TECHNICAL NOTES

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No. 743

LOCAL INSTABILITY OF COLUMNS WITH I-, Z-, CHANNEL, AND RECTANGULAR-TUBE SECTIONS

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SUMMARY

Charts are presented for the coefficients in the formulas for the critical compressive stress at which cross-sectional distortion begins in thin-wall columns of I-, Z-, channel, and rectangular-tube sections. The energy method of Timoshenko was used in the theoretical calculations required for the construction of the charts. The deflection equations were carefully selected to give good accuracy.

The calculation of the critical compressive stress at stresses above the elastic range is briefly discussed. In order to demonstrate the use of the formulas and the charts in engineering calculations, two illustrative problems are included.

INTRODUCTION

In the design of compression members for aircraft, whether they be stiffeners in stressed-skin structures or struts in trussed structures, the allowable stress for the member is equal to the lowest strength corresponding to any of the possible types of failure. In references 1 and 2, all types of column failure are classed under two headings:

(a) Primary, or general, failure.

(b) Secondary, or local, failure.

Primary, or general, failure of a column is defined as any type of failure in which the cross sections are translated, rotated, or both translated and rotated but not distorted in their own planes (fig. 1). Secondary, or local, failure of a column is defined as any type of failure in which the cross sections are distorted in their own planes but not translated or rotated (fig. 2). Consideration is given in this paper only to local failure.

One of the factors to be considered in a study of local failure is the critical compressive stress at which the cross section begins to distort. This critical stress can usually be given in coefficient form. In two previous papers, coefficients are given for the rectangular tube (reference 3) and for columns of channel section and Z-section (reference 4). The purpose of the present paper is to summarize the results embodied in references 3 and 4 and also to present coefficients that will permit the critical compressive stress to be computed for an I-section column.

The calculations required to evaluate the coefficients plotted in the charts were made by the energy method of Timoshenko (reference 5). Because the calculations are long and were made as a part of a more extended study of local failure in thin-metal columns, they have been omitted from this paper.

Bernard Rubenstein, formerly of the N.A.C.A. staff, performed a large part of the mathematical derivations required for the preparation of this paper.

CHARTS

The calculation of the critical compressive stress at which cross-sectional distortion begins in any thin-wall column is, in reality, a problem in the buckling of thin plates, proper consideration being given to the interaction between the adjacent plates composing the cross section. Each of the sections considered in this paper consists of two basic plate elements. For example, in the I-section these elements are the flange plate and the web plate.

Timoshenko gives the critical stress for a rectangular plate under edge compression in the following form (reference 6, p. 605):

$$f_{cr} = \frac{k\pi^2 E t^2}{12(1-\mu^2)b^2} \qquad (1)$$

where

- E is tension-compression modulus of elasticity for the material.
- μ, Poisson's ratio for the material.
- t, thickness of the plate.
- b, width of the plate.
- k, a nondimensional coefficient dependent upon the conditions of edge support and the dimensions of the plate.

This equation can be used to calculate the critical compressive stress at which cross-sectional distortion begins in columns with the sections considered in this paper. If t and b are the thickness and the half-width of the flange, respectively, for an I-section, then the restraining effect of the web, whether positive, negative, or zero, is included in the coefficient k. On the other hand, if t and b are the thickness and the width of the web, a different set of values for k is obtained. It is therefore necessary to decide whether t and b in the equation for the critical stress shall refer to the flange plate or the web plate. Each form has advantages in certain cases. In this report, both forms will be given, either of which may be used to calculate the critical stress.

As applied to I-section columns, these two forms are:

$$f_{cr} = \frac{k_F \pi^2 E t_F{}^2}{12(1 - \mu^2) b_F{}^2} \qquad (2)$$

and

$$f_{cr} = \frac{k_W \pi^2 E t_W{}^2}{12(1 - \mu^2) b_W{}^2} \qquad (3)$$

In these formulas, either of which may be used,

t_F and t_W are the thicknesses of the flange and the web plates, respectively.

k_F and k_W, nondimensional coefficients dependent on the shape of the cross section. (See figs. 3 and 4.)

b_F is the half-width of the flange. (See sketches on figs. 3 and 4.)

b_W, width of the web.

The values of k_F and k_W, as calculated by the previously mentioned energy method, are listed in tables I and II. These calculated values were plotted against b_W/b_F and b_F/b_W, respectively, to obtain the curves shown in figures 3 and 4.

Equations (2) and (3) are given on figures 3 and 4, respectively, with f_{cr} replaced by f_{cr}/η. The nondimensional coefficient η allows for the effect of stress beyond the elastic range and is discussed in a later section.

The relation between k_F and k_W for a given I-section column is sometimes of interest; it is obtained by equating the right-hand sides of equations (2) and (3), which gives

$$k_F = \left(\frac{b_F}{b_W}\right)^2 \left(\frac{t_W}{t_F}\right)^2 k_W \qquad (4)$$

Equations (2), (3), and (4) also apply to columns of channel-section and Z-section, for which the values of k_F and k_W are given in figures 5 and 6. It is important to note that, in these cases, b_F refers to the total width of the flange. The values of k_F and k_W used to draw the curves in figures 5 and 6 are listed in tables III and IV, respectively.

For the case of the rectangular tube, let the cross-sectional dimensions of the tube be as shown in the sketch on figure 7. It will always be possible to select the sides of width b and h so that $\frac{b}{h} \leq 1$. With this choice of sides, the critical compressive stress for the tube is

$$f_{cr} = \frac{k\pi^2 E t_h^2}{12(1 - \mu^2)h^2} \qquad (5)$$

in which k may be read from figure 7. The values of k used to draw these curves are listed in table V.

The k-coefficients given herein apply to columns in which the material is both elastic and isotropic. Steel, aluminum alloys, and other metallic materials usually satisfy these conditions provided that the material is not stressed beyond the elastic range. When a material is stressed beyond the proportional limit in one direction, it is no longer elastic and is probably no longer isotropic. In a later part of the paper a method is presented for calculating the critical stress when the columns are loaded beyond the proportional limit.

DEFLECTION EQUATIONS

The plate elements that make up the sides of the columns treated fall into two classes, namely:

(a) Plates restrained along both edges, as the web of a channel or as the sides of the rectangular tube.

(b) Plates restrained along only one edge, as one flange of the channel section or the half-flange of the I-section.

