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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 708

A SIMPLIFIED METHOD FOR THE CALCULATION OF

AIRFOIL PRESSURE DISTRIBUTION

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Washington
May 1939

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SUMMARY

A method is presented for the rapid calculation of
the pressure distribution over an airfoil section when
the normal-force distribution and the pressure distribu-
tion over the "base profile" (i.e., the profile of the
same airfoil were the camber line straight and the re-
sulting airfoil at zero angle of attack) are known. This
note is intended as a supplement to N.A.C.A. Reports Nos.
631 and 634 wherein methods are presented for the calcu-
lation of the normal-force distribution over plain and
flapped airfoils, respectively, but not of the pressures
on the individual surfaces.

Base-profile pressure-coefficient distributions for
the usual N.A.C.A. family of airfoils, which are also
suitable for several other commonly employed airfoils, are
included in tabular form. With these tabulated base-
profile pressures and the computed normal-force distribu-
tions, pressure distributions adequate for most engineer-
ing purposes can be obtained.

INTRODUCTION

A method is given in reference 1 for computing the
chordwise pressure distribution over both the upper and
the lower surfaces of an airfoil. In this method, a per-
fect nonviscous fluid was assumed and, consequently, the
agreement between the integrated forces and moments and
those obtained from experimental observations was in many
cases inadequate. In the work reported in reference 2,
the effect of viscosity was accounted for by an adjustment
of the mean camber line of the airfoil and the agreement
was improved. Both methods, however, are too laborious
for practical use even though the results might be consid-
ered adequate for design purposes.

A subsequent analysis of theory and experimental data
(references 3 and 4) resulted in a series of charts where-
by the normal-force distributions, but not the pressures
on the individual surfaces, could be obtained for ordinary
airfoils and airfoils with flaps. In this method, the
magnitudes of the various component distributions that are
added to obtain the normal-force distribution are deter-
mined from experimental force and moment coefficients.
When the calculated normal-force distribution is inte-
grated, it then yields normal-force and moment coeffi-
cients that must agree in magnitude with the corresponding
experimental coefficients.

The present paper, which is a supplement to refer-
ences 3 and 4, gives a method of utilizing the calculated
normal-force distribution to obtain the pressures on the
individual surfaces of an airfoil. Again, the resulting
pressure distributions when integrated must yield normal-
force and moment coefficients that agree with those ob-
tained by experiment.

This method requires, in addition to a knowledge of
the chordwise normal-force distribution over the airfoil
section (as may be obtained from references 3 and 4), the
pressure distribution over the "base profile" of the air-
foil section (i.e., the profile of the same airfoil were
the camber line straight and the resulting airfoil at zero
angle of attack). The method is applicable, to date, to
normal airfoils and to airfoils with plain trailing-edge
flaps. The base-profile pressure distributions for the
N.A.C.A. family of airfoils (as well as for the Clark Y
and the Göttingen 398) calculated by the method of refer-
ence 1 are given in this report.

THEORY OF THE METHOD

Let it be assumed that the chordwise distribution of
the filaments of the bound vortices within an airfoil sec-
tion are located along the mean camber line and that the
curvature of the mean camber line is slight so that the
induced velocities on the upper and the lower surfaces at
any given distance, x, behind the leading edge of the
airfoil are equal in magnitude but opposite in sign.

The velocities on the upper and the lower surfaces of
the airfoil at any point, x, behind the leading edge are,
respectively,

$$u_U = u' + \Delta u$$
$$u_L = u' - \Delta u$$
$$\tag{1}$$

where Δu is the induced velocity and u' is assumed to be the velocity that the base profile would experience at the point x behind the leading edge under the same stream conditions.

If p_f be designated as the surface pressure at x on the base profile, then, by Bernoulli's equation,

$$p_f = \frac{\rho U^2}{2} - \frac{\rho (u')^2}{2}$$

or

$$p_f = q - q \left(\frac{u'}{U}\right)^2$$

Hence

$$\frac{u'}{U} = \sqrt{s_7} \qquad u' = U\sqrt{1 - P_f} \tag{2}$$

where U is the stream velocity and P_f is the pressure coefficient, p_f/q, where q is the stream dynamic pressure.

Combining equations (1) and (2) gives

$$u_U = U\sqrt{1 - P_f} + \Delta u \qquad \frac{u_U}{U} = \sqrt{s_7} + \frac{\Delta u}{U} \tag{3}$$

$$u_L = U\sqrt{1 - P_f} - \Delta u$$

The pressures on the upper and the lower surfaces of the airfoil at x are then, respectively,

$$p_U = q - \frac{\rho u_U^2}{2} = q - \frac{\rho}{2} (U\sqrt{1 - P_f} + \Delta u)^2$$
$$p_L = q - \frac{\rho u_L^2}{2} = q - \frac{\rho}{2} (U\sqrt{1 - P_f} - \Delta u)^2$$
$$\tag{4}$$

Now, if p is the normal force experienced by the airfoil at x, then, an upward force being considered positive,

$$p = -p_U + p_L$$

Then substituting equations (4), canceling, and rearranging gives

$$p = 2\rho \ U \ \Delta u \sqrt{1 - P_f}$$

Hence

$$\Delta u = \frac{\frac{1}{4}\left(\frac{p}{q}\right) U}{\sqrt{1 - P_f}} = \frac{\frac{1}{4} P \ U}{\sqrt{1 - P_f}} \qquad \qquad (5)$$

Substituting equation (5) in (4) and rearranging gives

$$\left.\begin{array}{l}P_U = \dfrac{p_U}{q} = 1 - \dfrac{\left(1 - P_f + \frac{1}{4} P\right)^2}{(1 - P_f)} \\[6mm] P_L = \dfrac{p_L}{q} = 1 - \dfrac{\left(1 - P_f - \frac{1}{4} P\right)^2}{(1 - P_f)}\end{array}\right\} \qquad (6)$$

Thus, if the normal-force distribution over an airfoil P and the pressure distribution over the base profile P_f are known, then pressure coefficients for the upper and the lower surfaces may be calculated.

