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No. 455

COMPARISON OF THREE METHODS FOR CALCULATING

THE COMPRESSIVE STRENGTH OF FLAT AND SLIGHTLY

CURVED SHEET AND STIFFENER COMBINATIONS

By Eugene E. Lundquist
Langley Memorial Aeronautical Laboratory

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SUMMARY

This report gives a comparison of the accuracy of
three methods for calculating the compressive strength of
flat sheet and stiffener combinations such as occur in
stressed-skin or monocoque structures for aircraft. Of
the three methods based upon various assumptions with re-
gard to the interaction of sheet and stiffener, the method
based upon mutual action of the stiffener and an effective
width of sheet as a column gave the best agreement with
the results of tests.

An investigation of the effect of small curvature re-
sulted in the conclusion that the compressive strength of
curved panels is, for all practical purposes, equal to the
strength of flat panels except for thick sheet where non-
uniform curvature throughout the length of the panel may
cause the strength of a curved panel to be as much as 10
to 15 per cent less than the strength of a corresponding
flat panel.

INTRODUCTION

In the recent literature on the strength of stressed-
skin or monocoque structures for aircraft, several methods
are suggested for calculating the compressive strength of
sheet and stiffener combinations. The purpose of this re-
port is to compare the accuracy of the methods suggested
for flat sheet and stiffener combinations and to investi-
gate the effect of small curvature on the compressive
strength of a curved sheet and stiffener combination. For
convenience of reference, the methods have been designated
A, B, and C.

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Method A is that recommended by Professor Joseph S. Newell (references 1 and 2), who has cooperated in the preparation of this report by providing the National Advisory Committee for Aeronautics with the results of compression tests made at the Massachusetts Institute of Technology on sheet and stiffener combinations. Methods B and C are logical developments from the remarks of Dr. Theodor von Karman in reference 3.

In order that the application of each method may be clearly understood, examples illustrating each are given in Appendices A, B, and C, respectively.

DESCRIPTION OF METHODS   A, B, AND C

Method A.- Method A consists of adding to the ultimate load carried by the stiffener when tested alone the ultimate load carried by the sheet when tested with the unloaded edges supported in V-shaped grooves. The load carried by the sheet may be determined either from special tests or from the data given in reference 4.

This method assumes complete independence of action of sheet and stiffeners, except that the stiffeners are assumed to give simple support to the sheet.

Method B.- Method B consists of adding to the ultimate load carried by the stiffener when tested alone the load carried by an effective width of sheet subjected to the same stress as the stiffener. The equation for the effective width of sheet as derived by von Karman in reference 3 is

$$2w = k \sqrt{\frac{E}{\sigma}} \, t \tag{1}$$

where $2w$,   effective width of sheet, in.   (See fig. 1)

E,   modulus of elasticity, lb. per sq.in.

σ,   stress in the stiffener, lb. per sq.in.

t,   thickness of sheet, in.

k,   a constant, see Appendix B.

This method assumes complete independence of action of the stiffeners but not of the sheet.

Method C.- Method C assumes the stiffener and effective width of sheet to behave as a column which fails by bending normal to the plane of the sheet. The moment of inertia and slenderness ratio of the combination of stiffener and effective width of sheet are calculated and the area of the combination is multiplied by the stress for a column of these proportions.

This method assumes no independence of action of the sheet and stiffener but rather a mutual action of the two.

COMPARISON OF OBSERVED AND PREDICTED COMPRESSIVE LOADS

FOR FLAT SHEET AND CHANNEL STIFFENER COMBINATIONS

In Table I are tabulated observed and predicted compressive loads for the panels shown in Figure 2 tested with flat ends. The loads observed in test were obtained from Newell and the predicted loads were calculated as outlined in Appendices A, B, and C.

The observed and predicted loads recorded in Table I are plotted in Figure 3. It will be observed that for all panel lengths with thin sheet and for short panels with thick sheet there tends to be little difference between the loads predicted by the three methods and that the observed and predicted loads tend to be in good agreement. For long panels with thick sheet, the loads predicted by methods A and B err on the unsafe side by an amount which increases with increase in both length of panel and thickness of sheet, but the loads predicted by method C agree very well with those observed in tests.

Because the tests were made with indefinite end conditions (flat ends), any detailed consideration of small differences between observed and predicted loads is not justified. Conclusions will therefore be drawn with regard to large differences only.

Of the three methods for predicting the compressive strength of flat sheet and stiffener combinations, method C gives the best general agreement between observed and predicted loads for the specimens tested. For specimens

with thin sheet where the load carried by the sheet is small as compared with the load carried by the stiffeners, or for specimens of short length with thick sheet where the stress in the stiffener at failure approaches the compression yield point for the material, the compressive strength of a flat sheet and stiffener combination is predicted equally well by methods A, B, and C. For long specimens where the stress at failure depends on the slenderness ratio, l/ρ, of the combination of stiffener and effective width of sheet, the use of methods A and B must be restricted because they do not properly describe the behavior of the combination. The logical restrictions to be placed on these methods are: For method A, the stiffeners shall be of such proportions that when tested with the sheet they fail at stresses that approach the compression yield point for the material in the sheet, or the sheet shall be of such thickness that it carries only a small percentage of the load carried by the stiffener; and for method B, the slenderness ratio l/ρ of the stiffener shall not be changed appreciably by consideration of the effective width of sheet.

In order to establish definite limits within which methods A and B may be used it is necessary to specify the permissible error. As it is beyond the scope of this report to specify the permissible error, it is recommended that method C be used except where it has been found by experience that the accuracy of method A or B is sufficient.

EFFECT OF SMALL CURVATURE ON THE COMPRESSIVE STRENGTH

OF A SHEET AND STIFFENER COMBINATION

In a stressed-skin wing or monocoque fuselage the sheet is usually curved instead of flat. It is therefore desirable to consider the effect of small curvature on the compressive strength of a sheet and stiffener combination.

In Figure 4 are plotted the results of compression tests on curved panels of the type shown in Figure 2, the data of which are given in Table I of reference 2. The curved panels, as in the case of the flat panels, were tested with flat ends and prior to curvature were of the same dimensions as the flat panels. In Figure 4 it will be observed that for large values of the radius/thickness ratio, some of the results of tests on curved panels plot

below the horizontal dashed lines for flat panels. In reference 1 it is explained that a part of the reduced strength may have been caused by failure to obtain uniform curvature throughout the length of the panel with the result that the elements of the sheet between stiffeners were eccentrically loaded. However, in reference 3 it is stated that, for small curvature with the stiffeners located on the concave side of the sheet, it is to be expected that the compressive strength of a curved panel would be less than the strength of a corresponding flat panel because the effect of curvature is such as to reduce the moment of inertia of the combination of stiffener and effective width of sheet.

In an effort to determine the quantitative effect of small curvature, the following equation was derived for the moment of inertia of the combination of stiffener and effective material in the sheet when the sheet is either flat or curved; in the derivation of this equation it was assumed that the thickness of the sheet was small compared to the dimensions of the stiffener and that the curvature was sufficiently small that the sine of the angles involved could be approximated by the angles themselves:

$$I = I_{stiff.} + I_{sheet} + \frac{A \times 2wt}{A + 2wt} [z \pm y]^2 \qquad (2)$$

where I,   moment of inertia of an individual stiffener and the effective material in the sheet about an axis through the centroid of the combination parallel to the sheet in.[4]

$I_{stiff.}$,   moment of inertia of the stiffener about its centroidal axis parallel to the sheet in.[4]

$I_{sheet} = \dfrac{k_1 w^5 t}{R^2}$,   moment of inertia of the effective material in the sheet about an axis through its centroid parallel to the sheet, in.[4]

A,   area of stiffener, sq.in.

$2w$,   effective width of sheet, in.   (See fig. 1.)

t,   thickness of sheet, in.

z,   distance from the middle surface of the
        sheet at the stiffener to the centroid
        of the stiffener, in.

$y = \dfrac{k_2 w^2}{R}$,   distance from the middle surface of the
        sheet at the stiffener to the centroid of
        the effective material in the sheet, in.

R,   radius of curved sheet, in.

In equation (2) the plus sign before y in the last term
should be used when the stiffener is located on the convex
side of the sheet and the minus sign when on the concave
side. The constants K_1 and K_2 in the equations for
I_{sheet} and y depend on the stress distribution. When
the sheet is flat the moment of inertia of the combina-
tion of stiffener and effective material in the sheet is
the same regardless of whether the effective material is
distributed along the sheet or assumed to be concentrated
near the stiffener; i.e., independent of distribution.
However, when the sheet is curved the moment of inertia
is dependent on the distribution of the effective material
in the sheet. Therefore, when calculating the moment of
inertia of the combination of stiffener and effective
width of sheet, when the sheet is curved, the effective
material in the sheet must be considered to be distribut-
ed in proportion to the stress distribution. In Figure 5
values of K_1 and K_2 are given for several assumed dis-
tributions of the effective material in the sheet. (Cases
I, II, III, and IV.)

