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A METHOD OF CALCULATING THE PERFORMANCE OF CONTROLLABLE PROPELLERS WITH SAMPLE COMPUTATIONS

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[redacted]

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A METHOD OF CALCULATING THE PERFORMANCE ~~OF CONTROLLABLE~~ PROPELLERS, WITH SAMPLE COMPUTATIO[redacted].

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SUMMARY

This paper contains a series of calculations showing how the performance of controllable propellers may be derived from data on fixed-pitch propellers given in N.A.C.A. Technical Report No. 350, or from similar data.

Sample calculations are given which compare the performance of airplanes with fixed-pitch and with controllable propellers. The gain in performance with controllable propellers is shown to be largely due to the increased power available, rather than to an increase in efficiency. Controllable propellers are of particular advantage when used with geared and with supercharged engines.

A controllable propeller reduces the take-off run, increases the rate of climb and the ceiling, but does not increase the high speed, except when operating above the design altitude of the previously used fixed-pitch propeller or when that propeller was designed for other than high speed.

INTRODUCTION

The rapid refinement of airplanes and engines has been accompanied by an increasing demand for a more flexible type of propeller, particularly because the resulting higher airplane speeds have necessitated the use of high pitch settings with a resultant sacrifice in take-off and climbing performance.

The reduced low-speed performance of a fixed high-pitch propeller operating on a high-speed airplane is due principally to two causes: (1) The drop in engine speed and power between the high speed and standing condition is greater for high-speed airplanes than for low-speed airplanes; (2) the blades are stalled during the beginning of the take-off run resulting in a severe loss of take-off thrust.

These difficulties may be overcome by the use of a controllable propeller which may be adjusted in flight by the pilot to a suitable pitch setting. The same purpose may also be accomplished by the use of an automatic propeller whose blades accommodate themselves to the most favorable pitch settings for the various conditions of operation.

The controllable propeller has only recently been developed into a practicable form. Stimulated by the growing need for such a propeller, several manufacturers have now produced controllable propellers of sufficiently satisfactory design to be acceptable to conservative airline operators and airplane manufacturers. The increased interest in their performance characteristics has resulted in numerous requests for test data on them.

Since the controllable propeller is merely the equivalent of a series of fixed-pitch propellers of different pitches, it should be clear that its performance may be calculated from propeller data already available. It is the purpose of this report to show how such calculations may be made, and by the use of several examples give a quantitative indication of the benefit which may be derived from the use of controllable propellers on airplanes of various types.

PERFORMANCE CALCULATIONS

The propeller data used in this report were taken from the results of wind-tunnel tests of a full-size propeller operating in conjunction with several engine-fuselage combinations. These data, given in reference 1, constitute the most extensive and reliable information available on this subject. A set of propeller curves taken from this reference is reproduced in figure 1. In this figure, propulsive efficiency η and V/nD are plotted against C_s with blade angle β as the parameter. The nondimensional speed-power coefficient C_s is defined by $C_s = \sqrt[5]{\frac{\rho V^5}{P n^2}}$ in which V is air speed, P is input power,

n is propeller revolution speed per unit time, and ρ is the mass density of the air.

The C_s charts are particularly useful in selecting a propeller for a given airplane. If the design engine revolution speed, engine power, and air speed of the airplane are known, or assumed, one may calculate C_s and subsequently from the charts obtain V/nD, blade angle, and efficiency for the design condition. The diameter may then be calculated from the V/nD thus obtained. The details of these calculations are fully explained in reference 1, and need not be recounted here.

Propeller performance as used in this report means the ability of a propeller to convert the full-rated power of the engine into thrust power at all flight velocities. It is represented by a curve of full-throttle thrust horsepower available ($t.hp._a$) against air speed and is the most practical basis upon which propellers may be compared.

An additional curve of thrust horsepower required ($t.hp._r$) against air speed is necessary for comparing the performances of an airplane equipped with various types of propellers. This curve is calculated from design and performance data by a method described later.

Thrust Horsepower Available

Fixed-pitch propeller.- One difficulty in calculating the performance of a fixed-pitch propeller is caused by the variation of propeller revolution speed with air speed. Since both V/nD and C_s involve the factor n, an indirect method of some length is usually required for such calculations.

It is convenient in making these computations to have a table of C_s values for air speeds and revolution speeds likely to be encountered by the propeller. Before such a table can be made, however, it is necessary to have a full-throttle power curve for the engine, obtained either by test or by empirical methods. In the illustrative examples given in this report the power curves for the unsupercharged engines were computed from the relation $\dfrac{b.hp.}{(b.hp.)_o} \cong \dfrac{r.p.m.}{(r.p.m.)_o}$ where $(b.hp.)_o$ and $(r.p.m.)_o$ are the rated power and speed of the engine. This relation is fairly accurate throughout the usual flight range of engine speeds for unsupercharged engines but may be considerably in error if used for supercharged engines. Actual test results are necessary for problems involving supercharged engines.

With this information available the values of C_s mentioned above may be calculated and may be conveniently put in the form of table I.

Air speeds are selected at small intervals throughout the flight range and for each air speed selected three values of engine speed $(r.p.m.)_1$ are chosen in the range where the actual engine speed is likely to be. A value of C_s is found in the C_s table for each value of $(r.p.m.)_1$ and, since the blade angle is known, the V/nD for each value of $(r.p.m.)_1$ may be found from the charts in reference 1. With this value of V/nD and known values of V and D a second value of $(r.p.m.)_2$ may be calculated for each chosen $(r.p.m.)_1$. If $(r.p.m.)_1$ and $(r.p.m.)_2$ are plotted against C_s as in figure 2, the intersection of the two curves determines the actual value of r.p.m. and C_s for this particular air speed. With a little practice a quick mental calculation will suffice to determine the common point of the two curves and plotting becomes unnecessary.

Now that the correct value of r.p.m. is known, the corresponding $b.hp._a$ may be easily found, and also since the correct C_s and blade angle are known the efficiency may be found from the charts and $t.hp._a$ calculated.

Controllable propeller.- In general, the optimum performance of a controllable propeller is obtained when the pitch is controlled so as to permit the engine to develop maximum permissible engine revolution speed and power at all flight velocities. For the examples given in this report, the permissible limits of engine speed and power will be taken as the rated speed and power of the engine. The fact that the engine revolution speed and the power are held constant at all air speeds greatly simplifies performance calculation for the controllable propeller since C_s may be quickly and directly calculated for each air speed. With the diameter of the fixed-pitch propeller as a basis for selection, the diameter for the controllable propeller may be chosen, having due respect for high tip speeds.

Since the air speed, revolution speed, and diameter are known, the V/nD ratio for each air speed may be calculated. On the propeller charts in reference 1, the known values of V/nD and C_s determine β and η. The thrust horsepower available is then the product of the efficiency and the rated horsepower of the engine.

This method of propeller-performance calculation applies to the automatic propellers only, where the blade angle is continuously adjusted to maintain constant engine speed. When the propeller installation provides for only two or three blade-angle settings within the flight range and is manually controlled, the performance may be calculated as for a fixed-pitch propeller in the air-speed intervals between blade-angle changes.

Hypothetical controllable-pitch-and-diameter (C.P.& D.) propeller.- Theory and experiment have shown that the propeller diameter that is best for one particular air speed is not the best for all air speeds. For optimum performance, then, not only the blade angle but the diameter must be changed for each air speed.

It is interesting for comparative purposes to calculate the performance of a hypothetical propeller whose blade angle and diameter may be set at their ideal values for each condition of flight. Such a propeller has a performance that cannot be exceeded by any other propeller of similar blade form.

The performance of a C.P. & D. propeller was calculated for each of the example airplanes in this report. Since such a propeller is purely hypothetical in nature, the high-tip-speed losses, which would inevitably occur at low air speeds where the diameter is large, were intentionally neglected. In actual propeller applications the effect of high tip speeds must not be overlooked. Information regarding this subject is found in reference 2.

The values of C_s for the C.P.& D. propeller are the same as for the controllable propeller, but the values of V/nD and blade angle for each speed are found from the propeller charts at the intersection of the C_s ordinate and the broken line designated "maximum efficiency for C_s." Values of V/nD and blade angles on this line give efficiencies falling on the envelope of the efficiency curves. The efficiency and thrust horsepower available are found as for the controllable propeller. The best diameter for each velocity is determined from the known values of V/nD, V, and n by simple substitution.

Thrust Horsepower Required

The thrust horsepower required by an airplane may be calculated from the equation

$$t.hp._r = \frac{\rho f V^3}{1100} + \frac{2\ W^2}{\pi\ \rho\ b_e^2\ 550\ V}$$

in which the first term is the parasite power required and the second is the induced power required. The terms in this equation have the following significance:

W, weight in lb.

V, air speed in ft./sec.

f, a parasite area, in sq.ft., defined by $f = D_p/q$

D_p, parasite drag in lb.

q, dynamic pressure, lb./sq.ft.

ρ, air density in slugs per cu.ft.

b_e^2, an effective span squared equal to $e(kb)^2$

b, span in feet

k, Munk's span factor

e is a term described in reference 3, page 20, as an airplane efficiency factor the value of which usually lies between 0.75 and 1.0, depending upon the cleanness and other characteristics of the airplane. Its purpose is to make allowance for differences in actual conditions from ideal, such as nonelliptical span loading and variable parasite drag coefficient.

The parasite area f must not be confused with the commonly used "equivalent flat plate area" which is equal to $f/1.28$. The equivalent parasite area f may be determined by a summation of the drags of the component parts of the airplane plus an allowance for interference, but if the high speed of the airplane is known it is much easier and more accurate to solve the $t.hp._r$ equation for f as follows:

$$f = \frac{\left(b.hp._o \times \eta_{max} - \dfrac{2\,W^2}{\pi\,\rho\,b_e^2\,V_{max}\,550}\right)1100}{\rho\,V_{max}^3}$$

With this value of f the $t.hp._r$ equation may be rewritten

$$t.hp._r = K\,V^3 + K_1/V$$

Table II gives the results of the solution of this equation throughout the speed range for the first example airplane.

Airplane Performance Characteristics

The high speed and climb of the example airplanes were determined from the curves of $t.hp._r$ and $t.hp._a$ in the usual way, which will not be described here.

The take-off runs were calculated using Diehl's take-off run equation, the development of which is given in reference 4. This equation for still air is

$$S = \frac{K_s\,V_s^2}{\dfrac{T_1}{W}} \quad \text{where} \quad \frac{T_1}{W} = \frac{T_o}{W} - \mu$$

and S, the take-off run in feet

μ, coefficient of friction between wheels and ground

W, gross weight in pounds

T_o, static thrust of propeller

V_s, take-off speed in miles per hour

K_s, a factor the value of which may be determined from figure 3 where K_s is plotted against

$$\frac{T_F}{T_1} = \left(\frac{\frac{T_V}{W} - \frac{D}{L}}{\frac{T_0}{W} - \mu}\right)$$

in which T_V is the thrust at take-off speed V_s obtained by multiplying the thrust horsepower at V_s by $375/V_s$; D/L is the reciprocal of $(L/D)_{max}$ and may be calculated as follows.

