NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 1150

# A RELAXATION PROCEDURE FOR THE STRESS ANALYSIS OF A CONTINUOUS BEAM-COLUMN ELASTICALLY RESTRAINED AGAINST DEFLECTION AND ROTATION AT THE SUPPORTS

By Pai C. Hu and Charles Libove

## SUMMARY

A method of stress analysis is presented for a continuous beam-column supported by deflectional and rotational springs. The principal feature of the method is the use of a relaxation procedure to determine the deflections and rotations of the supports. The shears and moments at the supports and between the supports can then be calculated with the aid of simple equations and graphs. An example is presented to illustrate the use of the method.

## INTRODUCTION

The usual simplifying assumptions made in the stress analysis of a continuous beam-column are that the supports are unyielding and that they provide no resistance to rotation. In actual structures the supports are always somewhat flexible. A simple way of taking the flexible resistance of the supports into account is to assume that each support consists of two independent elastic springs - one, a deflectional spring and the other, a rotational spring. This idealization is shown in figure 1. The values of the spring stiffnesses depend upon the nature of the supports and must be calculated or estimated by the designer.

Even after this simplified representation of the supports has been made, the usual methods of analysis are quite inadequate except in special cases. The unknown yielding of the supports makes the usual type of moment distribution described in references 1 and 2 not readily applicable, and solution by the differential-equation method becomes exceedingly involved as the number of spans increases.

The method of the present paper is proposed as a practicable procedure for the stress analysis of an elastically supported continuous beam-column having supports that may be represented by deflectional and rotational springs. A relaxation process is used to determine the deflections and rotations of the supports and the deflection curve, shears, and bending moments are then calculated with the aid of simple equations and graphs. The method is applicable only when the stresses are below the proportional limit. In all other respects it is quite general. However, tables and graphs to facilitate the use of the method have been prepared only for the more important cases involving axial compression and spans with uniform flexural stiffness.

A simple problem is solved to illustrate the use of the method.

## SIGN CONVENTIONS

In the present paper the sign conventions are as follows: Deflection is positive downward. Rotation is positive clockwise. Lateral loads and external forces, except the resisting force of a deflectional spring, are positive downward. The resisting force of a deflectional spring is positive upward. Externally applied moments, except the resisting moment of a rotational spring, are positive clockwise. The resisting moment of a rotational spring is positive counterclockwise. The internal shears and moments acting upon the ends of a single span (spring supports excluded) are positive downward and clockwise, respectively. In the figures of the present paper deflections, rotations, forces, and moments are always shown in their positive directions.

## SYMBOLS

<u>General</u>.- The following symbols apply throughout the present paper:

P axial load (parallel to undeflected axis of beam-column)

L length of span

E Young's modulus of elasticity

I moment of inertia of cross-sectional area about neutral bending axis

$j = \sqrt{EI/P}$

| | |
|---|---|
| $L/j$ | nondimensional measure of axial load in span $\left(L\sqrt{P/EI}\right)$ |
| $C$ | deflectional stiffness of support, force per unit deflection |
| $K$ | rotational stiffness of support, moment per radian |
| $x$ | distance of point on span from left end of span |
| $y$ | deflection of point on loaded beam-column from its position before loading |
| $\Delta$ | deflection of point on span with respect to left end of span; positive downward |
| $S$ | shear |
| $M$ | moment |

Single span.- The following symbols apply to a single axially loaded span free at one end and clamped at the other end. (For examples, see figs. 5 and 6.) In general, a symbol $C$ stands for a force per unit deflection or rotation, and a symbol $K$ stands for a moment per unit deflection or rotation. The subscript $\delta$ indicates that an effect (force or moment) associated with deflection of the free end is being considered. The subscript $\theta$ indicates that an effect associated with rotation of the free end is being considered. The subscript $F$ indicates that the effect denoted by the symbol occurs at the free end, and the subscript $C$ indicates that the effect is at the clamped end. Those symbols with the subscript $L$ apply to a span extending to the left from the free end (for example, see fig. 5), and those with the sub-script $R$ apply to a span extending to the right from the free end. (For example, see fig. 6.)

| | |
|---|---|
| $\delta$ | deflection at free end |
| $\theta$ | rotation at free end |
| $C_{F\theta_{L(\mathrm{or}\ R)}}$, $K_{F\theta_{L(\mathrm{or}\ R)}}$ | force and moment required at free end to produce unit rotation $(\theta = 1)$ and zero deflection $(\delta = 0)$ at free end |

