TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 860

THE CRITICAL COMPRESSION LOAD FOR A UNIVERSAL TESTING MACHINE WHEN THE SPECIMEN IS LOADED THROUGH KNIFE EDGES

By Eugene E. Lundquist and Edward B. Schwartz

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

Washington
September 1942

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 860

THE CRITICAL COMPRESSION LOAD FOR A UNIVERSAL TESTING MACHINE WHEN THE SPECIMEN IS LOADED THROUGH KNIFE EDGES

By Eugene E. Lundquist and Edward B. Schwartz

SUMMARY

The results of a theoretical and experimental investigation to determine the critical compression load for a universal testing machine are presented for specimens loaded through knife edges. The critical load for the testing machine is the load at which one of the loading heads becomes laterally unstable in relation to the other. For very short specimens the critical load was found to be less than the rated capacity given by the manufacturer for the machine. A load-length diagram is proposed for defining the safe limits of the test region for the machine.

Although this report is particularly concerned with a universal testing machine of a certain type, the basic theory which led to the derivation of the general equation for the critical load,

$$P_{cr} = \alpha L$$

can be applied to any testing machine operated in compression where the specimen is loaded through knife edges. In this equation, L is the length of the specimen between knife edges and α is the force necessary to displace the upper end of the specimen unit horizontal distance relative to the lower end of the specimen in a direction normal to the knife edges through which the specimen is loaded.

INTRODUCTION

During the past several years, questions have arisen regarding the stability of universal testing machines. It has been observed that, during a compression test, the part of the machine which bears on the top end of the specimen sometimes moves horizontally in relation to the part which applies load to the bottom end of the specimen. Doubts naturally arise as to the value of compression-test data when this sidewise displacement occurs. (See fig. 1.)

If the ends of the compression specimen are, in effect, welded or rigidly attached to the machine, a horizontal displacement of the top end of the specimen with respect to the bottom end involves an elastic distortion of both the specimen and the machine. For this condition of the ends of the specimen, the critical load at which the horizontal movement occurs for a centrally positioned specimen is given by an equation that involves the elastic properties of both the testing machine and the specimen. If, however, the ends of the compression specimen have knife edges or frictionless pins through which the load is applied, this critical load is given by an equation that involves only the elastic properties of the testing machine.

The present report is concerned with an experimental and theoretical investigation of the more simple case in which the load is applied through knife edges.

DERIVATION OF THE CRITICAL COMPRESSION LOAD FOR THE TESTING MACHINE

Consider the testing machine shown in figure 2. The structural elements of this machine involved in considerations of stability are shown diagrammatically in figure 3.

In the following derivation, the specimen is assumed to be centrally positioned in and centrally loaded by the testing machine. It is also assumed that the specimen is loaded through knife edges and that the specimen is sufficiently strong not to buckle before the critical load for the machine is reached.

The weighing platen of the testing machine against which the upper end of the specimen bears is connected by tension rods to the base structure, which contains the ram and loading platen against which the lower end of the specimen bears. The upper end of the specimen is therefore elastically restrained against horizontal movements relative to the lower end of the specimen. Let the following symbols be defined as:

α force necessary to displace upper end of specimen unit horizontal distance relative to lower end of specimen in a direction normal to knife edges through which specimen is loaded

W external horizontal force acting at upper end of specimen to deflect the two tension rods equally

P compressive load on specimen applied by testing machine

L length of specimen between knife edges

In order to derive the equation for the critical compression load of the testing machine, assume that the machine applies a compressive load P to the specimen. In addition, assume that an external horizontal force W is also applied at the upper end of the specimen. Under the action of these forces the upper end of the specimen will be displaced horizontally an amount δ relative to the lower end of the specimen. The total side force H, which results in this deflection δ, is the sum of the external horizontal force W and the horizontal component of the force in the specimen. Thus,

$$H = W + P\frac{\delta}{L} \tag{1}$$

From the definition of α

$$H = \alpha\delta \tag{2}$$

Hence,

$$\alpha\delta = W + P\frac{\delta}{L} \tag{3}$$

from which

$$\delta = \frac{W}{\alpha - \frac{P}{L}} \tag{4}$$

In the derivation of equation (4) the external horizontal force W was introduced for purposes of analysis. In the case under consideration of a specimen centrally positioned in and centrally loaded by the testing machine, there is no external force W acting. For the case of $W = 0$, equation (4) shows that a deflection δ is possible only if the denominator, $\alpha - \frac{P}{L}$ is zero. In a perfectly centered column test, the upper end of the specimen can therefore move horizontally in relation to the lower end of the specimen when

$$\alpha - \frac{P}{L} = 0 \tag{5}$$

The value of P that satisfies this equation is the critical compression load for the testing machine,

$$P_{cr} = \alpha L \tag{6}$$

The value of α is a function of the dimensions of the testing machine, the position of the weighing platen, and the load P through its effect on the stiffness of the tension rods. Because all materials have finite values for the modulus of elasticity, the value of α will always be finite. An important conclusion to be drawn from equation (6) is therefore that, as the length L of the specimen approaches zero, the critical compression load at which the testing machine becomes unstable also approaches zero, even though α will be larger when L is small than when L is large.