Determination of $(L/D)_{max}$

The maximum value of the ratio L/D may quite easily be found from the $t.hp._r$ equation, which may be written

$$t.hp._r = \frac{D\ V}{375} = K\ V^3 + \frac{K_1}{V}, \quad \text{from which}$$

$$D = 375\ K\ V^2 + 375\ \frac{K_1}{V^2}$$

where V is in miles per hour. In order to find the minimum drag the first derivative of D with respect to V must be equated to zero.

$$\frac{dD}{dV} = 2 \times 375\ KV - (2 \times 375 \times K_1)/V^3 = 0, \quad \text{so that}$$

$$2 \times 375\ KV = (2 \times 375\ K_1)/V^3$$

and $K\ V^3 = K_1/V$, which shows that the air speed for D_{min} and therefore $(L/D)_{max}$ is the air speed at which the induced power required is equal to the parasite power required.

Then at $(L/D)_{max}$

$$V = \sqrt[4]{\frac{K_1}{K}}$$

$$t.hp._r = 2 \times K_1/V$$

$$\text{drag} = (\text{t.hp.}_r \times 375)/V$$

$$\text{lift} \cong \text{weight}$$

$$(L/D)_{max} = \text{weight}/\text{drag}$$

Static Thrust

The static thrust of propellers similar in plan form to the one used in reference 1 may be found from figure 4, where $\frac{T_o D}{Q}$ is plotted against blade angle β. The engine torque Q may for unsupercharged engines be taken as the full-throttle torque of the engine calculated from its rated power and speed. For supercharged engines Q should be taken as the actual engine torque at the beginning of the take-off run. The curve in figure 4 is a mean of the $\frac{T_o D}{Q}$ curves for the propeller-fuselage arrangement shown in reference 1 and may be used with fair accuracy for any of them. More complete data with regard to static thrust of propellers may be found in reference 5.

The blade angles of the controllable propellers at take-off were the ones that permitted the engine to turn at full rated speed. The blade angles and diameters of the C.P.& D. propellers for take-off were those that gave the greatest static thrust with the engine operating at rated power and speed.

EXAMPLES

Table III is a summary of pertinent data regarding the four airplanes used as examples in this report. The characteristics $(L/D)_{max}$ and f were calculated by the methods previously given. The airplane efficiency factor e was chosen for each airplane roughly in proportion to its aerodynamic cleanness, except for airplane no. 2 where previous tests had shown this coefficient to be approximately 1.0. The remaining characteristics were taken from published data.

Airplane no. 1 is a low-winged 7-passenger transport with retractable landing gear; no. 2 is a gull-winged monoplane with a well-streamlined water-cooled engine installation; no. 3 is a 14-passenger trimotored monoplane transport; and no. 4 is a pursuit-type single-place biplane with a supercharged air-cooled engine. The engine, which has a critical altitude of 6,000 feet, has a manifold pressure regulator for sea-level operation.

Performance Calculation for Airplane No. 1

Since airplane no. 1 is a high-speed transport, the propeller will be selected for its high-speed qualities.

$$C_s \text{ for high speed at sea level} = \frac{0.638 \times V}{(\text{b.hp.})^{1/5} \times \text{r.p.m.}^{2/5}}$$

$$\text{and for this example} = \frac{0.638 \times 211}{(525)^{1/5} \times (1900)^{2/5}} = 1.88.$$

In figure 1, the maximum efficiency obtainable with this C_s is 0.865 with a blade angle of 28° and a V/nD of 1.088. The best diameter for the high-speed condition is then $\dfrac{211 \times 88}{1900 \times 1.088} = 9.00$ feet.

This airplane being a monoplane, Munk's span factor will be 1 and since it is a clean airplane e will be taken as 0.9. The equivalent parasite area will be

$$f = \frac{\left(525 \times 0.865 - \dfrac{2 \quad (5200)^2}{\pi\rho(42.8)^2 \times 0.9 \times 211 \times 1.467 \times 550}\right) 1100}{\rho \quad (211 \times 1.467)^3} =$$

6.74 square feet.

With this value of f the t.hp._r equation may be written $\text{t.hp.}_r = K\,V^3 + K_1/V = 0.0000458V^3 + 4920/V$ at sea level; V is in miles per hour. In table II are given the results of the solution of this equation for values of the air speed in the flight range.

The performance of the fixed-pitch propeller was calculated by the method previously given. Table IV and figure 2, which indicate the procedure, are self-explanatory. For this airplane the performance of a 9-foot and a 10-foot controllable propeller and a C.P.& D. propeller was computed. The results of all the calculations made up to this point are listed in table V and are plotted in figures 5 and 6.

The high speed of the airplane is obtained from the intersection of the $t.hp._r$ and the $t.hp._a$ curves, and the rate of climb is calculated from the excess t.hp. in the usual way. Figure 7 is a plot of the rate of climb at various air speeds for this airplane.

The static thrust of the propeller and the $(L/D)_{max}$ of the airplane must be known before take-off calculations can be made. Table V gives the results of static-thrust calculations for the various propellers of airplane no. 1, and is sufficiently clear to need no further explanation.

At $(L/D)_{max}$ the velocity $= \sqrt[4]{\frac{K_1}{K}}$

$$= \sqrt[4]{\frac{4920}{0.0000458}} = 101.5 \text{ miles per hour}$$

$$t.hp._r = 2 \times 4920/101.5 = 97$$

$$\text{lift} \cong \text{weight} = 5{,}200 \text{ pounds}$$

$$\text{drag} = 97 \times 375/101.5 = 358 \text{ pounds}$$

$(L/D)_{max} = 5200/358 = 14.5$ with the landing gear retracted.

The drag of a retractable landing gear when extended is usually quite high, so that if the drag at $(L/D)_{max}$ is assumed to be increased by a third when the gear is down the maximum L/D ratio will be approximately 11.

The take-off runs with the various propellers may now be calculated. For the fixed-pitch propeller,

T_V = 1,151 pounds

μ is assumed in all examples to be 0.05

V_s = 75 m.p.h. from table III

T_o = 920 pounds, from table IV

$$D/L = \frac{1}{11} = 0.091$$

$$T_1/W = 0.127$$

$$T_F/T_1 = 1.02$$

$$K_s = 0.033, \quad \text{from figure 3}$$

The take-off run for the fixed-pitch propeller is then

$$S = \frac{0.033 \times \overline{75}^2}{0.127} = 1,455 \text{ feet}$$

The take-off runs with the controllable propellers were also calculated by Diehl's method, although for such propellers the method is less rigorously correct. The results of these calculations are given in table VII.

Table VIII is a summary of the performance characteristics of airplane no. 1 when equipped with various types of propellers.

Airplanes Nos. 2, 3, and 4

For the sake of brevity the detailed calculations for airplanes 2, 3, and 4 will not be included in this report. However, the results of the calculations are found in figures 8 to 14 and tables IX to XIII. The performance of airplane no. 4 was computed for three altitudes: sea level, 6,000 feet (critical altitude for the engine), and 20,000 feet. An individual set of performance curves and a summary for each altitude are given in figures 11, 12, and 13, and tables XI, XII, and XIII. The rates of climb are plotted against altitude in figure 14. The intersection of a straight line passing through the rate-of-climb points, with the altitude axis, determines approximately the ceiling of the airplane.

The time-to-climb curves, which are also plotted in figure 14, were calculated from the following equation

$$T = \frac{H}{C_0}\left[\log_e \frac{1}{1 - \frac{h}{H}}\right]$$

where T is the time to climb to altitude h, H the ceiling, and C_0 the initial rate of climb.

Gearing

The effect of gearing upon the performance of fixed and controllable propellers is shown in figure 15. In this example it was assumed that a 450-horsepower engine having a variable gear ratio was mounted on an airplane having a top speed of 180 miles per hour. The performance of both a fixed-pitch and a controllable propeller was calculated for each of three gear ratios. The diameters and blade angles of the fixed-pitch propellers were chosen so as to give the greatest speed, whereas the diameters of the controllable propellers, in accordance with the information obtained from airplane no. 1, were chosen considerably larger than that giving the highest speed yet not large enough to be affected by high tip speeds. Neither the increase in propulsive efficiency with propeller body-diameter ratio nor the loss of power due to gearing was considered. In an actual case these would tend to balance each other.

DISCUSSION AND RESULTS

The curves in figures 6 and 10 indicate that the difference in performance between a controllable and a fixed-pitch propeller of equal diameter is largely due to the maintenance of engine speed and power by the controllable propeller rather than to the difference in their propulsive efficiencies.

In nine examples calculated, four of which are given in this report, the difference between the propulsive efficiencies of controllable and fixed-pitch propellers of equal diameters was quite small, the greatest difference being in example 1 (fig. 6). In two examples the efficiency of the fixed-pitch propeller was actually greater than that of the controllable or even the C.P.& D. propeller throughout part of the flight range.

Figures 5 and 6 indicate that with airplanes similar to no. 1 the use of a controllable propeller of larger diameter than that for best high-speed performance increases the all-round performance except for a negligible loss of high speed. This advantage would be largely offset if, due to the use of the larger propeller, high tip-speed losses were involved. It should be clear, then, that slow-speed or geared engines have an advantage in airplanes of this type.

The engines of airplane no. 3 having a rated speed of 2,100 r.p.m. required the use of small-diameter propellers in order to avoid high tip speeds. A considerably better performance could be obtained if a moderately geared engine and larger propellers were used.

Airplane no. 2 with its high top speed and geared engine appears to be admirably suited for the adaptation of a controllable propeller. The slow-turning propeller shaft permits the use of a large-diameter propeller, thus obtaining a performance nearly equal to that of the hypothetical controllable-pitch-and-diameter propeller, and undoubtedly much better than a smaller diameter propeller on an ungeared engine of equal power.

Airplane no. 4, with its slow-turning supercharged engine, provides the greatest possibilities for the use of a controllable propeller. It will be noted in figures 10, 11, and 12 that the drop in engine speed, and therefore in engine power, with the fixed-pitch propeller is exceptionally high. This effect is due to both the high speed of the airplane, and also to the fact that with a supercharged engine the power falls off with engine speed more rapidly than with an unsupercharged engine. It is evident therefore that a controllable propeller is a very valuable asset on high-speed airplanes using supercharged engines.

Although it is generally considered that a controllable propeller has but small effect in the high speed and cruising speed of an airplane it has been shown in reference 6 that, due to a better correlation between manifold pressure and revolution speed, the cruising speed of an airplane equipped with a supercharged engine may in certain cases be materially increased by the use of a controllable propeller. The example cited in this reference of a large twin-engine transport showed an increase in cruising speed of $5\frac{1}{2}$ percent when the fixed-pitch propellers were replaced by controllable propellers.

As the critical altitude of a supercharged engine is increased the sea-level performance of a fixed-pitch propeller designed for critical altitude decreases. This decrease is due to the fact that with the manifold pressure set to allow maximum permissible cylinder pressure the propeller designed for critical altitude holds the engine below its rated speed and power at sea level. For this reason a controllable propeller is equally valuable at altitudes below critical altitude as at altitudes above critical altitude. It will be noted from the curves for the examples given that the greatest range of blade angle for ordinary flight covers about 7° for the airplane with the supercharged engine and considerably less for the other three.

The performance of a controllable propeller having but two pitch settings was computed for airplane no. 2. The performance summary for this airplane given in table IX shows that its sea-level performance in take-off, climb, and high speed when equipped with the 2-pitch-setting propeller is very nearly as good as its performance when equipped with a propeller having a large number of pitch settings.

