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LOADS IMPOSED ON INTERMEDIATE FRAMES OF STIFFENED SHELLS

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SUMMARY

The loads imposed on intermediate frames by the curvature of the longitudinals and by the diagonal-tension effects are treated. A new empirical method is proposed for analyzing diagonal-tension effects. The basic formulas of the pure diagonal-tension theory are used, and the part of the total shear S carried by diagonal tension is assumed to be given by the expression

$$S_{DT} = S\left(1 - \frac{\tau_0}{\tau}\right)^n$$

where τ_0 is the critical shear stress, τ the total (nominal) shear stress, and $n = 3 - \sigma/\tau$ where σ is the stress in the intermediate frame. Numerical examples illustrate all cases treated.

INTRODUCTION

The structural elements of stiffened shells may be divided into two main classes: strength elements and form elements. The strength elements primarily develop the stresses necessary to hold the external loads in equilibrium; the form elements primarily serve to give the desired shape to the structure and to maintain this shape as long as possible when the loads increase. The longitudinal stiffeners and the skin of a fuselage, for example, are strength elements and the intermediate frames are form elements.

No sharp line of demarcation, of course, exists between the two kinds of elements. If the strength members have any tendency to change their shape under load, then the form members will develop stresses resisting further deformation of the strength members. Consequently, the form members might also be called secondary strength members. The loads imposed by the strength members on the form members are the subject of the present paper. An appendix gives numerical examples for the application of the principles developed in the paper.

GENERAL ASSUMPTIONS

The stiffened shell, on the stressed-skin structure, employs two distinct types of strength element: the relatively compact longitudinal stiffener and the relatively thin sheet. The same two components are characteristic of the plate girder, which has been extensively employed for a long time in civil engineering. The plate girder may be considered as a two-dimensional form and the stiffened shell, as a three-dimensional form of stressed-skin structure.

Civil engineers have established the custon of assuming that the flanges of plate girders take most or all of the normal stresses due to bending and that the web takes all the shear. Measurements have shown that the actual stresses are sometimes distributed in a quite irregular manner and that, therefore, the maximum stresses do not agree very well with the calculated stresses. The assumptions mentioned have nevertheless been accepted as a satisfactory basis for design in civil engineering and they have been adopted for aeronautical design. Refinements have been made, however, in establishing formulas for the effective width of sheet that may be considered to work in conjunction with the stiffener in carrying normal stresses.

If the normal stresses due to bending are known, the shearing stresses in the sheet can be calculated. It is customary to assume that the ordinary engineering theory of bending applies, which gives for the normal stress and the shear stress, respectively,

$$\sigma = \frac{My}{I} \tag{1}$$

$$\tau = \frac{SQ}{bI} \tag{2}$$

This theory neglects shear deformation and is sufficiently accurate for engineering work when applied to beams of solid cross section. When the theory is applied to stiffened shells of round or elliptical cross section, such as fuselages, the errors in normal stresses may amount to 5 or 10 percent. When the theory is applied to wide shallow box beams, such as wings, the errors may be as much as 30 percent. The design of the intermediate frames rests, however, on the average rather than the peak values of the normal stresses. For most practical purposes, the use of the engineering theory of bending is believed to be sufficiently accurate for the primary analysis of the shell that must be made to obtain the design loads in the intermediate frames. In order to obtain reliable design loads on the shell itself, however, it will often be necessary to refine the calculation by taking into account the shear deformation of the skin.

The shear stresses due to torsion may be calculated by the formula

$$\tau = \frac{T}{2At} \tag{3}$$

This formula applies strictly only to a shell of constant cross section loaded by shear stresses at the ends. It is probably sufficiently accurate for design purposes at all cross sections where intermediate frames are located.

It is impossible to overemphasize that all theories of stress analysis are of limited applicability. It cannot be urged too often that the stress analyst study the basic assumptions underlying the theories in order to become acquainted with their limitations.

PLATE-GIRDER THEORY

For convenience of reference, the term "plate girder" will be used to denote a girder with a plate web designed so that the web will not buckle under shear loads until the design load is reached. As already pointed out, such a girder may be considered as a special case of stressed-skin structure, the intermediate frames of the shell being represented by uprights or web stiffeners in the plate girder.

The girder recommends itself by its simplicity as a basis for discussing the necessary principles; the application of these principles to shell structures will then follow as a natural extension.

The Plate Girder with Parallel Flanges

In a plate girder with parallel flanges (fig. 1), no intermediate "frames" or uprights are required. Uprights may be used, if desired, to increase the critical buckling stress of the web but they will not be subjected to loads until the web buckles. By the definition of the term "plate girder" used in the present paper, this buckling does not occur until the design load is reached or passed.

The Plate Girder with Curved (or Inclined) Flanges

The plate girder with curved flanges (fig. 2) is the general case of the girder with inclined flanges. In all the following discussions, the girders will be assumed to be symmetrical about the longitudinal axis, at least as far as inclination of the flanges is concerned.

In a girder with inclined flanges, the flange forces carry part of the shear load. The vertical component V of each flange force is given by the formula

$$V = \frac{M}{h_o} \tan \delta \tag{4}$$

The horizontal component H is given by

$$H = \frac{M}{h_o} \tag{5}$$

The shear force S_W in the web is given by

$$S_W = P - 2V = P\frac{h_P}{h_o} \tag{6}$$

Uprights are not indispensable in practice because the plate web is capable of taking some transverse normal stresses, although these stresses are neglected in the design. If uprights are used, they may be analyzed by assuming the curved flanges to be replaced by straight flanges kinked at each upright, as will be shown presently.

The Plate Girder with Kinked Flanges

In a girder with kinked flanges, it is necessary to use uprights at each kink if the web is to carry only shear (fig. 3(a)). The horizontal components H_1 and H_2 of the flange forces just to the left and to the right of the kink will be equal. The vertical components V_1 and V_2, however, will differ by an amount ΔV_L. Application of formula (4) gives

$$\Delta V_L = V_2 - V_1 = \frac{M}{h_o} (\tan \delta_2 - \tan \delta_1) \qquad (7)$$

The force ΔV_L must be absorbed by an upright to be converted into change of shear force in the web (fig. 3(b)). One force ΔV_L acts on the top and one on the bottom of the upright. The total force exerted on the upright must be held in equilibrium by the difference between the web shears S_{W1} and S_{W2}:

$$2\Delta V_L = S_{W1} - S_{W2} \qquad (8)$$

By formula (6)

$$S_{W1} = P \frac{h_{P1}}{h_o} \quad \text{and} \quad S_{W2} = P \frac{h_{P2}}{h_o}$$

Therefore

$$2\Delta V_L = S_{W1} - S_{W2} = P \frac{h_{P1} - h_{P2}}{h_o} \qquad (9)$$

It can easily be shown that the values of ΔV_L obtained from equation (7) and from equation (9) are numerically equal.

Application of Plate-Girder Theory to Shells

Figure 4 shows the cross section of a fuselage, as well as the side view and the plan view of a short section of the fuselage at either side of an intermediate frame. The curved outlines of the actual fuselage have been replaced by straight lines between frames.

As indicated in the figure, the stringer direction lines at any one cross section are assumed to meet in a common apex on the axis of the fuselage. The angle of inclination between a stringer and the fuselage axis is denoted by δ_V when seen in the projection on the vertical plane and by δ_H when seen in the projection on the horizontal plane.

Any two corresponding stringers, for instance, the upper longeron U and the lower longeron L of figure 4, together with the intervening skin may be considered as a two-flange girder; the entire shell may be considered as a superposition of several such individual girders. The shear carried in the web of each individual girder is given by formula (6), if P is understood temporarily to refer to the share of the total load carried by each individual girder. Now, the factor h_P/h_o is the same for all stringers at any one section. The shear carried by the skin (or web) at any cross section of the fuselage is therefore

$$S_W = P \frac{h_P}{h_o} \qquad (10)$$

where P is again the total load, and the factor h_P/h_o may be determined from any one stringer, provided only that the proper plane of projection is used, namely, the plane parallel to the load.

The horizontal load in any stringer is assumed to be given by formula (1). The shear load per inch perimeter of the shell skin is therefore obtained by applying formulas (2) and (10) as

$$\tau t = \frac{PQ}{2I} \frac{h_P}{h_o} \qquad (11)$$

(It should be noted that, in the application of formula

(2), the width b is taken as twice the actual skin thickness measured normal to the skin. The use of the "slant thickness" is incorrect because the shear must be tangential to the median line.) At the frame (fig. 4), the difference between the shear forces just to the left and just to the right of the frame

$$\Delta\tau t = (\tau t)_1 - (\tau t)_2 = \frac{PQ}{2I} \frac{h_{P1} - h_{P2}}{h_o} \tag{12}$$

acts as a distributed shear load on the frame (fig. 5(a)). This distributed load is held in equilibrium by the differences between the vertical components of the stringer forces

$$\Delta V_L = \frac{My}{I} A \ (\tan \delta_{V2} - \tan \delta_{V1}) \tag{13}$$

where A is the cross-sectional area of the stringer under consideration; y, its distance from the neutral axis; and I, the moment of inertia of the shell cross section. These forces ΔV_L are also shown in figure 5(a)

In the horizontal projection, the stringers are also kinked and therefore exert transverse forces on the frame

$$\Delta T_L = \frac{My}{I} A \ (\tan \delta_{H2} - \tan \delta_{H1}) \tag{14}$$

The resulting system of horizontal loads acting on the frame is shown in figure 5(b). The analysis of the frame for these load systems may be made by any desired method.

The skin was assumed to be directly connected to the bulkhead frame. If the frame is connected only to the stringers and not to the skin, the formulas will still give the average forces correctly, but there will be force concentrations resulting in higher maximum stresses. This fact would influence, for instance, the design of the skin; the design of the frame, however, will not be materially influenced because it depends on the integrated effects of the applied forces.

THE EFFECTS OF BEAM CURVATURE

Beams may be curved for two reasons: They may be built with initial curvature or they may be bent into a curved shape by the applied loads. It will be assumed that the curvature is small.

Curvature may be thought of as a succession of many small kinks, and it has been previously shown that there is a vertical component of the flange force at each kink, as given by formula (7). If the curvature of the two flanges is of opposite sign, as in the cases previously treated, the vertical components of the two flange forces act in the same direction and must be balanced by shear forces. If the curvature of the two flanges is of the same sign (fig. 6), however, the vertical components will oppose each other and will give rise to transverse loads. These distributed transverse loads will cause compression in the shear webs and bending in the stringers between frames. The frames act as supports to the stringers and furnish the concentrated reactions to the transverse loads on the stringers. The frames are therefore in compression, as indicated in figure 6(b).

If the radius of curvature R of the beam is assumed to be large compared with the depth of the beam, the distributed transverse force per unit run of span acting on any stringer is

$$\frac{V}{\Delta x} = \frac{F}{R} \qquad (15)$$

where F is the axial force on the stringer. If Δx is made equal to the frame spacing d, the resulting expression

$$V = \frac{Fd}{R} \qquad (16)$$

gives the concentrated force exerted by any given stringer on the frame.

