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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 609

CONSIDERATIONS AFFECTING THE ADDITIONAL WEIGHT REQUIRED IN MASS BALANCE OF AILERONS

By W. S. Diehl
Bureau of Aeronautics, Navy Department

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SUMMARY

This paper is essentially a consideration of mass balance of ailerons from a preliminary design standpoint, in which the extra weight of the mass counterbalance is the most important phase of the problem. Equations are developed for the required balance weight for a simple aileron and this weight is correlated with the mass-balance coefficient. It is concluded the location of the c.g. of the basic aileron is of paramount importance and that complete mass balance imposes no great weight penalty if the aileron is designed to have its c.g. inherently near to the hinge axis.

INTRODUCTION

As the performance of an airplane is increased a point is reached at which unbalanced mass forces become intolerable owing to their effects on aerodynamic forces. The simplest and at the same time the most important example of this effect is found in an aileron having its center of gravity behind the hinge axis. Such an aileron has an inertia moment about its hinge axis when the wing rolls. This inertia moment is directly proportional to the product of inertia for the aileron obtained by summation of the products of each increment of weight into its distances from the hinge axis and from the roll axis.

If the inertia moment about the hinge axis is great enough, it will cause the aileron to lag behind the wing in roll or bending. This lag produces a change in wing lift tending to increase roll or bending. When the periods of the wing and the aileron approach resonance a destructive flutter may occur.

It is becoming increasingly necessary to investigate and, if possible, to eliminate the more important conditions tending to cause flutter. Among these conditions mass unbalance in the ailerons is generally the most important single factor. It is indeed fortunate that the designer can secure a large measure of immunity from wing flutter by the use of counterweights properly disposed and so proportioned to give zero product of inertia for the ailerons. It is unfortunate that many designers still regard the mass counterbalance as a pure dead load, but this attitude is perhaps due to lack of information on the basic factors involved.

The calculations forming the basis of this paper were started in an attempt to find how much weight penalty was involved in complete mass balance for a series of simple ailerons. The relations as derived appeared to be sufficiently general to justify a more complete investigation of this phase of the subject than had originally been intended. It is believed that the final results give the designer a definite physical basis for interpreting products of inertia and balance coefficients.

PRODUCT OF INERTIA FOR SIMPLE AILERON

The product of inertia for an aileron is obtained by summation of the product wxy where x and y are the coordinates chordwise and spanwise of the increment of weight w. In practice the aileron may be divided into areas for which the weight and c.g. may be calculated. The positive direction of x is aft of the hinge, and the positive direction of y is outboard.

For a simple uniform aileron, as shown in figure 1, the product of inertia is readily calculated. The moment about the hinge axis is obviously in the opposite direction to the motion so that

$$H = + \int_{y_1}^{y_2} wxy \; dy \qquad (1)$$

where w is the weight per unit length. The total aileron weight will therefore be $W_C = w\ (y_2 - y_1)$ and

$$H = \frac{wx}{2} (y_2^2 - y_1^2)$$

$$= \frac{W_c x}{2} (y_1 + y_2) \qquad (2)$$

Since $\frac{1}{2} (y_1 + y_2) = y$, the average span of the aileron, the product of inertia for a uniform aileron is

$$H = W_c \, x \, y \qquad (3)$$

where W_c is the total aileron weight, and x and y are the coordinates of the aileron c.g. with respect to the hinge axis and to the rolling axis.

MASS-BALANCE COEFFICIENT

The Army (reference 1) and the Department of Commerce (reference 2) have employed a nondimensional coefficient of mass balance defined by

$$C_B = \frac{H}{W_c \, S_c} \qquad (4)$$

where H is the product of inertia of the surface having a total weight W_c and area S_c. The Army specifications call for a value of C_B less than 0.05, and the Department of Commerce specifications call for a value of C_B less than 0.08 for airplanes whose high speed is over 150 miles per hour. The physical significance of these values will be investigated.

