TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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SIMPLE FORMULA FOR ESTIMATING AIRPLANE CEILINGS.

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Summary.

The absolute ceiling of an average airplane is given to a close approximation by

$$H \doteq 19000 \log_{10} \frac{10000}{\left(\frac{W}{HP}\right)^2 \left(\frac{W}{S}\right)}$$

where

H = absolute ceiling, in feet

$\left(\frac{W}{HP}\right)$ = "power loading," based on full load and normal B.H.P.

and

$\left(\frac{W}{S}\right)$ = wing loading, based on full load.

The aeronautical engineer often has occasion to estimate the absolute ceiling of an airplane for which a detailed performance calculation is out of the question. In such cases it is customary to use either empirical performance charts or formulae. The performance charts which are given in several of the recent works on aerodynamics and performance are satisfactory so long as the airplane under consideration does not depart too far from the average in its characteristics. The formulae, with one exception, are no better. This exception is developed by Kann in Technische Berichte 1-6 and is of the form

$$H = C_1 \log_{10} \frac{C \dfrac{C_L{}^3}{C_D{}^2} \eta^2}{\left(\dfrac{W}{HP}\right)^2 \left(\dfrac{W}{S}\right)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

where

H = absolute ceiling

η = propeller efficiency

$\left(\frac{W}{HP}\right)$ = power loading $\left(\frac{\text{gross load}}{\text{normal BHP}}\right)$

$\left(\frac{W}{S}\right)$ = wing loading $\left(\frac{\text{gross load}}{\text{wing area}}\right)$

C_L = absolute lift coefficient

C_D = absolute drag coefficient

C_1 & C_2 = constants.

The chief criticism of this formula is that it is too complicated for extensive use.

Experience has shown that the terms $\left(\frac{C_L{}^3}{C_D{}^2}\right)$ and η^2 may be neglected without seriously affecting the results given by the formula. That is, we may write

$$H = K_1 \log_{10} \frac{K_2}{\left(\dfrac{W}{HP}\right)^2 \left(\dfrac{W}{S}\right)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2)$$

and determine appropriate values of the constants K_1 and K_2 by calculation or from reliable performance data. Obviously K may be arbitrarily assigned any value which will cause the term within the brackets to assume a reasonable value. The average value of $\left(\frac{W}{HP}\right)^2 \left(\frac{W}{S}\right)$ is of the order of 1000 and it is desirable that the log term be about unity.

$$K_2 = 10000 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

fulfills this requirement and the formula may be written:

$$H = K \log_{10} \frac{10000}{\left(\frac{W}{HP}\right)^2 \left(\frac{W}{S}\right)} \quad \ldots\ldots\ldots\ldots\ldots \quad (2a)$$

K has been determined for a number of airplanes of all types and is found to have an average value of 19000 with individual values ranging from 17500 to 21000. In most cases, however, the value is quite close to 19000 as shown by Table 1, which contains a few representative determinatioms of K.

In any particular case for which the performance is known, the formula is quite accurate in predicting the effect of small changes in W., B.H.P., or S. To do this, Equation (2a) is solved for the exact value of K by substitution of the original data and this value is used with the new values of $\left(\frac{W}{HP}\right)$ or $\left(\frac{W}{S}\right)$ to obtain the new ceiling.

The absolute ceiling for any common values of $\left(\frac{W}{HP}\right)$ and $\left(\frac{W}{S}\right)$ may be read directly from Fig. 1 which is based on Equation (2a) with K = 19000.

Table 1.

Value of constant K in

$$H = K \log_{10} \frac{10000}{\left(\frac{W}{HP}\right)^2 \left(\frac{W}{S}\right)}$$

Airplanes	$\left(\frac{W}{HP}\right)$	$\left(\frac{W}{S}\right)$	H	K	Remarks
JN-4D	22.6	5.72	9250	17300	Landplane
DH-4	11.20	7.62	19200	19800	"
VE-7	11.60	7.36	18900	19200	"
MB-3	7.00	8.4	25500	18400	"
M 80	8.80	12.3	19900	19500	"
Nieuport "Nighthawk"	6.60	7.77	29000	19800	"
Fokker D-VII	8.47	8.36	25000	20400	"
Martinsyde Scout	7.50	6.95	26800	19000	"
N9-H	18.30	5.54	14500	19500	Float seaplane
R-9L	10.9	7.10	20000	18500	" "
HA-2	10.40	8.00	20500	19200	" "
F-5-L	18.10	9.30	9500	18500	Boat seaplane
DT-1	17.20	9.75	10200	19000	" "
NC-5	21.30	9.70	6000	17000	" "

