Structures Res. Div. Files

927

RESTRICTED

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 927

DETERMINATION OF STRESS-STRAIN RELATIONS FROM "OFFSET" YIELD STRENGTH VALUES

By H. N. Hill
Aluminum Company of America

Washington
February 1944

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 927

DETERMINATION OF STRESS-STRAIN RELATIONS FROM "OFFSET" YIELD STRENGTH VALUES

By H. N. Hill

The shape of the stress-strain curve for a material is sometimes of considerable interest to the designing engineer. This is particularly true when he is dealing with members or elements subjected to compression, which become unstable at stresses beyond the elastic range. It is obvious that the two compressive properties commonly recorded (modulus of elasticity and yield strength) are insufficient to define the shape of the stress-strain curve.

Ramberg and Osgood (reference 1) have used the following three-parameter equation for expressing the relation between stress and strain for stresses up to a value slightly greater than the yield strength of the material:

$$e = \frac{S}{E} + K\left(\frac{S}{E}\right)^n \qquad (1)$$

where

e unit strain

S unit stress

E Young's modulus

and

K and n constants for a given curve

Ramberg and Osgood have evaluated the constants K and n in terms of two secant yield strength values determined for slopes of 0.7E and 0.85E.

Since yield strength values determined by the offset method are much more commonly used than the secant yield strengths, it may be of interest to note that the constants K and n in equation (1) can be readily evaluated in terms of two offset yield strength values. Consider two yield strength values:

$$S_1 \text{ at an offset of } d_1$$

and

$$S_2 \text{ at an offset of } d_2$$

The equation for the deviation of the curve from the initial modulus line can be written

$$d = K\left(\frac{S}{E}\right)^n \tag{2}$$

from which

$$\log d = \log K + n \log\left(\frac{S}{E}\right) \tag{2a}$$

Substituting S_1 and d_1, and S_2 and d_2 into equation (2a) gives two simultaneous equations in K and n, which when solved for n yield the relation

$$n = \frac{\log\left(\dfrac{d_2}{d_1}\right)}{\log\left(\dfrac{S_2}{S_1}\right)} \tag{3}$$

From equation (2), K can be expressed

$$K = \frac{d_2}{\left(\dfrac{S_2}{E}\right)^n} \quad \text{or} \quad K = \frac{d_1}{\left(\dfrac{S_1}{E}\right)^n} \tag{4}$$

Substituting equation (4) into equation (1) gives for the equation of the stress-strain curve

$$e = \frac{S}{E} + d_2\left(\frac{S}{S_2}\right)^n \quad \text{or} \quad e = \frac{S}{E} + d_1\left(\frac{S}{S_1}\right)^n \tag{1a}$$

the value for n being given by equation (3).

A logical offset value to use for determining yield strength, in addition to the commonly used value of 0.002 (d_2), would be half this value or 0.001 (d_1). This offset value will locate a point on the stress-strain curve, between the elastic range and the conventional yield strength value, which is in the region of the curve which is of greatest significance in plastic buckling. Substituting values of 0.001 and 0.002 for d_1 and d_2, respectively, the stress-strain curve can be expressed

$$e = \frac{S}{E} + 0.002\left(\frac{S}{S_2}\right)^n \tag{1b}$$

in which

$$n = \frac{0.301}{\log\left(\frac{S_2}{S_1}\right)} \tag{3a}$$

RATIO OF STRESS TO EFFECTIVE MODULUS

In problems dealing with buckling at stresses beyond the elastic range of the material, the significant feature of the stress-strain curve is the relation between the stress and the slope of the curve at that stress. The slope of the curve is commonly called the tangent modulus (E_T).

Formulas that give critical buckling stresses for elastic action are applicable to buckling in the plastic range of the material, if the Young's modulus term (E) is replaced by the proper effective modulus term (E_F). Theoretically, the value for reduced or effective modulus (E_F) will always be greater than the corresponding value for tangent modulus (E_T). (See reference 2.) Experience has indicated (reference 3) that the use of the tangent modulus in the Euler equation for columns gives calculated values for critical stress that are in reasonable agreement with test results. It is therefore probably somewhat conservative and not illogical to assume, for design purposes, that the effective modulus can be represented by the corresponding value of tangent modulus. If this assumption is made, the comparative buckling strength of various aluminum alloys in the plastic range can be readily determined from the equations for their stress-strain curves.

In general, any buckling equation can be expressed in the form

$$\frac{S}{E_F} = CD \tag{5}$$

where

S critical stress

E_F effective modulus corresponding to the stress S

C coefficient depending on the type of member and on the nature of the loads and restraints acting on the member or element

and

D a function of the dimensions of the piece

In determining critical stresses in the plastic range it is therefore convenient to have a curve for the material, showing stress (S) plotted against the ratio of stress to effective modulus $\left(\frac{S}{E_F}\right)$, or, on the basis of the assumption previously stated, against the ratio of stress to tangent modulus $\left(\frac{S}{E_T}\right)$. The comparative buckling strength of different materials can be determined by a comparison of such curves.