The deflection equation assumed for plates of class (a) is

$$w = \left[4A \frac{y}{b}\left(1 - \frac{y}{b}\right) + B \sin \frac{\pi y}{b}\right] \sin \frac{n\pi x}{L} \qquad (6)$$

and, for class (b),

$$w = \left\{C \frac{y}{b} - \frac{D}{3.889}\left[\left(\frac{y}{b}\right)^5 - 4.963\left(\frac{y}{b}\right)^4 + 9.852\left(\frac{y}{b}\right)^3 - 9.778\left(\frac{y}{b}\right)^2\right]\right\} \sin \frac{n\pi x}{L} \qquad (7)$$

where

w is deflection normal to plate.

A, B, C, and D, arbitrary deflection amplitudes.

y, coordinate across plate, measured from one edge.

b, width of member.

n, number of half-waves in length L.

x, coordinate in direction of length L.

L, length of member.

For the channel, the Z-, and the I-sections, the values of C and D for the flanges may be expressed in terms of A and B for the web through the conditions that the corner angles are maintained during buckling and that the moments at each corner are in equilibrium. The values of B/A and L/n are then adjusted to make the critical stress a minimum. The same considerations apply to the rectangular tube, except that two equations of the form of (5) are used, one for each pair of walls.

The foregoing deflection equations used in the energy solution were carefully selected. A comparison of the exact values of k with the values of k for the rectangular tube as given by the energy method reveals that the energy values are less than 1 percent in error. (See reference 3.) The values of k for the other sections are also believed to be correct to the same order of magnitude. This belief is justified because, in the limiting cases for which exact solutions are available, the precision is within these limits. In addition, other problems in which these deflection equations have been used gave a precision better than 1 percent.

DISCUSSION OF CHARTS

Figures 3 to 7 give the computed values of the k-coefficients plotted against b_W/b_F, b_F/b_W, or b/h. When the webs are narrow in comparison with the flanges, the instability occurs first in the flanges. As b_W/b_F increases, a point is reached where the webs become the weaker part of the cross section. For example, this point is clearly discernible in figure 3 where two of the curves break sharply at certain values of b_W/b_F.

CRITICAL STRESS FOR LOADING BEYOND THE PROPORTIONAL LIMIT

In the elastic range, the critical compressive stress for an ordinary column that fails by bending is given by the Euler formula. Beyond the proportional limit marking the upper end of the elastic range, the reduced slope of the stress-strain curve requires that an effective modulus $\overline{E}$ be substituted for Young's modulus E in the Euler formula. The value of $\overline{E}$ is sometimes written

$$\overline{E} = \tau E \tag{8}$$

The value of the nondimensional coefficient τ varies with stress. By the use of the double-modulus theory of column action, theoretical values of τ can be obtained from the compressive stress-strain curve for the material (reference 6, p. 572, and references 7 and 8). Tests show that, in practice, theoretical values of τ, derived on the assumption that no deflection occurs until the critical load is reached, are too large. It is therefore best, for practical use, to obtain the value of τ from the accepted column curve for the material, as will be shown later. The values of τ thus obtained take into account the effect of imperfections that cause deflection from the beginning of loading as well as other factors that may have a bearing on the strength.

For cross-sectional distortion of a thin-wall column, the critical compressive stress is given by the basic equation (1). Beyond the proportional limit, the critical compressive stress is given by this equation with an effective modulus ηE substituted for E, or:

$$f_{cr} = \eta \frac{k\pi^2 E t^2}{12(1 - \mu^2)b^2} \tag{9}$$

In the absence of adequate test data, the value of the nondimensional coefficient η cannot be definitely established. It is reasonable to suppose, however, that η and τ are related in some way.

Various equations relating η and τ have been suggested. The discussion of reference 3 points out that, when η is considered to be a function of τ, the equation for η will depend upon the manner of evaluation of τ. If τ is determined from the stress-strain curve on the assumption that no deflection takes place until the critical load is reached, the effect of deflections from the beginning of loading must be separately considered. If τ is determined, however, from the accepted column curve for the material in the manner outlined in the illustrative problems of reference 3, part, if not all, of this effect is automatically considered.

A careful study of the theory and of such experimental data as are available indicates that a conservative assumption is

$$\eta = \frac{\tau + 3\sqrt{\tau}}{4} \tag{10}$$

provided that τ is evaluated by use of the accepted column curve for the material. This equation will probably need to be modified as more test data become available.

Now τ is itself a function of the stress f_{cr}. Hence, η is a function of f_{cr}. Consequently, equation (9) cannot be solved directly for f_{cr}. If the equation is divided by η, however, then f_{cr}/η is given directly by the geometrical dimensions of the cross section.

Thus, for the I-section column, equations (2) and (3) become

$$\frac{f_{cr}}{\eta} = \frac{k_F \pi^2 E t_F^2}{12(1 - \mu^2) b_F^2} \tag{11}$$

and

$$\frac{f_{cr}}{\eta} = \frac{k_W \pi^2 E t_W^2}{12(1 - \mu^2) b_W^2} \tag{12}$$

For the columns of channel and Z-section, equations (11) and (12) also hold, except that b_F is the entire width of the flange.

For the rectangular tube

$$\frac{f_{cr}}{\eta} = \frac{k\pi^2 E t_h^2}{12(1 - \mu^2)h^2} \tag{13}$$

The values of the k-coefficients are read as usual from figures 3 to 7. Thus, f_{cr}/η is determined for the cross section under consideration from equation (11), (12), or (13). The value of f_{cr} is then determined from f_{cr}/η.

RELATION BETWEEN f_{cr} AND f_{cr}/η

Assume that the value of η is given by equation (10). As stated previously, τ and hence η depend upon the critical stress. Although theoretically the values of τ and hence of η can be obtained from the stress-strain curve, they are best obtained from the accepted column curve for the material.

The equations that show the variation of τ with stress for 24ST aluminum alloy that just meets the requirements of Navy Department Specification 46A9a (tensile yield strength, 42,000 pounds per square inch) are given in part I of reference 9. In order to show how similar equations can be derived for any other material, equations will be derived from the column formulas given in reference 9 for 24ST aluminum alloy.

