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By W. A. Wildhack
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SUMMARY

The theoretical basis of calculation of pressure drop in tubing is reviewed briefly. The effect of pressure drop in connecting tubing upon the operation and indication of aircraft instruments is discussed. Approximate equations are developed, and charts and tables based upon them are presented for use in designing installations of altimeters, air-speed indicators, rate-of-climb indicators, and air-driven gyroscopic instruments.

INTRODUCTION

Gyroscopic instruments used in aircraft are commonly driven by air, suction being supplied by either venturi tubes or vacuum pumps. For reliable performance, the rotors of such instruments should be operated at or above a specified speed. The suction-speed characteristics of the instruments and of the suction sources are usually known. In order to predict the performance of instruments in any particular installation or to plan a satisfactory installation, it is necessary to know also the pressure drop in the connecting tubing for various flows.

The indications of aircraft instruments actuated by air pressure alone, such as altimeters, air-speed indicators, and rate-of-climb indicators may be influenced by pressure lag in the tubing connecting them to the static or pitot heads. When the altitude of the airplane is changing, the air pressure at the pitot and static heads is also changing, and therefore air must flow to or from the instruments through the connecting tubing in order to maintain pressure equilibrium. Similarly, when the air speed is changing, flow of air occurs in the tubing connecting the pitot head to the diaphragm capsule of the air-speed indicator. While this air flow is in progress, a pressure drop exists between the two ends of the tubing, resulting in a lag of pressure at one end behind that at the other end. It is therefore desirable in installing such instruments to select a large enough size of tubing to make the pressure lag small. Also in interpreting flight test results, it may be important to be able to compute what the pressure lag is under particular conditions.

It is the purpose in this paper to develop graphical methods of presenting the relations existing between the various factors that determine the pressure drop in the lines and the pressure lag in the instruments. This report has been prepared with the financial assistance of the National Advisory Committee for Aeronautics.

THEORETICAL

The use of dimensional analysis offers the easiest approach to the general problem of flow of fluids in tubing. A clear and complete account of this method of analyzing physical problems may be found in reference 1; and equations applying specifically to this problem, with experimental data and other references, are given in reference 2. Briefly, the application of the method consists in forming dimensionless combinations of the various physical quantities that may be involved in the problem. Some functional relation exists between these dimensionless combinations but, in general, its exact form must be determined by some other means, usually by experiment.

In the development of the equations the following general notation will be used; other symbols will be defined when used in the text.

Symbol	Name of quantity
μ,	viscosity.
ρ,	density.
V,	velocity.
D,	tube diameter.
L,	tube length.

Symbol	Name of quantity
P,	pressure.
P', $\frac{dP}{dL}$,	pressure gradient along tube
Q,	volume rate of flow.
M,	mass rate of flow.
$\frac{DV\rho}{\mu}$, R.N.,	Reynolds Number.
T,	absolute temperature.
H,	altitude
t,	time.
R,	time rate of change of pressure.

Experiment verifies the supposition that the factors which influence pressure drop caused by the flow of a fluid in a straight tube are the viscosity, density, and mean velocity of the fluid, and the length, bore, and roughness of the tubing. Since experiments on the flow of fluids in commercial brass and copper tubing show the latter to act practically like smooth pipe, the roughness factor will be omitted from consideration. Five independent quantities are left, from which may be formed two dimensionless combinations. The functional relation may be expressed in the following form:

$$\frac{DP'}{\rho V^2} = f\left(\frac{DV\rho}{\mu}\right) \qquad (1)$$

The form of the function must be determined by experiment. It cannot be simply expressed in a single equation, since it is applied to two distinct types of flow - laminar and turbulent - but is given quite exactly for values of Reynolds Number $DV\rho/\mu$, up to about 2,500, by

$$\frac{DP'}{\rho V^2} = 32\left(\frac{DV\rho}{\mu}\right)^{-1} \qquad (2)$$

The flow in this case is laminar.

For values of Reynolds Number between about 3,000 and 100,000, the function for commercial tubing closely follows the equation

$$\frac{DP'}{\rho V^2} = \frac{1}{6}\left(\frac{DV\rho}{\mu}\right)^{-1/4} \tag{3}$$

Here the flow is turbulent.

These equations apply rigorously only for straight tubing. The effect of ordinary curvatures, where the radius of the bend is not less than, say, 20 times the radius of the tube, is negligible for very small Reynolds Numbers but may become very large for flows near the critical value of the Reynolds Number at which the flow changes from laminar to turbulent. In the domain of turbulent flow, the effect is again small.

Preliminary computation shows that in practice, for instruments which are actuated by pressure alone, the conditions of installation are usually such that the Reynolds Number is of the order of a few hundred while in gyroscopic instrument installations the usual Reynolds Number is well above the critical value. The formulas given will therefore hold approximately for tubing in aircraft, if there are no kinks or bends of small radius and no reduction of the cross-sectional area. A simple criterion will be developed later for determining whether the flow in any given case is laminar or turbulent.

PRESSURE LAG IN LINES OF AIRCRAFT INDICATORS, ALTIMETERS, AND RATE-OF-CLIMB INDICATORS

The expression for the pressure lag is derived from equation (2) which may be solved for P' and integrated over a length L, giving as the pressure drop:

$$P_1 - P_2 = 32\,\frac{VL\mu}{D^2} \tag{4}$$

In making this integration it is assumed: (a) that the flow is isothermal, and (b) that the pressure drop in the tubing is small compared to the pressure at either end. These assumptions are quite permissible in the usual installations of pressure-type instruments where the tubes are of metal and the maximum pressure drops are of the order of one percent of the atmospheric pressure. Since $Q = \frac{\pi D^2 V}{4}$, this may be written in the more familiar form:

$$\frac{P_1 - P_a}{L} = \frac{128\ Q\ \mu}{\pi\ D^4} \qquad (5)$$

This is Poiseuille's equation for laminar flow.

An installed air-speed indicator or altimeter may, for the purposes of analysis, be considered to consist of a chamber connected to a region of changing pressure by a line of tubing. The rate of change of air pressure in the chamber will depend on the rate of air flow through the tube. For simplicity, a constant rate of change of pressure at the open end of the tube will be assumed.

Let P_c be the pressure in chamber

C, volume of chamber

P_a, initial value of the ambient pressure at the outlet end of the tube

R, time rate of change of the ambient pressure at the outlet end of the tube

t, time, measured from the beginning of change

The pressure difference between the two ends of the tube, or pressure lag, at any time t, will be $P_a + Rt - P_c$. Then by equation (5),

$$P_a + Rt - P_c = \frac{128 Q\ \mu\ L}{\pi\ D^4} \qquad (6)$$

The mass rate of flow $Q\rho$, is also the rate of change of the mass of air in the chamber $\frac{d}{dt}(C\rho)$; and since ρ is proportional to the pressure,

$$Q = \frac{C}{P_a}\frac{dP_c}{dt}$$

if Q is measured at the pressure P_a, and the volume of the tube is neglected. Substituting in equation (6) for Q

$$P_a + Rt - P_c = \frac{128\ \mu\ LC}{\pi\ D^4 P_a} \frac{dP_c}{dt} \qquad (7)$$

For brevity, set

$$\frac{128\ \mu\ LC}{\pi\ D^4\ P_a} = \lambda \qquad (8)$$

Equation (7) can then be written

$$\frac{dP_c}{dt} + \frac{1}{\lambda} P_c = \frac{1}{\lambda} P_a + \frac{1}{\lambda} Rt \qquad (9)$$

If λ is considered as a constant, a solution of this differential equation is:

$$P_c = P_a + Rt - \lambda R + K e^{-t/\lambda} \qquad (10)$$

and since $P_c = P_a$ when $t = 0$, the constant $K = \lambda R$. Setting the pressure lag $P_a + Rt - P_c = \Delta P$, equation (10) becomes

$$\Delta P = \lambda R (1 - e^{-t/\lambda}) \qquad (11)$$

Ordinarily the tubing is chosen so that λ, called the lag factor, is of the order of one second or less, in which case the exponential or transient term for all practical purposes will vanish within a few seconds. The steady state will be attained in most maneuvers. It is therefore usually necessary to consider only steady state conditions. Weems (reference 3) has considered the more general case where the pressure does not change linearly with time.