According to the theory of bending, the bending curvature is given by

$$\frac{1}{R} = \frac{M}{EI} \qquad (17)$$

The stringer force F is given by

$$F = \frac{Mc}{I} A \qquad (18)$$

where A is the cross-sectional area of the stringer. Substituting from equations (17) and (18) into equation (16) and using the fundamental relation (1) gives

$$V = \left(\frac{M}{I}\right)^2 \frac{Acd}{E} = \sigma^2 \frac{A}{E} \frac{d}{c} \qquad (19)$$

This load is normally very small; but it may become of some importance in very shallow flexible beams, being inversely proportional to the beam depth. It should be noted that the effect is proportional to the square of the applied bending moment, a fact which may be important, for instance, in connection with strain-gage tests at low loads.

The loads V may become quite important when the intermediate frame is open; this case occurs, for example, at a cockpit cut-out when the fuselage is subjected to the fin-and-rudder load.

It might be noted in passing that, whenever the forces V become important for bulkhead design, the distributed transverse loads from which they arise become important on account of the local bending which they cause in the stringers.

DIAGONAL-TENSION THEORY

The Plane Beam in Pure Diagonal Tension

The diagonal-tension beam is so well-known to aeronautical designers that no long discussion will be given. It will be sufficient to recall that the web of the beam (fig. 7) develops a series of parallel folds inclined at an angle α to the axis of the beam. The stress in the web sheet is assumed to be pure tension along the lines of the folds. Uprights are necessary, as in a truss with tension diagonals, to keep the flanges separated against the tendency of the diagonals to pull the flanges together.

The angle of folds α is determined by the formula

$$\sin^2 \alpha = \sqrt{a^2 + a} - a \tag{20}$$

with

$$a = \frac{1 + \frac{ht}{2A_F}}{\frac{dt}{A_U} - \frac{ht}{2A_F}} \tag{21}$$

where A_F is the cross-sectional area of the flange and A_U is that of the upright. For the practical range of construction, the calculated value of α is around 42°. On the assumption of rigid flanges and uprights, the theory gives $\alpha = 45°$, which is a convenient value for practical use and is slightly conservative for the design of the uprights with which this paper is concerned. For uprights inclined to the flanges at an angle β, the assumption of rigid edge members gives $\alpha = \beta/2$, which is probably always used in design work.

The load on the upright is given by the formula

$$V_U = P \frac{d}{h} \tan \alpha = \tau td \tan \alpha \tag{22}$$

for the case of figure 7. The derivation of the equations given may be found in reference 1. The application of these formulas to shells with plane walls is well enough known to obviate a detailed discussion.

The Shell with Curved Walls in Pure Diagonal Tension

In curved diagonal-tension fields, such as side walls of fuselages, the calculation of the angle of folds α becomes more complex. The formulas are given in reference 2 but, unfortunately, it is impossible to give a single formula comparable with equation (20) that is applicable to a curved field. The assumption of rigid flanges and uprights made for plane fields may also be made for curved fields, the longitudinals and the rings being assumed rigid in order to obtain a practical approximation to the angle α.

The calculation of the loads imposed by the skin on the ring of a curved diagonal-tension field is also more complex than the calculation of the load imposed on the upright of a plane beam. For simplicity, the case of uniform diagonal tension around the circumference will be first considered. This case occurs in a shell in torsion.

First of all, the circumferential component of the tension in the skin must be counterbalanced by hoop compression in the ring. This relation is exactly analogous to the relation between skin tension and upright force in the plane beam, and formula (22) again applies.

The circumferential tension is transmitted to the ring by radial pressure. If the ring touches the skin, this transfer is continuous and there are no additional effects. If the ring does not touch the skin and receives its load through the longitudinals, then the radial pressure inward is concentrated at the longitudinals (fig. 8(a)). By application of formula (15), it will be found that this curved beam is statically equivalent to the straight beam shown in figure 8(b). Under the assumed conditions of uniformity, the part of the ring between longitudinals is therefore in the condition of a beam built in at both ends and loaded by a uniformly distributed load. If P_r is the radial load exerted by one longitudinal on the ring, the maximum bending moment M on the ring will be

$$M = \frac{P_r\, h}{12}$$

occurring where the ring is loaded by the longitudinal. Now the load P_r is given by elementary statics as

$$P_r = V_U \frac{h}{R} = \tau\, td \tan \alpha \frac{h}{R}$$

Therefore

$$M = \tau\, td \tan \alpha \frac{h^2}{12\, R} \tag{23}$$

where h and R are measured on the circle of contact between longitudinals and ring.

These equations hold strictly only when there is an infinite number of longitudinals. They are, however, sufficiently accurate for practical purposes if the number of longitudinals n distributed around the circle is greater than 6.

Formula (22) applies to any shape of cross section, as long as the assumption of uniform tension along the circumference is fulfilled. Formula (23), of course, applies strictly only if the radius R is constant between longitudinals.

If the ring is open, the case is analogous to that of a plane diagonal-tension beam with stiffeners on only one side of the web. The pull of the sheet causes an eccentricity moment acting throughout the length of the ring, or upright,

$$M = V_U\, e = \tau\, td \tan \alpha\, e \qquad (24)$$

where e is the distance between the centroid of the ring, or upright, and the line of action of the sheet tension. In a plane web, e will be constant along the length of the upright. In a curved web, however, the folds in the sheet will leave the original plane of the sheet and will finally lie along the chords from longitudinal to longitudinal, and e will be variable.

Eccentricity moments will also arise if the sheet tension and with it the force V_U varies along the circumference of the ring, or along the length of the upright. Consider, for instance, a shell as indicated schematically in figure 9; assume that the skin has buckled into diagonal-tension fields in the panels next to the neutral axis but not in the panels next to the extreme fibers. Under this condition, eccentricity moments as shown in the figure will act on the ring (assuming the load to act downward), each moment being again expressed by formula (24). In the general case, however, when there is diagonal tension in each panel, the moment is caused only by the difference between successive forces V_U, or

$$M = e(V_{U_n} - V_{U_{n-1}}) \qquad (25)$$

starting the count at the extreme fiber. If symmetry exists about both axes, the moment at A is

$$M_A = M \frac{2\theta}{\pi} \tag{26a}$$

and the moment at B is

$$M_B = M\left(1 - \frac{2\theta}{\pi}\right) \tag{26b}$$

The total moments at A and B are obtained by summing up the individual contributions from each panel point, or for each longitudinal.

The theory of the preceding paragraph is based on the assumption that the circumferential component of the diagonal tension in a given panel does not affect the circumferential tension in the panels adjacent to it on the circumference. Actually, these circumferential tensions in the skin can directly equalize themselves to some extent around the circumference. At the limit, when they equalize themselves completely, the hoop compression V_U in the frame will be uniform around the circumference and the eccentricity moments given by equation (25) will disappear.

The Incomplete Diagonal-Tension Field

Practical experience has proved that the design formulas based on the assumption of pure diagonal tension are too conservative in many cases. It was found that the critical buckling stress of the sheet has to be exceeded many times before the state of stress in the sheet approaches reasonably closely to the assumed condition of pure tension. In relatively heavy sheets, and particularly in curved sheets, the critical buckling stress is often exceeded only a few times at the design load, so that the assumptions of the theory are not very well fulfilled.

The main reason for the discrepancy is obviously the fact that the sheet continues to carry part of the load in shear after it has buckled. Borrowing an assumption sometimes made in structures with compression members, Wagner and Ballerstedt (reference 2) and others therefore proposed to assume that the sheet carries the critical shear stress as shear even after buckling and that only the excess over the critical stress is converted into diagonal tension.

A more detailed study led Schapitz (references 3 and 4) to propose a theory of the "incomplete diagonal-tension field." He considers a skin-stringer panel loaded by normal forces along one axis and by shear forces. In sections parallel to the normal forces, the stress conditions are assumed to be uniform. In transverse sections, however, the normal stress is assumed to vary continuously from tension along the center line through zero to compression in the longitudinal stiffeners. The second principal stress is assumed to be a compression equal to the critical shear stress.

The law of stress variation is assumed only qualitatively. A characteristic value determining the quantitative variation is then chosen so that the results of the analysis give the best possible agreement with tests. A second characteristic value enters into the picture for curved diagonal-tension fields, so that there is ample possibility of adjusting the theory to fit the facts.

Proposed New Theory of Incomplete Diagonal-Tension Field

The present paper is concerned with only one single item of the diagonal-tension theory, namely, the loads imposed on the uprights or the transverse stiffeners. A detailed discussion of diagonal-tension theories would therefore be out of place. Although the theory of Schapitz has much to recommend it on the basis of wide applicability, it is thought that a somewhat simpler theory built up on different assumptions will be suitable for the present restricted purpose and perhaps for a wider field of application.

The assumptions underlying the proposed theory are the following:

1. The angle of folds is given by the curve in figure 10, obtained as follows: In reference 2 are given curves of α against $\frac{d}{R}\sqrt{\frac{E}{\tau}}$ calculated first for closely spaced longitudinals, then for closely spaced rings. The longitudinals and the rings being assumed rigid ($a = 0$ in reference 2), two curves are obtained for α. These two curves intersect, and the single curve of figure 10 was obtained by using the branches of the two

curves that give the higher values of α and fairing over the break near the intersection. Experimental evidence to be discussed later furnished the basis and justification for this procedure.

2. The shear force S is carried partly as shear S_S and partly as diagonal tension S_{DT}

$$S = S_S + S_{DT} \tag{27}$$

The part carried as diagonal tension is given by the expression

$$S_{DT} = S\left(1 - \frac{\tau_o}{\tau}\right)^n \qquad (\text{for} \quad \tau > \tau_o) \tag{28}$$

where τ is the actual nominal shear stress (i.e., total shear force divided by sheet area) and τ_o is the critical buckling stress, so that τ/τ_o is the factor by which the buckling stress is exceeded. The exponent n is determined from tests. With $n = 1$, the Wagner-Ballerstedt assumption is obtained. On the basis of the Wagner-Lahde tests (reference 5), it appears safe to set

$$n = 3 - \frac{\sigma_U}{\tau} \tag{29}$$

where σ_U is the compressive stress in the upright. It will be permissible, as far as present knowledge indicates, to use $n = 3$ if $\frac{\sigma_U}{\tau} < \frac{1}{2}$ and $n = 2$ if $\frac{\sigma_U}{\tau} > \frac{1}{2}$. Equation (29) is based on the assumption that the stress in the longitudinals caused by diagonal tension is small. This assumption is valid for most practical cases; in the tests reported in reference 5, this consideration was taken into account by making the longitudinals extremely heavy.

The physical meaning of equation (28) is that the shear stress continues to increase after buckling if $n > 1$. This assumption is the main difference between the proposed new theory and the older theories, which assumed $n = 1$, that is, the shear stress to remain equal to the buckling stress.

The theory expressed by formulas (28) and (29) at present rests chiefly on tests made of plane diagonal-tension fields. It is possible that, for curved fields, a more complicated relation should be used. The experimental evidence for curved fields, admittedly very scanty, does not appear to indicate the immediate necessity of introducing further complications.

Application of Proposed New Theory to Shells

On the basis of the proposed theory of the incomplete diagonal-tension field, the procedure of stress analyzing a shell would be as follows:

For each panel, the total nominal shear stress is calculated by means of formulas (2) and (3).

The angle of folds α is determined from figure 10.

