TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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THE ESTIMATION OF THE RATE OF CHANGE OF YAWING MOMENT WITH SIDESLIP

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Washington
February 1938

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SUMMARY

Wind-tunnel data are presented on the rate of change of yawing moment with sideslip for tests of 9 complete airplane models, 20 fuselage shapes, and 3 wing models with various combinations of dihedral, sweepback, and twist. The data were collected during a survey of existing information, which was made to find a reliable method of computing the yawing moment due to sideslip. Important errors common to methods of computation used at present appear to be due to large interference effects, the investigation of which will undoubtedly require an extensive program of systematic wind-tunnel tests. At present it is necessary to place considerable reliance on past design experience in proportioning an airplane so as to obtain a reasonable degree of directional stability.

INTRODUCTION

Theoretical studies of lateral stability (reference 1) have shown that the rate of change of yawing-moment coefficient with angle of sideslip $dC_n/d\beta$, is one of the more important factors influencing the lateral-stability characteristics of an airplane. At present there exists no dependable method of computing this factor from the dimensions of an airplane. Several methods of estimating its approximate value are in use but they have proved to be inaccurate when the results are compared with those from wind-tunnel tests. In an attempt to devise a reliable method of determining the value of the derivative for an airplane in the course of design, a study has been made of all available wind-tunnel data on the subject.

During the survey, the results of wind-tunnel tests of 127 airplane models were analyzed. The models embraced a wide variety of designs, including such diverse types as racing seaplanes and troop and cargo carriers. In spite of the large number of test results available, no satisfactory method of estimating $dC_n/d\beta$ was developed because the data offered little opportunity for the study of interference effects. Indications are that the interference effects between components of the airplane may change the yawing moment for the combination by an amount equal to the sum of the yawing moments obtained when the components are tested separately. An extensive program of correlated wind-tunnel tests will probably be required to permit the isolation and analysis of these interference effects.

In the absence of an accurate method of estimation, certain of the more useful data collected during the study are presented as an aid to the designer in judging the value of $dC_n/d\beta$ for complete airplanes or component parts.

PRESENTATION OF DATA

As theoretical considerations indicate that the value of $dC_n/d\beta$ should be only slightly dependent on the angle of attack, the greater portion of the test data presented is only for low angles of attack. Figure 1 has been included to show the variation of $dC_n/d\beta$ with angle of attack α, for eight complete airplane models. From the figure it can be seen that, although the variation of $dC_n/d\beta$ with α is appreciable in the normal-flight range, the magnitude of the effect is not large except at angles of attack above the stall. The yawing moments were measured about an axis normal to the relative wind. The data of figure 1, and also the rest of the aerodynamic data presented, were obtained from wind-tunnel tests made at Reynolds Numbers in the neighborhood of 200,000.

Table I presents yawing-moment data obtained from wind-tunnel tests of nine airplane designs (fig. 2). Two of the types were tested, each with two tail arrangements. All the models were tested both complete and with the empennage removed. The table gives the value of $dC_n/d\beta$, where β is measured in radians, for the complete models and also the increment of $dC_n/d\beta$ contributed by the vertical tail surfaces $dC_{n_t}/d\beta$, as determined from the difference between the results of the tests of the complete models and the tests of the models without empennage. The proportions of the models listed in the table may be determined from the given values of wing span b, wing aspect ratio b^2/S_w, ratio of vertical tail area to wing area S_t/S_w, ratio of fuselage side area to wing area S_f/S_w, ratio of the distance between the rudder hinge and the airplane center of gravity to wing span l_t/b, ratio of over-all fuselage length to wing span l_f/b, aspect ratio of vertical tail surfaces h_t^2/S_t, ratio of over-all fuselage length to maximum depth of fuselage l_f/d_f, and ratio of distance between the airplane center of gravity and the fuselage nose to over-all fuselage length x_1/l_f. The height of the vertical tail surfaces h_t, used in the calculation of aspect ratio, does not include the fuselage.

The results of tests of a wide variety of fuselage shapes (fig. 3) are given in table II. The values of the rate of change of lateral-force coefficient with angle of sideslip, $dC_Y/d\beta$, and of $dC_n/d\beta$, given in table II, are of necessity based on the side area and over-all length of the fuselage rather than on wing area and span. For all the fuselage shapes, the yawing-moment data are given about an axis located a distance 0.30 l_f back of the fuselage nose. On the basis of average airplane proportions, the coefficients used in this table are about five times as large as corresponding coefficients based on wing area and span. The fact that all the fuselage shapes tested, except Hull No. 10, have unstable (negative) values of $dC_n/d\beta$ is predicted by the theory of yawed streamlined bodies (reference 4). Hull No. 10 had considerable vertical fin area built in at the rear. (See fig. 3.)

