TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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THE PROBLEM OF THE TURBO-COMPRESSOR.

By René Devillers,
Ingénieur de L'École Supérieure D'Aéronautique.

Extract from
"The Internal Combustion Engine".

Translated from the French
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In terminating the study of the adaptation of the engine to the airplane, we will examine the problem of the turbo-compressor, the first realization of which dates from the war; this will form an addition to the indications already given on supercharging at various altitudes.

We will begin by giving the experimental results at present determined relative to the influence on the power of a back pressure at the exhaust greater or less than the pressure in the admission manifold.

This subject is of great importance for the application of the turbo-compressor worked by the exhaust gases. As a matter of fact, a compressor increasing the pressure in the admission manifold may be controlled by the engine shaft by means of multiplication gear or by a turbine operated by the exhaust gas. Assuming that the increase of pressure in the admission manifold is the same in both cases, the pressure in the exhaust manifold would be greater in the case in which the compressor is worked by the exhaust gas and there would result a certain reduction of engine power which we must be able to calculate. On the other hand, if the compressor is controlled by the engine shaft, a certain fraction of the excess power supplied is utilized for the rotation of the compressor. In order to compare the two systems, it is therefore necessary to determine the value of the reduction of power due to back pressure when the turbine is employed.

THE INFLUENCE OF BACK PRESSURE AT THE EXHAUST ON THE POWER.

A series of tests has been made with a 220 HP engine, 1,600 r.p.m. water circulation, 12 cylinders 114 x 139, volumetric ratio 4.7. At the outlet of the exhaust pipe this engine was fitted with a throttle valve, so that the positive back pressure could be regulated as desired. The negative back pressures were obtained by connecting the exhaust with a depression chamber. In these tests the air admitted was at the temperature and pressure of the atmosphere, the carburetter inlets opening direct into the surrounding air.

The general conclusions drawn from these tests were as follows:

1st. If the pressure in the inlet manifold be raised above the pressure in the exhaust manifold, the effect is an increase of power in direct proportion to the excess of inlet pressure over exhaust pressure. It is practically independent of the speed of the engine.

2nd. If the pressure in the inlet manifeld be maintained at the normal atmospheric pressure on the ground, the exhaust pressure being below it, as in the case of the compressor worked by the crankshaft, the percentage of increase of power with respect to that on the ground is given pretty accurately by 18p, p being the excess of pressure in the inlet manifold over the pressure in the exhaust manifold, expressed in kilograms per square centimeter. It should be remarked that the results vary slightly according to the type of engine and even with engines theoretically identical. The reason is probably to be found in differences in the regulation of the valve gear.

3rd. A positive back pressure at the exhaust reduces power more than a corresponding negative back pressure increases it. As the back pressure increases, the effect becomes rapidly more marked. The mean results of tests made on two engines of the same type, gave in percentage a loss of power equal to $18p + 24p^2$, p being the excess of pressure in the exhaust over that in the admission, expressed in kilograms per square centimeter.

Tests on engines of other types have shown that results differ notably with different engines.

The very marked effect of a positive back pressure, even a comparatively small one, probably arises from the fact that, when the exhaust pressure is appreciably greater than the admission pressure, non-escaped exhaust gas remains under pressure in the cylinder. When the admission valve opens, this gas deteriorates the mixture in the admission manifold. In the extreme case in which the carburetor might be affected by such a counter-stroke, the effect might be to expel the gasoline.

The effect produced depends partly on the compression ratio and on valve adjustment, but more especially on the shape of the inlet manifold.

4th. Increase of pressure in the inlet manifold or of negative back pressure in the exhaust, improves the volumetric efficiency by reducing the proportion of exhaust gas remaining in the cylinder. The consumption of gasoline per h.p./hour may be slightly reduced in consequence.

Thus, for a normal consumption of 230 grammes, we have noted a specific consumption of 220 grammes with a back pressure of -0 kg. 600.

A positive back pressure has the contrary effect. A back pressure of +0.500 kg. raised the specific consumption to 250 grs.

5th. Increase of pressure in the admission manifold increases thermic efficiency and reduces the proportion of heat carried off by the water circulation. This point is of great interest in designing radiators for supercharged engines.

We may admit that the total amount of heat to be evacuated by the radiator is constant for all negative back pressures, the ratio of this quantity to the calorific equivalent of the power diminishing as the negative back pressure increases.

Fig. 393 (p. 875 of original text) gives the results of tests made on two engines of the same type.

APPLICATION. - Let us consider an engine fitted with a compressor which, at all altitudes, maintains the pressure in the admission manifold equal to the normal pressure on the ground level. With the foregoing figures, the power developed will be greater than the power on the ground level by 18p % of this power, p being the difference between the pressure in the admission and exhaust manifolds, expressed in kilograms per square centimeter.

The Rateau device, presenting a turbine driven by the exhaust gases, provides for the maintenance of pressure in the admission and exhaust manifolds at the pressure on the ground level, thus keeping up the power of the engine. In this case, the power developed by the engine at a given altitude will be less by 18p % of its value than the power which would be developed at the same altitude if the engine were fitted with a compressor operated by multiplication gear.

We will consider the 220 HP engine used in the tests previously mentioned, and the altitude of 4,000 meters for which the relative atmospheric pressure is 0.61. With the turbine worked by exhaust gas, the engine would give a horsepower of 220, equal to that on the ground level. With the compressor worked by multiplication gear, which also maintains the intake pressure at the level of the pressure on the ground, we shall have a negative back pressure at the exhaust $p = 1 - 0.61 = 0.39$, whence a gain of power of $18 \times 0.39 = 7$ % of the power on the ground level, that is, $0.07 \times 220 = 15$ horsepower. The power developed would thus be 235 HP instead of 220 as in the first case. To obtain the effective power, we must deduct that required for the control of the compressor, that is, 14 horsepower according to measurements made on the bench.

The available power is thus the same in both cases and no reason indicates a priori that one solution is better than the other.

THE PROBLEM OF THE TURBO-COMPRESSOR. - The object of the Rateau turbo-compressor, realized during the war, is to re-establish in the carburetor, up to an altitude of 5,000 meters, the atmospheric pressure on the ground level, the exhaust gases being evacuated into a chamber in which the pressure is practically the same as on the ground level. The conditions of flight near the ground will thus be re-established up to 5,000 meters.

We will call p_0 the atmospheric pressure on the ground level, and p_z the atmospheric pressure at the altitude Z. The exhaust gas escaping into a chamber in which the pressure is the same as on the ground level, the remaining calories are utilized for turning a single-rotor, axial flow turbine, the exhaust gas expanding in this driving turbine from the pressure p_o to the pressure p_z.

The turbine is coupled directly to a centrifugal compressor which compresses the air charging the carburetor, making it pass from the surrounding pressure p_z to the atmospheric pressure on the ground level, p_o.

IN SHORT, the energy corresponding to the expansion of the exhaust gas from p_o to p_z through the driving turbine, is utilized in a centrifugal compressor to make the inlet air pass from a pressure of p_z to a pressure p_o.