The accepted column formulas for 24ST aluminum alloy are given by equations (8) and (9) of reference 9. These equations are:

For $41{,}200 > f_{cr} > 19{,}600$ lb./sq. in.,

$$f_{cr} = 43{,}700 \left(1 - 0.00752 \frac{L}{\rho}\right) \tag{14}$$

For $f_{cr} < 19{,}600$ lb./sq. in.,

$$f_{cr} = \frac{105200000}{(L/\rho)^2} \tag{15}$$

For the same member, the critical stress given by the accepted column curve must equal the critical stress given by the Euler formula with an effective modulus $\bar{E} = \tau E$ substituted for Young's modulus E, or

$$f_{cr} = \frac{\pi^2 \tau E}{(L/\rho)^2} = \tau \frac{105200000}{(L/\rho)^2} \tag{16}$$

If equations (14) and (15) are solved for L/ρ, the following expressions are obtained:

For $41,200 > f_{cr} > 19,600$ lb./sq. in.,

$$L/\rho = \frac{43700 - f_{cr}}{328.6} \tag{17}$$

For $f_{cr} < 19,600$ lb./sq. in.

$$\frac{L}{\rho} = \sqrt{\frac{105200000}{f_{cr}}} \tag{18}$$

Substitution of these values of L/ρ in equation (16) gives:

For $41,200 > f_{cr} > 19,600$ lb./sq. in.,

$$\tau = \frac{f_{cr}}{8925}\left(1,224 - \frac{f_{cr}}{25700}\right)^2 \tag{19}$$

For $f_{cr} < 19,600$ lb./sq. in.,

$$\tau = 1 \tag{20}$$

By the use of equations (19) and (20), the value of τ can be established for assumed values of f_{cr}. The values of τ obtained can then be substituted in equation (10) to obtain the corresponding values of η. If the assumed values of f_{cr} are divided by the corresponding values of η, a curve of f_{cr} against f_{cr}/η can be plotted. In figure 8, several such curves are given for 24ST aluminum alloy for different assumed relations between η and τ. When the value of f_{cr}/η has been obtained by use of equation (11), (12), or (13), the value of f_{cr} is read from one of the curves of figure 8.

The ultimate strength of a thin-wall column will, in general, be higher than the load at which cross-sectional distortion begins. At stresses approaching the yield point of the material, the critical load and the ultimate load approach the same value. No attempt has been made in this paper to discuss the ultimate strength of a thin-wall column; the solution for the critical load logically precedes the solution for the ultimate load.

ILLUSTRATIVE PROBLEMS

It is desired to calculate the critical compressive stress at which cross-sectional distortion begins in two I-section columns constructed of 24ST aluminum alloy:

Column A	Column B
b_F = 1 in.	b_F = 2 in.
b_W = 2 in.	b_W = 4 in.
t_F = 0.1 in.	t_F = 0.2 in.
t_W = 0.1 in.	t_W = 0.1 in.

Solution for Column A

$$\frac{b_W}{b_F} = \frac{2}{1} = 2$$

$$\frac{t_W}{t_F} = \frac{0.1}{0.1} = 1$$

From figure 3, $k_F = 0.662$

For this material, $E = 10.66 \times 10^6$ lb. per sq. in.

and $\mu = 0.3$.

From equation (11),

$$\frac{f_{cr}}{\eta} = \frac{0.662 \times \pi^2 \times 10.66 \times 10^6 \times (0.1)^2}{12(1 - 0.3^2) \times (1)^2} = 63{,}800 \text{ lb./sq. in.}$$

From the solid curve of figure 8, f_{cr} = 33,000 lb./sq. in.

Solution for Column B

$$\frac{b_F}{b_W} = \frac{2}{4} = 0.5$$

$$\frac{t_W}{t_F} = \frac{0.1}{0.2} = 0.5$$

From figure 4, $k_W = 6.82$

From this material, $E = 10.66 \times 10^6$ lb. per sq. in.

and $\mu = 0.3$

From equation (12),

$$\frac{f_{cr}}{\eta} = \frac{6.82 \times \pi^2 \times 10.66 \times 10^6 \times (0.1)^2}{12(1 - 0.3^2) \times (4)^2} = 41,100 \text{ lb./sq. in.}$$

From the solid curve of figure 8, $f_{cr} = 28,800$ lb./sq. in.

CONCLUSIONS

1. The critical compressive stress at which cross-sectional distortion begins in a thin-wall column of channel, Z-, or I-section is given by either of the following equations:

$$f_{cr} = \eta \frac{k_F \pi^2 E t_F{}^2}{12(1 - \mu^2) b_F{}^2}$$

or

$$f_{cr} = \eta \frac{k_W \pi^2 E t_W{}^2}{12(1 - \mu^2) b_W{}^2}$$

where E and μ are Young's modulus and Poisson's ratio for the material, respectively.

b_F is the half-width of the flange for the I-section, or the total width of the flange for the channel and Z-sections. (See sketches on figs. 3 to 6.)

b_W, width of the web.

t_F and t_W are the thicknesses of the flange and the web plates, respectively.

k_F and k_W, nondimensional coefficients read from figures 3 to 6.

η is a nondimensional factor taken so that ηE gives the effective modulus of flange and web at stresses beyond the elastic range.

For the rectangular tube,

$$f_{cr} = \eta \frac{k\pi^2 E t_h^2}{12(1 - \mu^2)h^2}$$

where

h and t_h are the width and thickness, respectively, of the long side of the rectangular-tube section.

k, nondimensional coefficient read from figure 7.

2. At stresses beyond the elastic range, the value of the effective modulus ηE for local buckling of thin-wall columns will depend upon tests. In the absence of such tests, however, it is reasonable to assume that η is a function of τ, where τE is the effective modulus of an ordinary column at stresses beyond the elastic range. A careful study of theory and of such experimental data as are available indicates that it is safe to assume

$$\eta = \frac{\tau + 3\sqrt{\tau}}{4}$$

provided that τ is evaluated by use of the accepted column curve for the material.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 13, 1939.

REFERENCES

1. Lundquist, Eugene E., and Fligg, Claude M.: A Theory for Primary Failure of Straight Centrally Loaded Columns. T.R. No. 582, N.A.C.A., 1937.

2. Lundquist, Eugene E.: On the Strength of Columns That Fail by Twisting. Jour. Aero. Sci., vol. 4, no. 6, April 1937, pp. 249-253.

3. Lundquist, Eugene E.: Local Instability of Symmetrical Rectangular Tubes under Axial Compression. T.N. No. 686, N.A.C.A., 1939.

4. Lundquist, Eugene E.: Local Instability of Centrally Loaded Columns of Channel Section and Z-Section. T.N. No. 722, N.A.C.A., 1939.

5. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., 1936, p. 324, art. 62.