EVALUATION OF α

Theoretical evaluation.- On the assumption that α depends only on the bending properties of the tension rods, the derivation in the appendix shows that, for $P > 0$,

$$\alpha = \frac{P}{b} \frac{1}{1 - \dfrac{1}{mb \coth mb + \dfrac{b}{a}(ka \coth ka - 1)}} \tag{7}$$

For $P = 0$, this equation reduces to

$$\alpha_o = \frac{6\,EI_b}{b^3\left(1 + \dfrac{I_b a}{I_a b}\right)} \tag{8}$$

In equation (7),

$$k = \sqrt{\frac{P}{2EI_a}} \tag{9}$$

and

$$m = \sqrt{\frac{P}{2EI_b}} \tag{10}$$

The lengths a and b are defined in figure 4. In expressions (9) and (10) the quantities $2EI_a$ and $2EI_b$ are the combined bending stiffnesses of the portions of the two tension rods of lengths a and b, respectively. In the derivation of equation (7), it was assumed that the tension rods were simply supported at their lower ends as well as at the flexure-plate support and that the tension force in each rod was $P/2$.

It can be seen from figure 4 that, for a given position of the loading platen, b is equal to a constant plus the length of specimen L. Equation (7) therefore gives α in terms of the variables P and L. The theoretical curves determined by equation (7) are plotted in figure 5.

Experimental evaluation.- For the experimental determination of α, a compression-test set-up was used, with the addition of a horizontal side load applied to the weighing platen symmetrically with respect to the tension rods. The resulting horizontal deflections were measured by the average of dial gage readings on each tension rod. The specimens were steel bars of rectangular cross section, supported on knife edges and having buckling loads above 100,000 pounds, the capacity of the machine. Specimen lengths were measured from knife edge to knife edge.

Before a test run was made, the specimen was centered in the machine by shifting the grooved bars in which the knife edges were seated. The dial-gage readings when the specimen was loaded to 100,000 pounds with zero applied side load were taken as a measure of the accuracy of centering, and the centering process was continued until further shifting of the grooved bars produced no further reduction in these readings.

After the specimens had been centered, they were subjected to axial loads from 0 to 100,000 pounds in increments of 10,000 pounds, with two values of side load W.

If these two side loads are designated W_1 and W_2 and the corresponding deflections are designated δ_1 and δ_2, the value of α for a given axial load P is obtained from equation (3) with W replaced by $(W_2 - W_1)$ and δ replaced by $(\delta_2 - \delta_1)$. Thus, the experimental values of α were computed from the equation

$$\alpha = \frac{(W_2 - W_1) + P \dfrac{(\delta_2 - \delta_1)}{L}}{(\delta_2 - \delta_1)} \tag{11}$$

The experimental values so determined are plotted in figure 5.

Correlation of the theoretical and experimental values of α.- It may be observed in figure 5 that, for any given length of specimen, the variation of α with P is essentially linear, both for the theoretical values of α calculated by equation (7) and for the experimental values calculated by use of equation (11). The failure to obtain perfect agreement between the theoretical and experimental values of α is caused by the idealized assumptions made in the derivation of equation (7) and the great difficulty of selecting proper dimensions from the actual machine for substitution in an equation derived for an idealized machine. The fact that both the theoretical and experimental values of α vary linearly with P suggests that a straight line drawn through the experimental points for each specimen length would give the true value of α for any load P. The true values of α are therefore assumed to be given by the straight-line equation

$$\alpha = \alpha_0 + nP \tag{12}$$

where the intercept α_0 and the slope n are established for each specimen length by the straight line drawn through the experimental points in figure 5.

EVALUATION OF P_{cr}

The critical compression load for the testing machine is given by equation (6). According to equation (12) the value of α is a function of the load P on the testing machine. The value of α when the load on the testing machine is equal to the critical load is therefore given by equation (12) with P replaced by P_{cr}. Substitution of this value of α in equation (6) gives

$$P_{cr} = (\alpha_o + nP_{cr})L \tag{13}$$

from which

$$P_{cr} = \frac{\alpha_o L}{1 - nL} \tag{14}$$

The values of α_o and n in equation (14) depend upon the dimensions and elastic properties of the testing machine and the specimen length L. For a particular testing machine, the dimensions and elastic properties are fixed. For any machine, therefore, equation (14) gives the critical compression load as a function of specimen length L. The values of P_{cr} established from equation (14) by use of the experimental data (table I) are therefore plotted as ordinates against L as abscissas in figure 6. As previously mentioned, P_{cr} approaches zero as L approaches zero. When L becomes large, P_{cr} approaches a constant value asymptotically.

DISCUSSION

During the useful life of a testing machine, there will probably be some, if not many, compression tests made in which the specimens are not perfectly centered in the machine. In such cases the loads on the machine are eccentrically applied. As the load on the specimen is increased, the effect of this eccentric loading of the machine is to cause a horizontal movement of the upper end of the compression specimen relative to its lower end from the beginning of loading.

