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No. 469

A SUMMARY OF DESIGN FORMULAS FOR BEAMS HAVING THIN WEBS IN DIAGONAL TENSION

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A SUMMARY OF DESIGN FORMULAS FOR BEAMS HAVING THIN WEBS IN DIAGONAL TENSION

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SUMMARY

This report presents an explanation of the fundamental principles and a summary of the essential formulas for the design of diagonal-tension field beams, i.e., beams with very thin webs, as developed by Professor Wagner of Germany.

INTRODUCTION

The necessity for designing structures to the smallest possible weight for a given load factor has forced airplane designers to deviate materially in some instances from construction practices that have become standard in older branches of engineering. Diagonal-tension field beams are one example of this trend away from established practice.

Diagonal-tension field beams are a special development of plate girders in which the shearing force is small compared with the depth of the girder, so that the required web thickness is very small. Such a thin web would buckle before it reached the ultimate shearing stress. In structural engineering, this buckling is prevented by attaching stiffeners to the web. In many aeronautical structures, however, the web is so thin that an excessive number of stiffeners would be required to develop a high shearing stress before buckling. Therefore, a different solution of the problem has been attempted. The flanges of the beam are connected by a number of struts which act not as web stiffeners, but as flange spacers. The web is thus left free to buckle, the basic idea being that the web after buckling cannot carry the shear in the beam by developing shearing stresses, but can and does carry the shear by developing tensile stresses in the direction of the diagonal buckles or folds; hence the name "diagonal-tension field beams."

The choice between the plate girder with a web safe against buckling and a diagonal-tension field beam depends on the relative magnitudes of shearing force and depth of beam. Using as a criterion the "index value" $K = \frac{\sqrt{S}}{h}$, where S is the total shear in pounds and h the depth of the beam in inches, Wagner has estimated (reference 1, p. 3) that a diagonal-tension field beam is probably preferable if K is less than about seven, while a plate girder with a shear-resistant web is preferable if K is more than about eleven. In the intermediate region there is little choice between the two.

Beams with an index value K of less than seven are frequently found in aircraft structures. Instances are found elsewhere than among beams in the narrower sense of the word. The theory can also be applied to the shear skin of monocoque fuselages, hulls, and floats; to the skin of metal-covered wings, when the skin is used to take the shear loads due to drag or torsion; and to the bulkheads for monocoque wings, fuselages, floats, and hulls. Attention is called to the fact that the use of a thin web may be of advantage in truss-type assemblies because the lateral support which the web contributes to the compression members may more than compensate for the increase in weight due to the use of the web.

The theory of diagonal-tension field beams has been treated by Professor Wagner, of Danzig, Germany, and his publications have been made available to the American designer in several N.A.C.A. Technical Memorandums. (See references 1 - 5.) These translations, however, are difficult to follow and contain some errors. Consequently, the present report has been prepared to explain the fundamental principles of diagonal-tension field beams, or "Wagner beams" as we shall call them for brevity, and to give the formulas essential to the design of such beams. No attempt has been made to present the derivation of the equations. Any person interested in the theoretical aspects of the subject may refer to the original articles or their translations.

FUNDAMENTAL PRINCIPLES

When a frame as shown in figure 1 is loaded by a force P, the diagonal D_1 will be in tension and the diagonal D_2 in compression. If D_2 is a very slender column it will buckle when P has reached some definite small value, and if P be increased beyond this value, D_1 will take all of the increase in shear in the panel. The diagonal D_2 will continue to carry a load about equal to its buckling load, but when P becomes very large, the load in D_2 will become negligible as compared with the load in D_1.

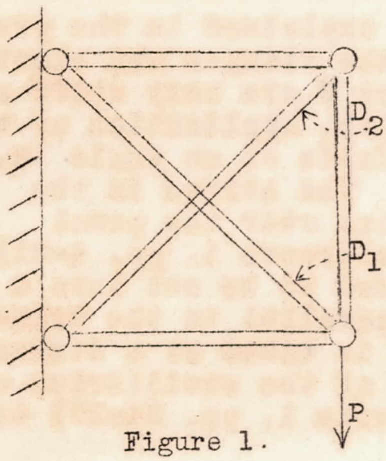

Figure 1.

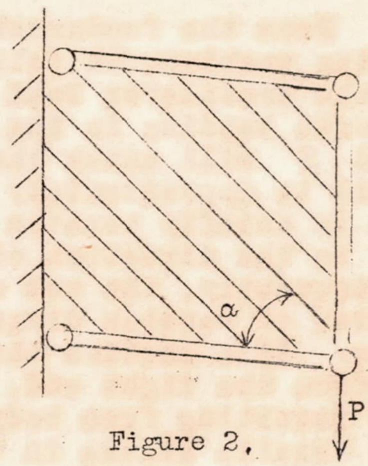

Figure 2.

If the frame is converted into a beam by replacing the diagonals with a very thin web, a similar argument applies. The compression stresses in the direction of D_2 fold the web into corrugations as indicated in figure 2, and the shear in the panel is carried by tensile stresses in the direction of D_1. Such a panel with the web in diagonal tension constitutes the fundamental unit of the Wagner beam. If the panel is square, such as is shown in figure 1, it is quite obvious that the folds will form at an angle of approximately 45°. If the panel is a rectangle, the direction of the folds is not so obvious, but theory shows that it will still be approximately 45°, provided that all edge members are stiff. The introduction of additional struts in the panel (fig. 3) does not change the direction of the folds if these struts are parellel to the original end struts (reference 1, p. 10).

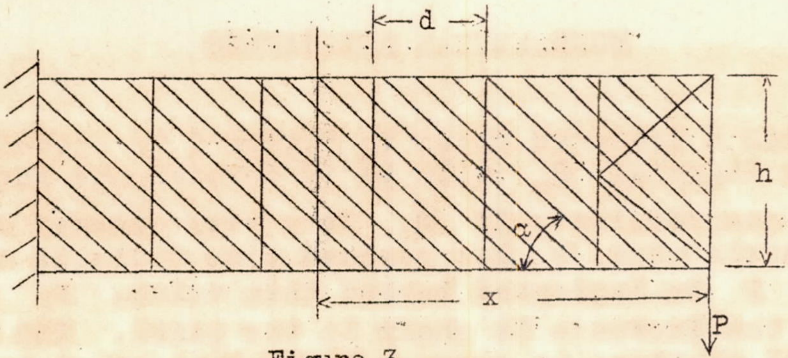

Figure 3.