The problem being thus stated, it remains to be seen whether the energy which can be collected by the turbine is sufficient to obtain the integral compression of the air admitted to the carburetor.

EXPANSION OF THE EXHAUST GASES THROUGH A CONVERGING NOZZLE.

We would recall the principles relating to the flow of gas through a nozzle. The flow of gas through a non-diverging nozzle depends only, outside of the coefficient of contraction, on the ratio p/P of the leeward pressure p to the windward pressure P.

1st. $p/P > 0.52$.

If the leeward pressure p is greater than 0.52 P, the rate of flow depends on the ratio of the pressures and increases as this ratio decreases. The pressure of gas at the outlet of the nozzle is equal to the leeward pressure.

2nd. $p/P = 0.52$.

If the leeward pressure p is equal to 0.52 P, the rate of flow reaches its maximum value at the velocity of sound in the gas considered for the pressure 0.52 P. The pressure at the outlet is equal to the leeward pressure .

In the two cases just considered, the gas may expand completely in a converging nozzle.

3rd. $p/P < 0.52$

If the leeward pressure p is less than 0.52 P, the expansion cannot descend lower than the limit value 0.52 P. The pressure of gas at the neck is greater than the leeward pressure p and is equal to 0.52 P. At its outflow, the fluid then penetrates into a medium of which the pressure p is less than its own pressure 0.52 P.

There results a sudden expansion and the stream of gas issues making a loud noise.

The rate of flow is equal to the velocity of sound in the fluid, and the output depends entirely on the windward pressure P.

In order that there may be complete expansion in this case, the nozzle must be first converging, then diverging, and the speed at the outlet may then reach considerable values.

APPLICATION TO THE TURBO-COMPRESSOR. - The diffuser of the Rateau turbine is formed of converging nozzles; the above considerations may, therefore, be applied to it.

The ratio p_z/p_0 is practically equal to 0.52 at an altitude of 5,000 meters. Under these conditions, assuming that windward pressure must remain constant and equal to the atmospheric pressure p_0 on the ground level, there will be two cases of functioning to consider.

1st. FROM 0 to 5,000 METERS: $p/P = p_z/p_0 > 0.52$. - The exhaust gases expand completely through the diffuser. The speed and output of gas, as well asthe efficiency of the turbine, increase with altitude.

2nd. ABOVE 5,000 METERS: $p/P = p_z/p_0 < 0.52$. - On leaving the diffuser the exhaust gas is expanded only up to a pressure of 0.52P. Complete expansion only takes place after the passage of the fins at the turbine exhaust valve.

This part of the expansion is lost, efficiency is lowered. In this case we shall either take a windward pressure equal to p_0, and, consequently, incomplete expansion, or we shall be satisfied with a windward pressure $P = \frac{p_z}{0.52}$ in order to keep complete expansion. In any case, the power supplied by the engine will be reduced at altitudes above 5,000 meters.

CONCLUSION. - With the single-rotor turbine and converging nozzle the exhaust gas can be completely utilized at ground level pressure only up to an altitude of 5,000 meters.

For higher altitudes, a converging-diverging nozzle must be used, involving either turbines of greater rim speeds or the adoption of a multiple rotor turbine.

ESTIMATION OF RECOVERABLE ENERGY IN EXHAUST GASES. - We will take the case of complete expansion when $p_z/p_0 = 0.52$, that is, at the altitude of 5,000 meters, and will estimate the available energy corresponding to this expansion for 1 kilogram of gas.

We will call:

T_1 THE ABSOLUTE TEMPERATURE OF THE GAS IN FRONT OF NOZZLE, that is, the temperature in the exhaust manifold. From measurements taken direct, we may admit a temperature of 700° C, whence $T_1 = 973$.

T_2, THE ABSOLUTE TEMPERATURE TO REAR OF NOZZLE.

γ, RATIO OF SPECIFIC HEATS; as the mean value for expansion we will adopt the figure 1.293.

By the adiabatic law we have:

(1) $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_o}\right)^{\frac{\gamma - 1}{\gamma}} = 0.52^{0.226} = 0.863$$

From this we deduce $T_2 = 840$, whence a fall in temperature $T_1 - T_2 = 133°$ C.

The kinetic energy brought to the rotor by the gas after expansion in the nozzle is $\frac{V^2}{2g}$, V being the velocity of the gas. If C is the specific heat of the gas at constant pressure, we know, by formula (44) of Chapter II on Thermodynamics, that this available energy corresponding to the adiabatic expansion considered, is expressed in calories.

(2) $$Q = C(T_1 - T_2).$$

We will take for C the value of the specific heat of the exhaust gas at 600° C, that is, C = 0.3075, whence :

(3) $$Q = 0.3075 \times 133 = 41 \text{ calories.}$$

Admitting a loss of 5%, there remains an energy of 0.95 x 41 = 39 calories or 425 x 39 = 16,575 kilogrammeters.

Consequently, an output of 1 kilogram of gas per second would give rise to a power of $\frac{16.575}{75}$ = 221 horsepower.

If we now take ρ_t as the efficiency of the turbine, the energy available on the shaft for actuating the compressor will be, in kilogrammeters:

(4) $$E = 16{,}575 \ \rho_t.$$

The fall in calories being 39, the rate of flow of the gas at the outlet of the nozzle will be given by the formula:

(5) $$x^2 = 2g \times 425 \times 39$$

whence

(6) $$x = 91.3 \sqrt{39} = 570 \text{ meters per second.}$$

Counting on a loss of speed of about 2% by friction, we finally obtain a speed of 558 meters per second.

THE COMPRESSION STRESS OF THE AIR ADMITTED TO THE CARBURETOR.

We know that 15 grammes of air are required for consuming 1 gramme of gasoline. In order to obtain 1 kilogram of exhaust gas we must therefore have $\frac{15}{16} = 0.940$ kgs. of air. Let us assume a leakage of 10% in the pipes. For 1 kilogram of exhaust gas, we must thus compress 1.034 kgs. of air.

The compression, assumed to be adiabatic, is effected by means of a centrifugal fan.

The initial absolute pressure and temperature of the air are those of the atmosphere, that is, p_z and T_z; the final pressure and temperature are p_o and T_o. p_o should be practically equal to the atmospheric pressure on the ground level.

We will take $\gamma = 1.4$ as ratio of the specific heats C/c. We shall have:

(7) $$\frac{T_o}{T_z} = \left(\frac{p_o}{p_z}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{p_o}{p_z}\right)^{0.29}$$

For the altitude of 5,000 meters previously considered, we may admit a surrounding temperature of -13° whence

$$T_z = 273 - 13 = 260.$$

Then by substituting $\frac{1}{0.52}$ for $\frac{p_o}{p_z}$ we have:

(8) $$T_o = 1.21\ T_z = 315$$

that is, a temperature of 12° C. We thus obtain an increase of temperature:

(9) $$T_o - T_z = 55^{\circ}$$

Let us now take as a basis 1 kilogram of air. The kinetic energy $\frac{x^2}{2g}$ of the air at the outlet of the centrifugal fan is transformed into pressure. By equation (43) of Chapter II on Thermodynamics, the evolution being assumed adiabatic, the energy corresponding to this kinetic energy is, in calories:

(10) $$Q' = C(T_o - T_z)$$

C = 0.24 being the specific heat of the air at constant pressure.