In order to show the effect of an initial deflection δ_o, which produces the same general effect as an eccentric load on the machine, equation (4) is written in the form

$$\delta = \frac{\frac{W}{\alpha}}{1 - \frac{P}{\alpha L}} \tag{15}$$

By definition,

$$\delta_o = \frac{W}{\alpha_o} \tag{16}$$

By use of this relation and equations (12) and (14), equation (15) becomes

$$\delta = \delta_o\left(\frac{1}{1 - \dfrac{P}{P_{cr}}}\right) \tag{17}$$

In equation (17) the quantity

$$\frac{1}{1 - \dfrac{P}{P_{cr}}}$$

is a magnification factor because it shows how the initial deflection δ_o is magnified by the load P. In order that the effects of an initial displacement not be greatly magnified, the magnification factor must be small in each compression test. The magnification factor will be small if the value of P, the compression load on the specimen, is small in comparison with P_{cr}. For the proper use of the testing machine, the compression load on the specimen should therefore not approach too closely the critical load for the machine. The load on the specimen should also not exceed the rated capacity given by the manufacturer for the machine. These two conditions establish a test region ABC in figure 7 to which all knife-edge compression tests should be confined. The particular location of the point B is established by the maximum magnification factor considered permissible. Because, for very short specimens, P_{cr} for the testing machine will always be less than the rated capacity, the engineer using a testing machine must take the responsibility for its proper use. This responsibility may include the definition of the safe limits of the test region determined by the location of point B on the load-length diagram for the testing machine.

In the case of long specimens, it is the responsibility of the manufacturer to build into the testing machine a value for P_{cr} several times in excess of the rated capacity of the machine. Tension rods of large cross section and a machine of sturdy construction will insure a high value for P_{cr} and will narrow the range of short specimen lengths for which the user of the machine must exercise judgment. A testing machine similar in design to the one shown in figure 2 but with the addition of another set of flexure-plate supports above the lowest level of the loading platen should be much more dependable for compression testing of short specimens. The foregoing recommendations to improve the stiffness of the testing machine naturally suggest that reasonably close fits between parts are also desirable.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va.

APPENDIX

DERIVATION OF THE THEORETICAL VALUE OF α

By definition α, for a testing machine, is the force necessary to displace the upper end of the specimen unit horizontal distance relative to the lower end of the specimen in a direction normal to the knife edges through which the specimen is loaded. In the derivation that follows, it is assumed that the value of α depends only on the bending properties of the tension rods and that the tension rods are simply supported at their lower ends and at the flexure-plate supports.

Figure 4 shows the elastic curve of the two tension rods considered as a single member. If a positive moment M is one that produces compression in the fibers to the left of the neutral axis and x and y are positive in the directions shown in figure 4, the differential equation of the elastic curve of the two tension rods is

$$2EI \frac{d^2y}{dx^2} = -M \qquad \text{(A 1)}$$

where $2EI$ is the combined flexural rigidity of the two rods. For the case shown in figure 4, there are two different expressions for the bending moment and two different values of flexural rigidity corresponding to the two portions a and b of the tension rods. Equation (A 1) must therefore be written for each portion. The values of flexural rigidity for the portions a and b will be $2EI_a$ and $2EI_b$, respectively. If the deflections for the two parts are denoted by y_a and y_b, the differential equation becomes, for $-a < x < 0$

$$2EI_a \frac{d^2y_a}{dx^2} = -P(\delta - y_a) + R_1 x + H(b - x) \qquad \text{(A 2)}$$

and for $0 < x < b$

$$2EI_b \frac{d^2y_b}{dx^2} = -P(\delta - y_b) + H(b - x) \qquad \text{(A 3)}$$

From equilibrium of moments, the reaction

$$R_1 = \frac{H(a + b) - P\delta}{a} \qquad \text{(A 4)}$$

With this value of R_1, equation (A 2) may be written

$$2EI_a \frac{d^2y_a}{dx^2} = Py_a + (Hb - P\delta)\left(1 + \frac{x}{a}\right) \qquad \text{(A 5)}$$

With the notation

$$k^2 = \frac{P}{2EI_a} \qquad \text{(A 6)}$$

and

$$m^2 = \frac{P}{2EI_b}$$

Equations (A 5) and (A 3) may be written in the form

$$\frac{d^2y_a}{dx^2} - k^2y_a = \frac{(Hb - P\delta)}{P} k^2\left(1 + \frac{x}{a}\right) \qquad \text{(A 7)}$$

$$\frac{d^2y_b}{dx^2} - m^2y_b = -m^2\delta + \frac{H}{P}m^2(b - x) \qquad \text{(A 8)}$$

The solutions of these differential equations are

$$y_a = A \cosh kx + B \sinh kx - \frac{(Hb - P\delta)}{P}\left(1 + \frac{x}{a}\right) \qquad \text{(A 9)}$$

$$y_b = A' \cosh mx + B' \sinh mx + \delta - \frac{H}{P}(b - x) \qquad \text{(A 10)}$$

The four constants of integration A, B, A', and B' are established by the following end conditions:

$$(y_a)_{x=o} = 0 \qquad \text{(A 11)}$$

$$(y_a)_{x=-a} = 0 \qquad \text{(A 12)}$$

$$(y_b)_{x=o} = 0 \qquad \text{(A 13)}$$

$$\left(\frac{dy_a}{dx}\right)_{x=o} = \left(\frac{dy_b}{dx}\right)_{x=o} \qquad \text{(A 14)}$$

With A, B, A', and B' evaluated, equation (A 10) becomes

$$y_b = \frac{1}{P}(Hb - P\delta) \cosh mx + \frac{1}{Pm}\left[\frac{1}{a}(Hb-P\delta)(ka \coth ka-1) - H\right] \sinh mx - \frac{H}{P}(b-x)+\delta \qquad \text{(A 15}$$

