TECHNICAL NOTES

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AERODYNAMIC HEATING AND THE DEFLECTION OF DROPS BY AN OBSTACLE IN AN AIR STREAM IN RELATION TO AIRCRAFT ICING

By Arthur Kantrowitz
Langley Memorial Aeronautical Laboratory

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SUMMARY

Two topics of interest to persons attempting to apply the heat method of preventing ice formation on aircraft are considered. Surfaces moving through air at high speed are shown, both theoretically and experimentally, to be subject to important aerodynamic heating effects that will materially reduce the heat required to prevent ice.

Numerical calculations of the paths of water drops in an air stream around a circular cylinder are given. From these calculations, information is obtained on the percentage of the swept area cleared of drops.

INTRODUCTION

The NACA has been conducting a program to develop a method of using engine-exhaust heat to prevent or to remove ice formations on aircraft. The chief object of this paper is to provide information that will be useful to persons attempting to apply this method.

Two subjects are included in the present investigation:

1. The temperature rise of surfaces exposed to boundary-layer friction has been studied. This natural temperature rise is expected materially to reduce the amount of heat required to keep a surface in high-speed flight above the freezing temperature.

2. The paths of water drops in an air stream disturbed by the presence of a circular cylinder have been numerically calculated. The calculation of these paths leads to information

on the percentage of the swept area cleared of drops by the circular cylinder. This information should be of assistance in estimating the heat required to overcome the supercooling of drops that strike the wing.

AERODYNAMIC HEATING

In viscous fluid flow the losses in mechanical energy appear as heat. If all the losses in mechanical energy in a fluid element are assumed to appear as heat in <u>that</u> element, then the total energy of the element will remain constant. A surface exposed to boundary-layer friction, but otherwise thermally isolated, would then be expected to reach a temperature higher than the outside air by an amount determined by the assumed relation that the excess temperature energy of the air stopped at the surface equals the mechanical energy of the outside air.

The validity of the preceding assumption can be examined for cases where the boundary layer is thin. The total energy per unit mass of an element of mass of the air is $\frac{u^2}{2} + Jc_pT$, where u is the velocity in the direction of flow; J, the mechanical equivalent of heat; c_p, the specific heat at constant pressure; and T, the absolute temperature.

It will be shown that the total energy is unchanged by changes in pressure and, under certain conditions, by viscosity and heat conductivity. If viscosity is neglected,

$$\rho u \, du = -dp$$

where p is the pressure and ρ is the density of the air. For perfect gases this expression becomes (if heat conductivity is neglected)

$$d\left(\frac{u^2}{2} - Jc_pT\right) = 0$$

In a thin boundary layer the contribution due to the heat conductivity k is $Jk\frac{\partial^2 T}{\partial y^2}$ and that due to the viscosity μ is $\mu\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right)$, where y is the coordinate perpendicular to the surface. The total energy contribution from these sources per unit time is

$$\frac{dE}{dt} = Jk\frac{\partial^2 T}{\partial y^2} + \mu\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right) \tag{1}$$

When the fluid enters the boundary layer, these two contributions are related if the fluid started from a uniform stream because, in this case, $\frac{u^2}{2} + Jc_pT = \text{constant}$ holds initially. Thus, equation (1) becomes <u>initially</u>

$$-Jk\frac{\partial^2}{\partial y^2}\left(\frac{u^2}{2Jc_p}\right) + \mu\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right) = \frac{dE}{dt}$$

or

$$\left(\frac{c_p\mu}{k} - 1\right)\frac{\partial^2}{\partial y^2}\left(\frac{u^2}{2}\right) = \frac{dE}{dt}\frac{c_p}{k}$$

Now, if $c_p\mu/k$, which is called σ or the Prandtl number, is unity, dE/dt is zero and $\frac{1}{2}u^2 + Jc_pT$ will remain constant.

This analysis is also valid for turbulent boundary layers if μ includes the eddy viscosity and k the eddy heat conduction. For turbulent boundary layers, then, σ will be closer to unity because the eddy viscosity is equal to the eddy heat conduction divided by the specific heat, as in Reynolds' analogy.