The data obtained from wind-tunnel yaw tests of several types of airfoils are given in table III. The values of $dC_n/d\beta$ listed are based on a yawing-moment axis passing through the quarter-chord point at the center section of the wing. For the tests of wing twist, the airfoils had a uniform rate of twist along the semispan such that the wing-tip incidence differs from the center-section incidence by an amount defined as the angle of twist. The angle of twist was such that the wing tips had washout.

Table IV shows the effect of deflecting flaps on the value of $dC_n/d\beta$ for nine complete airplane models and for one case of a wing alone.

DISCUSSION

During the course of the survey of factors affecting the value of $dC_n/d\beta$, certain pertinent points were noticed. These points are presented in relation to the increment of $dC_n/d\beta$ contributed by the vertical tail surfaces, $dC_{n_t}/d\beta$; the increment contributed by the fuselage, $dC_{n_f}/d\beta$; and the increment contributed by the wing cellule, $dC_{n_w}/d\beta$.

Factors Affecting $dC_{n_t}/d\beta$

For all practical purposes, if the angle of sideslip β, is limited to small values, the value of $dC_{n_t}/d\beta$ is

$$\frac{dC_{n_t}}{d\beta} = \frac{S_t}{S_w}\,\frac{l_t}{b}\,\frac{dC_{c_t}}{d\beta}$$

where S_t is the area of the vertical tail surfaces, l_t is the distance from the rudder hinge to the airplane center of gravity, and C_{c_t} is the cross-wind force coefficient for the tail, based on S_t. Since $dC_{c_t}/d\beta$ is analogous to the rate of change of lift with angle of attack, $dC_L/d\alpha$, for an airfoil, the problem of determining $dC_{n_t}/d\beta$ becomes one of determining the slope of the lift curve for the vertical tail surfaces. Data presented in references 7 and 8 indicate that the value of $dC_{c_t}/d\beta$ will not be affected by airfoil section for airfoils of the symmetrical type normally used for tail surfaces. Reference 9 indicates that the effect of tail upper contour (corresponding to wing-tip shape) will be small and may be neglected for aspect ratios usually encountered in vertical tail surfaces.

The determination of the effective aspect ratio of the vertical tail surfaces is difficult, primarily because of the flow interference caused by other portions of the airplane. The location, size, and shape of the horizontal surfaces appear to have a marked influence on the magnitude of this interference effect. An analysis of the data given in table I indicates that the most efficient arrangement is the one in which the vertical surfaces are placed as high as possible above the horizontal surfaces. Undoubtedly, the location of the vertical tail area below the horizontal surfaces would be equally effective. The poorest arrangement appears to be the one in which the horizontal surfaces are located in a median position.

In addition to the change in effective aspect ratio caused by interference effects, various parts of the airplane may also cause a reduction in dynamic pressure at the vertical tail surfaces. These two interference effects are naturally difficult to separate, but together they may change the effectiveness of the vertical tail surfaces as much as 65 percent.

Factors Affecting $dC_{n_f}/d\beta$

Values of $dC_{n_f}/d\beta$ are plotted against the ratio l_f/d_f in figure 4 for all the models listed in table II except Hull No. 10. Data for that model were omitted because of the unusually large side area at the rear of the hull. The coefficient C_{n_f} of figure 4 is based on S_f and l_f. It is seen that, in general, the value of $dC_{n_f}/d\beta$ has a tendency to become less negative for larger values of l_f/d_f. Although many other factors, such as fuselage nose shape, windshields, etc., undoubtedly have an important effect on the value of $dC_{n_f}/d\beta$, their influence could not be determined from the data used in the study.

Factors Affecting $dC_{n_w}/d\beta$

The effect of wing-tip plan form and elevation shape on the value of $dC_{n_w}/d\beta$ has been treated in reference 5. Although the results published in reference 5 show that there is a large percentage change in the value of the derivative with changes in wing-tip plan form and elevation shape, the numerical change involved in comparison with $dC_n/d\beta$ for a complete airplane is of minor importance.

Reference 5 also indicates that, for zero dihedral, aspect ratio has no effect on $dC_{n_W}/d\beta$.

The effects of dihedral, sweepback, and twist on $dC_{n_W}/d\beta$ at low angles of attack were determined from data given in table III. The pertinent data for dihedral and sweepback are plotted in figure 5. For both the rectangular and the Army tips, the effect of dihedral is approximated by

$$\frac{\partial}{\partial\Gamma}\left(\frac{dC_{n_W}}{d\beta}\right) = -0.00079$$

where Γ is the dihedral angle in degrees and β is the angle of sideslip in radians. Additional data given in reference 5 show that this relationship should vary slightly with lift coefficient, as is predicted by theory. For sweepback with rectangular tips

$$\frac{\partial}{\partial\Lambda}\left(\frac{dC_{n_W}}{d\beta}\right) = 0.00111$$

where Λ is the angle of sweepback in degrees. The increments of $dC_{n_W}/d\beta$ due to dihedral or sweepback are to be added algebraically to the value of $dC_{n_W}/d\beta$ for the wing with no dihedral or sweepback. The test data indicate that wing twist has a negligible effect on $dC_{n_W}/d\beta$.