ACCURACY AND LIMITATIONS OF THE METHOD

At the outset, it is necessary to justify the assumptions made in the development of the present method. To this end, the theoretical pressure distributions over the N.A.C.A. 4412 airfoil section at two angles of attack were calculated by the method of reference 1 and are plotted in figures 1 and 2. The theoretically exact normal-force distributions were determined from the pressure distributions for each case. The pressure distribution over the base profile (N.A.C.A. 0012 at zero angle of attack) was also calculated by the method of reference 1. By means of the normal-force and the base-profile pressure distributions, the pressure distributions were then calculated by equations (6) and are plotted for comparison in figures 1 and 2. Similar calculations were made for the N.A.C.A. 6512 and 23018 at $c_l = 0$ and $c_l = 1.0$; the resulting pressure distributions are shown in figures 3 to 6. The

N.A.C.A. 6512 airfoil is representative of the more highly cambered airfoils; the N.A.C.A. 23018, of fairly thick airfoils; and the N.A.C.A. 4412, of the more commonly employed airfoils. The agreement between the theoretical distributions and those calculated by equations (6) for the several airfoils investigated is good, as shown by the distributions in figures 1 to 6.

When this method is applied to the calculation of airfoil pressure distributions, the accuracy of the calculation will depend principally on:

(1)   The accuracy of the calculation of the chordwise normal-force distribution over the airfoil.

(2)   The accuracy of the base-profile pressure distribution.

(3)   The thickness of the boundary layer over the airfoil.

The close agreement between the theoretical pressure distributions of figures 1 to 6 shows that, for airfoils of normal profile, thickness, and camber, equations (6) are sufficiently accurate. For an airfoil with a plain flap deflected through a large angle, the basic assumption that the curvature of the mean camber line is small is disregarded so that the accuracy of equations (6) is accordingly less. The same statement is, of course, true for highly cambered airfoils. Again, the assumption that the bound vorticity may be considered as distributed along the mean camber line, although it has been shown to be permissible for airfoils of normal thickness, will probably be less accurate for the thicker profiles.

For plain airfoils, the accuracy of the method for the calculation of the normal-force distribution given in reference 3 is, as a rule, well within 5 percent; for airfoils with flaps, the accuracy of the method given in reference 4 is, as a rule, within 10 percent of the true normal-force distribution.

The base-profile pressure distribution calculated by the method of reference 1 will probably be sufficiently accurate for design purposes for all base profiles in common use. The base-profile pressure distributions for the N.A.C.A. family of airfoils, as well as for the Clark Y

and the Göttingen 398, were calculated by the method of
reference 1 and are given in table I.

That the thickness of the boundary layer is a factor
concerning the accuracy of this method is apparent when
it is seen that not only is the base profile effectively
changed when the boundary layer is thickened but also that
the effective shape of the mean camber line and therefore
the effective position of the bound vortices are altered
when the boundary layer is thicker on one surface of the
airfoil than the other (as for example when the angle of
maximum lift is approached). This inaccuracy, which af-
fects the distribution of pressure between the upper and
the lower surfaces of the·airfoil, exists in spite of the
fact that the effect of the boundary layer on the normal-
force distribution is partly taken care of by the methods
of references 3 and 4.

A consideration of the several factors would lead one
to expect that the method would be very accurate for thin
airfoils with small amounts of camber and would be less
accurate for thick highly cambered airfoils and particu-
larly for airfoils with ordinary trailing-edge flaps. That
these expectations are fulfilled is shown in figures 7 to
14, where the computed and the experimental pressure dis-
tributions for several airfoils (one with a flap) are
given. In every case, the normal-force distributions were
determined by the methods of references 3 and 4 and the
base-profile pressure distributions given in table I were
used. The experimental pressure distributions over the
N.A.C.A. 4412 airfoil (figs. 7 to 10) were obtained from
reference 2 and those over the N.A.C.A. 23012 (figs. 11 to
14) were obtained from reference 5.

One limitation should be noted although it is of neg-
ligible importance. Equations (6) cannot be used to pre-
dict the pressure at the most forward point of the airfoil
because at this point the slope of the airfoil profile is
infinite and, even though the normal force approaches zero,
the solution must, of course, be indeterminate at this
point. The nose point should therefore be omitted from
practical calculations.

<center>CONCLUSIONS</center>

When the base-profile pressure-coefficient distribu-
tions given in table I and the normal-force distributions
computed by the methods of references 3 and 4 are used,

the pressure distributions calculated by the method of this note are adequate for most engineering purposes. The computed pressure distributions not only agree in form with experiment but the integrated normal forces and moments agree in magnitude with experiment.

Langley Memorial Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., April 3, 1939.

REFERENCES

1. Theodorsen, Theodore: Theory of Wing Sections of Arbitrary Shape. T.R. No. 411, N.A.C.A., 1931.

2. Pinkerton, Robert M.: Calculated and Measured Pressure Distributions over the Midspan Section of the N.A.C.A. 4412 Airfoil. T.R. No. 563, N.A.C.A., 1936.

3. Jacobs, Eastman N., and Rhode, R. V.: Airfoil Section Characteristics as Applied to the Prediction of Air Forces and Their Distribution on Wings. T.R. No. 631, N.A.C.A., 1938.

4. Allen, H. Julian: Calculation of the Chordwise Load Distribution over Airfoil Sections with Plain, Split, or Serially Hinged Trailing-Edge Flaps. T.R. No. 634, N.A.C.A., 1938.