Upon application of equation (2) to the curved panels
with channel stiffeners tested by Newell, the following
tables may be constructed:

12-inch panel with 0.33-inch sheet;
(σ = 22,200 lb. per sq.in., $2w$ = 36.9t)
Area of stiffener - - - - - - - - 0.0566 sq.in.
Area of combination - - - - - - - 0.096   sq.in.
Moment of inertia of stiffener - - 0.00144 in.^{4}

R (in.)	$\dfrac{R}{t}$	I (in.^{4})			
		Case I	Case II	Case III	Case IV
∞	∞	0.00214	0.00214	0.00214	0.00214
80	2,420	.00214	.00213	.00213	.00212
50	1,515	.00213	.00212	.00212	.00211

18-inch panel with 0.052-inch sheet;
(σ = 8,200 lb. per sq.in., 2w = 60.8t)

Area of stiffener - - - - - - - - 0.0566 sq.in.
Area of combination - - - - - - - 0.221 sq.in.
Moment of inertia of stiffener - - 0.00144 in.[4]

R (in.)	$\frac{R}{t}$	I (in.[4])			
		Case I	Case II	Case III	Case IV
∞	∞	0.00284	0.00284	0.00284	0.00284
80	1,540	.00276	.00271	.00269	.00269

From these tables it may be concluded that the reduction in the moment of inertia of the combination of stiffener and effective width of sheet at large values of the radius/thickness ratio is small (less than 6 per cent) and hence accounts for but a small part of the observed reduction in strength of a curved over that for a flat panel in Figure 4. Consequently the reduced strength of curved panels at large values of the radius/thickness ratio must be caused, as Newell suggested, by failure to obtain uniform curvature throughout the length of the panel. Because the percentage of the total load carried by the sheet increases with increase in sheet thickness, the percentage reduction in load caused by nonuniform curvature will also increase with increase in sheet thickness and this conclusion is in accordance with the test data plotted in Figure 4.

For small values of the radius/thickness ratio the increased stability of the curved sheet, which is small for large values of the radius/thickness ratio and was neglected in the preceding discussion, becomes appreciable and the stiffeners, together with their effective widths of sheet, can no longer be assumed to behave as independent columns supported in the plane of the sheet. However, in stressed-skin wings and the larger monocoque fuselages the radius/thickness ratio R/t, will approach or exceed 1,200. When such is the case, the individual panels may be assumed to be flat for purposes of strength calculation and the reduction in strength of curved panels allowed for by an arbitrary factor determined from the data plotted in Figure 4. It is doubtful if this factor will need to be greater than 10 or 15 per cent in any case of practical importance.

DISCUSSION

In the compression tests on flat sheet and channel
stiffeners failure occurred, for the longer lengths, by
bending of the stiffener and sheet normal to the plane of
the sheet in a manner similar to primary failure in col-
umns. The sheet buckled between stiffeners but no mention
was made in references 1 or 2 of failure having occurred
by buckling of the sheet between rivets attaching sheet to
stiffeners or by local wrinkling of the outstanding legs
of the stiffeners. Consequently, the conclusions drawn
from the results of the tests on panels with channel stiff-
eners should also apply to panels with any type of stiff-
ener that fails by bending normal to the plane of the
sheet. A few of these stiffener sections are shown in
Figure 6.

Where two lines of rivets are required to attach the
sheet to the stiffener (A, B, C, D, F, G, and H, fig. 6)
the area of that portion of the sheet between the two riv-
et lines should be added to the area of the stiffener.
However, if the width of the sheet between the two rivet
lines is greater than $2w$, the effective width outside
the rivet lines, then an area of only $2wt$ should be add-
ed for the part inside the rivet lines.

For stiffeners that fail other than by bending nor-
mal to the plane of the sheet in a manner similar to pri-
mary failure in columns, the sheet may or may not alter
the strength of the stiffener. If the stiffener fails lo-
cally by wrinkling of a thin part, then the load carried
by the stiffener remains unchanged provided the effective
width of sheet has not altered the slenderness ratio l/ρ,
of the stiffener to such an extent that failure occurs by
bending in a plane normal to the plane of the sheet at a
stress below that for local failure. For a stiffener
which fails by twisting when tested alone (see reference
5), the strength of the stiffener is increased by the
sheet which provides resistance to twisting.

Unless properly proportioned, stiffeners such as shown
in Figure 7 may fail by the outstanding part buckling par-
allel to the plane of the sheet. (See fig. 8.)

It is appreciated that the ideal design of a stiff-
ened panel to carry compression is one in which failure
is equally likely to occur in each of the many possible

ways. However, it is probably best to proportion the panel so that local failure in thin stiffeners, rivets, and connections does not occur before primary failure of the panel as a whole by compression or bending normal to the plane of the sheet. Consequently, the designer should test several lengths of the particular stiffener proposed for use when riveted to a sheet in order to study its behavior and proportion it so that maximum stiffness is obtained normal to the plane of the sheet without the possibility of local or secondary failure in the stiffener. For such a stiffener the conclusions reached in this report with regard to the accuracy of methods A, B, and C may be considered to apply.

Attention is called here to the possible errors which may result from the construction of curves of strength plotted against percentage reinforcement. Strictly speaking, such curves apply only for the particular type, size, and length of stiffener for which the curves are constructed. If used for other stiffeners than for the one constructed, appreciable errors may result.

CONCLUSIONS

1. For stiffeners that do not fail locally but rather fail by bending of the stiffener and sheet normal to the plane of the sheet in a manner similar to primary failure in columns, method C, which is based upon a mutual action of sheet and stiffener, gives the best agreement between observed and predicted loads and is followed in order of accuracy by methods B and A.

2. The limits within which methods A and B may be used are dependent upon the permissible error. As it is beyond the scope of this report to specify the permissible error, it is recommended that method C be used except where it has been found by experience that the accuracy of method A or B is sufficient.

3. For large values of the radius/thickness ratio ($R/t = 1,200$ or more), the compressive strength of curved panels is, for all practical purposes, equal to the strength of flat panels except for thick sheet where nonuniform curvature throughout the length of the panel may

cause the strength of a curved panel to be as much as 10
to 15 per cent less than the strength of a corresponding
flat panel.

Langley Memorial Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., March 2, 1933.

APPENDIX A

In Newell's calculations of the load carried by a
flat sheet and stiffener combination (references 1 and 2),
that portion of the sheet which lies outside the rivet
lines of the two edge stiffeners was neglected. Examina-
tion of the observed loads in Table I and Figure 3 indi-
cates that this portion of sheet cannot be neglected be-
cause if the load carried by one stiffener and the sheet
between stiffeners is subtracted from the load carried by
the panels with two stiffeners, the remaining load is, in
almost every case, greater than the load carried by the
one remaining stiffener. Therefore, if the loads calcu-
lated by method A are to be comparable with those calcu-
lated by methods B and C, as outlined in Appendices B and
C, respectively, this additional load should be included.

As there are no tests similar to the tests of refer-
ence 4 from which to obtain the ultimate load carried by
a plate with one of the unloaded edges free (not support-
ed in a V-shaped groove), the load carried by the sheet
outside the rivet lines of the two edge stiffeners is cal-
culated according to the method of an effective width of
sheet outlined in Appendix B, where σ is the yield point
stress in compression, assumed in this case to be 33,000
pounds per square inch. (See reference 6.) The assump-
tion that the effective width of sheet outside the rivet
lines of the two edge stiffeners carries the ultimate load
obtained by multiplying the area by the yield point stress
is consistent with the fundamental assumption of method A
that the sheet between stiffeners carries the ultimate
load when tested with the unloaded edges supported in V-
shaped grooves.

Examples Illustrating the Use of Method A as Applied

to Flat Sheet and Channel Stiffeners

Length of specimen, 6 inches; thickness of sheet, 0.019 in.

Number of stiffeners	2	3	4
Load carried by stiffeners (1,550 lb. each, references 1 and 2)	3,100	4,650	6,200
Load carried by sheet (Between adjacent stiffeners, 350 lb., fig. 9 of reference 4)	350	700	1,050
(Outside of rivet lines on edge stiffeners, calculated as outlined in Appendix B with σ = 33,000 lb. per sq.in.)	250	250	250
	3,700	5,600	7,500

Length of specimen, 18 in.; thickness of sheet, 0.052 in.