For $x = b$, y_b equals the deflection δ. With these values of x and y_b inserted, equation (A 15) can be solved for H/δ. By definition (see equation (2)), $H/\delta = \alpha$; equation (A 15) then reduces to

$$\alpha = \frac{P}{b} \cfrac{1}{1 - \cfrac{1}{mb \coth mb + \frac{b}{a}(ka \coth ka - 1)}} \qquad \text{(see equation (7)).}$$

For $P = 0$, equation (7) becomes

$$\alpha_o = \frac{6EI_b}{b^3\left(1 + \frac{I_b a}{I_a b}\right)} \qquad \text{(See equation (8).)}$$

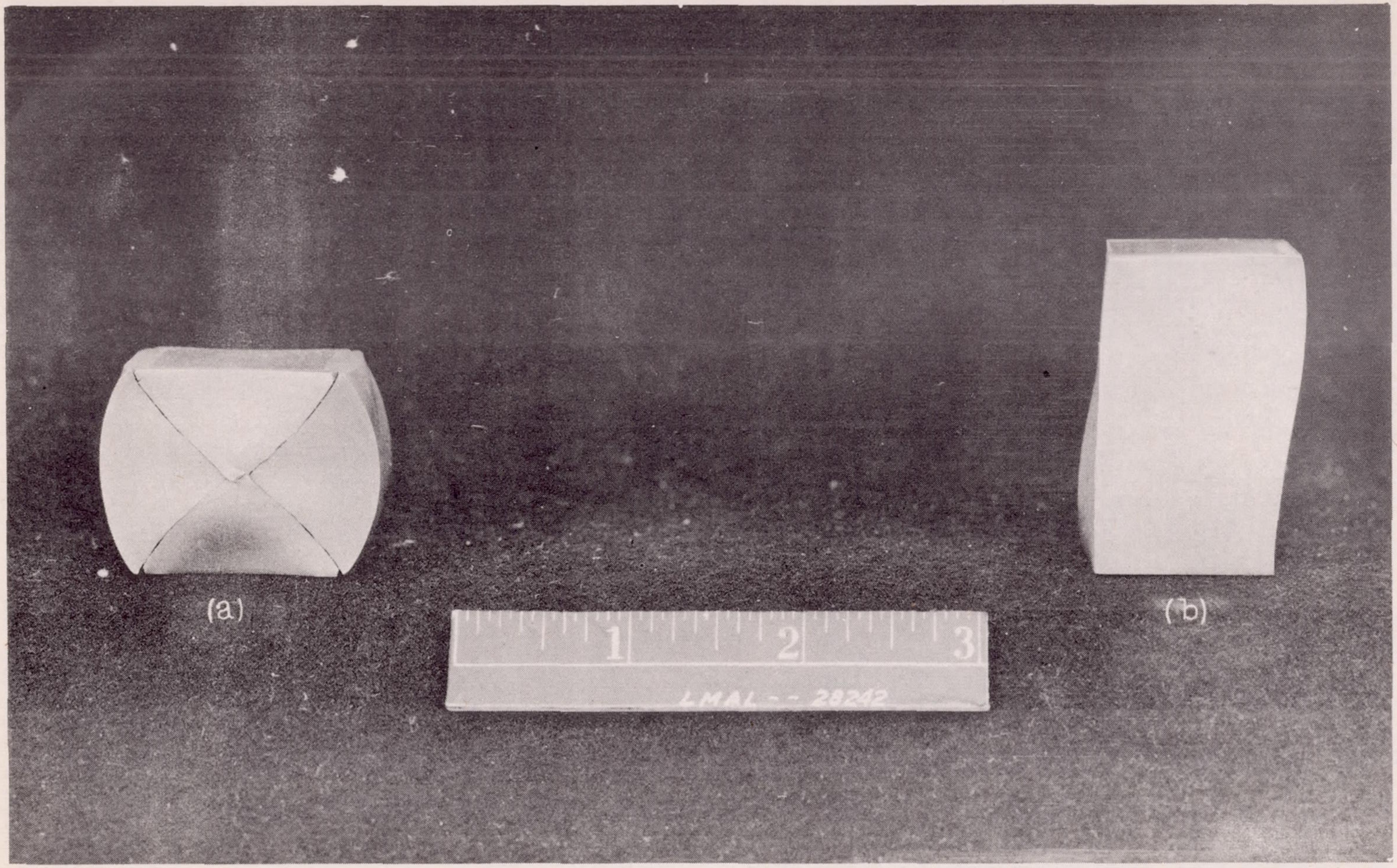
TABLE I

VALUES OF P_{cr} ESTABLISHED BY

THE EXPERIMENTAL DATA

L (in.)	α_0 (kips/in.) (a)	n ($in.^{-1}$) (a)	1-nL	$\alpha_0 L$ (kips)	$P_{cr}=\frac{\alpha_0 L}{1-nL}$ (kips)
0	---	---	1	0	0
5.750	21.96	0.06952	.6003	126.3	210
10.875	11.76	.04704	.4884	127.9	262
26.875	3.26	.02688	.2776	87.6	316
36.875	1.82	.02136	.2123	67.1	316
50.875	.92	.01676	.1473	46.8	318

[a]Values read from a large scale drawing of figure 5. The unestablished values for L = 0 are finite.