APPLICATION OF PRINCIPLES TO A CANTILEVER BEAM

From the fundamental principles explained in the preceding section, it follows that, if the flanges and struts of the cantilever beam shown in figure 3 are very stiff as regards bending in the plane of the web, application of the load P will cause the web to form folds at an angle α, which is approximately equal to 45°. The stress in the web is chiefly tension which is uniform over the panel and in the direction of the folds (reference 1, pp. 4-21); consequently, the web may be considered to be cut into a number of tension diagonals by cuts parallel to the wrinkles. If a section through the beam is taken at a distance x from the right end, consideration of the equilibrium of the resulting free body shows (reference 1, pp. 24-27) that the tensile stress in the web is

$$f = \frac{2P}{ht} \; \frac{1}{\sin 2\alpha} \qquad (1')$$

where t is the thickness of the web, and that the forces in the tension and compression flanges are

$$H_{T,C} = \pm \frac{Px}{h} - \frac{P}{2} \cot \alpha \qquad (2')$$

where the second term is due to the horizontal component of the web tension. The vertical component of the web tension, acting along a length d of the flange, gives the force in the struts

$$V = - P \frac{d}{h} \tan \alpha \qquad (3')$$

Theoretical calculations have shown that α is usually a few degrees less than 45° (reference 1, p. 22). Observation of test beams has shown that the unavoidable irregularities in material, riveting, etc., cause deviations from the theoretical value of α. Consequently, it is sufficiently accurate for design to assume the convenient value $\alpha = 45^\circ$. The preceding formulas therefore become

$$f = \frac{2P}{ht} \tag{1}$$

$$H_{T,C} = \pm \frac{Px}{h} - \frac{1}{2} P \tag{2}$$

$$V = - P \frac{d}{h} \tag{3}$$

The spacing of the struts in a Wagner beam should normally vary between one sixth and one half the depth of the beam. If the spacing of the struts becomes greater than the depth of the beam, α may become much less than 45°. A conservative procedure in this case is to compute the forces in the tension flange and in the struts with $\alpha = 45^\circ$, the force in the compression flange and the stress in the web with $\alpha = \alpha' = \tan^{-1} \frac{h}{d}$. In general, such wide spacing is very impractical and should be avoided unless strength is a minor consideration.

THE GENERAL CASE OF A BEAM WITH PARALLEL FLANGES

In the general case of a beam with parallel flanges, the struts have an inclination β and loads P_n are introduced at points other than at the end of the beam (figs. 4a and 4b).

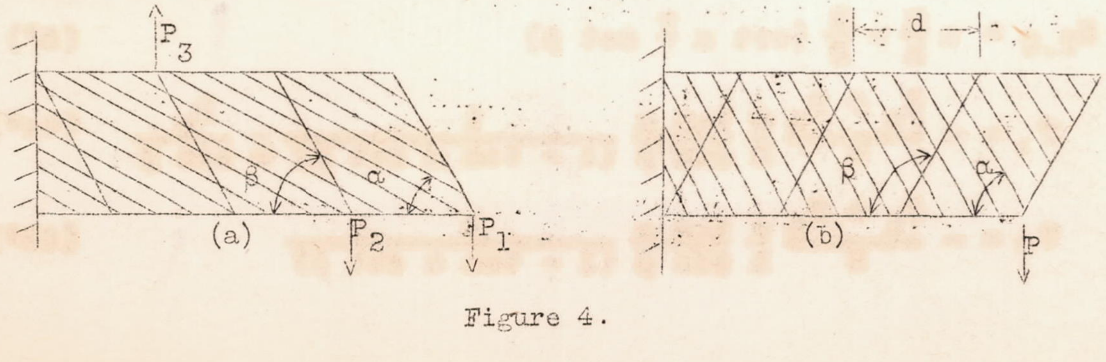

Figure 4.

Since angle sections or other open sections of small bending stiffness are often used for struts, the formulas are derived under two assumptions: I - struts with infinite stiffness against bending in the plane of the web; and II - struts with no bending stiffness. The effects of finite bending stiffness of the flanges will also be considered.

<u>I - Beam with struts of infinite bending stiffness in</u> **plane of web.-** If the struts are rigid and well riveted to the web, the web tension is constant in any bay between two struts and changes by a constant amount proportional to P_n at any strut where a load P_n is introduced (fig. 5). Wherever such a load is introduced the force in the strut varies linearly from V_1 to V_2 throughout the length of the strut.

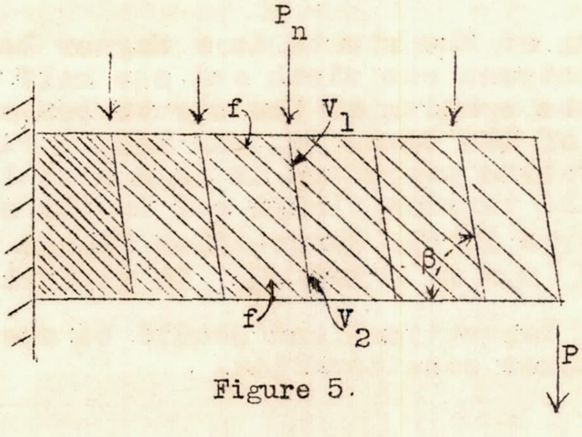

Figure 5.