If the inner and outer ends of the aileron are as before, at y_1 and y_2 the aileron chord is t and the weight per unit length is w, and the aileron weight and area are

$$W_c = w (y_2 - y_1) \qquad (5)$$

$$S_c = t (y_2 - y_1) \qquad (6)$$

Substituting equations (2), (5), and (6) into equation (4) gives

$$C_B = \frac{\frac{wx}{2}\left(y_2^2 - y_1^2\right)}{w\,t\,(y_2 - y_1)\,(y_2 - y_1)}$$

$$= \frac{1}{2}\,\frac{x}{t}\,\frac{(y_2 + y_1)}{(y_2 - y_1)} \qquad (7)$$

If $y_1 = r\,y_2$, then

$$C_B = \frac{1}{2}\left(\frac{x}{t}\right)\left(\frac{1 + r}{1 - r}\right) \qquad (8)$$

The variation of (x/t) with C_B and r is given in table I and on figure 2. Since the value of x/t would probably be about 0.25 for a simple aileron without aerodynamic balance, figure 2 indicates that a value of $C_B =$ 0.08 represents approximately 75 percent reduction in mass forces and $C_B = 0.05$ represents approximately 85 percent reduction. The location of the aileron c.g. can be moved well forward by careful design in a Frise aileron. Since C_B varies directly with the value of x/t, the weight that must be incorporated in the mass counterbalance must vary linearly with the value of x/t. The value of the added weight may be expressed as a function of C_B, x/t, and r as will be shown.

SIMPLE MASS BALANCE

The product of inertia of the simple aileron may be reduced to any desired value by means of a counterbalance attached ahead of the hinge axis, and preferably as far outboard as practicable. If such a weight ΔW is located at the outer tip and at a distance k ahead of the hinge axis, as shown in figure 1, its negative product of inertia is

$$H_W = -\,\Delta W\,k\,y_2 \qquad (9)$$

The resultant product of inertia for the system is obtained by adding equations (2) and (9)

$$H_r = \frac{W_c x}{2}\,y_2\,(1 + r) - \Delta W\,k\,y_2 \qquad (10)$$

Substituting equations (6) and (10) into equation (4) gives:

$$C_B = \frac{\frac{W_c x}{2} y_2 (1 + r) - \Delta W \, k \, y_2}{W_c \, t \, y_2 \, (1 - r)}$$

from which

$$C_B = \frac{1}{2} \frac{x}{t} \frac{(1 + r)}{(1 - r)} - \frac{\Delta W}{W_c} \frac{k}{t} \frac{1}{(1 - r)} \qquad (11)$$

When $\Delta W = 0$ equation (11) is identical with equation (8).

Equation (11) may be solved for $(\Delta W/W_c)$ giving

$$\frac{\Delta W}{W_c} = \frac{1}{2} \frac{x}{k} (1 + r) - C_B \frac{(1 - r)}{\left(\frac{k}{t}\right)} \qquad (12)$$

Table II contains calculations for $\Delta W/W_c$ to give zero C_B for a series of ailerons in which the span and initial c.g. are varied. The initial value of C_B for the unbalanced aileron, designated as C_{B_o}, is also given. The value of C_{B_o} increases rapidly, as may be seen by inspection of equation (8), when the span of the aileron is reduced without moving the outer tip. However, this increase does not mean that an increase in actual weight of the counterbalance will be required for a short-span aileron. Since the mass distribution is assumed uniform, it is found that the actual weight of the required counterbalance is approximately proportional to the aileron weight as shown by figure 3, in which the relative values of W and ΔW are plotted.

Figure 4 is a plot of $\Delta W/W$ as a function of x/k and r. This curve clearly shows the importance from a weight standpoint of a low value of x and a large value of k.

In the foregoing analysis it has been assumed that actual weights are to be used. For strict accuracy it would be necessary to consider the apparent mass of the air carried with the aileron and the actual additional mass carried in the form of paint, rain, or ice. These factors are quite real so that a negative "apparent" product of inertia may be required to give actual complete mass balance in service.

It should be noted that while the analysis has been concerned only with the relations involved in the elementary case of a single concentrated balance weight, this does not mean that a large concentrated balance weight is permissible. Aside from any consideration of the undesirable torsional effects of a large concentrated counterbalance on a long aileron, the conclusion is inescapable that good design must incorporate a close approximation to inherent static balance.