(a) Tested in 1,200,000-pound-capacity testing machine. Critical load not reached. Maximum load, 293,000 pounds.

(b) Tested in 300,000-pound-capacity testing machine. Critical load reached. Maximum load, 219,000 pounds.

Figure 1.- Comparison of results of flat-end compression tests of identical steel specimens made in two different machines.

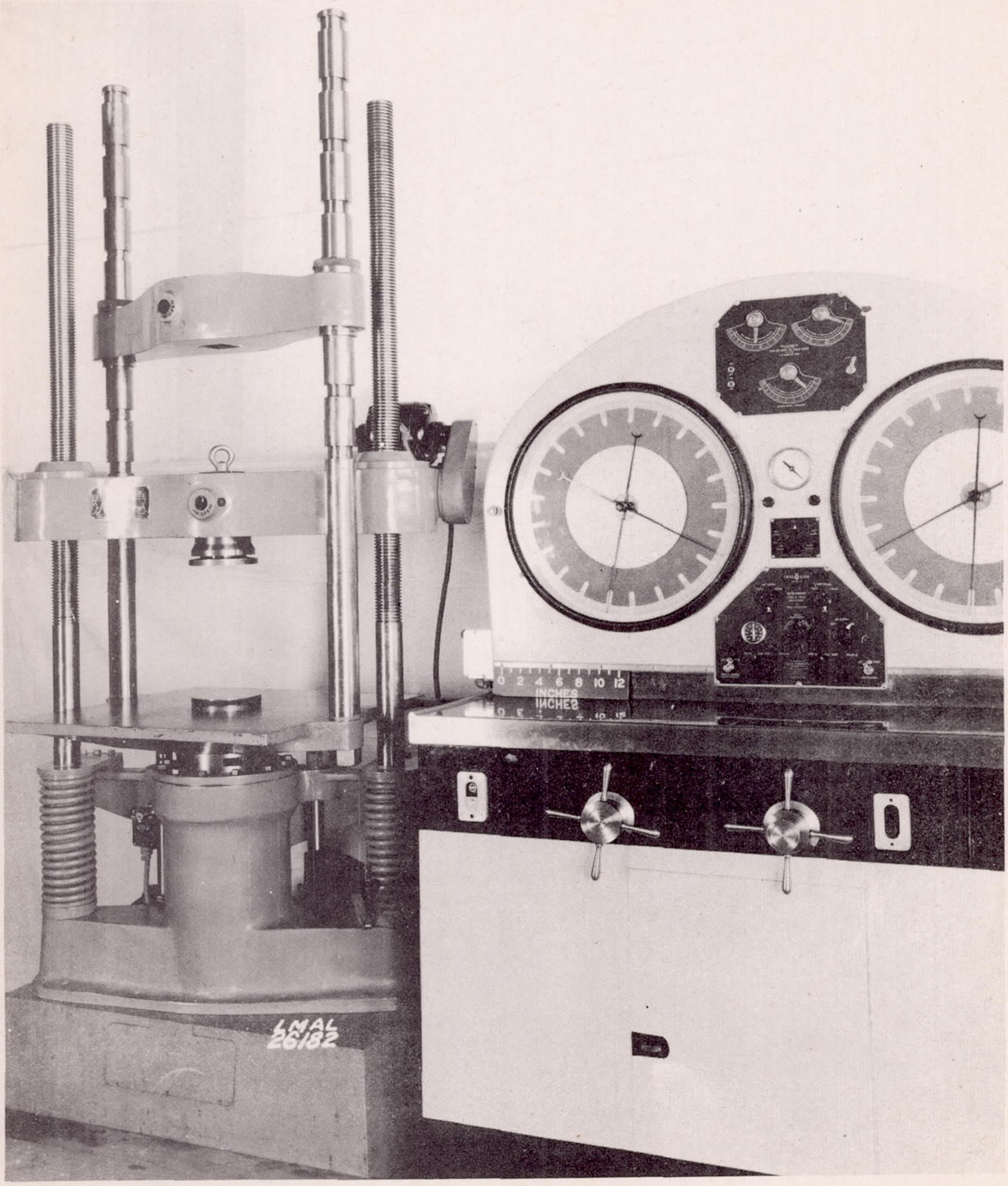


Figure 2.- The 100,000-pound-capacity universal testing machine in the NACA structures laboratory.