5. Wenzinger, Carl J., and Delano, James B.: Pressure Distribution over an N.A.C.A. 23012 Airfoil with a Slotted and a Plain Flap. T.R. No. 633, N.A.C.A., 1938.

TABLE I

VALUES OF $(1 - P_f)$ FOR N.A.C.A. FAMILY OF AIRFOIL SECTIONS

| Station (percent c)	Thickness (percent c)								
	6	9	[a]12	15	18	21	25	30	35
1.25	1.118	1.075	1.008	0.932	0.857	0.775	0.699	0.620	0.573
2.5	1.190	1.232	1.257	1.250	1.216	1.170	1.105	1.061	.982
5	1.219	1.313	1.395	1.460	1.505	1.529	1.543	1.550	1.553
7.5	1.222	1.330	1.425	1.513	1.601	1.675	1.760	1.828	1.899
10	1.220	1.330	1.432	1.534	1.635	1.737	1.861	2.000	2.115
15	1.209	1.316	1.428	1.536	1.647	1.760	1.914	2.106	2.300
20	1.192	1.298	1.407	1.518	1.634	1.749	1.905	2.106	2.309
25	1.180	1.277	1.380	1.490	1.604	1.715	1.870	2.070	2.271
30	1.165	1.256	1.352	1.453	1.560	1.667	1.813	2.001	2.193
35	1.150	1.233	1.321	1.413	1.510	1.603	1.740	1.905	2.075
40	1.137	1.210	1.290	1.372	1.457	1.540	1.656	1.805	1.958
45	1.122	1.189	1.256	1.328	1.401	1.473	1.570	1.702	1.835
50	1.109	1.167	1.225	1.288	1.351	1.410	1.492	1.605	1.711
55	1.094	1.146	1.195	1.248	1.300	1.350	1.417	1.509	1.597
60	1.081	1.125	1.167	1.208	1.250	1.293	1.343	1.411	1.484
65	1.070	1.104	1.138	1.171	1.202	1.239	1.279	1.326	1.378
70	1.056	1.082	1.107	1.132	1.157	1.182	1.211	1.240	1.271
75	1.041	1.059	1.074	1.091	1.105	1.122	1.141	1.160	1.180
80	1.021	1.032	1.041	1.048	1.053	1.059	1.069	1.074	1.085
85	1.002	1.002	1.001	1.000	.998	.995	.990	.982	.972
90	.981	.970	.955	.941	.929	.911	.890	.860	.829
95	.949	.922	.895	.865	.834	.807	.764	.712	.663

[a]Clark Y; Göttingen 398.