Assuming that the exponential term is negligible, equation (11) becomes:

$$\frac{\Delta P}{R} = \lambda \qquad (12)$$

Now it is seen that λ has the dimensions of time; it is, in fact, the time interval by which the pressure in the instrumemt lags behind the pressure at the open end of the tube. If the same units of pressure are used for ΔP and R, and if R is expressed in terms of the pressure change per second, λ will be in seconds.

These relations are shown graphically in figure 1, where an assumed linear increase of the ambient pressure with time is represented by the upper line, and the variation of pressure within the instrument is shown by the lower line. The pressure difference between the two lines at any given time is the pressure drop in the tubing. The time difference between the two lines at a given pressure is $\lambda (1 - e^{-t/\lambda})$, or simply λ after the two lines have become parallel, which occurs a short time after the pressure increase begins. The slope of the upper line is the rate of change in the pressure, denoted by the constant R, and the relation $\lambda R = \Delta P$ is evident from the figure.

Lag factor.- The lag factor is defined by equation (8):

$$\lambda = \frac{128 \mu LC}{\pi D^4 P_a} \qquad (8)$$

in which μ and P_a are the viscosity and pressure of the air, respectively; L and D are the length and internal diameter of the tubing; and C is the volume of the chamber or chambers to which the tubing is connected.

It is seen that λ is inversely proportional to the air pressure P_a, and hence increases with altitude. The ratios of the lag factor at various altitudes to that at 5,000 feet, are shown in table I where P_5 is the pressure at an altitude of 5,000 feet, P_H is the pressure at altitude H, and P_o is the pressure at sea level. Other ratios may be computed from pressure-altitude tables (reference 4).

TABLE I

Altitude, H	0	2,000	5,000	10,000	15,000	20,000
Pressure, P	760	706.6	632.3	522.6	446.3	349.1
Lag ratio, $\frac{P_5}{P_H}$	0.83	0.90	1.00	1.21	1.41	1.80
Pressure ratio, $\frac{P_o}{P_H}$	1.0	1.08	1.20	1.46	1.70	2.18

Also, since C is the total volume of the individual chambers of the instruments to which the tubing is connected, the lag factor, and therefore the lag in indication, will depend on the number and types of instruments which are connected to the line. Approximate values of the volumes of instruments in use at the present time may be taken as follows:

Instrument	Chamber volume
Sensitive altimeter	225 cm^3
Rate-of-climb indicator (new pioneer)	225 cm^3
Air-speed indicator (static side)	160 cm^3
Air-speed indicator (pitot side)	30 cm^3

Table II gives the ratios of the total volume of various instrument combinations C, to the volume of one altimeter C_a.

TABLE II

Altimeters	Instrument combinations: Air-speed indicators	Instrument combinations: Rate-of-climb indicators	C/C_a
1	-	-	1.0
1	1	-	1.7
1	1	1	2.7
2	1	-	2.7
2	2	-	3.4
-	1 (static side)	-	.70
-	1 (pitot side)	-	.15

For convenience in making computations, equation (8) may be written:

$$\lambda = \lambda_{5a} \frac{P_5}{P_H} \frac{C}{C_a} \tag{13}$$

where λ_{5a} is the value of λ for one altimeter at an altitude of 5,000 feet. The values of the last two terms may now be obtained from tables I and II. In figure 2 the value of λ_{5a} is plotted against length of line for several sizes of tubing. The internal diameters of the tubes are assumed to be 0.152, 0.305, and 0.457 centimeter for 1/8-, 3/16-, and 1/4-inch nominal sizes of tubing, respectively. The viscosity of air is taken as 180×10^{-6} poise; P_5 equals 8×10^5 dynes per cm^2; and C_a is taken as 225 cm^3, the volume of one altimeter. In using the chart it must be kept in mind that the value of λ for a combination of instruments is the indicated value for one altimeter as given by the chart, multiplied by the appropriate factors in tables I and II. For combinations of instruments other than those given, the factor is the total volume in cubic centimeters divided by 225.

No consideration has thus far been given to the effect of the volume of the tubing on the lag factor. Theoretically, the value of C in equations (8) and (13) should include one-half the volume of the tubing in addition to the volumes of the instrument chambers. However, since the uncertainty in the lag factor due to uncertainty in other quantities, such as the tube diameter, the effect of bends in the tubing, etc., may be greater than the correction for the volume of the tube, the correction usually need not be made except for extremely long tubes of the larger sizes.

<u>Experimental determination of lag factor</u> - (a) <u>Theory</u>.- The lag factor λ, may be obtained experimentally for any combination of line and instruments by subjecting the instruments and line to an initial pressure differential and noting the time required for the pressure differential to fall to $1/e$ of its initial value. This can be shown as follows:

Let P_a be the ambient pressure

P_1, initial instrument pressure (when $t = 0$)

P, instrument pressure at time t

$P_a - P$, differential pressure at the two ends of the tube

Then, as in equation (7)

$$\frac{dP}{dt} = (P_a - P) \frac{\pi D^4}{128 \mu L} \frac{P_a}{C} \tag{14}$$

where C is the volume of the part of the instruments connected to the line.

In the usual notation

$$\frac{dP}{P_a - P} = \frac{dt}{\lambda} \tag{15}$$

This gives, upon integration, since $P = P_1$ when $t = 0$

$$(P_a - P) = (P_a - P_1) e^{-t/\lambda} \tag{16}$$

$$\text{If } t = \lambda, \quad \frac{P_a - P}{P_a - P_1} = \frac{1}{e} = 0.365 \tag{17}$$

$$\text{If } t = 2\lambda, \quad \frac{P_a - P}{P_a - P_1} = \frac{1}{e^2} = 0.135 \tag{17a}$$

The time required for the differential pressure P_a-P to change from $(P_a - P_1)$ to $0.365 \ (P_a - P_1)$ is λ and to change to $0.135 \ (P_a - P_1)$ is 2λ. The value of λ thus determined may be multiplied by the ratio of the air pressure at which it was determined to the air pressure at any other altitude to obtain the value of λ for the other altitude.