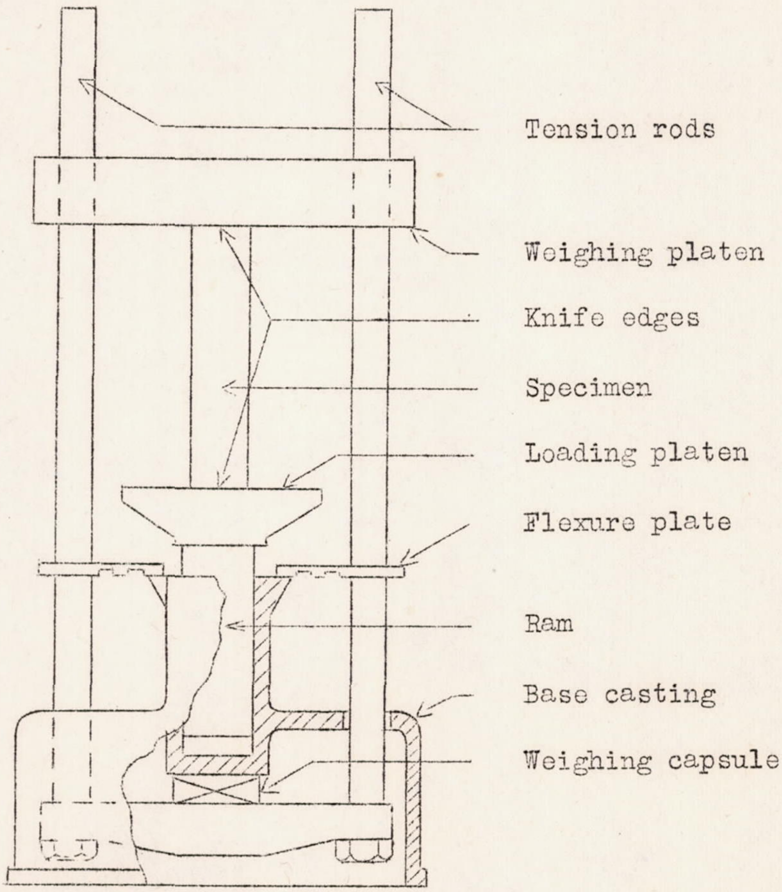


Figure 3.- The structural elements of a 100,000 - pound - capacity universal testing machine.

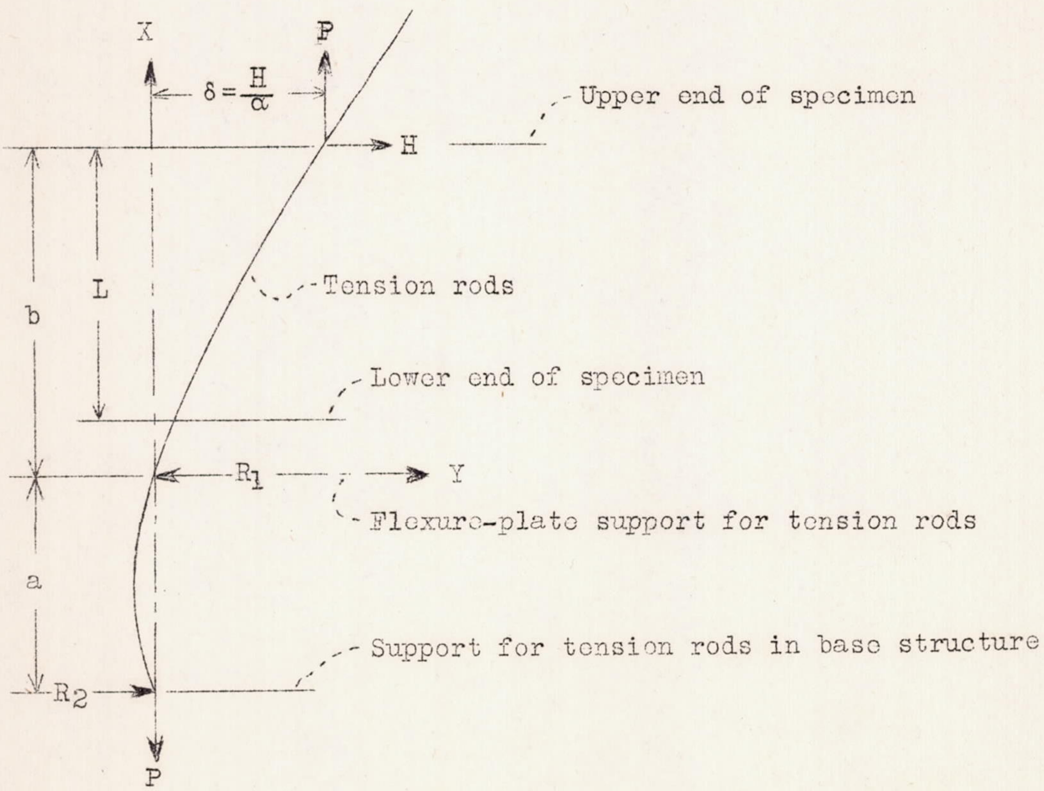


Figure 4.- Tension rods in the deflected position.

