TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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NOTES ON THE DESIGN OF LATTICED COLUMNS SUBJECT TO LATERAL LOADS.

By Charles J. McCarthy,
Bureau of Aeronautics, Navy Department.

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The increasing interest in the use of metal for the construction of aircraft makes timely a discussion of the problems and difficulties to be met in the design of efficient compression members. No rational column formula has yet been developed which gives results which are sufficiently precise for the design of airplane members, and consequently it is necessary to fall back upon experimental testing. In order to derive the maximum benefit from experiments, however, it is necessary that the experiments be guided by theory, and it is the object of this paper to suggest a method of procedure by means of which the data needed to modify existing formulae may be obtained with a minimum of tests.

Although it is common in wing construction to find wing beams continuous over several supports, for the sake of simplicity this discussion will be limited to that of a simple column supported at both ends and subjected to uniformly distributed loads perpendicular to its axis and to end loads either axially or eccentrically applied.

Ideal Columns.

The failing strength of a perfectly straight homogeneous column with pinned ends in which the compressive load is exactly axially applied is expressed by Euler's formula:

$$\frac{P'}{A} = \pi^2 E \left(\frac{K}{L}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (1)$$

Where P' is the critical end load;
A is the cross sectional area of the column;
E is the modulus of elasticity of the material;
$\frac{L}{K}$ is the least ratio of length to radius of gyration.

It should be kept in mind that the critical load calculated from the above formula is the end load required to buckle the strut and that for loads smaller than this the ideal column remains perfectly straight. It is apparent also that the column will fail elastically as soon as the stress at the ends reaches the elastic limit of the material. Consequently the curve of ultimate stress vs. L/K for an Euler column has the form of the right hand curve of Fig. 1.

If now instead of being axially applied, the end load has an eccentricity, h, bending stresses are introduced which increase the stresses in the fibers of the column and decrease the magnitude of the load which will cause failure. In the case of a practical strut, variations in the shape and thickness of the section, initial curvature and other imperfections have the effect of giving an eccentricity to the end load.

The equation for the maximum intensity of stress under these conditions is given by Morley as*

$$f_t = \frac{P}{A}\left(1 + \frac{hd}{2K^2} \sec \frac{L}{2}\sqrt{\frac{P}{EI}}\right) \qquad (2)$$

Where P is the end load applied;
A is the cross sectional area of the column;
h is the eccentricity, i.e., the distance from the point of application of the load to the centroid of the section;
d is the depth of the section in the plane of bending;
k is the radius of gyration in the plane of bending.

This formula may be expressed (appoximately) as follows:**

$$f_t = \frac{P}{A}\left[1 + \frac{\frac{1.2\ hd}{2K^2}}{1 - \frac{PL^2}{\pi^2 EI}}\right] \qquad (3)$$

which may be simplified by substituting the Euler load, P' for the expression $\frac{\pi^2 EI}{L^2}$.

$$\text{Thus,}\quad f_t = \frac{P}{A} + \frac{P}{A}\left(\frac{P'}{P' - P}\right)\left(\frac{0.6\ hd}{K^2}\right) \qquad (4)$$

Failure occurs when f_t reaches the elastic limit of the material in compression, f_c.

It will be noted in equation (4) that as P approaches P' the ratio $\frac{P'}{P' - P}$ approaches infinity.

The curves of Fig. 1, which are taken from Morley, are of interest as they show how end stress at failure is affected by vary-

* Morley. Strength of Materials 1916, p. 276.

** Ibid. p. 276.

ing eccentricities and varying values of L/K.

Another condition to be considered is the combination of the axial loads with forces perpendicular to the axis of the strut. The deflection of the strut which is produced by the lateral loads has the effect of making the axial loads eccentric with a consequent increase in the maximum bending moment in the strut. The total bending moment is the sum of an infinite series, the first two terms of which are the bending moment due to the lateral load, and that of the product of the axial load by the deflection of the column under the lateral loads. For a uniformly distributed lateral load of w per unit length the exact equation for the maximum bending moment at the center of the column, M_o under the combined loading, is given by *

$$M_o = \frac{wEI}{P} \left(\sec \frac{\pi}{2} \sqrt{\frac{P}{P'}} - 1 \right) \qquad (5)$$

This may be more conveniently expressed by Perry's approximate formula:

$$M_o = M \left(\frac{P'}{P' - P} \right) \qquad (6)$$

where M is the maximum bending moment due to the lateral loads alone, and the other symbols have the same significance as before.

