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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 351

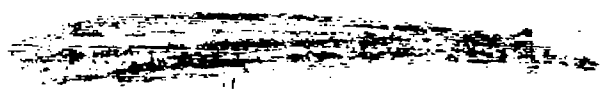
AN ACCURATE METHOD OF MEASURING THE MOMENTS OF INERTIA
OF AIRPLANES

By M. P. Miller
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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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AN ACCURATE METHOD OF MEASURING THE MOMENTS
OF INERTIA OF AIRPLANES.

By M. P. Miller.

S u m m a r y

This note contains a description of an improved apparatus and procedure used by the National Advisory Committee for Aeronautics for determining the moments of inertia of airplanes. The method, based on the pendulum theory, is similar to that previously used, but a recent investigation of its accuracy has resulted in the improvements described herein. The error, when using the new apparatus and procedure, has been found to be of the order of 1 per cent. Sample data and calculations for an airplane are included in an appendix.

I n t r o d u c t i o n

In studying the motion of an airplane in rotational flight, the dynamic forces and couples must be considered. To determine these forces and couples when the motion is defined requires a knowledge of the momental ellipsoid. Thus, it has become necessary to develop an accurate means of determining the ellipsoids of inertia of airplanes used in a study of rotational flight, particularly spinning.

A method used by the National Advisory Committee for Aeronautics for determining momental ellipsoids of airplanes has been previously described in Reference 1. This method involves swinging the body (airplane) as a pendulum. Since the method was first developed the apparatus and procedure have been considerably improved. In previous swinging tests, the apparatus comprising the pendulum system was very susceptible to secondary oscillations, which seriously affected the accuracy of the results. The apparatus described here has been designed to eliminate as far as possible the errors due to secondary oscillations, and has been carefully tested by swinging a body of known moment of inertia. These tests have established the fact that the accuracy attained is satisfactory.

A detailed description of the present apparatus and method and sample data for the Doyle O-2 airplane are given in this paper.

It ain't an ellipsoid!

Apparatus and Procedure

The quantities needed for the determination of the ellipsoid of inertia of an airplane by the method described herein are the moments of inertia about the three body axes and a fourth axis lying in the XZ-plane. Each of these moments of inertia is calculated from data obtained by swinging the airplane, mounted on a swinging gear, as a pendulum with the axis about which the moment of inertia is desired coincident with or parallel to the axis of oscillation. The moment of inertia of the pendulum,

consisting of the airplane and the swinging gear, about the axis of oscillation, is calculated directly from the data obtained by swinging. The moment of inertia of the swinging gear about this axis is subtracted from this value, leaving only the moment of inertia of the airplane about the axis of oscillation. When the axis about which the moment of inertia is desired is not coincident with the axis of oscillation, the moment of inertia is transferred to it by a further calculation.

For convenience in handling the airplane two types of pendulums are used, - a compound for determining the moments of inertia about the X, Y, and X-Z axes, and a bifilar torsion type for determining that about the Z~~X~~ axis. For each type of pendulum the moment of inertia about the axis of oscillation is calculated from the weight, period, and dimensions. The dimensions required depend upon the type of pendulum. The length from the axis of oscillation to the center of gravity of the compound pendulum and the length and distance between the bifilars of the torsion pendulum are needed. The types of pendulums are illustrated in Figures 1 and 2.

The swinging gear upon which the airplane is suspended is constructed as a separate unit which is sufficiently rigid to hold its shape when suspended alone for a determination of its moment of inertia by swinging. It is adjustable so that it can be adapted to the different types of pendulums and to different sizes of airplanes. The weight and dimensions of all

the members which compose the swinging gear are known so that it is possible, by noting the members used and their arrangement, to calculate the weight and center of gravity location of the gear for any arrangement.

When used as a compound pendulum, the swinging gear consists essentially of a cradle, knife-edges, and tie rods assembled as shown in Figure 1. The cradle is a rectangular frame made of two I_x beams for supporting the airplane and two light angle irons for spacers. The length of the spacers is adjustable to permit the handling of different sizes of airplanes. The knife-edges (Fig. 3) provide a definite axis about which the pendulum oscillates with very little friction. They are mounted on a track so that their spacing can also be varied when necessary. The tie rods are used to join the cradle to the knife-edges. The length and arrangement of the pendulum is varied by the use of different combinations of tie rods.

When used as a bifilar torsion pendulum, the swinging gear consists of the same essential parts as before with the addition of two universal joints and a spacer at the lower ends of the two vertical fibers, assembled as shown in Figure 2. The universal joints are specially constructed as shown in Figure 4, to allow rotation within limits in every direction. They are used to provide definite points of oscillation at the lower ends of the vertical fibers. The spacer between these joints prevents a change in the distance between the vertical fibers when the pendulum is oscillating.

As the center of gravity of the airplane is the origin of the axes about which moments of inertia are to be found, its location is determined before any swinging is done. The method used for the location of the center of gravity is based on the principle that the center of gravity of a body suspended from a single pivot point lies on a vertical line through the point of suspension. In its simplest form the method consists of suspending the airplane from two successive points in the XZ-plane. The points of attachment are chosen so that the airplane is inclined with the nose up for one suspension and nose down for the other. In each case the projection of a plumb line from the point of suspension is drawn on the side of the fuselage. A transit set up with its optical axis in a plane perpendicular to the XZ-plane of the airplane is used for projecting the plumb lines. The intersection of the two lines locates the vertical and longitudinal position of the center of gravity. Its lateral position is assumed to be in the plane of symmetry.

In practice it is not usually convenient to follow the simple method outlined above, due to the difficulty in finding suitable points of attachment on the airplane which do not endanger the structure. A satisfactory method employing the use of the swinging gear assembled as a compound pendulum is therefore usually followed. In this case the plumb line for the entire mass, airplane and swinging gear, is found as previously described and a correction made for the mass of the swinging gear.