6. Timoshenko, S.: Strength of Materials. Pt. II. D. Van Nostrand Co., Inc., 1930.

7. Osgood, William R.: Column Curves and Stress-Strain Diagrams. Res. Paper No. 492, Bur. Standards Jour. Res., vol. 9, no. 4, Oct. 1932, pp. 571-582.

8. Osgood, William R.: The Double-Modulus Theory of Column Action. Civil Eng., vol. 5, no. 3, March 1935, pp. 173-175.

9. Osgood, William R., and Holt, Marshall: The Column Strength of Two Extruded Aluminum-Alloy H-Sections. T.R. No. 656, N.A.C.A., 1939.

TABLE I

Calculated Minimum Values of

k_F for I-Sections by the Energy Solution

(μ = 0.3)

$\frac{b_W}{b_F}$ \ $\frac{t_W}{t_F}$	k_F		
	0.5	1	2
0	1.288	1.288	1.288
.20	-	1.019	-
.40	.623	-	1.193
.60	-	.852	-
.80	.567	-	1.147
1.00	-	.783	-
1.20	.536	-	1.127
1.40	-	.735	-
1.60	.508	-	1.118
1.70	.500	-	-
1.80	.491	.688	-
1.85	.486	-	-
1.90	.473	-	-
2.00	.427	.662	1.113
2.40	.295	.597	1.110
2.80	.216	.520	1.105
3.20	.165	.440	1.093
3.40	-	-	1.062
3.60	.130	.367	1.015
3.80	-	-	.955
4.00	.105	.306	.893
5.00	.067	.200	.623
6.67	.037	.112	.364

TABLE II

Calculated Minimum Values of

k_W for I-Sections by the Energy Solution
($\mu = 0.3$)

$\frac{b_F}{b_W}$ \ $\frac{t_W}{t_F}$	k_W		
	0.5	1	2
0	4.00	4.00	4.00
.050	6.05	4.49	4.06
.100	6.46	4.82	4.09
.150	6.61	4.98	4.04
.200	6.68	5.01	3.89
.250	6.72	4.89	3.57
.278	6.73	4.75	3.29
.313	6.75	4.50	2.80
.357	6.77	4.08	2.17
.417	6.79	3.44	1.60
.500	6.82	2.65	1.11
.526	6.84	-	-
.556	6.37	2.23	-
.588	5.78	-	-
.625	5.20	-	.715
.714	-	1.44	-
.833	3.09	-	.406
1.000	-	.784	-

TABLE III

Calculated Minimum Values of k_F

for Channel and Z-Sections by the Energy Solution
($\mu = 0.3$)

$\frac{b_W}{b_F}$ \ $\frac{t_W}{t_F}$	k_F		
	0.5	1	2
0	1.288	1.288	1.288
.200	-	1.111	-
.400	.695	-	1.234
.600	-	.962	-
.800	.621	-	1.204
1.00	-	.892	-
1.20	.576	-	1.193
1.40	-	.836	-
1.60	.528	-	1.188
1.75	.506	-	-
1.80	.499	.770	-
1.82	.493	-	-
1.90	.455	-	-
2.00	.410	.730	1.187
2.20	.338	-	-
2.40	.284	.629	1.188
2.80	.208	.521	1.190
3.20	.159	.423	1.192
3.40	-	-	1.178
3.60	.125	.345	1.103
3.80	-	-	1.021
4.00	.101	.284	.940
4.40	.083	.236	.799
4.80	-	.199	.681
5.20	.059	.170	.587
5.60	-	.146	.508
6.00	.044	.127	.444

TABLE IV

Calculated Minimum Values of k_W

for Channel and Z-Sections by the Energy Solution

(μ = 0.3)

$\frac{b_F}{b_W}$ \ $\frac{t_W}{t_F}$	k_W		
	0.5	1	2
0	4.00	4.00	4.00
.050	5.46	4.26	4.03
.100	6.02	4.45	4.04
.130	6.19	-	-
.167	6.31	4.58	4.00
.179	-	4.59	3.98
.192	6.38	4.60	3.97
.208	-	4.59	3.92
.227	6.43	4.58	3.86
.250	6.46	4.54	3.76
.263	-	-	3.68
.278	6.49	4.47	3.57
.294	-	-	3.40
.313	6.51	4.33	3.05
.357	6.53	4.08	2.33
.417	6.54	3.62	1.71
.455	6.55	-	-
.500	6.56	2.92	1.19
.526	6.56	-	-
.548	6.57	-	-
.556	6.47	2.50	-
.571	6.20	-	-
.625	5.41	-	.761
.714	-	1.64	-
.833	3.32	-	.429
1.000	-	.892	-

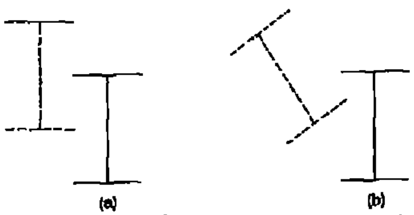
TABLE V

Calculated Minimum Values of k

for Rectangular Tube by the Energy Solution

(μ = 0.3)

$\frac{b}{h}$ \ $\frac{t_b}{t_h}$	k		
	0.5	1	2
0	7.01	7.01	7.01
.050	5.13	6.45	-
.075	4.88	-	-
.100	4.72	6.09	6.85
.125	4.62	-	-
.200	4.43	5.68	6.73
.300	4.31	5.45	6.65
.400	4.22	5.29	6.61
.500	4.11	5.16	6.59
.525	4.08	-	-
.550	4.04	-	-
.575	4.00	-	-
.590	3.97	-	-
.600	3.95	5.03	6.57
.610	3.92	-	-
.625	3.89	-	-
.650	3.81	-	-
.675	3.64	-	-
.700	3.38	4.87	6.57
.800	2.58	4.66	6.57
.900	2.03	4.37	6.57
1.000	1.64	4.00	6.58



(a) Translated (b) Translated and rotated

Figure 1.- Displacements of the cross section in primary, or general, failure of a column.

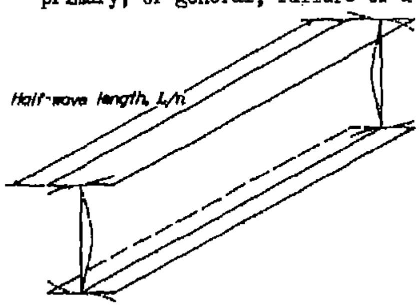


Figure 2.- Displacements of the cross section in secondary, or local, failure of a column.