The critical shear stress is calculated. From available data on structures simulating aircraft construction (references 3 and 6), it appears that the formula

$$\tau_o = 0.1\ E\ \frac{t}{R} + 5E \left(\frac{t}{h}\right)^2 \left[1 + 0.8 \left(\frac{h}{d}\right)^2\right] \qquad (30)$$

may be expected to give good average values for the critical shear stress. If the values given by formula (30) are multiplied by 0.75, reasonable assurance may be had that the resulting value is conservative. In formula (30), the notation of figure 10 is used, and it is assumed that $d > h$. If $h > d$, these two letters must be interchanged in (30).

The portion τ_{DT} of the total shear stress τ that is carried as diagonal tension is calculated with the help of formula (28). In the first analysis, the exponent n

must be assumed. In the final analysis, n is calculated by using formula (29), σ_U being replaced by the value obtained from the first analysis.

With the angle α and the stress τ_{DT} known, the forces and the moments on the ring can be calculated by using the previously discussed theory of the pure diagonal-tension field.

Experimental Checks

Angle of folds.- The tests by Limpert (reference 7) give an excellent check for the curve of figure 10. In these tests, the angle α was measured at various stages of loading, and the experiments clearly showed the increase of α with increasing shear stress predicted by the theory.

In the torsion tests made by Schapitz on complete shells (reference 3), the strain measurements were made at two load stages, the lowest one being well beyond the buckling load. The calculated angles α for these two load stages closely bracket the observed angle.

In the tests made by Thorn (reference 6), the average observed angle is 26.8° and the observed maximum is 34°. The average angle calculated from figure 10 is 26.2°, if the load acting on the specimens is assumed to be the critical load. The average calculated angle is about 38°, if the load is assumed to be the load used in the last stage of the strain-gage test. The test report does not state at what loads the angles were measured, so that no direct comparison can be made. It is improbable that the actual changes in angles were as large as predicted by theory; consequently, the calculated angles are probably too large for these tests and the resulting calculated frame stresses are high, that is, conservative.

Stresses in frames.- Strain measurements on frames are described in references 3 and 6. For the tests described in reference 6, the proposed method of analysis gave conservative results, the observed stresses being as low as 50 percent of the calculated stresses. For the tests reported in reference 3, the method was not conservative. The cylinders used in these tests had been used previously for bending tests, leaving permanent deformations. For this reason, and possibly others, the observed critical stress was low, in one case as low as 57 percent of that calculated by formula (30). The worst disagreement in the frame stresses occurs in the cylinder with the smallest ratio R/t (= 740). In this cylinder it was necessary to multiply the buckling stress from formula (30) by 0.6 to reach agreement between the calculated and the observed frame stresses. In the cylinder with a value of R/t of about 1,000, a factor of 0.75 had to be used. In the tests reported in reference 6 where R/T was between 2,000 and 3,000, no correction factor was needed for applying formula (30). The effect of previous loading is apparently important if R/t is low.

Concluding Remarks on Diagonal-Tension Theory

The proposed new diagonal-tension theory will probably be generally conservative for the design of intermediate frames if the conservative values of the angle α from figure 10 and conservative values of τ_0 are used. Less conservative values for α and τ_0 may give better agreement with the facts in some cases, but it is impossible to predict such cases with the present knowledge. Unlike the theory of the plate girder presented in the first part of this paper, the theory of diagonal-tension action may be expected to undergo continual changes and refinements for some time. The stress analyst should constantly endeavor to keep abreast of such developements.

Langley Memorial Aeronautical Laboratory,
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APPENDIX

Example 1

Find the loads acting on the intermediate ring A-A of the fuselage shown in figure 11. Assume the rings to be riveted to the skin.

Compute first the properties of the cross section, assuming the sheet to be fully effective in taking bending stresses.

Total stringer area $16 \times 0.120 = 1.92$ sq. in.

Total skin area πDt $= \underline{2.01}$ sq. in.

Total area 3.93 sq. in.

Effective thickness $t_e = \dfrac{3.93}{\pi D} = 0.03125$ in.

$$I = \pi R^3 t_e = 785 \text{ in.}^4$$

$Q = 2R^2 t_e \sin\theta$ (See fig. 12.)

$$\frac{I}{Q} = \frac{\pi R}{2 \sin\theta}$$

The bending moment at ring A-A is

$$M = 1{,}000 \times 144 = 144{,}000 \text{ in.-lb.}$$

The maximum fiber stress is therefore

$$\sigma = \frac{MR}{I \cos\delta} = \frac{144000 \times 20}{785} = 3{,}670 \text{ lb./sq. in.}$$

(The angle δ between the stringer and the axis is so small that $\cos\delta \approx 1$.)

Estimating the buckling stress of the sheet as 20 percent higher than the theoretical compressive buckling stress,

$$\sigma_{crit} = 1.2 \times 0.363\, E\, \frac{t}{R} = 3{,}500 \text{ lb./sq. in.}$$

and the maximum stress is close enough to the critical stress to justify the assumption that the sheet is fully effective.

The shear stress in the panels next to the neutral axis $\left(\theta = \frac{\pi}{2}\right)$ is

$$\tau = \frac{PQ}{2tI} = \frac{1000 \times 2}{2 \times 0.016 \times \pi \times 20} = 995 \text{ lb./sq. in.}$$

The critical shear stress is, by formula (30),

$$\tau_o = 0.1 \times 10^7 \times \frac{0.016}{20} + 5 \times 10^7 \times \left(\frac{0.016}{7.85}\right)^2 \left[1 + 0.8 \left(\frac{7.85}{12}\right)^2\right]$$

$$= 800 + 279 = 1{,}079 \text{ lb./sq. in.}$$

which is above the shear stress actually existing, so that there will be no diagonal-tension effects.

The shear forces applied by the skin to the ring, which are the differences between the skin shear forces in the adjacent panels, are next found by formula (12)

$$\Delta\tau t = \frac{PQ}{2I} \frac{h_{P1} - h_{P2}}{h_o}$$

$$= \frac{1000 \sin \theta}{\pi R} \frac{26.3 - 20.8}{40}$$

$$\Delta\tau t = 2.19 \sin \theta$$

where the values of h_o, h_{P1}, and h_{P2} were obtained from figure 13, which is drawn to an exaggerated vertical scale.

The shear force transmitted in each panel to the ring is obtained by multiplying the shear intensity τt by the developed width of the panel

$$\frac{\pi D}{16} = 7.85 \text{ in.}$$

so that

$$S_a = 2.19 \times 7.85 \times \sin 11.25^\circ = 3.35 \text{ lb.}$$

$$S_b = 17.2 \times \sin 33.75^\circ = 9.57 \text{ lb.}$$

$$S_c = 17.2 \times \sin 56.25^\circ = 14.31 \text{ lb.}$$

$$S_d = 17.2 \times \sin 78.75^\circ = 17.00 \text{ lb.}$$

Next, the vertical forces exerted by the longitudinals on the ring are computed by formula (13). Table I gives all the data and is self explanatory.

In a similar manner, the horizontal forces acting on the ring are obtained by using formula (14). The factors ($\tan \delta_{H2} - \tan \delta_{H1}$) can be obtained from the factors ($\tan \delta_{V2} - \tan \delta_{V1}$) by inspection in the case of a circular ring. Table II gives the details of calculating the horizontal forces. Figure 14 shows the two systems of forces graphically.

TABLE I

Calculation of Vertical Loads on Ring

Stringer	y (in.)	$\frac{My}{I}$ (lb./sq.in.)	$\frac{My}{I}A$ (lb.) (a)	$\tan \delta_{V2}$ (b)	$\tan \delta_{V1}$ (c)	$(\tan \delta_{V2} - \tan \delta_{V1})$	ΔV_L (lb.)
1	20.0	3,670	903	0.0667	0.0476	0.0191	17.25
2	18.48	3,390	835	.0616	.0440	.0176	14.70
3	14.14	2,590	637	.0471	.0337	.0134	8.53
4	7.65	1,404	345	.0255	.0182	.0073	2.52
5	0	0	0	0	0	0	0

[a] A = area of stringer + area of effective skin

$= 0.120 + 0.126 = 0.246$

[b] $\tan \delta_{V2} = \frac{y}{25 \times 12}$ [c] $\tan \delta_{V1} = \frac{y}{35 \times 12}$

TABLE II

Calculation of Horizontal Loads on Ring

Stringer	$\frac{My}{I}A$ (lb.)	$(\tan \delta_{H2} - \tan \delta_{H1})$	ΔT_L (lb.)
1	903	0	0
2	835	.0073	6.08
3	637	.0134	8.54
4	345	.0176	6.08
5	0	.0191	0

Example 2

Figure 15(a) shows the cross section of a wing beam at the first bulkhead from the root. The bulkhead spacing d is 50 in.; the bending moment M at the bulkhead is 1,500,000 in.-lb. Find the forces on the bulkhead caused by beam curvature.

The flange forces are

$$F = \frac{M}{h} = \frac{1500000}{10} = 150,000 \text{ lb.}$$

The compressive stress is

$$\sigma_c = \frac{F}{A_c} = \frac{150000}{4.2} = 35,700 \text{ lb./sq. in.}$$

The forces V acting on the compression side are, by formula (19),

for each flange

$$V = 35,700^2 \times \frac{0.6}{10^7} \times \frac{50}{4.32} = 885 \text{ lb.}$$

and for each stringer

$$V = 35,700^2 \times \frac{0.5}{10^7} \times \frac{50}{4.32} = 738 \text{ lb.}$$

This load is uniformly distributed along each stringer. The resulting maximum bending moment on each stringer is

$$M = \frac{Vd}{12} = \frac{738 \times 50}{12} = 3,080 \text{ in.-lb.}$$

occurring at the bulkhead.

The tensile stress is

$$\sigma_t = \frac{F}{A_t} = \frac{150000}{3.2} = 46,900 \text{ lb./sq. in.}$$

The forces V acting on the tension side are, by formula (19), for each flange

$$V = 46,900^2 \times \frac{0.6}{10^7} \times \frac{50}{5.68} = 1,160 \text{ lb.}$$

and for each stringer

$$V = 46,900^2 \times \frac{0.5}{10^7} \times \frac{50}{5.68} = 966 \text{ lb.}$$

This load is uniformly distributed along each stringer. It causes at the bulkhead a bending moment

$$M = \frac{Vd}{12} = \frac{966 \times 50}{12} = 4,030 \text{ in.-lb.}$$

The total area of each stringer is 0.5 sq. in. Assuming the depth of the stringer to be 1 in., the section modulus will probably be less than 0.2 in.^{3}; the bending stress will therefore be more than

$$\sigma = \frac{4030}{0.2} = 20,150 \text{ lb./sq. in.}$$

It is obvious that the bulkhead spacing of 50 in. should be reduced. For actual design, it should be borne in mind that the vertical loads on the flanges are actually distributed along the shear web over a distance d of 50 in.

Figure 15(b) shows the forces acting on the bulkhead.

Example 3

Figure 16 shows the dimensions of a small sport-plane fuselage at the cockpit. The fin-and-rudder load stresses the longerons to $\sigma = 30,000$ lb./sq. in. Find the size of open ring required at section C - C to prevent caving in of the cockpit. Disregard stresses carried in the skin.