This second method makes use of the fact that the two parts of the body, the airplane and the gear, exert equal and opposite

moments about the axis of the knife-edges. The angles assumed by the gear when assembled as in Figure 5, and subjected to varied moments about the knife-edge axis, are determined without the airplane in place. The moments are applied by hanging known weights on one side of the cradle. By this procedure a calibration showing the moment corresponding to any position of the gear is obtained. The airplane is then mounted on the gear with the X-axis parallel to and equidistant from the I-beams and in such a position that the angle (α) assumed by the cradle is about 12 to 15 degrees (Fig. 6). The moment of the gear about the knife-edge axis is then found from the calibration, and since the moments of the gear and airplane are equal in magnitude, the moment of the airplane is thus obtained. The horizontal distance between the center of gravity of the entire mass and the center of gravity of the airplane is found by dividing this moment by the weight of the airplane. A vertical line drawn on the side of the fuselage at the above calculated distance from the plumb line will then pass through the center of gravity of the airplane. The fore-and-aft position of the airplane relative to the gear is then changed so that the inclination of the gear is approximately as great as before, but in the opposite direction, and a second vertical line is drawn through the center of gravity. As by the first method the intersection of these two lines locates the vertical and longitudinal position of the center of gravity of the airplane. A check is

obtained by moving the airplane until the gear is level. A plumb line through the knife-edge axis should then pass through the intersection of the two lines previously established.

After the center of gravity of the airplane has been found, the actual swinging tests are made. For these tests the airplane must be mounted on the swinging gear cradle so that its center of gravity lies on a line passing through the center of the cradle and normal to the plane of the I-beams. This position remains unchanged throughout the tests, as all the necessary changes in the gear are made with the tie rods, their attachment to the knife-edges, and the spacing of the knife-edges. With the airplane thus mounted and the cradle attached to the knife-edges so that the Y-axis is parallel to the knife-edge axis and the XY-plane horizontal (Fig. 5), the apparatus is in readiness for a determination of the moment of inertia about the Y-axis. If the center of gravity of the airplane is determined and finally checked by the method employing the use of the swinging gear, the final check on the center of gravity location leaves the airplane suspended as just described.

As previously mentioned, the quantities necessary for the determination of the moment of inertia of a compound pendulum about its axis of oscillation are the weight, period of oscillation, and length from the knife-edge axis to the center of gravity of the pendulum. The weight is the total of the airplane and swinging gear weights. To determine the period, the pendulum

is given an oscillation of small magnitude and the time for fifty or more oscillations found with a stop watch. The length from the knife-edge axis to the center of gravity is found from the respective weights and center of gravity locations of the swinging gear and airplane. The distance from the knife-edge axis to the center of gravity of the gear as well as the gear's weight is calculated from the weights and dimensions of the component members. The distance from the knife-edge axis to the center of gravity of the airplane is found by measuring the difference of elevation between them by means of a transit.

From the moment of inertia of the pendulum about the knife-edge axis, the moment of inertia of the airplane about its Y-axis is found by subtracting the moment of inertia of the gear and transferring the remainder to the Y-axis. The moment of inertia of the gear is found by removing the airplane and swinging the gear alone. For convenience, swinging the gear alone for all types of suspensions is postponed until all the swinging tests with the airplane in place are completed.

The moments of inertia about the X and XZ-axes are found in a similar manner, but with the changes necessary to make the desired axis parallel to the axis of oscillation. For the X-axis suspension, the tie rods are connected to the knife-edges as shown in Figure 1, so that the X-axis is parallel to the knife-edge axis and the XY-plane is horizontal. For the XZ-axis suspension, an inclined axis in the XZ-plane is made parallel to

the axis of oscillation by adding short tie rods to either the front or rear members so that the XY-plane is rotated about the Y-axis. The angle of inclination of the X-axis is measured with a protractor level

The arrangement of the swinging gear members necessary to form a bifilar suspension for the determination of the moment of inertia about the Z-axis is shown in Figure 2. The fibers are made vertical by spacing the knife-edges as necessary. The XY-plane is horizontal and the Z-axis is made coincident with the axis of oscillation (by locating the center of gravity midway between the vertical fibers). The pendulum is given a torsional oscillation and the period found as before for fifty or more oscillations of small magnitude. The length of the fibers and the distance between them are measured with a tape. The moment of inertia of the gear is again found by swinging it with the airplane removed.

D i s c u s s i o n

The method of obtaining the moments of inertia of an airplane described herein has been found to give very satisfactory results. The accuracy was established by swinging tests conducted with a homogeneous body having a regular section whose moment of inertia could be computed. The body selected had approximately the same weight and moment of inertia as a small airplane. It was found by comparing the calculated moment of

inertia with those obtained from swinging tests that an accuracy within 1 per cent can be obtained consistently. This accuracy, however, depends not only on the construction of the gear but also upon its dimensions, careful operation, and the precision with which all measurements are made.

Although the pendulum dimensions are governed somewhat by the size and type of airplane to be swung, it has been found by tests that they should also be governed as far as possible by other considerations. The compound pendulum should be kept short so that the moment of inertia of the airplane about an axis through its center of gravity will be a large percentage of the total moment of inertia of the pendulum about the axis of oscillation. Pendulum lengths of approximately 4 to 6 feet have given satisfactory results with airplanes weighing up to 5000 pounds. In tests of the bifilar pendulum with varied lengths of the vertical fibers and a fixed distance between them, it was found that the results were accurate only when the length of the vertical fibers was approximately equal to or greater than the distance between them. It has been found satisfactory and convenient in swinging various airplanes to place the vertical fibers about 8 feet apart.

The oscillations of both the compound and bifilar pendulums should have a small amplitude because the pendulum formulas only apply when the assumption is valid that $\sin \theta = \tan \theta = \theta$ (where $\theta =$ one-half the angle of oscillation). In practice

this angle need not exceed 1 degree.

The precision of the measurement of length of the compound pendulum depends primarily upon the accurate location of the center of gravity of the airplane. If it is not located accurately the pendulum dimensions will be in error even though subsequent measurements are very precise. This emphasizes the necessity for extreme care in locating and checking the position of the center of gravity.

An accurate measurement of the period should be obtained without any difficulty. The average of five or more readings of fifty complete oscillations each will give the required precision.

To minimize personal errors, two observers should be used. To eliminate gross errors, two swingings should be made about each axis with slightly different pendulum lengths. A change in length of a few inches is sufficient for this purpose. The average of the two values is taken as the correct moment of inertia. In addition, the airplane should be swung in both nose-up and nose-down attitudes, as this affords a check on the position of the principal axes of inertia of the airplane. Data from both nose-up and nose-down swingings should give identical positions of the principal axes.