If Z equals the section modulus the maximum fiber stress due to bending equals

$$f_t = \frac{M_o}{Z} = \frac{M}{Z} \left(\frac{P'}{P' - P} \right) \qquad (7)$$

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* Morley. p. 282.

The error introduced by this approximation amounts to less than 3 percent for ratios of P to P' up to 0.9.

Combining equations (4) and (7) results in a general formula for the maximum intensity of stress in a perfectly straight column of homogeneous material with pin ends, loaded with a uniformly distributed transverse load, which, acting alone, would produce a maximum bending moment M; and in addition, an end load, P which is applied a distance, h from the centroid of the section in the plane of bending.

$$f_t = \frac{P}{A} + \frac{P}{A}\left(\frac{P'}{P' - P}\right)\left(\frac{0.6\ hd}{K^2}\right) + \frac{M}{Z}\ \frac{P'}{P' - P} \qquad (8)$$

This formula is an approximation, but is sufficiently precise when the ratio of P to P' does not exceed 0.9. For higher values the formulae of equations (2) and (5) are recommended. It should be noted here that P' in equation (8) is introduced merely as a substitute for the expression $\frac{\pi^2 EI}{L^2}$ and its value is not limited by the strength of the material at the elastic limit, as is the case when calculating the strength of a "Euler" strut, as has been explained in connection with equation (1).

Failure of the column may be expected to occur when the total fiber stress f_t reaches the elastic limit of the material in compression.

Latticed Columns.

The above formula, equation (8), has been derived for a column of homogeneous material, but may be applied to one built up of longitudinal members or flanges which are laced together with lattice bars if attention is paid to the fact that the individual flanges act independently as little columns of length equal to the lattice spacing. The maximum fiber stress of equation (8) should be limited to the end stress which the flange will carry as a pin-ended column whose length equals the lattice spacing. It is not correct to base the design of a lattice column on the assumption that the column is homogeneous and then limit the spacing of the lattices such that the $\frac{L}{K}$ of each flange between the points of attachment of lattices does not exceed the $\frac{L}{K}$ of the column as a whole. This procedure leaves no margin to allow for the increase in stress in the flange due to its acting as an independent column between lattices.

Another point to be noted is that when the column is acting as a beam the flanges receive their load from the lattices, and the flange as a whole acts approximately along its centroidal axis. In calculating the section modulus, Z, therefore, it will be more nearly representative of the true condition if the "extreme fiber distance", Y, is measured from the centroid of the flange instead of taking one-half the depth of the column. This amounts practically to assuming that the stress is uniformly distributed over the flange section.

Application of Theory to Practical Columns.

Many attempts have been made to develop a rational formula which will properly express the state of stress in a practical column, but this has not yet been accomplished. Paaswell says * in commenting on a recent paper on the subject: "Briefly, a column is an engineering structure subjected to a compressive force of a determinate character and to a flexure absolutely indeterminate and unpredictive with any mathematical certainty. This of course refers to columns presumably axially loaded. The introduction of flexural stresses occurs in a manner which can only form a matter of conjecture."

Chew** classifies imperfections which may reduce the strength of an actual column as follows:

"1. Initial stresses in material due to manufacture.
2. Variation in strength of component parts of section.
3. Crookedness of component parts.
4. Crookedness of whole member.
5. Local stresses due to details and shop work.
6. Accidental eccentricity.
7. Deflection caused by the foregoing imperfections."

Basquin*** too has gone into the problem of developing a formula for the design of columns which will take separate account of the stresses to be anticipated in the actual column due to crookedness, probable eccentricities, etc., but the tests on which his work has been based were not extensive enough to warrant the general application of his conclusions to design.

It has been found, furthermore, that a built-up column as regards bending action does not act as a perfect unit. Fig. 2 is

* Proc. ASCE, January, 1922.
** Proc. Am. Soc. Civil Engrs., May, 1911.
*** Basquin on Columns Journal W.S.C.E., 1911.