Since the equation for laminar flow was assumed in this development, care must be taken to have the initial pressure difference small enough so that the flow in the tubing will be laminar. Criteria for determining the type of flow may be developed from equation (2)

$$\frac{DP'}{\rho V^2} = \frac{D}{\rho V^2} \frac{\Delta P}{L} = 32 \left(\frac{DV\rho}{\mu}\right)^{-1} \tag{2}$$

Multiplying both sides by $\left(\frac{DV\rho}{\mu}\right)^2$,

$$D^3 \frac{\Delta P}{L} = 32 \frac{\mu^2}{\rho} \left(\frac{DV\rho}{\mu}\right) = 32 \frac{\mu^2}{\rho} \text{ R.N.} \tag{18}$$

The critical, or transition value of R.N. may be taken conservatively as 2,000. When allowance is made for the choice of units usual in aeronautics and numerical values are inserted for ρ and μ, equation (18) yields as the condition for laminar flow that:

$$D^3 \frac{\Delta P}{L} \text{ must be less than } 9.4 \times 10^{-5} \qquad (19)$$

when D is in inches and $\frac{\Delta P}{L}$ is the pressure drop in inches of mercury per foot of tubing.

For 3/16-inch tubing (inside diameter 0.12 inch), $\frac{\Delta P}{L}$ must be less than 0.054 inch of mercury per foot for laminar flow. The change in indication of the altimeter corresponding to this pressure, near sea level, is about 50 feet. In experimental determination of the lag factor λ, the initial change of reading of the altimeter should not exceed 50 feet per foot of length of the static tube for 3/16-inch tubing.

For 1/4-inch tubing (inside diameter 0.18 inch), the initial change of reading of the altimeter in feet should not exceed 15 times the length of the static tube in feet. If the initial pressures are higher than the values so computed, the value of λ obtained will be somewhat too large.

<u>(b) Experimental procedure.</u>- The lag factor λ_s for the system comprising the static line and the instruments connected thereto may be determined simply by applying to the open end of the static line a suction sufficient to change the altimeter reading by 500 feet, releasing the suction, and noting the time required for the altimeter indication to decrease 318 feet. This time is λ_s. If λ_s is small, it may be determined more accurately as half the time required for the altimeter to decrease its indication 435 feet. Alternatively, the time lag factor for the system can be determined by applying a suction to the static line sufficient to obtain a reading of 100 miles per hour or knots on the air-speed indicator, releasing this suction, and noting the time for the reading to decrease to 61 miles per hour or knots. This time is λ_s. One-half of the time for the reading to decrease to 35 miles per hour or knots also gives λ_s.

The lag factor for the pitot pressure line λ_p, may be found by applying a pressure to it sufficient to cause the air-speed indicator to read 100 miles per hour or knots, and determining the time required for the indication to decrease to a reading of 35 miles per hour or knots after the pressure is released. λ_p is one-half of this time.

The system must be leaktight in making these tests. The factor so found is for the altitude at which the test is made; to secure its value at other altitudes, use table I (p. 7).

Lag in altimeter indication.- For the altimeter,

$$\frac{\Delta P}{R} = \lambda_s \tag{20}$$

where λ_s is the lag factor for the system consisting of the static tube and the instruments connected to it. In order to obtain the lag in indication of the altimeter ΔI_a, in feet, equation (20) may be written

$$\frac{\Delta P}{R} = \frac{\Delta I_a}{\frac{dH}{dt}} = \lambda_s \tag{21}$$

or

$$\Delta I_a = \lambda_s \frac{dH}{dt} \tag{22}$$

since the air pressure varies nearly linearly with the altitude H for short intervals. The term dH/dt is the rate of climb in feet per second.

For any service installation the value of the lag in indication ΔI_a at any altitude and rate of climb may be computed by formula (22). The value of λ_s can be computed by equation (13), using the factors from table I and table II and the chart in figure 2, or it may be determined experimentally for a given installation by the method described in the preceding section.

Lag in air-speed indication.- The approximate equation for the pressure difference developed by the pitot-static head is

$$p = K \tfrac{1}{2} \rho V^2 \tag{23}$$

where p is the pressure difference, ρ is the air density, V is the true air speed, and K is a constant of proportionality depending on the choice of units. The type of air-speed indicator in general use operates on this differential pressure, and is calibrated so that the indicated air speed is equal to the true air speed only when the air density is that corresponding to sea-level pressure. Thus the indicated air speed I, for steady conditions at any altitude is related to the true air speed V by the relation

$$I = V\sqrt{\frac{\rho}{\rho_o}} \qquad (24)$$

where ρ_o is the air density at sea level.

The effect of pressure lag on the indications of the air-speed indicator is somewhat complicated. First, part of the lag in indication depends upon the difference between two pressure lags - one on the static side of the instrument, one on the pitot side. The lag factors for the two sides λ_s and λ_p, respectively, usually differ because of the different volumes connected to the instrument end of the lines. Second, the rate of change of pressure is also usually different on the two sides, since the pitot pressure is affected by changes in either altitude or air speed, while the static pressure is affected only by changes in altitude. Third, the change in indication of the air-speed indicator corresponding to a given small change in differential pressure is not the same for all air speeds, since the indication is proportional to the square root of the pressure difference, as shown by a comparison of equations (23) and (24).

Let I, as defined by equation (24), be called the "true indicated air speed"; denote the indication of the instrument, when a pressure drop occurs in either or both of the lines, by I_1; and designate the differential pressures corresponding to I and I_1 as p and p_1, respectively, and the air pressures in the pitot and static heads by P_p and P_s, respectively. Since the indication of the instrument depends on the pressure differential across it, which is that developed at the pitot-static head minus the line drops, the equation connecting I and I_1 may be derived from expressions for p and p_1. These may be written as follows, using equation (12):

$$p = P_p - P_s \qquad (25)$$

$$p_1 = P_p - \lambda_p \frac{d}{dt}(P_s + p) - \left(P_s - \lambda_s \frac{dP_s}{dt}\right) \qquad (26)$$

From these equations:

$$p_1 = p - \lambda_p \frac{d}{dt} (P_s + p) + \lambda_s \frac{dP_s}{dt} \qquad (27)$$

Eliminating V from equations (23) and (24),

$$p = \frac{K \rho_o}{2} I^2 \qquad (28)$$

Similarly, $$p_1 = \frac{K \rho_o}{2} I_1^2$$

Substituting for p and p_1 in equation (27),

$$\frac{K\rho_o}{2} I_1^2 = \frac{K\rho_o}{2} I^2 - \lambda_p \frac{dP_s}{dt} - \lambda_p \frac{K\rho_o}{2} \frac{dI^2}{dt} + \lambda_s \frac{dP_s}{dt} \qquad (29)$$

Differentiating, substituting, and rearranging,

$$I^2 - I_1^2 = \frac{2}{K\rho_o} (\lambda_p - \lambda_s) \frac{dP_s}{dt} + 2\lambda_p I \frac{dI}{dt} \qquad (30)$$

The term $I^2 - I_1^2$ may be factored into $(I - I_1)$ $(I + I_1)$ and since $I - I_1$, which is the lag in indication ΔI, will be small compared with I or I_1:

$$I^2 - I_1^2 = 2I \, \Delta I \qquad (31)$$

Making use of this approximate relation:

$$\Delta I \doteq \frac{\lambda_p - \lambda_s}{K \rho_o I} \frac{dP_s}{dt} + \lambda_p \frac{dI}{dt} \qquad (32)$$

Still another substitution helps to make the equation somewhat more usable. The rate of change of pressure is related to the rate of change of altitude by the approximate relation, based on an isothermal atmosphere:

$$\frac{dP_s}{dt} = - k_1 P_s \frac{dH}{dt} \qquad (33)$$

where, for an air temperature of 0° C., $k_1 = 3.8 \times 10^{-5}$ when the pressure is expressed in millimeters of mercury, and altitude is in feet.