Fig. 1
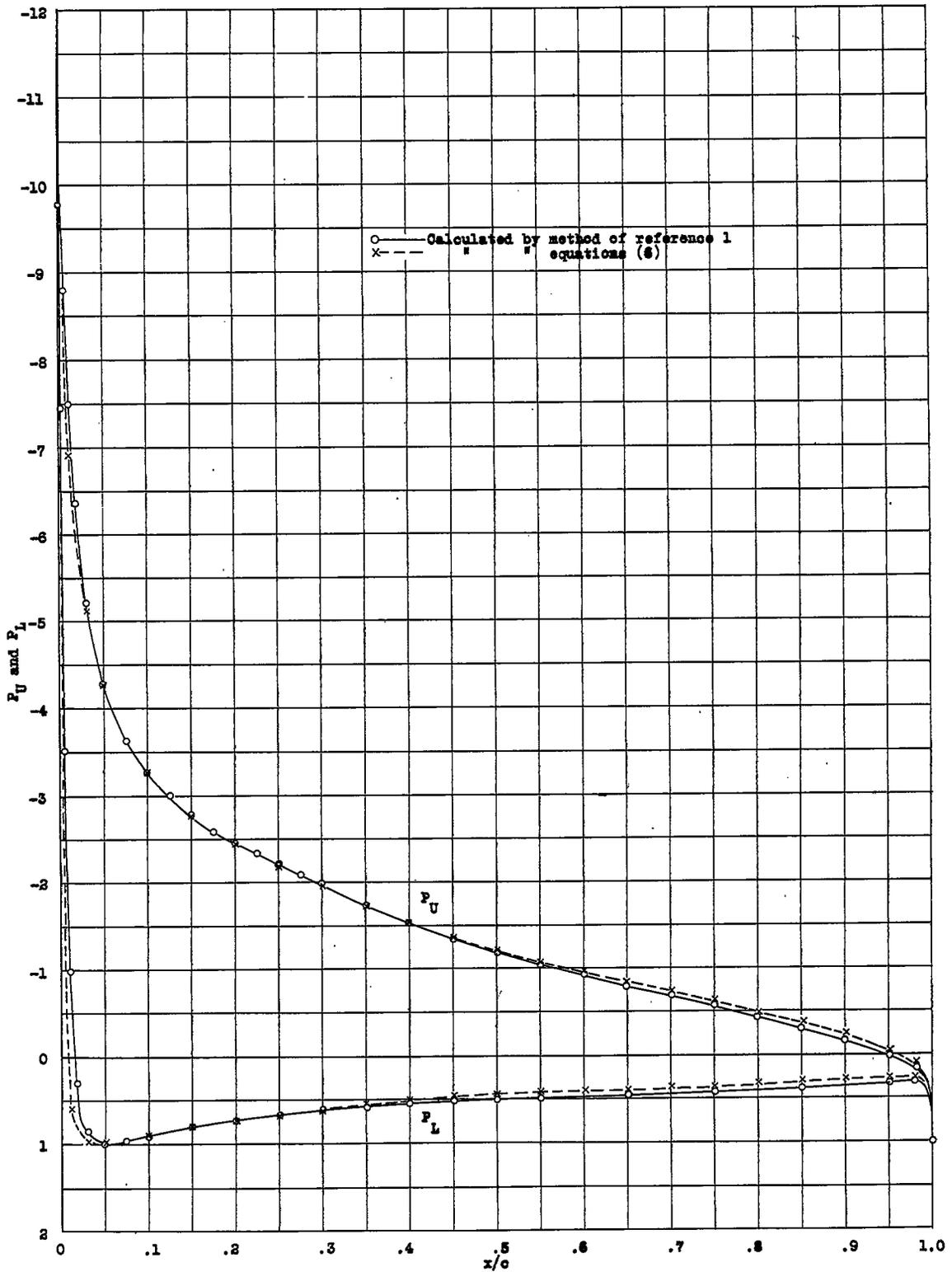
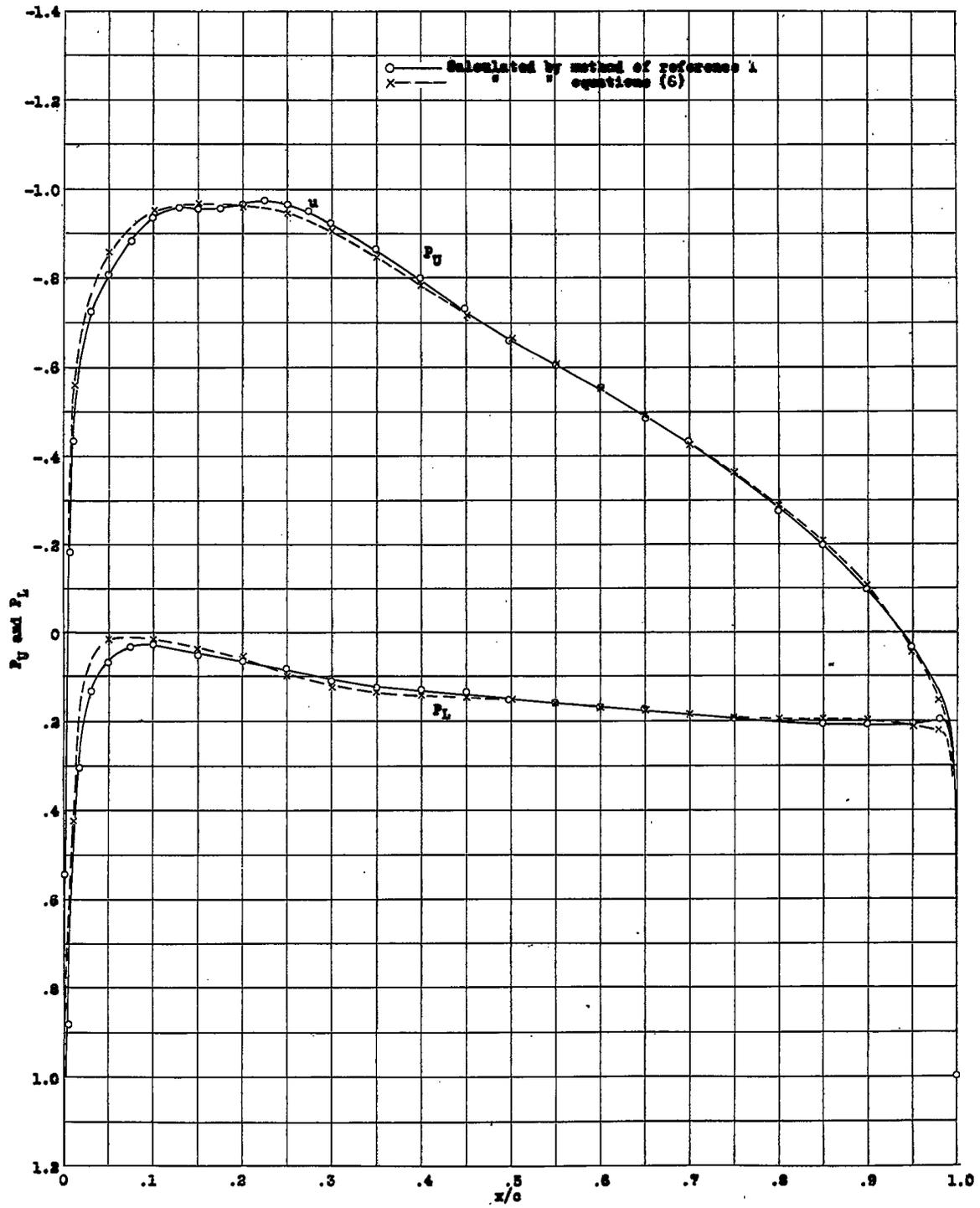Figure 1.- Theoretical pressure distribution. N.A.C.A. 4412 airfoil; α , 13° $57'$.