The formulas for the case under discussion are (reference 4, pp. 7 and 8, and reference 1, pp. 33 and 34):

$$f = \frac{2S}{ht} \frac{1}{\sin 2\alpha\,(1 - \tan\alpha \cot\beta)} \qquad (4')$$

$$H_{T,C} = \pm \frac{M}{h} - \frac{S}{2}(\cot\alpha \mp \cot\beta) \qquad (5')$$

$$V_1 = -\frac{S_L + S_R}{2} \frac{d}{h} \frac{\tan\alpha}{\sin\beta} \frac{1}{(1 - \tan\alpha \cot\beta)} \mp \frac{P_n}{\sin\beta} \qquad (6a')$$

$$V_2 = -\frac{S_L + S_R}{2} \frac{d}{h} \frac{\tan\alpha}{\sin\beta} \frac{1}{(1 - \tan\alpha \cot\beta)} \qquad (6b')$$

where S and M are the shear and moment, respectively, at the section considered, S_L and S_R are the shears in the bays on the left and on the right of the strut considered, and P_n is the external load applied at the strut.

Figure 4 indicates how the angles α and β are measured. If there is any doubt as to whether the acute or the obtuse value of β should be used, a diagram of the beam should be drawn and the tension diagonals sketched in for each panel, their slope depending upon the direction of the shear in the panel. The angle α is always acute and can be taken equal to $\beta/2$ unless the struts are spaced too far apart. If the angle α', determined by a tension diagonal from panel point to panel point (e.g., P_2 to P_3 in fig. 4a) becomes less than $\beta/2$, then the angle α' should be used in place of α for computing the stresses in the web and in the compression flange, while the angle $\alpha = \beta/2$ should be used for computing the stresses in the tension flange and in the strut.

In formula (6a'), the negative sign for P_n must be used if the load P_n causes compression in the strut and the positive sign of P_n causes tension. The maximum force in the strut is given either by (6a') or (6b'), depending upon the sign of P_n, and it occurs at the junction of the strut with that flange which would be cut first by an arrow flying in the direction of the force P_n.

II - Beams with struts of zero bending stiffness in plane of web.- A better general approximation to actual conditions is probably obtained by assuming the struts to have negligible bending stiffness in the plane of the web. Under this condition, the folds are not interrupted where they cross the struts (fig. 6) and the web stress is constant along the full length of any tension diagonal.

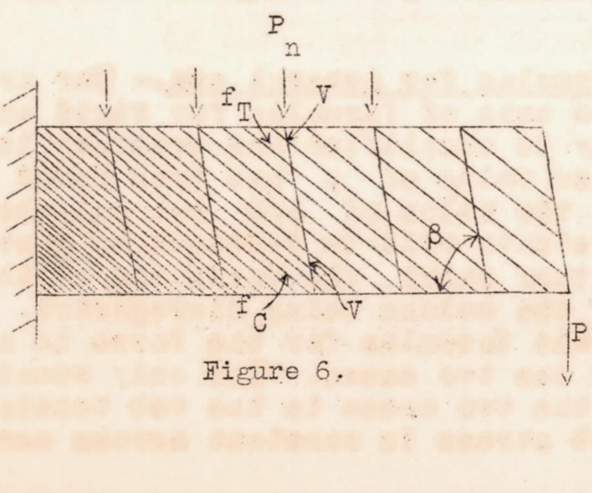

Figure 6.

Thus, at any section of the beam taken parallel to the struts, the web stress varies throughout the depth of the beam. This case has been solved under the assumption that all struts are loaded, and that the loads P_n are proportional to the spacing of the struts. Under this assumption the force in the strut is constant throughout its length.

The formula for the forces in the flanges is the same as under the assumption of rigid struts. The formulas for the web tension at the strut and for the forces in the struts become for this case (reference 4, p. 9),

$$f_{T,C} = (S_L + S_R) \frac{1}{ht \sin 2\alpha \, (1 - \tan\alpha \cot\beta)} \mp \frac{1}{2} \frac{P_n}{dt \sin^2 \alpha} \qquad (7')$$

$$V = -\frac{(S_L + S_R)}{2} \frac{d}{h} \frac{\tan\alpha}{\sin\beta} \frac{1}{(1 - \tan\alpha \cot\beta)} \mp \frac{P_n}{2 \sin\beta} \qquad (8')$$

If loads P_n are applied over only a portion of the beam and are approximately proportional to the spacing of the struts, the formulas can be used as good approximations in the middle part of the loaded region of the beam. On the borders of this region, or in general at any place where the loads P_n are not proportional to the spacing of the struts, each case must be given special consideration, as indicated in the last example of the appendix.

III - Formulas for general use.- For practical purposes, the two sets of formulas for rigid and for flexible struts may be simplified and combined into one set. When the proper value of β has been found as explained in section I, the value of $\beta/2$ can be substituted for α. Furthermore, struts will be designed in most cases for the average load they carry, the variation of this load along the length of the column being disregarded. With this simplification, the formulas for the force in a strut become identical for the two cases. The only remaining difference between the two cases is the web tension; for rigid struts the web stress is constant across sections parallel to the struts; for flexible struts, the web tension varies linearly between the two values given by formula (4b) across such sections of the beam.

The general formulas therefore become

$$f = \frac{2S}{ht} \cot \frac{\beta}{2} \text{ (for stiff struts)} \tag{4a}$$

$$f = \frac{(S_L + S_R)}{ht} \cot \frac{\beta}{2} \mp \frac{P_n}{2\, dt \sin^2 \frac{\beta}{2}} \text{ (for flexible struts)} \tag{4b}$$

$$H_{T,C} = \pm \frac{M}{h} - \frac{S}{2} \left(\cot \frac{\beta}{2} \mp \cot \beta\right) \tag{5}$$

$$V_n = - \frac{(S_L + S_R)}{2} \frac{d}{h} \mp \frac{P_n}{2 \sin \beta} \tag{6}$$

(for choice of sign in equation (6) see note regarding equation (6a').)

The decision as to whether a given strut should be considered as being rigid, very flexible, or of some intermediate stiffness must be left to the judgment of the designer. In general, it can be said that even struts of closed section do not approach the theoretical condition of rigidity very closely.

<u>IV - The effects of small bending stiffness of the flanges</u>.- The tension in the web causes bending stresses in the flanges (fig. 7) which are superposed on the longitudinal stress caused by H_T or H_C.