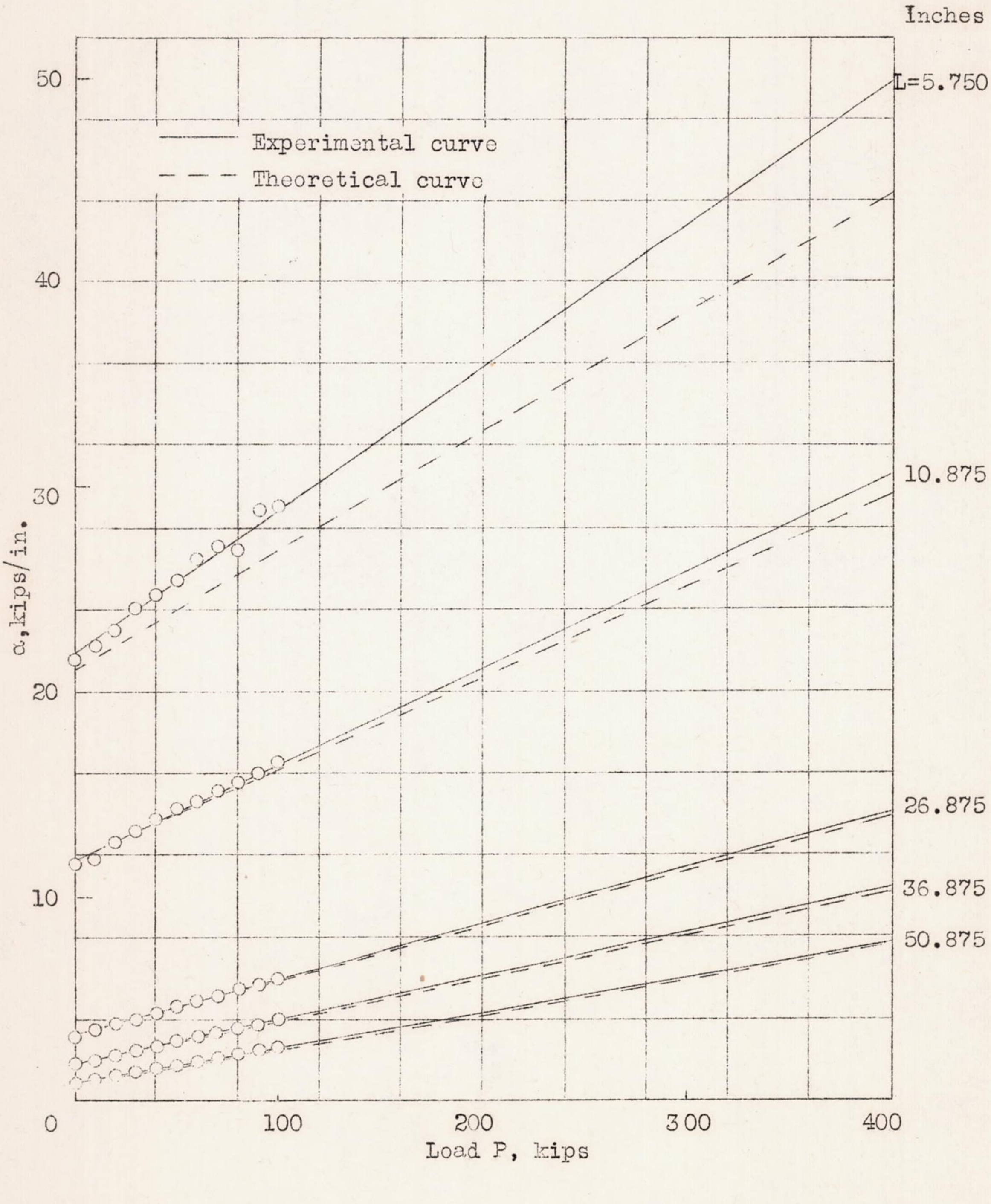


Figure 5.- Theoretical and experimental values of α.