Number of stiffeners	2	3	4
Load carried by stiffeners, lb. (880 lb. each, references 1 and 2)	1,760	2,640	3,520
Load carried by sheet, lb. (Between adjacent stiffeners, 2,700 lb., fig. 9 of reference 4)	2,700	5,400	8,100
(Outside of rivet lines on edge stiffeners, calculated as outlined in Appendix B with σ = 33,000 lb. per sq.in.)	1,290	1,290	1,290
	5,750	9,330	12,910

APPENDIX B

In reference 3 von Karman derived an equation for the effective width of sheet which may be considered to act with the stiffener and carry the same stress as the stiffener. This equation was derived from consideration of the buckling of a flat plate simply supported at the edges and subjected to compressive forces on two opposite edges.

$$\frac{\sigma}{E} = \frac{4 \pi^2 t^2}{12 (1 - \mu^2) (2w)^2}$$

$$2w = \frac{2 \pi t}{\sqrt{12 (1 - \mu^2)}} \sqrt{\frac{E}{\sigma}}$$

or for $\mu = 0.3$

$$2w = 1.90 \sqrt{\frac{E}{\sigma}} t \qquad (3)$$

Consequently, if it be assumed that the effect of riveting the sheet to the stiffeners is such as to give simple support at the edges of the sheet, von Karman's equation for an effective width of sheet is only applicable for that portion of the sheet between adjacent stiffeners.

For that portion of the sheet which lies outside the rivet lines of the two edge stiffeners an equation similar to von Karman's may be derived by consideration of the buckling of a plate simply supported along three edges, free on the fourth edge, and subjected to compressive forces on the two opposite supported edges. (Equation 199, reference 7.)

$$\frac{\sigma}{E} = \frac{0.506 \pi^2 t^2}{12 (1 - \mu^2) w^2}$$

$$w = \frac{0.712 \pi t}{\sqrt{12 (1 - \mu^2)}} \sqrt{\frac{E}{\sigma}}$$

or for $\mu = 0.3$

$$w = 0.68 \sqrt{\frac{E}{\sigma}} t \qquad (4)$$

As the results of the tests of reference 4 indicate that the coefficient in equation (3) is not likely to exceed 1.70 (fig. 3, reference 3) this value will be used for sheet riveted to stiffeners and the coefficient in equation (4) reduced accordingly to (1.70/1.90) X 0.68, or 0.60. The reason for choosing the highest value of the coefficient obtained from tests is: (a) the coefficients plotted in Figure 3 of reference 3 are somewhat low because it was assumed that the yield point in compression was the same as the yield point in tension, whereas in reality it is somewhat lower (reference 6), and (b) the effect of riveting the sheet to a stiffener is such as to cause the coefficient to be increased over that for a plate with simply supported edges.

When equation (4) with a coefficient of 0.60 instead of 0.68 is applied to the test panels of Figure 2 and the width w exceeds 0.375 inch, the actual width of sheet outside the rivet lines, then a width of 0.375 inch should be used instead of the calculated width. A similar argument also applies to equation (3) with a coefficient of 1.70; if the width 2w exceeds the width between stiffeners, then the width between stiffeners should be considered as effective instead of the calculated width.

Examples Illustrating the Use of Method B as Applied

to Flat Sheet and Channel Stiffeners

Preliminary Calculations

Length of specimen, inches	6	18
Load carried by stiffener, lb. (references 1 and 2)	1,550	880
Area of stiffener, sq.in. (approximate)	0.0566	0.0566
Stress in stiffener, lb. per sq.in. (approximate)	27,400	15,500
Modulus of elasticity, lb. per sq.in. (assumed)	10.5×10^6	10.5×10^6
t, inch	.019	.052

Effective width of sheet, in.
  Between stiffeners, $2w = 1.70 \sqrt{\dfrac{E}{\sigma}}\, t$ .631   2.30
  Outside of edge stiffeners,

   $w = 0.60 \sqrt{\dfrac{E}{\sigma}}\, t$ .223   .375

  but not to exceed 0.375 in.

Final Calculations

Length of specimen, 6 inches; thickness of sheet, 0.019 in.

Number of stiffeners   2   3   4

Load carried by stiffeners, lb.
  (1,550 lb. each, references
  1 and 2)   3,100 4,650 6,200

Load carried by sheet
  (Between adjacent stiffeners,
  0.631 X 0.019 X 27,400 = 328)   330   660   990

  (Outside of rivet lines on edge
   stiffeners, 2X0.223X0.019X27400=
   232)   230   230   230
                                                ───── ───── ─────
                                                3,660 5,540 7,420

Length of specimen, 6 inches; thickness of sheet, 0.052 in.

Number of stiffeners   2   3   4

Load carried by stiffeners, lb.
  (880 lb. each, references
  1 and 2)   1,760 2,640 3,520

Load carried by sheet
  (Between adjacent stiffeners,
  2.30 X 0.052 X 15500 = 1857)   1,860 3,720 5,580

  (Outside of rivet lines on edge
   stiffeners, 2X0.375X0.052X15500=
   606)   610   610   610
                                                ───── ───── ─────
                                                4,230 6,970 9,710

APPENDIX C

Examples Illustrating the Use of Method C as Applied

to Flat Sheet and Channel Stiffeners

Before an attempt is made to calculate the compressive load for a flat sheet and stiffener combination by the use of method C, it is advantageous to have for ready reference a column curve for the material and also curves showing the variation of area and slenderness ratio of the combination of stiffener and effective width of sheet with the effective width of sheet. These curves for the channel stiffeners used in Newell's tests, together with curves for the moment of inertia and radius of gyration, are given in Figures 9 to 13, inclusive. The column curve for flat end specimens (fig. 12) has been constructed from the results of Newell's flat end tests on the channel stiffener. It is admitted that the extrapolation for values of l/ρ greater than 112.5, the largest value for which a test was made, is open to some question but is, undoubtedly, approximately correct up to values of $l/\rho = 140$ or 150.

It is appreciated that when the sheet is riveted to stiffeners the same column curve does not strictly apply; first, because with flat ends the end conditions are indefinite and subject to change during test, and second, because the form factor for the combination of stiffener and effective width of sheet differs from that for the stiffener alone. (It is assumed that failure in either case occurs by bending of the stiffener in a plane normal to the sheet.) However, for purposes of comparison it will be assumed that the effect of changes in end conditions and form factor are of no consequence.

Following is the procedure for calculating the load carried by one channel stiffener and the effective width of sheet.

1. Assume a stress at failure; lacking other information, assume the stress corresponding to the slenderness ratio of the stiffener.

2. Calculate the effective width of sheet in terms of the sheet thickness using equation (1).

3. With the effective width of sheet determined, obtain the slenderness ratio for the combination of stiffener and effective width of sheet from Figure 11.

4. With the slenderness ratio determined, obtain the stress at failure from Figure 12.

5. If the stress at failure thus determined agrees with the assumed stress at failure, multiply this stress by the area of the combination of stiffener and effective width of sheet as determined from Figure 13 to obtain the load carried by the combination. If the stress at failure thus determined does not agree with the assumed stress at failure, then assume a new stress and repeat the calculation.

In Table II the procedure outlined above is employed for calculating the load carried by the 6-inch panels with 0.019-inch sheet and the 18-inch panels with 0.052-inch sheet.

It will be noted by inspection of Table II that the calculated loads for the stiffener and effective width of sheet do not change appreciably after the second trial. Consequently, it is unnecessary in any practical case to carry the calculations to the degree of refinement indicated.

Upon addition of the loads calculated for the end and intermediate stiffeners, the loads carried by the 6-inch specimen with 0.019-inch sheet are:

Two stiffeners   2 × 1,810   = 3,620 lb.

Three stiffeners   (2×1,810) + 1,860   = 5,480 "

Four stiffeners   (2×1,810) + (2×1,860) = 7,340 "

and for the 18-inch specimens with the 0.052-inch sheet

Two stiffeners   2 × 1,650   = 3,300 lb.

Three stiffeners   (2 × 1,650) + 1,790   = 5,090 "

Four stiffeners   (2×1,650) + (2×1,790) = 6,880 "

In view of the fact that the panels shown in Figure 2 were loaded in a testing machine where deformation is presumed to be uniform on all stiffeners, the assumption that the edge and intermediate stiffeners carry the maximum loads calculated in Table II regardless of the wide differences in stress may be slightly in error. However, the error is small in any case because the calculated loads do not change appreciably with stress.