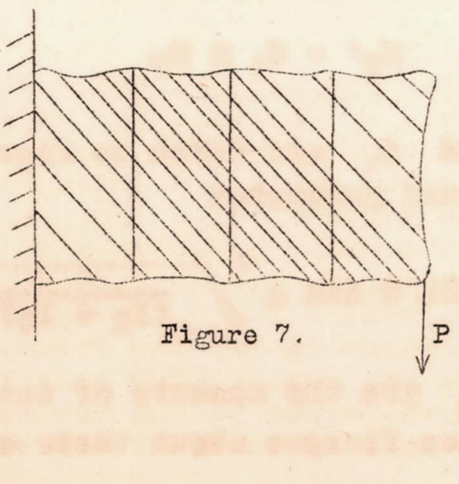

Figure 7.

The normal component of the web tension being considered as a uniform load and the flange as a beam continuous over the struts as supports, the maximum bending moment in the flange occurs at the strut and has the magnitude

$$M_F = \frac{S\ d^2}{12\ h} \tan \alpha \qquad (9')$$

This expression is sufficiently exact for calculating the secondary stresses caused by bending of the flanges in any Wagner beam of normal proportions (reference 5, p. 34); i.e., in a beam where the struts are spaced from one sixth to one half the depth of the beam.

If the bending stiffness of the flanges is not infinite and the spacing of the struts is increased, a point is reached where only a part ψ d of the web is in tension (fig. 8).

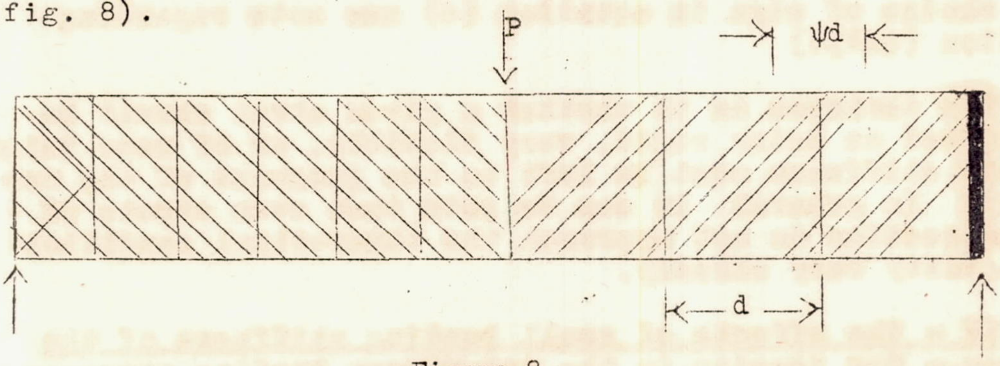

Figure 8.

This causes a reduction in M_F to M_F', where

$$M_F' = C_1 \times M_F \qquad (10')$$

The factors ψ and C_1 are given in figure 9 as functions of the nondimensional parameter

$$\omega d = 1.25\ d \sin \alpha \sqrt[4]{\frac{t}{(I_T + I_C)\ h}} \qquad (11')$$

where I_T and I_C are the moments of inertia of the tension and compression flanges about their own centroidal

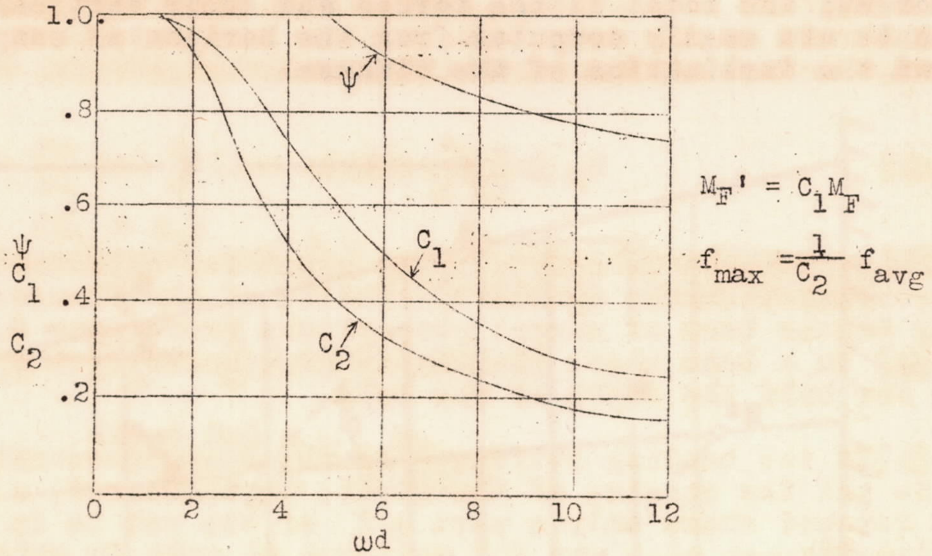

Figure 9.

axes. When only a portion of the web is in tension, equation (1'), (4'), or (7') gives the average stress. The maximum web stress is

$$f_{max} = f_{avg} \times \frac{1}{C_2}$$

where C_2 is a factor given by figure 9. (See reference 5, pp. 33-37, for equations (10'), (11'), and (12').)

At the end of the beam, or at any point where an external load is applied, a bending moment analogous to M_F is induced in the struts. Either these members must be made sufficiently strong to withstand the bending moments or diagonal members must be used in adjacent bays. Figures 3 and 8 show some of the possible solutions.

THE CASE OF THE BEAM WITH NONPARALLEL FLANGES

In structural design, it is generally assumed that in a beam with nonparallel flanges the forces in the flanges are in the direction of the flanges. Hence, equation (2) or (5) gives only the horizontal component of the flange forces; the total flange forces and their vertical components are easily computed from the horizontal components and the inclination of the flanges.