The ratio of stress to tangent modulus can be expressed

$$\frac{S}{E_T} = \frac{S}{\frac{dS}{de}} = S\frac{de}{dS} \tag{6}$$

The term de/dS can be evaluated by differentiating equation (1b). Equation (6) then becomes

$$\frac{S}{E_T} = \frac{S}{E} + 0.002\, n \left(\frac{S}{S_2}\right)^n \tag{6a}$$

In order to demonstrate the effect of the shape of the stress-strain curve on the buckling strength in the plastic range, consider two alloys having the properties shown in the following table:

Alloy	Yield strength at 0.002 offset, S_2 (lb/sq in.)	Yield strength at 0.001 offset, S_1 (lb/sq in.)	$\frac{S_2}{S_1}$	n
A	50,000	43,000	1.163	4.59
B	50,000	48,000	1.042	16.8

Both alloys have a modulus of elasticity of 10,000,000 pounds per square inch. Figure 1 shows stress-strain curves and curves of S against S/E_T for these two alloys. A comparison of the curves indicates that for stresses below about 20,000 pounds per square inch the behavior of both materials is essentially elastic. Essentially elastic action continues in alloy B up to a stress of about 35,000 pounds per square inch. For stresses between 20,000 pounds per square inch and 45,000 pounds per square inch alloy B has lower values of S/E_T and consequently greater buckling strength than alloy A. Above 45,000 pounds per square inch, however, the buckling strength of alloy A is greater than that of alloy B.

It is evident that whereas a knowledge of the yield strength (stress at 0.002 offset) of a material is inadequate to define the shape of the stress-strain curve, the determination of an additional yield strength value corresponding to some other offset value, together with the Young's modulus of the material may provide sufficient additional information for evaluating the stress-strain relation and consequently for determining the buckling resistance of the material in the plastic range.

Aluminum Research Laboratories,
Aluminum Company of America,
New Kensington, Pa., October 14, 1943.

REFERENCES

1. Ramberg, Walter, and Osgood, William R.: Description of Stress-Strain Curves by Three Parameters. T.N. No. 902, NACA, 1943.

2. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., pp. 156, 384.

3. Templin, R. L., Sturm, R. G., Hartman, E. C., and Holt, M.: Column Strength of Various Aluminum Alloys. Tech. Paper No. 1, Aluminum Res. Lab., Aluminum Co. of Am., 1938.

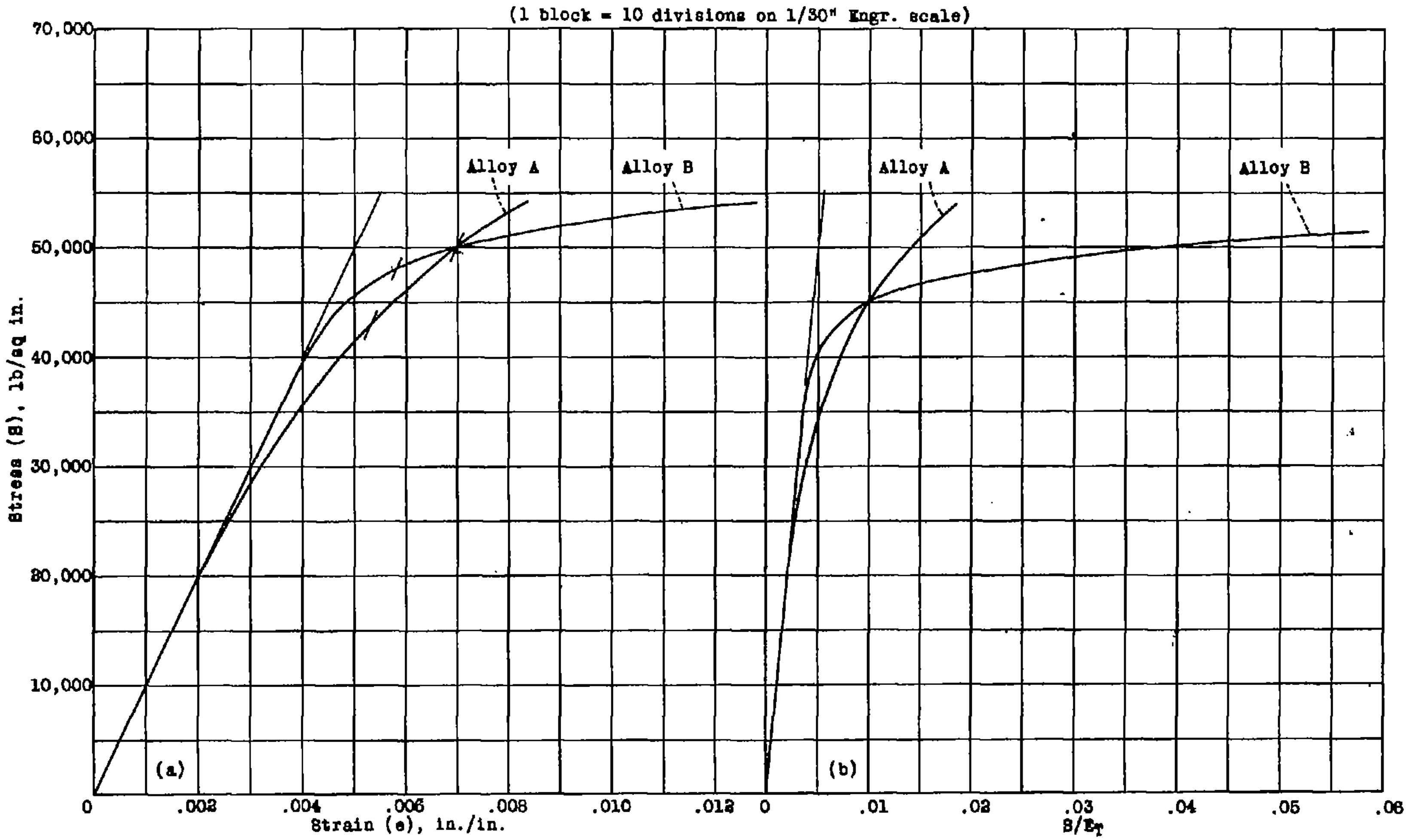

Figure 1.- Stress-strain and stress- S/E_T curves for two alloys having the same modulus and the same yield strength (0.002 offset), but with different stress-strain curves.