Q' represents the compression work of the fan; it is, with the figures assumed, 0.24 x 55 = 13.2 calories, that is, 425 x 13.2 = 5,610 kilogrammeters.

But, as the weight of the air to be considered per kilogram of exhaust gas is actually 1.034 kg., we shall have to consider the energy.

(11) $$E' = 1.034 \times 5{,}610 : 5{,}800 \text{ kilogrammeters.}$$

With an output per second of 1 kilogram of exhaust gas giving 221 horsepower, the power utilized for compressing the equivalent weight of the air admitted would be $\frac{5{,}800}{75} = 77.4$ HP.

If ρ_v stands for the efficiency of the fan, the energy required for its rotation will be, in kilogrammeters:

(12) $$E = \frac{5{,}800}{\rho_v}$$

REQUIRED EFFICIENCY OF THE TURBO-COMPRESSOR SET. - Making expression (4), energy supplied by the turbine, equal to expression (12), energy absorbed by the fan, we have finally:

(13) $$16{,}575 \quad \rho_t = \frac{5{,}800}{\rho_v}$$

The product $\rho = \rho_t \rho_v$ is the overall efficiency of the system We thus obtain the required condition:

(14) $$\rho = \rho_t \rho_v = \frac{5{,}800}{16{,}575} = 0.35.$$

This requires the turbine to maintain an efficiency of 0.64, assuming that the compressor has an efficiency of 0.55.

It appears that at the present time we cannot count on a turbine efficiency, ρ_t, greater than 0.53, and therefore, with a compressor efficiency $\rho_v = 0.55$, not on an overall efficiency of more than 0.29.

In the tests made on the Bréguet 14 A_2 with Renault 300 HP engine, it was not found possible to re-establish ground level pressure in the carburetor at an altitude of 5,000 meters. It has been shown by calculation that, in order to obtain this result, very careful adjustment is necessary and also special care in reducing losses to a minimum.

Still placing ourselves in the conditions of an altitude of 5,000 meters, let us now seek the value of the pressure p_o which we may hope to obtain in the carburetor with an overall efficiency $\rho = 0.29$ of the set corresponding, say, to $\rho_t = 0.53$ for the turbine and to $\rho_v = 0.55$ for the fan.

We will still assume the same available energy of 16,575 kilogrammeters in the exhaust gas per kilogram of gas consumed. The force utilised for compressing the corresponding weight, 1.034 kg., of air admitted, will be 0.29 x 16,575 = 4,800 kilogrammeters, that is, $\frac{4,800}{1.034}$ = 4,650 kilogrammeters or $\frac{4,650}{425}$ = 10.94 calories per kilogram of air.

T_o being the final absolute temperature of the air and T_z its initial temperature equal to the atmospheric temperature, the energy utilized in compression is, in calories, $C(T_o - T_z)$, C = 0.24 representing the specific heat of the air at constant pressure. From this we can immediately deduce the rise in the adiabatic temperature :

(15) $$T_o - T_z = \frac{10.94}{0.24} = 45.6^\circ.$$

As before we will take a surrounding temperature of -13° giving T_z = 260, whence:

(16) $$\frac{T_o}{T_z} = 1 + \frac{45.6}{260} = 1.175$$

The ratio between the final pressure p_o and the initial pressure p_z, which is the atmospheric pressure , is then given by the adiabatic formula in which γ = 1.4 is the ratio of specific heat:

(17) $$\frac{p_o}{p_z} = \left(\frac{T_o}{T_z}\right)^{\frac{\gamma}{\gamma - 1}} = 1.175^{3.5} = 1.758$$

The atmospheric pressure at 5,000 meters being 407 millimeters of mercury, we shall have re-established in the carburetor a pressure of only 407 x 1.758 = 716 millimeters of mercury instead of the ground level pressure equal to 760 millimeters of mercury.

Assuming that a pressure of 760 millimeters has been maintained to windward of the turbine, we shall have a positive back pressure at the exhaust of 760 - 716 = 41 millimeters of mercury, that is, 0.06 kg. per square centimeter. From what we have seen in studying back pressure at the exhaust, there is a resulting loss of engine power of about 1.2% with respect to the power it would supply if the pressure at the exhaust valve were 716 millimeters as at the inlet valve.

GENERAL FORMULA OF THE TURBO-COMPRESSOR. - In order to summarize the preceding statements, we will deduce from them the general formula of the turbo-compressor theory.

Notation:

T_1, Absolute temperature of exhaust gas to windward of the diffuser.

T_2, Absolute temperature of exhaust gas to leeward of the diffuser.

p_o, Pressure of exhaust gas to windward of the diffuser, to be maintained at ground level pressure.

p_z, Pressure of exhaust gas to leeward of the diffuser, equal to atmospheric pressure, expansion being assumed complete.

T_z, Absolute temperature of atmospheric air.

T_o, Absolute temperature of air after compression.

P_o, Pressure of air after compression, to be maintained equal to pressure on ground level.

ρ_t, Efficiency of turbine.

ρ_v, Efficiency of fan.

$\rho = \rho_t \; \rho_v$, Efficiency of turbo - compressor set.

Taking 1.293 as ratio of the specific heats of the exhaust gases and 1.4 for the inlet air, we have:

(18)
$$\frac{T_2}{T_1} = \left(\frac{p_z}{p_o}\right)^{0.236}$$

(19)
$$\frac{T_o}{T_z} = \left(\frac{p_o}{p_z}\right)^{0.29}$$

The specific heat C at constant pressure is taken as equal to 0.3075 for exhaust gas and to 0.24 for inlet air.

At the outlet of the nozzle the energy of 1 kilogram of exhaust gas completely expanded is:

(20)
$$C(T_1 - T_2) = 0.3075\, T_1 \left[1 - \left(\frac{p_z}{p_o}\right)^{0.226}\right]$$

The energy utilized for the compression of the corresponding weight, 1.034 kg., of inlet air, is of the value:

(21)
$$1.034C(T_o - T_z) = 1.034 \times 0.24T_z \left[\left(\frac{p_o}{p_z}\right)^{0.29} - 1\right]$$

Putting the energy supplied by the turbine as equal to the energy absorbed by the fan, we have:

(22)
$$\rho_t 0.3075T_1 \left[1 - \left(\frac{p_z}{p_o}\right)^{0.226}\right] = \frac{0.248T_z}{v}\left[\left(\frac{p_o}{p_z}\right)^{0.29} - 1\right]$$

whence , finally, taking $x = \frac{p_o}{p_z}$:

$$(23) \qquad \rho = \rho_t \; \rho_v = 0.81 \; \frac{T_z}{T_1} \; \frac{x^{0.226}\,(x^{0.29} - 1)}{x^{0.226} - 1}$$

Such is the general formula giving the required efficiency ρ.