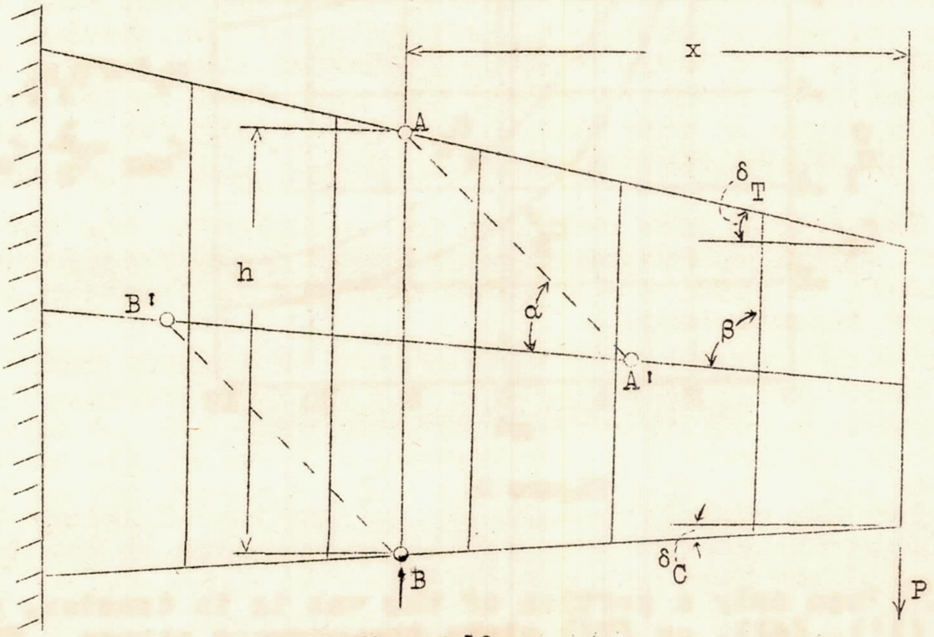

Figure 10.

The vertical components of the flange forces carry a part of the shear. Accordingly, the shear S_W carried by the web is the difference between the total shear S and the vertical components of the flange forces:

$$S_W = S - \frac{M}{h} (\tan \delta_T + \tan \delta_C) \qquad (13')$$

(See fig. 10.) This shear S_W is used to calculate the web tension and the force in the struts, using the formulas given for beams with parallel flanges (reference 4, pp. 1-6).

The web stress thus computed is the stress at the center line of the beam. It varies along the depth of the beam even though there are no intermediate loads applied at the struts. Since the stress is constant along any tension diagonal, the web stresses at points A and B may be obtained by drawing the tension diagonals through them, making an angle α with the center line of the beam, and calculating the stresses for points A' and B'. The same method applies for a beam with intermediate loads if the struts have small bending stiffness, provided that the loading of the beam near the section investigated conforms to the assumption underlying formula (7'); viz, that the loads are proportional to the spacing of the struts. If the struts have large bending stiffness, the tension may be considered constant in any bay and equal to the average tension given by equation (1) or (4), using for h the average height of the bay.

The method here outlined for calculating the forces in Wagner beams with nonparallel flanges is only approximate; it should be used with caution when the inclination of the flanges becomes large.

DEFLECTIONS OF WAGNER BEAMS

For the computation of the deflection of Wagner beams, the following approximate method is proposed by the author until further data are obtained:

(1) Calculate the bending deflection of the beam by standard beam-deflection formulas.

(2) Calculate the shear deflection of the web in the following manner:

Imagine the beam replaced by a frame consisting of the beam flanges, diagonals inclined at the angle α, and vertical struts regardless of whether the struts in the Wagner beam are vertical or inclined.

Assume the diagonals to be under a stress equal to f and compute the deflection of the substitute frame due to elongation of the diagonals only.

(When the frame is divided into panels in the manner prescribed there will usually be a short odd panel left at the end, but this panel of odd size can be neglected in the calculation of the shear deflection.)

(3) Add the bending deflection and the shear deflection.

If the method proposed is applied to a cantilever beam such as the one shown in figure 3, the following formula is obtained for the deflection at the end of the beam:

$$D = \frac{P l^3}{3EI} + \frac{4Pl}{Eth} \qquad (14')$$

where I is the moment of inertia of the beam.

EXPERIMENTAL CHECK OF ACCURACY OF FORMULAS

The results of strain-gage measurements on a beam with parallel flanges and vertical struts are given in reference 3. The experimental results check the calculated values within about 5 percent for the stresses in the web and in the flanges. The experimentally obtained stresses in the strut are much smaller than the calculated stresses. This discrepancy is probably due to the fact that the actual inclination of the folds differs from the assumed inclination. Examination of formulas (1'), (2'), and (3') will show that an error in α affects the force in the strut much more than it affects the stresses in the web or those in the flanges.

It may be mentioned here that Professor Wagner suggests the use of $\alpha = 40^\circ$. This is indeed a better average value, but attention has already been called to the fact that the inclination of the folds is never quite regular. Furthermore, the gain in the average accuracy of computing the force on the strut is only of academic interest, since the allowable stress for the struts is very uncertain. The use of $\alpha = 45^\circ$ in preference to $\alpha = 40^\circ$ is therefore recommended because it is simpler to use and more conservative.

The formulas for the cantilever beam with parallel and rigid flanges, closely spaced vertical struts, and a single load can be derived with very few basic assumptions. Any complication such as inclined struts, inclined flanges, or intermediate loads necessitates additional assumptions and decreases the probable accuracy of the formulas. However, it is believed that all the formulas are sufficiently accurate for airplane design purposes as long as the proportions of the beam are not abnormal.

Formula (14') errs on the unsafe side. For loads up to about 10 percent of the yield-point load, calculated deflections should be multiplied by 4/3. For higher loads, much higher correction factors may be necessary, but the experimental evidence is insufficient to warrant any recommendations.

THE DESIGN OF WAGNER BEAMS

Omitting problems of detail design which are best met in the shop, this discussion will confine itself to allowable stresses. It seems advisable to deal with the problem first from a simple but "theoretical" point of view. Later it will be pointed out that practical considerations may require considerable modification of the "theoretical" allowable stresses.

If the design is to be based on the ultimate strength, the allowable stress for the web and the tension flange should be the ultimate tensile strength of the material. If the design is to be based on the yield strength, the yield-point stress would, of course, be substituted.

The allowable stress for the compression flange depends on the shape of cross section, the lateral support of flange, etc., considerations which are beyond the scope of this report and will not be discussed here.