Theory indicates that the value of $\frac{\partial}{\partial\Lambda}\left(\frac{dC_{n_W}}{d\beta}\right)$ is dependent on the lift coefficient. In addition, the effect of sweepback may be considerably different for other than rectangular tips. Values of $dC_Y/d\beta$ are given for the airfoils listed in table III to permit the calculation of $dC_n/d\beta$ about an axis other than through the quarter-chord point of the center section, if so desired.

Insufficient data are available to study the effect of other factors of probable importance in determining the value of $dC_{n_W}/d\beta$, such as wing section, biplane arrangements, etc. Also, no conclusions can be drawn as to the influence of one factor on the effect of another. Comparison of test results given in table III for one wing with combined dihedral, sweepback, and twist with data for the same wing without dihedral, sweepback, or twist indicates that the effects are not additive.

The effect of the interference between the wing and fuselage is another factor that cannot be determined from the data available at present. Unpublished results of wind-tunnel tests made by the N.A.C.A. of a flying-boat model, for the wing and hull separately and in combination, indicate that this effect may equal the summation of the moments of the wing and of the fuselage tested separately.

Effect of Flaps

Study of the data listed in table IV gives conflicting indications but, in general, deflecting the flaps increases the value of $dC_n/d\beta$. It should be noted that, when flaps are deflected, they not only affect the value of $dC_n/d\beta$ through their effect on the wing and the interference between the wing and the fuselage, etc., but may also increase the blanketing of the vertical tail surfaces to a considerable extent. For this reason, the effect of flaps is likely to be extremely variable for different designs.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 6, 1938.

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TABLE I. Values of $\frac{dC_n}{d\beta}$ for Complete Airplanes and for Vertical Tail Surfaces

Model[1]	Angle of attack (deg.)	b (ft.)	$\frac{b^2}{S_w}$	$\frac{S_t}{S_w}$	$\frac{S_f}{S_w}$	$\frac{l_t}{b}$	$\frac{l_f}{b}$	$\frac{h_t^2}{S_t}$	$\frac{l_f}{d_f}$	$\frac{x_1}{l_f}$	$\frac{dC_n}{d\beta}$	$\frac{dC_{n_t}}{d\beta}$
A	1.00	2.625	3.225	0.0640	0.301	0.493	0.806	1.305	4.59	0.301	0.0530	0.0664
B	-.75	2.875	5.719	.1365	.652	.564	.905	1.589	5.40	.377	.1210	.0929
C	.75	2.011	3.442	.0451	.223	.486	.696	1.753	5.82	.255	.0229	.0391
D	1.00	2.125	3.790	.0547	.211	.431	.622	1.496	5.17	.278	.0239	.0447
E	1.00	2.125	3.790	.0646	.212	.438	.631	1.402	5.24	.274	.0220	.0428
F	1.00	2.125	3.790	.0679	.211	.428	.622	1.602	5.17	.278	.0430	.0646
G	.00	3.562	7.998	.0825	.429	.364	.669	1.373	5.71	.353	.0696	.0556
H	-.50	2.438	5.895	.0876	.378	.435	.687	1.118	5.44	.296	.0480	.0678
I	-2.00	2.250	3.377	.0513	.326	.492	.697	1.660	4.03	.294	.0322	.0509
J	1.75	2.562	4.132	.0776	.338	.473	.646	2.174	4.25	.279	-.0046	.0525
K	1.75	2.562	4.132	.1040	.343	.488	.660	1.876	4.34	.273	.0079	.0650
L	3.80	3.917	3.514	.0711	.196	.455	.854	[2]2.472	9.56	.374	.0590	.0649

[1]All data from tests at Washington Navy Yard except model L, for which the data were taken from reference 2.

[2]Based on one fin and rudder.

TABLE II

Values of $dC_n/d\beta$ for Fuselages

(All data were obtained at zero angle of attack. Note that the coefficients C_Y and C_n are based on S_f and l_f instead of on S_W and b.)