According to formula (19), the forces V arising from bending curvature are

$$V = 30,000^2 \times \frac{0.1}{10^7} \times \frac{17}{10} = 15.3 \text{ lb.}$$

The maximum bending moment in the ring will be

$$M = 15.3 \times 37 + 15.3 \times 7 = 673 \text{ in.-lb.}$$

Since prevention of caving in of the cockpit is an essential condition for safety, the allowable stress will be kept fairly low, say 30,000 lb./sq. in. The required section modulus will then be

$$Z = \frac{M}{\sigma} = \frac{673}{30000} = 0.0224 \text{ in.}^3$$

If standard angles are used, a $1 \times 1 \times \frac{3}{32}$ angle will be just sufficient.

Example 4

The fuselage of example 1 is subjected to a transverse load of $P = 5{,}000$ lb. and a torque of $T = 40{,}000$ in.-lb. acting simultaneously. Find the maximum effects caused by diagonal tension on the ring.

Obviously the maximum effects will occur where the shear stress reaches its maximum, in the panels adjacent to the neutral axis.

The load being higher than in Example 1, the sheet will no longer be fully effective in carrying normal stresses, so that Q and I change. The ratio I/Q does not change, however, if the same simplifying assumptions are made on the distribution of the material. Under the assumption used for Example 1,

$$\frac{I}{Q} = \frac{\pi R^3 t_e}{2 R^2 t_e \sin \theta} = \frac{\pi R}{2 \sin \theta}$$

or, for the neutral axis,

$$\frac{I}{Q} = \frac{\pi R}{2} = 31.42$$

The nominal shear stress in panel 4-5 is therefore

$$\tau = \frac{SQ}{2tI} = \frac{5000}{2 \times 0.016 \times 31.42} = 4{,}970 \text{ lb./sq. in.}$$

The nominal shear stress due to the torque is

$$\tau = \frac{T}{2At} = \frac{40000}{2 \times 1256.6 \times 0.016} = 995 \text{ lb./sq. in.}$$

The total nominal shear stress is therefore

$$\tau = 4,970 + 995 = 5,965 \text{ lb./sq. in.}$$

To find the angle of folds α, calculate

$$\frac{h}{R}\sqrt{\frac{E}{\tau}} = \frac{7.85}{20}\sqrt{\frac{10^7}{5965}} = 16.07$$

From figure 10, $\alpha = 26.4^\circ$

The critical shear stress is

$$\tau_o = 1,079 \text{ lb./sq. in.} \quad \text{(See Example 1)}$$

This value is the critical stress that may be expected as average for a number of panels. For any given panel, it may be lower, and the designer may wish to add some factor of safety beyond that already provided in the design requirements. For the present example, the computed value of τ_o will be used. The portion of the total shear stress carried as diagonal tension is, by formula (28),

$$\tau_{DT} = 5,965 \left(1 - \frac{1079}{5965}\right)^n$$

Estimating $n = 2$,

$$\tau_{DT} = 5,965 \times 0.819^2 = 4,000 \text{ lb./sq. in.}$$

The compressive force on the ring is then, by formula (22),

$$V_U = 4,000 \times 0.016 \times 12 \times 0.496 = 381 \text{ lb.}$$

and the stress is

$$\sigma_U = \frac{381}{0.100} = 3,810 \text{ lb./sq. in.}$$

The ratio

$$\frac{\sigma_U}{\tau} = \frac{3810}{5965} = 0.64$$

giving

$$n = 3 - 0.64 = 2.36$$

which gives, in second approximation,

$$\tau_{DT} = 5,965 \times 0.819^{2.36} = 3,725 \text{ lb./sq. in.}$$

giving

$$\sigma_U = 3,560 \text{ lb./sq. in.,} \qquad n = 2.40$$

$$\tau_{DT} = 5,965 \times 0.819^{2.40} = 3,690 \text{ lb./sq. in.}$$

which may be taken as the final value. The compressive force in the ring becomes

$$V_U = 3,690 \times 0.016 \times 12 \times 0.496 = 351 \text{ lb.}$$

and the stress

$$\sigma_U = \frac{351}{0.100} = 3,510 \text{ lb./sq. in.}$$

In Example 1, the skin was assumed to be riveted to the rings. Under this condition, there will be no bending moments in the ring due to diagonal tension except eccentricity moments. For the sake of illustration, however, the bending moments that would exist if the skin did not touch the rings will be computed. According to formula (23), these moments would be

$$M = V_U \frac{h^2}{12R} = 351 \times \frac{7.85^2}{12 \times 20} = 90.1 \text{ in.-lb.}$$

which is somewhat conservative because h and R were measured on the skin contour, lacking detail data.

Assuming the ring to be 1 in. deep, the section modulus Z would be less than 0.05, since $A = 0.100$. Therefore the bending stress

$$\sigma_B > \frac{90.1}{0.05} \quad \text{or} \quad \sigma_B > 1{,}802 \text{ lb./sq. in.}$$

Before the eccentricity moments acting at longitudinal 4 can be calculated, the diagonal-tension load in panel 3-4 must be calculated. The ratio

$$\frac{I}{Q} = \frac{\pi R}{2 \sin 56.25^\circ} = \frac{31.42}{1 \times 0.831} = 37.82$$

Therefore the shear stress due to the transverse load

$$\tau = \frac{5000}{2 \times 0.016 \times 37.82} = 4{,}130 \text{ lb./sq. in.}$$

and the total shear stress

$$\tau = 4{,}130 + 995 = 5{,}125 \text{ lb./sq. in.}$$

Then

$$\frac{h}{R}\sqrt{\frac{E}{\tau}} = 17.33 \qquad \alpha = 25.7^\circ$$

The part of the shear stress carried by diagonal tension is

$$\tau_{DT} = 5{,}125 \left(1 - \frac{1079}{5125}\right)^{2.40} = 2{,}910 \text{ lb./sq. in.}$$

where n was taken from the last step of the calculation for panel 4-5 as a first approximation. This computation gives

$$V_U = 2{,}910 \times 0.016 \times 12 \times 0.481 = 269 \text{ lb.}$$

Assuming the eccentricity between skin and ring to be 0.5 in., the eccentricity moment acting on longitudinal 4 will be, by formula (25),

$$M = 0.5\ (351 - 269) = 41 \text{ in.-lb.}$$

In order to evaluate completely the ring stresses at longitudinal 5, it would be necessary to calculate also the eccentricity moments at longitudinals 2 and 3 and to add their effect at longitudinal 5 by using formula (26a).

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*Some of these papers give simple formulas and curves for the designer's use. They imply, however, assumptions of very questionable accuracy; for instance, they may assume that the angle of the diagonal-tension folds is 45°, which is too inaccurate for fuselages.

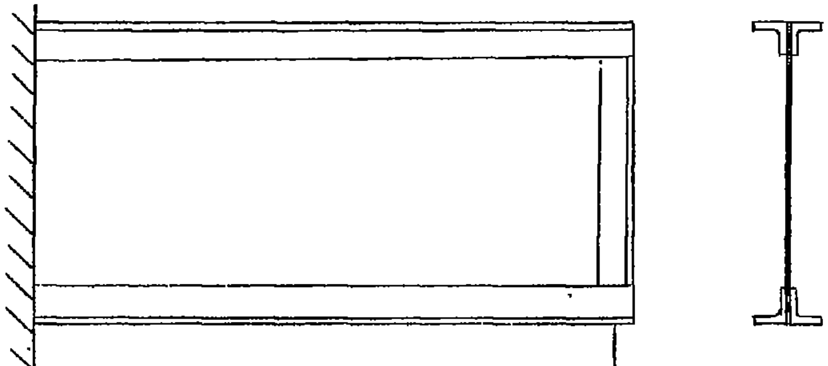


Figure 1.- Plate girder with parallel flanges.

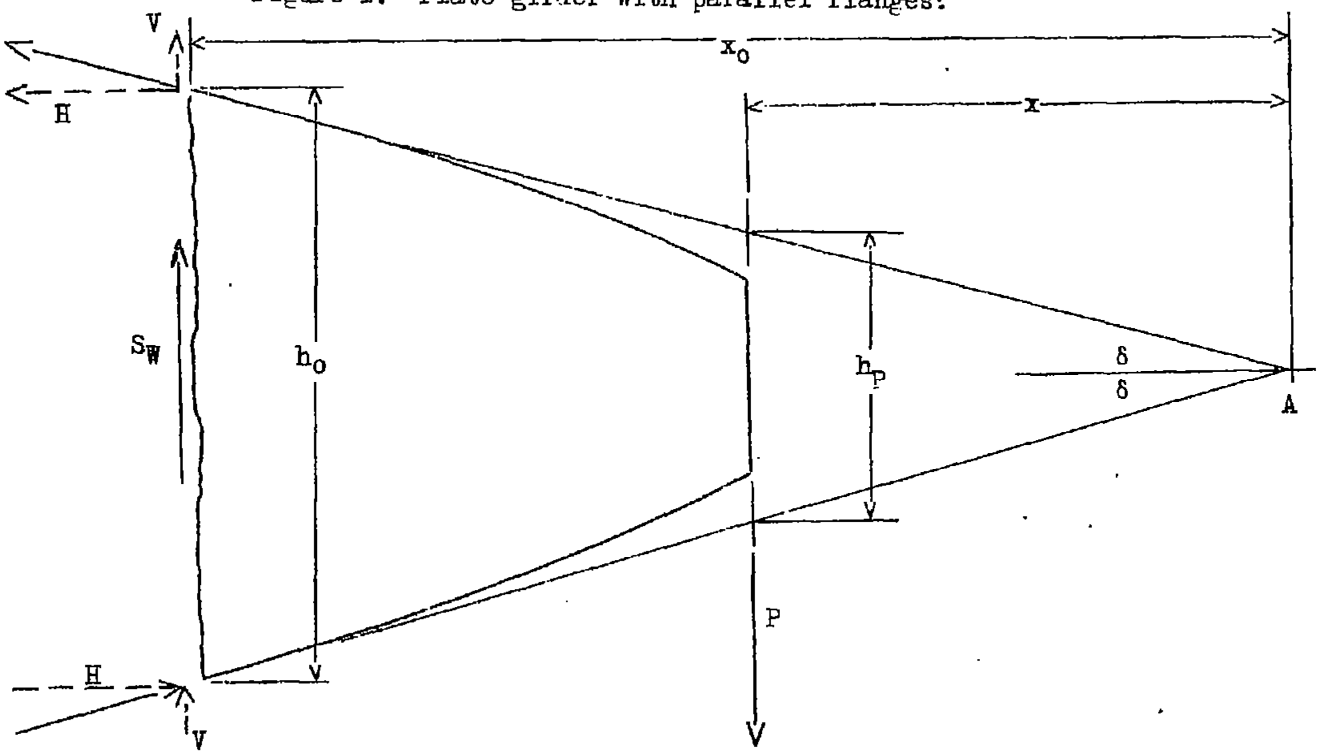


Figure 2.- Free-body sketch of plate girder with curved flanges.

