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THE DETERMINATION OF DOWNWASH.

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Lt. Walter S. Diehl,
Bureau of Construction & Repair, U.S.N.

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Introduction.

It is obvious that, in accordance with Newton's second law, the lift on an aerofoil must be equal to the vertical momentum communicated per second to the air mass affected. Consequently a lifting aerofoil in flight is trailed by a wash which has a definite inclination corresponding to the factors producing the lift. It is thought that sufficient data, theoretical and experimental, are now available for a complete determination of this wash with respect to the variation of its angle of inclination to the originating aerofoil and with respect to the law which governs its decay in space.

Munk's Formula for Downwash.

Although it has long been known that the angle of downwash ϵ, as observed at a given point behind the aerofoil, is directly proportional to the lift of the aerofoil (Br. A.C.A. R. & M. No.196) and inversely proportional to the aspect ratio (Lanchester "Aerial Flight" Vol. 1, Chapter VIII, Br. A.C.A. R. & M. No.161), Munk (Technische Berichte III-1) seems to have been the first to propose a quantitative solution. He asserts that ϵ must be represented as the product of some constant and the angle of attack as expressed by the formula (Betz, T.B.I-4),

$$\alpha = \frac{57.3}{\pi} \cdot 2 L_c \left(\frac{S}{b^2}\right)$$

where b is the span, L_c is the lift coefficient and S is the area of the aerofoil. The formula for downwash then becomes

$$\epsilon = c \cdot \alpha = \frac{57.3c}{\pi} \cdot 2L_c \left(\frac{S}{b^2}\right)$$

the value of the constant c being determined by experiment. The formula as given applies to monoplanes but may be applied to multiplanes, according to Munk, by the introduction of another constant k which reduces the span b to the span of the equivalent monoplane.

The values of c were determined for several models by photographing a series of streamers. Owing to the lack of certain vital data, the results have not been included in this study, but the conclusions are given instead. It appears that the equation, as given above, is not general. The values of c vary somewhat more than is allowable for a "constant". No attempt was made to determine the variation of c with aspect ratio, nor was any allowance made for the inevitable dying out or the wash effect in space. It appears, however, that the angle of downwash is substantially constant over about eight-tenths of the span, with sudden changes near the tips.

N. P. L. Formula.

The most comprehensive series of tests on downwash, which have been published, are those by Sandison, Glauert, and Jones (Br. A.C.A. R. & M. No.426). In this investigation the variation of downwash was determined in space for a number of points behind, above, and below the trailing edge of the aerofoil. It was found that, in accordance with hydrodynamic theory,* the angle of downwash decreases exponentially with the distance from the aerofoil (a biplane in this case) and might be expressed by the empirical formula

$$\epsilon = \epsilon_0 \; 10^{-0.05\,\xi - 0.08\,\zeta}$$

where

ξ is the distance behind the wing in chord lengths,

ζ is the distance below the chord of the upper wing in terms of the gap,

and ϵ_0 is a constant for any given arrangement.

This appears to have been the first attempt to express the variation of ϵ_0 from point to point. With a satisfactory law for the variation of ϵ_0 it would have been complete.

Derivation of a Comprehensive Downwash Formula.

It is now possible to derive a comprehensive downwash formula based on the Göttingen theoretical and the N.P.L.

* See Lanchester, Aerial Flight, Vol. I, Chap. III.

empirical formulae. It is known definitely that downwash varies exponentially with distance from the trailing edge. The plotted results of N.P.L. investigations, which show this variation vertically and horizontally, are given in Figs. 1 and 2, respectively. The data in Fig. 1 have been replotted on a logarithmic scale in Fig. 3, with the vertical distance from the trailing edge expressed in chord lengths plus one chord length* as abscissa and angles of downwash as ordinates. It is found that for a given angle of attack, the angles of downwash at various vertical distances from the trailing edge lie on a straight line. The lines corresponding to the various angles of attack are all parallel and have a slope of $-13^\circ \pm 0.5^\circ$. This indicates that the variation of angle of downwash with vertical distance from the trailing edge can be represented by an equation of the form:

$$\epsilon = c_2 \, (y + 1)^n,$$

where c_1 = a constant,

y = the vertical distance, of the point under consideration, in chord lengths, from the trailing edge,

and $n = \tan(-13^\circ \pm 0.5^\circ)$

$= -0.23 \pm 0.01.$

In a similar manner the data from Fig. 2 have been plotted in Fig. 4. The points again fall near parallel straight lines but their slope, $-21^\circ \pm 0.5^\circ$, is steeper than that in Fig. 3. The indicated variation of the angle