C o m p u t a t i o n s

The following formulas are used in calculating the moments of inertia and the dimensions of the momental ellipsoid from the data obtained by swinging the airplane as a pendulum. *N.B. see fig 7.*

Moments of Inertia about the Body Axes.— The moment of inertia of a compound pendulum is

$$I_C = \frac{W T^2 L}{4 \pi^2},$$

where

I_C = the moment of inertia about the axis of oscillation in slugs feet²,

W = the weight in pounds,

T = the period of one complete oscillation in seconds,

L = the distance from the axis of oscillation to the center of gravity in feet.

The moment of inertia of the airplane about an axis through its center of gravity equals the moment of inertia of the entire pendulum about the axis of oscillation minus the moment of inertia of the swinging gear about the same axis minus the additional moment of inertia due to the displacement of the center of gravity of the airplane from the axis of oscillation.

$$I = \frac{W_1 T_1^2 L_1}{4 \pi^2} - \frac{W_2 T_2^2 L_2}{4 \pi^2} - \frac{W_3 L_3^2}{g}$$

$$L_1 = \frac{W_2 L_2 + W_3 L_3}{W_1}$$

- where I = the moment of inertia of the airplane about an axis through the C.G. and parallel to axis of oscillation,
- W_1 = the total weight of the airplane and swinging gear,
- W_2 = the weight of the swinging gear,
- W_3 = the weight of the airplane,
- T_1 = the period of airplane and swinging gear when swung together,
- T_2 = the period of the swinging gear when swung alone,
- L_1 = the distance from the axis to the C.G. of the airplane and swinging gear,
- L_2 = the distance from the axis to the C.G. of the gear,
- L_3 = the distance from the axis to the C.G. of the airplane.

The moment of inertia of a bifilar pendulum is

$$I_t = \frac{W T^2 A^2}{16 \pi^2 L}$$

- where I_t = the moment of inertia about a vertical axis midway between the vertical fibers,
- W = weight,
- T = the period of a complete oscillation,
- A = the distance between the vertical fibers,
- L = the length of the vertical fibers.

The moment of inertia of the airplane about an axis through its C.G. and coincident with the axis of oscillation for the bifilar pendulum equals the moment of inertia of the pendulum about the axis of oscillation minus the moment of inertia of the

gear about the same axis.

$$I = \frac{W_1 T_1^2 A^2}{16 \pi^2 L} - \frac{W_2 T_2^2 A^2}{16 \pi^2 L}$$

where I = the moment of inertia about a vertical axis through the C.G. of the airplane,

W_1 = the total weight of the airplane and swinging gear,

W_2 = the weight of the swinging gear,

T_1 = the period of the airplane and swinging gear when swung together,

T_2 = the period of the swinging gear when swung alone.

Location of the Principal Axes of the Ellipsoid of Inertia.--

Since the airplane is symmetrical about the XZ-plane, the transverse axis will be one of the principal axes of the ellipsoid.

The other two principal axes which lie in the plane of symmetry are located by

$$\tan 2 \tau = \frac{2 E}{I_z - I_x}$$

where $E = \frac{I_x \cos^2 \theta + I_z \sin^2 \theta - I_{xz}}{\sin 2 \theta}$

τ = the angle between the principal axis and the X-axis of the airplane,

θ = the angle between the X and XZ-axes,

E = the product of inertia about the body axes,

I_x = the moment of inertia about the X-axis,

I_z = the moment of inertia about the Z-axis,

I_{xz} = the moment of inertia about the XZ-axis.

When measuring angles the positive quadrant is the one formed by the rearward directed X-axis and the upward directed Z-axis. The sign of the product of inertia (E) determines in which quadrants, formed by the X- and Z-axes of the airplane, the principal axes lie. When (E) is zero the principal axes coincide with the body axes of the airplane.

Principal Moments of Inertia.— The moment of inertia about the transverse axis is one of the principal moments of inertia.

$$I_{x'} = I_x \cos^2 \tau - E \sin 2\tau + I_z \sin^2 \tau$$

$$I_{z'} = I_z \cos^2 \tau + E \sin 2\tau + I_x \sin^2 \tau$$

$$I_{y'} = I_y$$

where $I_{x'}$, $I_{z'}$, $I_{y'}$ = the moments of inertia about the principal axes of the airplane,

I_y = the moment of inertia about the Y-axis.

R e f e r e n c e

1. Green, M. W. : Measurement of the Moments of Inertia of Full Scale Airplanes. N.A.C.A. Technical Note No. 265, 1927.

A p p e n d i x

Following are sample data and calculations for determining the ellipsoid of inertia of the Doyle O-2 airplane:

Moments of Inertia:

Compound Pendulum -

The moment of inertia about the X-axis is given by

$$I_x = \frac{W_1 T_1^2 L_1}{4 \pi^2} - \frac{W_2 T_2^2 L_2}{4 \pi^2} - \frac{W_3 L_3^2}{g}$$

The experimental data obtained by swinging the airplane are:

	<u>Short suspension</u>	<u>Long suspension</u>
W_1	1688 pounds	1693 pounds
W_2	300 "	305 "
W_3	1388 "	1388 "
L_1	5.45 feet	6.30 feet
L_2	3.83 "	4.59 "
L_3	5.80 "	6.67 "
T_1	3.22 seconds	3.31 seconds
T_2	2.96 "	3.02 "

from which

$$\begin{aligned} I_x \text{ (short suspension)} &= \frac{1688 \times (3.22)^2 \times 5.45}{39.48} - \\ &\quad \frac{300 \times (2.96)^2 \times 3.83}{39.48} - \frac{1388 \times (5.80)^2}{32.15} \\ &= 710 \text{ slugs feet}^2, \end{aligned}$$

and

$$I_x \text{ (long suspension)} = \frac{1693 \times (3.31)^2 \times 6.30}{39.48} - \frac{305 \times (3.02)^2 \times 4.59}{39.48} - \frac{1388 \times (6.67)^2}{32.15}$$

$$= 715 \text{ slugs feet}^2,$$

*793 - given
about 100% the way*

the average of which (713 slugs feet²) is used in further calculations.

The moment of inertia about the Y-axis and the XZ-axes are calculated in a similar manner from data obtained with the compound pendulum.