For any one altitude it appears that two pitch settings may be sufficient; however, when an airplane rises above the design altitude of the propeller the propeller speed drops and a third and fourth pitch setting may be highly desirable. In general then, for airplanes which do a large part of their flying at nearly constant altitude a propeller having two pitch settings may suffice, whereas for airplanes which operate through wide ranges of altitude, especially for those with supercharged engines, a multipitch setting or automatic propeller is preferable.

From figure 15 it appears that for certain types of airplanes the effect of gearing on the performance of controllable propellers is distinctly advantageous, whereas the effect upon the performance of a fixed-pitch propeller is practically negligible.

CONCLUSIONS

1. The relative efficiencies of a fixed-pitch propeller and a controllable propeller of the same diameter account for only a small part of the difference in performance between the two propellers.

2. The primary benefit of a controllable propeller comes from the fact that a controllable propeller permits

the engine to maintain the maximum allowable revolution speed and power at all air speeds.

3. A secondary advantage comes from the fact that with a controllable propeller the use of a larger diameter than that giving best high speed increases the all-round performance of the airplane except for a negligible loss in high speed.

4. A geared engine is desirable for use with a controllable propeller.

5. The controllable propeller is particularly desirable for use with supercharged engines.

6. The change of blade angle necessary to maintain constant engine speed throughout the flight range is from 4° to 8°.

7. A controllable propeller reduces the take-off run, increases the rate of climb and ceiling, and above the critical altitude of the engine it increases the high speed.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 30, 1933.

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Table I

Airplane No. 1. Values of C_s

r.p.m.	2,000	1,900	1,800	1,700	1,600	1,500
$(r.p.m.)^{2/5}$	20.95	20.50	20.10	19.63	19.15	18.70
b.hp.	552	525	497	468	442	413
$(b.hp.)^{1/5}$	3.54	3.505	3.47	3.425	3.385	3.335
V (m.p.h.)	Values of C_s					
50	0.430	0.444	0.457	0.474	0.492	0.511
60	.516	.532	.549	.570	.590	.614
75	.645	.665	.686	.711	.738	.766
100	.860	.887	.915	.949	.984	1.022
125	1.075	1.11	1.14	1.185	1.23	1.28
150	1.29	1.33	1.37	1.42	1.48	1.53
175	1.51	1.55	1.60	1.66	1.72	1.79
200	1.72	1.77	1.83	1.90	1.97	2.04
225	1.93	2.00	2.06	2.13	2.21	2.30

Table II

Airplane No. 1. Thrust Horsepower Required at Sea Level

Air speed		Parasite power required	Induced power required	Total power required
V(m.p.h.)	V^3	KV^3 (t.hp.)	K_1/V(t.hp.	$t.hp._r$
225	11,400,000	522	22	544
200	8,000,000	366	25	391
175	5,370,000	246	28	274
150	3,370,000	155	33	188
125	1,950,000	89	39	128
100	1,000,000	46	49	95
75	422,000	19	66	85

Table III

Airplane Characteristics

No.	Type	Weight (lb.)	Span (ft.)	$(b.hp.)_o$	$(r.p.m.)_o$	High speed (m.p.h.)	Take-off speed (m.p.h.)	e	$(L/D)_{max}$	Munk's k	f (sq.ft.)
1	High-speed transport	5,200	42.8	525	1,900	211	75	0.9	14.5	1	6.74
2	Military observation	4,865	45.7	600	2,450 geared 7:5	191	67	1.0	12.5	1	10.44
3	Trimotored transport	13,515	78.0	3x450	2,100	148	75	.85	9.5	1	44.1
4	Pursuit bi-plane	3,730	larger 31.5	610 at 6,000 ft.	1,900	193	65	.85	8.6	1.15	11.75

Table IV

Determination of $b.hp._a$ for Fixed-Pitch Propeller

Diameter 9 ft. $\beta = 28^{\circ}$

Airplane no.1

(r.p.m.) (assumed)	C_s (table 3)	V/nD (fig.1)	$(r.p.m.)_2$ (calculated)	r.p.m. (actual)	C_s (actual)	η	$b.hp._a$	t.hp.
			Air speed 225 m.p.h.					
2000	1.93	1.111	1975					
1900	2.00	1.142	1925	1950	1.96	0.870	538	468
1800	2.06	1.167	1890					
			Air speed 200 m.p.h.					
2000	1.72	1.013	1925					
1900	1.77	1.037	1885	1870	1.79	.860	517	445
1800	1.83	1.066	1835					
			Air speed 175 m.p.h.					
1900	1.55	.927	1847					
1800	1.60	.951	1800	1800	1.60	.840	497	418
1700	1.66	.982	1740					
			Air speed 150 m.p.h.					
1800	1.37	.827	1785					
1700	1.42	.855	1720	1750	1.394	.807	483	390
1600	1.48	.888	1655					
			Air speed 125 m.p.h.					
1800	1.14	.692	1765					
1700	1.185	.718	1700	1700	1.185	.752	470	353
1600	1.23	.747	1635					
			Air speed 100 m.p.h.					
1800	.915	.557	1755					
1700	.949	.577	1692	1685	.954	.647	465	301
1600	.984	.598	1635					
			Air speed 75 m.p.h.					
1800	.686	.418	1755					
1700	.711	.434	1690	1670	.719	.495	461	229
1600	.738	.451	1625					
			Air speed 60 m.p.h.					
1800	.549	.336	1745					
1700	.570	.348	1680	1650	.580	.393	456	179
1600	.590	.362	1620					
			Air speed 50 m.p.h.					
1800	.457	.280	1745					
1700	.474	.293	1670	1640	.485	.320	453	145
1600	.492	.302	1620					

Table V

Performance Figures for Controllable and Fixed Pitch Propellers

Airplane no. 1

V	C_s		V/nD			β (deg.)				η				$t.hp._a$				r.p.m.		Best diam. ft.	$t.hp._r$
(m.p.h.)	C.P.	F.P.	C.P. & D.	C.P. 9-ft.	C.P. 10-ft.	C.P. & D.	C.P. 9-ft.	C.P. 10-ft.	F.P. 9-ft.	C.P. & D.	C.P. 9-ft.	C.P. 10-ft.	F.P. 9-ft.	C.P. & D.	C.P. 9-ft.	C.P. 10-ft.	F.P. 9-ft.	C.P. all diams.	F.P. 9-ft.		
50	0.444	0.477		0.257	0.231		23	16.2	28	0.502	0.400	0.478	0.320	261	210	250	145	1900	1640		
60	.532	.573		.309	.278		23.2	16.2	28	.556	.452	.541	.393	292	237	284	179	1900	1650		
75	.665	.715	0.300	.386	.347	9.4	23.3	16.3	28	.628	.546	.620	.495	330	287	325	229	1900	1670	11.55	85
100	.887	.954	.442	.515	.463	15	23.7	17	28	.717	.665	.716	.647	376	349	376	301	1900	1685	10.48	95
125	1.11	1.185	.587	.644	.578	18.7	24.2	18	28	.780	.755	.778	.752	410	396	408	353	1900	1700	9.85	128
150	1.33	1.394	.732	.772	.694	21.9	25	19.2	28	.822	.807	.818	.807	431	424	429	390	1900	1750	9.50	188
175	1.55	1.60	.876	.901	.810	24.5	26.1	20.5	28	.844	.841	.837	.840	443	441	440	418	1900	1800	9.25	274
200	1.77	1.79	1.02	1.03	.925	27	27.6	21.9	28	.862	.860	.837	.860	453	451	440	445	1900	1870	9.1	391
225	1.99	1.96	1.165	1.159	1.04	29.4	29.2	23.6	28	.875	.874	.838	.870	460	459	440	468	1900	1950	8.95	544

Table VI

Determination of Static Thrust

Airplane no.1

Propeller	β (deg.)	$\frac{T_o D}{Q}$	D (ft.)	Q (lb.-ft.)	T_o (lb.)
1. C.P. & Diam.	11	19.35	11.2	1455	2515
2. C.P.	16	16.6	10	1455	2415
3. C.P.	23	9.7	9	1455	1568
4. F.P.	28	5.7	9	1455	920

$$Q = (\text{hp.} \times 5250)/\text{r.p.m.}$$

Table VII

Airplane No. 1. Determination of Take-Off Run

Propeller	T_o (lb.)	T_v (lb.)	T_1/W	K_s	S (ft.)	Ratio $\left(\frac{S}{S_o}\right)$
1. C.P.& D	2,515	1,660	0.434	0.050	646	0.44
2. C.P. 10 ft.	2,415	1,625	.415	.0495	669	.46
3. C.P. 9 ft.	1,568	1,430	.251	.041	916	.63
4. F.P. 9 ft.	920	1,151	.127	.033	1455=S_o	1.00

Table VIII

Airplane No. 1. Performance Summary

Propeller	High speed (m.p.h.)	Ratio $\left(\frac{V}{V_o}\right)$	Max. rate of climb (ft./min.)	Ratio $\left(\frac{C}{C_o}\right)$	Take-off run (ft.)	Ratio $\left(\frac{S}{S_o}\right)$
1. C.P.& D.	211	1.00	1,825	1.29	646	0.44
2. C.P. 10 ft.	209	.99	1,825	1.29	669	.46
3. C.P. 9 ft.	211	1.00	1,690	1.19	916	.63
4. F.P. 9 ft.	211=V_o	1.00	1415=C_o	1.00	1445= S_o	1.00

Table IX

Airplane No. 2. Performance Summary

Propeller	High speed (m.p.h.)	Ratio $\left(\frac{V}{V_o}\right)$	Max. rate of climb (ft./min.)	Ratio $\left(\frac{C}{C_o}\right)$	Take-off run (ft.)	Ratio $\left(\frac{S}{S_o}\right)$
1. C.P.& D.	191	1.00	2,230	1.24	377	0.56
2. C.P. 10.5 ft.	190	.995	2,200	1.23	414	.62
3. F.P. 10 ft.	191=V_o	1.00	1795=C_o	1.00	674= S_o	1.00
4. C.P. 10.5 ft. (2 pitch settings)	190	.995	2,190	1.22	430	.639

Table X

Airplane No. 3. Performance Summary

Propeller	High speed (m.p.h.)	Ratio $\left(\frac{V}{V_0}\right)$	Max. rate of climb (ft./min.)	Ratio $\left(\frac{C}{C_0}\right)$	Take-off run (ft.)	Ratio $\left(\frac{S}{S_0}\right)$
1. C.P.& D	149	1.00	1,460	1.29	681	0.71
2. C.P. 8.75 ft.	149	1.00	1,310	1.15	766	.80
3. F.P. 8.75 ft.	$149=V_0$	1.00	$1135=C_0$	1.00	$952=S_0$	1.00

Table XI

Airplane No. 4. Performance Summary
(Sea level)

Propeller	High speed (m.p.h.)	Ratio $\left(\frac{V}{V_0}\right)$	Max. rate of climb (ft./min.)	Ratio $\left(\frac{C}{C_0}\right)$	Take-off run (ft.)	Ratio $\left(\frac{S}{S_0}\right)$
1. C.P.& D	182	1.024	2,670	1.30	290	0.61
2. C.P. 10 ft.	182	1.024	2,620	1.28	314	.66
3. F.P. 10 ft.	$178=V_0$	1.00	$2050=C_0$	1.00	$478=S_0$	1.00

Table XII

Airplane No. 4. Critical Altitude
(6,000 ft.)