The value of $K\rho_o$ in equation (32) is 18.4×10^{-4} when I is in miles per hour and P is in millimeters of mercury.

When these numerical values are inserted, equations (32) and (33) combine to give the general form of the expression for lag:

$$\Delta I = 0.021\ (\lambda_s - \lambda_p)\ \frac{P_s}{I}\ \frac{dH}{dt} + \lambda_p\ \frac{dI}{dt} \qquad (34)$$

The two terms in the right-hand member of equation (34) may be called, respectively, the "climb" and the "acceleration" terms. It should be remembered that λ_s and λ_p refer to the lag factors for a particular installation, in which one or more air-speed indicators and altimeters may be connected to the lines, and are given by equations (8) or (13).

Table III gives the values of the climb and acceleration terms, as well as the total lag in indication in an average installation for various assumed conditions. The lags are computed for values of λ_s and λ_p of 0.6 and 0.1 second, respectively.

The lag factor for the pitot line is much less than that for the static line because the volume at the instrument end of the pitot line is much smaller than that at the end of the static line while the tubing size is usually the same for both lines. In table III the ratio of the lag factors, that is, of the volumes, for the two lines was assumed to be 1 to 6. In actual installations the volume at the end of the pitot line is likely to be relatively even less than given by this ratio. As indicated in table III, under most conditions of use the lag factor λ_p may be neglected without great loss in accuracy. In this case equation (34) becomes

$$\Delta I = 0.021\ \lambda_s\ \frac{P_s}{I}\ \frac{dH}{dt} \qquad (35)$$

TABLE III

Lag in Indication of Air-Speed Indicator

for Representative Conditions

$$\Delta I = 0.021\ (\lambda_s - \lambda_p)\ \frac{P_s}{I}\ \frac{dH}{dt} + \lambda_p\ \frac{dI}{dt} = C + A$$

λ_s = 0.6 second λ_p = 0.1 second

	Assumed conditions						
	Indicated air speed I m.p.h.	Air pressure P_s mm Hg	Rate of climb $\frac{dH}{dt}$ f.p.s.	Air speed acceleration $\frac{dI}{dt}$ m.p.h. per second	Climb term C m.p.h.	Acceleration term A m.p.h.	Total lag ΔI m.p.h.
Take-off	60	760	0	10	0	1	1
Just after take-off	80	760	30	10	3	1	4
Steady climb	150	600	30	0	1	0	1
Steady climb	150	300	15	0	0.3	0	0.3
Level flight	*	*	0	10	0	1	1
Descent	200	600	-30	10	-1	1	0
Start dive	200	500	-350	40	-9	4	-5
Steady dive	400	600	-400	0	-6	0	-6
Zoom after dive	300	600	50	-20	1	-2	-1
Landing	60	760	-15	-10	-2	-1	-3

A negative ΔI means that the indication is greater than the true value.

*The lag in this case is the same for all values of I and P_s.

For an approximation which is sometimes useful in computing lags at altitudes below 10,000 feet, equation (35) may be simplified by expressing the rate of change of altitude in terms of the air speed and the angle of climb, neglecting the quantity $\sqrt{\frac{P_5}{P_s}} - 1$. The error in the computed lag due to this approximation is less than 5 percent at 2,000 feet, zero at 5,000 feet, and less than 10 percent at 10,000 feet. This simplified expression is

$$\Delta I = 21 \lambda_5 \sin \theta \qquad (35a)$$

where ΔI is the lag in miles per hour, λ_5 is the lag factor of the installation for an altitude of 5,000 feet, and θ is the angle of climb or dive, measured positively upward from the horizontal.

In diving, the negative value of θ gives a negative lag, which means that the air-speed indication is too high; a positive lag, in climbing, means that the indication is low.

The lag in indication of an installed air-speed indicator can be computed by formula (34) or (35) for any condition of use, provided that the lag factors are known. These may be computed as described in the section on "Lag Factor" using the chart given in figure 2, together with the appropriate value for the volume factor from table II and altitude factor from table I, or they may be determined experimentally by the method previously described.

Lag in rate-of-climb indication.- The third pressure instrument in more or less common use is the rate-of-climb indicator. This instrument utilizes a pressure differential across a porous plug or fine tube produced by the lag phenomena already described. Thus its operation depends only on the rate of change of pressure.

As shown in figure 1, the rate of change of pressure at the instrument end of the static line is equal to that at the open end a short time after the beginning of the change. Thus the static line and the other instruments connected to the rate-of-climb indicator have only a transient effect upon its indication. Therefore, for quantitative measurement of a constant rate of climb, the accuracy of the rate-of-climb indicator is unaffected by the other instruments and line. When used as a null indicator for level flight, however, the sensitivity of the instrument is slightly reduced owing to the increased time lag of the pressure in the case behind the pressure at the open end of the static tube.

As the lag constant of the instrument itself when not connected to a line is rather large, from 2.5 to 5 seconds, and the added lag due to the line and other instruments is usually less than one second, the decrease in sensitivity is not very important.

Computation of lag in indication.- For convenience in computing the lag in indication of installed instruments, the procedure is summarized here, and the necessary formulas are repeated.

1. Determine λ_s:

(a) By calculation;

$$\lambda_s = \lambda_{5_a} \frac{P_5}{P_H} \frac{C_s}{C_a} \tag{13}$$

Obtain λ_{5_a} from figure 2; $\frac{P_5}{P_H}$ from table I; $\frac{C_s}{C_a}$ from table II; or,

(b) By experiment;

λ_s = time for pressure differential in instrument and tube to decrease to $1/e$ of its initial value. See section on "Experimental Determination of Lag Factor."

2. For altimeters:

$$\Delta I = \lambda_s \frac{dH}{dt} \tag{22}$$

where ΔI is the lag in feet and $\frac{dH}{dt}$ the rate of climb in feet per second.

3. For air-speed indicators:

(a) $$\Delta I = 0.021 \lambda_s \frac{P_s}{I} \frac{dH}{dt} \tag{35}$$

where ΔI is the lag in miles per hour, I is the indicated air speed in miles per hour, P_s is the static pressure in millimeters of mercury, and dH/dt is the rate of climb in feet per second; or,

(b) as a good approximation below 10,000 feet,

$$\Delta I = 21\ \lambda_{5_a} \frac{C_s}{C_a} \sin \theta \qquad (35a)$$

where ΔI is the lag in indication in miles per hour and θ is the angle of climb.

The negative values of ΔI obtained in descent merely mean that the indicated air speed or altitude is too great. Positive values of ΔI mean that the indications are too low.

The above formulas hold good only if the connections and instruments are airtight. If the instruments leak appreciably the pressure in the instrument cases may follow the pressure in the cockpit more closely than that at the outer end of the static tube.