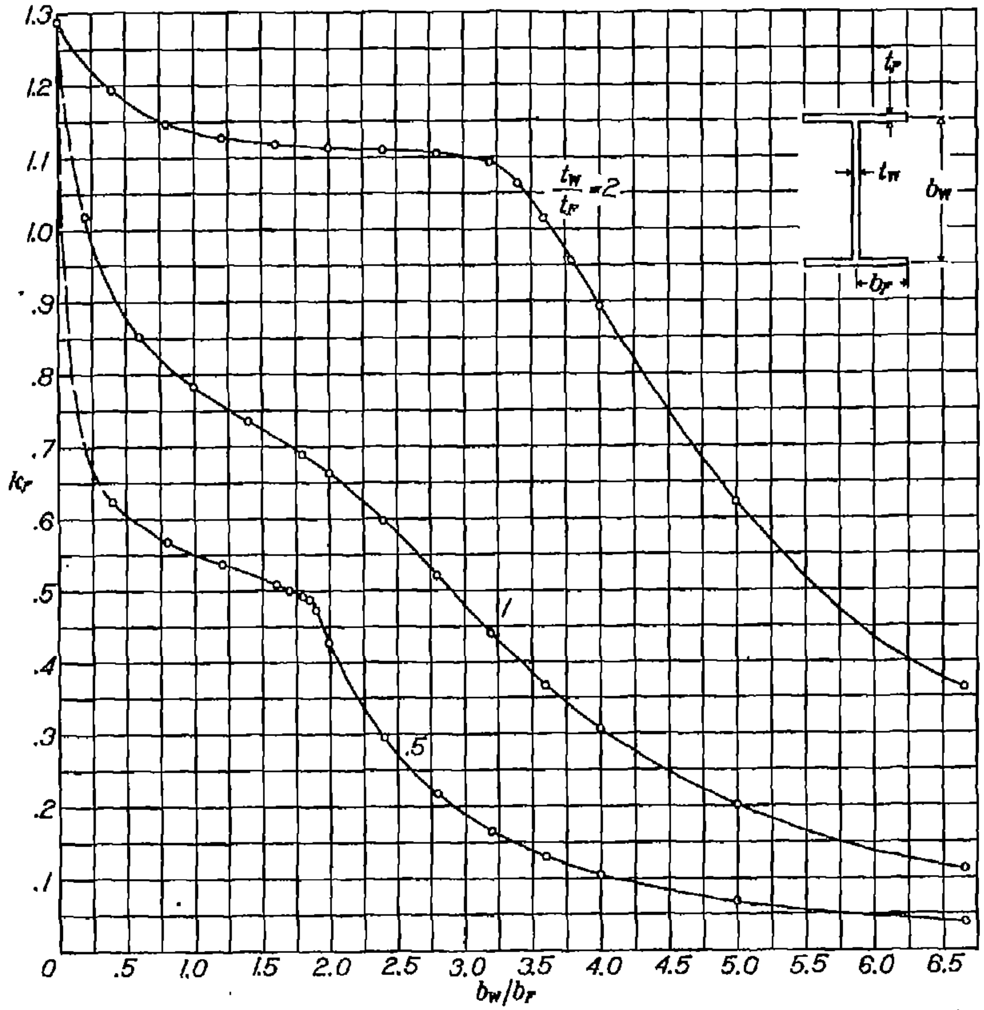


Figure 3.- Minimum values of k_F for centrally loaded columns of I-section ($\mu = 0.3$)

$$\frac{f_{cr}}{\eta} = \frac{k_F \pi^2 E t_F^2}{12(1 - \mu^2) b_F^2}$$

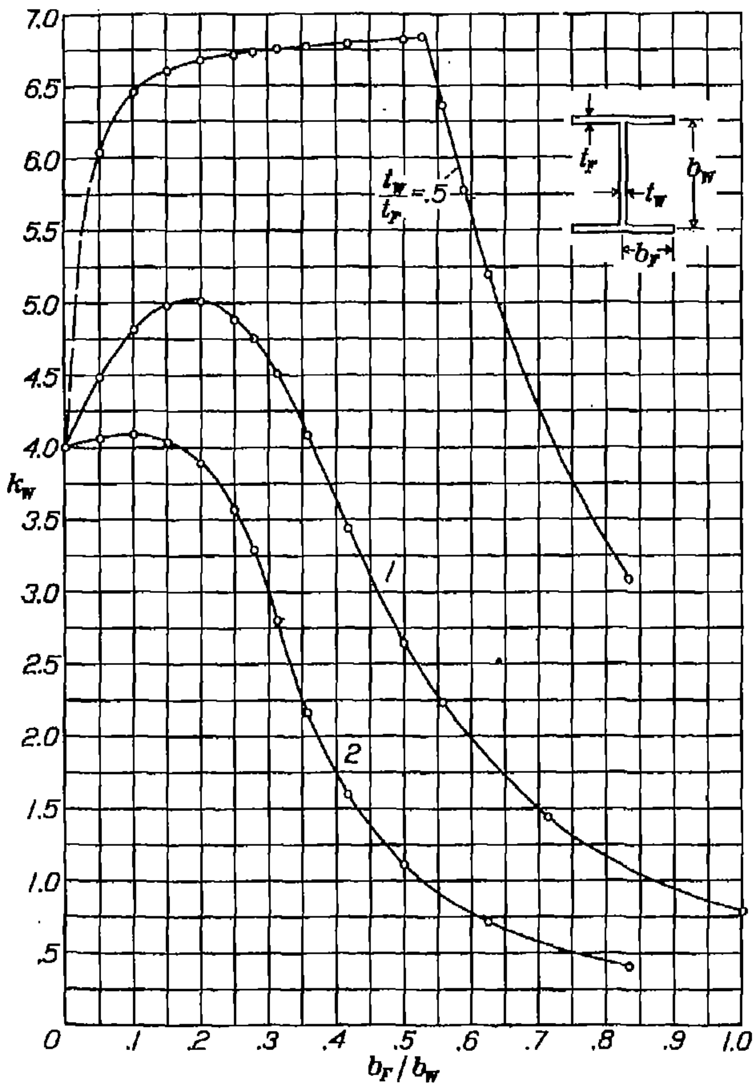


Figure 4.- Minimum values of k_w for centrally loaded columns of I-section (μ=0.3)