taken from the comments of Prof. H. F. Moore, of the University of Illinois*, and gives the results of a series of tests conducted at the University of Illinois to determine the ratio of computed to actual fiber stress in the cross section of members built up of channels, fastened together with different types of lacing. Quoting Prof. Moore, "Short column sections (all of the same length) were tested as beams with flexure in a plane parallel to the plane of the lacing. Assuming integrity of action of cross section, the extreme fiber stresses in a test beam were calculated for various loads, and the actual fiber deformations developed under these loads were measured by means of a strain gauge, and the actual fiber stresses, determined from the observed elongations and compressions, were indicated by the strain gauge. In Fig. 2 is shown the variation of flexural efficiency with computed fiber stress for various column sections. In a column of usual length in structures ($\frac{l}{r}$ = 50 to 75), the compressive stress is the principal stress in the column and the flexural stress is not very high; so in comparing the flexural efficiencies of different column sections the efficiencies under low flexural stresses are most significant. The superiority of the double-laced section with rivets at the crossing of the bars is evident; the efficiency of this section at low stress proved to be the same as the efficiency of a pair of channels tested in flexure in a plane parallel to the plane of their webs. The low efficiency of channels connected by means of batten plates is

* Illinois University Bulletin No. 40.

noteworthy as is the very low efficiency of two channels connected by non-overlapping bars with only one rivet for each end of a bar. In each test piece approximately the same weight of lacing material was used, and all tests were in duplicate. Each test was loaded symmetrically at two points of the span, and the spans were the same for all test pieces."

Major Nicholson has also observed, in a series of tests on metal girders designed for airplanes, that the deflections of latticed girders under transverse loading exceeded those of similar girders with solid webs.*

The weight of other authorities whose opinions are in the same vein might be added, but those quoted above should be sufficient to indicate the difficulties to be encountered in attempting to calculate the distribution of stress in compression members. In ordinary structural design these difficulties are sometimes circumvented by the device of limiting the calculated maximum intensity of stress due to the combination of end and side loads to the allowable end stress on the strut as a simple pin-ended column. This procedure is illustrated in the design of a large derrick boom which has been worked out in detail by M. C. Bland in a paper entitled "Investigation of Stresses in Derricks."** This procedure is conservative, and while it probably gives results which are quite satisfactory for structural work where a slight excess in the weight of a member is not a serious matter, it is not sufficiently precise for general use in the design of

* The Development of Metal Construction in "Aircraft Engineering," London, March 12, 1920.
** Trans. ASCE, 1920.

airplane girders, particularly when the end load is relatively small compared with the transverse load. As the magnitude of the end load approaches zero, the column becomes a simple beam, but according to the above method the criterion for the maximum intensity of fiber stress is still the limiting stress on the member as a pin-ended column.

We are thus forced to the conclusion that for the design of compression members the theoretical formulae must be reinforced and modified by experiments on the particular type of column which is to be used. The most hopeful procedure is to select a formula such as equation (8) and by a series of careful experiments on full-size columns, determine the factors which must be introduced into this formula to make it fit the actual members. Referring to equation (8), it will be noted that there are two quantities, P' and h, to which modifying factors could be applied.

As has been stated previously, P' in this formula is merely a shorthand expression of the quantity $\frac{\pi^2 EI}{L^2}$. Now the only quantity in this expression to be determined experimentally is the E, which represents the modulus of elasticity of the built-up member. This can be easily found by measuring the deflection of the column when loaded as a simple beam by a transverse load concentrated at the center, and solving for E in the well-known deflection formula $S = \frac{1}{48} \frac{WL^3}{EI}$. The procedure may be improved, however, by retaining $\bar{E}$ as the modulus of elasticity of the material of which the column is built and introducing a coefficient C into the formula: thus $S = \frac{1}{48} \frac{WL^3}{CEI}$. C may be looked upon as the "form factor" for the section, and represents the ratio of the stiffness of the actual column to that of a solid theoretical column of the same material. This coefficient could then be applied to the calculation of P', but it will be preferable to introduce C into equation (8) and use the modulus of elasticity of the material in calculating P'.

The term h, may be considered as being the sum of the known eccentricity of the application of the load to ends of the column H, and an equivalent eccentricity which represents the overall "constructional" eccentricity of the actual column, that is, the sum of the imperfections of the actual strut and is designated by e. To find e, it is necessary to build and test as pin-ended struts with axial loads, a number of full-sized columns of varying lengths of the type to be used. These test specimens should of course be built as far as possible to the same quality of workmanship and straightness as will be followed in the construction of the columns or beams to be used in the airplane itself. A column formula may be plotted from the results of these tests and e calculated from the relation

$$\frac{0.6\ ed}{K^2} = \left(f_c \frac{A}{P_o} - 1\right) \left(\frac{CP' - P_o}{CP'}\right)^* \qquad (9)$$

Where f_c = the compressive elastic limit of the material for homogeneous struts and for latticed struts the limiting unit end stress on an individual flange of a length equal to the lattice spacing,

* *From Morley, p. 276.*

$\frac{P_o}{A}$ = the observed ultimate end stress as a pin-ended column.