It is readily shown that the distribution $u^2/2 + Jc_pT = \text{constant}$ is the condition for no heat transfer from the surface because in that case

$$\left[\frac{\partial T}{\partial y}\right]_{y=o} = -\frac{1}{Jc_p}\left[\frac{\partial}{\partial y}\frac{u^2}{2}\right]_{y=o} = -\frac{1}{Jc_p}u\left[\frac{\partial u}{\partial y}\right]_{y=o} = 0$$

A surface exposed to a boundary layer (in a fluid for which $\sigma = 1$) but otherwise thermally isolated will therefore reach a temperature $T_0 + u_0{}^2/2Jc_p$. (T_0 and u_0 are free-stream values.) This result has been previously obtained for compressible laminar flow along a flat plate (reference 1).

For air, σ is 0.769 (at 0° C) for laminar boundary layers and, in this case, $\frac{\partial^2}{\partial y^2} \frac{u^2}{2}$ is positive; hence dE/dt is negative and a rise in temperature of the stopped fluid somewhat less than the adiabatic rise $(u_0{}^2/2Jc_p)$ would be expected. Pohlhausen (see reference 2, p. 627) calculated the temperature rise for the case $\sigma \neq 1$ and obtained approximately $\sigma^{\frac{1}{2}}(u_0{}^2/2Jc_p)$; and for $\sigma = 0.769$, his results give 88 percent of the adiabatic rise. For the turbulent boundary layer, σ is closer to unity and a somewhat larger temperature rise should be expected.

The NACA 24-inch high-speed wind tunnel (reference 3) offers an excellent opportunity to test these theoretical conclusions for two reasons: First, high speeds result in large, easily measured temperature differences; and second, the test-section air has been expanded from atmospheric conditions. Thus, the determination of the difference between surface temperature and atmospheric temperature will give a direct measure of the difference between the surface temperature and the temperature the surface would have had if it had experienced the adiabatic temperature rise. This difference being much smaller than the actual temperature rise, a considerable gain in accuracy is effected by this method.

Tests were therefore conducted in the NACA 24-inch tunnel of an airfoil prepared as shown in figure 1. Platinum wires 1 foot long having a diameter of 0.002 inch were embedded in balsa inserts in a 5-inch-chord aluminum-alloy NACA 0012T airfoil. The balsa inserts were painted and the paint was rubbed to a smooth surface exposing the wires. The wires were so placed that one would be in the laminar boundary-layer region (at 10 percent of the chord) and one in the turbulent layer (at 70 percent of the chord).

The surface temperatures were determined by measuring the resistance of the wires, which were calibrated after the wing was tested. Atmospheric temperatures were found before and after each test point was taken by determining the wire resistances at a very low tunnel speed (30 to 40 mph).

In order to check the method of measurement, a wire was inserted at the stagnation point of an NACA 0018 airfoil and the temperatures were measured over the range of speeds. As was anticipated, no significant difference in temperature between the stagnation point and the atmosphere was found to exist.

The results for the NACA 0012T airfoil at 0° angle of attack are plotted in figure 2. The temperature rises are nearly equal to the adiabatic, as was expected. The fact that the wire in the laminar flow gave a considerably lower temperature than the one in the turbulent flow is chiefly due to the fact that the pressure at 0.10c was lower than at 0.70c. This lower pressure produces a greater temperature drop in the fluid just outside the boundary layer and therefore, even if the same fraction of this drop were recovered across the boundary layer, a lower temperature would be expected at the lower pressure point.

In order to obtain a comparison of the test results with theory, the temperature rises across the boundary layer were computed. The local pressures were obtained by calculating the low-speed pressure distribution by Theodorsen's method (reference 4) and correcting for compressibility by multiplying these pressures by $1/\sqrt{1 - M^2}$. Here M, the Mach number, is the ratio of the velocity in the test section to the velocity of sound in the test section. The local Mach number and the temperature of the air outside the boundary layer were then calculated from this pressure distribution for the laminar and the turbulent stations. The percentage of the local temperature drop outside the boundary layer that was recovered in the boundary layer is shown in figure 3. It should be pointed out that the points plotted for local Mach number of the order of 1.0 or greater have little meaning because the preceding method of calculating the local pressure is invalid in these cases.

The temperature rises across the laminar boundary layer are only slightly smaller than those across the turbulent layer. The scatter of the points in the turbulent layer was also greater than in the laminar layer. It was thought that perhaps the boundary layer at the 70-percent station might be laminar part of the time. This possibility was eliminated by noting that a strip of No. 80 emery placed just back of the wire at 0.10c had no effect on the temperature rise of the wire at the 0.70c station.