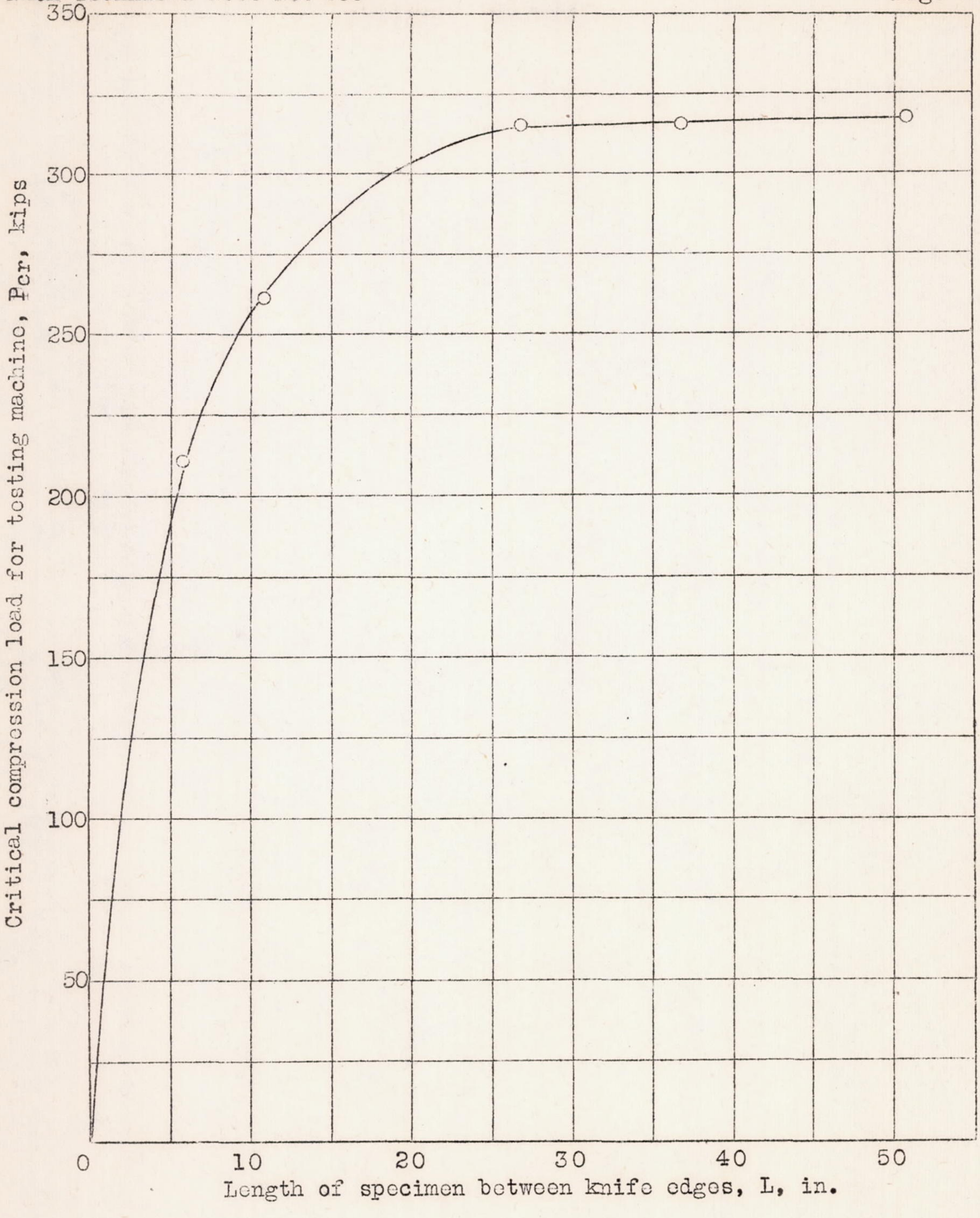


Figure 6.- Critical compression load for the testing machine plotted against length of specimen.

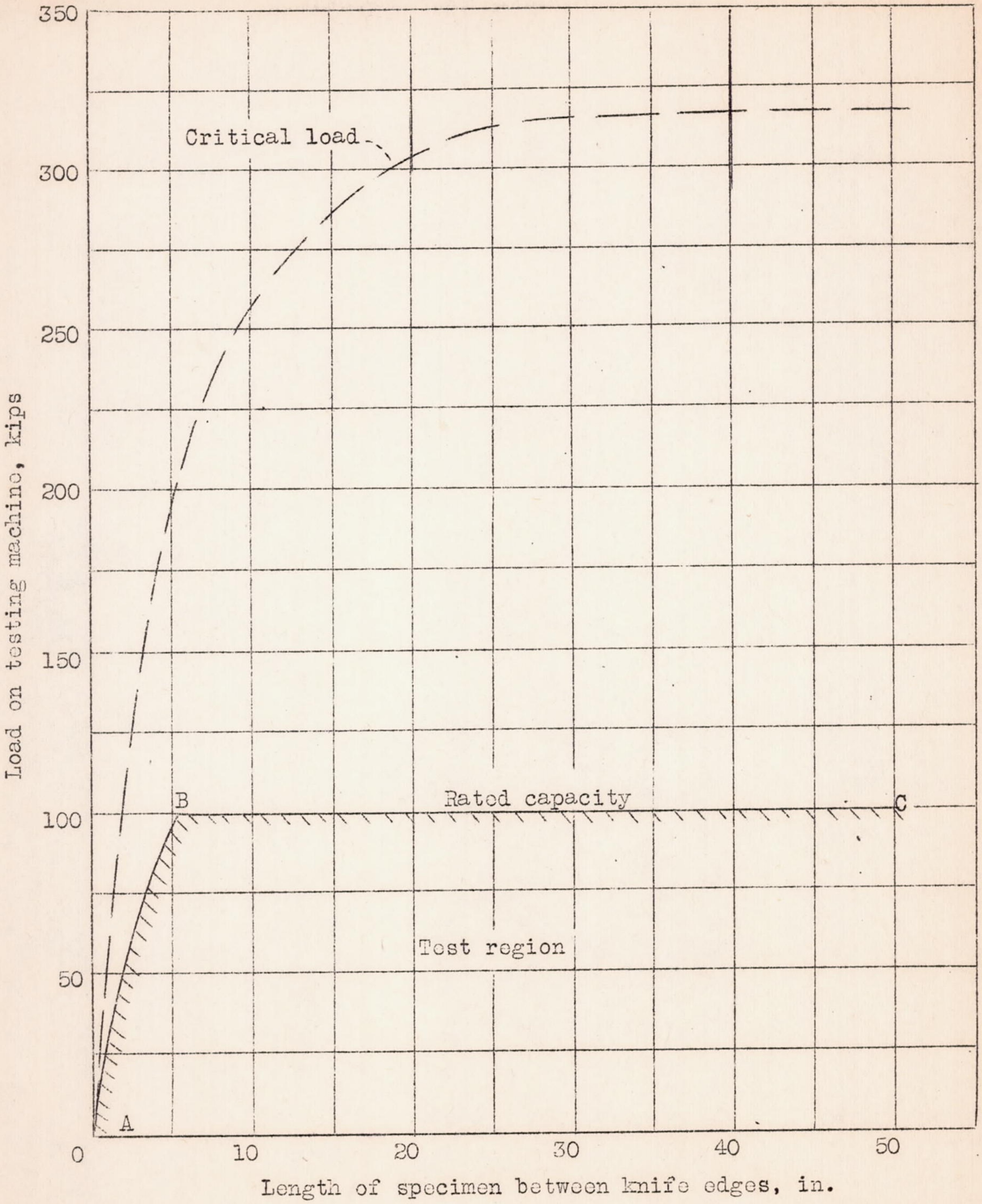


Figure 7.- Load-length diagram for the testing machine.