Selection of tubing size.- The procedure to be followed in determining the size of tubing that should be used to obtain a desired performance is summarized below:

1. Determine the value of λ_{5_a} corresponding to the allowable lag ΔI_a, in the indication of the altimeter for a maximum rate of climb or dive dH/dt.

$$\lambda_{5_a} = \frac{\Delta I_a}{\frac{dH}{dt}} \frac{P_H}{P_5} \frac{C_a}{C_s} \qquad (21a)$$

The values of $\frac{P_H}{P_5}$ and $\frac{C_a}{C_s}$ may be obtained from tables I and II.

2. Determine the value of λ_{5_a} corresponding to the the allowable lag ΔI, in the indication of air speed in miles per hour. The following form of equation (35) is convenient:

$$\lambda_{5_a} = 0.075\ \Delta I \frac{I}{\frac{dH}{dt}} \frac{C_a}{C_s} \qquad (35b)$$

where I is the indicated air speed in miles per hour, and dH/dt is the rate of climb or dive in feet per second, at which ΔI is not to exceed the given value. The reciprocal of the ratio C_a/C_s may be obtained from table II.

3. Enter figure 2 with the smaller of the two values of λ_{5_a} obtained above, and with the length of tubing, to obtain the proper size of tubing.

As an example, assume that an altimeter, an air-speed indicator, and a rate-of-climb indicator are connected to a static head by a 20-foot line. The lag in indication of the altimeter at 700 millimeters of mercury and at a rate of descent of 30 feet per second is not to exceed 20 feet. The lag in the air-speed indication is not to exceed two miles per hour at an air speed of 50 miles per hour, a rate of descent of 15 feet per second, and at an air pressure of 760 millimeters of mercury.

1. For the altimeter:

$$\lambda_s = \frac{\Delta I_a}{\frac{dH}{dt}} = \frac{20}{30} = 0.67 \text{ second}$$

$$\lambda_{5_a} = \lambda_s \frac{P_H}{P_5} \frac{C_a}{C_s} =$$

$$0.67 \times \frac{700}{632} \times \frac{225}{225 + 225 + 160} = 0.27 \text{ second}$$

2. For the air-speed indicator:

$$\lambda_{5_a} = 0.075 \; \Delta I \frac{I}{\frac{dH}{dt}} \frac{C_a}{C_s}$$

$$= 0.075 \times 2 \times \frac{50}{15} \times \frac{1}{2.7} = 0.19$$

3. Entering figure 2 with 0.19 second and 20 feet, it is seen that tubing having an internal diameter of 0.11 inch is satisfactory, or practically, 3/16-inch tubing should be used.

A table showing the tubing length for various arbitrary values of lag in indication and tubing sizes is given in reference 5.

PRESSURE DROP IN LINES OF SUCTION-DRIVEN INSTRUMENTS

For the instruments which operate on a steady flow of air, the flow will usually be turbulent. For this condition, equation (3) is to be used to compute the pressure drop in the lines.

$$\frac{DP'}{\rho V^2} = \frac{1}{6}\left(\frac{DV\rho}{\mu}\right)^{-1/4} \tag{3}$$

Solving for P', the pressure drop per unit length,

$$P' = \frac{1}{6}\,\mu^{1/4}\,\rho^{3/4}\,V^{7/4}\,D^{-5/4} \tag{36}$$

or, since the mass flow $M = \rho V \frac{\pi D^2}{4}$, V can be eliminated, giving

$$P' = \frac{1}{6}\left(\frac{4}{\pi}\right)^{7/4}\mu^{1/4}\,\rho^{-1}\,D^{-19/4}\,M^{7/4} \tag{37}$$

Over the range of temperatures encountered in practice,

$$\rho = \rho_o\,\frac{P}{P_o}\,\frac{T_o}{T}, \quad \text{and} \quad \mu = \mu_o\,\frac{T}{T_o} \tag{38}$$

where the subscripts refer to conditions at 20° C. and normal atmospheric pressure, 760 millimeters of mercury. Substituting,

$$P' = \frac{dP}{dL} = \frac{1}{6}\left(\frac{4}{\pi}\right)^{7/4}\frac{P_o}{P}\,\frac{\mu_o^{1/4}}{\rho_o}\left(\frac{T}{T_o}\right)^{5/4}\frac{M^{7/4}}{D^{19/4}} \tag{39}$$

or, separating variables,

$$P\,dP = \frac{1}{6}\left(\frac{4}{\pi}\right)^{7/4}P_o\,\frac{\mu_o^{1/4}}{\rho_o}\left(\frac{T}{T_o}\right)^{5/4}\frac{M^{7/4}}{D^{19/4}}\,dL \tag{40}$$

Integrating over the length of the tube, assuming isothermal flow,

$$\frac{1}{2}\left(P_1^2 - P_2^2\right) = \frac{1}{6}\left(\frac{4}{\pi}\right)^{7/4} P_0 \frac{\mu_0^{1/4}}{\rho_0}\left(\frac{T}{T_0}\right)^{5/4} \frac{M^{7/4}}{D^{19/4}} L \quad (41)$$

This equation gives the relation between the pressures P_1 and P_2 at the ends of the tube. The pressure drop $P_1 - P_2$, designated by ΔP, is not explicitly given by this equation, but we can factor $(P_1^2 - P_2^2)$ into $(P_1 - P_2)(P_1 + P_2)$ or $\Delta P (P_1 + P_2)$, and express equation (41) in a form that will permit a simple approximation:

$$\Delta P = \frac{1}{6}\left(\frac{4}{\pi}\right)^{7/4} \frac{2P_0}{P_1 + P_2}\frac{\mu_0^{1/4}}{\rho_0}\left(\frac{T}{T_0}\right)^{5/4} \frac{M^{7/4}}{D^{19/4}} L \qquad (42)$$

Now, when $P_1 = P_0$ and $T = T_0$, the term $\left(\frac{T}{T_0}\right)^{5/4} =$ 1 and the term $\frac{2P_0}{P_1 + P_2} = \frac{2P_0}{2P_0 - \Delta P}$ may be set equal to unity for values of ΔP small as compared with $2P_0$. As $P_0 = 760$ millimeters of mercury, neglecting this term will result in an error of only about 10 percent for a pressure drop of 150 millimeters of mercury. For this condition, then, we have the approximate equation:

$$\Delta P = \frac{1}{6}\left(\frac{4}{\pi}\right)^{7/4} \frac{\mu_0^{1/4}}{\rho_0} \frac{M^{7/4}}{D^{19/4}} L \qquad (43)$$

All equations given thus far are valid for any consistent system of units. It is customary in aeronautics to express pressure in inches of mercury, flow (F_s) in cubic feet per minute of air at 20° C. and 29.92 inches of mercury (760 mm of mercury), tube diameter in inches, and tube length in feet. When numerical allowance is made in the constant term for this choice of units, and the values of μ_0 (= 180×10^{-6} poise) and ρ_0 (= 0.0012 g/cm^3), are inserted, the working equation is obtained:

$$\Delta P = 9.8 \times 10^{-5} \frac{F_s^{7/4}}{D^{19/4}} L \qquad (44)$$

If a fixed value is taken for L, say 10 feet, and ΔP is plotted against F_s on a log-log chart, straight lines are obtained for various values of D. Figure 3 was prepared in this manner, and covers inside tube diameters from 0.2 to 1.0 inch, flows from 0.5 to 20 cubic feet per minute of air at 20° C. and 29.92 inches of mercury, and pressure drops from 0.1 to 2.0 inches of mercury per ten-foot length of tubing.