The temperature rises for high speeds are large enough to have an important effect on aircraft icing. In particular, airplanes traveling at speeds greater than 300 miles per hour would probably not experience the most severe icing conditions, which occur at atmospheric temperatures only a few degrees below freezing. Aerodynamic heating is also probably the reason for the lack of <u>any</u> ice formation on the outboard regions of propellers except under conditions of extreme cold and low tip speed.

If the ice formed and was slung off by centrifugal force, the shear strength of ice indicates that pieces less than one-eighth inch thick, which ought to be light enough to stick, should be found.

This phenomenon ought to make it easier to supply enough heat to prevent ice formation on high-speed airplanes because a considerable part of the necessary heat will be supplied by the boundary-layer friction.

PATHS OF WATER DROPS

The most dangerous ice formations are produced by the striking and the freezing of the supercooled water drop on the aircraft. Inasmuch as supercooling of these drops can remove an appreciable amount of heat, the fraction of the swept area that is cleared of drops by a wing should be of interest in the prevention of ice formation. Also, for cases where the drops freeze on impact, as in the formation of rime, a relation between the iced area and the maximum drop size can be obtained from a knowledge of the paths. In order to provide qualitative information on these and other points, numerical calculations of the drop paths in air flowing at 200 miles per hour around a circular cylinder 1 foot in diameter were made.

The equation of motion of a drop is

$$m \frac{dV}{dt} = - D(V - V_a)_1 \tag{6}$$

where m is the mass of the drop; V, its velocity; D, the drag of the drop in an air stream of velocity $(V - V_a)$, where V_a is the local air velocity and $(V - V_a)_1$ is a unit vector. The drag can be taken from data for spheres given by Zahm (reference 5) for a drop of diameter d as

$$D = \left\{ 28 \left[\frac{(V - V_a)d}{\nu} \right]^{-0.85} + 0.48 \right\} \left[\frac{1}{2} \rho (V - V_a)^2 \frac{\pi d^2}{4} \right] \tag{7}$$

The local air velocity around the forward part of a cylinder can be found from the potential function (reference 6, p. 30)

$$\psi = U \left(\frac{r^2 y}{x^2 + y^2} - y \right)$$

where U is the stream velocity; r, the cylinder radius; and the coordinates are measured from the center of the cylinder.

The velocities parallel and perpendicular to the stream direction are

$$\left. \begin{aligned} u &= \frac{\partial \psi}{\partial y} = U \left(\frac{r^2}{x^2 + y^2} - \frac{2r^2y^2}{(x^2 + y^2)^2} - 1 \right) \\ v &= - \frac{\partial \psi}{\partial x} = U \frac{2r^2xy}{(x^2 + y^2)^2} \end{aligned} \right\} \tag{8}$$

The equation of motion of the drop was then numerically integrated. A drop of a given size was assumed to be at a given ordinate y_0 and at an abscissa x_0 large enough that the initial acceleration was small; the drop was also assumed to have the free-stream velocity. The local air velocity V_a was then obtained from equation (8) and the air force and the acceleration from equations (7) and (6). The drop was assumed to move with this velocity and acceleration for a short time interval Δt and new coordinates and a new velocity were computed after this interval. The process was repeated until the drop either hit or clearly missed the cylinder. In this way, the maximum initial ordinate that a drop could have and still strike the cylinder was found. Thus, the area swept clear of drops was found and is plotted in figure 4. From the same calculations, the region of the cylinder struck by the drops was obtained and is plotted in figure 5. This region would be the area covered with ice if the drops were of uniform size and froze on contact.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 24, 1940.

REFERENCES

1. von Kármán, Th., and Tsien, H. S.: Boundary Layer in Compressible Fluids. Jour. Aero. Sci., vol. 5, no. 6, April 1938, pp. 227-232.

2. Goldstein, S.: Modern Developments in Fluid Dynamics. vol. II. Clarendon Press (Oxford), 1938.

3. Stack, John, Lindsey, W. F., and Littell, Robert E.: The Compressibility Burble and the Effect of Compressibility on Pressures and Forces Acting on an Airfoil. T.R. No. 646, NACA, 1938.