Propeller	High speed (m.p.h.)	Ratio $\left(\frac{V}{V_0}\right)$	Max. rate of climb (ft./min.)	Ratio $\left(\frac{C}{C_0}\right)$
1. C.P.& D.	195	1.00	2,735	1.51
2. C.P. 10 ft.	195	1.00	2,670	1.47
3. F.P. 10 ft.	$195=V_0$	1.00	$1815=C_0$	1.00

Table XIII

Airplane No. 4
(20,000 ft.)

Propeller	High speed (m.p.h.)	Ratio $\frac{V}{V_o}$	Max. rate of climb (ft./min.)	Ratio $\frac{C}{C_o}$	Ceiling	Ratio $\frac{H}{H_o}$	Climb in 10 minutes from critical altitude
1. C.P.& D.	182	1.04	1,240	2.26	31,600	1.20	-
2. C.P. 10 ft.	182	1.04	1,195	2.17	31,500	1.20	16,500 ft.
3. F.P. 10 ft.	$175=V_o$	1.00	$550=C_o$	1.00	$26300=H_o$	1.00	12,000 ft.

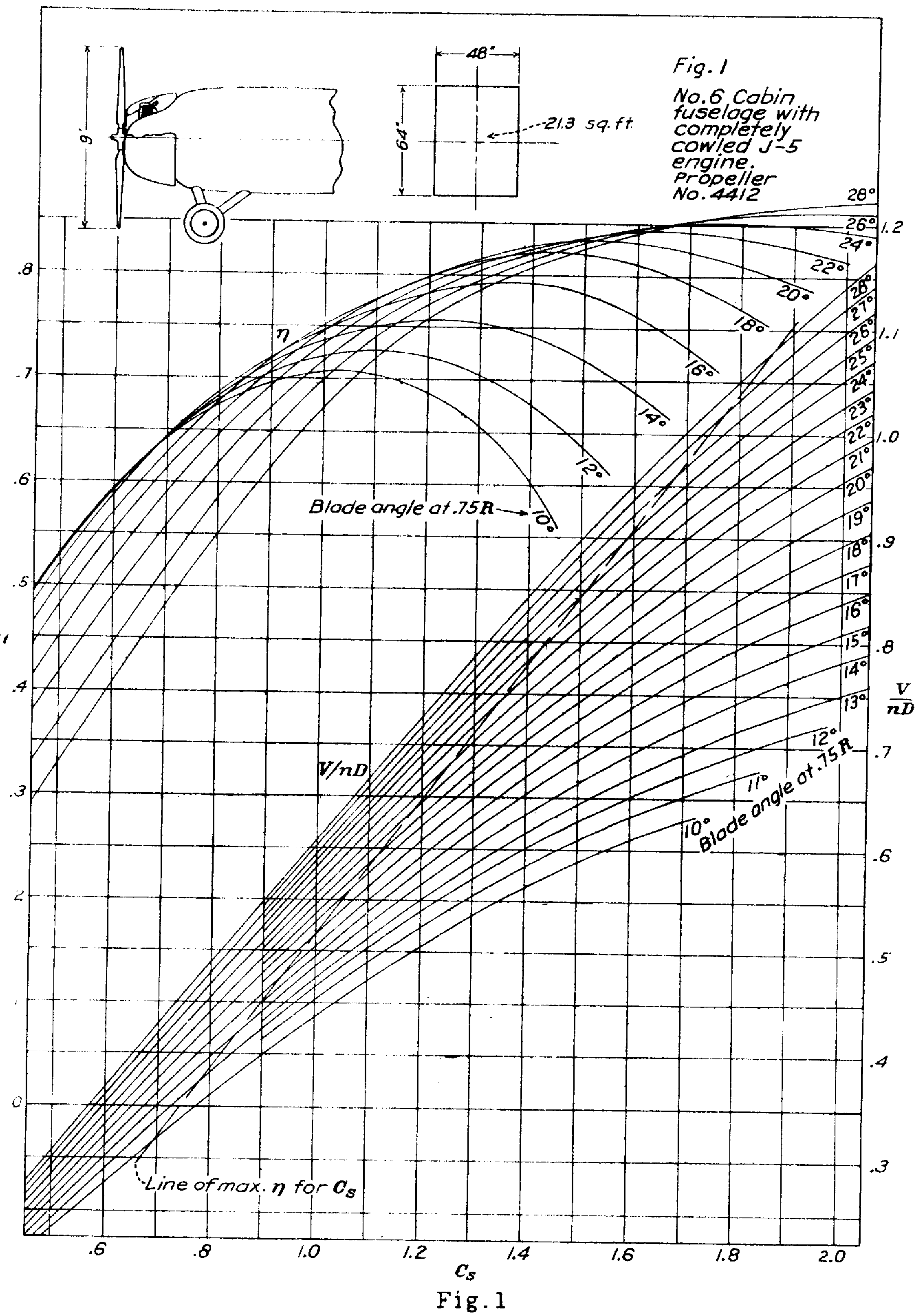
Fig. 1
No.6 Cabin fuselage with completely cowled J-5 engine. Propeller No.4412
48"
64"
9'
21.3 sq.ft.
η
V/nD
Blade angle at .75R
Blade angle at .75R
Line of max. η for C_s
$\frac{V}{nD}$
C_s
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0

Fig.1

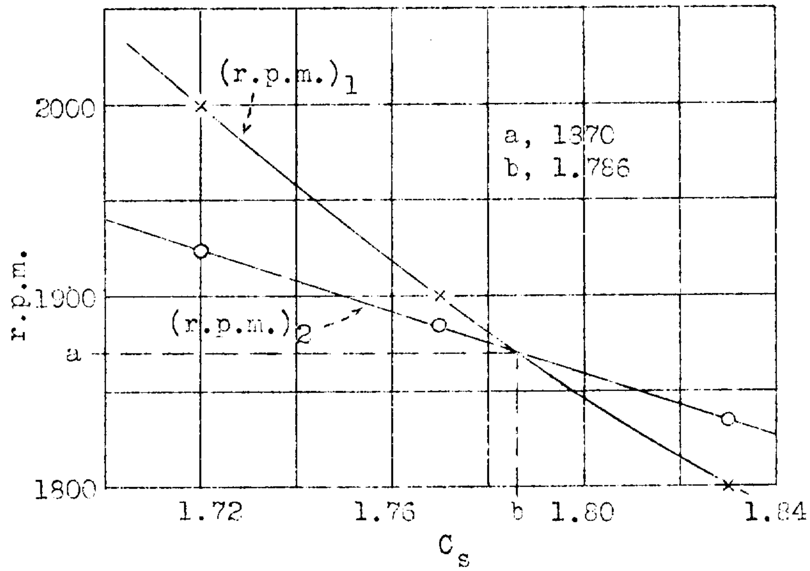

Figure 2.-Determination of actual r.p.m. and C_s for fixed-pitch propeller. Airplane no. 1. Air speed 200 m.p.h.

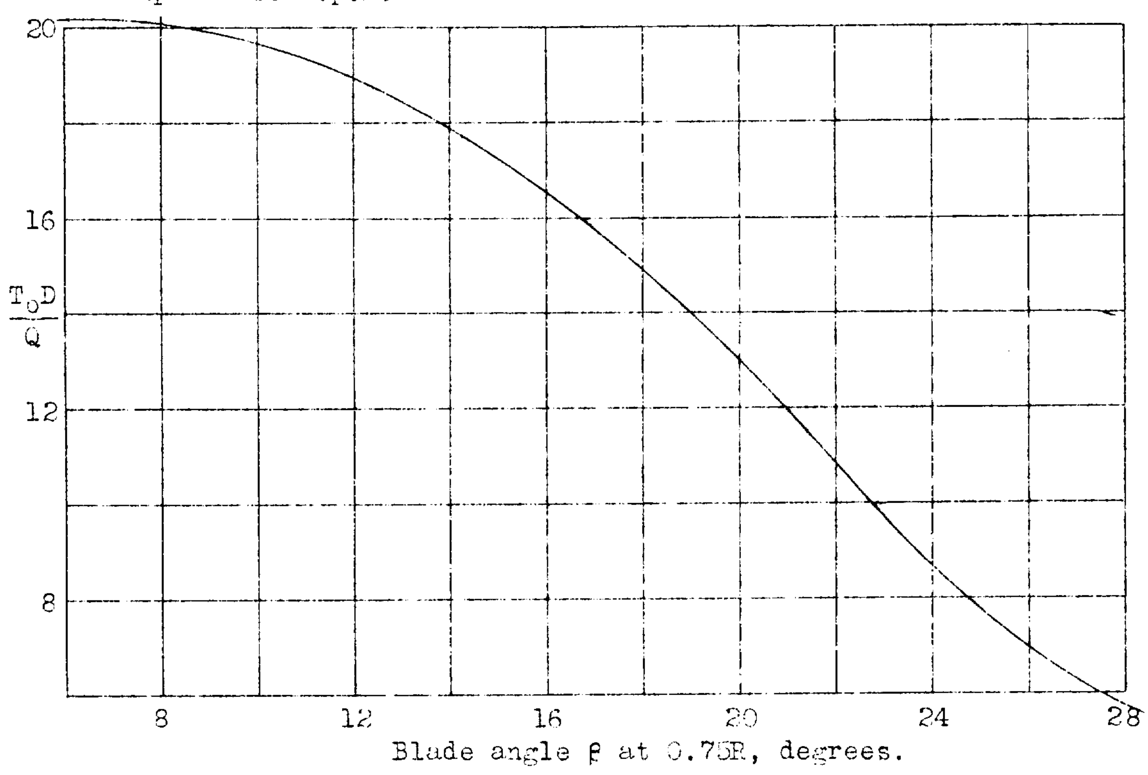

Figure 4.-Chart for the determination of the static thrust of a propeller.

