the ratio of the loss of head due to friction, to the velocity head in the experimental chamber. In design work, it is this last equation which may be applied to predict the tube efficiency, since the sum of the various losses in various parts of the tube can be estimated as a fraction of the total head, from empirical data and formulas.

The useful power of a propeller working under static conditions is usually defined as the product of the thrust and mean air velocity in the disc of rotation. The thrust has been shown to be very nearly equal to the product of the mean difference of pressure fore and aft of the disc and the area of the disc. The useful power may then be written

$$P_u = \Delta p_2 \ A_2 \ V_2$$

If ρ = constant,

$$P_u = \Delta p_2 \ A_1 \ V_1 \tag{7}$$

Equating (5) and (7) it appears that, where a wind tunnel propeller is used, the increment of head at the disc must be equal to the drop of head due to losses throughout the tube. If we designate the propeller efficiency by η_p

$$P_u = P_m \ \eta_p$$

But, since $P_m = E_s \eta_m$, we have

$$\eta_t = \frac{E_c}{E_c + E_s \eta_m \eta_p} \qquad (8)$$

It has been shown theoretically,* and verified experimentally, that for a propeller working under static conditions,

$$\eta p = \text{const.}$$

and

$$P_m \propto V^3$$

If, then, the conditions of flow permit the assumption of constant density throughout the tube

$$\frac{E_c}{P_m} = \text{const.}$$

$$\frac{E_c}{P_u} = \text{const.}$$

Therefore,

$$\eta = \text{const.}$$

and

$$\eta_t = \text{const.}$$

The overall efficiency η_o, however, cannot remain constant and independent of the velocity unless the efficiency of the motor remains constant. (Interesting and important discussions of the laws of similitude of wind tunnels and propellers can be found in the references given in the footnote.)

* See "The Design of Wind Tunnels and Wind Tunnel Propellers," by Edward P. Warner, F. H. Norton and C. M. Hebbert, N.A.C.A. Report No. 73: also "The General Theory of Blade Screws," by George de Bothezat, N.A.C.A., Report No. 29.

Using the foregoing definitions for the overall efficiency and the efficiencies of the separate units of a wind tunnel installation, it is seen that in each case, as the losses approach zero the efficiency approaches a numerical value of unity, and, vice versa, as the losses approach infinity the efficiency approaches zero. Two dissimilar wind tunnel installations may happen to have the same combined tube-fan efficiency; but by determining the tube efficiency of each, we can ascertain the possibility of improving the performance of one of the installations. The formulas thus allow one to obtain a satisfactory basis of comparison between the separate units.

As a result of the foregoing transformations we are able to write down the efficiencies in the following chronological order:

Tube efficiency--

$$\eta_t = \frac{1}{1 + \frac{P_u}{E_c}} = \frac{1}{1 + \frac{h_f}{h_c}}$$

Fan-tube efficiency--

$$\eta = \frac{1}{1 + \frac{P_m}{E_c}} = \frac{1}{1 + \frac{h_f}{h_c \eta_p}}$$

Overall efficiency--

$$\eta_c = \frac{1}{1 + \frac{E_s}{E_c}} = \frac{1}{1 + \frac{h_f}{h_c \eta_p \eta_m}}$$

A P P E N D I X.

The merits of a wind tunnel installation have been expressed in terms of the ratio of kinetic energy of the stream in the experimental chamber to the power consumed by the <u>complete installation</u>. This figure is generally termed the "Energy Ratio." It is therefore possible to write down the definition of overall efficiency in terms of the energy-ratio; thus, if the energy ratio is

$$R = \frac{E_c}{E_s}$$

then

$$\eta_o = \frac{E_c}{E_c + E_s} = \frac{1}{1 + \frac{E_s}{E_c}} = \frac{1}{1 + \frac{1}{R}} = \frac{R}{1+R}$$

The energy ratio (and as well, of course, the efficiency) should be computed upon the basis of the true wind speed in the tunnel; for, clearly, the kinetic energy of the stream is not equal to the product of the velocity head for standard conditions and the indicated velocity (where a calibrated gage is used) unless the indicated velocity happens to be the true velocity. However, in practice, when a 60°F. Standard is used, the error involved will usually be so small that thermodynamic corrections need not be applied.

The writer is indebted to Professor Edward P. Warner of the Massachusetts Institute of Technology for the following tabulation of the energy ratios of some well-known wind tunnels. An additional column of corresponding efficiencies is also given.

TABULATION.

Installation	Energy Ratio	Overall Efficiency (%)
Mass. Inst. of Tech. (4-ft.) N.P.L. type*	.48	32.5
Langley Field	1.82	64.5
Eiffel 2m.	1.35	57.5
Curtiss (7 ft.)	0.69	40.8**
Washington Navy Yard	0.88	46.9
Göttingen	1.09	52.1
Bureau of Standards (3 ft.)	3.04	75.2
McCook Field	3.65	78.5
Mass. Inst. of Tech. (4-ft.) Venturi type	1.31	56.7

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* This tunnel has recently been dismantled.

** Efficiency of fan-tube unit.