TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 197

SOME TABLES OF THE FACTOR OF APPARENT ADDITIONAL MASS.

By Max M. Munk.

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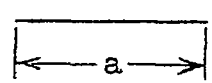
Summary

This note, prepared for publication by the National Advisory Committee for Aeronautics, is a collection of the tables of the factor of apparent mass that have been published up to now.

The theory of the motion of solids in a perfect fluid is of the greatest value for the study of most aerodynamic problems, and the additional apparent mass of an immersed solid is the most important characteristic for such theoretical numerical computations. It will therefore be helpful to have the most important values of the apparent mass - for some elementary cases - collected in a convenient form.

Two-dimensional flows.

1. Straight line, length = a

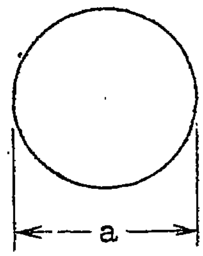


Additional apparent transverse mass = $\rho\, a^2\, \frac{\pi}{4}$

(ρ denotes the density of air.)

Source: Ref. 1.

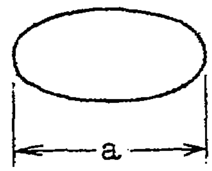
2. Circle, diameter = a



Additional apparent mass = $\rho\, a^2\, \frac{\pi}{4}$

Source: Ref. 1

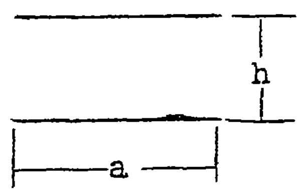
3. Ellipse, moving at right angles to a principal axis a.



Additional apparent mass = $\rho\, a^2\, \frac{\pi}{4}$

Source: Ref. 1

4. Pair of straight parallel lines, Length = a, distance = h. (No stagger.)



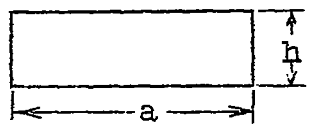
Additional apparent transverse mass = $\rho\, a^2 \cdot \frac{\pi}{4} \cdot k$

Table of k

h/a = 0	0.05	0.10	0.15	0.20	0.30	0.40	0.50
k = 1	1.123	1.212	1.289	1.353	1.462	1.550	1.626
h/a = 0.39	0.46	0.56	0.64	0.79	0.98	1.11	
k = 1.55	1.60	1.67	1.71	1.77	1.83	1.86	
h/a = 1.46	2.02	2.87	5.76	13.88	57.7	∞	
k = 1.91	1.94	1.94	1.99	1.99	2.00	2	

Source: Ref. 2 and 3

5. Rectangle, moving at right angles to its major side.



Sum of apparent mass and of the mass of the fluid displaced by the rectangle $= \rho\, a^2 \frac{\pi}{4} \cdot k$.

Table of k

h/a = 0	0.05	0.10	0.15	0.20	0.30	0.40	0.50
k = 1	1.156	1.271	1.374	1.471	1.664	1.835	2.000

Source: Ref. 2

Three-dimensional flows.

6. Ellipsoid of revolution.

Motion parallel to axis of revolution.
Additional apparent mass = ρ Volume $\cdot\, k_1$.

Motion at right angles to axis of revolution.
Additional apparent mass = ρ Volume $\cdot\, k_2$.

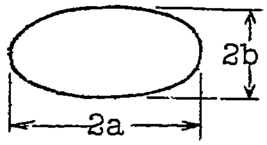
Rotation about longest diameter.
Additional apparent moment of inertia = $k' \cdot$ moment of inertia of displaced fluid.

Table of k_1, k_2 and k'.

$\frac{\text{Length}}{\text{Diameter}}$	k_2	k_1	k'
1.00	0.5	0.5	0
1.50	.621	.305	.094
2.00	.702	.209	.240
2.51	.763	.156	.367
2.99	.803	.122	.465
3.99	.860	.082	.608
4.99	.895	.059	.701
6.01	.918	.045	.764
6.97	.933	.036	.805
8.01	.945	.029	.840
9.02	.954	.024	.865
9.97	.960	.021	.883
∞	1.000	0	1.000

Source. Ref. 4

7. Elliptic disc,



Major axis = 2a, Minor axis = 2b.

Apparent additional transverse mass = $\rho\, a\, b^2 \cdot \frac{4}{3}\pi \cdot k$.

Table for k

b/a = 1	.899	.799	.695	.602
k = .637	.671	.704	.746	.781
b/a = .500	.399	.301	.250	.199
k = .826	.870	.912	.933	.952
b/a = .167	.125	.1011	0	
k = .964	.978	.984	1.000	

Source: Computed from Dr. L. B. Tuckerman's formula - Ref. 5.

Remarks on the Preceding Tables.

The application of the factors of apparent mass has been discussed in Ref. 6, 7 and 8. Only the apparent mass of elliptic discs does not occur there. This knowledge is of use in connection with the wing section theory.

It is customary to assume the flow around a wing section to be two-dimensional, and to compute the lift and moment therefrom. Since the moment is proportional to the difference of the apparent masses for the two principal directions, this assumption gives too large values. A wing of finite aspect ratio has a smaller maximum factor of apparent mass than a section of a wing of infinite aspect ratio of the same size.

Table 7 gives a good approximation of the correction factor, primarily for elliptic plan form, but also a good indication for other plan forms.

The same correction applies to the lift created. This follows from the fact that for ellipsoids the factors of apparent mass indicate also the maximum velocity of flow, and hence the magnitude of the velocity near the rear edge, neutralized by the circulation flow (Ref. 7 and 9).

Hence the relation between the moment and the lift is not greatly changed by the finite aspect ratio, but the effective angle of attack required for a certain lift becomes somewhat larger due to the finite aspect ratio.

List of References.

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3. W. M. Kutta: "Ueber ebene Circulation - Stroemungen nebst flugtechnischen Anwendungen." Muenchen, 1911.

4. Horace Lamb: "The Inertia Coefficients of an Ellipsoid Moving in Fluid." British Reports and Memo-oranda No. 623.

5. L. B. Tuckerman: "Inertia Factors of Ellipsoids for Use in Airship Design."

6. Max M. Munk: "The Aerodynamic Forces on Airship Hulls." N.A.C.A. Technical Report No. 184.

7. Max M. Munk: "Elements of the Wing Section Theory and of the Wing Theory." N.A.C.A. Technical Report No. 191.

8. Max M. Munk: "The Drag of Zeppelin Airships." N.A.C.A. Technical Report No. 117.

9. Max M. Munk: "Remarks on the Pressure Distribution over an Ellipsoid." N.A.C.A. Technical Note No. 196.