PRODUCT OF INERTIA IN FLEXURE

In deriving equation (2) it was assumed that the angular velocity and the angular acceleration were uniform along the span as in a simple roll. The general motion would involve a flexure or bending in which the deflection and hence the linear acceleration at any point varies as the square of its distance from the longitudinal axis. The "product of inertia" then becomes a "third moment" obtained by introducing a y^2 term in equation (2) or

$$H = + \int_{y_1}^{y_2} wx\, y^2\, dy$$

$$= \frac{wx}{3} \left[y_2{}^3 - y_1{}^3 \right] \tag{13}$$

Substituting equation (5) into equation (13) gives

$$H = \frac{W_c x}{3}\, y_2{}^2 \left[1 + r + r^2 \right] \tag{14}$$

where $r = y_1/y_2$. The product of inertia, or rather the third moment of a counterbalance weight at the outer tip will be

$$H = -\,\Delta W\, k\, y_2{}^2 \tag{15}$$

and the value of $\Delta W/W_c$ for $C_B = 0$ will be

$$\frac{\Delta W}{W_c} = \frac{1}{3}\,\frac{x}{k}\left[1 + r + r^2\right] \tag{16}$$

Mass unbalance in flexure is less severe than in roll. The average value of ΔW from equation (16) is about 80 percent of that from equation (12), the ratio varying from 70 percent for $r = 0.20$ to 90 percent for $r = 0.80$.

CONCLUSIONS

The conclusions reached during this study fall into two classes: Those concerned with weight saving and those concerned with the degree of mass balance that it is practicable to attain. Strictly speaking, however, the question of additional weight is involved in all considerations of the problem, and the conclusions may be listed without further classification:

1. If mass balance is to be obtained without undue additional weight, it is imperative to select an aileron type (e.g., the Frise) for which the c.g. tends to lie forward and then to attempt to get the c.g. as far forward as practicable by careful detail design.

2. The additional weight required in the counterbalance varies approximately as the aileron weight. (See fig. 3.) Hence, excess aileron span and chord are undesirable where complete mass balance is to be attained.

3. The actual mass-balance coefficient for an unbalanced aileron is unreliable as an indication of the additional weight required to attain complete mass balance.

4. A mass-balance coefficient of $C_B = 0.05$ represents an average reduction of between 60 percent and 90 percent in the mass effects as compared with a normal aileron having no counterbalance weights.

5. The attainment of complete mass balance $C_B = 0$ does not appear to impose any great weight

penalty if the aileron is designed to have its c.g. inherently near to the hinge axis.

Bureau of Aeronautics,
Navy Department, February 10, 1937.

REFERENCES

1. U.S. Army Air Corps, Matériel Division: Handbook of Instructions for Airplane Designers. Vol. I, sec. II, part V, pp. 653-655.

2. Bureau of Air Commerce, Department of Commerce: Airworthiness Requirements for Aircraft. Aeronautics Bulletin No. 7-A, 1934, section 30, pp. 28-29.

TABLE I

Location of Aileron c.g. as a Function of Aileron Span

and Mass-Balance Coefficient

Ratio inner span to outer span $\frac{y_1}{y_2} = r$	$\frac{1 - r}{1 + r}$	Location of aileron c.g. = x/t				
		C_B =	C_B =	C_B =	C_B =	C_B =
		0.05	0.08	0.12	0.20	0.40
0.20	0.667	.067	.107	.160	.267	.533
.30	.538	.054	.086	.129	.215	.430
.40	.428	.043	.069	.103	.172	.343
.50	.333	.033	.053	.080	.133	.267
.60	.250	.025	.040	.060	.100	.200
.70	.176	.018	.028	.042	.071	.141
.80	.111	.011	.018	.027	.044	.089

$$\left(\frac{x}{t}\right) = 2C_B \left(\frac{1 - r}{1 + r}\right)$$

TABLE II

Effect of Aileron Characteristics on
Initial Balance Coefficient and Required Mass
Counterbalance, Assuming $\frac{k}{t} = 0.5$