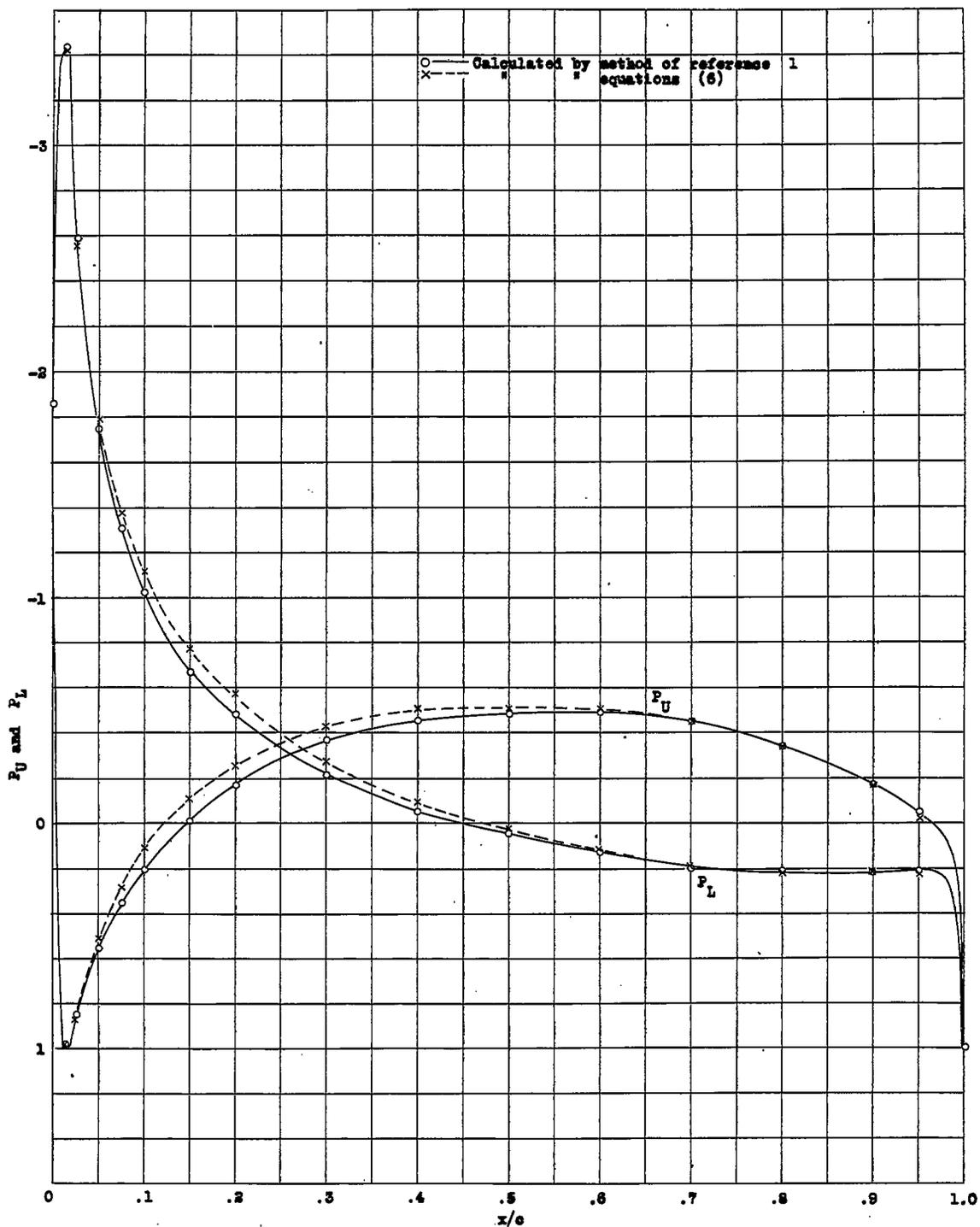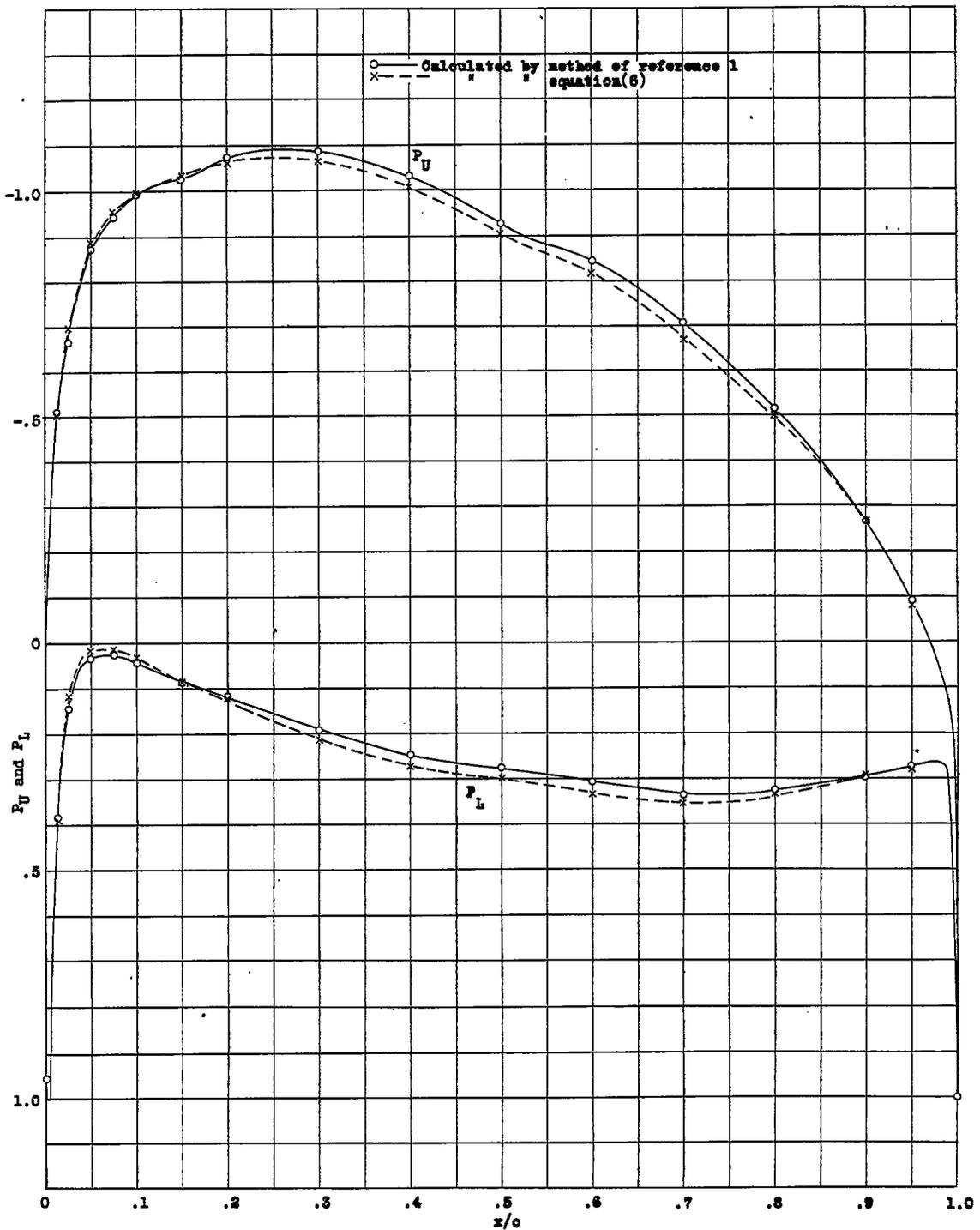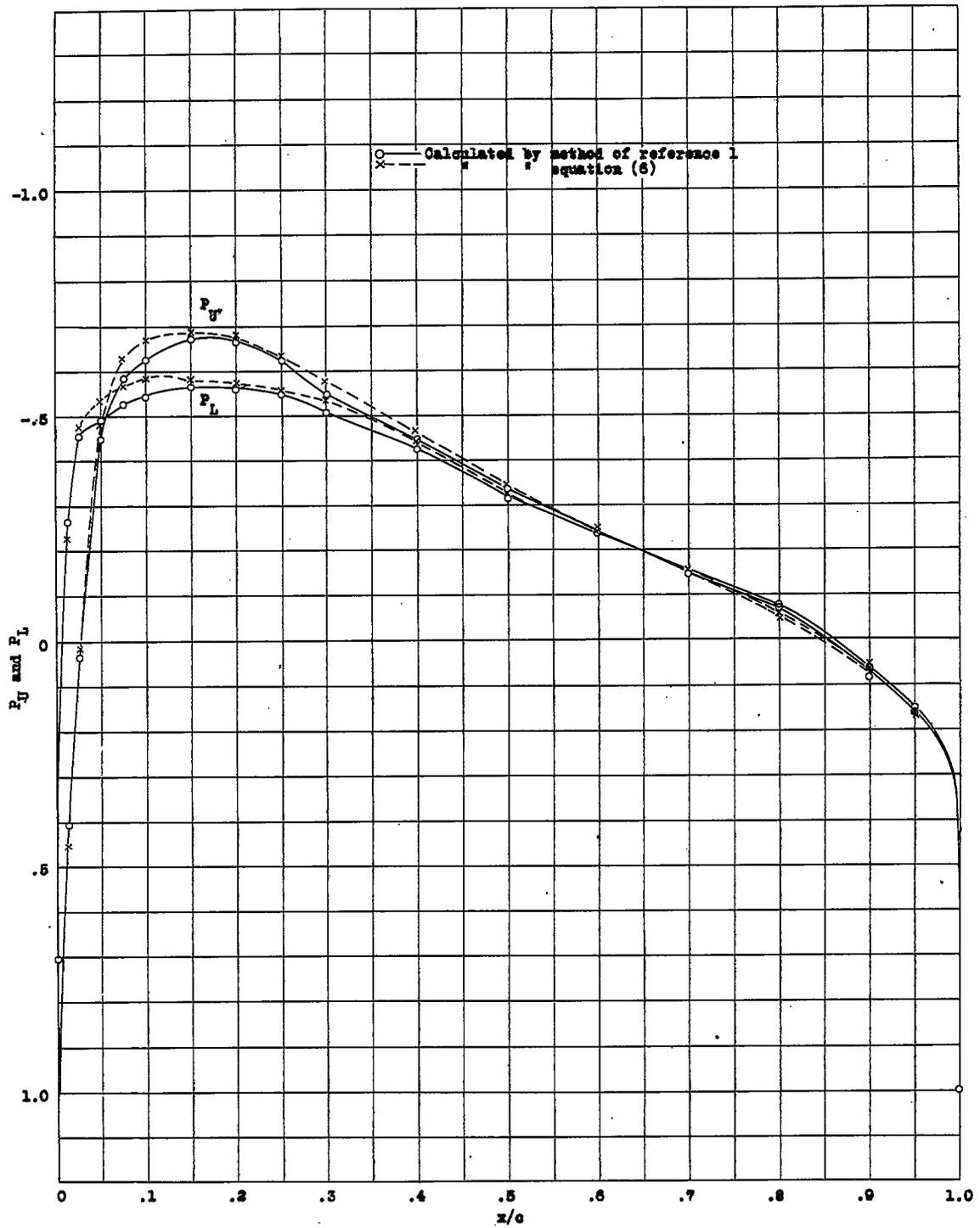Figure 2.- Theoretical pressure distribution. N.A.O.A. 4412 airfoil; α, 2° $2'$.