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* This is necessary so as to provide a finite value at the trailing edge.

of downwash with variation of horizontal distance from the trailing edge is of the form

$$\epsilon = c_2 \ldots (x + 1)^n$$

where c_2 = a constant,

x = the horizontal distance, of the point under consideration in chord lengths, from the trailing edge,

and

$n = \tan(-21^\circ \pm 0.5^\circ)$

$= -0.38 \pm 0.01.$

In order to eliminate the calculations involving fractional exponents the functions,

$$Z = (y + 1)^{-0.23}$$

and

$$Z = (x + 1)^{-0.38}$$

have been evaluated and plotted in Fig. 5.

Data from five series of downwash determinations have been plotted in Fig. 6, with angles of downwash as ordinates and lift coefficients as abscissae. The slope of the straight line, which passes through the points representing a series of tests, determines the value of $\Delta\epsilon/\Delta L_c$ for that particular arrangement and the point in space at which the observations were taken. The aspect ratio, the value of $\Delta\epsilon/\Delta L_c$, and the coordinates of the observation point are given, for each series of tests, in Table I.

It is evident from inspection of Fig. 6 that ϵ varies directly with lift coefficient. It has also been shown by data from the tests of Sandison, Glauert, and Jones (Br. A.C.A. R. & M. No.426) how ϵ varies in space. Munk's equation indicates that ϵ varies inversely as the aspect ratio, n. Therefore the angle of downwash should be given by

$$\epsilon = \frac{K}{N} \cdot (x+1)^{-0.38} \cdot (y+1)^{-0.23} \cdot L_c$$

where K is a constant, numerically equal to $\Delta\epsilon/\Delta L_c$ at the trailing edge of a wing of aspect ratio unity.

The value of K is determined for each of the five series of tests which are plotted in Fig. 6, by substituting the proper values for the functions of x and y and for the aspect ratio n. The procedure is indicated by the headings of columns in Table I.

It is found that K is substantially constant, varying from 164 to 176; a single exception of 158 corresponds to a series of tests on a biplane arrangement, the wings of which were equipped with flaps and represent abnormal conditions. It therefore appears that the angle of downwash can be represented to a good approximation by

$$\epsilon = \frac{170}{n} \cdot (x+1)^{-0.38} \cdot (y+1)^{-0.23} L_c$$

$$\epsilon = \frac{170}{n} \cdot F_x \cdot F_y \cdot L_c$$

F_x and F_y being the values of the functions of x and y which are given in Fig. 5.

The validity of this formula is obviously confined to that range of angle of attack or lift coefficient in which the air flow about the aerofoil is not abnormally turbulent.

Application of the Downwash Formula.

The chief use of a downwash formula is the calculation of the aerodynamic angle of attack of the horizontal tail surfaces. For this purpose a reference point is taken on the leading edge of the horizontal tail surfaces and the values of ϵ obtained from the formula. The aerodynamic angle of attack of the tail surfaces will then be

$$\alpha_t = \alpha - \beta - \epsilon$$

where α is the angle of attack of the wing and β is the acute angle between the chord lines of the wings and horizontal tail surfaces, considered positive (in the equation) if the tail is set at a less apparent angle than the wings.

The data from tests seem to indicate that in case of a biplane the maximum angle of downwash occurs in the horizontal plane midway between the two wings. The effect is so slight, however, that the above method may be used, referring the coordinates of the reference point to the nearest wing (preferably to the no lift line), with the assurance that the results so obtained will be as precise as it is practicable to calculate them with the data now available.

TABLE I.