r	$\frac{x}{t} = 0.05$		$\frac{x}{t} = 0.10$		$\frac{x}{t} = 0.15$		$\frac{x}{t} = 0.20$		$\frac{x}{t} = 0.25$		$\frac{\left(\frac{\Delta W}{W}\right)}{C_{B_o}}$	$\frac{\Delta W}{\Delta W_o}$
	C_{B_o}	$\frac{\Delta W}{W_c}$	C_{B_o}	$\frac{\Delta W}{W_c}$	C_{B_o}	$\frac{\Delta W}{W_c}$	C_{B_o}	$\frac{\Delta W}{W_c}$	C_{B_o}	$\frac{\Delta W}{W_c}$		
0.20	.037	.06	.075	.12	.112	.18	.150	.24	.187	.30	1.60	1.28
.30	-	-	.093	.13	-	-	.186	.26	-	-	1.40	1.21
.40	.058	.07	.117	.14	.175	.21	.233	.28	.292	.35	1.20	1.12
.50	-	-	.15	.15	-	-	.30	.30	-	-	1.00	1.00
.60	.10	.08	.20	.16	.30	.24	.40	.32	.50	.40	.80	.85
.70	-	-	.28	.17	-	-	.57	.34	-	-	.60	.68
.80	.225	.09	.45	.18	.676	.27	.90	.36	1.12	.45	.40	.48

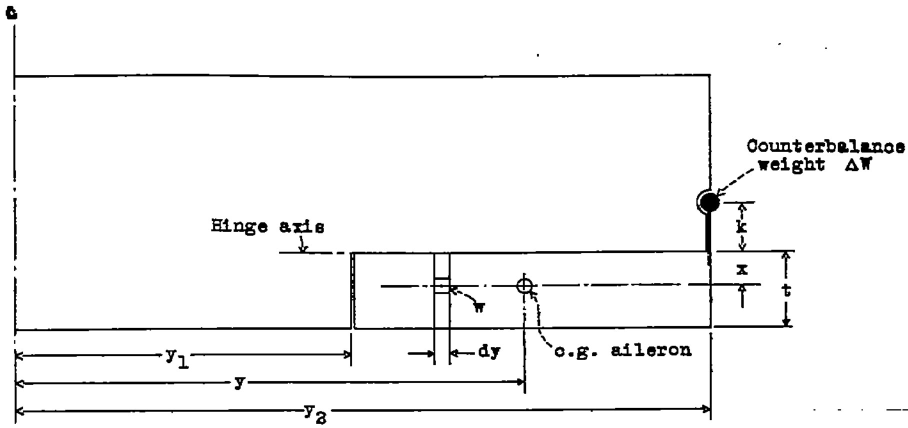

Figure 1.- Simple aileron.