$C_{C\theta_{L(\text{or } R)}}$, $K_{C\theta_{L(\text{or } R)}}$ — force and moment produced at clamped end by unit rotation $(\theta = 1)$ and zero deflection $(\delta = 0)$ of free end

$C_{F\delta_{L(\text{or } R)}}$, $K_{F\delta_{L(\text{or } R)}}$ — force and moment required at free end to produce unit deflection $(\delta = 1)$ and zero rotation $(\theta = 0)$ at free end

$C_{C\delta_{L(\text{or } R)}}$, $K_{C\delta_{L(\text{or } R)}}$ — force and moment produced at clamped end by unit deflection $(\delta = 1)$ and zero rotation $(\theta = 0)$ of free end

Two-span beam-column.- The following symbols apply to a two-span continuous beam-column with the ends clamped and the center joint supported by a deflectional and a rotational spring. (For example, see fig. 4.)

$\delta$ — deflection of center joint

$\theta$ — rotation of center joint

$C_\theta$, $K_\theta$ — external force and moment required at center joint to produce unit rotation $(\theta = 1)$ and zero deflection $(\delta = 0)$ of joint $\left(C_\theta = C_{F\theta_L} + C_{F\theta_R}\right.$ and $\left.K_\theta = K + K_{F\theta_L} + K_{F\theta_R}\right)$

$C_\delta$, $K_\delta$ — external force and moment required at center joint to produce unit deflection $(\delta = 1)$ and zero rotation $(\theta = 0)$ of joint $\left(C_\delta = C + C_{F\delta_L} + C_{F\delta_R}\right.$ and $\left.K_\delta = K_{F\delta_L} + K_{F\delta_R}\right)$

Other symbols are defined throughout the text where they are first used.

## CALCULATION OF DEFLECTIONS AND ROTATIONS OF SUPPORTS BY THE PROCEDURE OF FORCE AND MOMENT DISTRIBUTION

When an elastically supported continuous beam-column, such as that shown diagrammatically in figure 1, is gradually loaded all the supports deflect and rotate until a condition of stable equilibrium is reached. To solve for these deflections and rotations directly by means of the beam-flexure differential equation may be feasible for a beam-column supported at only two or three points. A direct solution for a beam-column supported at many points, however, usually involves a great, if not a prohibitive, amount of algebraic and numerical work. In order to effect a solution for this case, a relaxation procedure somewhat similar to moment distribution may be used.

At the outset all the joints are assumed to be "locked" or "frozen" against deflection and rotation, and the loads are applied. The fixed-end shears and moments produced at the span ends are then calculated. The locking force required at any joint is equal to the algebraic sum of the fixed-end shears at the joint; similarly, the locking moment is equal to the algebraic sum of the fixed-end moments at the joint. (Figs. 2 and 3 in the present paper and graphs III to X of reference 1 or figs. 14:14 to 14:21 of reference 2 can be used to calculate the fixed-end moments produced by several common types of loading in a span having uniform EI and constant axial compression. The fixed-end shears for any span can be calculated by applying the equations of statics to the span after the fixed-end moments have been determined.)

Then, at any joint a force $F$ (hereinafter called the "balancing force") equal but of opposite sign to the locking force and a moment $M$ (hereinafter called the "balancing moment") equal but of opposite sign to the locking moment are applied. The effect of applying these quantities $F$ and $M$ is to "unlock" or to release the joint. As a result of this unlocking the joint undergoes a vertical deflection $\delta$ and a rotation $\theta$ (see fig. 4) which are given by the following formulas identical to equations (A17) and (A18) in appendix A:

$$\delta = -\frac{C_\theta}{D} M + \frac{K_\theta}{D} F \qquad (1)$$

$$\theta = \frac{C_\delta}{D} M - \frac{K_\delta}{D} F \qquad (2)$$

The symbol $K_\theta$ stands for the total rotational stiffness of the joint, as used in ordinary moment distribution. The quantities $C_\theta$, $C_\delta$, and $K_\delta$ are other kinds of stiffness which have special significance for a joint with two degrees of freedom. These four stiffnesses are defined physically in the symbols and are given by the equations