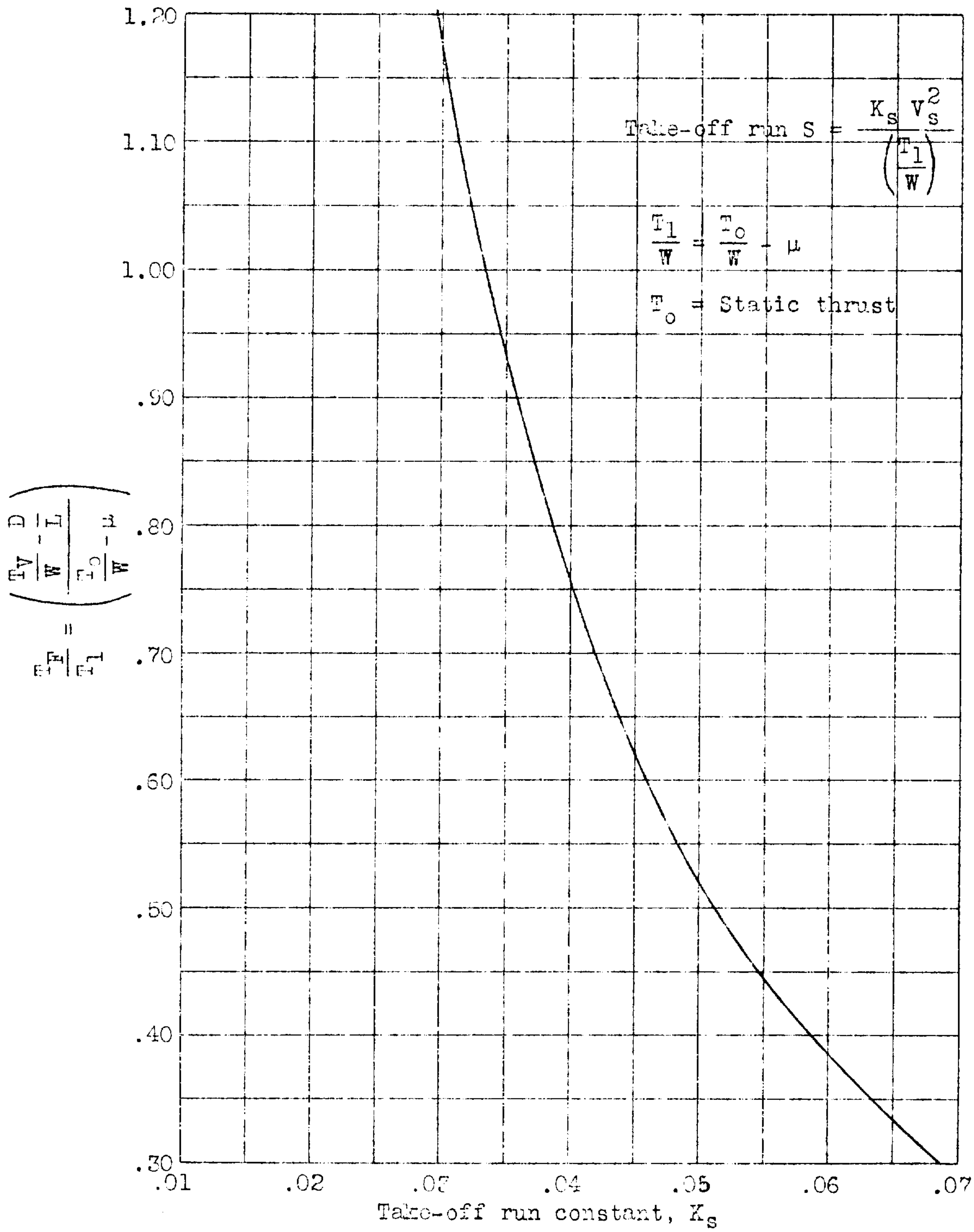

Figure 3.-Chart for the determination of the take-off run constant.

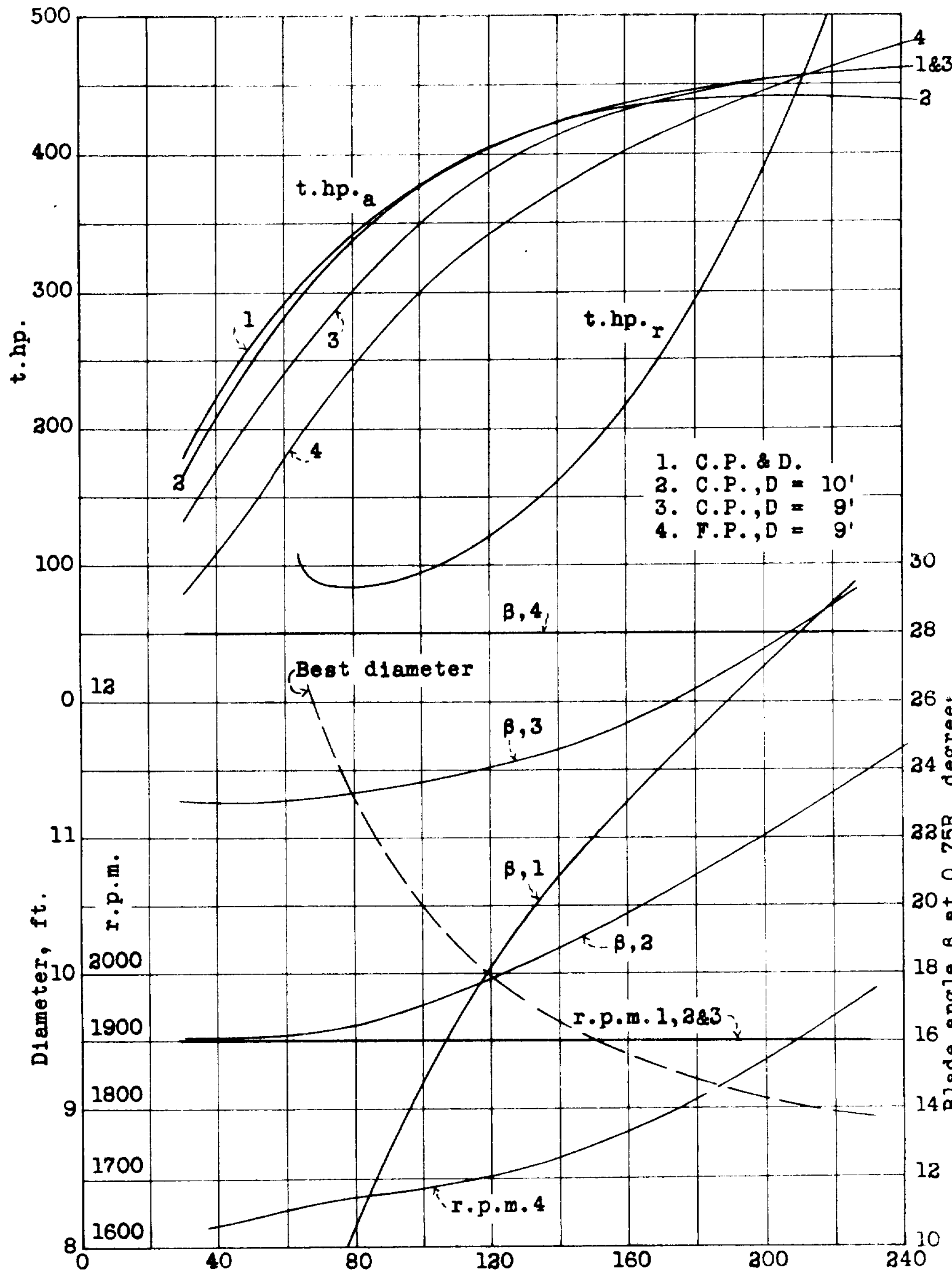

Figure 5.-Performance curves for controllable and fixed-pitch propellers. Airplane no. 1.

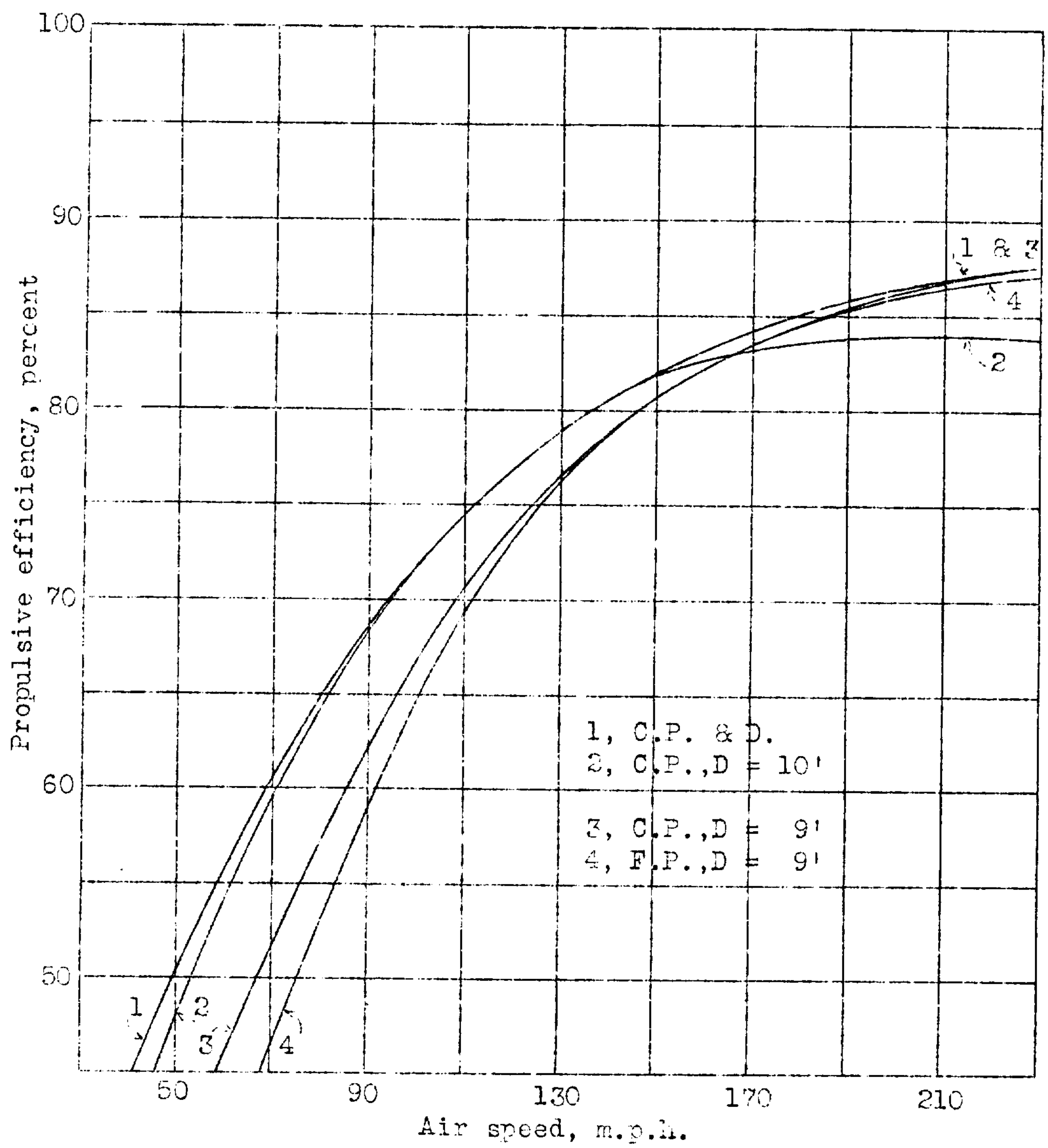

Figure 6.-Comparison of propulsive efficiencies for controllable and fixed-pitch propellers. Airplane no. 1.