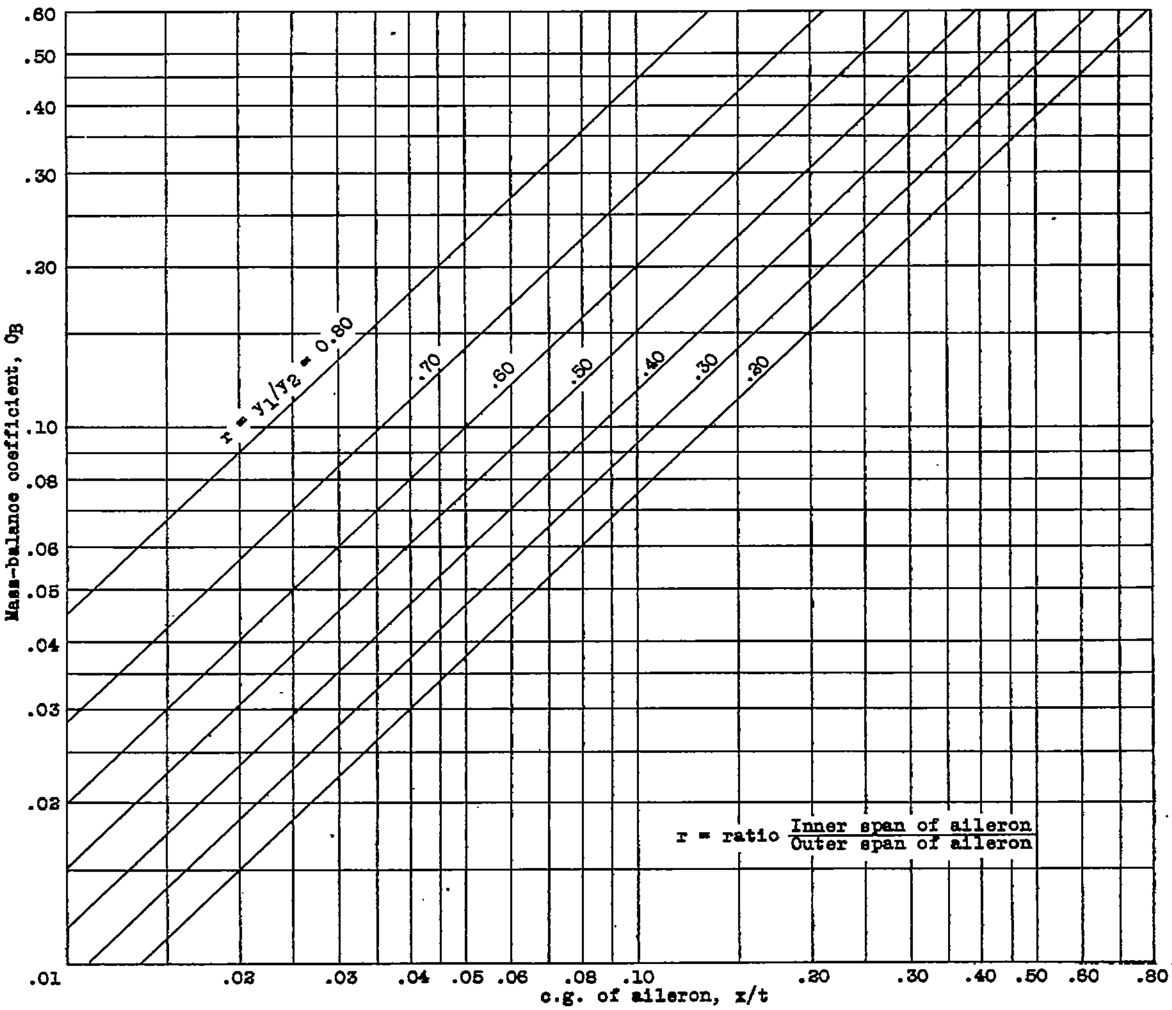

Figure 2.