Model	S_f (sq.ft.)	l_f (ft.)	$\frac{l_f}{d_f}$	$\frac{dC_Y}{d\beta}$	$\frac{dC_n}{d\beta}$
From Washington Navy Yard tests:					
MK-13	0.413	1.958	5.88	-0.308	-0.129
MK-14	.416	1.862	5.57	-.204	-.134
MK-15	.416	1.932	5.79	-.301	-.133
MK-15A	.326	1.694	6.23	-.300	-.083
MK-16	.429	1.957	5.97	-.076	-.175
MK-17	.413	1.759	6.07	-.159	-.125
MK-18	.425	1.834	6.32	-.179	-.131
MK-19	.409	1.849	5.62	-.153	-.151
MK-20	.282	1.513	5.67	-.083	-.139
MK-21	.316	1.590	5.44	-.171	-.132
MK-22	.453	1.908	5.93	-.130	-.162
MK-23	.446	1.792	5.56	-.115	-.173
MK-24	.532	1.670	5.92	-.109	-.114
From reference 3:					
Fuselage No. 1	1.219	2.995	6.33	-.176	-.137
Fuselage No. 2	1.468	3.373	5.61	-.160	-.116
Fuselage No. 3	1.182	3.197	6.85	-.157	-.100
Fuselage No. 4	1.268	3.281	6.66	-.452	-.058
Fuselage No. 5	1.193	3.544	7.70	-.278	-.108
Fuselage No. 6	1.300	2.953	4.94	-.560	-.188
Hull No. 7	.832	2.625	5.93	-.244	-.086
Hull No. 8	.781	2.953	7.79	-.253	-.105
Hull No. 9	.474	2.205	8.72	-.292	-.078
Hull No. 10	.783	2.748	7.61	-.530	.170

TABLE III

Values of $\frac{dC_n}{d\beta}$ for Airfoils

Wing shape	Angle of attack α (deg.)	Dihedral angle Γ (deg.)	Sweepback angle Λ (deg.)	Angle of twist (deg.)	$\frac{dC_n}{d\beta}$	$\frac{dC_Y}{d\beta}$
Rectangular plan form and tip; 0.93 $\frac{b}{2}$ dihedral; Clark Y section; aspect ratio 6 (from reference 5)	0	0	0	0	0.0046	-0.020
	0	5.0	0	0	.0000	-.049
	0	10.0	0	0	.0000	-.120
Do. but with Army tip	0	0	0	0	.0048	-.020
	0	2.0	0	0	.0017	-.020
	0	5.0	0	0	-.0046	-.049
	0	10.0	0	0	-.0014	-.092
	0	15.0	0	0	-.0077	-.192
Rectangular plan form and tip; 1.00 $\frac{b}{2}$ dihedral; Göttingen 387 section; aspect ratio 5 (from reference 6)	1.8	0	0	0	.0102	-.0464
	1.8	3.0	0	0	.0064	-.0590
	1.8	6.0	0	0	.0053	-.0728
	4.2	0	15.0	0	.0265	-.0573
	4.2	0	30.0	0	.0436	-.0665
	4.2	0	0	3.0	.0101	-.0482
	4.3	0	0	5.7	.0109	-.0499
	4.3	3.0	30.0	3.0	.0328	-.0797

TABLE IV

Effect of Flaps on $\frac{dC_n}{d\beta}$ for Complete Airplane

Model	Angle of attack (deg.)		$\frac{dC_n}{d\beta}$	
	Flaps up	Flaps down	Flaps up	Flaps down
1	9.9	10.3	0.0433	0.0622
2	11.8	12.2	.0335	.0312
3	13.5	13.6	.0972	.0923
4	9.9	10.0	.0816	.1126
5	11.1	12.2	.0579	.0539
6	10.9	9.3	.0685	.1011
7	10.9	11.3	.0149	.0550
8	9.5	8.0	.0257	.0492
9	8.0	8.0	.0026	.0063
9 (Wing only)	10.0	8.0	.0086	.0046