Bifilar Pendulum -

The moment of inertia about the Z-axis is given by

$$I_z = \frac{W_1 T_1^2 A^2}{16 \pi^2 L} - \frac{W_2 T_2^2 A^2}{16 \pi^2 L}$$

The experimental data obtained by swinging are:

	<u>Short suspension</u>	<u>Long suspension</u>
W ₁	1671 pounds	1671 pounds
W ₂	283 "	283 "
T ₁	3.76 seconds	3.84 seconds
T ₂	3.91 "	3.98 "
A	8.02 feet	8.02 feet
L	8.00 "	8.33 "

from which

$$I_z \text{ (short suspension)} = \frac{1671 \times (3.76)^2 \times (8.02)^2}{157.92 \times 8.00} - \frac{283 \times (3.91)^2 \times (8.02)^2}{157.92 \times 8.00}$$

$$= 979 \text{ slugs feet}^2,$$

$$\begin{aligned} \text{and } I_z (\text{long suspension}) &= \frac{1671 \times (3.84)^2 \times (8.02)^2}{157.92 \times 8.33} - \\ &\quad \frac{283 \times (3.98)^2 \times (8.02)^2}{157.92 \times 8.33} \\ &= 981 \text{ slugs feet}^2, \end{aligned}$$

1208, 980, 77, 1208

the average of which is 980 slugs feet².

The average value of the moment of inertia about each axis is as follows:

$$\begin{aligned} I_x &= 713 \text{ slugs feet}^2, \\ I_y &= 683 \quad " \quad " \\ I_z &= 980 \quad " \quad " \\ I_{xz} &= 732 \quad " \quad " \quad (\text{from nose-up swinging X-axis in-} \\ &\quad \text{clined 18 degrees 30 minutes),} \\ I_{xz} &= 748 \quad " \quad " \quad (\text{from nose-down swinging X-axis in-} \\ &\quad \text{clined 18 degrees 30 minutes),} \end{aligned}$$

Location of Principal Axes:

The product of inertia about the body axes is given by

$$E = \frac{I_x (\cos^2 \theta) + I_z (\sin^2 \theta) - I_{xz}}{\sin 2\theta}$$

where

	<u>Nose-up</u>	<u>Nose-down</u>
θ	18 degrees 30 minutes	-18 degrees 30 minutes
$\sin \theta$.3173	-.3173
$\cos \theta$.9483	+.9483
$\sin 2\theta$.6018	-.6018

and the moments of inertia are as given above.

From the nose-up swinging

$$E = \frac{713 (.9483)^2 + 980 (.3173)^2 - 732}{.6018} = 13.3$$

From the nose-down swinging

$$E = \frac{713 (.9483)^2 + 980 (-.3173)^2 - 748}{-.6018} = 13.3$$

The tangent of twice the angle between the principal axis and X-axis is given by

$$\begin{aligned} \tan 2\tau &= \frac{2E}{I_z - I_x} \\ &= \frac{2 \times 13.3}{980 - 713} = .0997, \end{aligned}$$

from which $\tau = 2$ degrees 51 minutes.

Principal Moments of Inertia:

The principal moments of inertia are given by

$$I_{x'} = I_x (\cos\tau)^2 - E \sin 2\tau + I_z (\sin\tau)^2,$$

$$I_{y'} = I_y,$$

$$I_{z'} = I_z (\cos\tau)^2 + E \sin 2\tau + I_x (\sin\tau)^2.$$

Then, since

$$\tau = 2 \text{ degrees } 51 \text{ minutes}$$

$$\sin \tau = .0497$$

$$\cos \tau = .9988$$

$$\sin 2\tau = .0996$$

and the other quantities are as previously calculated, it follows that

$$I_{x'} = 713 (.9988)^2 - 13.3 (.0996) + 980 (.0497)^2 = 710 \text{ slug}\cdot\text{feet}^2,$$

$$I_{y'} = 683 \text{ slug}\cdot\text{feet}^2,$$

$$I_{z'} = 980 (.9988)^2 + 13.3 (.0996) + 713 (.0497)^2 = 982 \text{ slug}\cdot\text{feet}^2,$$

The results of the swinging tests are represented graphically in Figure 7. This figure shows the trace of the intersection of the ellipsoid of inertia with the XZ-plane (the plane of symmetry). It is an ellipse constructed with $I_{x'}$ and $I_{z'}$ as the major and minor axes. The moments of inertia about the X, Z, and the two X-Z axes are plotted on the figure.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 15, 1930.