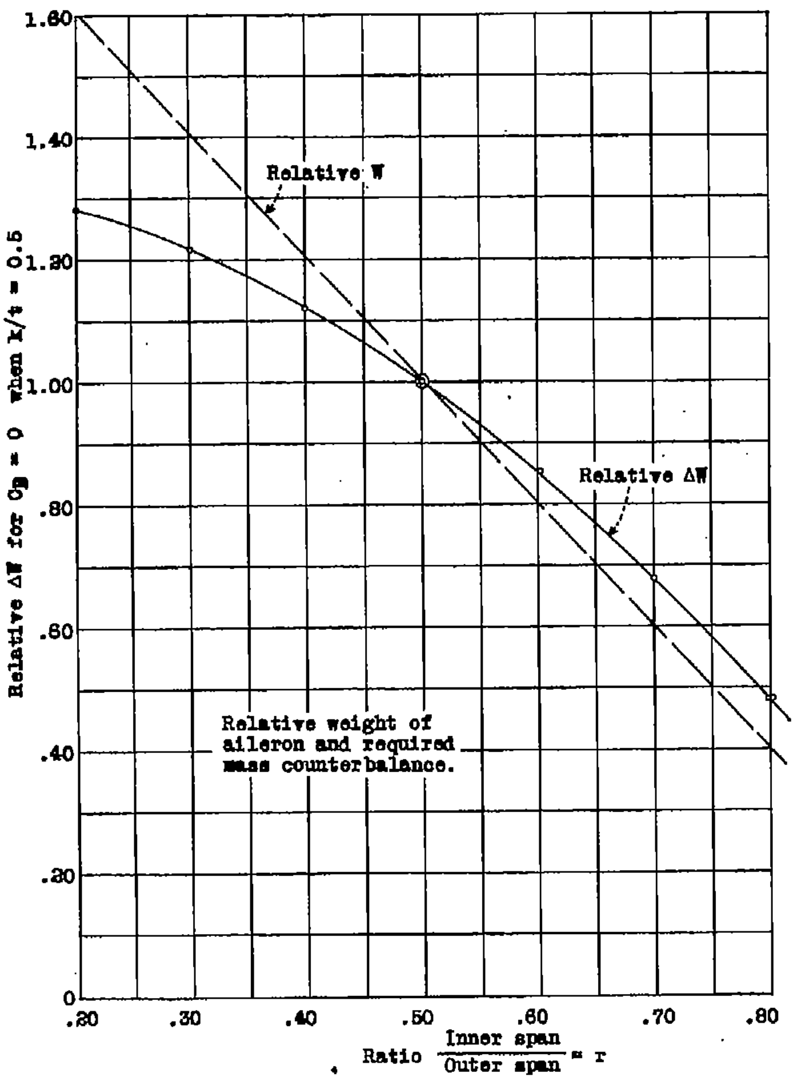

Figure 3

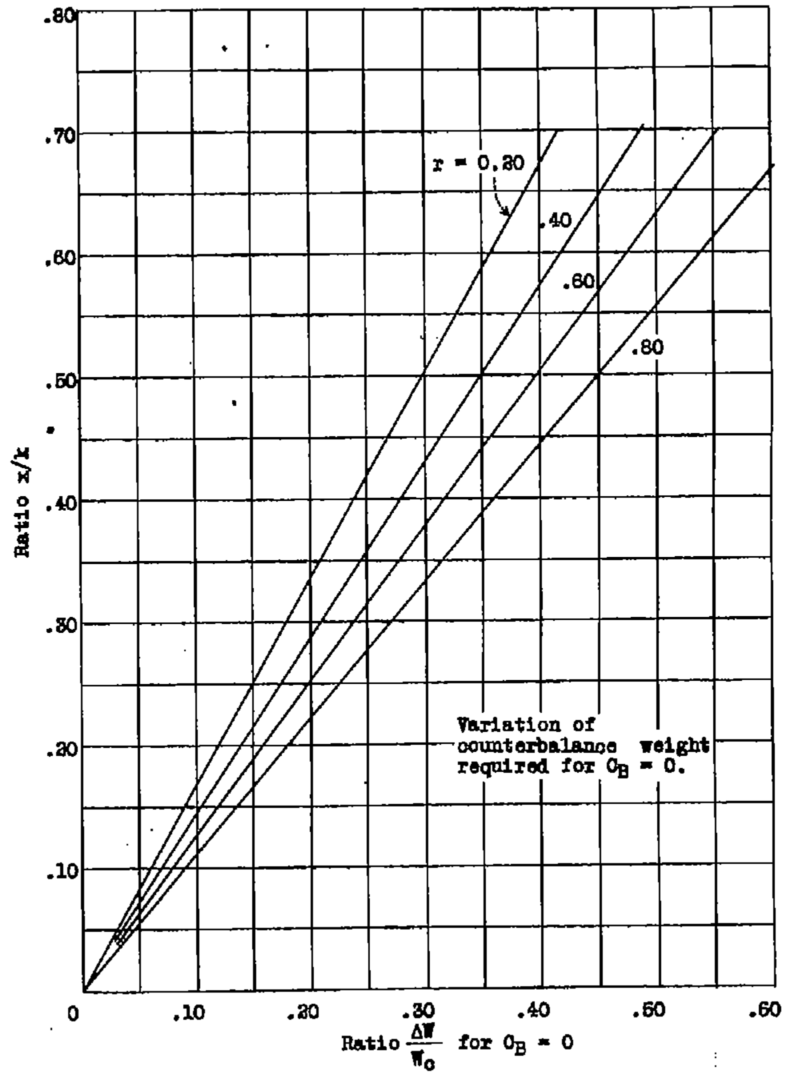

Figure 4