4. Theodorsen, Theodore: Theory of Wing Sections of Arbitrary Shape. T.R. No. 411, NACA, 1931.

5. Zahm, A. F.: Flow and Drag Formulas for Simple Quadrics. T.R. No. 253, NACA, 1927.

6. Glauert, H.: The Elements of Aerofoil and Airscrew Theory. The Univ. Press (Cambridge), 1926.

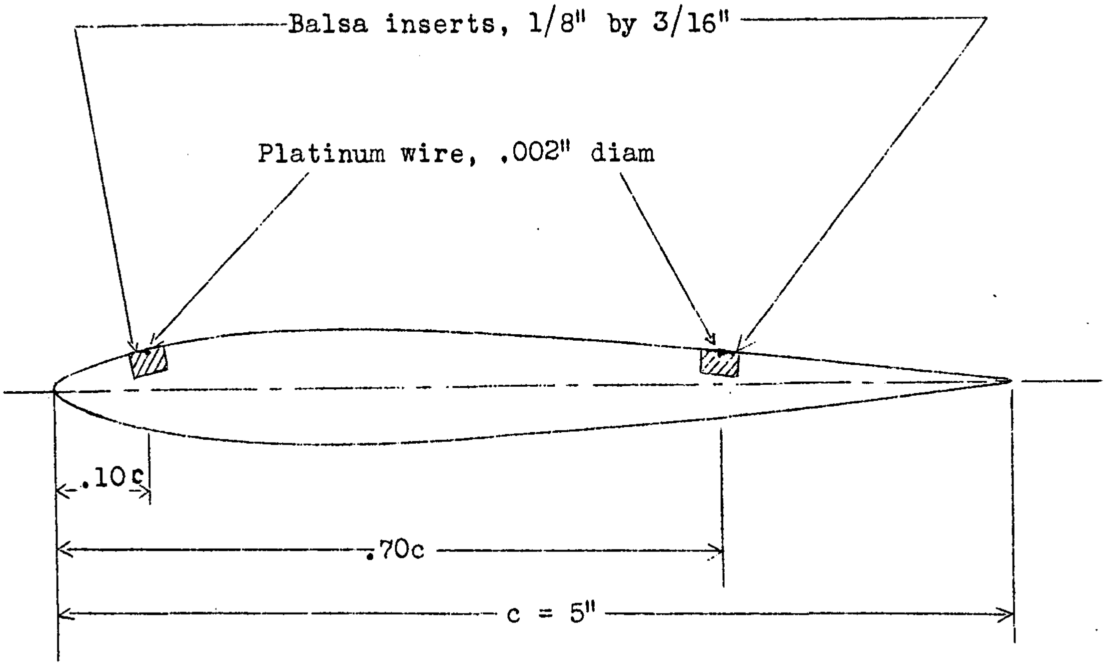

Figure 1.- Cross section of aluminum-alloy NACA 0012T airfoil used in tests.