Figure 3.- Theoretical pressure distribution. N.A.C.A. 6512 airfoil; α, -7° 4'; c_{l}, 0 .

Fig. 4

Figure 4.- Theoretical pressure distribution. N.A.C.A. 6512 airfoil; α, $1°$ $10'$; c_l, 1.0.

Figure 5.- Theoretical pressure distribution. N.A.C.A. 23018 airfoil; α, -1° 3' ; c_l , 0.
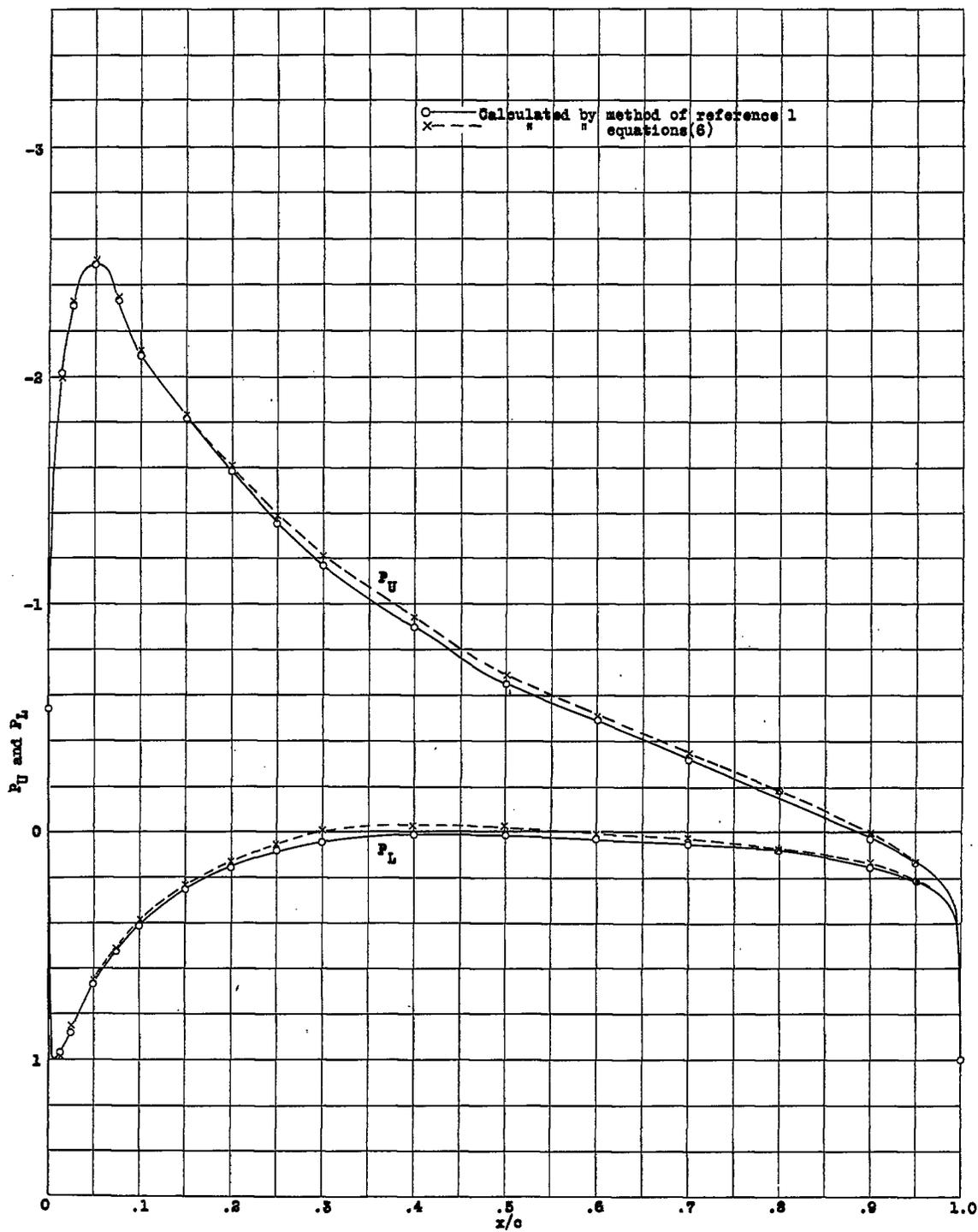
Figure 6.- Theoretical pressure distribution. N.A.C.A. 23018 airfoil; α, $6°$ $53'$; c_L, 1.0.
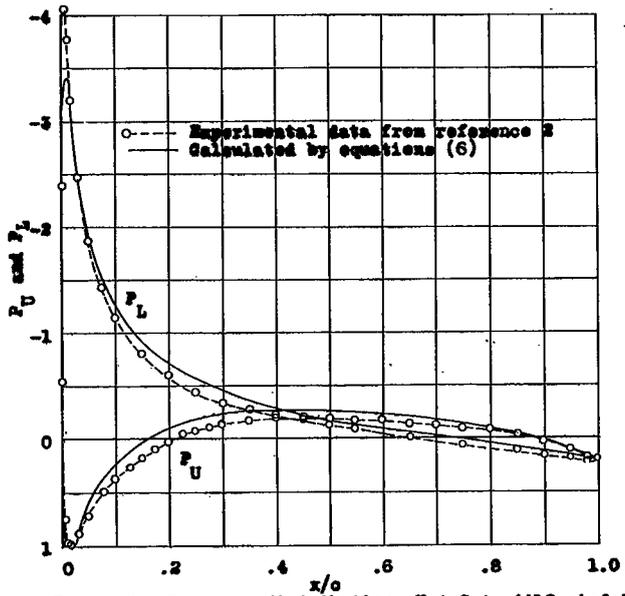
Figure 7.- Pressure distribution. N.A.C.A. 4412 airfoil; α, -8°.
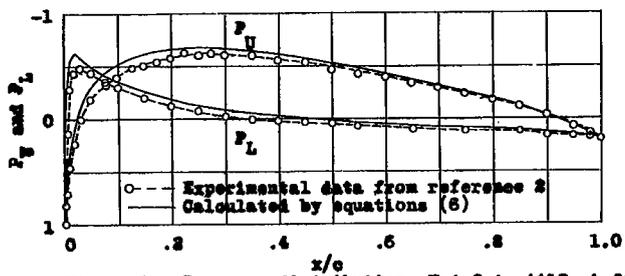
Figure 8.- Pressure distribution. N.A.C.A. 4412 airfoil; α, 0°.
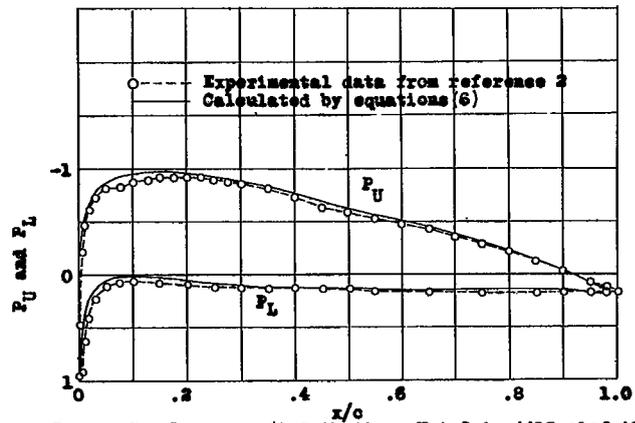
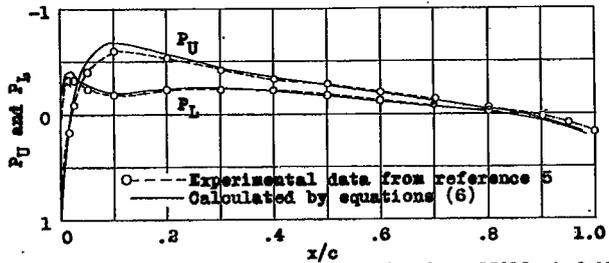Figure 9.- Pressure distribution. N.A.C.A. 4412 airfoil; α, 4°.
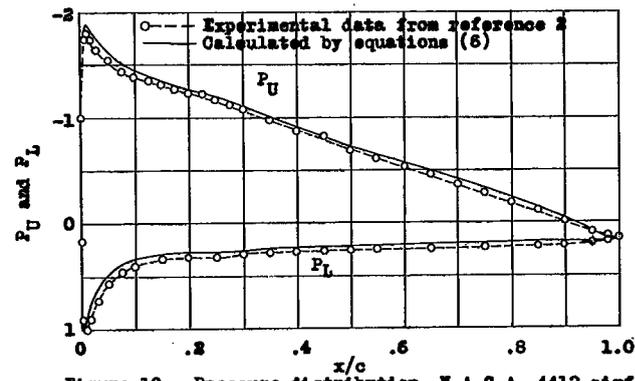
Figure 10.- Pressure distribution. N.A.C.A. 4412 airfoil; α, 8°.