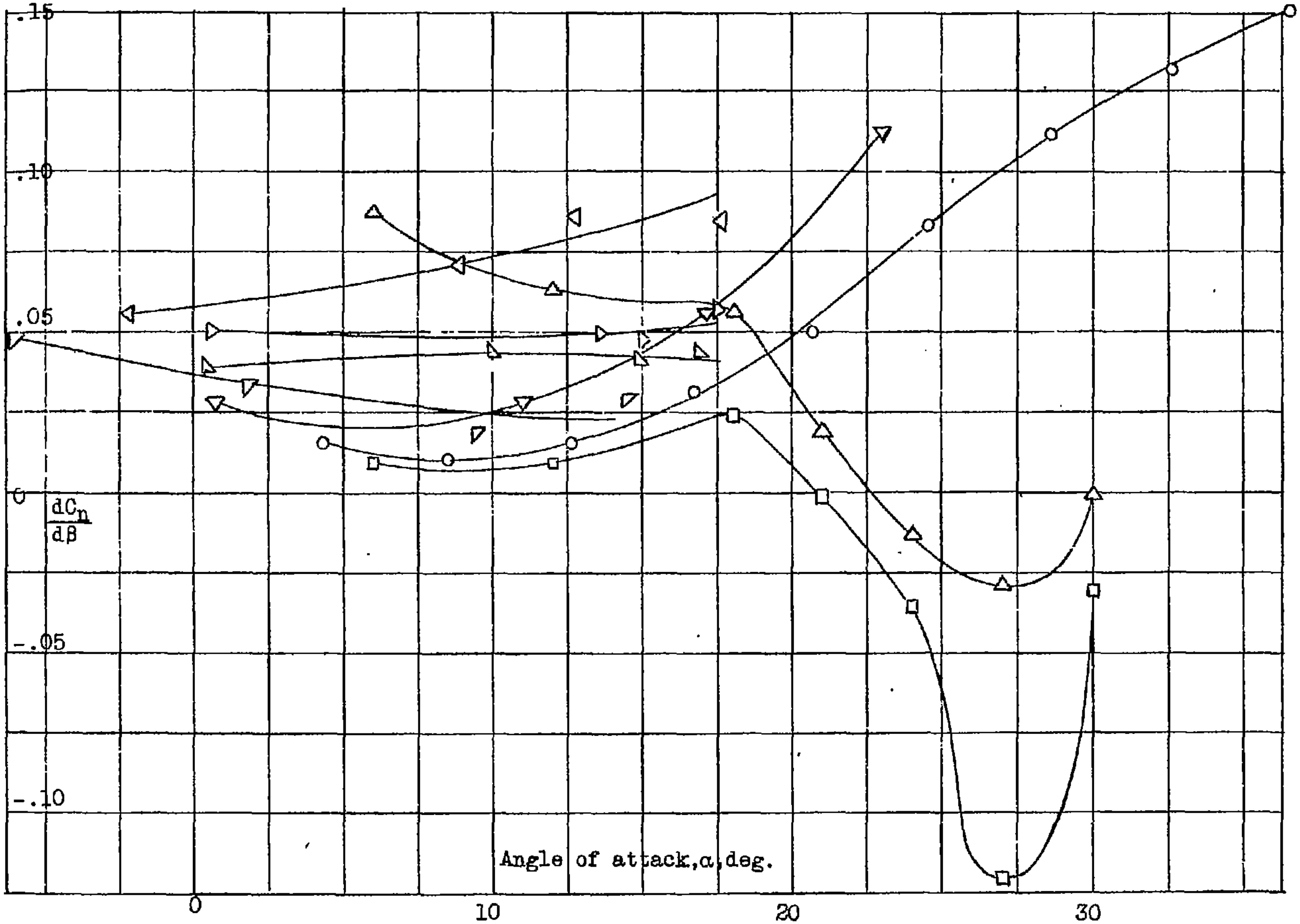

Figure 1.- Effect of angle of attack on rate of change of yawing-moment coefficient with sideslip.