P' = critical end load calculated from Euler's formula.

C = the form factor coefficient mentioned above (This factor does not appear in the formula as given by Morley.)

With latticed columns additional tests must be made of the strength of the individual flanges as pin-ended columns to determine the proper value of f_c to be used in the above formula.

The above equation (9) appears somewhat formidable, but it will be found from experiment in most cases that the eccentricity e, can be sufficiently expressed as a simple function of the length/or of the L/K of the column.

Introducing the above modifications, equation (8) may be re-written as

$$f_c = \frac{P}{A} + \frac{P}{A}\left(\frac{CP'}{CP' - P}\right)\left(\frac{0.6\ (e + H)\ d}{K^2}\right) + \frac{M}{Z}\left(\frac{CP'}{CP' - P}\right) \qquad (10)$$

Forces in the Bracing of Latticed Columns.

The forces which act upon lattice bars have been divided by Basquin into three classes:* "First, those introduced in the fabrication of the column;; second, those due to transverse shear caused by local bends in the column; and third, those due to transverse shear caused by general inclination of the column." The latter two conditions have been investigated in a series of careful extensometer tests by Talbot and Moore.** In case of a column built of two channels latticed together with flat bars and

* Journal W.S.C.E., 1913, p.493.

** "An Investigation of Built-Up Columns Under Load," University of Illinois Bulletin #40 of June 10th.

with an average end stress of 10,000 pounds per square inch, they conclude that: "It is evident from the tests that the relative stress in the two-channel members varies considerably from end to end and that the stress in the lattice bars also varies. It seems probable that the transverse shear developed may be traced largely to irregularities in outline, or at least that these irregularities may be expected to cover up other causes of stress in the lacing of centrally-loaded columns, if we include in such irregularities all unknown eccentricity. The futility of attempting to determine analytically the stresses in column lacing, using as a basis either a bending moment curve which varies from end to middle or an assumed deflection curve, is apparent from a study of the variation of stress in the columns of the tests and in that of the lattice bars."

It is necessary, nevertheless, to find some means of approximating the loads in the lattice members. The method most favored in the design of structural columns is to assume that the column is loaded with a uniformly distributed transverse load w, where w is the transverse load, which, considering the column as a simple beam, will produce a maximum fiber stress equal to the difference between the elastic limit of the material and the end unit stress allowed by the column formula. The vertical component of the load in the lattices at the ends of the column equals $\frac{wl}{2}$, which is assumed to be equally distributed between the lattices cut by a vertical plane normal to the axis of the girder.*

* Spofford - "The Theory of Structures," 1915, p. 303.

Alexander* has investigated the distribution of the shearing stresses in an ideal column, using equations which involve the true elastic curve of flexure of the column. His expression for the maximum shear may be put into the form

$$R = \frac{\pi Z}{L} \left(f_c - \frac{P'}{A} \right) \tag{11}$$

Where R = the shear at end of column;
Z = section modulus;
f_c = limiting stress on short column;
P' = Euler crippling load.

The constant π in the above equation is increased to 5 for actual struts to allow for longitudinal irregularities and slight imperfections in fitting and securing the lattice bars.

It may be noted that the assumption that the shear may be determined on the basis of the uniformly distributed lateral load mentioned above amounts to assuming a parabolic curve for the deflection of the column. This latter approximation gives a value of

$$R = \frac{4Z}{L} \left(f_c - \frac{P}{A} \right)$$

which exceeds the shear calculated by the more exact method but is less than that recommended by Alexander for practical columns. Until this subject has been more thoroughly investigated by experiment, it is recommended that the shear in lattice bars be calculated by Alexander's formula,

$$R = \frac{5Z}{L} \left(f_c - \frac{P'}{A} \right) \tag{12}$$

* Wm. Alexander, "Columns and Struts," 1912, Chap. X.

In view also of the approximate nature of this method, and the variations in shear due to local irregularities, etc., no attempt should be made to vary the strength of the latticing along the length of the column. Alexander points out that "the points where the deflection is a maximum and the shearing forces nil, are unknown and certain to be different in each strut," and concludes, "each case must be considered on its own merits. No general formula can be given for even the probable limits of reduction in shearing stresses."

