NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

MAILED

TO: Mr. Truscott

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 170

REDUCTION IN EFFICIENCY OF PROPELLERS DUE TO SLIPSTREAM.

By Max M. Munk.

From Technische Berichte, Volume III, No. 7.

December, 1923.

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By Max M. Munk.

In the slipstream behind a propeller there is a considerable amount of kinetic energy, which has been imparted to it by the engine without producing any corresponding propeller thrust. The increased absorption of power reduces the propeller efficiency. Attention has been previously directed to this question by Bendemann and Madelung (Technische Berichte, Volume II, No. 1, page 53) and other writers. Their contribution serves to verify the following simple method of calculating the reduction in the propeller efficiency due to the slipstream.

A Single Propeller.

Let T represent the propeller thrust (In case of need, this must first be estimated by assuming the usual efficiency), D the diameter of the propeller and q the head pressure. The latter is known to be $q = \left(\frac{V}{3.6}\right)^2 \frac{\rho}{2}$, in which V is the speed of flight in kilometers per hour and ρ is the density of the air (about 1/8 at sea-level). The speed of flight must also be provisionally estimated, if necessary.

We first calculate the "thrust coefficient" =

$$= \frac{\text{Propeller thrust}}{\text{Disk area of propeller} \times \text{head pressure}}$$

* From Technische Berichte, Vol. III, No.7, pp. 315-316. (1918)

$$C_T = \frac{T}{D^2 \frac{\rho}{4} q} \tag{1}$$

From this we obtain the reduction in the efficiency due to the slipstream

$$\epsilon = \frac{\sqrt{1 + C_T} - 1}{\sqrt{1 + C_T} + 1} \tag{2}$$

The usefulness of these formulas lies in the fact that with their help, we can calculate the loss of efficiency, when a propeller with a diameter D_1 is replaced by another with a Diameter D_2. The alteration in the efficiency with different diameters is caused entirely by the variation in the amount of power lost in the slipstream. The method, therefore, consists in estimating the reduction, for both diameters, by calculations based on the above two formulas. The difference is the change in efficiency due to the alteration in diameter. In exchanging two propellers, there are, to be sure, other differences in their working, which are due to the shape, peripheral velocity and particular differences between the two propellers. Any additional drop in the efficiency due to these differences has, however, nothing to do with the change of the diameter and may be avoided by correcting the design of the propeller.

Two Propellers Mounted in Tandem.

The above simple rule for calculating the losses due to the slipstream may be extended to the case where propellers of equal diameter and with approximately equal output, are mounted one behind the other, so that their axes coincide. The difference between the combined efficiency of the two propellers, in tandem and when working side by side, is of the greatest importance. The drop in efficiency is due entirely to the slipstream. In order to calculate the difference, it is necessary to find the difference between the slipstream losses of the two arrangements compared. Propellers arranged side by side have already been treated in the first section of this note. The following are the rules for the calculation of propellers in tandem.

From the thrust T, of one of the propellers (which, if necessary, must be provisionally estimated), we evaluate the equations, just as above

$$C_{T1} = \frac{T}{D^2 \; q \; \frac{\pi}{4}} \qquad (1) \qquad\qquad \epsilon_1 = \frac{\sqrt{1 + C_{T1}} - 1}{\sqrt{1 + C_{T1}} + 1} \qquad (2)$$

We then calculate

$$C_{T2} = C_{T1} \frac{1 + 2\epsilon_1}{1 + 4\epsilon_1} \qquad (3)$$

$$\epsilon_2 = \frac{\sqrt{1 + C_{T2}} - 1}{\sqrt{1 + C_{T2}} + 1} \qquad (4)$$

This is the loss in the combined efficiency of both propellers due to the slipstream. If, as is always the case, the two propellers rotate in opposite directions, it may be further assumed that the efficiency is improved about 1%, on account of the partial recovery of the rotational energy.

It is here assumed that the two tandem propellers are not geometrically similar. The front propeller must retain the same pitch as a single propeller under similar conditions. The proportional increase of incidence of the rear propeller must, on the other hand, be about twice as great as the proportional reduction of the combined efficiency.

Example.

We will assume that:

The power $P = 2 \times 300$ HP;

Diameter of the propeller $D = 3.2$ m (10.5 ft);

Velocity of flight $V = 160$ km (99.4 mi) per hour.

1. When the propellers are side by side.

Thrust for one propeller with the assumed efficiency of 70 per cent.

$$T = \frac{P \times 75 \times \eta \times 3.6}{V} = 354 \text{ kg } (780.44 \text{ lb}).$$

The head pressure amounts to

$$q = \left(\frac{160}{3.6}\right)^2 \times \frac{1}{16} = 123.6 \text{ kg/m}^2 \text{ } (25.32 \text{ lb/sq.ft}) \text{ (sea-level)}$$

$$C_T = \frac{T}{D^2 \frac{\pi}{4} q} = 0.357$$

$$\epsilon = \frac{\sqrt{1 + \lambda_T} - 1}{\sqrt{1 + \lambda_T} - 1} = 0.076$$

2. When the propellers are in tandem.

The efficiency is provisionally estimated at 65%. We then obtain, from the same formulas,

$$T = 329 \text{ kg } (725.32 \text{ lb})$$

$$C_T = 0.332$$

$$\epsilon_1 = 0.071.$$

Then further

$$C_{T2} = 2C_{T1}\frac{1 + 2\epsilon_1}{1 + 4\epsilon_1} = 0.591$$

$$\epsilon_2 = \frac{\sqrt{1 + C_{T2}} - 1}{\sqrt{1 + C_{T2}} + 1} = 0.115$$

The loss in efficiency (allowing for the rotational energy recovered with the tandem arrangement of propellers revolving in opposite directions) is, therefore,

$$\epsilon_2 - \epsilon_1 - 0.01 = 0.034,$$

The same calculation gives a loss in efficiency of about 0.054, when $V = 90$ km (55.92 mi) per hour.

The above calculation shows that, by arranging the propellers in tandem, some 4 to 5% of the engine power is lost to the airplane and that the rear propeller must, consequently, have twice this percentage (about 9%) additional incidence. In proportion to the thrust or useful power of the propeller, the loss appears still greater, approximating 6 to 7%. In the above instance, this loss, at a velocity of 160 km (99.42 mi) per hour, corresponds to an additional resisting surface of 0.4 m^2 (4.3 ft^2) and, at a velocity of 90 km (55.92 mi) per hour, it corresponds to a surface of about 2.2 m^2 (23.68 ft^2). The resisting surface saved by placing the engines in tandem will, however, generally be found to be less than this.

It is possible, however, that the tandem arrangement of the engines may be of material advantage to the structure as a whole, in which event the above formulas enable the economy of this arrangement to be examined.

Translated by
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