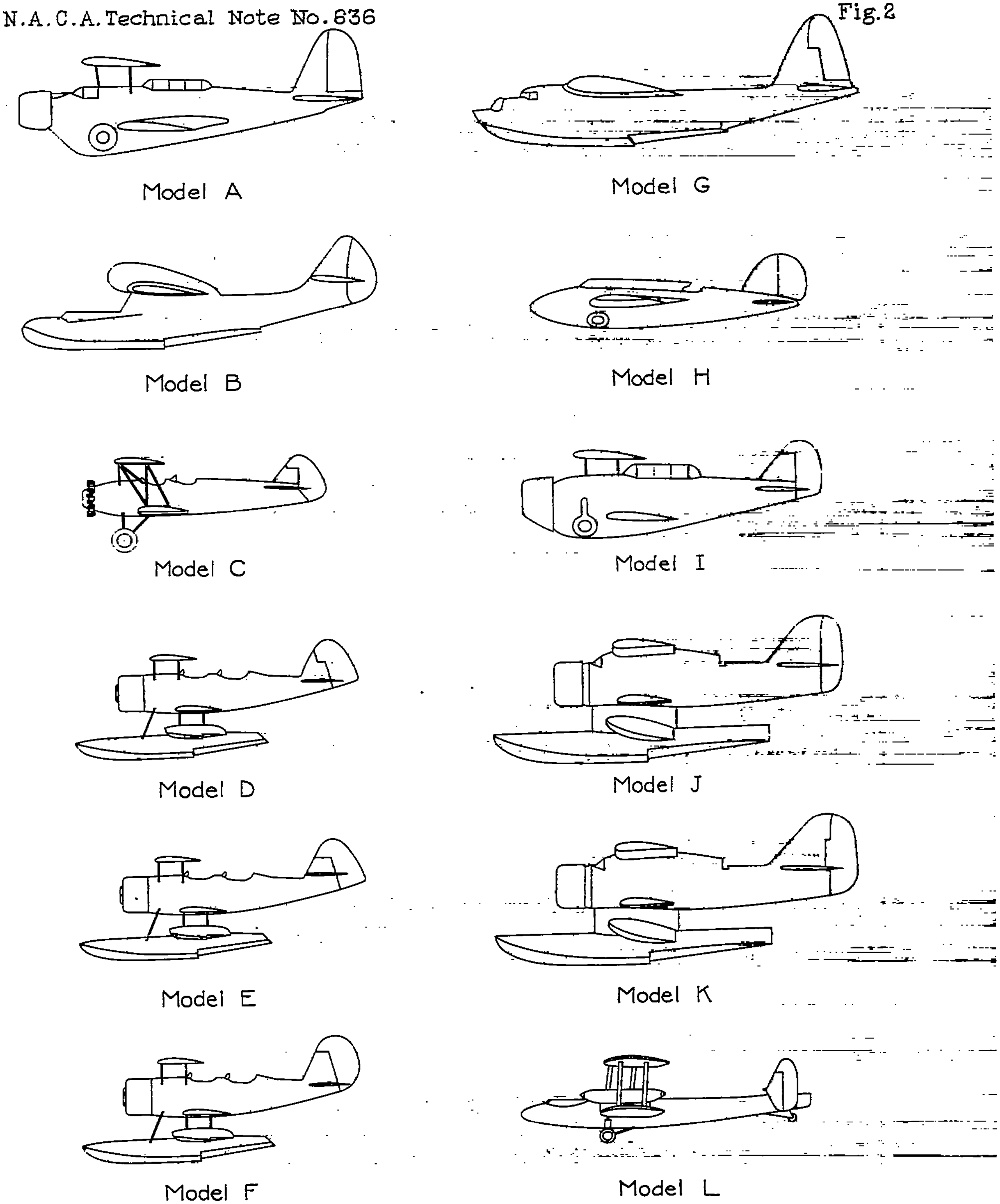

Figure 2.- Side elevations of models listed in table I.