It will be noted that the expression $\left(f_c - \frac{P'}{A} \right)$ in equation (12) would equal the fiber stress in an ideal strut due to flexure. To apply this formula to a practical strut, substitute for $\frac{P'}{A}$ the fiber stress permitted by the experimental formula. The resulting shear in the lattices will be 20 per cent in excess of that determined by the procedure given by Spofford.

To find the shear in the lattice bars of a strut under combined end and transverse loads, let the sum of the second and third terms of equation (10) equal f_b

$$\text{Then} \quad R = \frac{5Z}{L} \ (f_b) \tag{13}$$

This use of this formula is recommended.

<u>Illustrative Problem.</u>

To illustrate the application of this method of analysis an example will be worked out. The column chosen is of suitable proportions to be used as a portion of the wing beam of a large airplane. Its strength about the horizontal axis only will be investigated. Fig. 3 is a sketch of the column. It will be noted that the latticing on the top and bottom faces is entirely inside the flanges. While not the best design from a structural standpoint, it is desirable to facilitate sliding the wing ribs along the beam in the assembly of the wing panel.

Let P = the total end load including the factor of safety = 30,000 lbs.

M = maximum bending moment due to the uniformly distributed transverse loads = 81,600 inch pounds.

L = length of beam between points of inflexion = 144 inches.

A = area of flanges = 0.88 square inches.

I = moment of inertia = 6.9 $(\text{inches})^4$

K = radius of gyration of section about axis XX = 2.8 inches.

L/K = 144/2.8 = 51.

y = distance from centroid of section to centroid of flange = 2.8 inches.

Z = Section modulus = I/y = 2.46 inches cubed.

f_c = crippling end stress on one angle of flanges as a pin-ended column whose length equals the pitch of the lattices = 105,000 lbs. per sq. in.

$$P' = \frac{\pi^2 EI}{L^2}$$

P_o = failing load as a pin-ended column.

In the absence of experimental data on columns of this type,
~~curve A of Fi~~
curve A of Fig. 4 has been more or less arbitrarily chosen to represent the relation between L/K and the failing end stress.

C - the form factor coefficient has been assumed equal to 0.8.

Then $\frac{P_o}{A}$ = 58,000 lbs. per sq. in. from Fig. 4.

and $$CP' = \frac{0.8\,\pi^2 (30{,}000{,}000)\ (0.88 \times 2.8^2)}{(144)^2} = 79{,}000.$$

Find the "constructional eccentricity" of the column e from equation (9).

$$\frac{0.6\ e(d)}{(K)^2} = \left(\frac{105000}{58000} - 1\right)\left(\frac{79000 - 58000\ (.88)}{79000}\right) = 0.29$$

and e = 0.606" which is $\frac{1}{230}$ of the length of the column.

Substituting the above value of $\frac{0.6\ ed}{K^2}$ into equation (10) the maximum fiber stress in the flanges at a point of attachment of the lattice equals

$$f_t = \frac{P}{A} + \frac{P}{A}\left(\frac{CP'}{CP' - P}\right)\left(\frac{0.6\ ed}{K^2}\right) + \frac{M}{Z}\left(\frac{CP'}{CP' - P}\right) =$$

$$34100 + 34100\left(\frac{79000}{49000}\right)(0.29) + \frac{72600}{2.46}\left(\frac{79000}{49000}\right) =$$

$$= 103400 \text{ lbs. per sq. in.}$$

Since f_t is less than 105000 lbs. per sq. in., the area provided in the flanges is sufficient.

<u>Load in Lattice Members.</u>

By equation (13) the shear at the end of the column equals $R = \frac{5Z}{L}(f_b)$.

There f_b equals the maximum flexual fiber stress as expressed by the second and third terms of equation (8).

$$R = \frac{5\ (2.46)}{144}\ 69300 = 5920 \text{ lbs.}$$

Assuming the above shear distributed equally between the four lattices cut by a plane perpendicular to the longitudinal axis of the column, the total load in each lattice equals

$$\frac{5920}{4} \times \frac{7.25}{5.62} = 1905 \text{ lbs.}$$

Strength of Individual Lattices.

Assume that the lattice in compression is supported at the center by the adjacent lattice which is in tension and that the lattice fails as a pin-ended column whose length is equal to one-half the length of the lattice between centers of flange rivets.