$$K_\theta = K + K_{F\theta_L} + K_{F\theta_R} \tag{3}$$

$$C_\theta = C_{F\theta_L} + C_{F\theta_R} \tag{4}$$

$$C_\delta = C + C_{F\delta_L} + C_{F\delta_R} \tag{5}$$

$$K_\delta = K_{F\delta_L} + K_{F\delta_R} = C_\theta \tag{6}$$

where $C$ is the deflectional stiffness of the support at the joint in units of force per unit deflection and $K$ is the rotational stiffness of the support at the joint in units of moment per radian. The quantities $K_{F\theta_L}$, $K_{F\theta_R}$, $C_{F\theta_L}$, $C_{F\theta_R}$, $C_{F\delta_L}$, $C_{F\delta_R}$, $K_{F\delta_L}$, and $K_{F\delta_R}$ are component stiffnesses contributed to the joint by the two members entering the joint. They are defined in the symbols and can be readily evaluated with the aid of tables 1 and 2 for the special case of a span having uniform $EI$ and constant axial compression. The symbol $D$ represents a particular combination of the stiffnesses $K_\theta$, $C_\theta$, and $C_\delta$ and is defined by the equation

$$D = K_\theta C_\delta - C_\theta^2 \tag{7}$$

For the special case in which the rotational spring constant $K$ is infinite, no rotation of the joint occurs, and the deflection is given by the equation

$$\delta = \frac{F}{C_\delta} \tag{8}$$

For the case in which the deflectional spring constant $C$ is infinite, no deflection of the joint occurs, and the rotation is given by the equation

$$\theta = \frac{M}{K_\theta} \tag{9}$$

After deflection and rotation, the unlocked joint is in equilibrium under the balancing force $F$ and the balancing moment $M$. The joint is therefore balanced and is again locked, this time in its new equilibrium position.

In the course of the balancing, shears and moments are induced at the clamped ends of the two spans entering the joint. These shears and moments are shown on the free-body diagrams in figures 5 and 6. They may be calculated from the formulas

$$S_{C_L} = C_{C\theta_L}\theta + C_{C\delta_L}\delta \tag{10}$$

$$S_{C_R} = C_{C\theta_R}\theta + C_{C\delta_R}\delta \tag{11}$$

$$M_{C_L} = K_{C\theta_L}\theta + K_{C\delta_L}\delta \tag{12}$$

$$M_{C_R} = K_{C\theta_R}\theta + K_{C\delta_R}\delta \tag{13}$$

where

$S_{C_L}$ shear induced at clamped end of left-hand span (fig. 5)

$M_{C_L}$ moment induced at clamped end of left-hand span (fig. 5)

$S_{C_R}$ shear induced at clamped end of right-hand span (fig. 6)

$M_{C_R}$ moment induced at clamped end of right-hand span (fig. 6)

$\delta$ deflection of balanced joint; from equation (1) or (8)

$\theta$ rotation of the balanced joint; from equation (2) or (9)

The quantities $C_{C\theta_L}$, $C_{C\delta_L}$, $C_{C\theta_R}$, and $C_{C\delta_R}$ and $K_{C\theta_L}$, $K_{C\delta_L}$, $K_{C\theta_R}$, and $K_{C\delta_R}$ which appear in equations (10) to (13) can be calculated with the aid of tables 1 and 2 for the special case of a span having uniform $EI$ and constant axial compression. The shears and moments given by equations (10) to (13) represent additional locking forces and moments required at the neighboring joints to keep them in a locked condition and must be considered in computing the balancing forces and moments for the neighboring joints.

The balancing procedure just described, including the evaluation of the additional locking forces and moments at the neighboring joints, is successively repeated at all the joints except those which are locked in the actual structure. If the structure is stable under the given axial loading, a stage will be reached at which any additional locking forces and moments will be small enough to be neglected for the degree of accuracy desired. At this point the balancings may stop, and the final deflection and rotation of any support may be obtained by summing the deflections and rotations produced by all the individual balancings of that support or joint.

The essential operations will now be restated briefly as follows:

(1) The calculation of the deflection $\delta$ and rotation $\theta$ by use of equations (1) or (8) and equations (2) or (9) when a given joint is balanced

(2) The calculation of the shears and moments induced at the neighboring joints by use of equations (10) to (13).