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TABLE I

Observed and Predicted Compressive Loads on Panels of Aluminum Alloy Flat Sheet and Channel Stiffeners

Length of specimen (in.)	No. of stiffeners	(Load (lb.)) Thickness of sheet															
		0.019 (in.)					0.033 (in.)					0.052 (in.)				
		Predicted			Observed in test		Predicted			Observed in test		Predicted			Observed in test	
		A	B	C			A	B	C			A	B	C		
6	2	3700	3660	3620	3300	3100	5070	4770	4760	4400	4600	7090	6630	6500	7330	6950
	3	5600	5540	5480	5000	5300	7820	7310	7260	6300	7020	11340	10640	10440	11200	11150
	4	7500	7420	7340	6500	7100	10570	9850	9760	9080	9500	15590	14650	14380	14000	15000
12	2	3200	3110	3060	2890	2960	4570	4070	3920	4190	4030	6590	5760	5340	5350	5320
	3	4850	4710	4630	4500	3900	7070	6270	6030	6110	6025	10590	9320	8510	9720	9950
	4	6500	6310	6200	6390	6470	9570	8470	8140	8450	7570	14590	12880	11680	12500	13200
18	2	2360	2190	2120	2450	2300	3730	2890	2640	2200	2700	5750	4230	3300	3350	2920
	3	3590	3320	3210	3280	3270	5810	4520	4050	4300	3830	9330	6970	5090	4650	4890
	4	4820	4450	4300	4700	4200	7890	6150	5460	5900	5300	12910	9710	6880	6580	6660

TABLE II

Calculation of Load Carried by the Combination of Stiffener and Effective Width of Sheet According to Method C

(See Appendix C)

Length of specimen (in.)	Stiffener	Thickness of sheet (in.)	Assumed stress at failure (lb. per sq.in.)	Effective width of sheet (in.)	l/ρ	Stress at failure (lb. per sq.in.)	Area of stiffener and effective width of sheet (sq.in.)	Load carried by stiffener and effective width of sheet (lb.)	Remarks
6	edge	0.019	27,400	[a]28.4t	38	27,400	0.066	1,810	First trial
6	intermediate	.019	27,400	[b]33.3t	38	27,400	.068	1,860	First trial
18	edge	.052	15,500	[c]29.4t	131.5	11,900	.136	1,620	First trial
			11,900	[c]32.5t	134.5	11,400	.144	1,640	Second trial
			11,400	[c]33.0t	135.0	11,300	.146	1,650	Third trial
18	intermediate	.052	15,500	[b]44.3t	145.0	9,800	.176	1,720	First trial
			9,800	[b]55.7t	155.0	8,500	.207	1,760	Second trial
			8,500	[b]59.7t	158.0	8,200	.218	1,790	Third trial
			8,200	[b]60.9t	159.0	8,100	.221	1,790	Fourth trial

Effective width of sheet

[a] $\left[0.60 + \dfrac{1.70}{2}\right]\sqrt{\dfrac{E}{\sigma}}\,t = 1.45\sqrt{\dfrac{E}{\sigma}}\,t$

[b] $1.70\sqrt{\dfrac{E}{\sigma}}\,t$

[c] $0.375 + \dfrac{1.70}{2}\sqrt{\dfrac{E}{\sigma}}\,t = \left[7.2 + 0.85\sqrt{\dfrac{E}{\sigma}}\right]t$
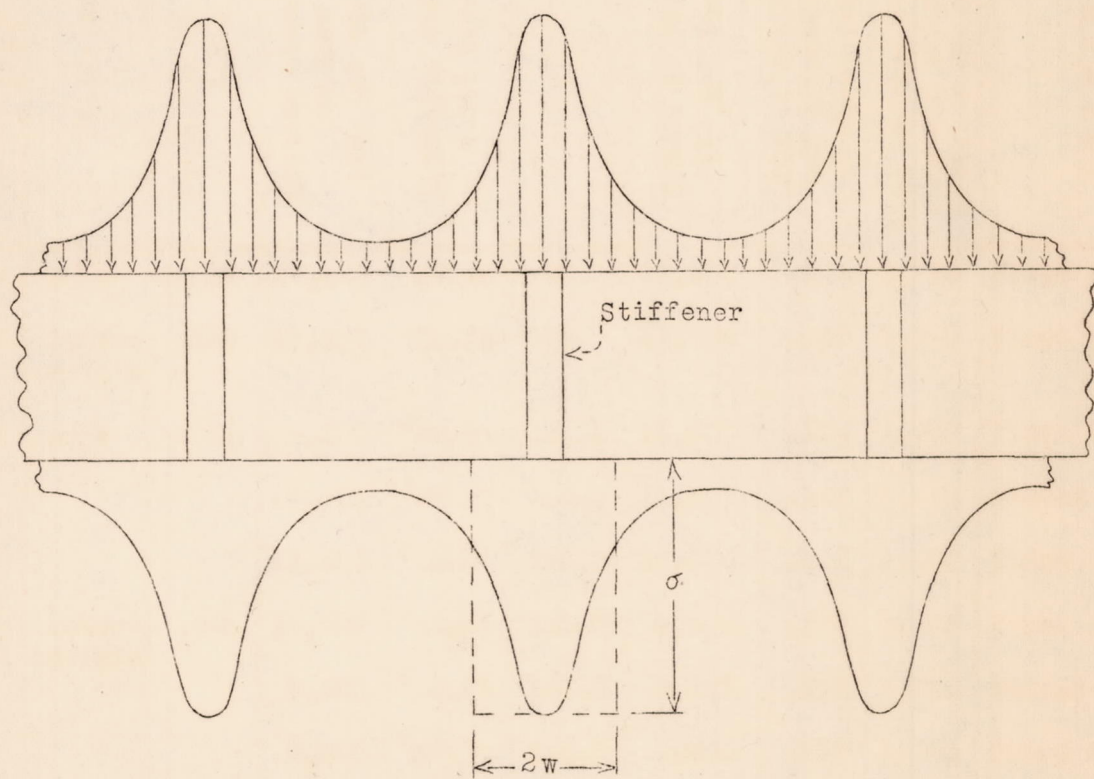
Figure 1.-Stress distribution in sheet after buckling has occurred be-
tween stiffeners.

20

21

12"

6"~12" or 18"

$\frac{3}{32}$ " dia. rivets, $\frac{3}{4}$ " pitch

A   A

$\frac{1}{2}$"

0.035"

$\frac{3}{4}$"

Section A-A

Curved panel
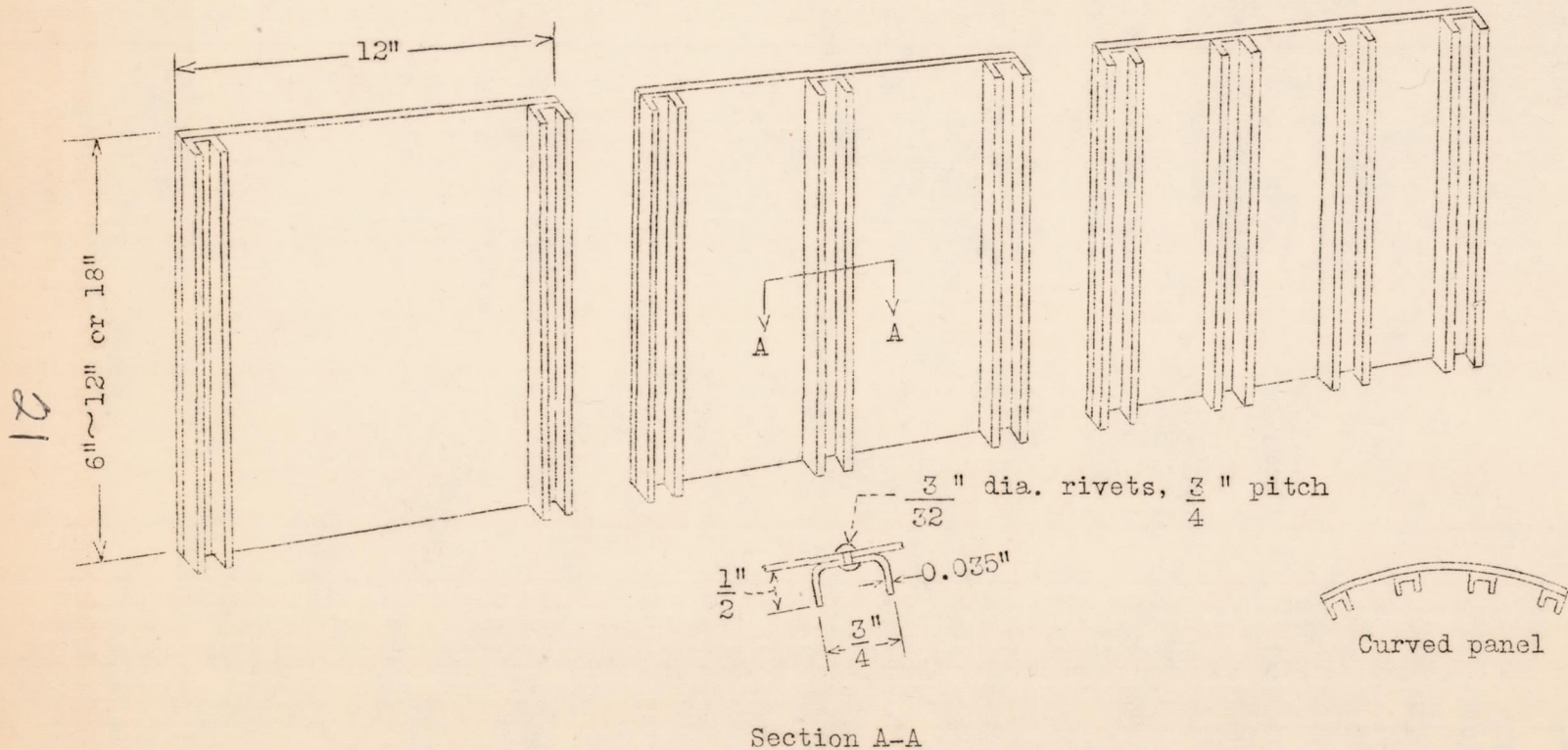
Figure 2.-Panels of aluminum-alloy flat sheet and channel stiffeners tested in compression with flat
   ends. (Reference 2)