DETERMINATION OF K IN THE EQUATION

$$\epsilon = \frac{K}{n} \cdot F_x \cdot F_y \cdot L_c$$

Source of Data	Aspect Ratio n	$\frac{\Delta \epsilon}{\Delta L_c}$	x	y	F_x	F_y	$\frac{\Delta \epsilon}{\Delta L_c}\left(\frac{1}{F_x \cdot F_y}\right)$	$\frac{\Delta \epsilon}{\Delta L_c}\left(\frac{K = 1}{F_x \cdot F_y}\right) \cdot n$
N C-1 Curtiss Tests	9.5	11.20	2.5	0	.625	1.00	17.9	170
R&M #196	6	15.20	2.3	0.55	.64	.90	26.4	158
R&M #426	6	14.70	3.0	0.61	.595	.89	27.8	167
R&M #426	6	16.00	3.0	0.43	.595	.915	29.3	176
R&M #515	7.73	11.8	2.6	0.6	.62	.895	21.3	164

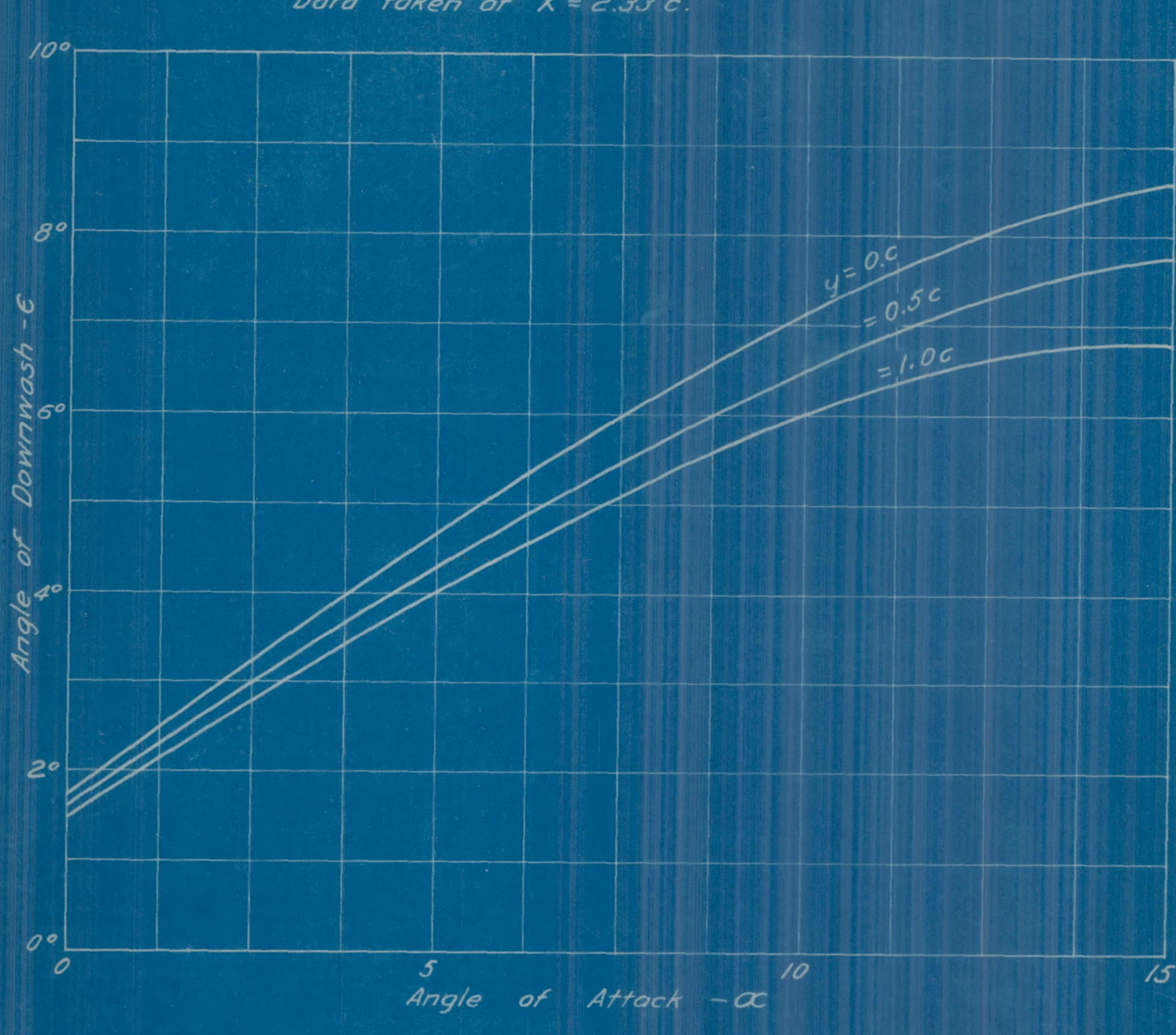
NOTE: There will be a slight decrease in the value of $\Delta\epsilon/\Delta L_c$ with increase in lift coefficient if the reference point is not fixed in space. This is caused by the change in the coordinates of the point with change in angle and the effect may easily be accounted for.