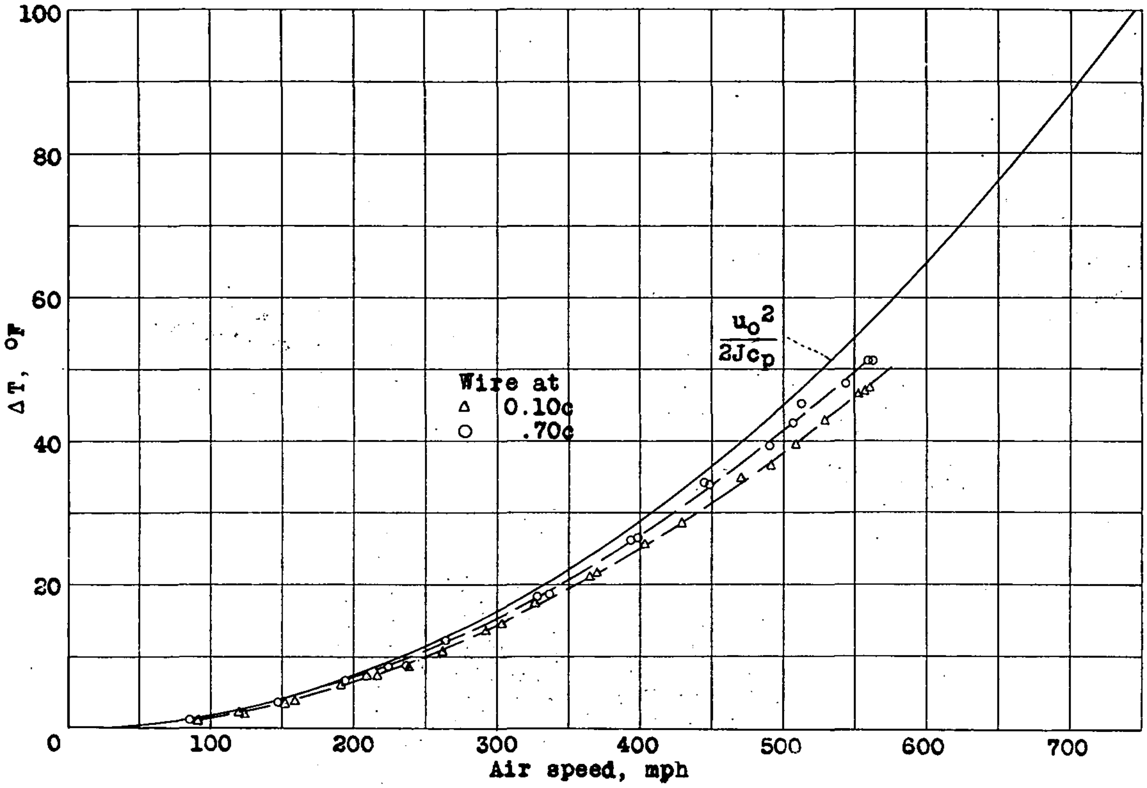

Figure 2.- Variation of observed temperature rise with air speed.

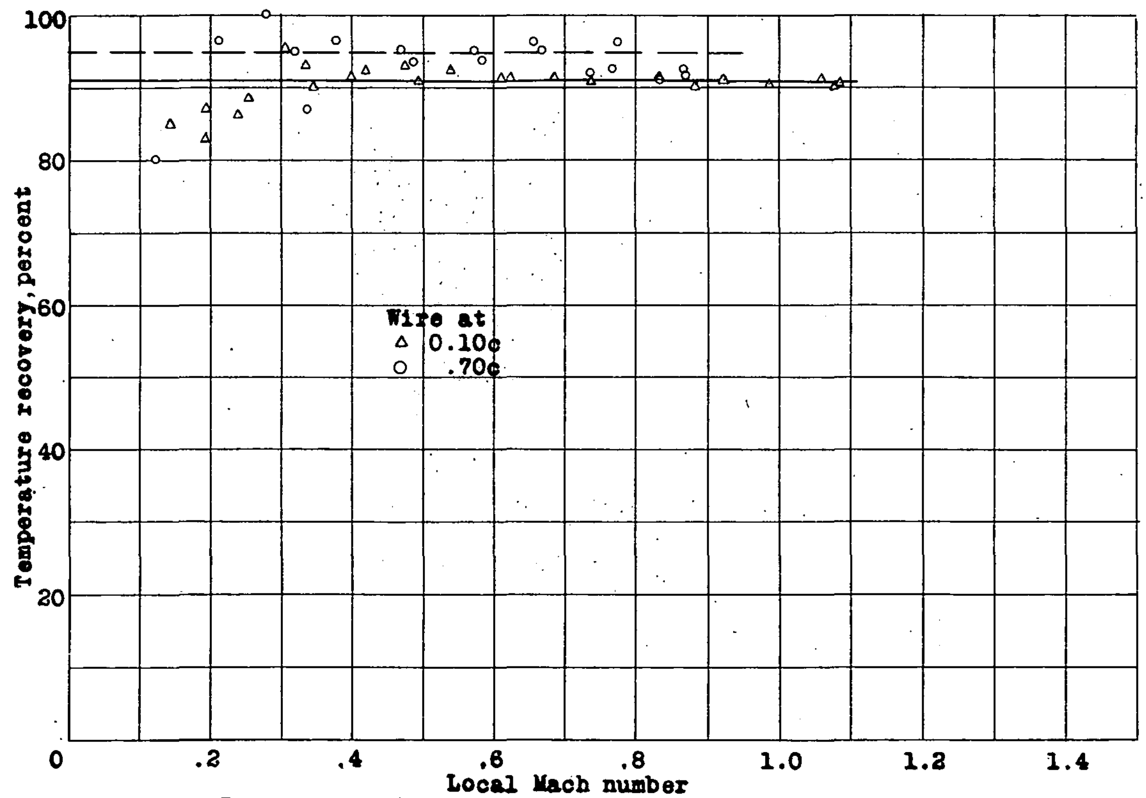

Figure 3.- Percentage of adiabatic rise recovered in the boundry layer.