$$\frac{f_{cr}}{\eta} = \frac{k_w \pi^2 E t_w^2}{12(1 - \mu^2) b_w^2}$$

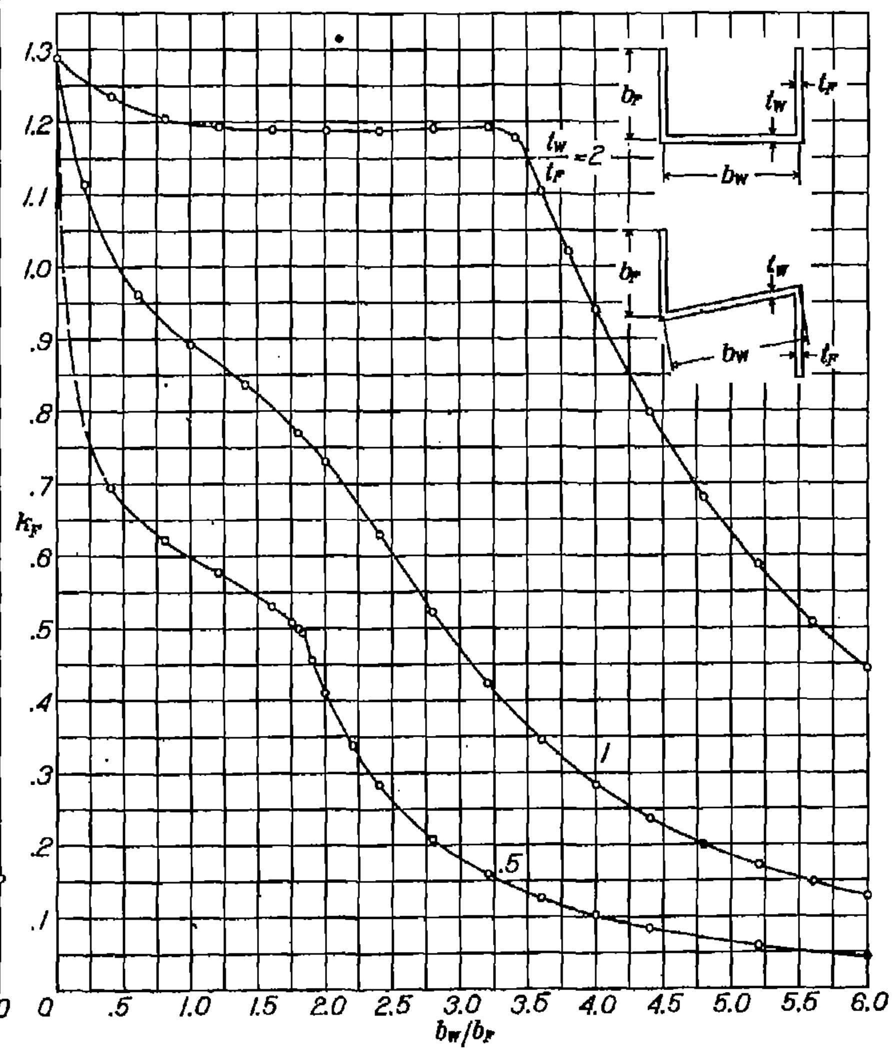


Figure 5.- Minimum values of k_F for centrally loaded columns of channel section and Z-section (μ= 0.3)

$$\frac{f_{cr}}{\eta} = \frac{k_F \pi^2 E t_F^2}{12(1 - \mu^2) b_F^2}$$

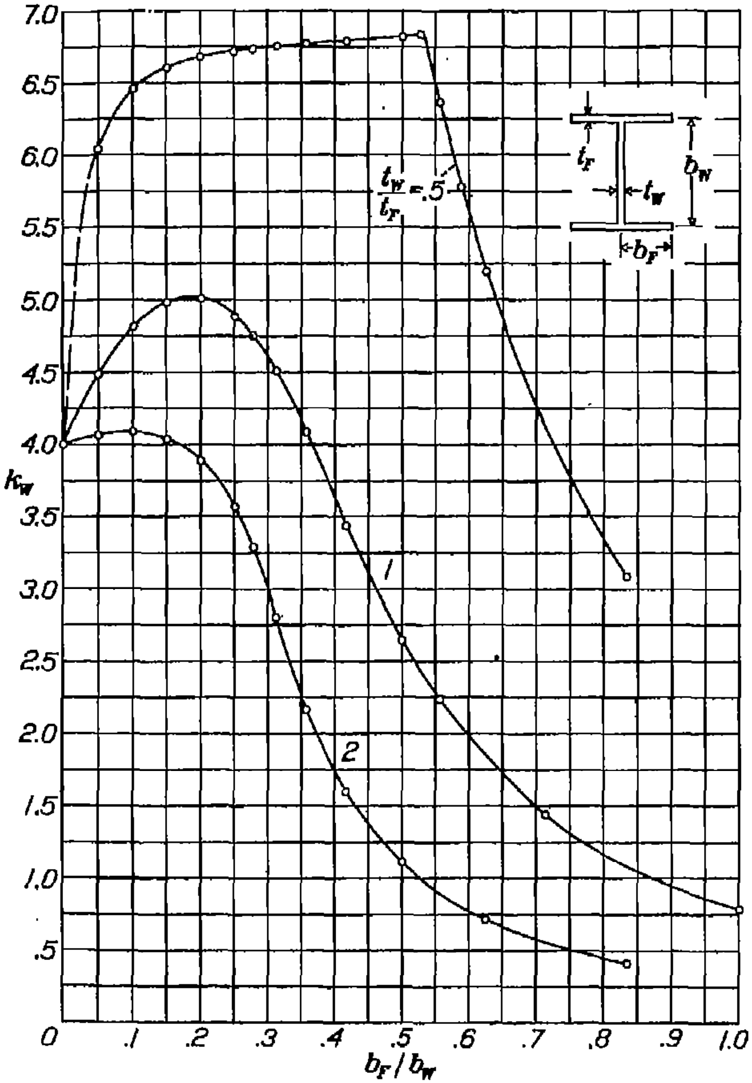


Figure 4.- Minimum values of k_W for centrally loaded columns of I-section (μ=0.3) $\frac{f_{cr}}{\eta} = \frac{k_W \pi^2 E t_W^2}{12(1 - \mu^2) b_W^2}$