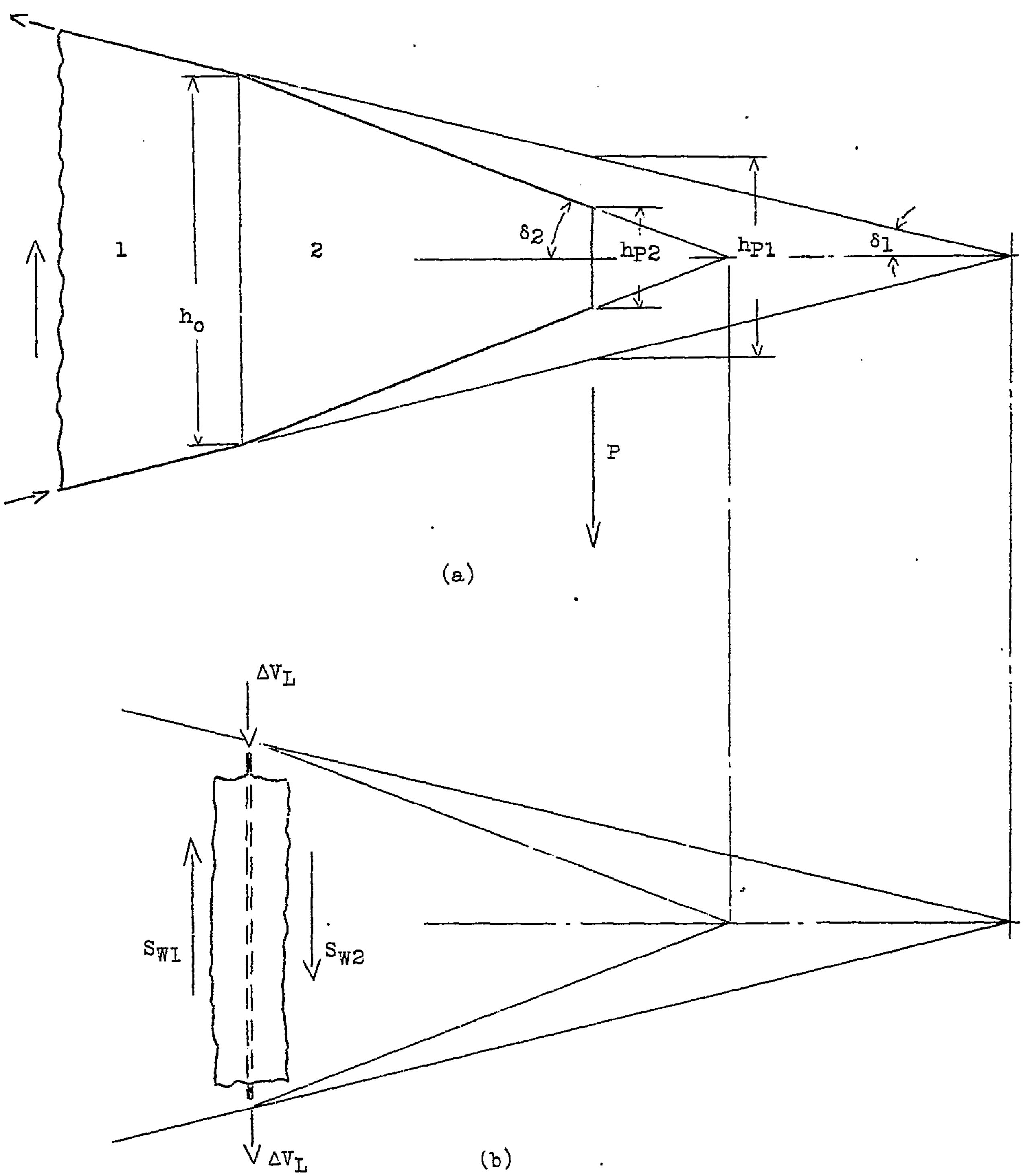


Figure 3.- Free-body sketch of plate girder with kinked flanges.

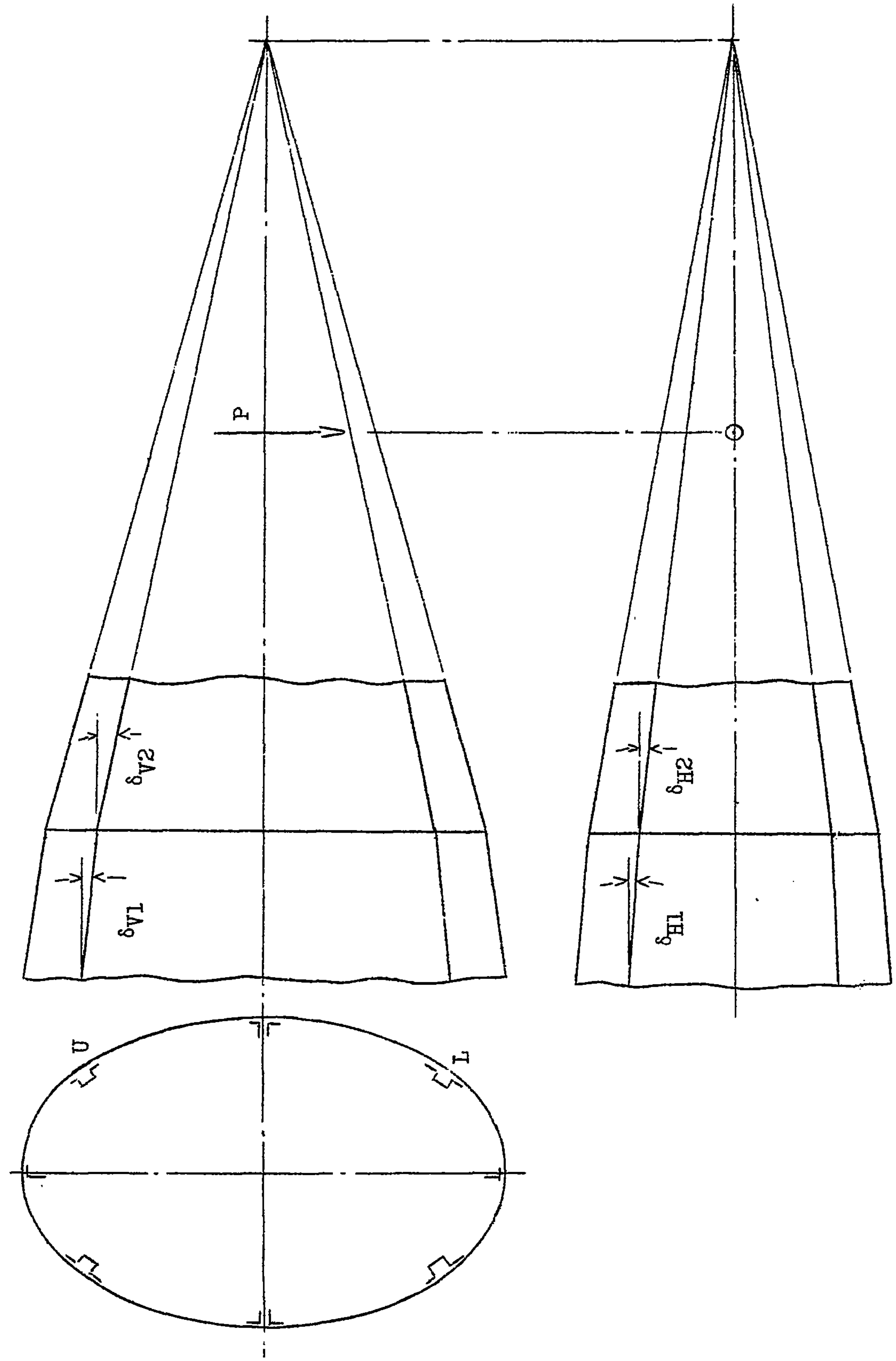


Figure 4.

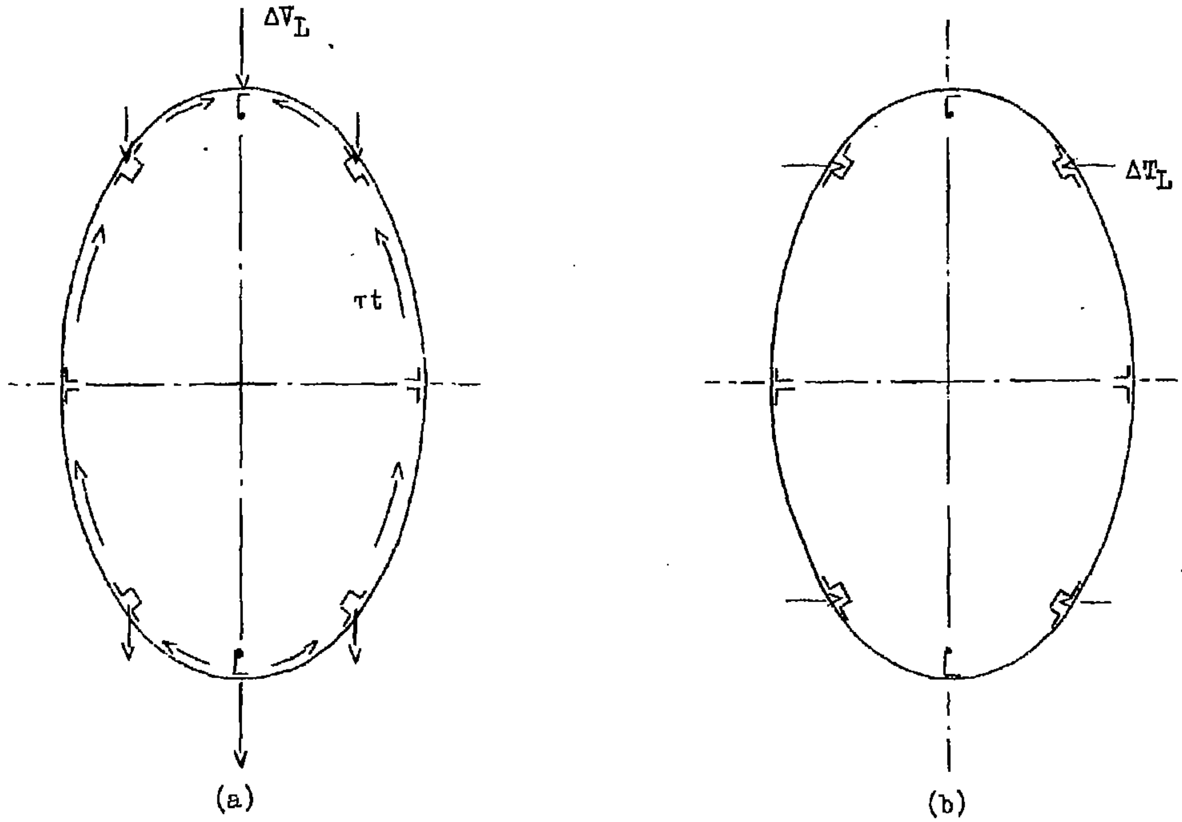


Figure 5.- Forces on fuselage ring.