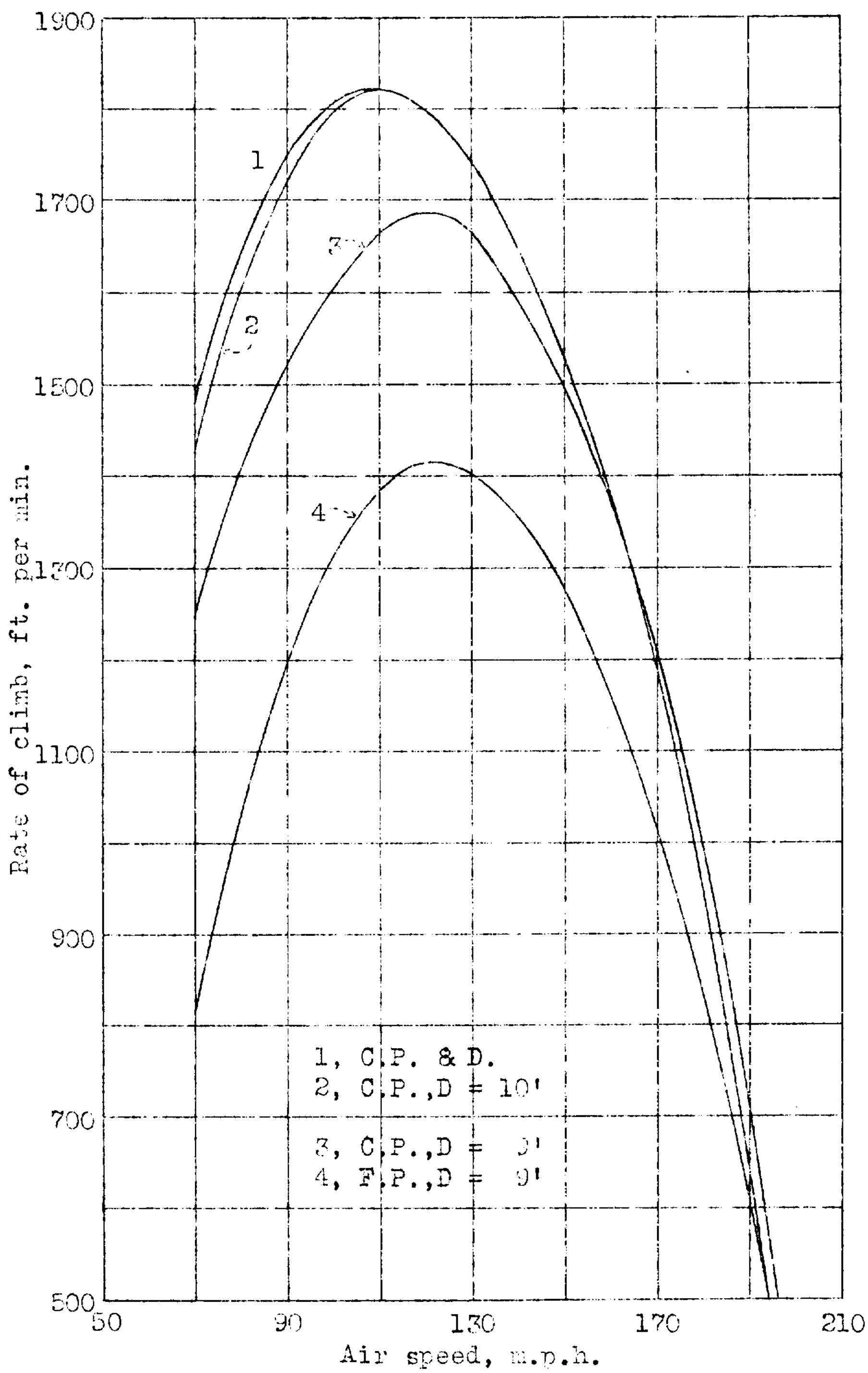

Figure 7.-Comparison of rates of climb for airplane equipped with controllable and fixed-pitch propellers. Airplane no. 1.

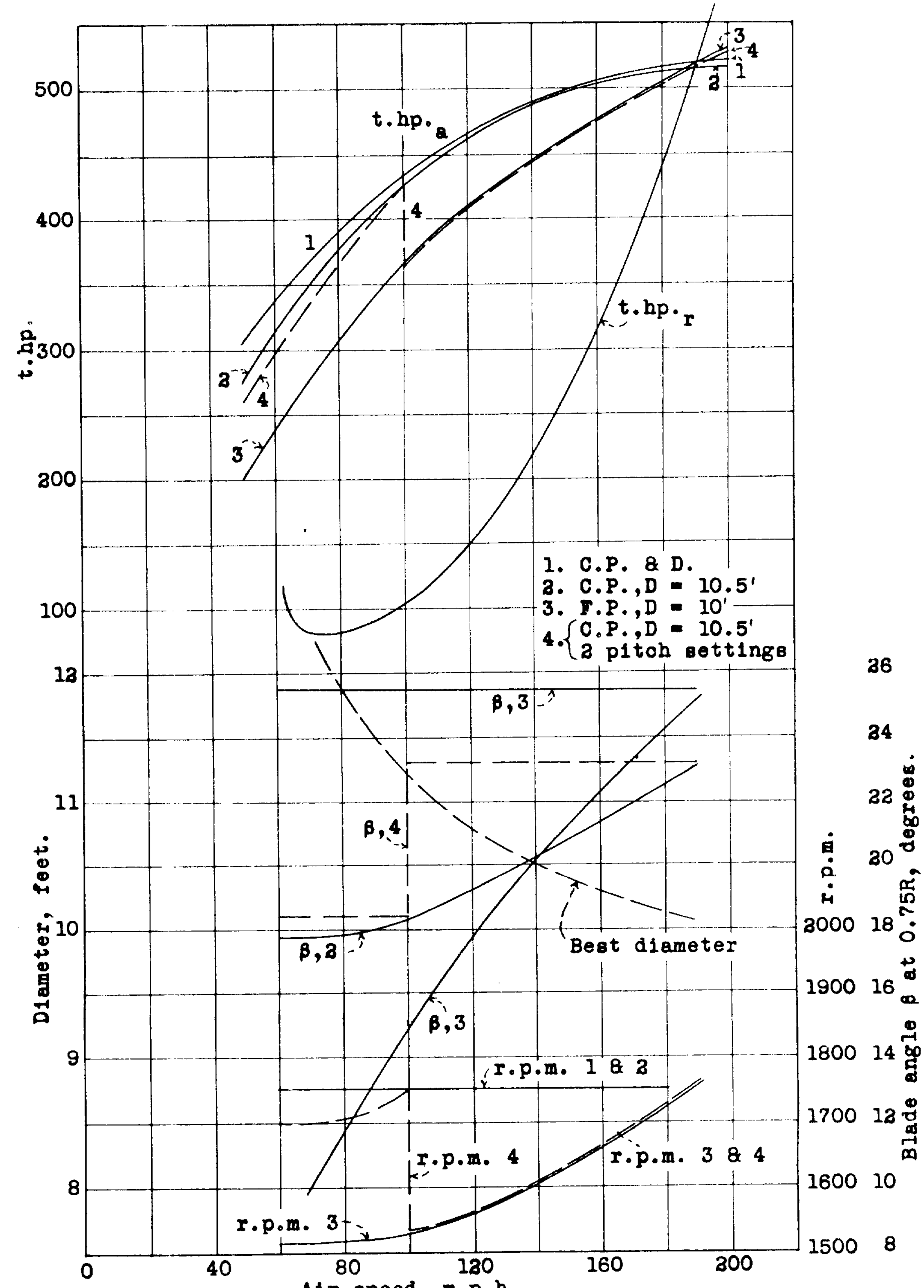

Figure 8.-Performance curves for controllable and fixed-pitch propellers. Airplane no. 2.

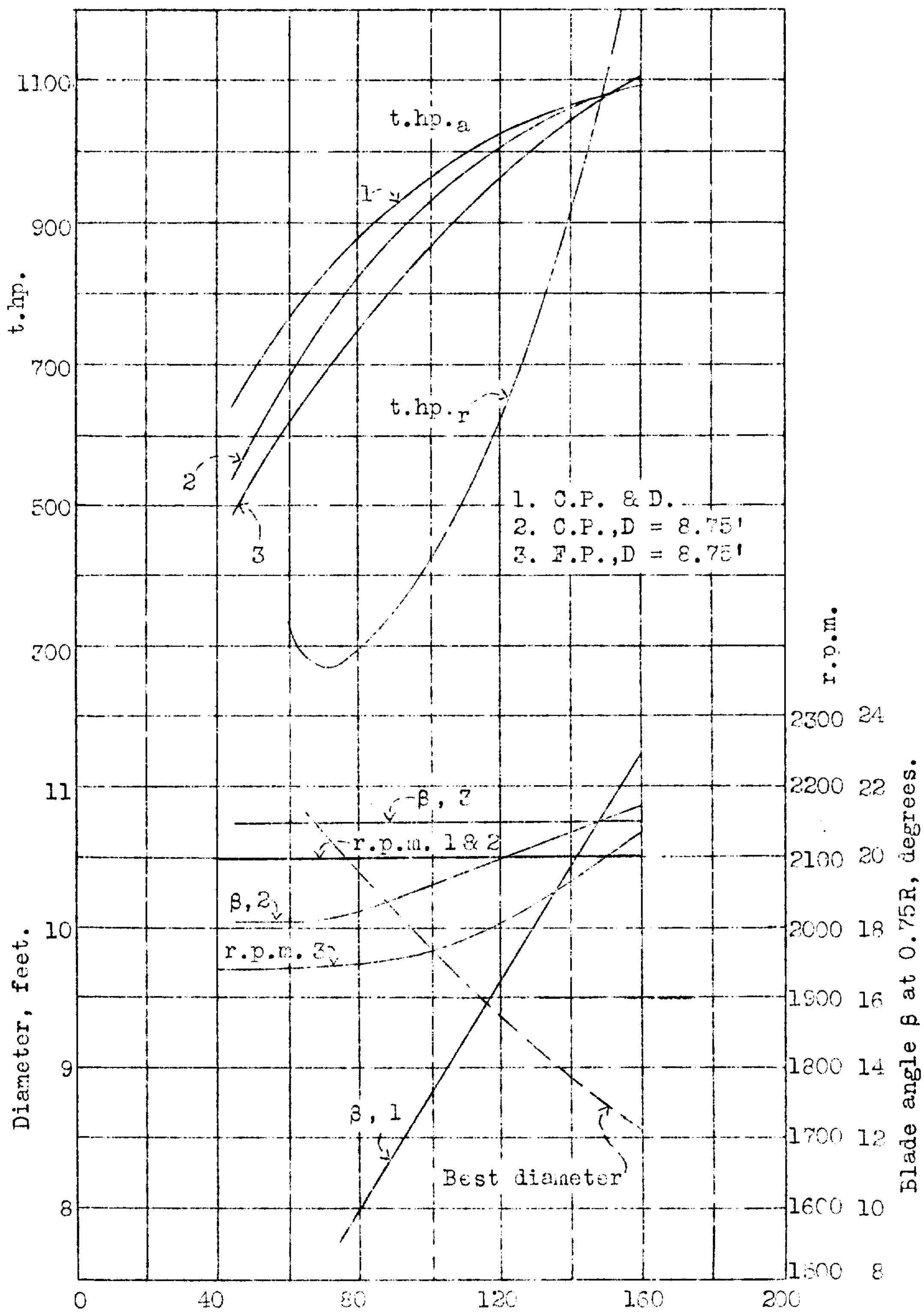

Figure 9.- Performance curves for controllable and fixed- pitch propellers. Airplane no. 3.