Taking as before :

$$x = \frac{1}{0.52} = 1.923, \qquad T_z = 260, \qquad T_1 = 973,$$

we find a required efficiency of 0.33 and, still assuming about 5% loss in the nozzle, we again find an efficiency of 0.35.

THE EFFICIENCY ρ_t OF THE TURBINE. - In the single rotor turbine which we are considering, the gas is completely expanded in the nozzle and possesses, as we have seen, a kinetic energy equal to the heat it has lost, and it is this energy which is transformed into mechanical force in the mobile rotor. THE NET EFFICIENCY ρ_t, is the ratio of the power on the crankshaft to that contained by the gas which has supplied it.

Let ρ_i be the INTERNAL EFFICIENCY or efficiency proper of the fins and ρ_e the EXTERNAL EFFICIENCY corresponding to the loss by friction of the disk on the gas and to loss in the thrust bearings; we shall have:

$$(24) \qquad \rho_t = \rho_i \; \rho_e$$

ESTIMATE OF THE INTERNAL EFFICIENCY ρ_i. - We would recall the graphical method invented by M. Rateau, which consists in plotting on the same diagram the triangle of speeds at the inlet and outlet of the rotor, taking as modulus of the length scale the absolute velocity V of the gas at the outlet of the nozzle.

We will call: (See Fig. 394, p.883 of original text)

U, the rim speed of the fins.
V, the velocity of the fluid at the outlet of the nozzles.
α, the angle of injection of the fluid or outlet angle of the diffuser.
β, the outlet angle of the fins.

Intersect the rotor by a median cylinder whose axis is the axis of rotation and develop this cylinder; the extremity B of the fin system falls on the straight line XX'. The nozzles throw out the gas at a velocity V which we will plot in magnitude and direction from the straight line XX' with which it forms the angle of injection α. We obtain the vector AB which will represent the unit of length. On the other hand, the mobile rotor is given a driving speed AC = U which we have laid off on the line XX'. The relative velocity V_1 of the gas with respect to the rotor is, in magnitude, direction, and sense, the resultant of V and -U.

It is represented by the vector CB to which the mobile fin should be tangent in order that there may be no shock at the inlet. ACB is the triangle of speeds at the inlet.

The losses by friction and by eddying in the fin system have the sole effect of reducing the relative speed V_1 by about 20%; at the outlet this speed is only $V_2 = 0.8\ V_1$, the condition of tangency at the inlet being assumed to be realized.

From C now lead the vector CD equal to V_2 and meeting XX' at the known outlet angle β .

Then project DB in DE in a direction parallel to XX'. The value of the internal efficiency is:

$$\rho_i = 2AC \times DE \tag{25}$$

the two lengths AC and DE being, as already said , estimated with V as unit of length.

For the turbine fitted on the Renault 300 HP engine, the speed of rotation provided for is 25,000 r.p.m. with a mean diameter of 0.158 m.

The rim speed U of the fins is thus 207 meters per second, and assuming for V the calculated value V = 558 meters/second, we find $AC = \frac{207}{558} = 0.37$. The angle of injection α is 22° and the outlet angle β 35°. The diagram gives us DE = 0.95, from which, finally, $\rho_i = 2 \times 0.37 \times 0.95 = 0.70$.

Taking an external efficiency ρ_e of 0.80, the net efficiency will be $\rho_t = \rho_i \ \rho_e = 0.56$. We may add that in the tests hitherto made we have not been able, owing to the care required to be taken of various organs, to keep a speed of 25,000 revolutions, and therefore the efficiency ρ_t was below the mark.

THE EFFICIENCY ρ_v OF THE CENTRIFUGAL FAN.

THE NET EFFICIENCY ρ_v of the centrifugal fan is the ratio of the power: employed on the crankshaft to that which would be required for obtaining compression pressure without loss and following the adiabatic cycle. In what follows we ignore the loss by radiation through the walls of the bodies of the apparatus.

We will call ρ_i THE INTERNAL EFFICIENCY corresponding to all losses resulting in additional heating of the air, and ρ_e THE EXTERNAL EFFICIENCY corresponding to leakages and friction in the thrust bearings.

The overall efficiency will therefore be:

$$\rho_v = \rho_i \ \rho_e \tag{26}$$

THE TRUE HEATING OF THE AIR AND INTERNAL EFFICIENCY ρ_i.

Call:

T_z, the initial absolute temperature of the air, equal to the atmospheric temperature.
P_z, the initial pressure of the air, equal to atmospheric pressure.
T_o, the theoretic absolute temperature of the air after compression, corresponding to the adiabatic cycle.
T'_o, the true absolute temperature of the air after compression.
P_o, the pressure of the air after compression.
$C = 0.24$, the specific heat of the air at constant pressure.
$\gamma = 1,4$, ratio of specific heats, therefore $\frac{\gamma - 1}{\gamma} = 0,29$.

As we have seen, if the cycle were rigorously adiabatic and without loss, the heating $\theta_a = T_o - T_z$ would be such that:

(27)
$$\frac{T_o}{T_z} = \left(\frac{P_o}{P_z}\right)^{0,29}$$

But by the principle of the conservation of energy, internal loss of energy is shown as an additional heating of the air, so that the true heating $\theta_r = T'_o - T_z$ is greater than the theoretic adiabatic heating θ_a:

(28)
$$\theta_r = \frac{\theta_a}{\rho_i}$$

ρ_i being a coefficient less than unity which is none other, as we shall show further on, than the internal efficiency.

We shall thus have:

(29)
$$T'_o - T_z = \frac{T_z}{\rho_i}\left[\left(\frac{P_o}{P_z}\right)^{0,29} - 1\right]$$

ρ_i is usually comprosed between 0,60 and 0,70; according to measurements made on the bench, 0.60 should be assumed for airplane centrifugal fans.

In the case of a fan operating at an altitude of 5,000 meters, the surrounding temperature being -13°C or $T_z = 260^\circ$ absolute, and $\frac{P_o}{P_z}$ being equal to $\frac{1}{0,52}$, we obtain adiabatic heating $\theta_a = 55^\circ$, giving a final temperature of 42°C. The real heating, still ignoring loss by radiation through bodies, will be $\theta_r = \frac{55}{0,60} = 92^\circ$, giving a final temperature of 79°C.

The mechanical force supplied to the crankshaft PER KILOGRAM OF AIR and reduced by external losses, leakages, and friction in the thrust bearings, is utilized for three purposes::

1st. To produce compression of the air, which, if it took place adiabatically without loss or radiation, would require an amount of energy in calories:

(30) $$Q' = C \quad \theta_a$$

This relation shows that the energy Q' is equivalent to the quantity of heat necessary for producing, at constant pressure, a rise in temperature equal to the adiabatic heating θ_a.