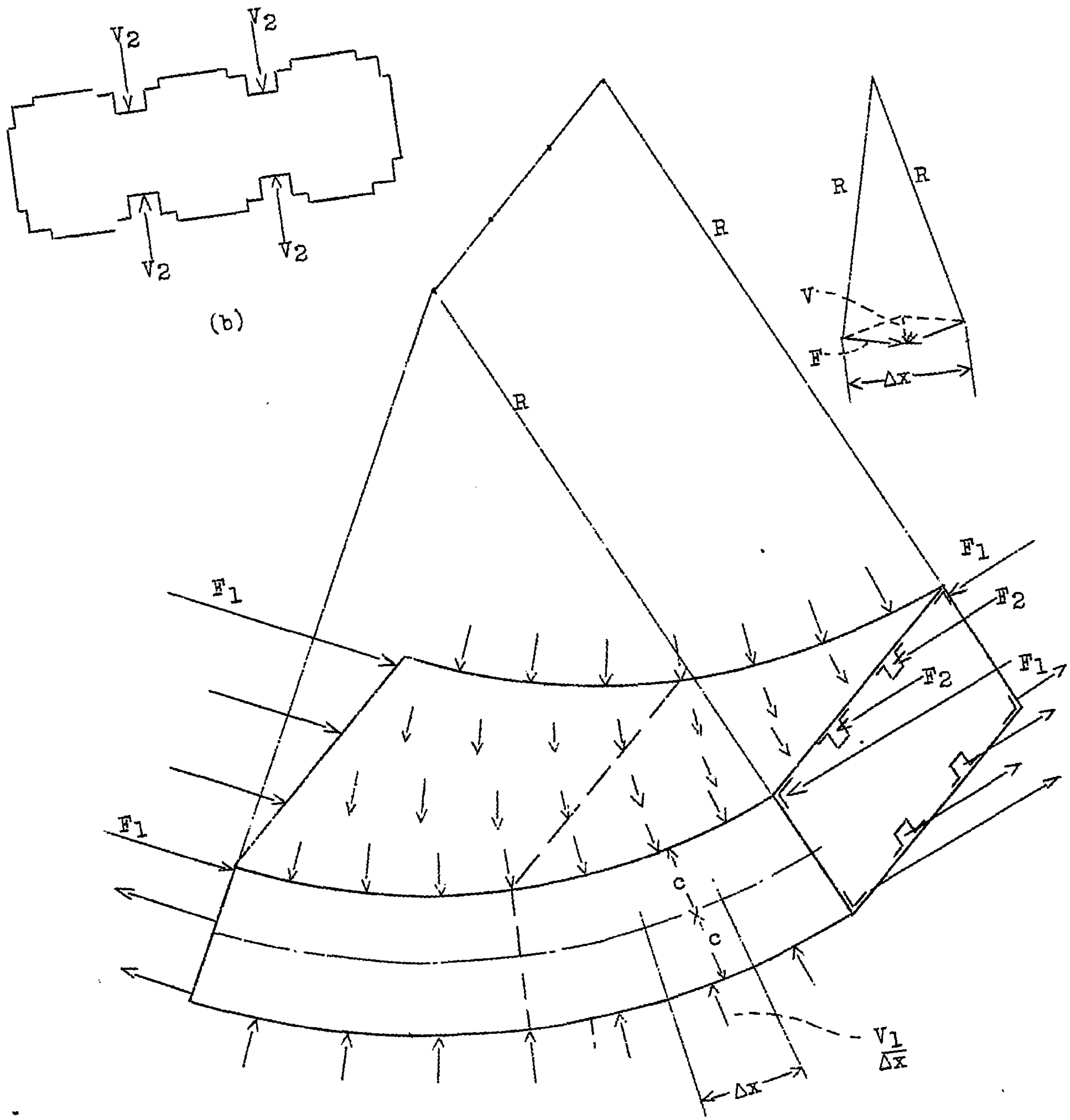


Figure 6.- Curved beam.

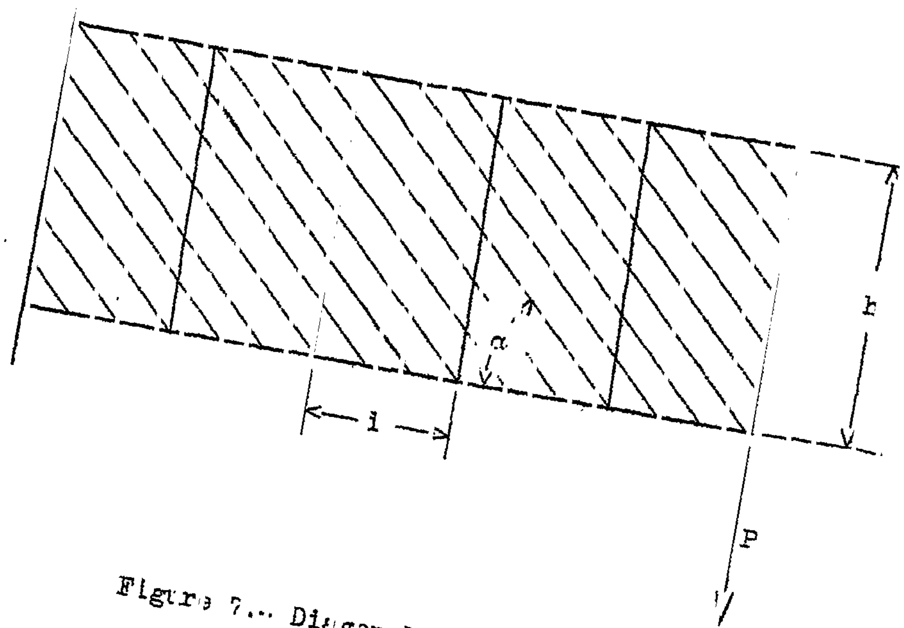


Figure 7.- Diagonal-tension beam.

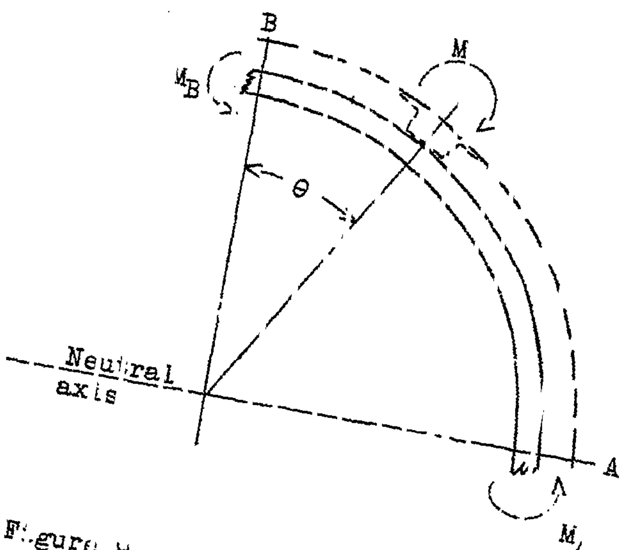


Figure 9.- Ring with eccentricity moments.

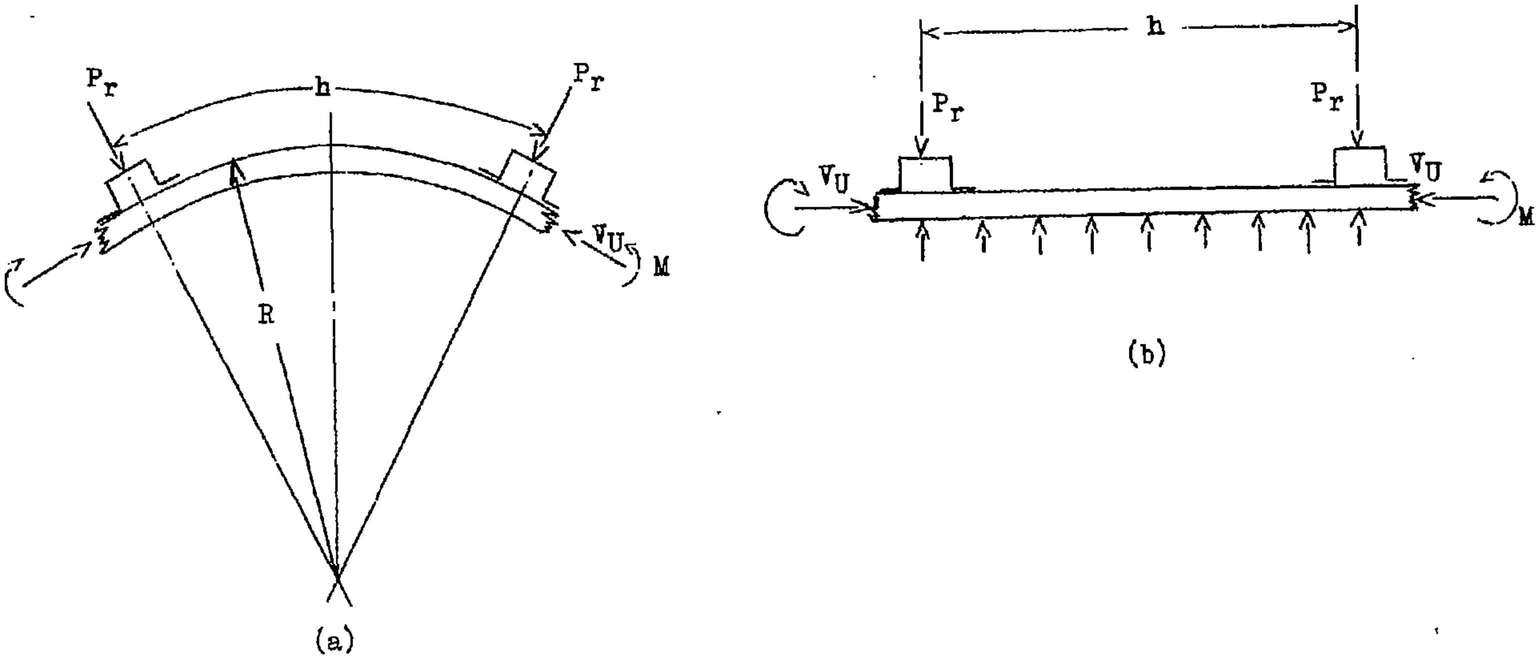


Figure 8.- Ring under radial load.