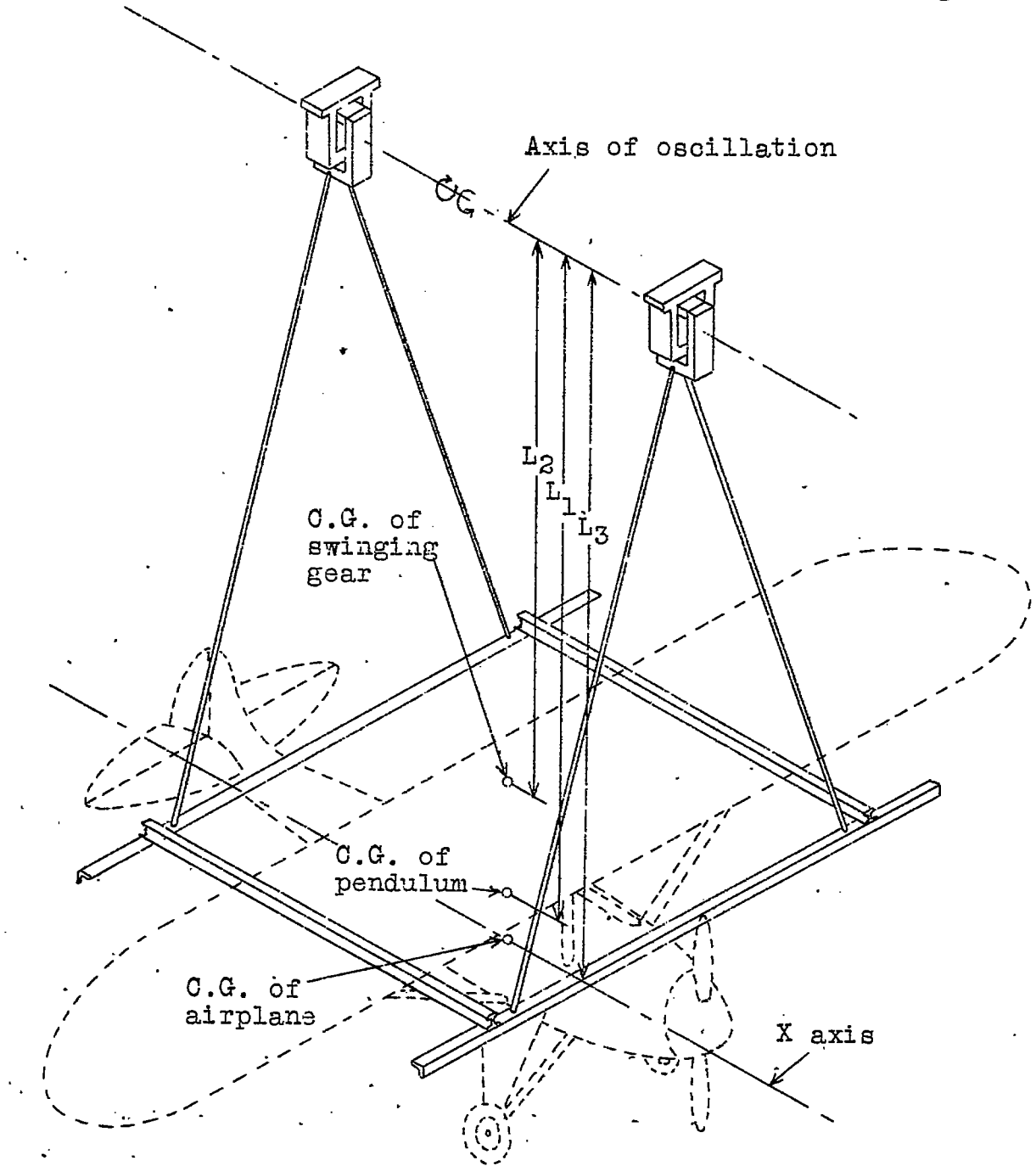


Fig.1 Airplane and swinging gear arranged for the determination of the moment of inertia about the X axis by the compound pendulum method.

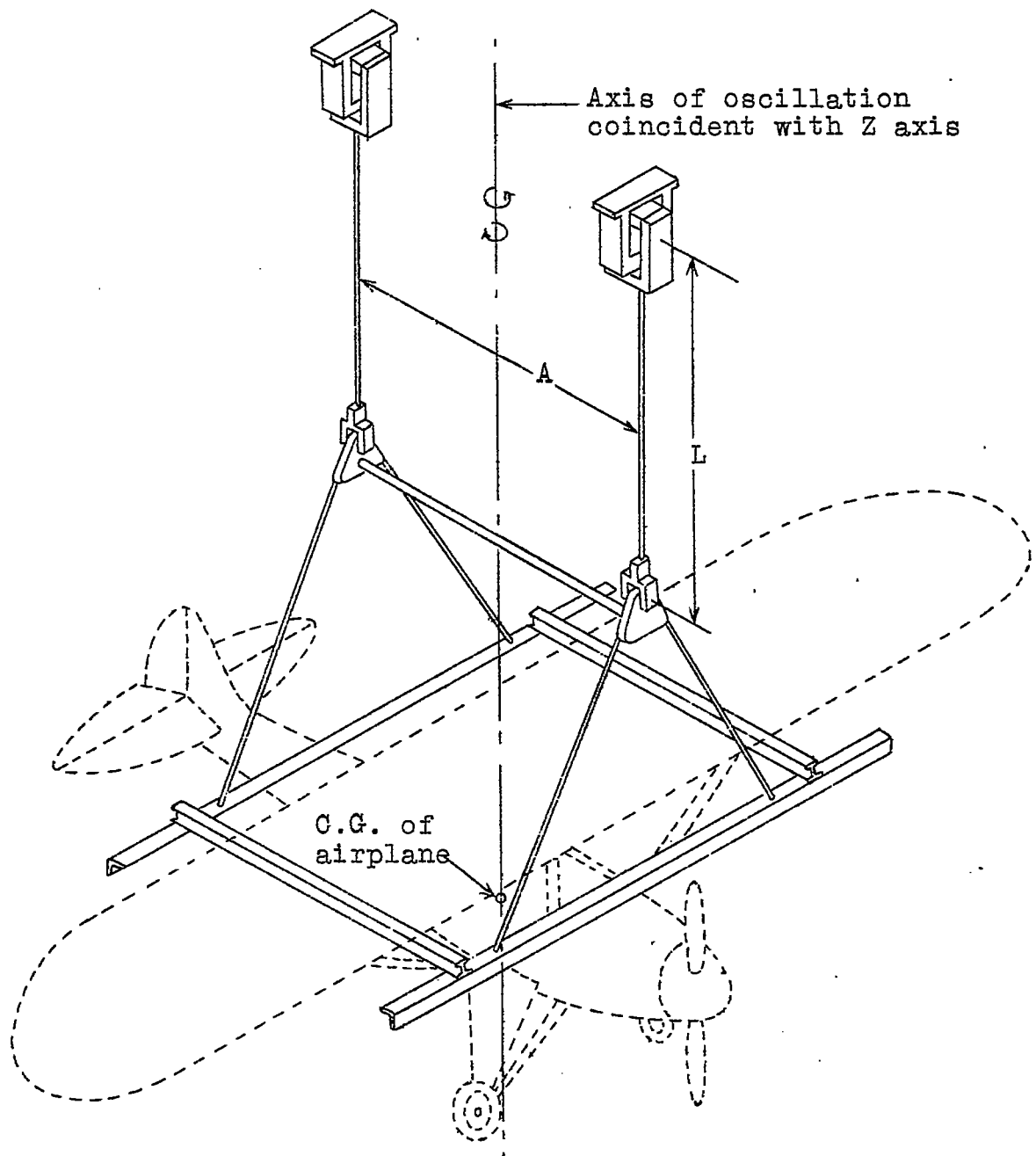


Fig.2 Airplane and swinging gear arranged for the determination of the moment of inertia about the Z axis by the bifilar torsion pendulum method.