The execution of step (1) requires that the balancing force F and the balancing moment M first be calculated. In calculating F and M for a joint that has not previously been balanced, consideration must be given to the fixed-end shears and moments at the joint as well as to the shears and moments that were induced by the balancing of neighboring joints. In calculating F and M for a joint that has previously been balanced, only those shears and moments that were induced at the joint since its last balancing need be considered.

The derivation of equations (1) to (13) is given in appendix A. The equations used to evaluate the various coefficients for tables 1 and 2 are also given in appendix A.

## DETERMINATION OF SHEARS AND MOMENTS AT THE SUPPORTS

After the deflections and rotations of the supports have been determined, the state of the beam-column is uniquely defined, and the shears and moments at the joints can be obtained from simple slope-deflection equations. Each equation expresses the final shear or moment at the end of any span as the sum of the following five parts:

(1) The fixed-end shear $S_F$ or fixed-end moment $M_F$

(2) Shear or moment produced by the deflection without rotation of the left end of the span

(3) Shear or moment produced by the rotation without deflection of the left end of the span

(4) Shear or moment produced by the deflection without rotation of the right end of the span

(5) Shear or moment produced by the rotation without deflection of the right end of the span

For a typical span jk, the final shear $S_{jk}$ and final moment $M_{jk}$ at the left end j can be written as

$$S_{jk} = (S_F)_{jk} + C_{F\delta_R}\delta_j + C_{F\theta_R}\theta_j + C_{C\delta_L}\delta_k + C_{C\theta_L}\theta_k \tag{14}$$

$$M_{jk} = (M_F)_{jk} + K_{F\delta_R}\delta_j + K_{F\theta_R}\theta_j + K_{C\delta_L}\delta_k + K_{C\theta_L}\theta_k \tag{15}$$

The shear $S_{kj}$ and the moment $M_{kj}$ at the right end k of the typical span jk are given by the equations

$$S_{kj} = (S_F)_{kj} + C_{C\delta_R}\delta_j + C_{C\theta_R}\theta_j + C_{F\delta_L}\delta_k + C_{F\theta_L}\theta_k \tag{16}$$

$$M_{kj} = (M_F)_{kj} + K_{C\delta_R}\delta_j + K_{C\theta_R}\theta_j + K_{F\delta_L}\delta_k + K_{F\theta_L}\theta_k \tag{17}$$

The coefficients of the $\delta$'s and $\theta$'s in equations (14) to (17) are readily calculable with the aid of tables 1 and 2.

An over-all check on the $\delta$'s and $\theta$'s and the span-end shears and moments can be made by applying the equations of static equilibrium to the joints as free bodies. If some of the joints are found to be not in equilibrium, a mistake in either the relaxation procedure or in the use of the slope-deflection equations (14) to (17) is indicated. The correct use of the slope-deflection equations can be checked by an investigation of the static equilibrium of the spans as free bodies.

DEFLECTIONS AND BENDING MOMENTS BETWEEN SUPPORTS

In order to obtain the bending moments at points between supports, the deflection curve of the beam-column must first be determined. The deflection curve for any span, say jk, can be obtained by superimposing the five following individual deflection curves:

(1) Deflection curve produced by lateral loading when both ends of the span are clamped

(2) Deflection curve produced by total deflection $\delta_j$ of left end when left end is restrained against rotation and right end is clamped

(3) Deflection curve produced by total rotation $\theta_j$ of left end when left end is restrained against deflection and right end is clamped

(4) Deflection curve produced by total deflection $\delta_k$ of right end when right end is restrained against rotation and left end is clamped

(5) Deflection curve produced by total rotation $\theta_k$ of right end when right end is restrained against deflection and left end is clamped

These five deflection curves can be obtained by use of figures 7 to 10 for spans having uniform EI and constant axial compression. The figures provided for the calculation of deflection curve (1) for both ends clamped consider only two types of lateral loading: a uniform load along the entire span (fig. 7) and a single concentrated load at the successive tenth points (figs. 8(a) to 8(e)). Most lateral loadings can be approximated by suitable combinations of these concentrated loads. The equations of the deflection curves, if desired, are given in appendix B.

After the deflection curve for a span has been obtained, the bending moment at any point on the span can be calculated. For a span in which the axial compression is constant, the bending moment at a point a distance x from the left end of the span is given by the expression

$$P\Delta + M_{jk} - S_{jk}x + M_{L.L.} \tag{18}$$

where

| | |
|---|---|
| $P$ | axial compression (parallel to undeflected axis of beam-column) |
| $\Delta$ | deflection of point with respect to left end of span (positive downward) |
| $M_{L.L.}$ | total moment about point under consideration of any lateral loads between point and left end of span (moment tending to cause compression on top fiber is considered positive) |

and $M_{jk}$ is obtained from equation (15) and $S_{jk}$ is obtained from equation (14).