Fig.2
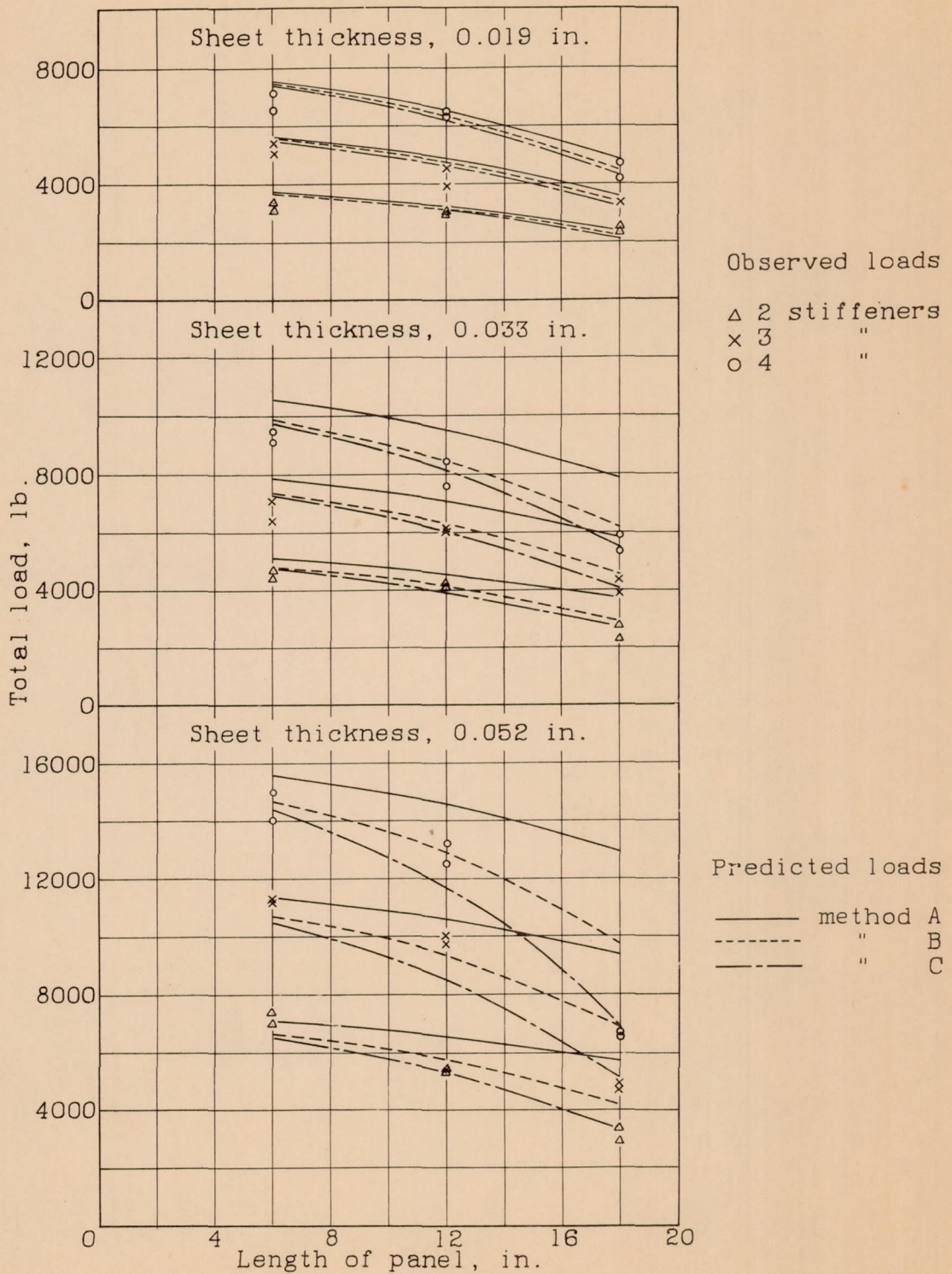
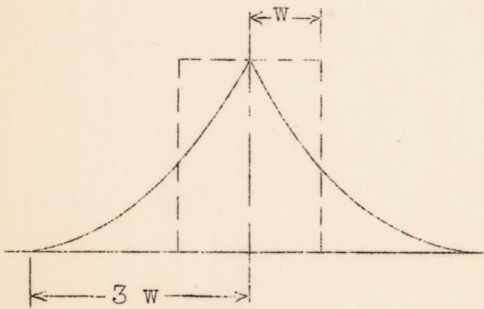Figure 3.-Comparison of observed and predicted compressive
loads for the flat panels of Figure 2
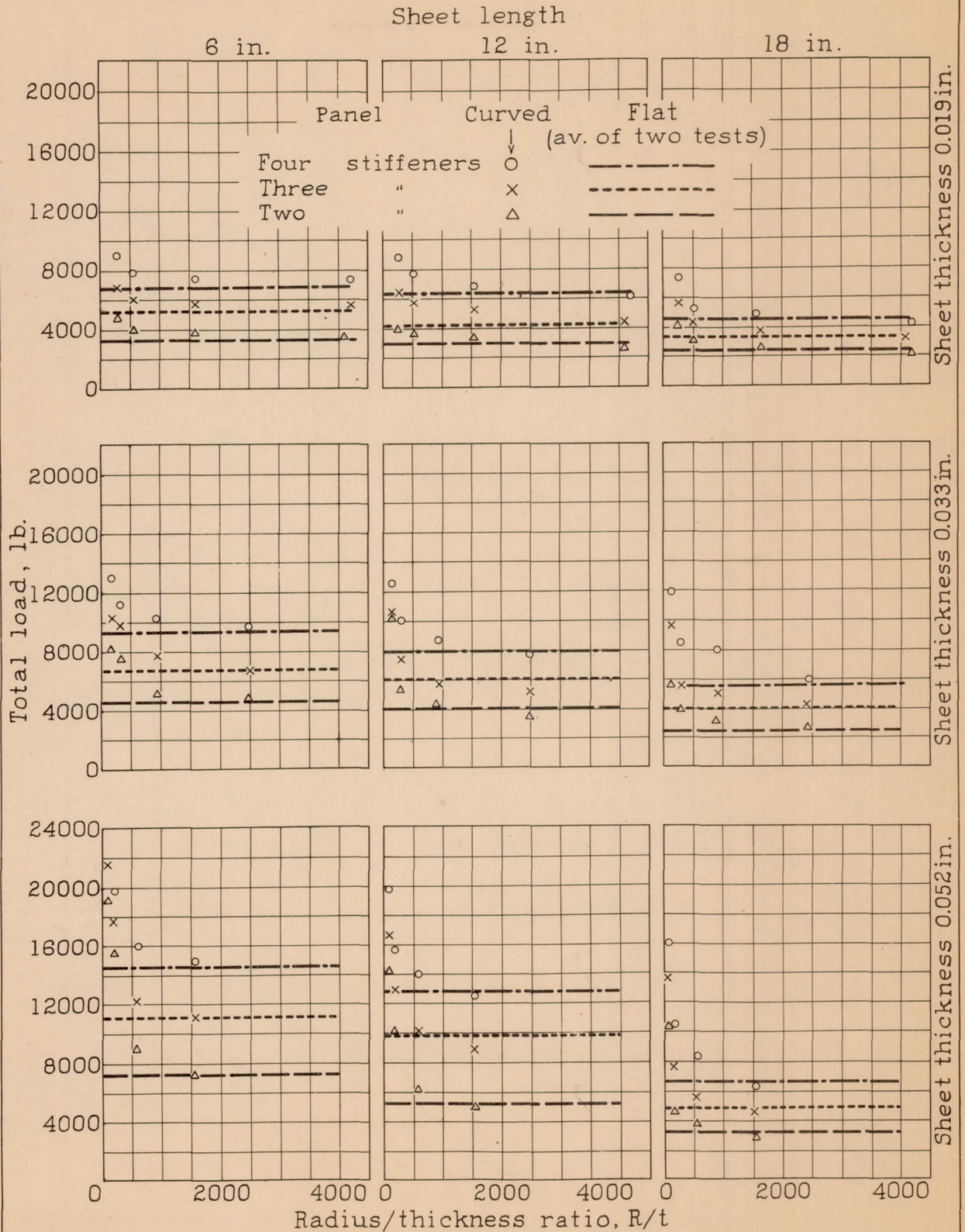
Figure 3.– Comparison of compressive strength of curved and flat panels of the type shown in Figure 2
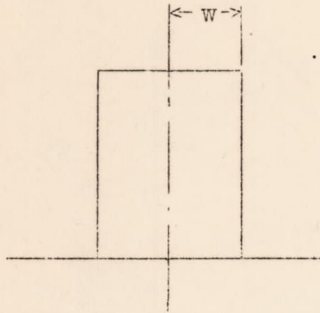
Case I
Rectangular
$k_1 = 0.0445$
$k_2 = 0.167$
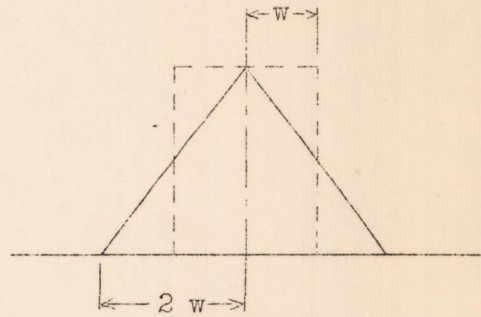
Case II
Triangular
$k_1 = 0.311$
$k_2 = 0.333$