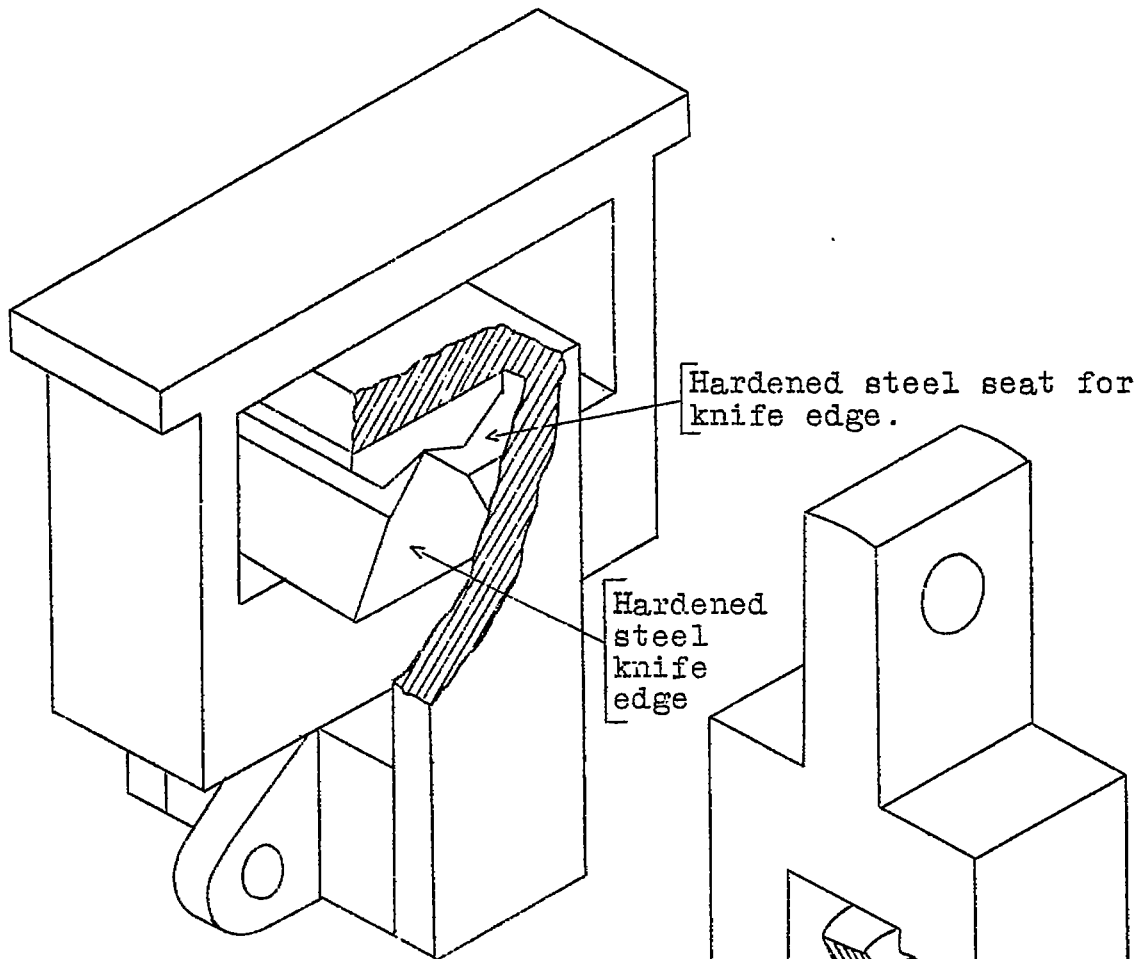


Fig.3 Knife edge.

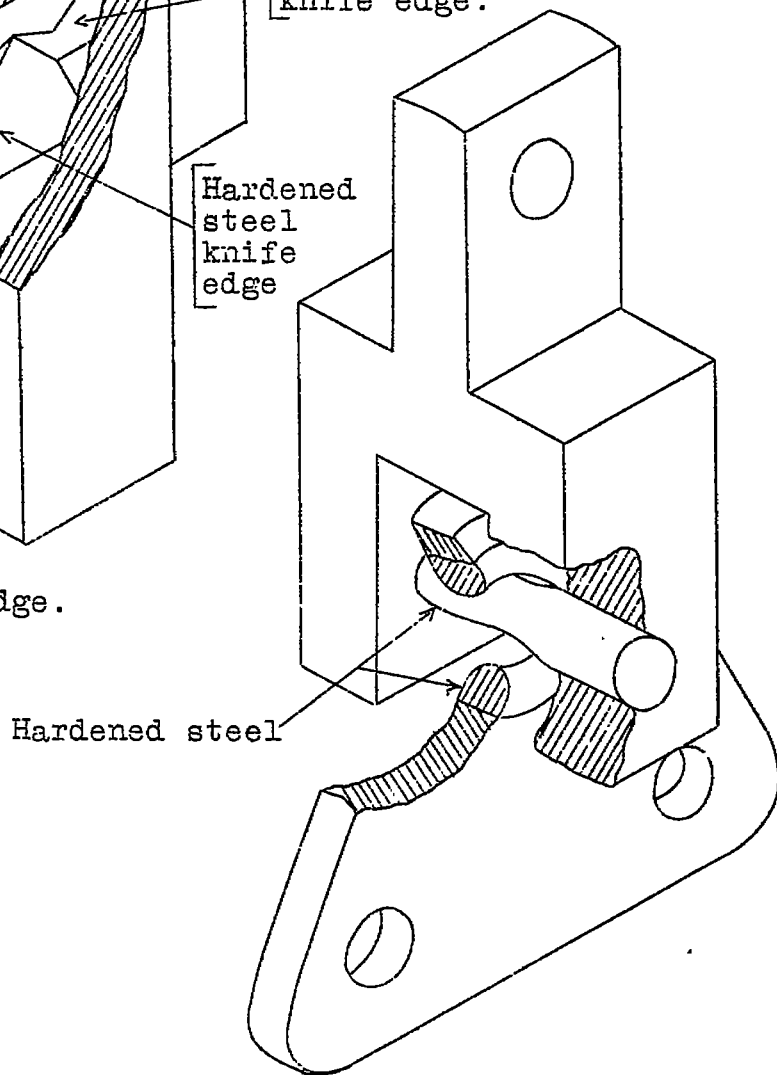


Fig.4 Universal joint.