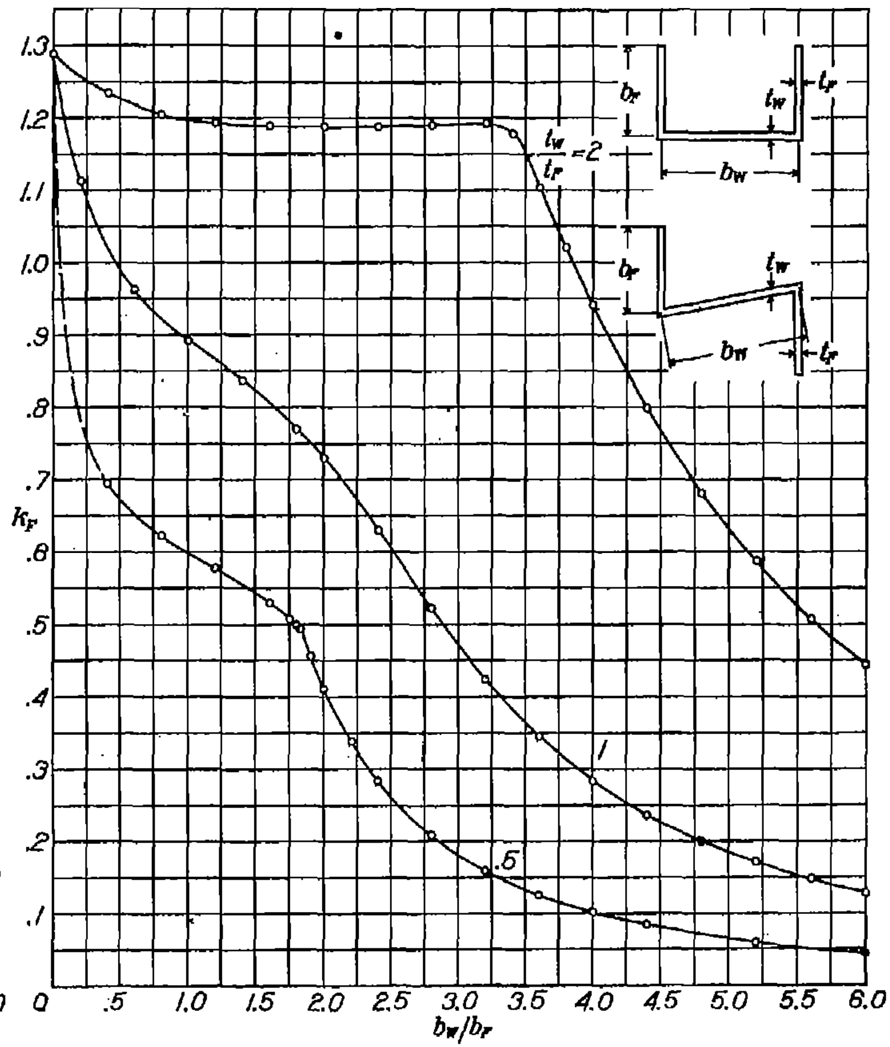


Figure 5.- Minimum values of k_F for centrally loaded columns of channel section and Z-section (μ= 0.3) $\frac{f_{cr}}{\eta} = \frac{k_F \pi^2 E t_F^2}{12(1 - \mu^2) b_F^2}$

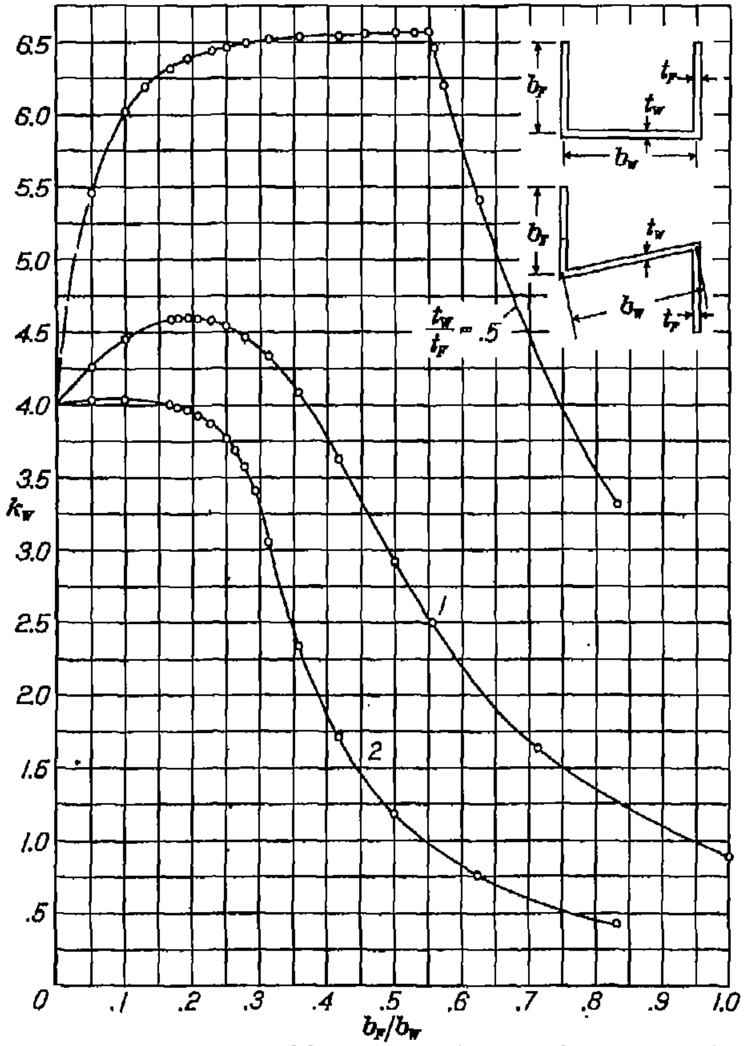


Figure 6.- Minimum values of k_W for centrally loaded columns of channel section and Z-section (μ = 0.3). $\frac{f_{cr}}{\eta} = \frac{k_W \pi^2 E t_W^2}{12(1 - \mu^2) b_W^2}$