The struts are, in effect, columns with lateral elastic support, since the tension in the web restrains the struts from buckling out of the plane of the web. By a series of calculations (reference 4, pp. 15-23), Professor Wagner has evaluated this effect on the theoretical buckling strength of the struts. On the further assumption (reference 4, p. 24) that two columns fail at the same stress if they have the same index value K, Professor Wagner's calculations yield a reduction factor C_3 (see fig. 11, computed from an approximation of the lower curve in fig. 27 of reference 4), which is a function of the parameter $\frac{d}{h\ (\cot\alpha - \cot\beta)}$, and by which the actual length l of the strut is multiplied in order to obtain a reduced length l

$$l' = C_3 l \qquad (15')$$

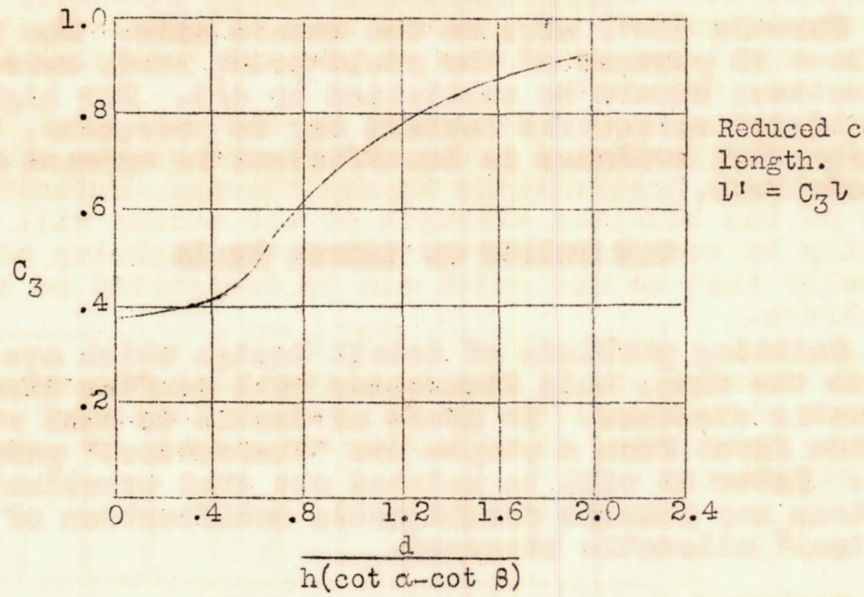

Figure 11.

With this reduced length l' and the actual cross section of the strut, the allowable stress for the strut may be computed (reference 4, p. 27) by standard formulas or obtained from column charts for pin-ended columns. The allowable load on the strut is then obtained by multiplying this allowable stress by the effective area, which is the sum of the area of the strut and an adjacent strip of the web. For duralumin, the effective width of this strip may be taken as $2w = 30\ t$; for stainless steel, $2w = 60\ t$ (reference 6),

The theoretical allowable stresses given may serve as a guide for design until additional practical experience has been gained. The following considerations should always be borne in mind, however, as they may necessitate appreciable changes in the allowable stresses.

1. The folds cause bending stresses which may lower the ultimate strength and the fatigue strength; the folds themselves may impair the performance of the airplane.

2. The wrinkles form at low loads and reach an appreciable size under normal flight conditions (fig. 12a taken from reference 2). If they appear on parts exposed

to view during flight (wing covering), they will engender a serious loss of confidence on the part of pilots and passengers even though the structure is perfectly safe. This consideration may perhaps seem unimportant, but the scanty experience available at present indicates that it may be very decisive.

3. The factor C_3 for the design of the struts is probably very conservative in most cases. Unfortunately, tests on the buckling strength of the struts will not ordinarily be very conclusive, since the buckling occurs so gradually that no one point can be designated as the point of failure.

In conclusion, it may be stated that the establishment of rules more comprehensive than those indicated will be possible only after considerable practical experience has been gained.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 1, 1933.

APPENDIX

Illustrative Problems

Problem 1.

Analyze the beam of figure 13. The material is duralumin; the allowable stress in the compression flange is assumed to be 26,000 pounds per square inch.

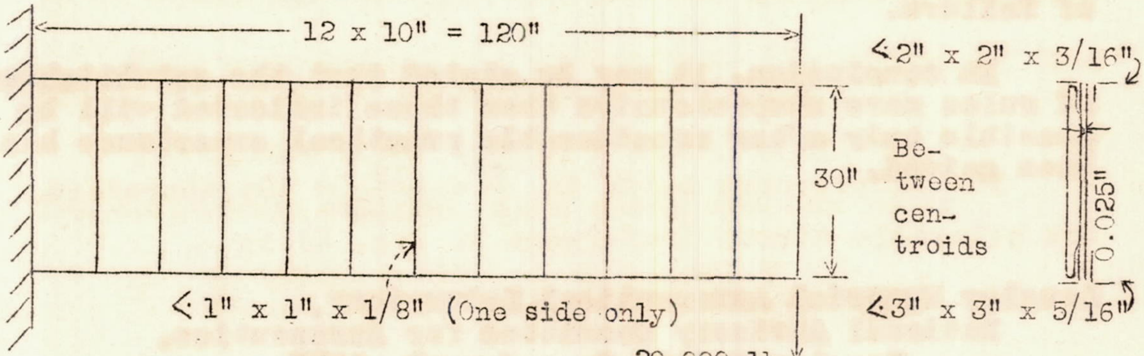

Figure 13.