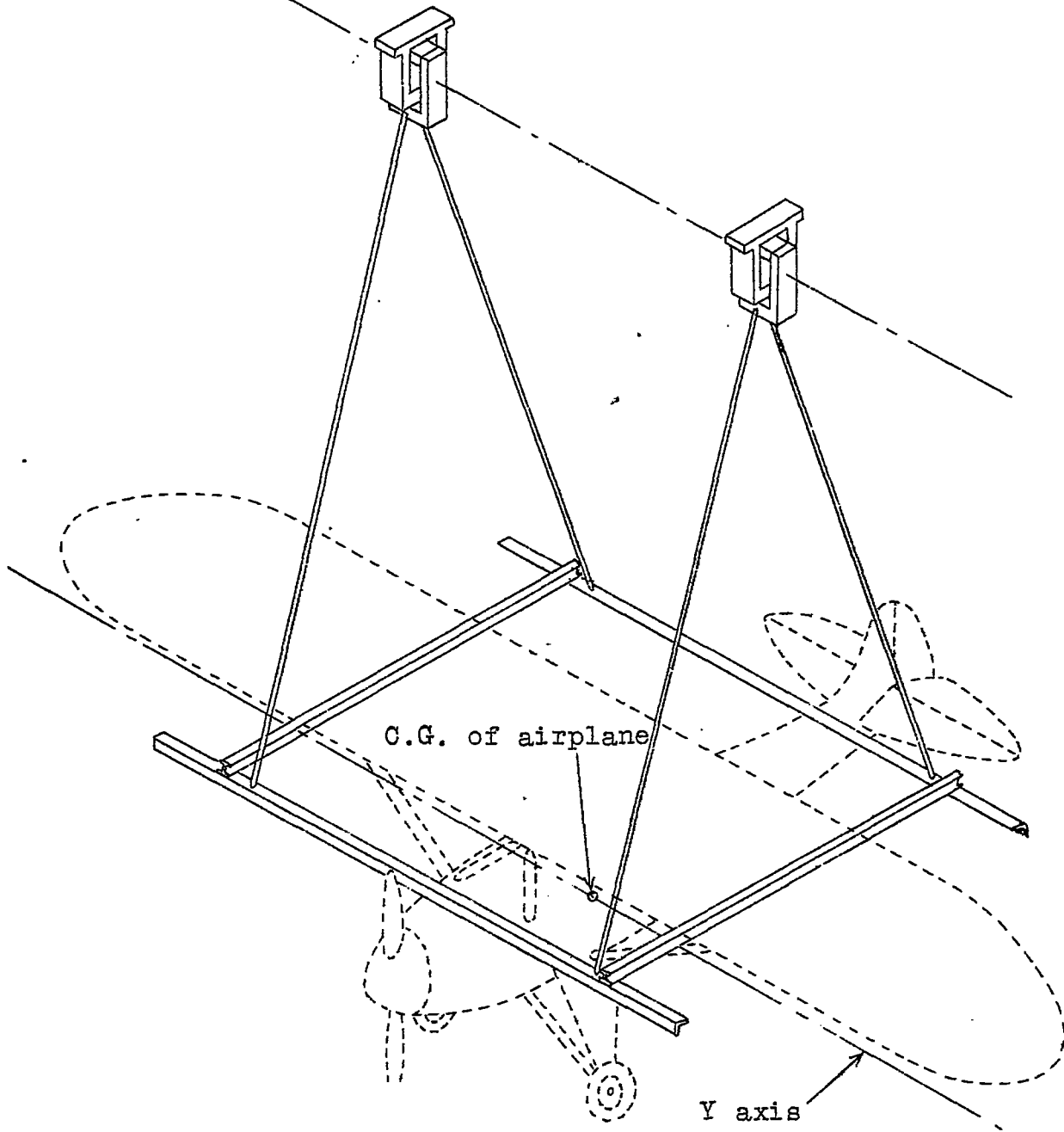


Fig.5 Arrangement for the determination of the moment of inertia about the Y axis.