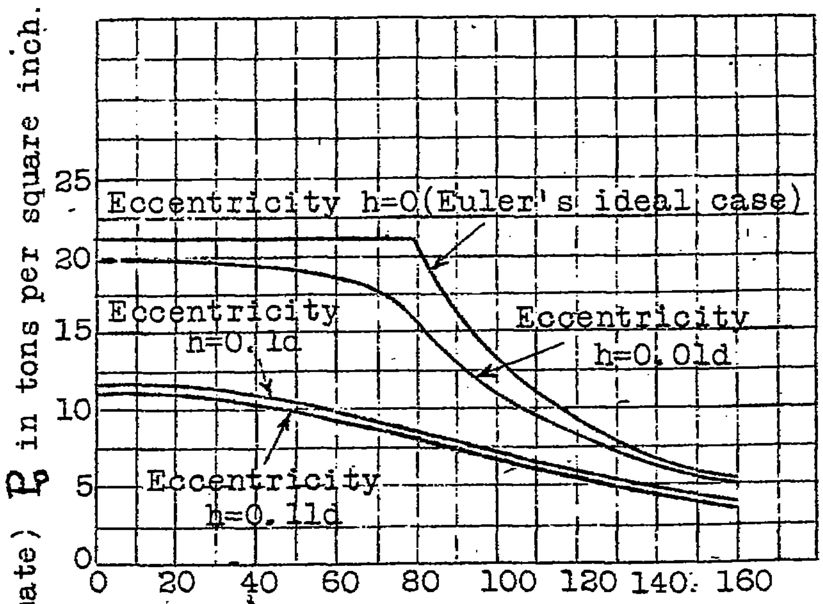
Area of section = 0.0325 sq.in.

Least radius of gyrations = 0.075 in.

$$L/K = \frac{3.4}{.075} = 45.$$

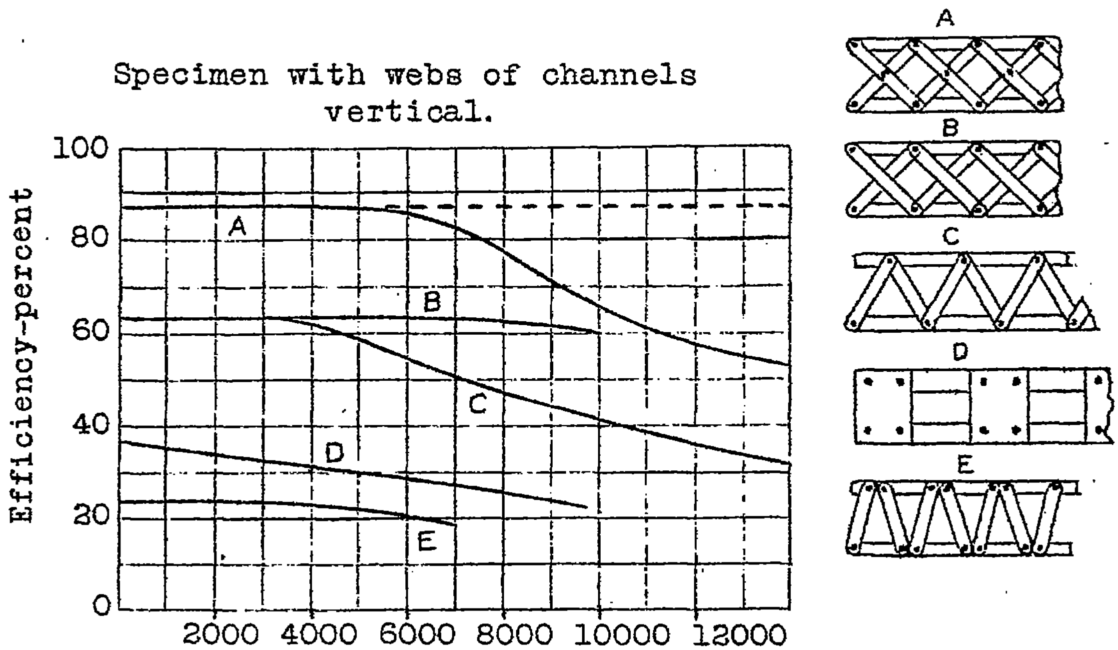
In the absence of test data on lattices as used in this design a column formula somewhat more conservative than Rankin's has been arbitrarily chosen. Using steel having an elastic limit of 100,000 lbs. per sq.in., the allowable P/A for an L/K of 45 equals 60,000 lbs. per sq.in.