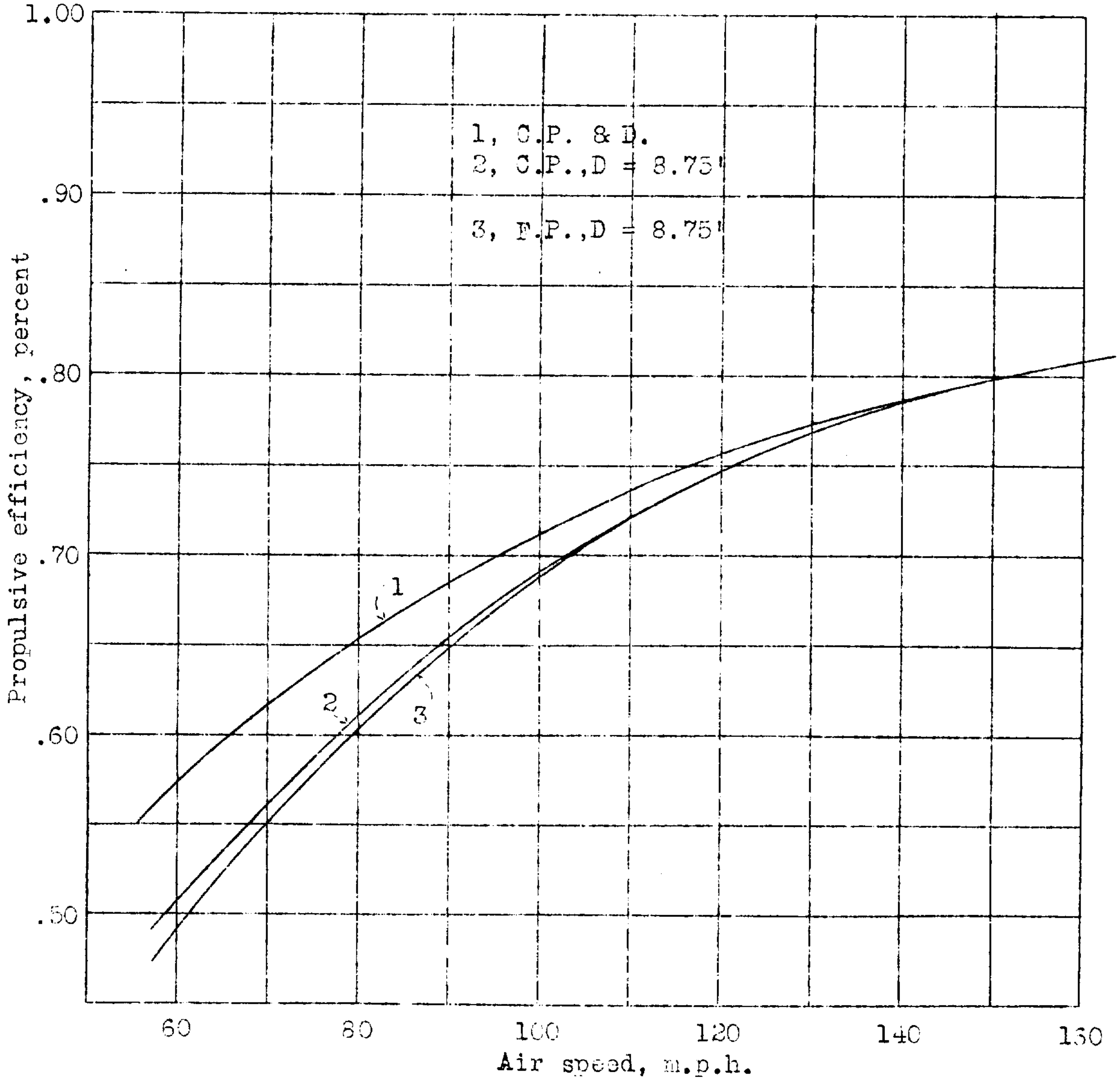

Figure 10.-Comparison of propulsive efficiencies for controllable and fixed-pitch propellers. Airplane no. 3.

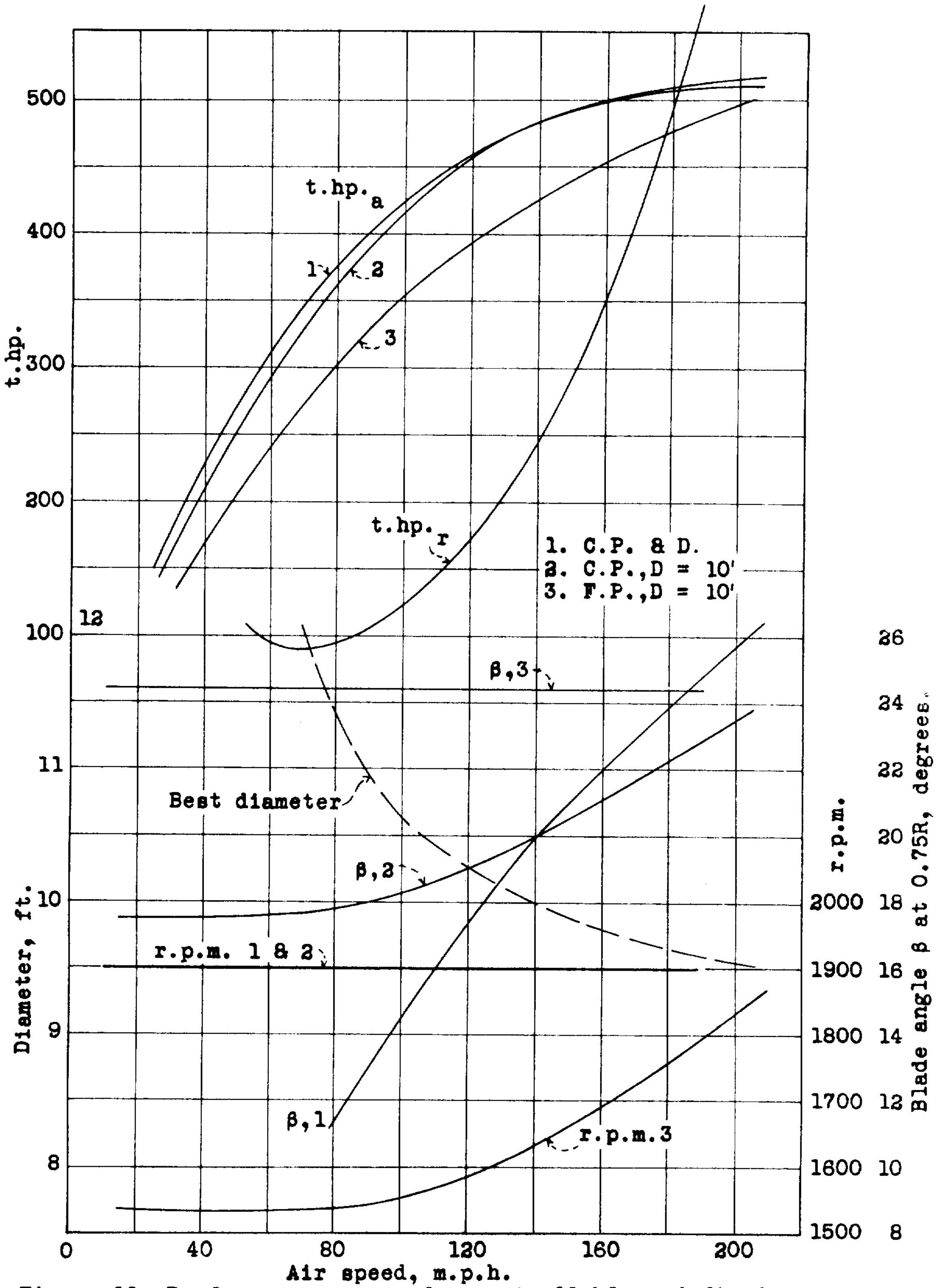

Figure 11.-Performance curves for controllable and fixed-pitch propellers at sea level. Airplane no. 4.

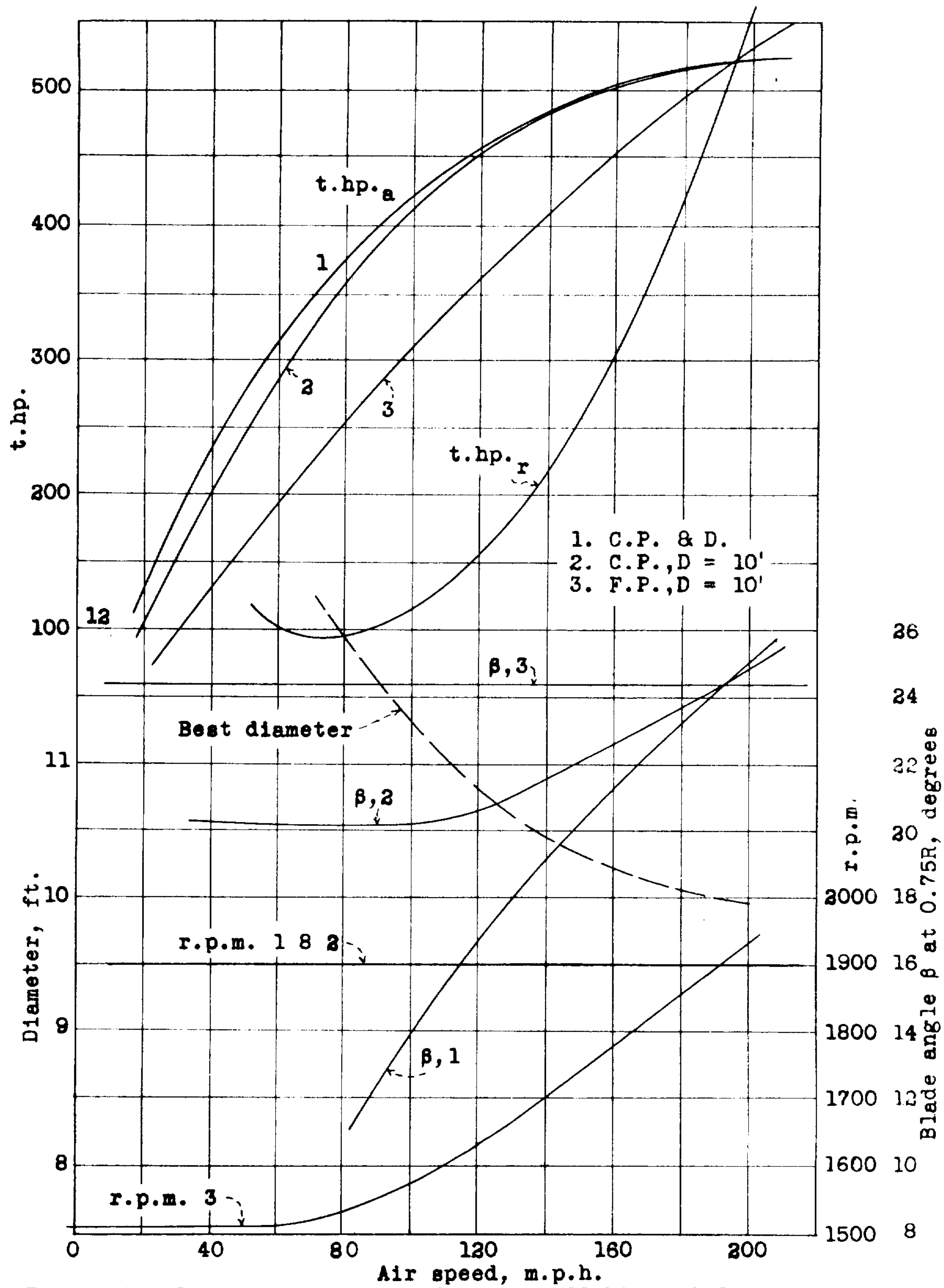

Figure 12. Performance curves for controllable and fixed-pitch propellers at 6.000 feet (critical altitude). Airplane no. 4