Correction factors.- As indicated in the development, figure 3 will give sufficiently accurate values of the pressure drop ΔP only when ΔP is small and P_1 is nearly equal to P_o. When these conditions are not fulfilled and accurate values of the pressure drop are required, equation (42) should be used. However, it will be shown later that for gyroscopic instruments, the pressure drop at or near zero altitude is sufficient to determine the size of tubing to be used.

For convenience in making more exact computations based on equation (42), a table and a chart are presented. If the total pressure drop in a given line, computed from figure 3, be designated by ΔP_c, the exact equation is:

$$\Delta P = \Delta P_c \left(\frac{T}{T_o}\right)^{5/4} \frac{2P_o}{P_1 + P_2} \qquad (45)$$

or,

$$\frac{P_1^2 - P_2^2}{2P_o} = \Delta P_c \left(\frac{T}{T_o}\right)^{5/4} \qquad (46)$$

In equation (46), P_1 is the air pressure at the instrument end of the line; P_2 at the pump end; $P_1 - P_2$ is then the pressure drop for the entire length of the line; T is the temperature of the air in absolute degrees centigrade; T_o equals 293 absolute degrees centigrade; and P_o equals 29.92 inches of mercury.

The value of the correction factor $(T/T_o)^{5/4}$ for various dentigrade temperatures t is given in table IV; the chart given in figure 4 furnishes a graphical means of solving equation (46) to obtain the true pressure drop. In the chart the ordinates are values of P_2 and the diagonals are values of P_1. The numbers on the vertical scale refer to both P_1 and P_2.

TABLE IV

Temperature Correction Factor, $\left(\frac{T}{T_o}\right)^{5/4}$

T_o = 293° absolute (20° C.)

t° C.	$\left(\frac{T}{T_o}\right)^{5/4}$	t° C.	$\left(\frac{T}{T_o}\right)^{5/4}$
-40	0.75	10	0.96
-30	.79	20	1.00
-20	.83	30	1.04
-10	.87	40	1.09
0	.92	50	1.14

The procedure in making a computation of the pressure drop $P_1 - P_2$ is as follows: ΔP_c is computed for the total length of tubing from the value obtained from figure 3 for a 10-foot length. The ratio $(T/T_o)^{5/4}$ is obtained from table IV. Enter the chart (fig. 4) with the value of the product $\Delta P_c\ (T/T_o)^{5/4}$ and P_1. The intersection of the diagonal representing P_1 with the vertical line representing the product determines the value of P_2 which is read on the ordinate scale. The procedure is illustrated by the dotted lines in figure 4. From the pressures P_1 and P_2 thus available, the true pressure drop $P_1 - P_2$, is then readily obtained.

Since the pressure drop varies inversely as nearly the fifth power of the diameter, a slight percentage uncertainty in the diameter will result in about five times as great a percentage uncertainty in the computed pressure drop. For this reason, one should not expect too exact agreement between computed and observed values of the pressure drop in ordinary tubing. Experiments have been made with copper, brass, and rubber tubing, which indicate that the agreement is usually within 10 or 15 percent when the nominal diameter is used.

It has been mentioned that, in determining the size of tubing to be used in operating gyroscopes in aircraft installations, it is usually unnecessary to determine the altitude corrections. The condition desired in this case is that the rotors of the gyroscope instruments should spin faster than a given minimum speed. It has been found by experiment that the gyroscope rotor speed is directly proportional to the volume flow through the instrument, and that the volume flow is determined by an equation of the form

$$\Delta P_i = k' \rho_i Q_i^2 \tag{47}$$

or

$$\Delta P_i = k'' P_i Q_i^2 \tag{48}$$

where P_i is the air pressure at the instrument outlet, ΔP_i the pressure drop through the instrument, and Q_i is the volume air flow measured at P_i.

In most installations a suction regulating valve is connected in the line near the instruments. This valve opens to admit air to the line when the suction exceeds a certain value, thus keeping the suction across the instruments nearly constant while allowing a large variation in flow to the suction source. It is seen from equation (48) that under this condition, with ΔP_i nearly constant, Q_i will increase with increasing altitude or decreasing pressure. As the rotor speed depends only on Q_i, it also will increase with altitude when the suction regulating valve is open.

When the suction developed by the source is not sufficient to open the regulating valve, the performance of the instrument will depend upon the characteristics of the suction source. The two most commonly used sources are venturi tubes and vacuum pumps of the displacement type. The characteristics of different venturi tubes vary so widely that quantitative computation for the general case is not feasible, but the qualitative effect of increasing altitude, with a constant indicated air speed, is to increase the volume flow through the instruments and therefore also the rotor speed. Unpublished data obtained by H. C. Sontag and D. P. Johnson have established this fact. For the vacuum pump, the volume flow as measured at the pump intake depends on only the pump speed. The volume flow when measured at the instrument is approximately the same as when measured at the pump intake, so that the speed of the rotors is nearly independent of altitude in this case.

Then, since in practice the rotor speeds will either increase with, or remain independent of, altitude it will be necessary in designing installations to consider sea-level conditions only, using the chart in figure 3.

National Bureau of Standards,
Washington, D. C., February 4, 1937.