The strength of the lattice = 60,000 x 0.0325 = 1950 lbs., which exceeds the required strength of 1905 lbs. The lattice design is therefore satisfactory.



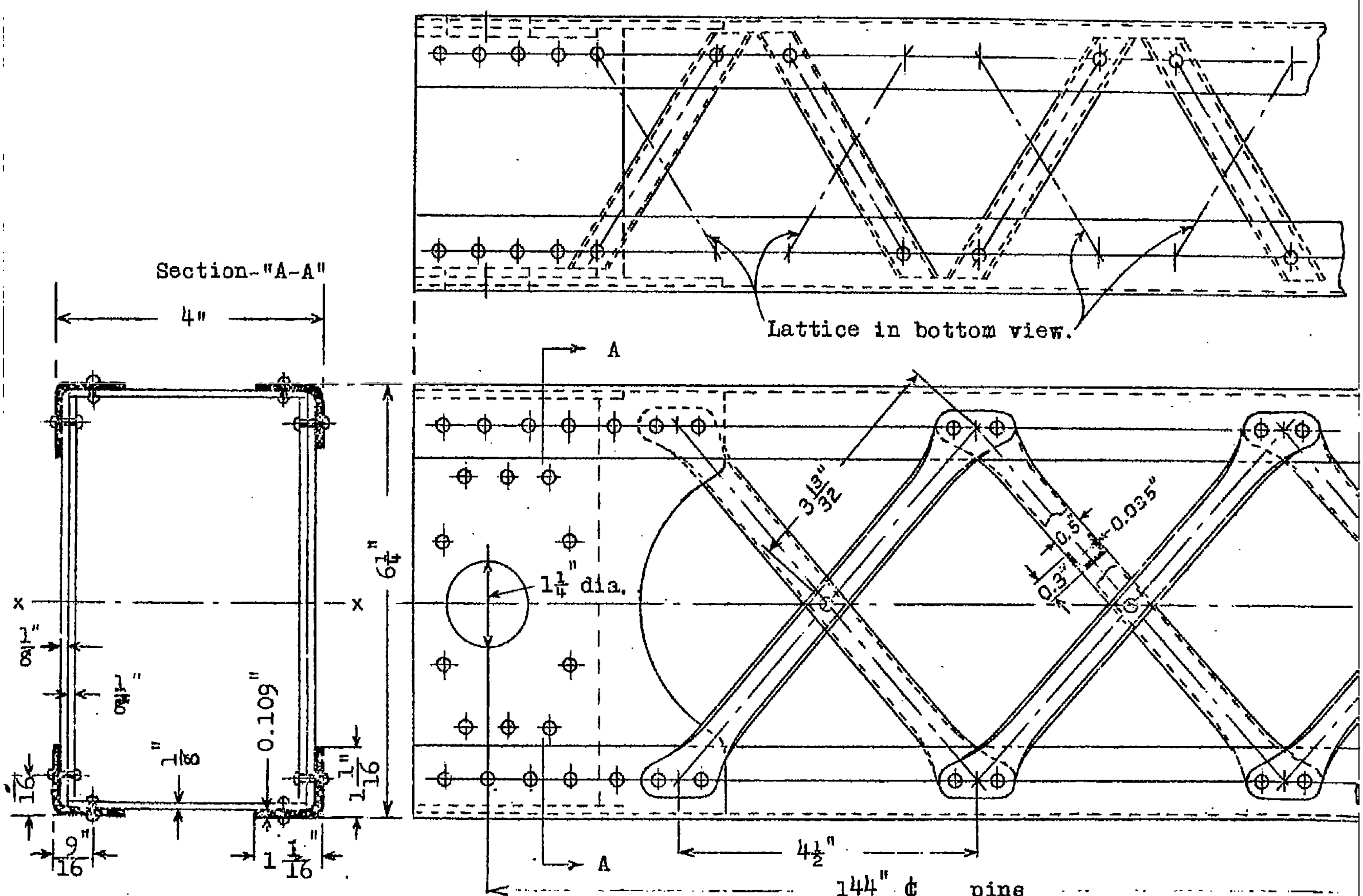
Ratio $\frac{l}{K}$ Mild steel strut of circular section. Morley-strength of materials-1916.

Fig. 1.



Computed fibre stress in flexure-lbs./sq.in.
Efficiency=(computed fibre stress)÷(observed fibre stress)

Fig. 2.