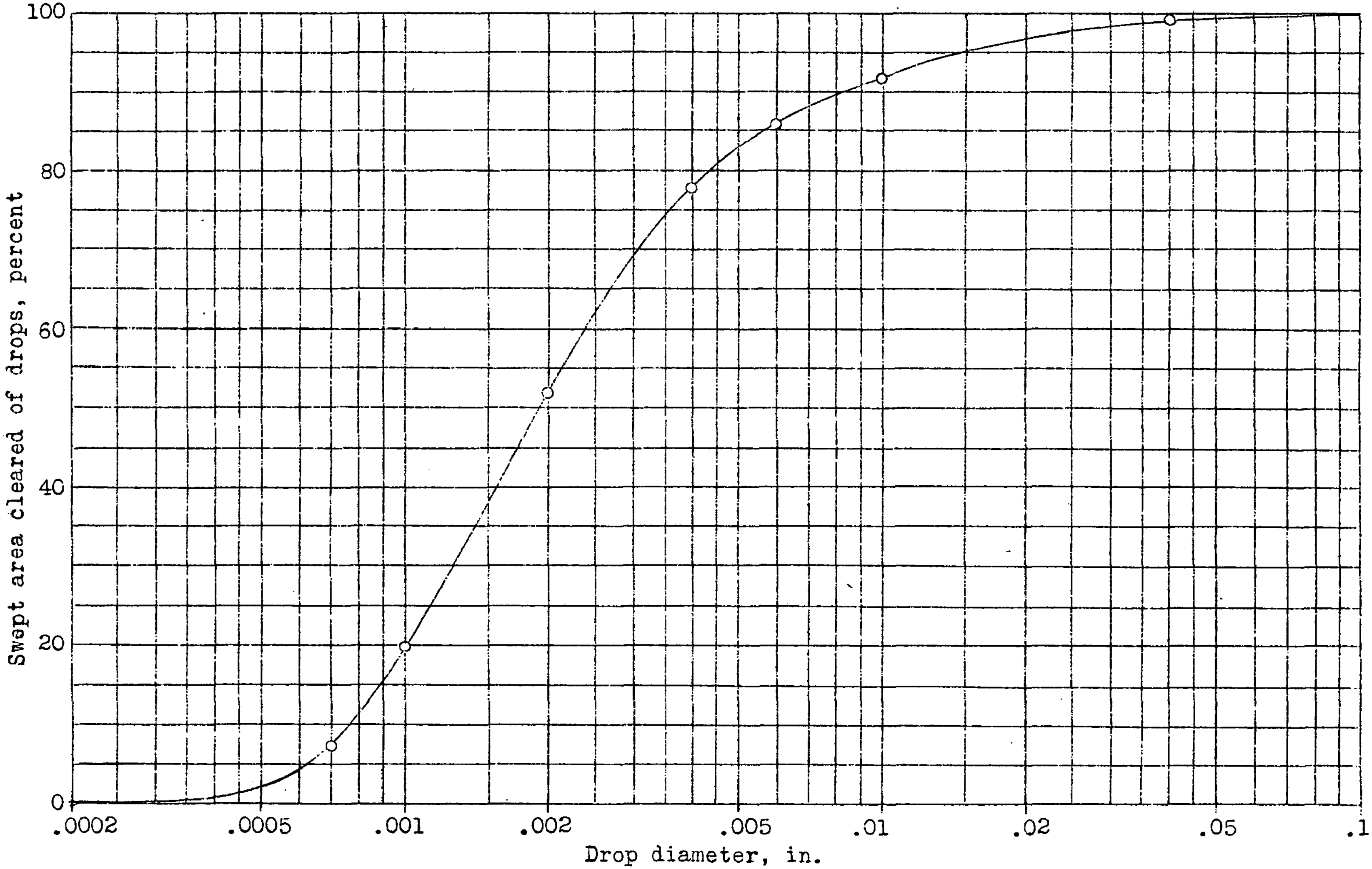

Figure 4.- Variation of swept area cleared of drops with drop diameter for a right circular cylinder.