The sign of the bending moment, as determined from expression (18), is consistent with the convention that bending moment tending to cause compression on the top fiber is positive.

## SHEAR DIAGRAM

The shear diagram for any span can be drawn in the conventional manner, the shear at any point being simply the algebraic sum of the left-end shear and all the vertical loads between the left end and the point under consideration. It should be noticed, however, that because of the presence of axial load the usual relationship between the shear and moment, namely $S = \frac{dM}{dx}$ with no distinction made between the vertical shear and the shear normal to the elastic curve, needs to be modified. This relationship is still correct if $S$ represents the shear normal to the elastic curve of the beam-column. The quantity $S$, as used in the present paper, however, represents the vertical shear, and the appropriate equation for it is

$$S = \frac{dM}{dx} - P\,\frac{dy}{dx} \qquad (19)$$

where $S$ is positive if it acts upward on the right-hand part of the span, $\frac{dM}{dx}$ is the slope of the bending-moment curve at the point, and $\frac{dy}{dx}$ is the slope of the deflection curve at the point. Equation (19) can be used to check the mutual consistency of the deflection curve and the bending-moment diagram.

## ILLUSTRATIVE EXAMPLE

Figure 11 shows a continuous beam-column free at the left end A and built in at the right end D with two intermediate supports B and C. The support at B is an unyielding hinge, and the support at C is a deflectional spring having a stiffness of 686.7 pounds per inch of deflection. Neither support includes a rotational spring. The cross-sectional moment of inertia I of the beam-column is 0.2 inch$^4$, and the modulus of elasticity E is 29,000,000 psi. Span AB has no axial load. Spans BC and CD are axially loaded with a compressive force of 8156.25 pounds, which was chosen to give these two spans a value of $L/j$ equal to 3. The lateral loading and dimensions are shown in figure 11.

In applying the method of force and moment distribution, the cantilever span AB could be thought of as an ordinary span supported at the left end by springs with zero stiffness. Labor will be saved however by regarding span AB as simply a loading device to provide a constant lateral force and moment just to the left of support B.

Deflections and rotations of the supports.- The tabular scheme for recording the force and moment distribution computations is shown in table 3. Each support is represented in the tabulation by a vertical line. Above each vertical line are written the necessary balancing equations for the deflection $\delta$ and rotation $\theta$ of the joint and the equations for shears and moments induced at the adjacent joints: equation (1) or (8) for $\delta$, equation (2) or (9) for $\theta$, and equations (10) to (13) for the induced shears and moments. Across the first horizontal line of the tabulation are written the fixed-end shears and fixed-end moments. One fixed-end moment is written on each side of the vertical line representing the joint. The fixed-end shears are written alongside the fixed-end moments but farther from the vertical line. The rest of the table is used to record the computations involved in the joint balancings. Each balancing of a joint requires the calculation of the following three sets of quantities which are recorded on separate horizontal lines of the table:

(1) The balancing force F and balancing moment M at joint

(2) The deflection $\delta$ and rotation $\theta$ of joint

(3) The shears and moments induced at neighboring joints

The induced shears are recorded in the same vertical lines as the fixed-end shears; similarly, the induced moments are recorded in the same columns as the fixed-end moments. After the calculated values of $\delta$ and $\theta$ have been recorded (item (2)), a short horizontal line is drawn under them. Item (3) is omitted in the final balancing when the induced shears and moments are assumed to be negligible.

In table 3, for purposes of illustration, a complete set of balancing, induced-shear, and induced-moment equations is written above joint C even though three of them $S_{C_L}$, $S_{C_R}$, and $M_{C_R}$ are not required in this particular problem. The equation for $\delta$ is omitted above joint B since the support at B is unyielding. Since span AB is considered to be merely a loading device, the quantities $S_{C_L}$ and $M_{C_L}$ do not exist for joint B and therefore their equations are omitted. The arithmetic involved in setting up the equations is given in detail in appendix C.