Section-"A-A"
4"
6¼"
x
x
0.109"
9/16"
1 1/16"
1 1/16"
A
A
Lattice in bottom view.
1¼" dia.
3 13/32"
0.5"
0.035"
0.3"
4½"
144" ℄ pins

Fig. 3.

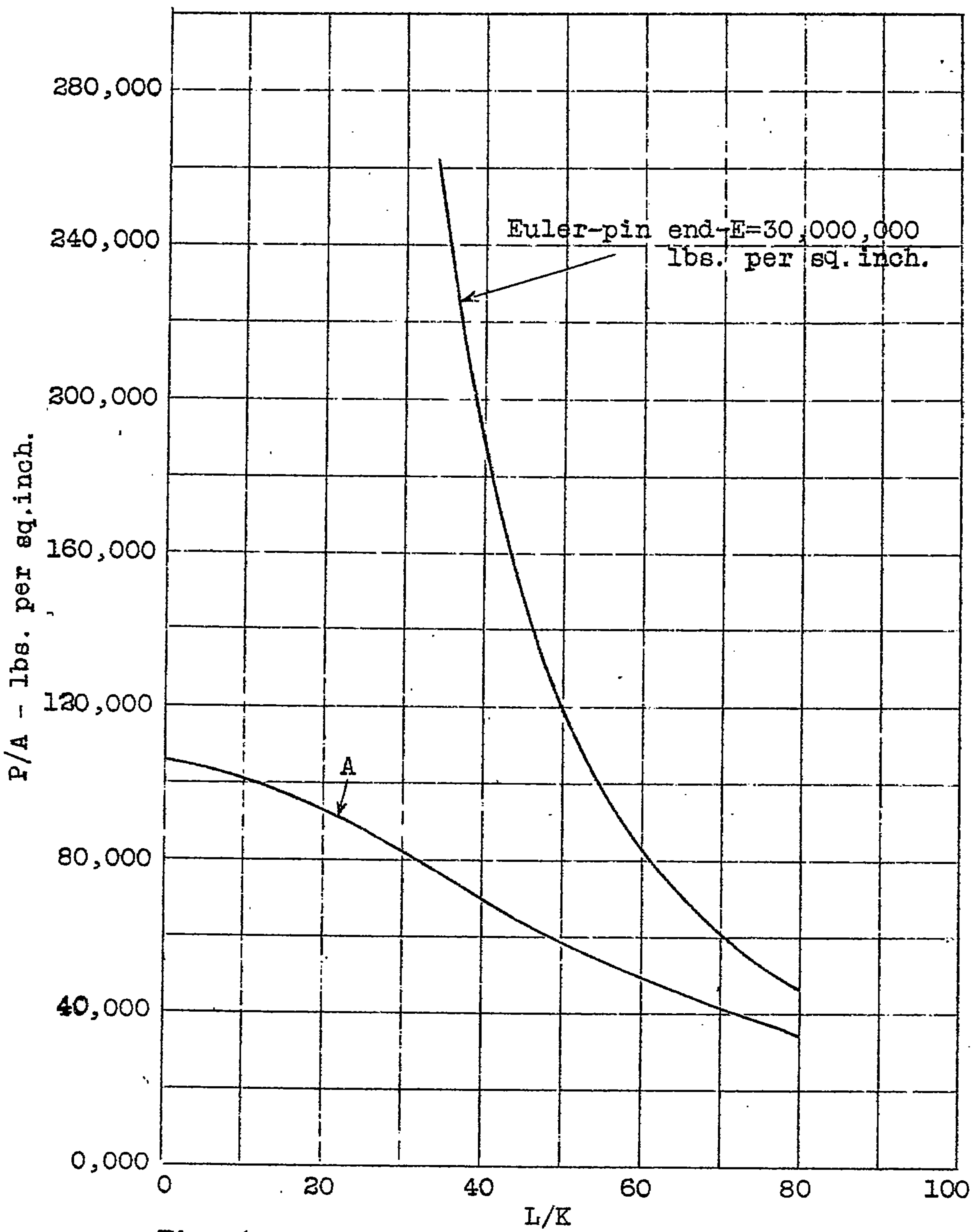

280,000
240,000
200,000
160,000
120,000
80,000
40,000
0,000
P/A - lbs. per sq.inch.
Euler-pin end-E=30,000,000 lbs. per sq.inch.
A
0
20
40
60
80
100
L/K

Fig. 4.