Case III
Parabolic
$k_1 = 0.752$
$k_2 = 0.450$
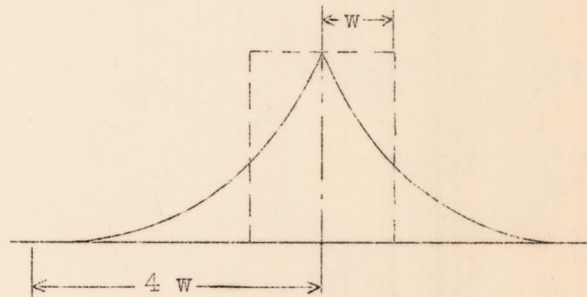
Case IV
Cubic
$k_1 = 1.260$
$k_2 = 0.533$

Figure 5.--Values of k_1 and k_2 for different assumed distributions of
the effective material in the sheet.

24

A

B
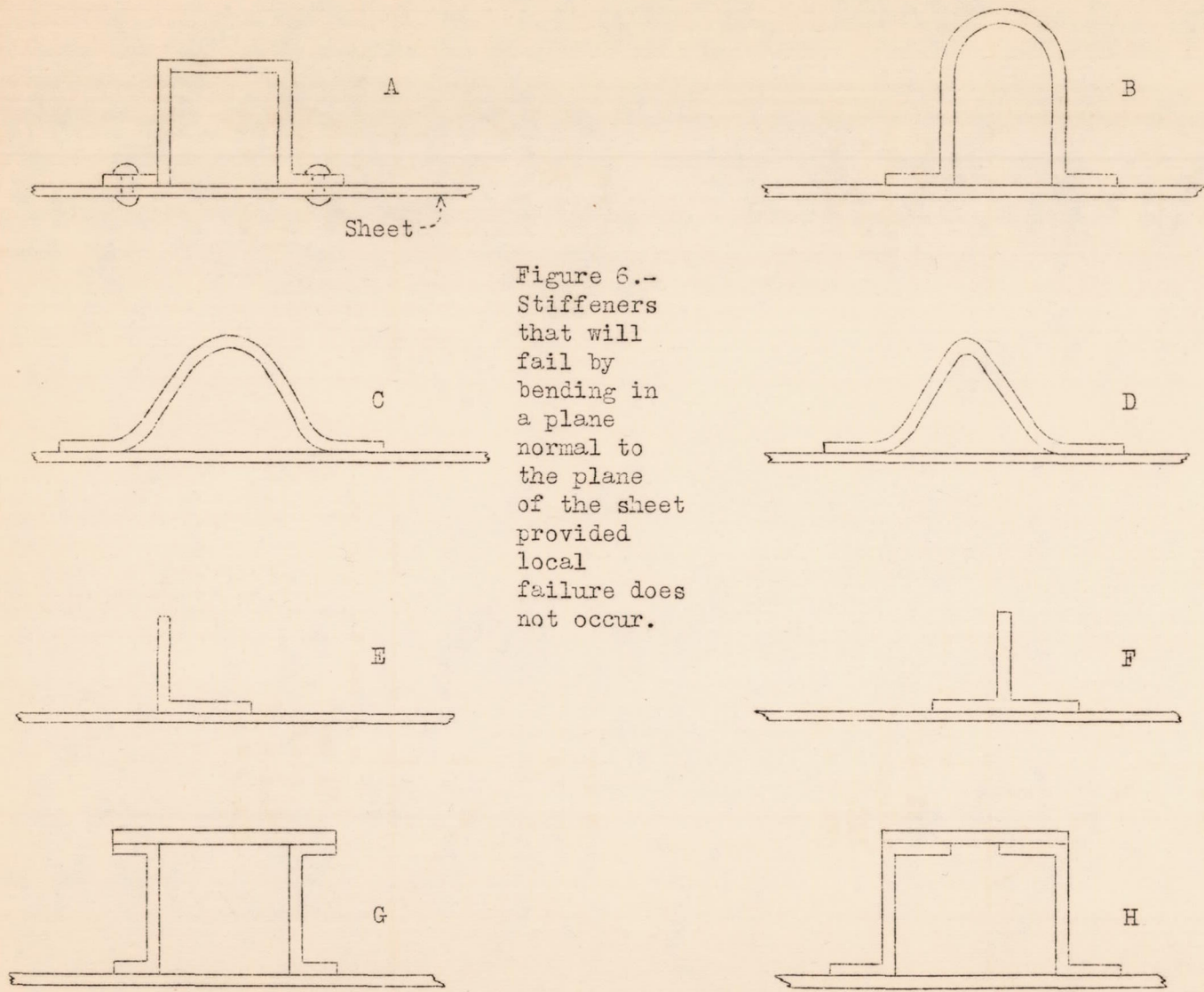
Sheet

Figure 6.-
Stiffeners
that will
fail by
bending in
a plane
normal to
the plane
of the sheet
provided
local
failure does
not occur.

C

D

E

F

G

H

Fig.6

A

Sheet

B

26

C

D
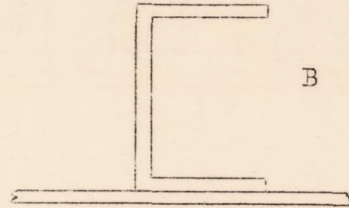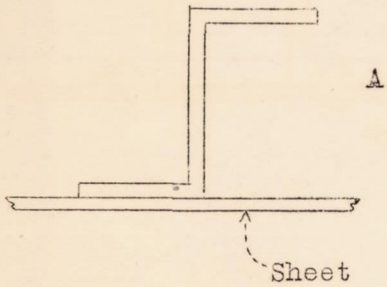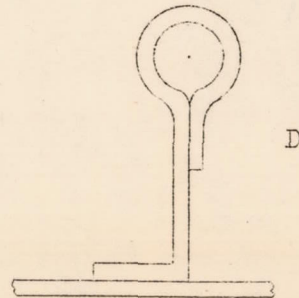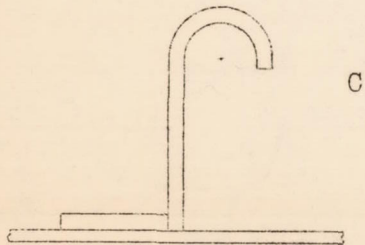
Figure 7.-Stiffeners that may fail by bending of the outstanding part in a plane parallel to the plane
of the sheet.