Let D = distance from trailing edge, T, to reference point P

θ = Inclination to horizontal of line TP

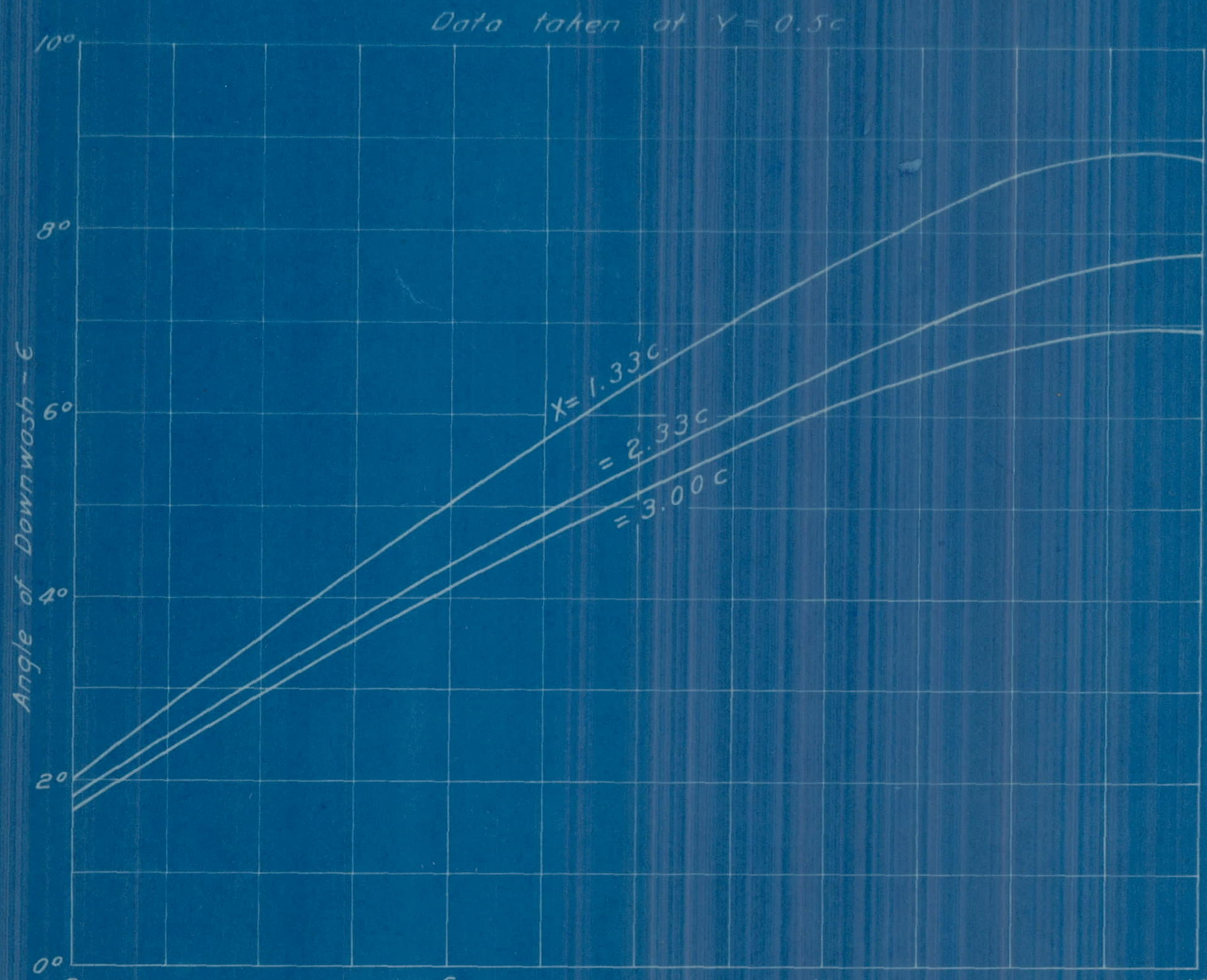
then $x = D \cdot \cos\theta$