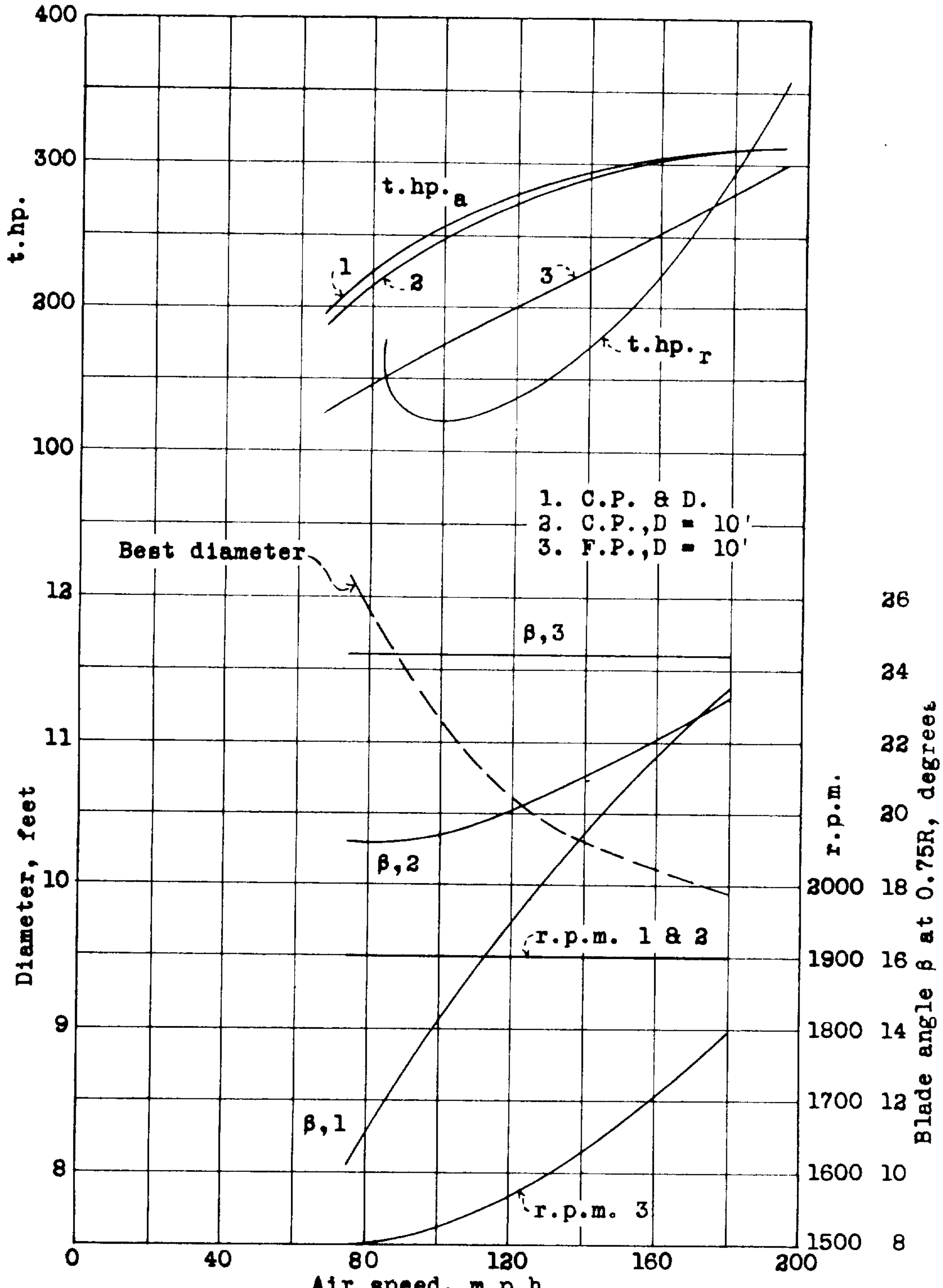

Figure 13. Performance curves for controllable and fixed-pitch propellers at 20,000 feet. Airplane no. 4.

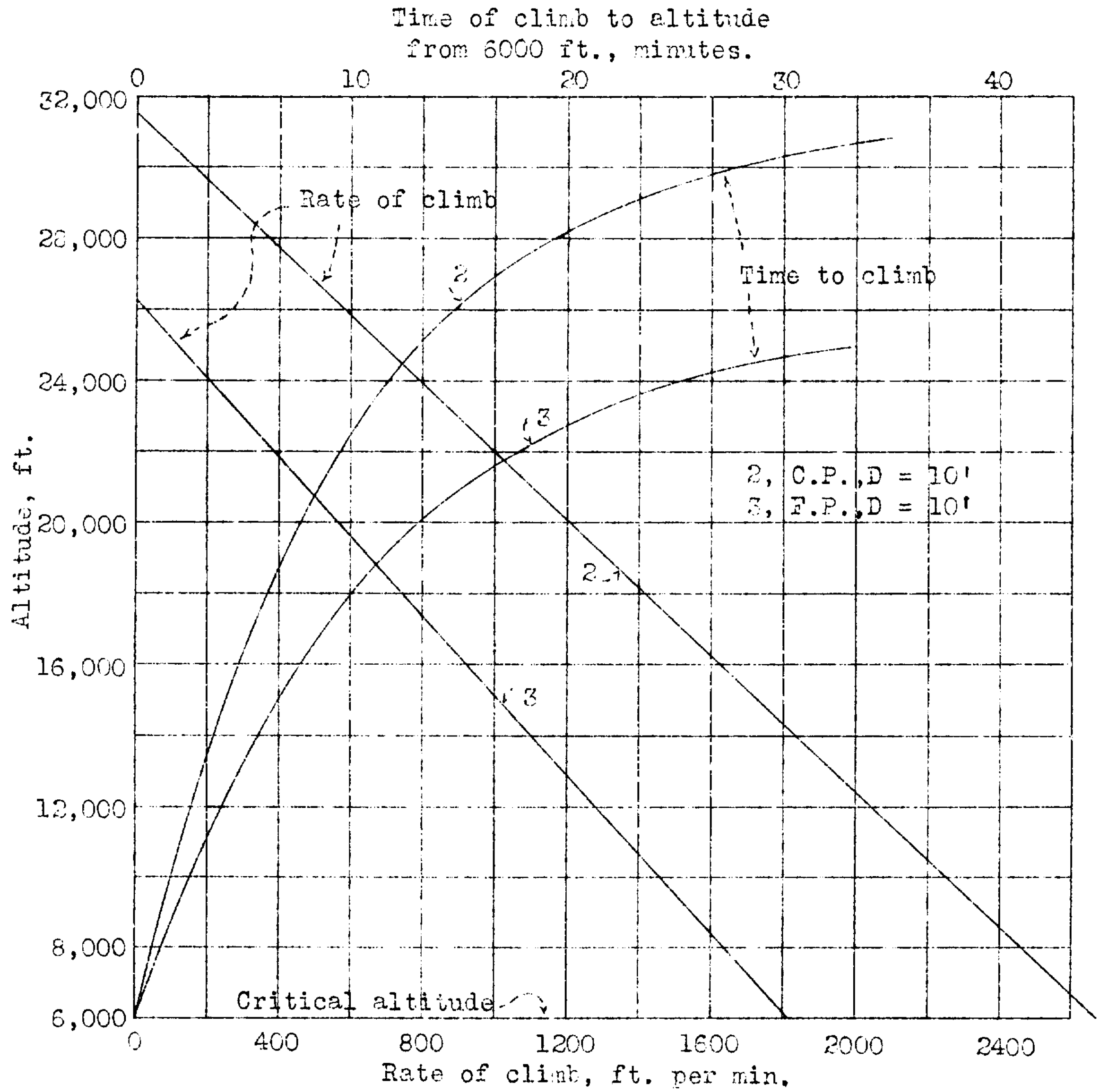

Figure 14.-Altitude performance of airplane equipped with controllable and fixed-pitch propellers. Airplane no. 4.

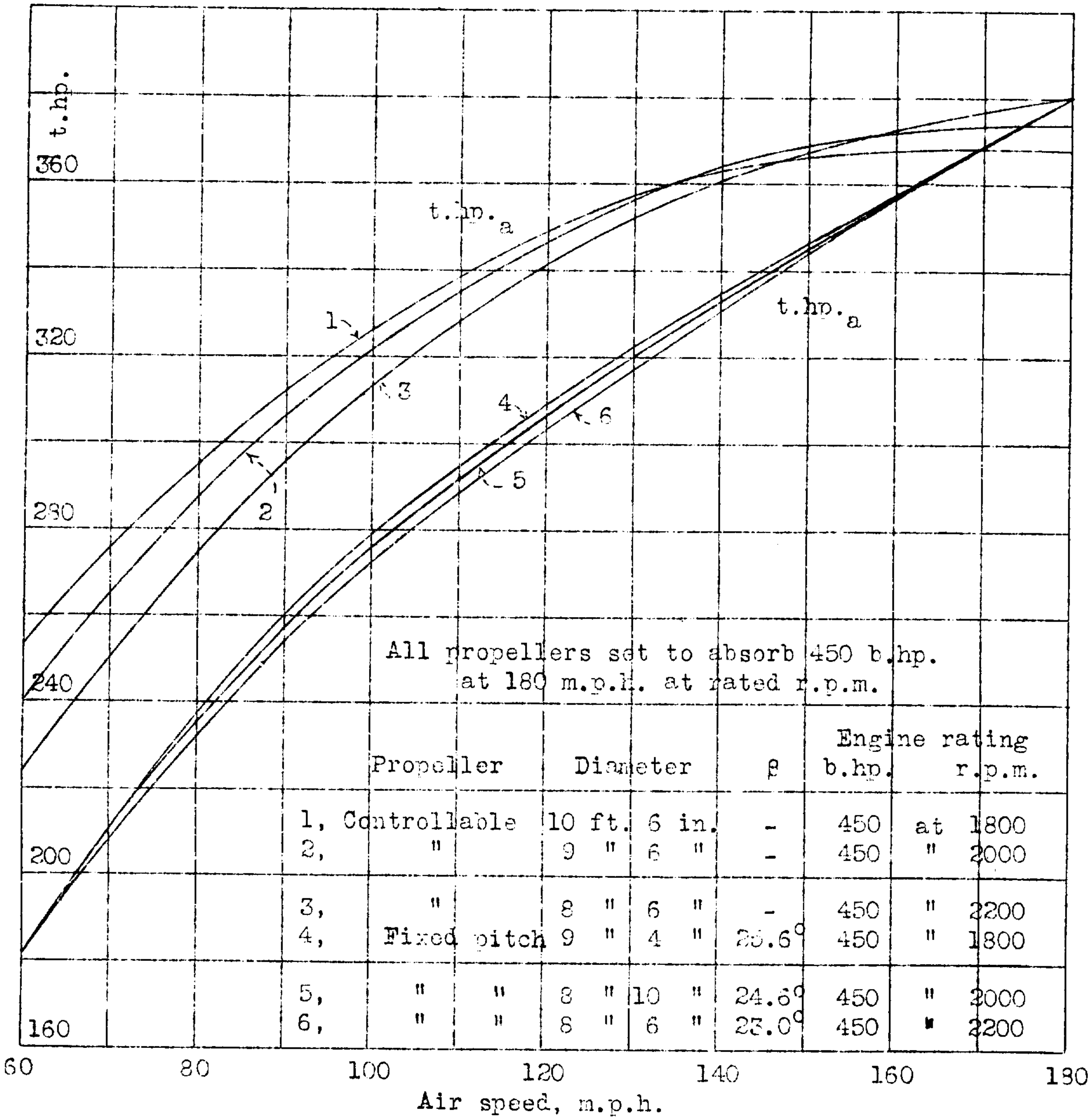

Figure 15.-Effect of gearing on the performance of fixed-pitch and controllable propellers.