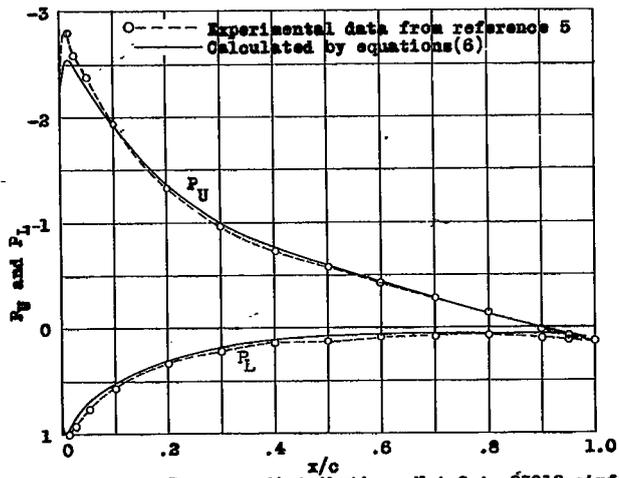Figure 11.- Pressure distribution. N.A.C.A. 23012 airfoil with 20-percent-chord plain flap; α, 0°;δ,0°.

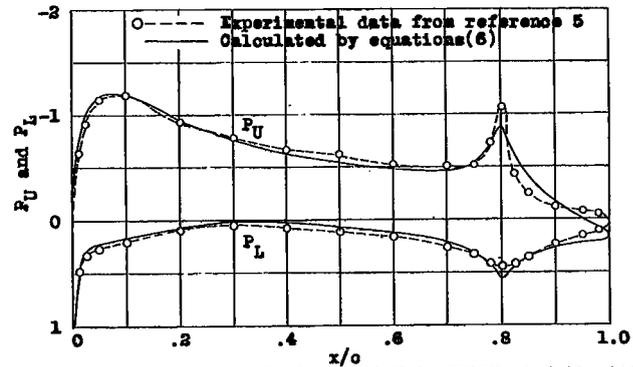Figure 12.- Pressure distribution. N.A.C.A. 23012 airfoil with 20-percent-chord plain flap; α, 8°; δ, 0°.

Figure 13.- Pressure distribution. N.A.C.A. 23012 airfoil with 20-percent-chord plain flap; α, 0°; δ, 15°.
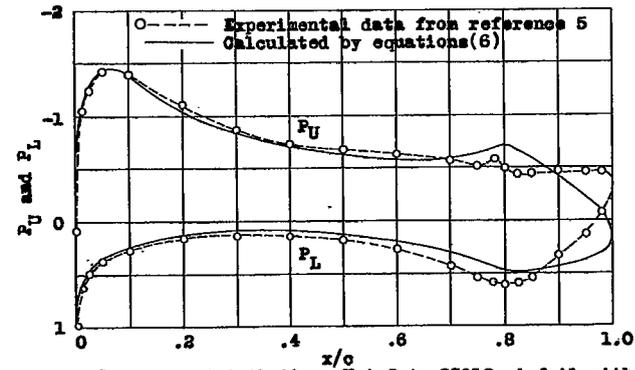

Figure 14.- Pressure distribution. N.A.C.A. 23012 airfoil with 20-percent-chord plain flap; α, 0°; δ, 30°.