The fixed-end moments shown in table 3 were taken from figure 12 of reference 1 in which a moment-distribution analysis was made of the same beam-column but on unyielding supports. The fixed-end shears were calculated by applying the equations of statics to the spans.

Joint C is balanced first. The locking force (the algebraic sum of the fixed-end shears) is -173.06 - 119.04 or -292.10 pounds. The balancing force F is recorded as the negative of the locking force, or 292.10 pounds. Similarly, the locking moment is the algebraic sum of the fixed-end moments, or 2229.2 inch-pounds, and the balancing moment M is the negative of the locking moment, or -2229.2 inch-pounds. From the formulas for $\delta$ and $\theta$ given for joint C in table 3, the movement of the joint for the first balancing is calculated and recorded as $\delta = 0.410847$ inch and $\theta = -5.85845 \times 10^{-3}$ radian. From the formula for $M_{C_L}$ above joint C the moment induced at joint B is calculated as -2899.15 inch-pounds. Because the formula for the rotation of joint B involves only the balancing moment M, the shear $S_{C_L}$ induced at joint B is not calculated; and since joint D will never be permitted to deflect or rotate, neither the shear $S_{C_R}$ nor the moment $M_{C_R}$ induced at joint D is calculated.

Joint B is balanced next. The locking moment (the algebraic sum of the fixed-end moments and the moment induced during the balancing of joint C) is 5000 - 6989.8 - 2899.15 or -4888.95 inch-pounds. The balancing moment M is therefore 4888.95. From the formula for $\theta$ given above joint B in table 3, the rotation of the joint is calculated as $\theta = 25.6968 \times 10^{-3}$ radian. From the formulas for $S_{C_R}$ and $M_{C_R}$ given for joint B the shear and moment induced at joint C during the balancing of joint B are calculated as -117.269 pounds and 4492.57 inch-pounds, respectively, and are recorded in their appropriate places near the vertical line representing joint C.

At this point one cycle of the procedure of force and moment distribution has been completed. Another cycle is begun by balancing joint C again. The locking force at joint C (the algebraic sum of the shears induced there since the last balancing of the joint) is -117.269 pounds. Similarly, the locking moment is 4492.57 inch-pounds. The balancing force F and the balancing moment M are then equal to 117.269 pounds and -4492.57 inch-pounds, respectively. By use of the formulas for $\delta$ and $\theta$ above joint C the additional movement caused by this second balancing of the joint is calculated as $\delta = 0.164942$ inch and $\theta = 11.8067 \times 10^{-3}$ radian. The moment induced at joint B is calculated as -2816.89 inch-pounds.

The successive balancings of joints C and B are continued until the shears and moments induced by the balancings are small enough to be neglected. At this point the process stops, and the final deflections and rotations are obtained by summing the deflections and rotations produced by the individual balancings. Four cycles of balancing are shown in table 3. The results obtained after 10 and 20 cycles are also given and compared with the exact results. The error after 10 cycles is seen to be only about 0.6 percent.

For this particular problem it was possible to calculate the exact results by means of the formula for the sum of an infinite geometric series, since a point was reached early in the process where the value $\delta$ or $\theta$ corresponding to any balancing was simply a constant factor (2816.89/4888.95) times the value of $\delta$ or $\theta$ for the preceding balancing. (In a more general problem, this fortuitous circumstance would not arise.) The calculation of the exact results is given at the end of appendix C.

The illustrative example just explained was adapted from a problem solved in reference 1. In reference 1 a moment-distribution analysis was made of the same structure, with support C assumed to be 0.8 inch below supports B and D. In order to make a problem that was suitable to the method of the present paper, the support at C was replaced by a deflectional spring. Furthermore, in order to provide a check on the calculations, the spring stiffness was so chosen as to give 0.8-inch deflection at joint C and was obtained by dividing the reaction at C, as calculated from the data given in the solution of reference 1, page 27, by the desired deflection of 0.8 inch. As shown in table 3, the exact value obtained for the deflection at C is 0.8 inch, which was the answer to be expected if the computations were correct.