Fig. 7
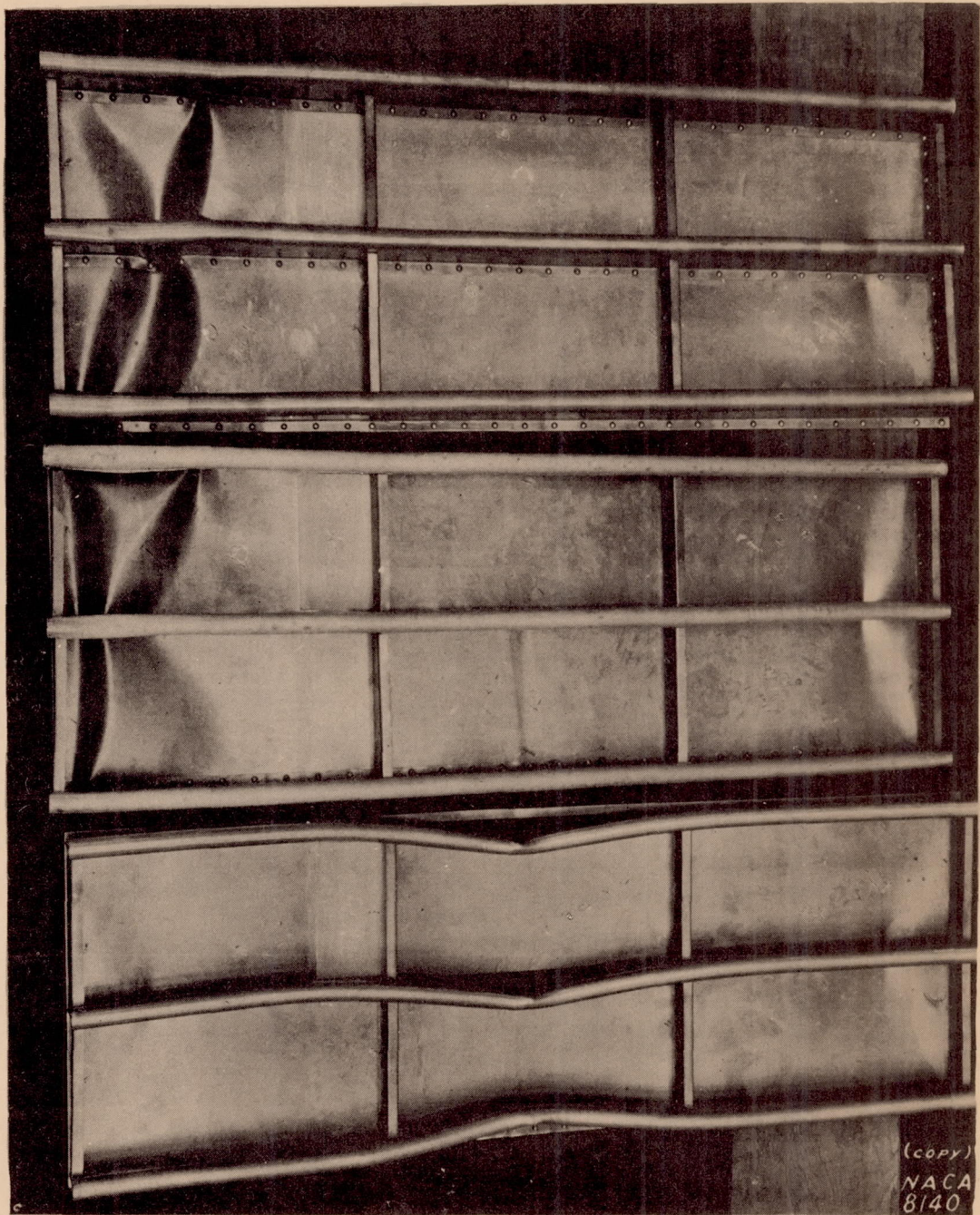
Figure 8.-
Photograph showing types of failure peculiar to stiffeners C and D of
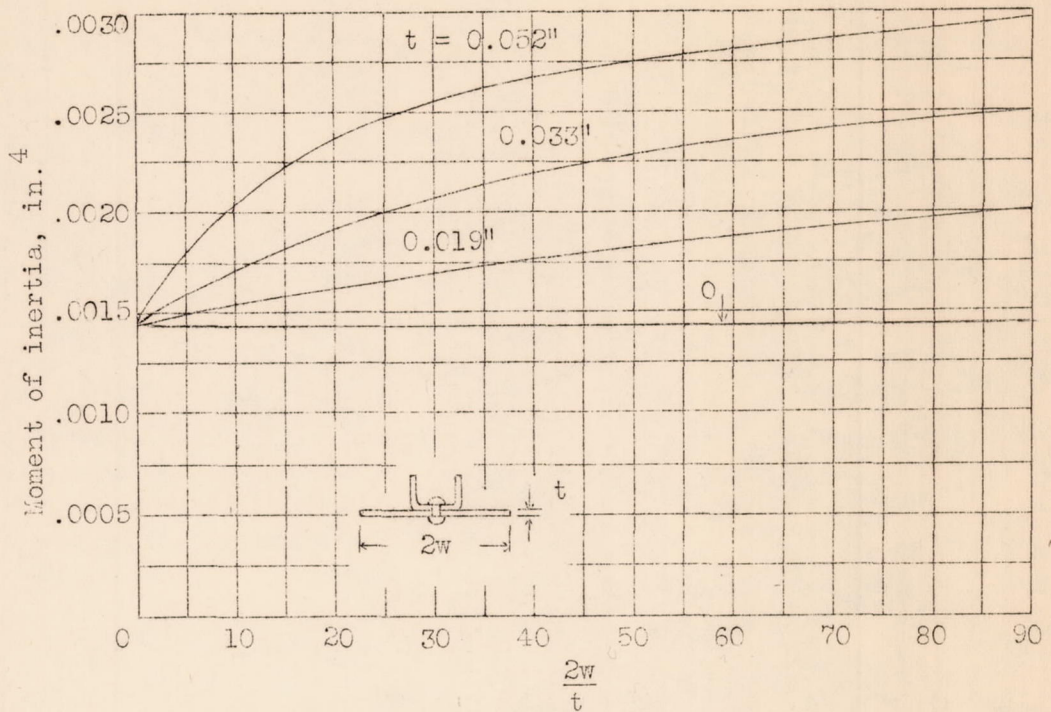Figure 7. (Courtesy of Navy Department, Bureau of Aeronautics.)

27

Figure 9.--Moment of inertia of combination of channel stiffener and
effective width of sheet about the centroidal axis of the
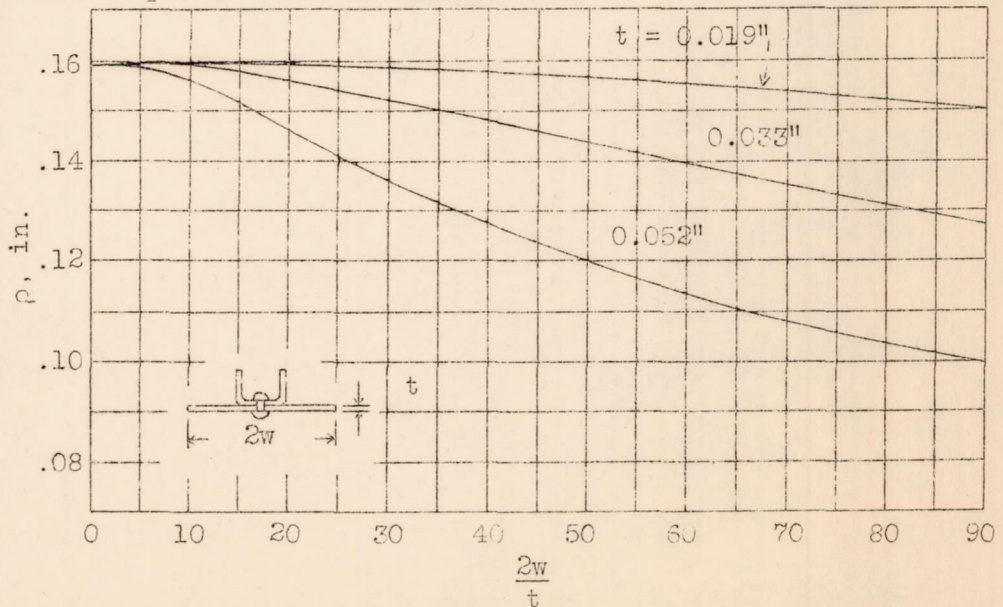combination parallel to the sheet.