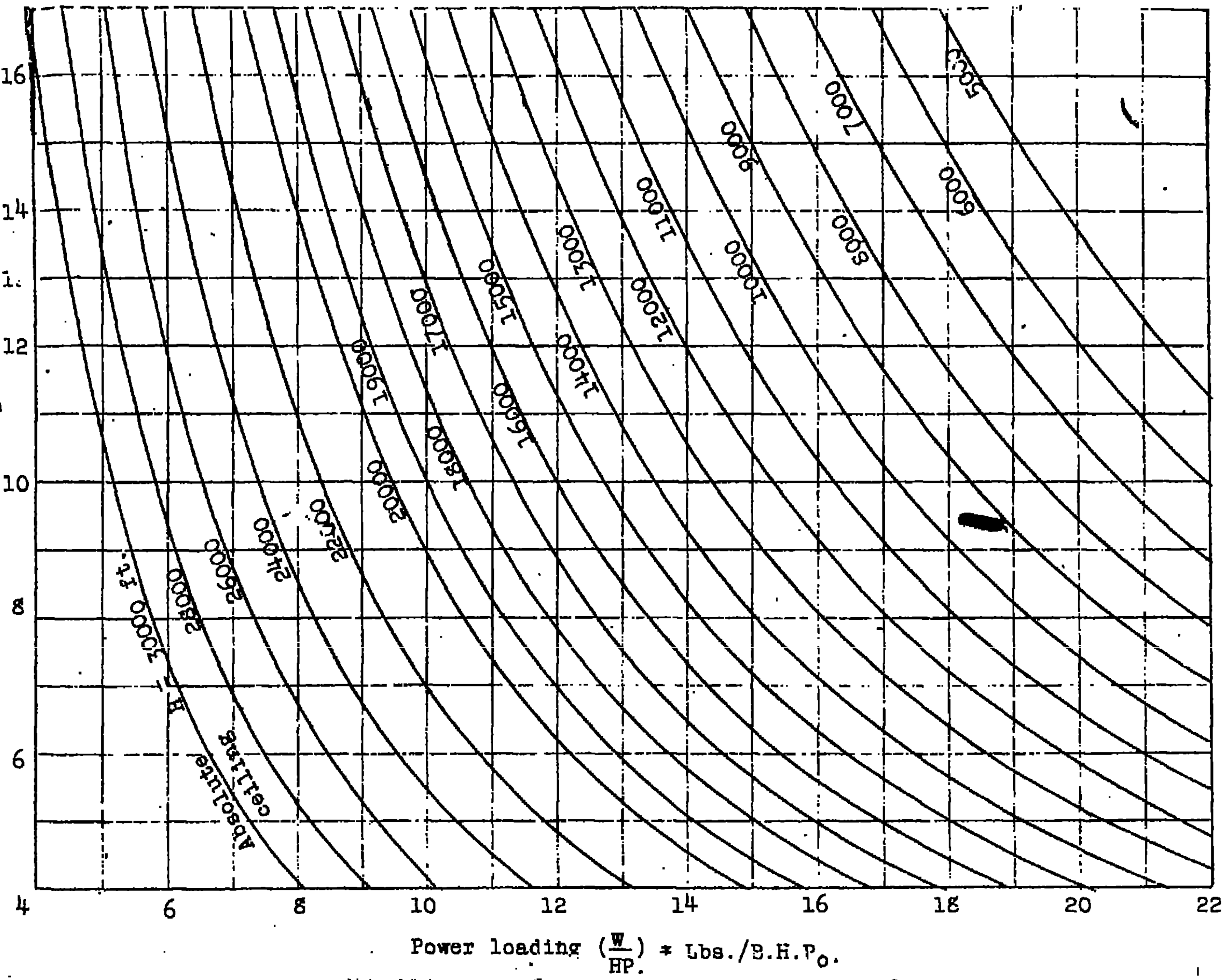
Wing loading (W/S) = Lbs./sq.ft.
Power loading (W/HP.) = Lbs./B.H.P₀.
4
6
8
10
12
14
16
18
20
22
30000 ft.
28000
26000
24000
22400
20000
19000
18000
17000
16000
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
H
Absolute ceiling

Fig. 1