$y = D \cdot \sin\theta$



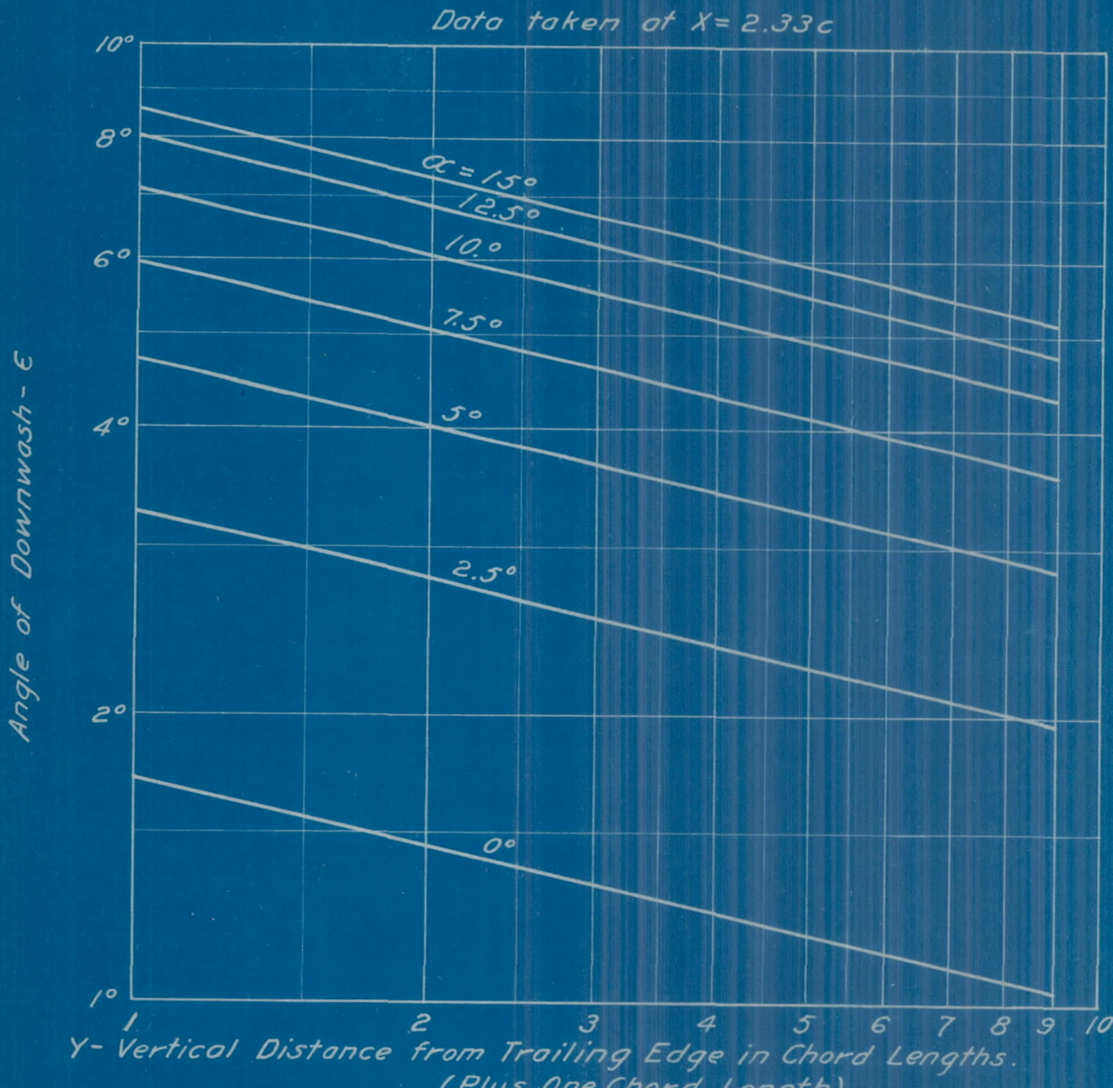
VARIATION OF ANGLE OF DOWNWASH WITH DEPTH Y BELOW UPPER WING. From R. and M. No. 426. Fig. 7.