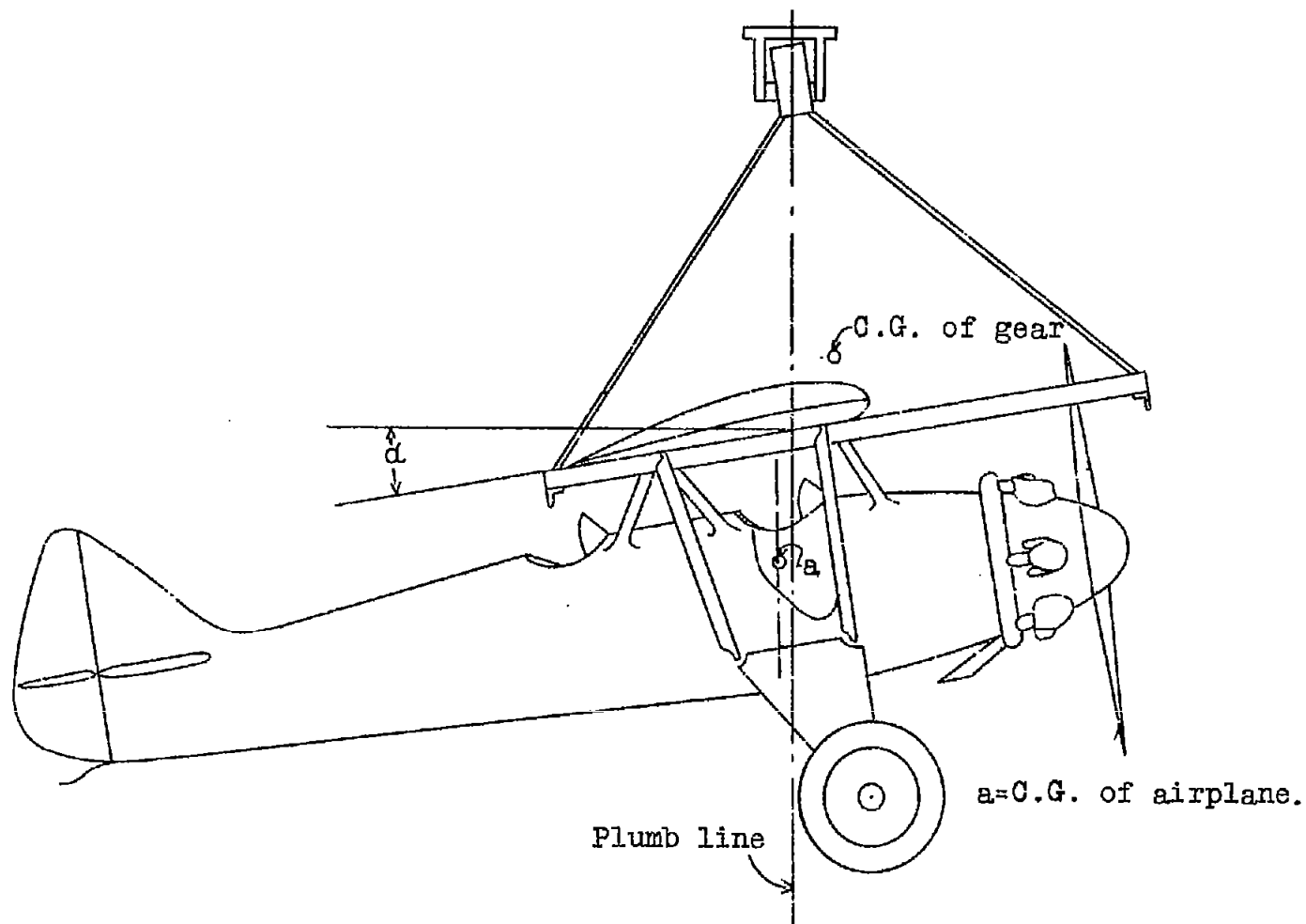


Fig.6 Position of the airplane when establishing line through the center of gravity.

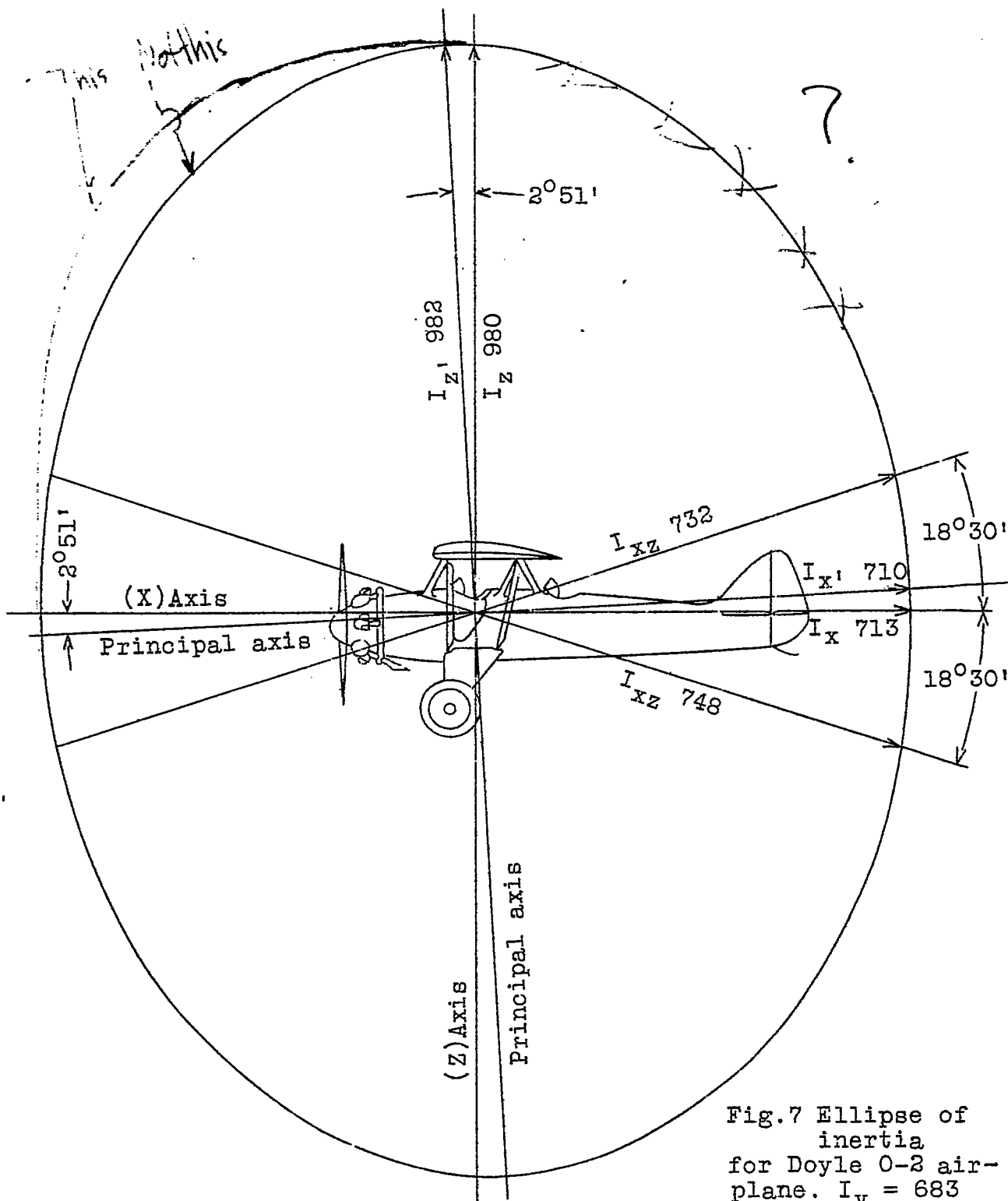


Fig. 7 Ellipse of inertia for Doyle O-2 airplane. $I_y = 683$

Who says this could be an ellipse?

Mr. Miller

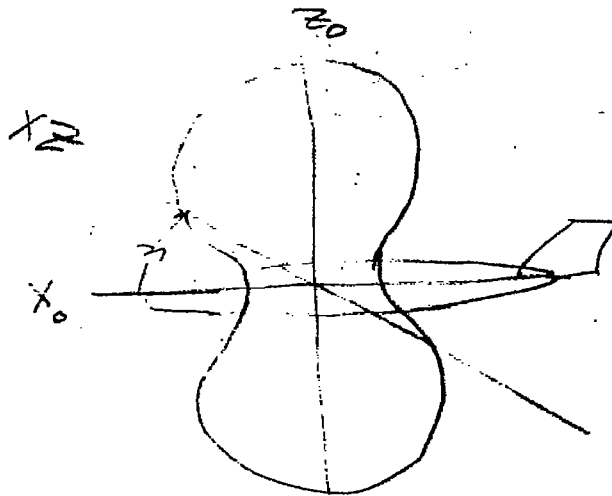
$$I_{xz} = I_{x_0} \cos^2 \eta + I_{z_0} \sin^2 \eta$$

I_{x_0} = I about principal x axis

I_{z_0} = I " " z axis

η = Angle between principal axis
and axis under consideration

I_{xz} = I about axis under consideration



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