Figure 10.--Radius of gyration of combination of channel stiffener
and effective width of sheet about the centroidal axis
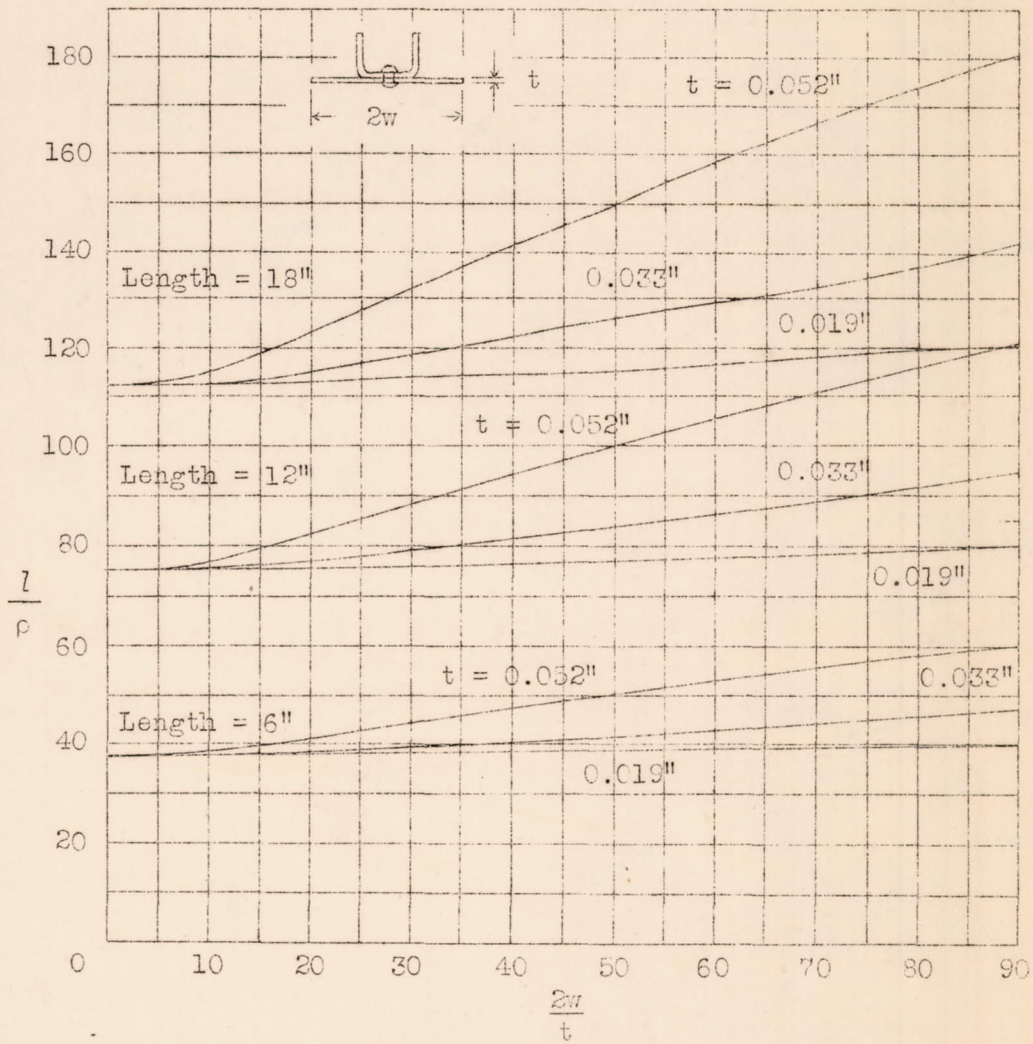of the combination parallel to the sheet.

28

Figure 11.-Slenderness ratio for combination of channel stiffener
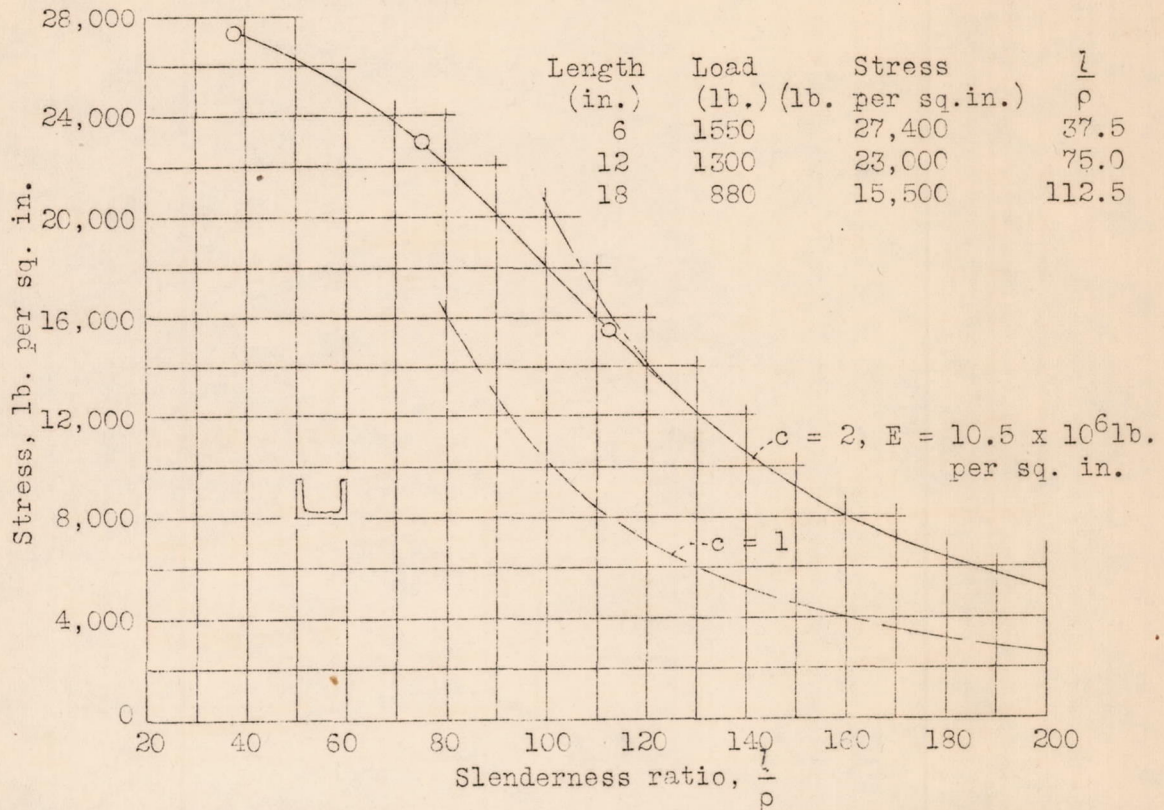and effective width of sheet.

29

Length (in.)	Load (lb.)	Stress (lb. per sq.in.)	$\frac{l}{\rho}$
6	1550	27,400	37.5
12	1300	23,000	75.0
18	880	15,500	112.5

$c = 2$, $E = 10.5 \times 10^6$ lb. per sq. in.

$c = 1$

Stress, lb. per sq. in.

Slenderness ratio, $\frac{l}{\rho}$

Figure 12.-Column curve for aluminum-alloy channel stiffener tested with flat ends.

$t = 0.052"$

$t = 0.033"$
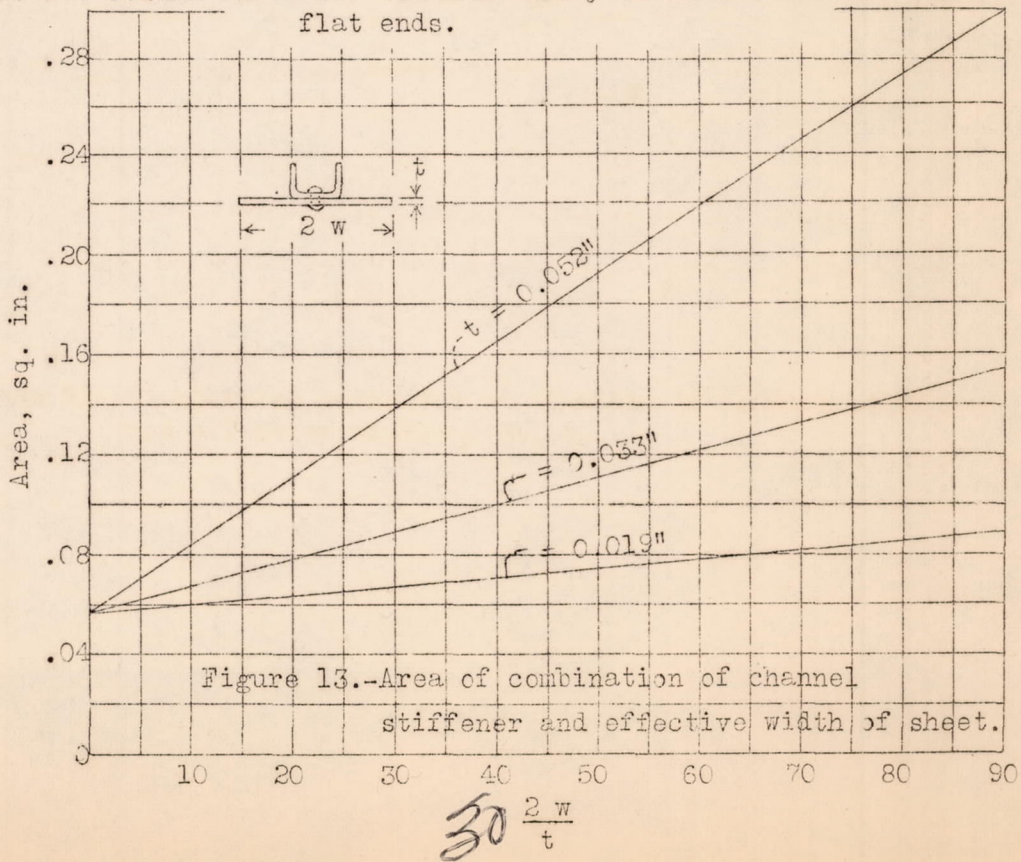
$t = 0.019"$

Area, sq. in.

Figure 13.-Area of combination of channel stiffener and effective width of sheet.

$\frac{2\,w}{t}$