Fig. 1.



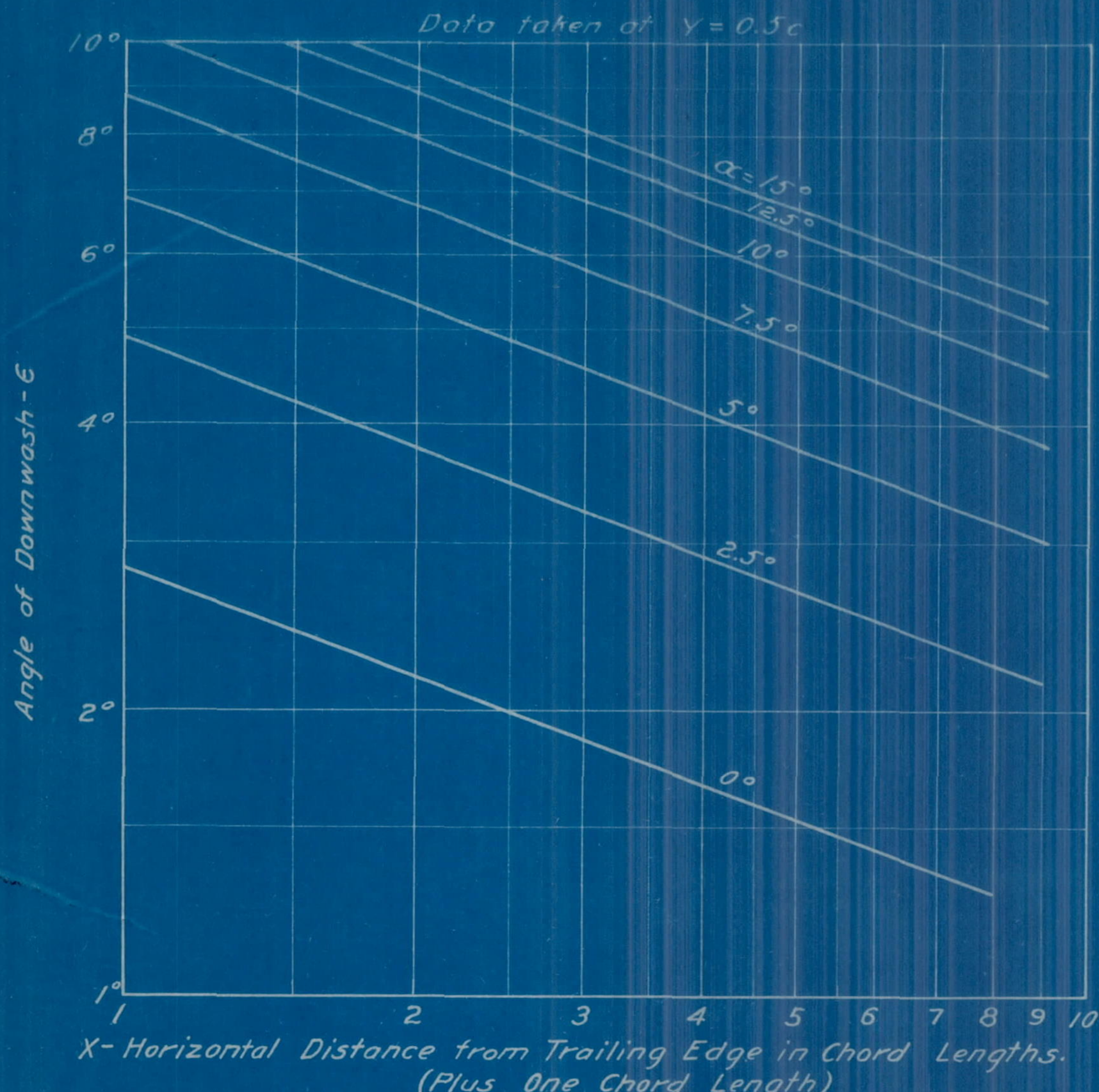
VARIATION OF ANGLE OF DOWNWASH WITH DISTANCE Y BEHIND WINGS. From Br. A.C.A. R. and M. No. 426. Fig. 7.