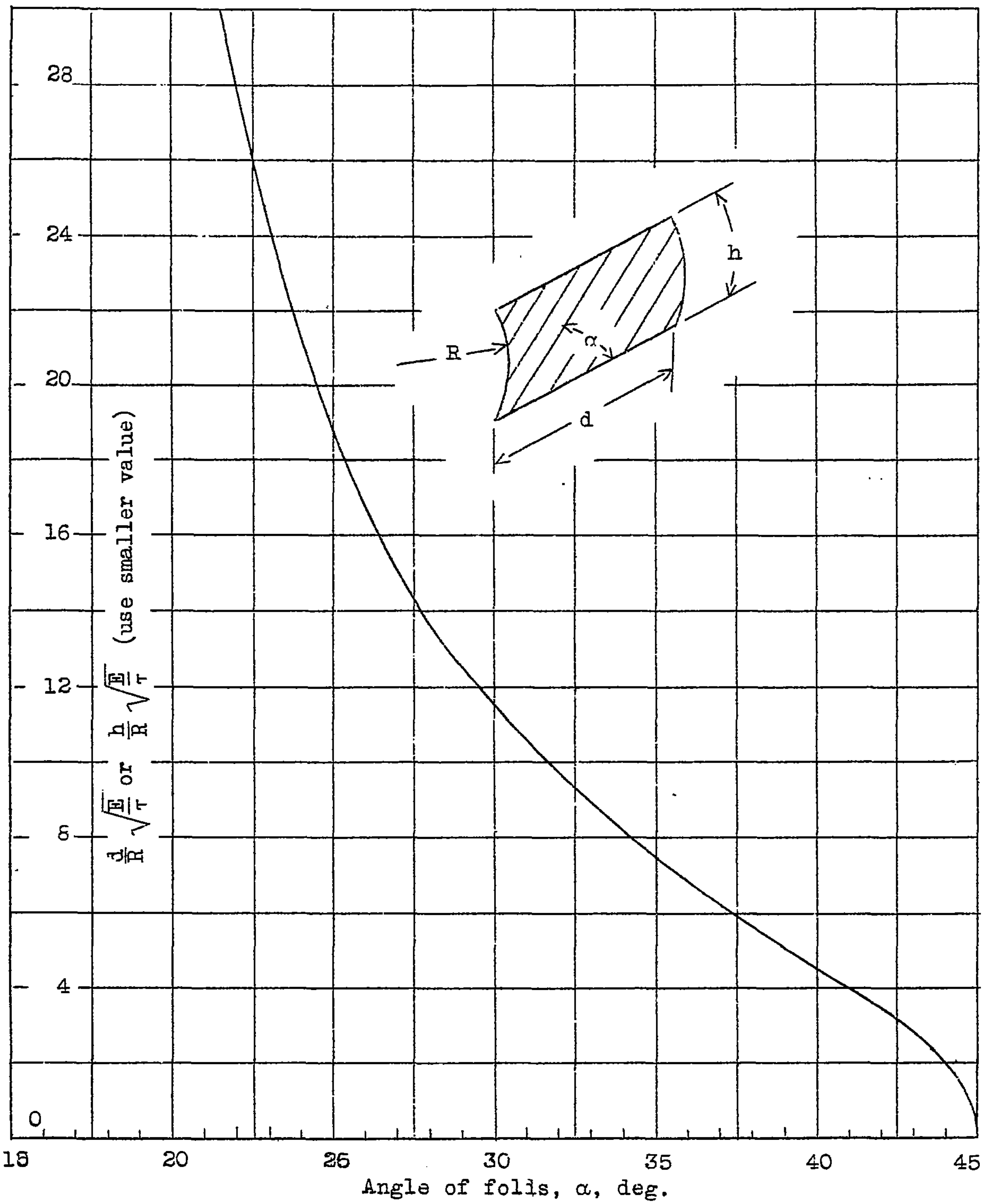


Figure 10.- Angle of folds in curved diagonal-tension fields.

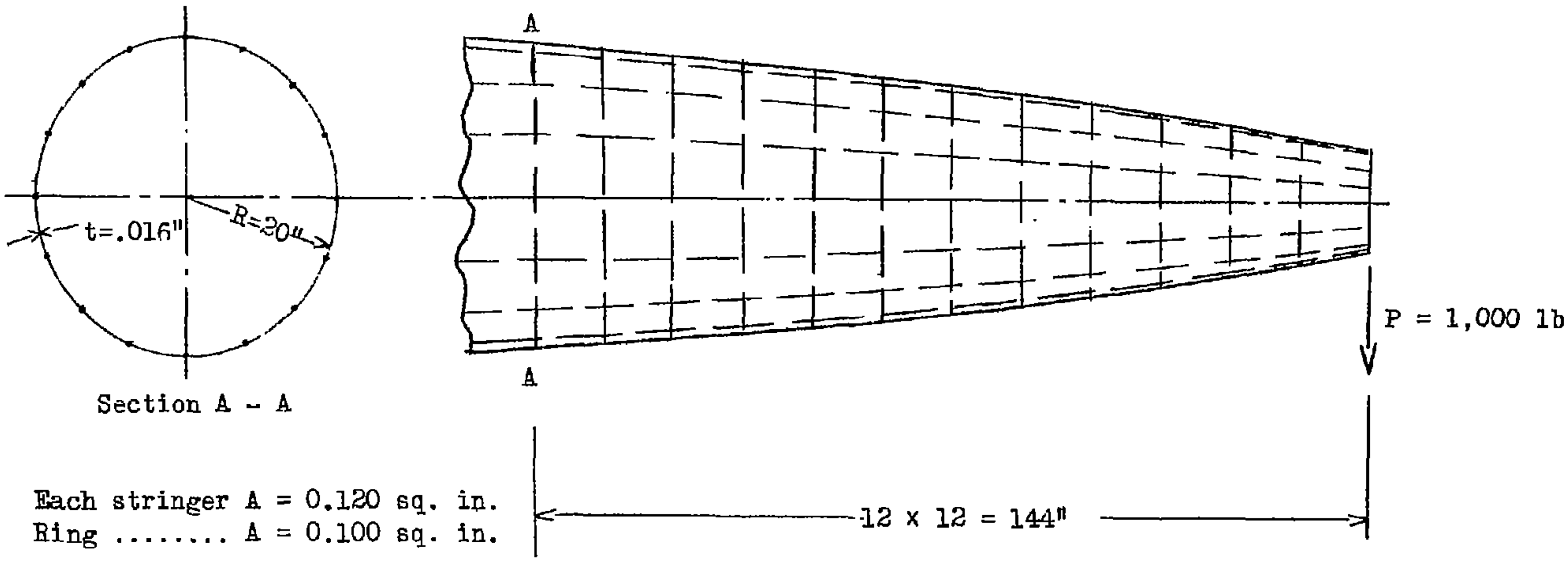

A
A
t=.016"
R=20"
Section A - A
P = 1,000 lb.
12 x 12 = 144"
Each stringer A = 0.120 sq. in.
Ring .... A = 0.100 sq. in.

Figure 11.

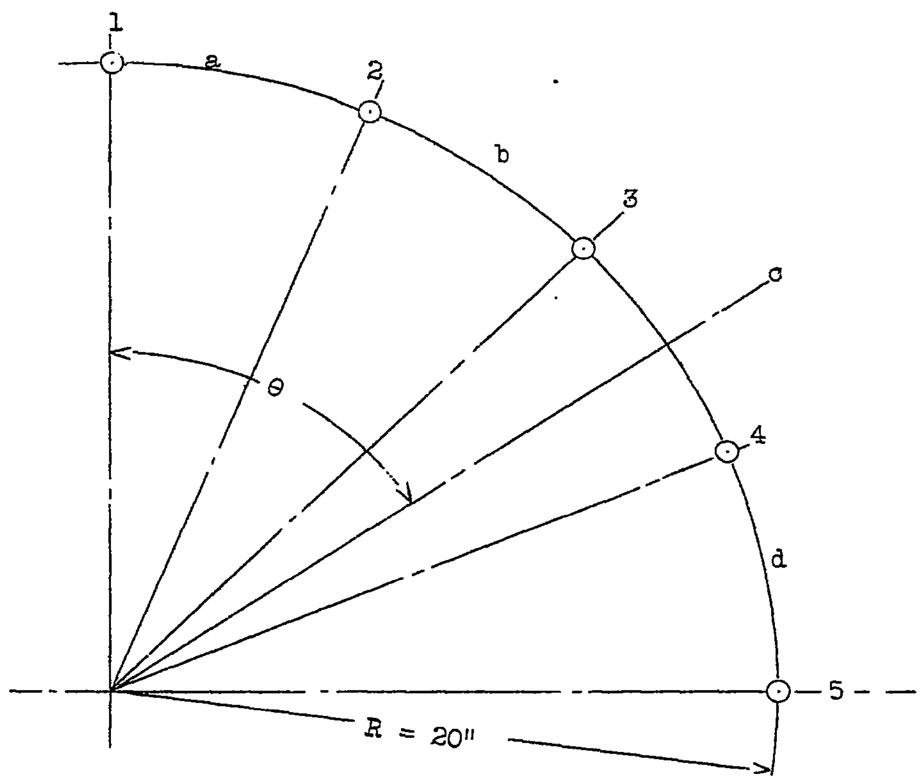


Figure 12.