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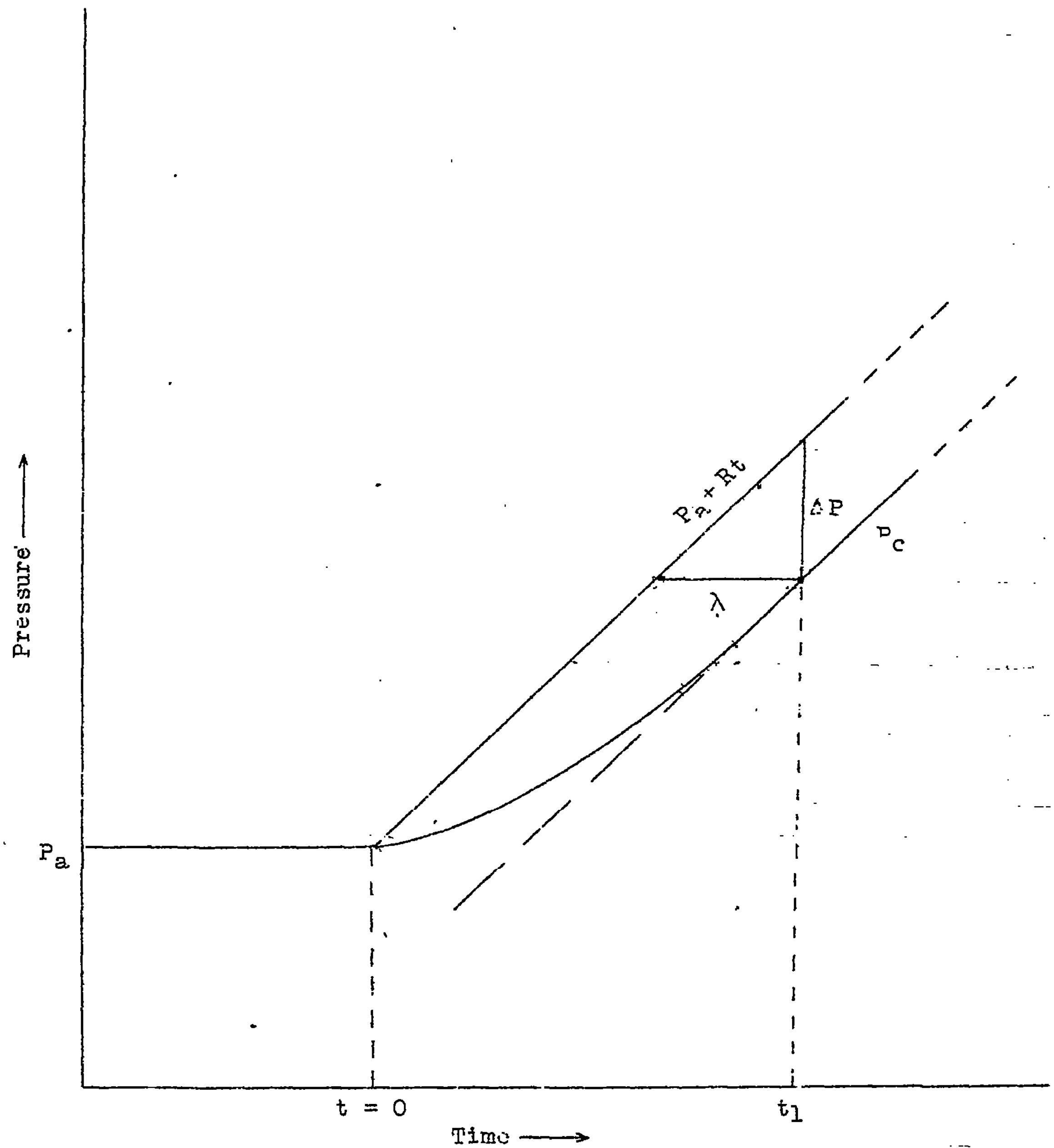

Figure 1.- Chart illustrating the relation between the pressure lag, ΔP, and the lag factor, λ, for a linear change of ambient pressure.

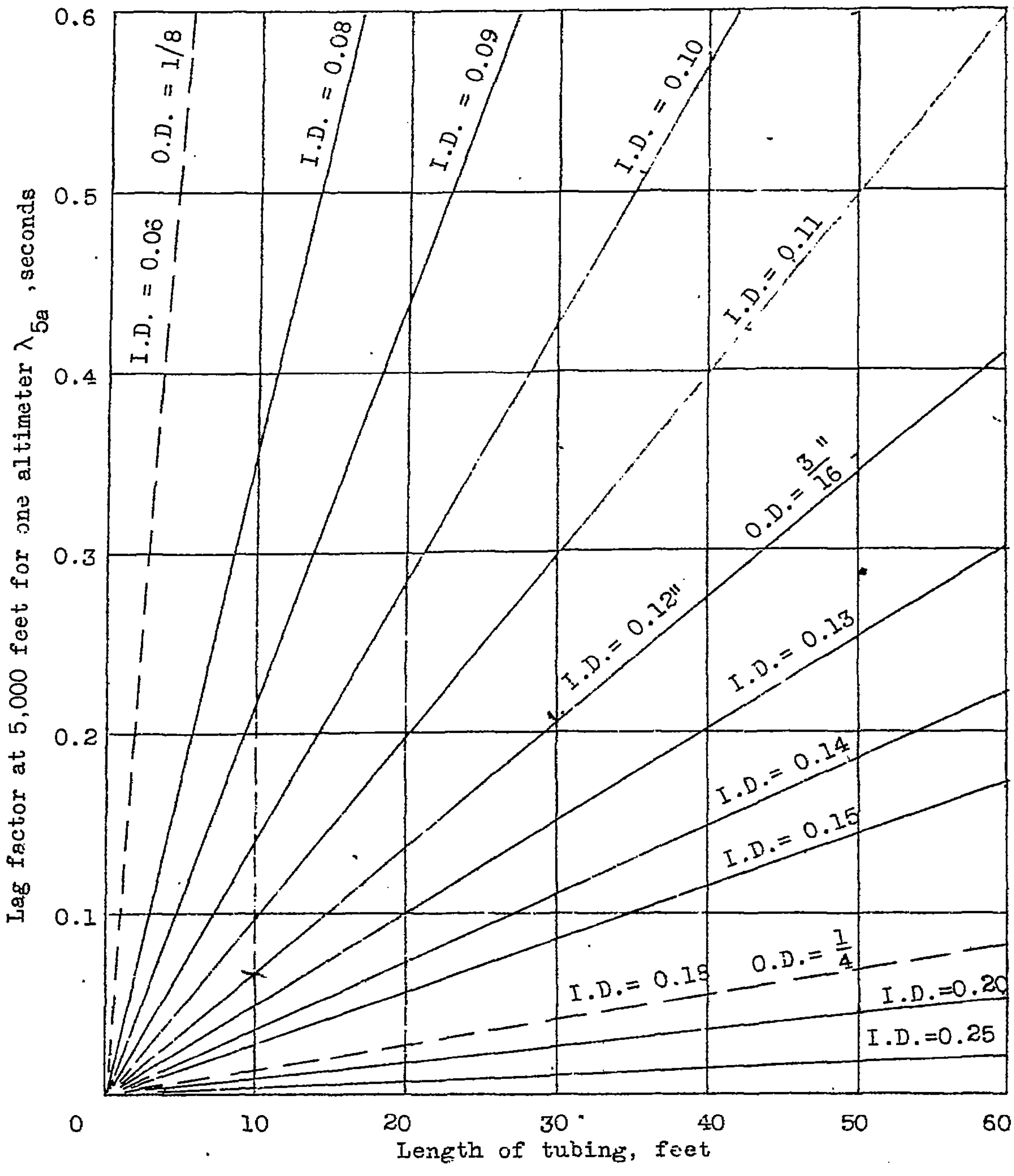

Figure 2.- Chart showing the relation between tube length and diameter and the lag constant for one altimeter at 5000 feet altitude. I.D. is the internal diameter of the tubing, O.D. the outside diameter, both in inches. To compute the lag constant for a combination of instruments, take λ_{5a} from the chart and multiply by the appropriate factor from Table II.