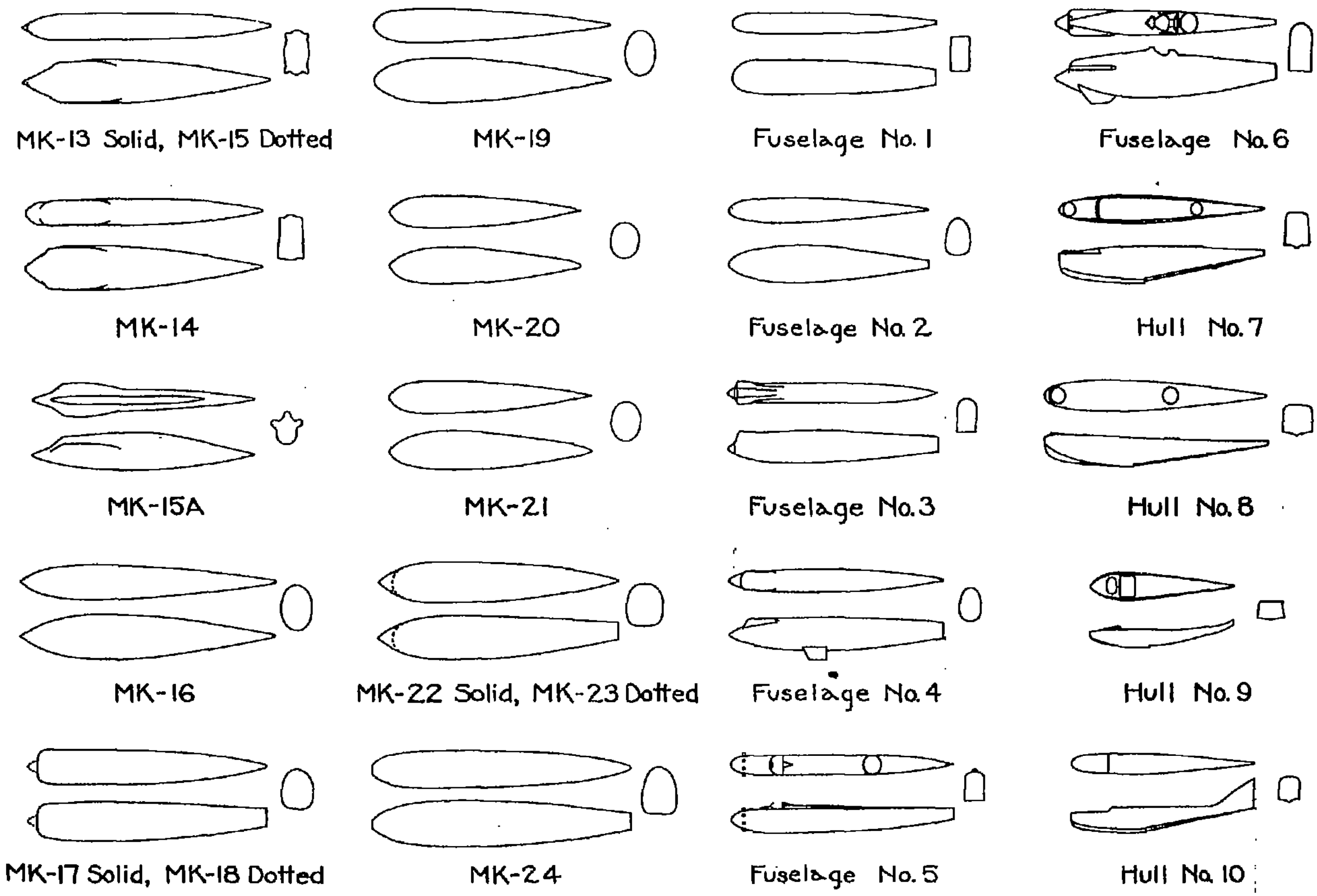

Figure 3.- Sketches of fuselages and hulls listed in table II.