Shears and moments at span ends.- The preceding computations have yielded the deflections and rotations necessary for calculating the shears and moments at the ends of spans BC and CD from equations (14) to (17). These shears and moments are found to be:

$$S_{BC} = -213.82 \text{ pounds}$$

$$M_{BC} = -5000.0 \text{ inch-pounds}$$

$$S_{CB} = -286.18 \text{ pounds}$$

$$M_{CB} = 5369.1 \text{ inch-pounds}$$

$$S_{CD} = -263.20 \text{ pounds}$$

$$M_{CD} = -5369.1 \text{ inch-pounds}$$

$$S_{DC} = -136.80 \text{ pounds}$$

$$M_{DC} = 1505.2 \text{ inch-pounds}$$

The final shear and moment at the right end of the cantilever AB are, from static considerations,

$$S_{BA} = -100 \text{ pounds}$$

$$M_{BA} = 5000 \text{ inch-pounds}$$

Bending-moment diagram for span BC.- The span BC is chosen for purposes of illustration. The deflection curve for the span is first obtained in the manner explained in the section of the present paper entitled "Deflections and Bending Moments between Supports." The computations for this deflection curve are given in table 4 and the deflection curve obtained is shown in figure 12. The bending-moment diagram for the span is then obtained by use of expression (18). The computations involved are summarized in table 5. The bending-moment diagram is shown in figure 13.

## DISCUSSION

### Scope of Method

The method of force and moment distribution is quite general in its applicability. In order to apply it to cases other than those considered in the present paper - such as axial tension, nonuniform EI, or nonuniform axial load within a span - it is only necessary to prepare tables and figures for these cases. The special case in which there is a sudden change in EI or axial load within a span can be handled by assuming the beam-column to be supported by springs of zero stiffness at the point of discontinuity. The span can then be regarded as two spans, each of which has uniform EI and constant axial load.

Thus far, application of the method has been restricted to beam-columns, the supports of which are such that the restraining force and restraining moment of a support are directly proportional to the deflection and rotation, respectively; that is,

$$\text{Restraining force} = C\delta$$

$$\text{Restraining moment} = K\theta$$

It can be shown, however, that the method is also applicable to a more general, mathematically possible case in which the restraining force and moment are linear functions of both the deflection and rotation; that is

$$\left.\begin{aligned} \text{Restraining force} &= C_1\delta + C_2\theta \\ \text{Restraining moment} &= K_1\delta + K_2\theta \end{aligned}\right\} \tag{20}$$

In order for the method to be applicable to this case, it is only necessary to revise equations (3) to (7) to read

$$K_\theta = K_2 + K_{F\theta_L} + K_{F\theta_R} \tag{21}$$

$$C_\theta = C_2 + C_{F\theta_L} + C_{F\theta_R} \tag{22}$$

$$C_\delta = C_1 + C_{F\delta_L} + C_{F\delta_R} \tag{23}$$

$$K_\delta = K_1 + K_{F\delta_L} + K_{F\delta_R} \tag{24}$$

$$D = K_\theta C_\delta - K_\delta C_\theta \tag{25}$$

The support consisting of two independent elastic springs is a special case of the more general type of support and occurs when $C_2$ and $K_1$ are both equal to zero.

The method of force and moment distribution is inapplicable if the stresses are above the proportional limit. In making a force and moment distribution analysis, the stresses are assumed to be below the proportional limit. The results obtained are correct only if the final stresses bear out this assumption.

## Procedure of Force and Moment Distribution

There is no definite order in which the joints of the beam-column must be balanced. In order to facilitate checking, however, and to minimize the possibilities for error, a definite order of balancing should be maintained. Not all the joints need be balanced the same number of times; small induced effects may be allowed to accumulate at a joint until the locking force and locking moment there are appreciable.

The problem of slow convergence or nonconvergence will sometimes arise. Nonconvergence is an indication that the structure is unstable under the axial loading. Slow convergence, accompanied by large induced shears and moments, is an indication that the structure is close to instability. When the convergence is slow, an alternate method of solution, based upon the slope-deflection equations (14) to (17) may be adopted. The internal shears and moments as given by equations (14) to (17) are combined with the external forces and moments and spring forces and moments at the joints in writing two equations of static equilibrium for each joint. The system of static-equilibrium equations is then solved simultaneously for the deflections and rotations at the joints.

## Principle of Superposition

The modified principle of superposition is of basic importance to the method of force and moment distribution, and in the explanation presented it was assumed to hold for a continuous beam-column supported by elastic deflectional and rotational springs. The fact that this principle does apply can be proved in a manner similar to that used in reference 3 for a single-span beam-column with hinged ends.