2nd. To heat the air at CONSTANT PRESSURE by the quantity $\theta_r - \theta_a$, the difference between the real and adiabatic heating. This energy corresponds to the transformation into heat of all the internal losses of the apparatus: its value is

(31) $$P = C(\theta_r - \theta_a)$$

3rd. To heat the exterior organs of the turbine by radiation and conduction.

If there is no cooling by injection or water circulation, the amount of energy thus lost is negligible as compared with those previously mentioned .

The total amount of energy supplied is therefore:

(32) $$Q' + P = C \ \theta_a + C(\theta_r - \theta_a) = C \ \theta_r$$

and the internal efficiency is, by definition:

(33) $$\rho_i = \frac{Q'}{Q' + P} = \frac{\theta_a}{\theta_r}$$

THE INTERNAL EFFICIENCY ρ_i IS THUS CLEARLY EQUAL TO THE RATIO BETWEEN THE ADIABATIC HEATING θ_a AND THE REAL HEATING θ_r.

Lastly, introducing the coefficient of external efficiency, ρ_e, we see that the force supplied to the shaft per kilogram of air will be, in calories:

(34) $$E = \frac{C \ \theta_a}{\rho_i \ \rho_e} = \frac{CT_z}{\rho_i \ \rho_e} \left[\left(\frac{p_o}{p_z} \right)^{0.29} - 1 \right]$$

In the conditions of operating of the airplane fan, we may take $\rho_e = 0.92$ to 0.95.

We thus see that with an internal efficiency of 0,60, we may practically count on a net efficiency $\rho_v = \rho_i \ \rho_e = 0.55$.

EXAMPLE OF A TURBO-COMPRESSOR ADAPTED TO AN ENGINE.

As an example we will take the Renault 300 HP 12 cylinders 125 x 150, on which the Rateau turbo-compressor was fitted. As before, we will examine the question for an altitude of 5,000 m., assuming for the turbine a net efficiency $\rho_t = 0.53$ and for the fan $\rho_v = 0.55$, that is, an overall efficiency of $\rho = \rho_t \rho_v = 0.29$.

We have seen that, under these conditions, we can re-establish in the carburetor a pressure of only 716 millimeters of mercury. The adiabatic rise of temperature given by formula (15) is $\theta_a = 45.6°$, from which, with an internal efficiency of 0.60 for the centrifugal fan, a real heating $\theta_r = \frac{45.6°}{0.60} = 76°$.

The initial absolute temperature T_z of the air being 260 at 5,000 meters, its final real temperature will be $T'_o = 260 + 76 = 336$, that is, 63° C.

Assuming that there is no air radiator, the inlet air will thus have a pressure of 716 millimeters of mercury at the carburetor and an absolute temperature of 336°. Under these conditions of pressure and temperature, the weight of a liter of air is:

(35) $$a = 1.293 \frac{716}{760} \times \frac{273}{336} = 0.99 \text{ gr.}$$

The total cylinder capacity of the engine is 22.1 liters; this is the geometrical volume taken in by suction in two revolutions. The speed of rotation being 1,600 revolutions, the geometrical volume taken in by suction per second is

$$\frac{22.1 \times 1.600}{2 \times 60} = 295 \text{ liters per second.}$$

We know that 1 gramme of air corresponds to $\frac{16}{15}$ grammes of carburetted mixture. Assuming a filling coefficient of 0.90, the output of exhaust gas per second will be :

$$\frac{16}{15} \times 0.90 \times 0.99 \times 295 = 280 \text{ grammes per second.}$$

We know that the power supplied to the turbine by an output of exhaust gas of 1 kilogram per second is 221 horsepower. The net efficiency of the turbine being 0.53, it will supply the crankshaft with:

$$221 \times 0.53 \times 0.280 = 33 \text{ horsepower.}$$

Steam tests made on the bench of the Rateau turbine fitted on the Renault engine now under consideration, have indicated a power on the crankshaft of about 40 horsepower.

We have already assumed that, per kilogram of carburetted gas, we should compress 1.034 kg. of air in order to make good the inevitable leakages in the manifolds. The centrifugal fan should therefore have an output of air per second of:

$$280 \times 1.034 = 290 \text{ grammes per second.}$$

Let us see whether, with 33 horsepower on the crankshaft, we really get a carburetor pressure of 716 millimeters of mercury. The atmospheric pressure at 5,000 meters being 407 millimeters of mercury, we shall have, for this rate of compression,

$$\frac{p_o}{p_z} = \frac{716}{407} = 1.758, \text{ whence } \left(\frac{p_o}{p_z}\right)^{0.29} = 1.1775.$$

By formula (34) we know that the energy in calories required on the fan shaft is:

$$(36) \qquad E = \frac{CT_z}{v}\left[\left(\frac{p_o}{p_z}\right)^{0.29} - 1\right]$$

Substituting 0.24 for C, 260 for T_z, 0.55 for $\frac{\rho}{v}$ and 1.1775 for $\left(\frac{p_o}{p_z}\right)^{0.29}$, we find E = 20.15 calories.

The horsepower required on the fan shaft for compressing 290 grammes of air per second will thus finally be:

$$20.15 \times 0.290 \times \frac{425}{75} = 33 \text{ h.p.}$$

CALCULATION OF THE SECTION OF THE DIFFUSER. - We assume that the weight of a liter of gas carburetted at 0° and 760 millimeters is the same as for non-carburetted gas under the same conditions of temperature and pressure, which comes to the same thing as ignoring the molecular expansion caused by combustion.

We know that 16 grammes of carburetted mixture correspond to 15 grammes of air. Under these conditions, at 0° and 760 mm., the weight of a liter of exhaust gas will be:

$$a_o = 1 \text{ gr } 293 \; \frac{16}{15} = 1.38 \text{ gr.}$$

The exhaust gas at the nozzle outlet is at an atmospheric pressure of 407 millimeters of mercury and at a temperature of $T_z = 840^\circ$ absolute , whence a weight per liter:

$$a = 1.38 \; \frac{407}{760} \times \frac{273}{840} = 0.24 \text{ gr.}$$

The output at the exhaust being 280 grammes per second, we conclude that the output in liters per second at the outlet of the nozzle will be:

$$\frac{280}{0.24} = 1.165 \text{ liters per second.}$$

But we have found that the velocity of the gas at that instant is 558 meters per second. The section s of the diffuser must thus be, in square centimeters:

$$s = 10 \frac{1.165}{558} = 20.8 \text{ square centimeters.}$$

The tests on the Bréguet 14 A_2 were made with a diffuser having a section of 20 square centimeters.

ENGINE POWER. - The Renault engine in question gives 340 h.p. on the ground level at 1,600 revolutions. Without back pressure at the exhaust valve, we may assume that the power remains proportional to the inlet pressure; therefore, in the conditions of operation studied and without back pressure, it would be:

$$T = 340 \frac{716}{760} = 320 \text{ h.p.}$$

In reality, there is a back pressure at the exhaust of 760 - 716 = 44 millimeters of mercury, that is, 0.06 kg. per square centimeter, reducing this power, as we have seen, by about 1.2%, which gives a corrected horsepower of 316.