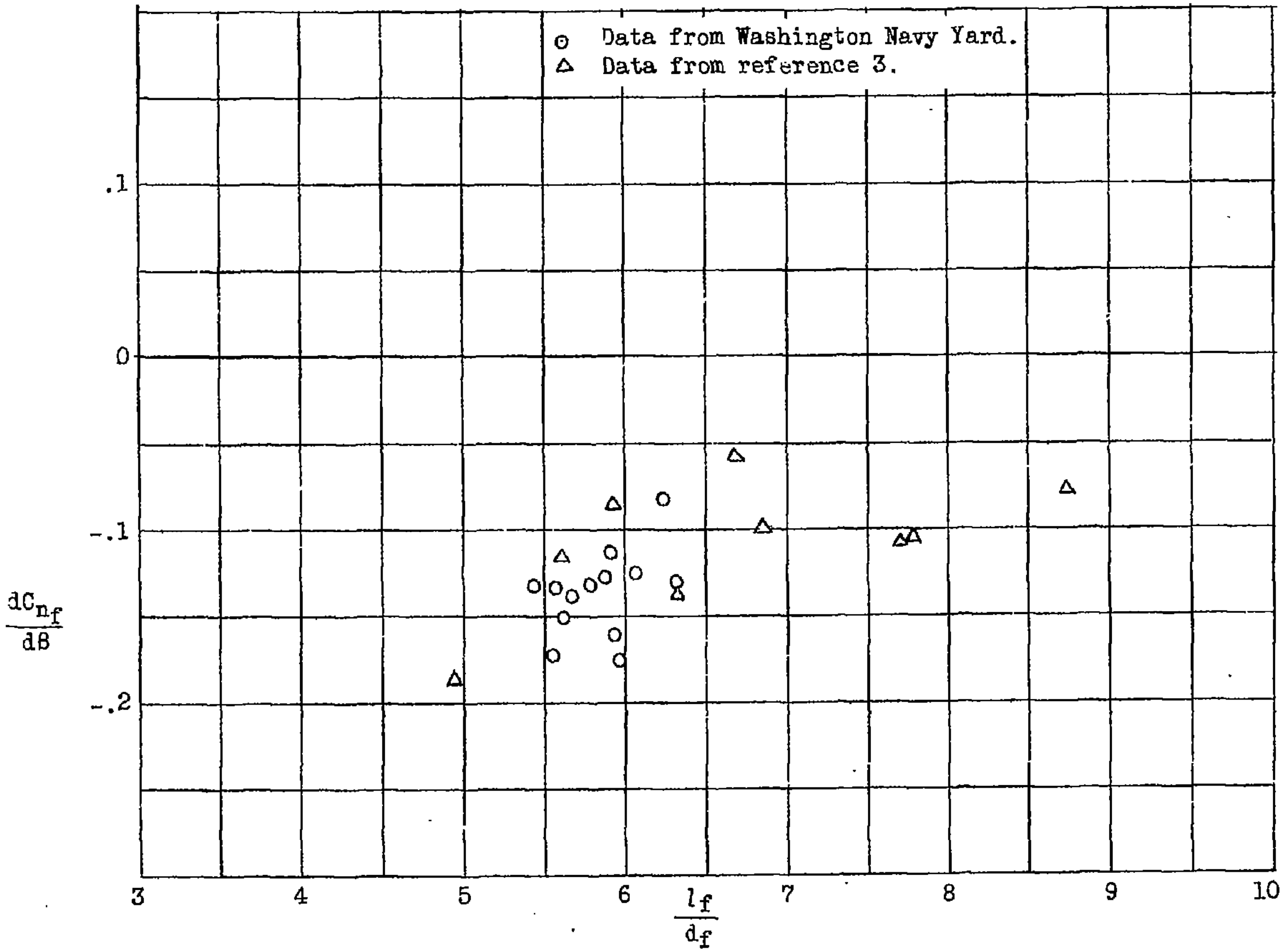

Figure 4.- Effect of fuselage length-depth ratio on rate of change of yawing-moment coefficient of fuselage with sideslip.

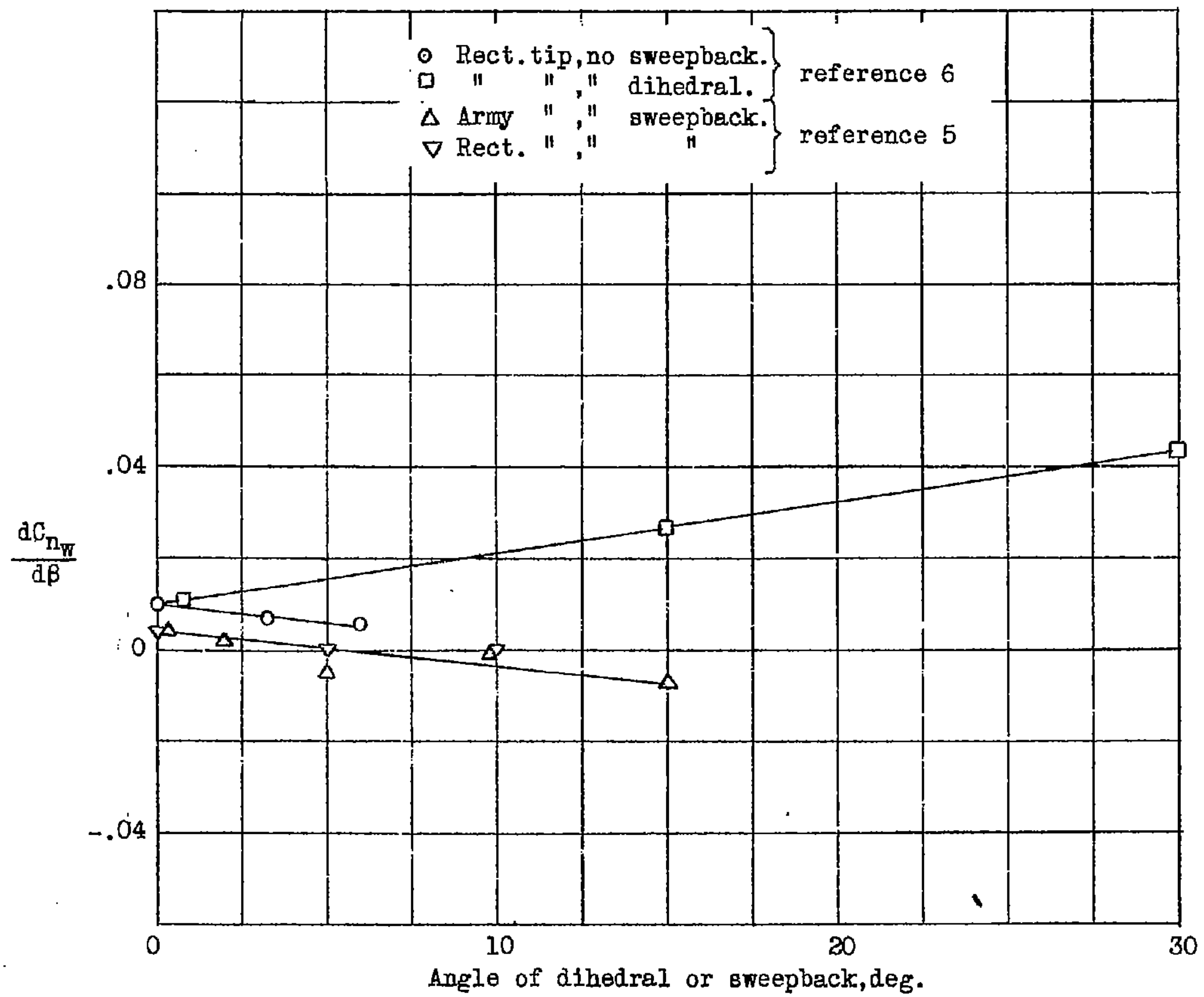

Figure 5.- Effect of dihedral and sweepback on rate of change of yawing-moment coefficient of wing with sideslip.