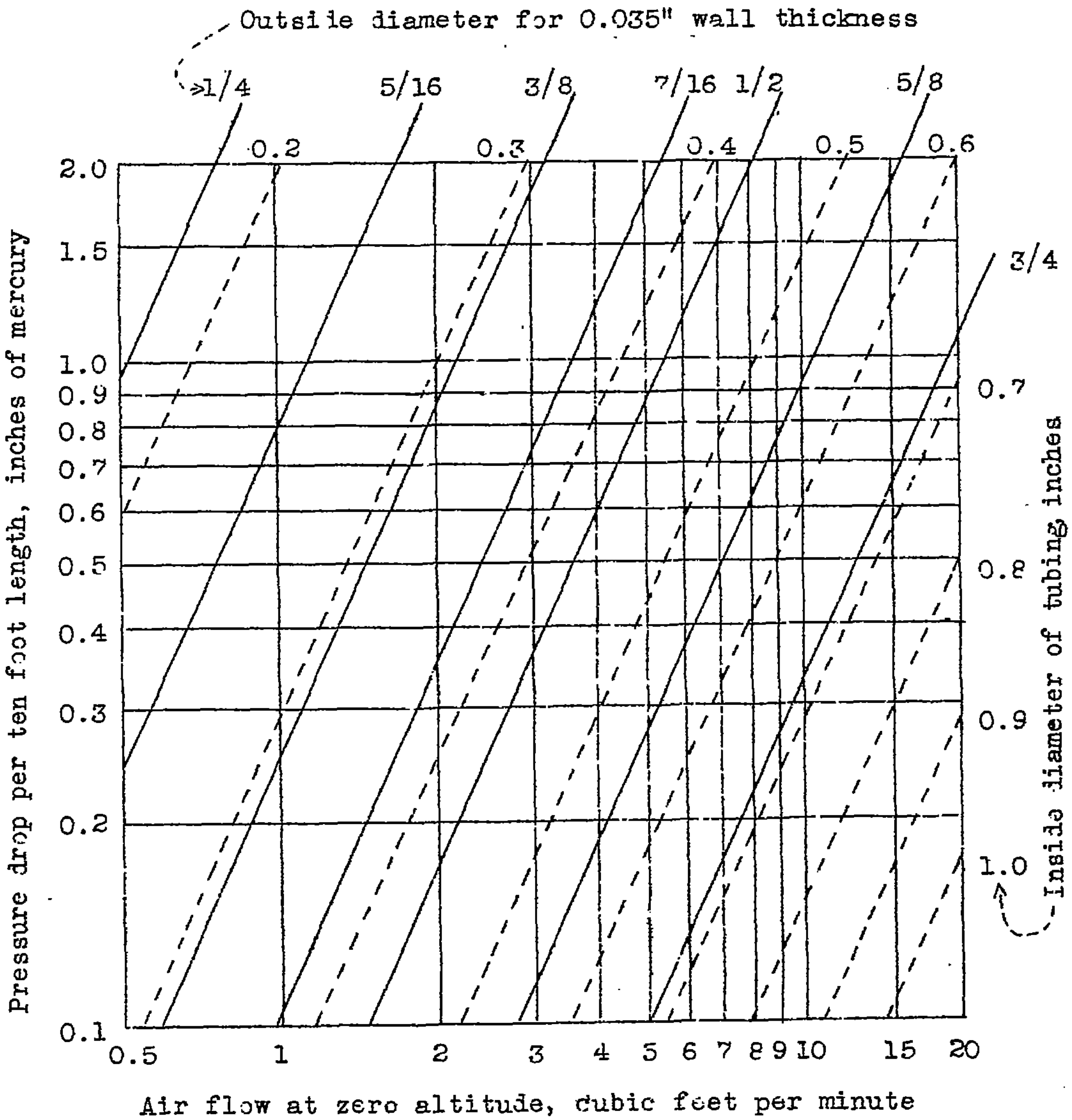

Figure 3.- Chart for computing the approximate pressure drop in smooth tubing at pressures near 30 inches of mercury.

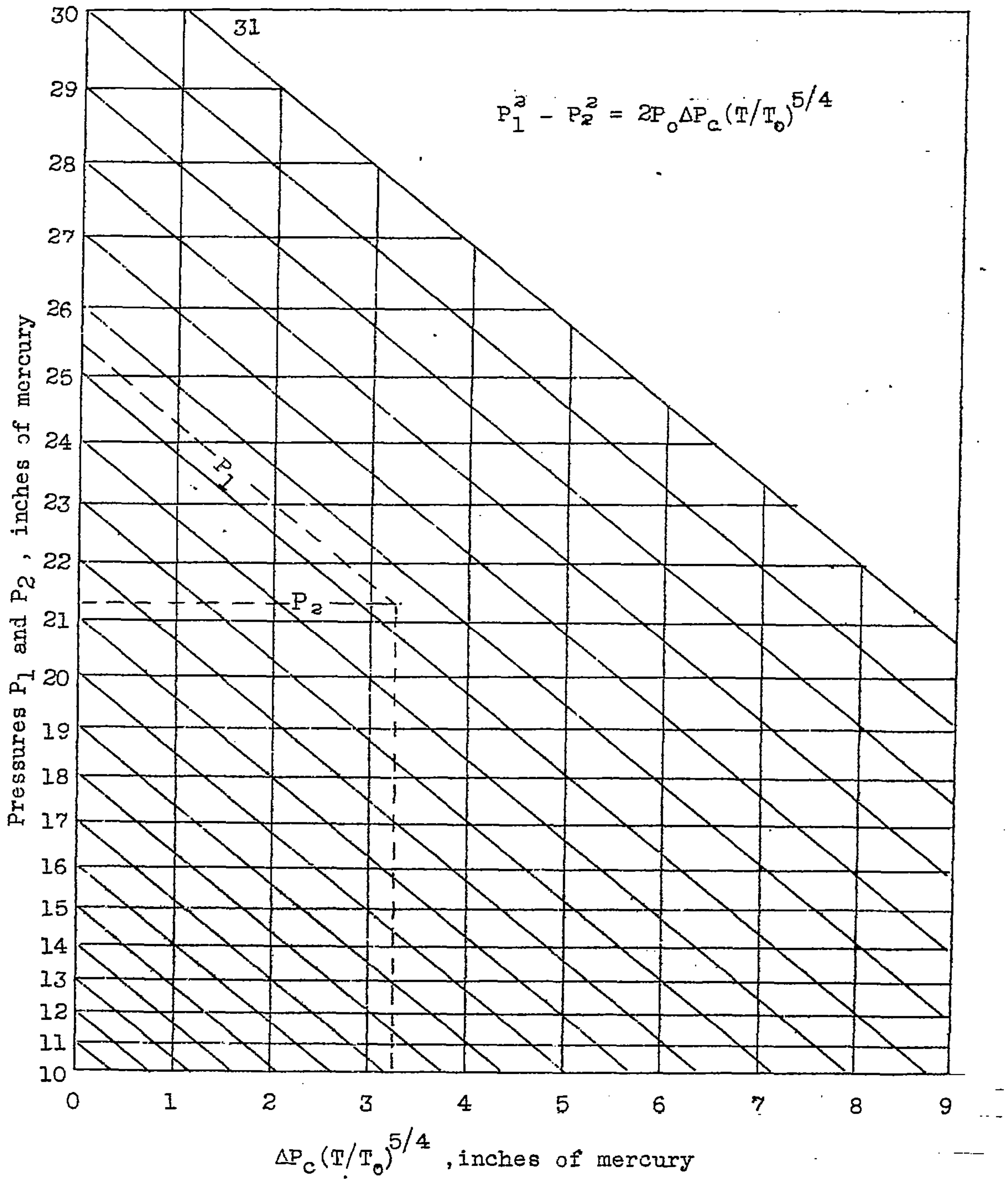

Figure 4.- Chart for obtaining true pressure drop. ΔP_c is computed by using the chart in Figure 3, $(T/T_o)^{5/4}$ is obtained from Table IV. Enter with the inlet pressure P_1 on the diagonals and $\Delta P_c (T/T_o)^{5/4}$ on the abscissa axis. The intersection indicates the value of P_2, which is read on the ordinate scale.