ADAPTATION OF A SUPERCHARGED ENGINE TO AN AIRPLANE.

Leaving for a moment the question of what manufacturers have succeeded in accomplishing up to date, we propose to bring out, in the following lines, as definitely and clearly as possible, what may be hoped for in the future from superchargers.

THE PROPELLER. - We take an altitude Z for which the propeller will be determined; n is the number of revolutions at that altitude.

The compressor, assumed to be perfect, enables the engine to work at altitude Z in the same conditions as on the ground level. Let T be the motive power developed, equal to that given on the ground level at the same speed n.

The ratio $\frac{V}{nD}$ will still be designated by γ, V being the speed of advance of the airplane and D the diameter of the propeller. Also, let β be the coefficient of power of the propeller in the atmospheric conditions of the ground level and δ the relative density of the air.

We shall have:

(37) $$T = \delta \beta n^3 D^5$$

with

(38) $$\gamma = \frac{V}{nD}$$

whence, by elimination of first D, then n:

(39) $$\frac{n^2 T}{\delta V^5} = \frac{\beta}{\gamma^5}$$

(40) $$\frac{\delta D^2 V^3}{T} = \frac{\gamma^3}{\beta}$$

As we indicated in the preceding chapter, we shall consider a family of propellers defined by the shape of the blade and differing from each other by the ratio of pitch to diameter. For each of them the values of the efficiency ρ in function of γ (Fig. 375, p. 820 of original text) have been determined experimentally. The envelope of the efficiency curves will define the points of functioning to be adopted.

We have seen that with the best propellers having a ratio of 0.075 to 0.08 between width of blade and diameter, this enveloping curve may practically be represented by the equation:

(41) $$\rho = 0.545 \sqrt[4]{10^2}$$

the speed V being expressed in kilometers per hour.

Assuming the law of proportionality of β to γ along this enveloping curve as given in the preceding chapter, we find that the efficiency attainable for each of these points and the corresponding diameter of the propeller are expressed as follows:

(42) $$\rho = 0.54 \sqrt[16]{\frac{\delta V^5}{n^2 T}}$$

(43) $$D = 1.04 \sqrt[4]{\frac{10^8 T}{\delta n^2 V}}$$

This theory shows that the equations relating to the propeller are the same as for an ordinary engine having a power $\frac{T}{\delta}$ on the ground level.

COMPARISON OF AN AIRPLANE WITH COMPRESSOR AND AN ORDINARY AIRPLANE OF THE SAME TYPE. - We will now compare two airplanes of the same type, one fitted with an ordinary engine and the other having the same engine with the addition of a supercharging device bringing the engine to ground level conditions at each altitude of flight considered.

The propeller of the supercharged engine will be determined for each altitude at a constant speed of rotation n, equal to that of the propeller of the ordinary engine. The power of the engine on ground level at this speed will be called T.

Write the general equations of the airplane, introducing a coefficient μ of decrease of power, equal to unity in the case of a supercharged engine. We shall have:

(44) $$P = \delta K_y S V^2 ;$$

(45) $$PBV = 75 \rho \mu T;$$

(46) $$\mu T = \delta \beta n^3 D^5 ;$$

(47) $$\gamma = \frac{V}{n D} .$$

As can be immediately verified, these equations show that the quantities:

$$\mu^2 \frac{\rho^3 \beta}{\gamma^5}, \quad \rho \mu \sqrt{\delta}\, D^3 \frac{\rho \beta}{\gamma^3} \quad \text{and} \quad \frac{V}{\mu \rho}$$

ARE THE SAME FOR THE TWO AIRPLANES WHEN THEY ARE FLYING AT THE SAME ANGLE OF ATTACK.

Therefore, for the non-supercharged engine, call:

μ_o, the coefficient of decrease of power for the altitude considered.
ρ_o, the efficiency of the propeller.
D_o, its diameter.
V_o, the speed of the airplane.
β_o and γ_o, the values of β and γ .

For the engine fitted with a compressor the same quantities become 1, ρ_1, D_1, V_1, β_1 and γ_1. The altitude of flight corresponding to the angle of attack considered is then characterized by the value δ_1 of the relative density of the air. We shall therefore have:

(48) $$\frac{\rho_1^3 \beta_1}{\gamma_1^5} = \mu_o^2 \cdot \frac{\rho_o^3 \beta_o}{\gamma_o^5}$$

(49) $$\delta_1 = \mu_o^2 \delta_o \frac{\rho_o^2}{\rho_1^2}$$

(50) $$D_1^2 = D_o^2 \frac{\rho_o \beta_o}{\gamma_o^3} \frac{\gamma_1^3}{\rho_1 \beta_1}$$

(51) $$V_1 = \frac{V_o}{\mu_o} \frac{\rho_1}{\rho_o}$$

Propellers being always determined for the points of efficiency of the envelope of the curve, $\frac{\rho^3\beta}{\gamma^5}$ and $\frac{\rho\beta}{\gamma^3}$ are functions of efficiency perfectly determined for all these points.

Knowing the value of $\frac{\rho^3\beta}{\gamma^5}$ in the case of the ordinary engine, equation (48) gives the new value suitable for flying at the same angle of attack and the same speed of rotation of the propeller with the supercharged engine.

From this we deduce the efficiency ρ_1 and $\frac{\rho_1 \beta_1}{\gamma_1^3}$

The following relations (49, 50 and 51) then determine the altitude of flight, the diameter of the propeller, and the speed of the airplane.

The problem is thus completely solved by taking as a basis the results given by an airplane of the same type fitted with an ordinary engine.

PRACTICAL FORMULAS. - We have seen that, for all the points used practically of the envelope of the efficiency curves of the best propellers, the efficiency ρ increases as the fourth root of γ and the coefficient of power β remains practically proportional to γ^5. Consequently, $\frac{\gamma^5}{\beta}$ varies as γ^4, that is, as ρ^{16}

Under these conditions formula (48) becomes:

(52) $$\rho_1 = \frac{\rho_o}{\rho_o^{0.154}}$$

Equation (49) giving the altitude of flight, then becomes:

(53) $$\delta_1 = \mu_o^{2.3}\delta_o$$

The coefficient of decrease of power μ_o is practically equal to the relative atmospheric pressure.

Taking δ and μ as equal to this pressure, we find the approximate correspondence:

(54) $$\mu_1 = \mu_o^{3.3}$$

The current barometric law

(55) $$Z = 18,400 \text{ Log } \frac{1}{\mu}$$

then gives the practical relation, sufficient for a first analysis, between the altitudes.

(56) $$Z_1 = 3.3\ Z_o$$

On the other hand, with the hypotheses made, $\frac{\gamma^3}{\beta}$ varies as γ^2, therefore as ρ^8.