The stress in the web is (formula (1))

$$f = \frac{2 \times 20000}{30 \times 0.025} = 53,300 \text{ lb./sq.in.}$$

The forces in the flanges are (formula (2))

$$H_{T,C} = \pm \frac{20000 \times 120}{30} - \frac{1}{2} \times 20,000$$

$$H_T = + 70,000 \text{ lb.}$$

$$H_C = - 90,000 \text{ lb.}$$

The stresses in the flanges are therefore

$$f_T = + \frac{70000}{2 \times 0.72} = 48,600 \text{ lb./sq.in.}$$

$$f_C = - \frac{90000}{2 \times 1.77} = - 25,400 \text{ lb./sq.in.}$$

The force on any strut is (formula (3))

$$V = - 20,000 \times \frac{10}{30} = - 6,670 \text{ lb.}$$

Since $\beta = 90^\circ$, $\alpha = 45^\circ$, and $\frac{d}{h} = 0.33$, figure 11 gives $C_3 = 0.40$; therefore the reduced column length (formula (15')) is

$$l' = 0.40 \times 30 = 12 \text{ in.}$$

The slenderness ratio is $\frac{L}{\rho} = \frac{12}{0.30} = 40$; therefore, the allowable stress (reference 7, fig. 6) is

$$F_C = 27,800 \text{ lb./sq.in.}$$

The effective width of sheet that acts with the strut is $2w = 30 \times 0.025 = 0.75$ in.; therefore, the total effective area

$$A_e = 0.23 + 0.75 \times 0.025 = 0.249 \text{ sq.in.}$$

and the allowable load

$$P_{allow} = 0.249 \times 27,800 = 6,920 \text{ lb.}$$

The maximum bending moment in the flanges due to the web tension is (formula (9'))

$$M_F = \frac{20000 \times 100}{12 \times 30} = 5,560 \text{ in.-lb.}$$

The maximum total stress in the tension flange is therefore

$$f_T = 48,600 + \frac{5560 \times 0.56}{0.54} = 54,360 \text{ lb./sq.in.}$$

The maximum total stress in the compression flange is

$$f_C = - 25,400 - \frac{5560 \times 2.15}{2.90} = - 29,510 \text{ lb./sq.in.}$$

This stress is above the allowable stress specified for the compression flange, but still below the yield point. In view of the fact that the specified maximum stress is based on considerations of buckling, the purely local and comparatively small overstress appears admissible.

Formula (11) gives

$$\omega d = 1.25 \times 10 \times 0.707 \sqrt[4]{\frac{0.025}{(0.54 + 2.90) \times 30}} = 1.1$$

Figure 9 shows that C_1 and C_2 are practically equal to unity for this value of ωd; there is consequently no reduction in M_F, and the web stress is uniform.

When calculating the deflection, the moment of inertia of the beam is computed approximately as

$$I = 3.54 \times 8.68^2 + 1.44 \times 21.32^2 = 923 \text{ in.}^4$$

The deflection formula (14') gives for low loads

$$D = \frac{4}{3}\left(\frac{20000 \times 120^3}{3 \times 10^7 \times 923} + \frac{4 \times 20000 \times 120}{10^7 \times 0.025 \times 30}\right)$$

$$= \frac{4}{3}(1.25 + 1.28) = 3.38 \text{ inches.}$$

Problem 2.

Given the beam of figure 13, but with a spacing $d = 20$ inches of the struts, calculate the stresses in the web and in the flanges.

The average stress in the web is, as in the preceding example,

$$f = 53,300 \text{ lb./sq.in.}$$

The direct stresses in the flanges also remain unchanged

$$f_T = 48,600 \text{ lb./sq.in.}$$

$$f_C = -25,400 \text{ lb./sq.in.}$$

The parameter ωd is twice that of the preceding example (since d is doubled)

$$\omega d = 2 \times 1.1 = 2.2$$

which gives $C_1 = 0.95$ and $C_2 = 0.90$.

The maximum web stress is therefore

$$f_{t\ max} = 53{,}300 \times \frac{1}{0.90} = 59{,}200 \text{ lb./sq.in.}$$

The maximum bending moment in the flange is (formulas (9') and (10'))

$$M_F' = 0.95 \times \frac{20000 \times 400}{12 \times 30} = 21{,}100 \text{ in.-lb.}$$

and therefore the bending stress in the flange

$$f_b = \frac{21100 \times 0.56}{0.54} = 21{,}900 \text{ lb./sq.in.}$$

or the maximum total stress in the tension flange at the inboard end

$$f_T = 48{,}600 + 21{,}900 = 70{,}500 \text{ lb./sq.in.}$$

$$f_C = -\ 25{,}400 - \frac{21100 \times 2.15}{2.90} = -\ 41{,}000 \text{ lb./sq.in.}$$

It will be necessary either to use stronger flanges or to reduce the spacing of the struts at the inboard end of the beam.

Problem 3.

Find the forces in the flanges, the forces on the struts, the reduced column length, and the stresses in the web for the beam shown in figure 14.

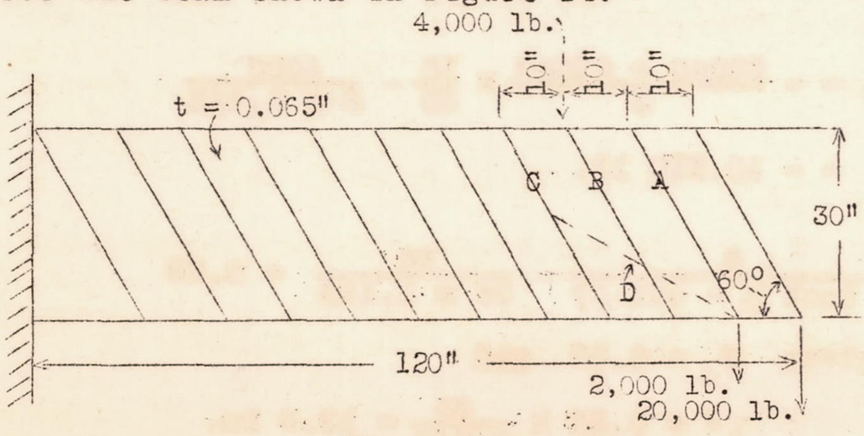

Figure 14.

The struts at the end and under the 4,000-pound load are to be considered as stiff; the other struts are to be considered as flexible.

The inclination of the struts is $\beta = 60°$; **therefore**, the inclination of the folds is $\alpha = \frac{\beta}{2} = 30°$. The moment at the inboard end is

$$M = 20,000 \times 120 + 2,000 \times 110 + 4,000 \times 82.7$$

$$= 2,950,000 \text{ in.-lb.}$$

The forces in the flanges at the inboard end (formula (5)) are

$$H_{T,C} = \pm \frac{2950000}{30} - \frac{26000}{2} (1.732 \mp 0.577)$$

$$H_T = + 83,360 \text{ lb.}$$

$$H_C = - 128,380 \text{ lb.}$$

the force in strut A is (formula (6'))

$$V_A = - \frac{20000 + 22000}{2} \times \frac{10}{30} + \frac{2000}{2 \times 0.866}$$

$$= - 5,850 \text{ lb.}$$

(Note that the second term has a positive sign for V_A and a negative sign for V_B. Cf. note on formula (6a).)