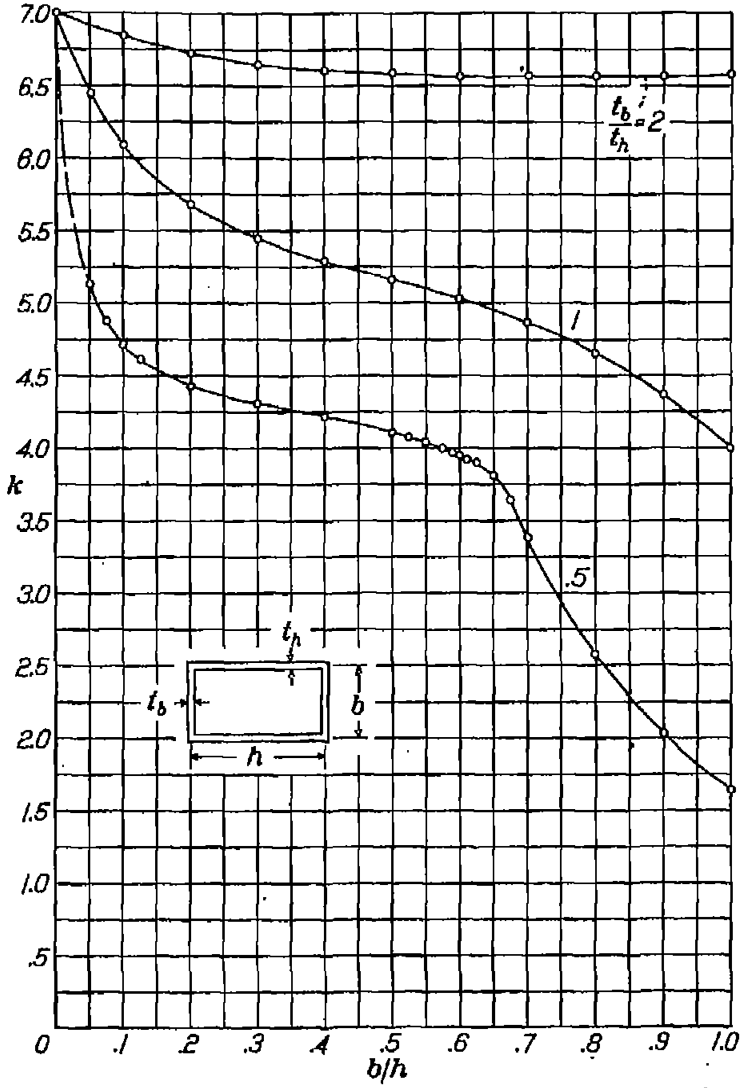


Figure 7.- Minimum values of k for a centrally loaded symmetrical rectangular tube (μ = 0.3). $\frac{f_{cr}}{\eta} = \frac{k \pi^2 E t_h^2}{12(1 - \mu^2) h^2}$

$\tau = \frac{\bar{E}}{E}$

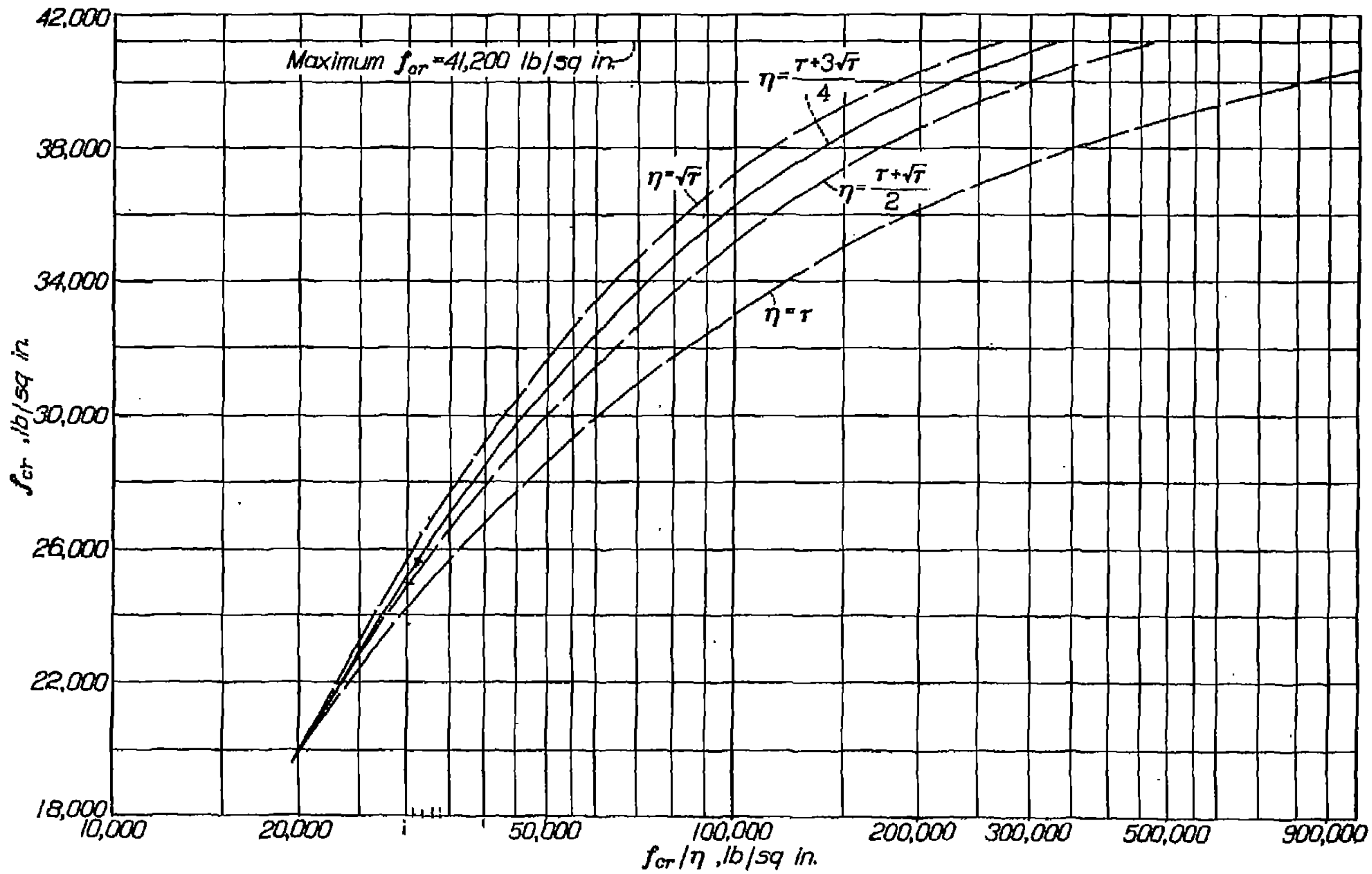


Figure 8.- Variation of f_{cr} with f_{cr}/η for 24ST aluminum alloy. When f_{cr} < 19,600 lb/sq in., η=1 and $f_{cr} = f_{cr}/\eta$.