Formula (50) indicating the correspondence between the diameters, becomes:

(57) $$D_1 = D_o \frac{\rho_1^{3.5}}{\rho_o^{3.5}} \qquad \frac{D_o}{\mu_o^{0.54}}$$

Lastly, the relation (51) between the speeds enables us to write:

(58) $$V_1 = \frac{V_o}{\mu_o^{1.154}}$$

As a conclusion we see that, for the airplane fitted with a compressor, THE EFFICIENCY OF THE PROPELLER, ITS DIAMETER, AND THEREFORE THE RATIO OF PITCH TO DIAMETER, WILL INCREASE WITH THE ALTITUDE OF FLIGHT. On the other hand, the speed V_o of the reference airplane diminishing much less rapidly than μ_o, THE SPEED OF THE AIRPLANE ALSO INCREASES WITH ALTITUDE.

Assuming that power can be maintained constant at this altitude, we see that THE CEILING WILL BE ABOUT 3.3 TIMES THAT OF THE AIRPLANE FITTED WITH AN ORDINARY ENGINE.

Moreover, if we exceed the angle of attack corresponding to flight at the ceiling, we know that the speed V_o, the efficiency ρ_o and the altitude Z_o will diminish. We should then obtain, for corresponding flight with the compressor, a speed V_1, an efficiency ρ_1 and an altitude Z_1 which would also gradually diminish.

Under these conditions, the maximum speed V_1 which cannot be exceeded, would be obtained for the angle of attack of flight at the ceiling.

The following Table gives the results which would be furnished by a perfect compressor, compared with those furnished by an airplane fitted with an ordinary engine.

REFERENCE AIRPLANE		AIRPLANE WITH COMPRESSOR.			
ALTITUDE Z_o	VALUE of μ_o	$\frac{\rho_1}{\rho_o} = \frac{1}{\mu_o^{0.154}}$	$Z_1 = 3.3\ Z_o$	$\frac{D_1}{D_o} = \frac{1}{\mu_o^{0.54}}$	$\frac{V_1}{V_o} = \frac{1}{\mu_o^{1.154}}$
meters			meters		
1,000	0.89	1.02	3,300	1.06	1.14
1,500	0.83	1.03	4,950	1.10	1.23
2,000	0.78	1.04	6,600	1.14	1.33
2,500	0.74	1.05	8,250	1.18	1.42
3,000	0.69	1.06	9,900	1.22	1.54
3,500	0.65	1.07	11,550	1.26	1.65
4,000	0.61	1.08	13,200	1.30	1.77
4,500	0.57	1.09	14,850	1.35	1.91
5,000	0.53	1.10	16,500	1.40	2.08
5,500	0.50	1.11	18,150	1.45	2.23
6,000	0.47	1.12	19,800	1.50	2.39
6,500	0.44	1.13	21,450	1.56	2.58
7,000	0.41	1.14	23,100	1.62	2.79

As example, we take a Brégust 14 A_2 fitted with a Renault 300 HP engine turning at 1,600 revolutions.

Tests with an ordinary engine gave the following results, the ceiling being at 6,000 meters.

ALTITUDE Z_o	SPEED V_o	ALTITUDE Z_o	SPEED V_o
meters	km/h	meters	km/h
0	180	4,000	163
1,000	177	5,000	155
2,000	174	6,000	140
3,000	170	(ceiling)	

By calculating as just indicated, the corresponding altitudes and speeds for the same airplane fitted with a perfect compressor and having the same speed of propeller, we obtain the following figures:

ALTITUDE Z_1	SPEED V_1	ALTITUDE Z_1	SPEED V_1	ALTITUDE Z_1	SPEED V_1	ALTITUDE Z_1	SPEED V_1
meters	km/h	meters	km/h	meters	km/h	meters	km/h
0	180	6,000	227	12,000	285	18,000	327
1,000	187	7,000	236	13,000	295	19,000	331
2,000	193	8,000	245	14,000	305	19,800	334
3,000	200	9,000	255	15,000	312	(ceiling)	
4,000	210	10,000	265	16,000	318		
5,000	220	11,000	275	17,000	323		

The diameter D_0 is 2.94 m., and the efficiency ρ_0 is about 0.75 at $Z_0 = 3,000$ meters. The efficiency ρ_1 will therefore practically be $1.06 \times 0.75 = 0.79$ at $Z_1 = 10,000$ meters and the propeller should have a diameter $D_1 = 1.22 \times 2.94 = 3.58$ m.

With a propeller of this dimension instead of 2.94 m., the engine when near the ground level will turn at only a low speed on account of the drag of the propeller. The power of the engine being proportional to its speed, the take-off and climb would certainly be difficult. On the contrary, above the normal altitude of flight the engine would race as altitude increases.

We thus see that the utilization of a propeller, determined for horizontal flight at a certain altitude , may provoke the following phenomena:

a) If the altitude chosen is too high, the airplane may not be able to get off the ground, or will do so with difficulty.

b) If this altitude is low enough to enable the airplane to get off the ground easily, the engine will race at high altitudes.

With a propeller of variable pitch and diameter we could obtain a uniform speed of rotation and best efficiency for each altitude. If absolutely necessary, we might utilize a propeller of variable pitch only, but should lose in efficiency. The tests made up to date on such propellers have not been satisfactory, although the mechanical realization of the problem does not seem to be insurmountable.

CONCLUSIONS. - It is only with a supercharged engine set designed for very high altitudes that the speed V_1 reaches interesting values of the order of 300 kilometers an hour. If, as in the case of the rateau turbo-compressor tested during the war, the ground level pressure is not re-established in the carburetor above 5,000 meters, no gain in speed is realized above that altitude. The best results obtained up to date in the flying tests at Villacoublay are as follows:

ALTITUDE	SPEED	
	WITHOUT SUPERCHARGING	WITH TURBO-COMPRESSOR
meters	km/h	km/h
3,000	170	192
4,000	163	200
5,000	155	198
6,000	140	180
7,000		160

We see that we are still far from the results which may be given by a perfect turbo-compressor with the best adapted propeller at 5,600 meters. The maximum speed realized at an altitude of 4,500 meters is 205 kilometers an hour, while the greatest speed obtainable at the altitude/is about 215 kilometers an hour.
5500 meters,

Before leaving this subject we may remark that the present day aviation engines, constructed during the war, present a certain security in operation because they are almost always utilized at high altitudes; this has the effect of reducing mean pressure and, consequently, the stress on the parts.

at increasing
If the torque were maintained constant/ altitude of flight, present day engines would not be strong enough to withstand the strain.

THE MAXIMUM DISTANCE WHICH CAN BE COVERED BY AN AIRPLANE WITH COMPRESSOR. - We will assume that the initial altitude of flight is that for which the engine is in the conditions of working on ground level, and that the flight is made AT CONSTANT ANGLE OF ATTACK.

From what has been established in the preceding chapter (p.845 of original text), the efficiency of the propeller, assumed to be of invariable shape, is constant, as well as the coefficient of power β, and the speed of advance V is proportional to the number of revolutions n.