The force in strut B is

$$V_B = - \frac{22000 + 26000}{2} \times \frac{10}{30} - \frac{4000}{2 \times 0.866}$$

$$= - 10,310 \text{ lb.}$$

$$\frac{d}{h\ (\cot \alpha - \cot \beta)} = \frac{10}{30 \times 1.155} = 0.29$$

which gives $C_3 = 0.39$ and

$$l' = 0.39 \times \frac{30}{0.866} = 13.5 \text{ in.}$$

The system of loads does not fulfill the assumption under which equation (4b) for the web stress with flexible struts is derived. Consequently, a special consideration is necessary in this case.

Assuming first that all struts are rigid, formula (4a) yields for the web stress -

in the end panel: $f = \frac{2 \times 20000}{30 \times 0.065} \times 1.732$

$= 35,600$ lb./sq.in.

in the second panel from the end: $f = 39,200$ lb./sq.in.

in the third panel (and all others): $f = 46,300$ lb./sq.in.

Considering strut A as flexible, equation (4b) gives for the web stress at strut A

$$f = \frac{22000 + 20000}{30 \times 0.065} \times 1.732 \mp \frac{2000}{2 \times 10 \times 0.065 \times 0.25}$$

$$= 37,400 \mp 6,150$$

$f_{max} = 43,550$ lb./sq.in.

$f_{min} = 31,250$ lb./sq.in.

The minimum web stress of 31,250 pounds per square inch at the upper end of A is probably too low, since if the 20,000-pound load were the only load acting there would be a uniform web tension of 35,600 pounds per square inch throughout the beam. This latter value should therefore be considered as the minimum web stress at strut A, occurring at the upper end.

The maximum web stress of 43,550 pounds per square inch occurs at the lower end of strut A, and should be used for design. Actually the stress may be less, in view of the argument given that the actual minimum stress at the upper end is probably more than the theoretical value.

If, for the purpose of saving weight, the thickness of the web is reduced in the end panel, a somewhat larger margin should be provided here than in the rest of the beam to take care of stress concentration due to flexibility of the end strut.

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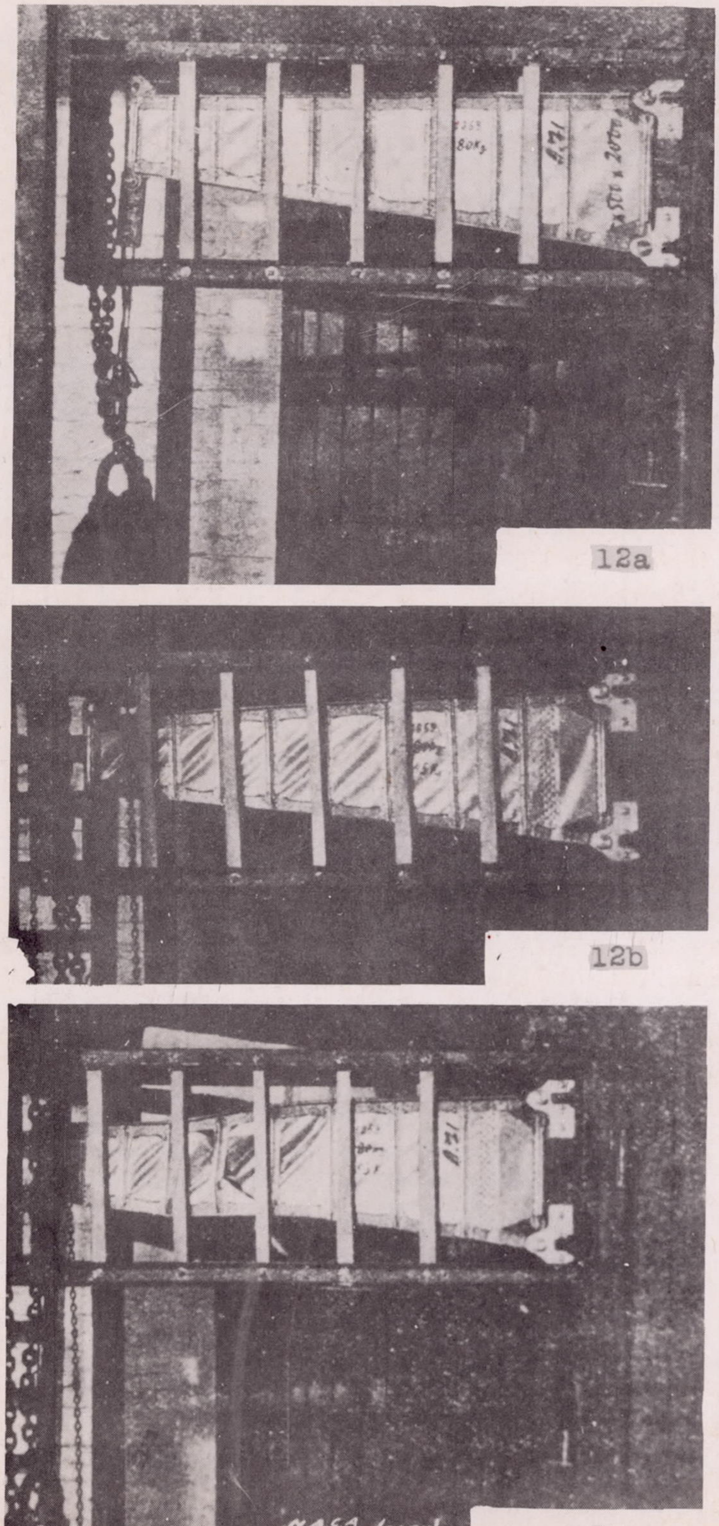

12a

12b

12c

Figure 12.- Cantilever Wagner beam with concentrated load at tip under test.