Fig. 2.



VARIATION OF ANGLE OF DOWNWASH WITH DEPTH BELOW WING.

Fig. 3.



VARIATION OF ANGLE OF DOWNWASH WITH DISTANCE BEHIND WING.

Fig. 4.

Plots of the Equations

$$Z = (x+1)^{-0.38}$$
$$Z = (y+1)^{-0.23}$$

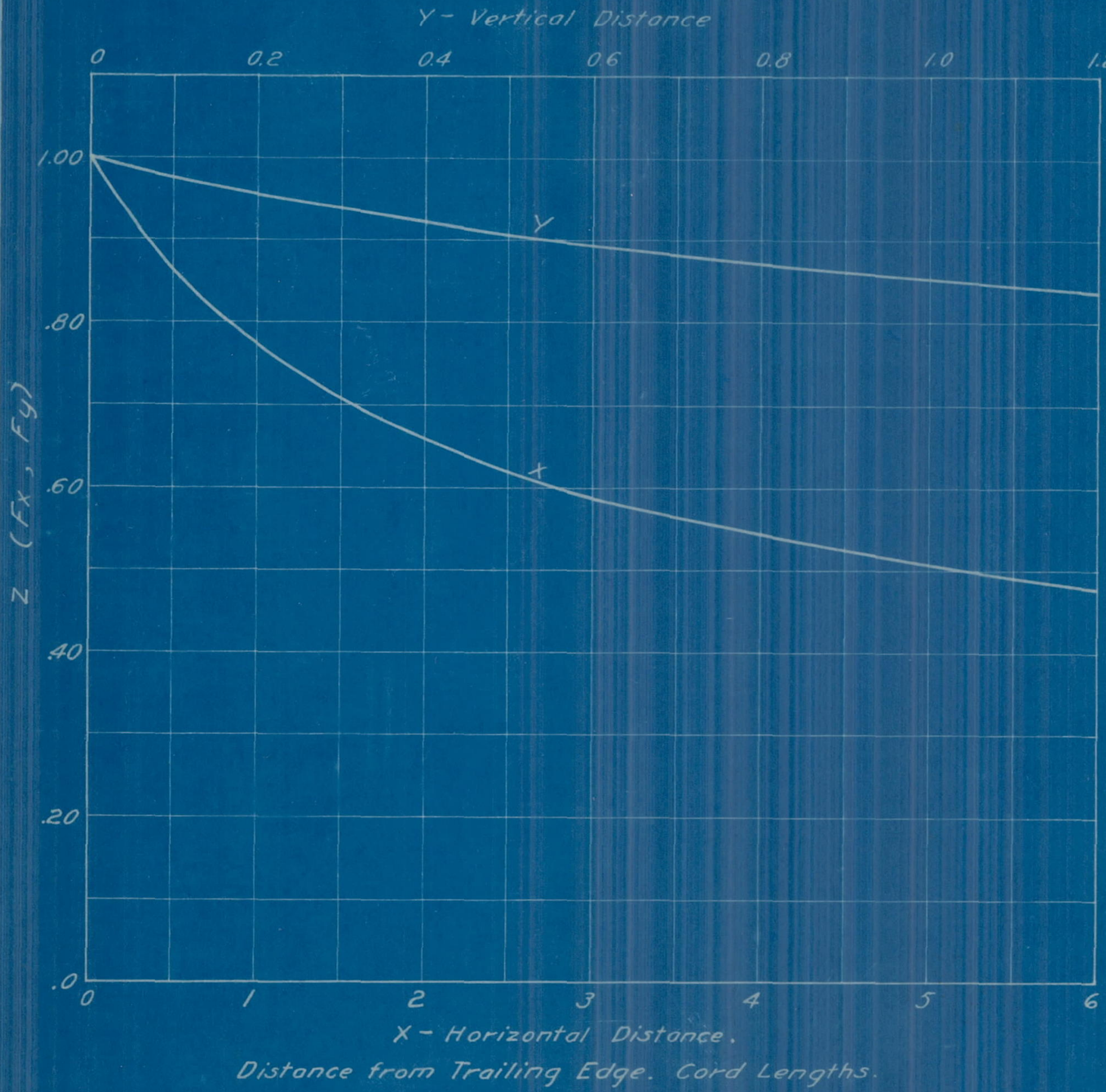
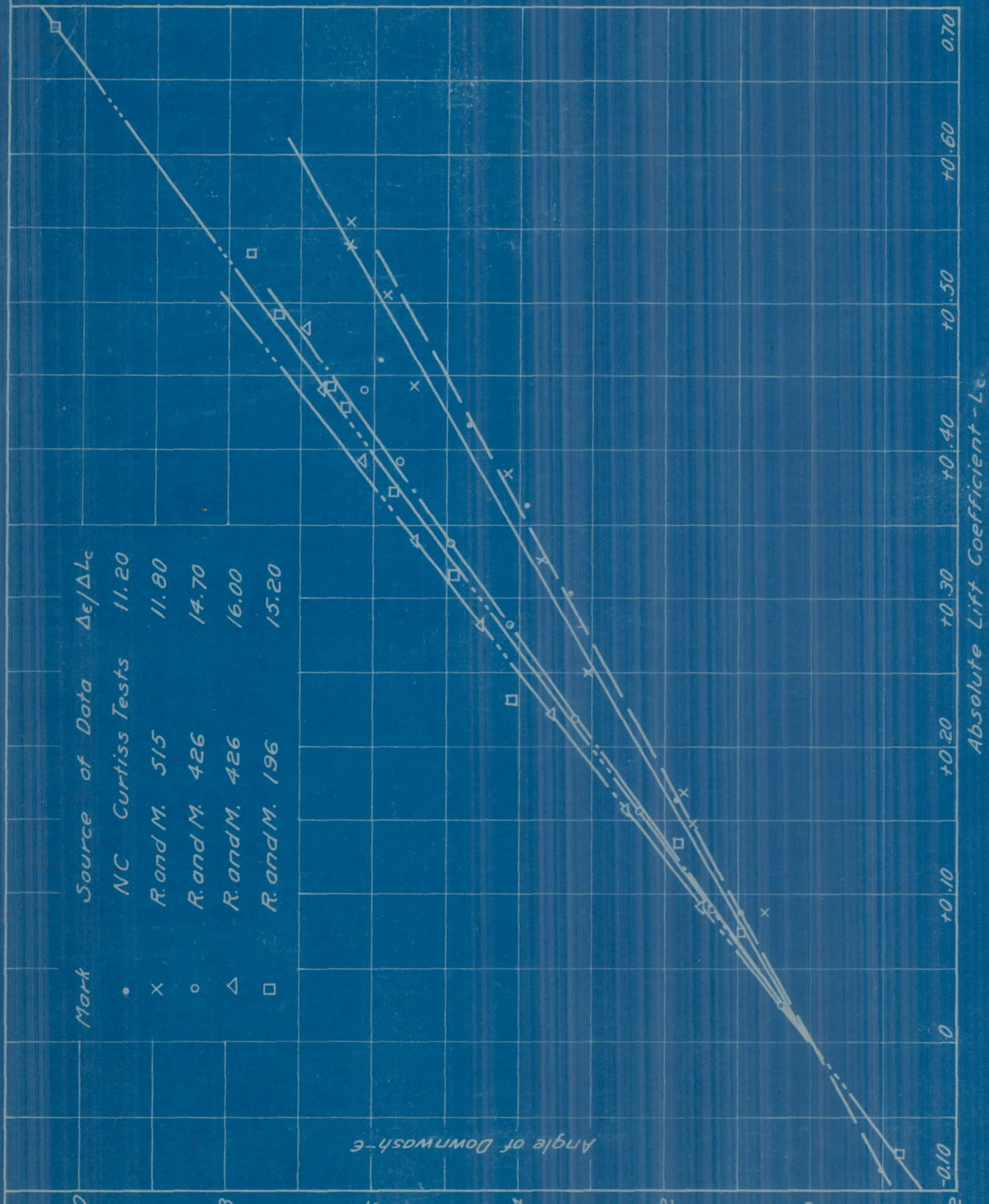


Fig. 5.



VARIATION OF THE ANGLE OF DOWNWASH WITH LIFT COEFFICIENT.

TEST DATA.

Fig. 6.