Formula (92) on p. 847 (of original text) established without any hypothesis as to the law of variation of power, is still applicable.

We may remark that, with respect to the similar ordinary airplane, we may obtain an appreciable gain on the distance covered, owing to the fact that the efficiency ρ is increased and that the specific consumption m must be smaller for an engine operating under ground level conditions than for one working at a pressure of reduced compression.

For an initial altitude of 3,000 meters for an ordinary airplane, the corresponding altitude for the second would be about 10,000 meters. Assuming a gain of 6% on efficiency and a reduction of 12% in specific consumption, we obtain a gain of 20% on the distance covered .

There are, as for the ordinary airplane, two methods of making the flight.

1st. If we wish to maintain the CONSTANT SPEED OF ROTATION n and, consequently, the speed of advance V, the power must, at each instant be reduced in proportion to the total weight. By the equation of lift, the altitude will be known at each instant by the proportionality of the total weight to the density of the air.

2nd. If we wish to maintain CONSTANT ALTITUDE, the speed V and the number of revolutions will be reduced as the square root of the total weight. As in the first case, the engine torque must be reduced in proportion to the weight, which will have the effect of lowering the power in the proportion of 3/2 of the weight.

Formulas (116) and (130) of the preceding chapter (pp. 855 and 858 of original text), relating to the duration of flight, are applied, unity being substituted for the coefficient of decrease of power at the initial altitude.

Let us then compare two airplanes, one ordinary, the other with compressor, having at the start the same weight of gasoline in proportion to the total weight, and assuming the same specific consumption m and the same weight per horsepower π . The first airplane begins its flight at a certain altitude where the coefficient of decrease of power is μ_o; the other takes the corresponding altitude at the same angle of attack. The duration of its flight is equal to that of the first airplane multiplied by the coefficient μ_o.

ATMOSPHERIC CONSTANTS AT HIGH ALTITUDES.

If we wish to determine the characteristics of the atmosphere at very high altitudes, we must abandon the law of Radau generally admitted for the decrease of temperature, this law being insufficient for altitudes above 6,000 or 7,000 meters.

By numerous soundings, Professor Gamba, Director of the Pavia Observatory, has found that up to 11,000 meters the temperature practically decreases according to a linear law of altitude. At altitude Z, the absolute temperature is T; according to these results, which agree with the most recent observations made in England and France, the THERMIC GRADIENT $G = \frac{dT}{dZ}$ would be constant up to 11,000 meters and practically equal to 6.5° per kilometer. Moreover, this law is, within a very little, that now accepted by English specialists.

Above 11,000 meters, the observations made lead to the assumption of constant temperature, that is, a zero gradient.

Admitting these laws, we find for pressures and densities simple analytical forms in function of the altitude which are very convenient for certain computations.

Let dP be the differential of pressure, dZ the differential of altitude , and a_z the specific weight of the air.

We have:

(59) $$- dP = a_z dZ$$

The law of perfect gases gives us:

(60) $$a_z = \frac{P}{RT}$$

where R = 29.27 for the air. We shall thus have:

(61) $$- \frac{dP}{P} = \frac{dZ}{RT}$$

Let T_o be the absolute temperature on the ground level. The gradient G being assumed constant, we have up to 11,000 m.:

(62) $$T = T_o - GZ$$

from which, by immediate integration:

(63) $$\mu = \frac{P}{P_o} = \left(1 - \frac{G}{T_o} Z\right)^{\frac{1}{GR}}$$

On the other hand, the ratio of densities is:

(64) $$\delta = \frac{P}{P_o} \frac{T_o}{T} = \left(1 - \frac{G}{T_o} Z\right)^{\frac{1}{GR} - 1} = \mu^{1 - GR}$$

Taking the meter and kilogram as units and assuming a ground level temperature of 15°C.: G = 0.0065; T_o = 288; R = 29.27; whence:

(65) $$\mu = \left(1 - \frac{0.0065}{288} Z\right)^{5.255}$$

(66) $$\delta = \left(1 - \frac{0.0065}{288} Z\right)^{4.255} = \mu^{0.81}$$

Above 11,000 meters the temperature is assumed to be uniform and of a value T_1. The pressure at 11,000 being P_1, formula (61) gives, by immediate integration:

(67) $$L \frac{P_1}{P} = \frac{Z - 11,000}{RT_1}$$

Taking $T_1 = 273 - 57 = 216$ and passing to common logarithms, we have:

(68) $$\text{Log} \frac{P_1}{P} = \text{Log} \frac{\mu_1}{\mu} = \frac{Z - 11,000}{14,600}$$

With a ground level temperature of 15° C the law of densities above 11,000 is:

(69) $$\delta = \frac{T_o}{T_1} \quad \frac{P}{P_o} = \frac{288}{216} \mu = 1.33 \mu$$

with

(70) $$\frac{a_z}{a_1} = \frac{P}{P_1}$$

As pointed out by M. Toussaint, it is interesting to compare the régime of temperatures in the atmosphere with the régime of the winds. Studying the results of numerous soundings, M. Ch. Maurain * has found that the mean velocity of the wind in clear weather increases regularly and in an almost linear fashion, from 5 meters per second at an altitude of 500 meters, up to 15.6 m/sec. at 11,000 m., then decreasing to about 8 m/sec. at 19,000 meters.

The following Table gives the atmospheric constants up to 20,000 meters, tabulated according to the foregoing hypotheses. It has been drawn up by M. Toussaint and has been proposed by him for the Standard Atmosphere.

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* Minutes of the Académie des Sciences, July 15th, 1919.

ATMOSPHERIC CONSTANTS AT HIGH ALTITUDES.

ALTITUDE Z	TEMPERATURE t	PRESSURE H	WEIGHT of liter of air a_z	$\mu = \frac{H}{760}$	$\delta = \frac{a_z}{a_0}$
meters	degrees C	mm of mercury	grammes		
0	15	760	1,225	1	1
1,000	9	673	1,112	0.886	0.906
2,000	2	596	1.008	0.784	0.823
3,000	- 5	526	0.907	0.692	0.740
4,000	- 11	462	0.820	0.608	0.665
5,000	- 18	405	0.735	0.533	0.600
6,000	- 24	354	0.660	0.465	0.540
7,000	- 31	308	0.588	0.405	0.480
8,000	- 37	267	0.525	0.351	0.429
9,000	- 44	230	0.467	0.303	0.382
10,000	- 50	198	0.413	0.2600	0.337
11,000	- 57	169	0.364	0.223	0.297
12,000	"	145	0.311	0.190	0.254
13,000	"	123	0.265	0.163	0.217
14,000	"	105	0.227	0.139	0.185
15,000	"	90	0.193	0.119	0.158
16,000	"	79	0.169	0.104	0.138
17,000	"	66	0.141	0.086	0.115
18,000	"	56	0.120	0.074	0.098
19,000	"	48	0.103	0.063	0.084
20,000	"	41	0.088	0.054	0.072