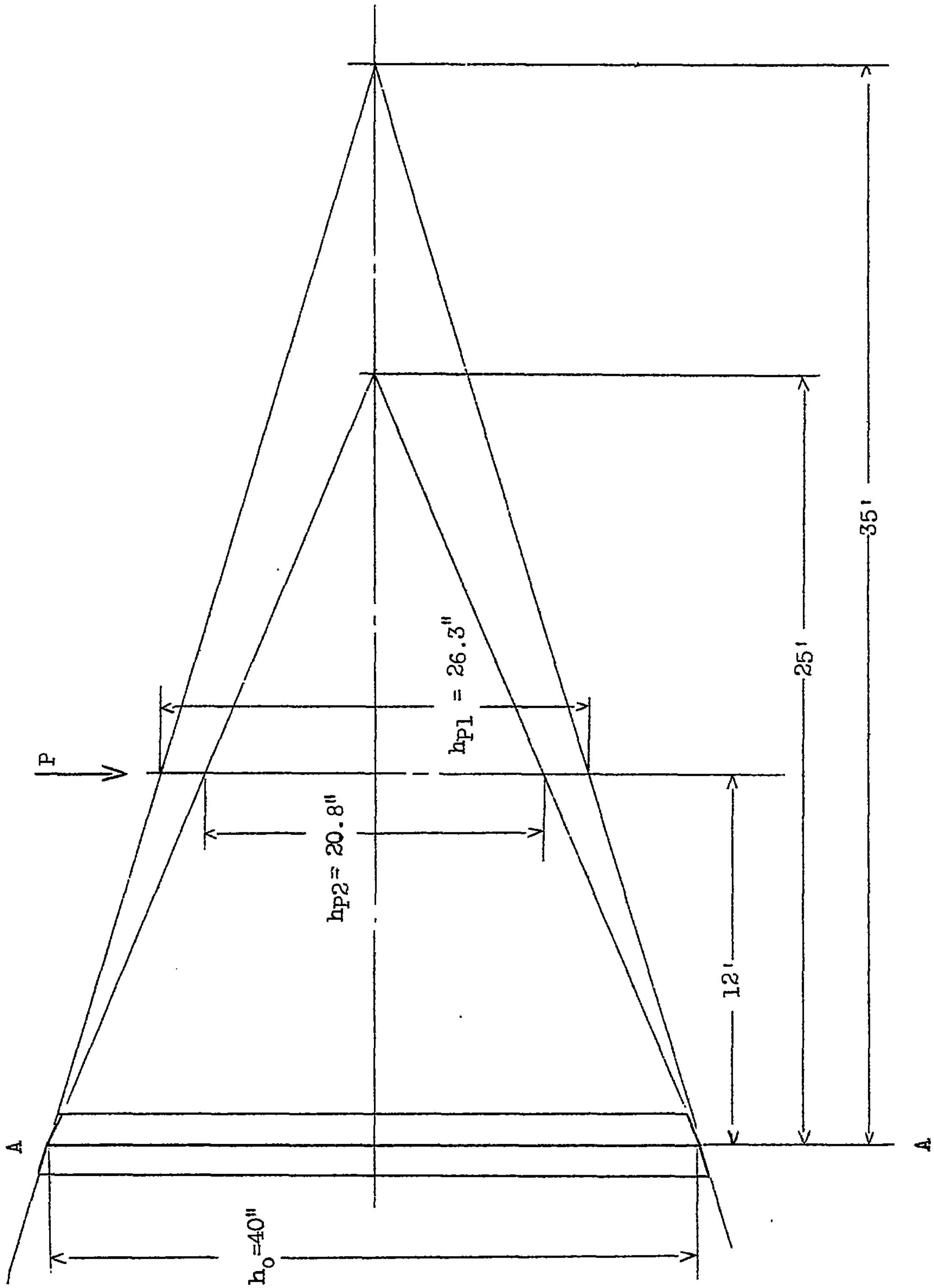
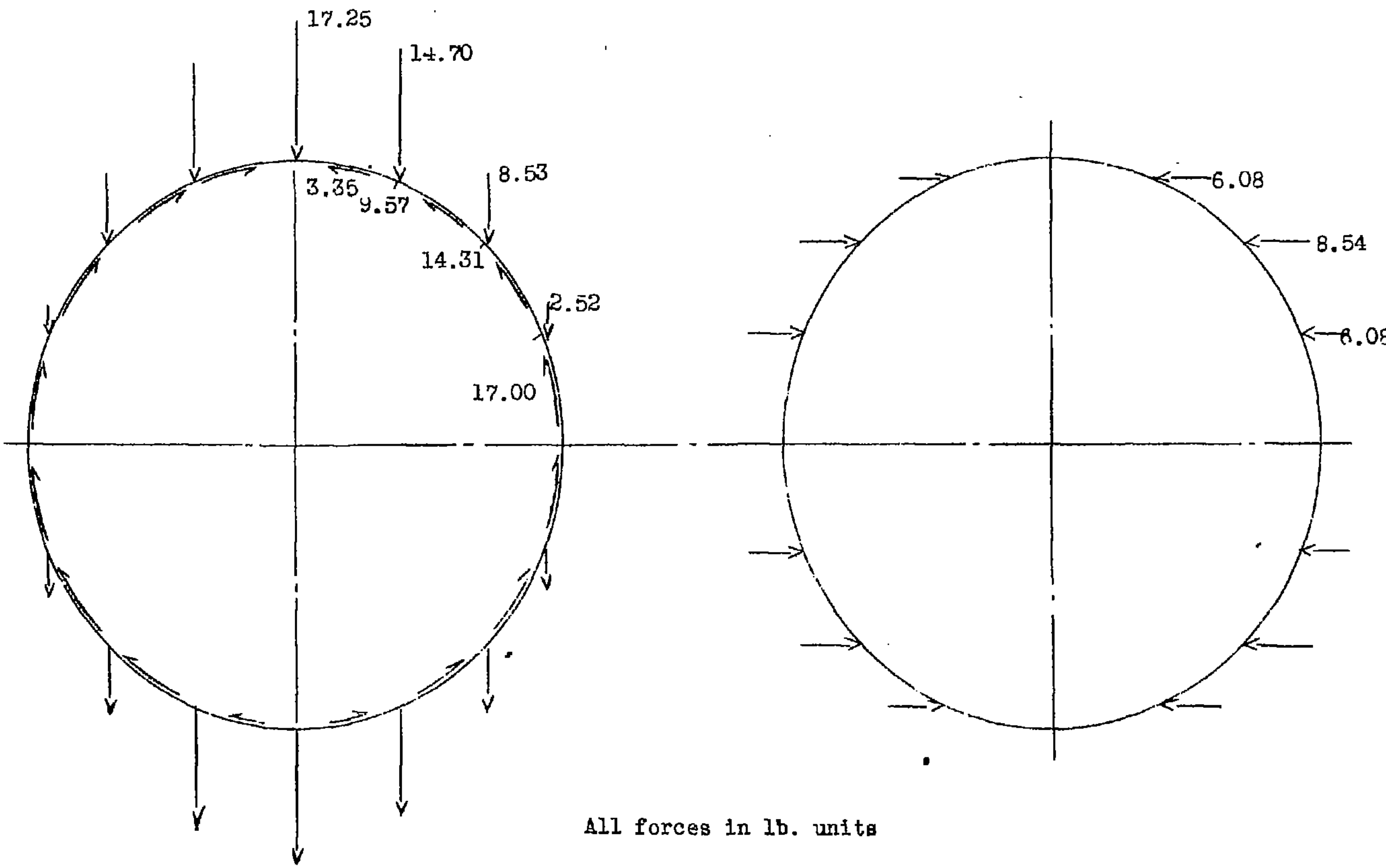


Figure 13.



All forces in lb. units

(a)

(b)

Figure 14.

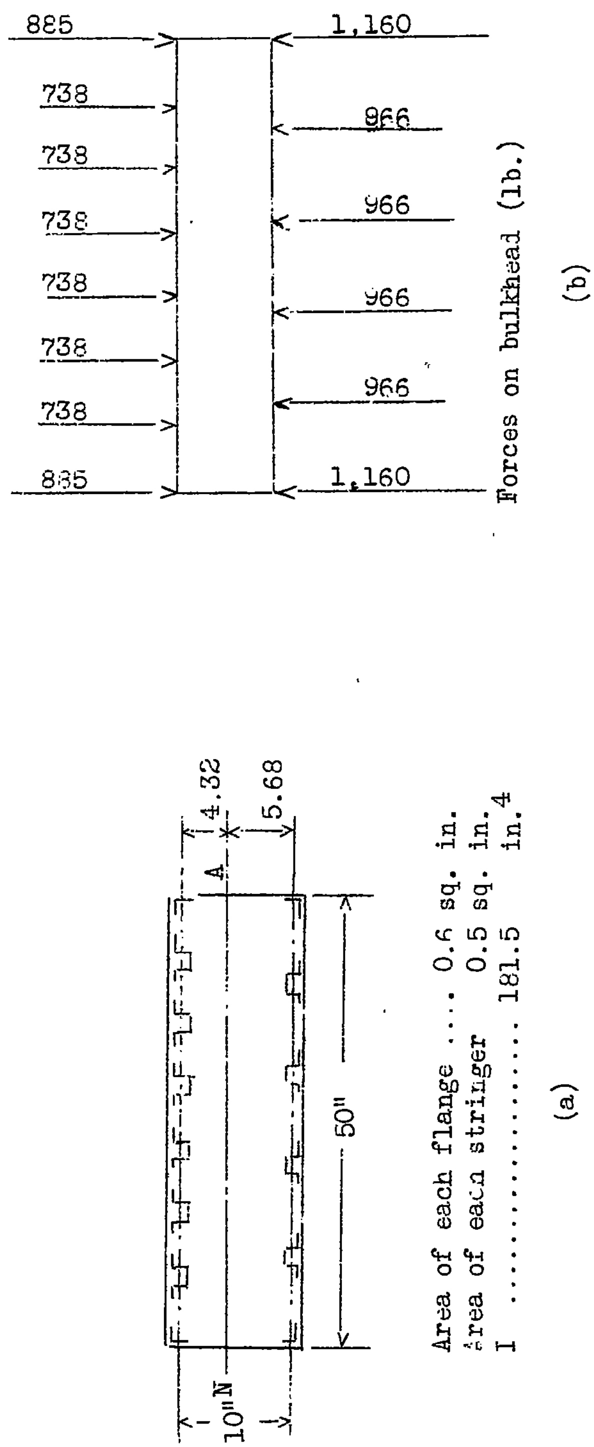

885
1,160
738
966
738
966
738
966
738
966
738
738
885
1,160
Forces on bulkhead (lb.)
(b)
4.32
5.68
A
50"
10"
N
Area of each flange .... 0.6 sq. in.
Area of each stringer 0.5 sq. in.
I 181.5 in.4
(a)

Figure 15.

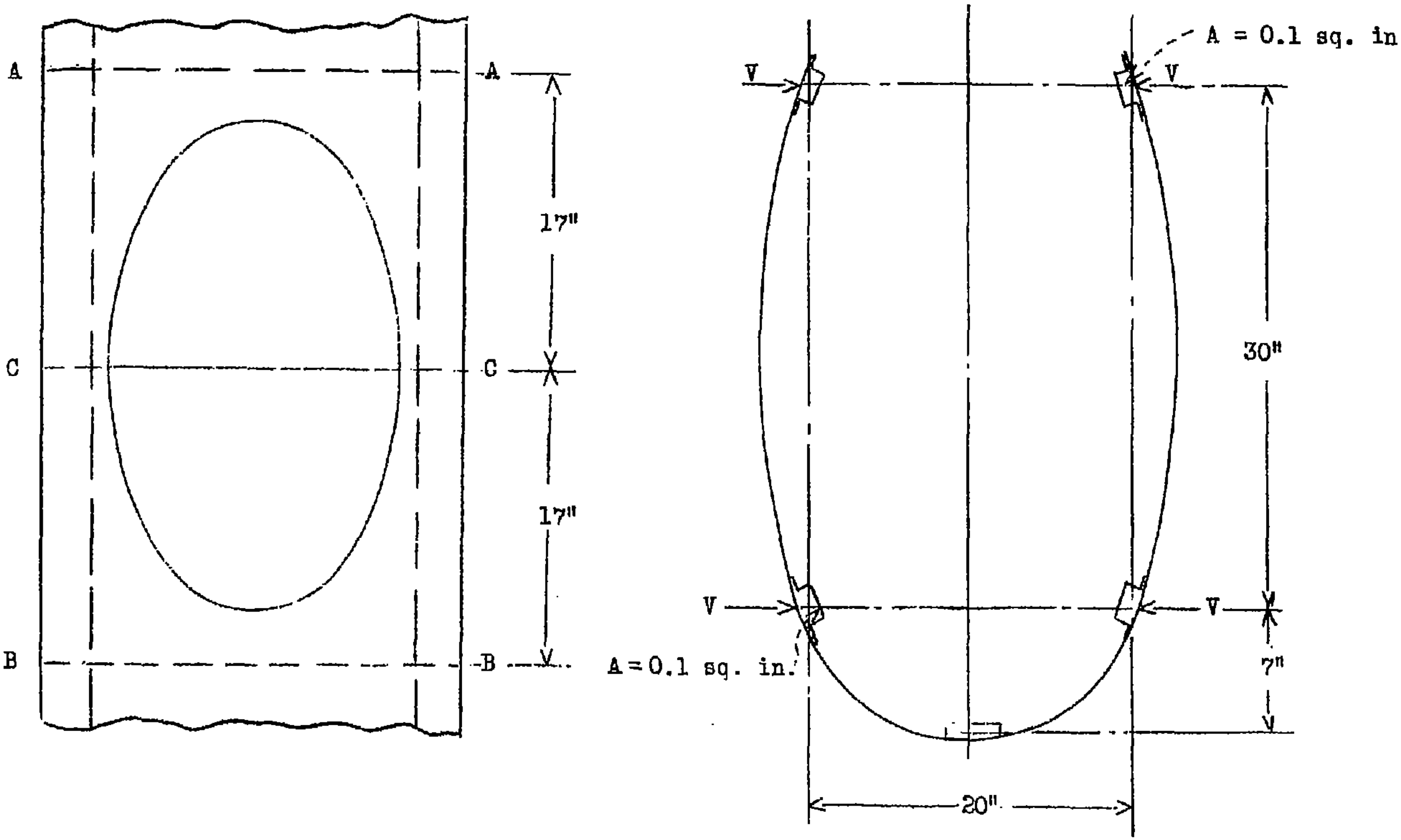


Figure 16