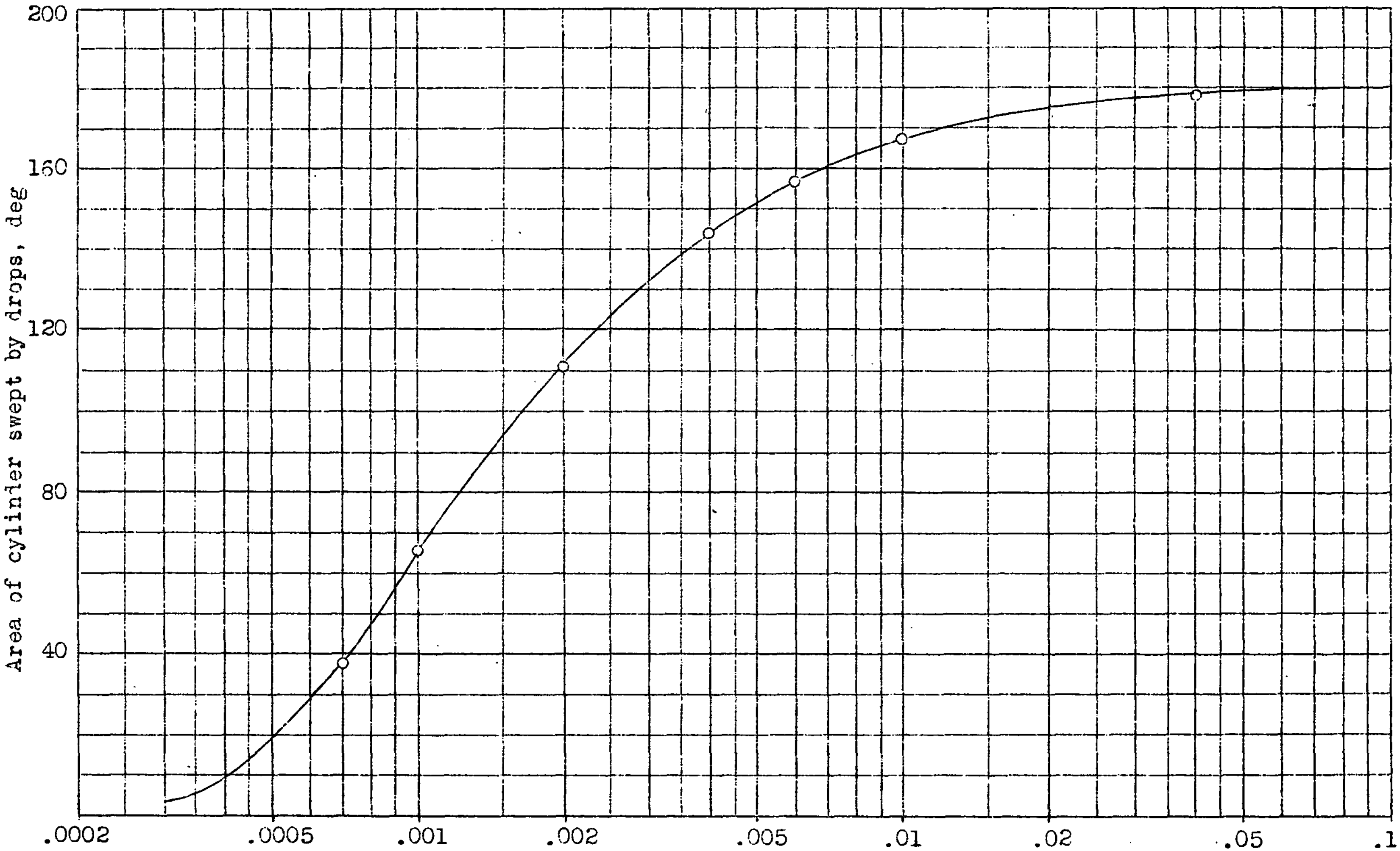

Figure 5.- Variation of area of right circular cylinder struck by drops with drop diameter.