The modified principle of superposition allows the original lateral loading to be resolved into any number of component loadings, each of which is applied separately in conjunction with the axial loads. The final deflections, rotations, shears, and bending moments in the beam-column may then be obtained by adding algebraically the effects of all the component loadings.

This principle is employed throughout the procedure of force and moment distribution. The locking of the joints at the start of the process is equivalent to applying to the beam-column a component loading which consists of the original lateral loads plus the locking forces and moments at the joints. The fixed-end shears and moments are calculated. This component loading is then removed, and the balancing force and moment applied at any joint together with the required locking forces and moments at the neighboring joints represent the second component loading. The effects of the second component loading - the deflection and rotation of the joint being balanced - are calculated. When this second component loading is added algebraically to the first, the original loading condition at the balanced joint is restored since the locking force and moment are canceled. Similarly, each balancing represents the restoration of the original lateral-loading conditions at a joint and the application of the required locking forces and moments at the neighboring joints. If the process is stopped when the locking forces and moments are negligibly small and all component lateral loadings are added algebraically, the resultant lateral loading will consist of the original lateral loads plus negligibly small locking forces and moments at some of the joints. By the principle of superposition, then, algebraic addition of the deflections and rotations produced by the individual balancings (each of which represents the application of a component loading) will yield the deflections and rotations produced by the original lateral loads plus the negligible locking forces and moments.

The principle of superposition is also important for stress analysis. It justifies the determination of the final deflection curve of any span by means of the algebraic addition of five individual deflection curves, as is done in the section entitled

"Deflections and Bending Moments between Supports." It was also employed in writing the slope-deflection equations (14) to (17).

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va. June 25, 1946

# APPENDIX A

## DERIVATION OF THE BALANCING, INDUCED-SHEAR, AND INDUCED-MOMENT EQUATIONS

Equations will be derived only for spans of uniform EI and constant axial compression.

Derivation of the equations for induced shear and moment.- In figure 4 a balancing force F and a balancing moment M are shown producing a deflection $\delta$ and a rotation $\theta$ at the center support of a two-span continuous beam-column clamped at the ends. The left-hand span and right-hand span of the beam-column are shown in figures 5 and 6, respectively.

By use of the ordinary beam-flexure differential equation, the equilibrium of internal and external moments about any section in the left-hand span (fig. 5) may be expressed mathematically as

$$EI\,\frac{d^2y}{dx^2} = -M_{C_L} - Py + S_{C_L}x \tag{A1}$$

The boundary conditions at $x = 0$ are $y = 0$ and $\frac{dy}{dx} = 0$ and at $x = L$ are $y = \delta$ and $\frac{dy}{dx} = \theta$.

By solving the differential equation and satisfying the boundary conditions and the equations of statics for the span as a whole, the following expressions are obtained for the end shears and moments in the left-hand span:

$$S_{F_L} = C_{F\theta_L}\theta + C_{F\delta_L}\delta \tag{A2}$$

$$M_{F_L} = K_{F\theta_L}\theta + K_{F\delta_L}\delta \tag{A3}$$

$$S_{C_L} = C_{C\theta_L}\theta + C_{C\delta_L}\delta \tag{A4}$$

$$M_{C_L} = K_{C\theta_L}\theta + K_{C\delta_L}\delta \tag{A5}$$

where

$$
\left.
\begin{aligned}
K_{F\theta_L}\left(\frac{L}{EI}\right) &= \frac{-\frac{L}{j}\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{F\delta_L}\left(\frac{L^2}{EI}\right) &= \frac{\left(\frac{L}{j}\right)^2\left(1 - \cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
C_{F\theta_L}\left(\frac{L^2}{EI}\right) &= K_{F\delta_L}\left(\frac{L^2}{EI}\right) \\
C_{F\delta_L}\left(\frac{L^3}{EI}\right) &= \frac{-\left(\frac{L}{j}\right)^3\sin\frac{L}{j}}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{C\theta_L}\left(\frac{L}{EI}\right) &= -K_{F\delta_L}\left(\frac{L^2}{EI}\right) - K_{F\theta_L}\left(\frac{L}{EI}\right) \\
K_{C\delta_L}\left(\frac{L^2}{EI}\right) &= K_{F\delta_L}\left(\frac{L^2}{EI}\right) \\
C_{C\theta_L}\left(\frac{L^2}{EI}\right) &= -K_{F\delta_L}\left(\frac{L^2}{EI}\right) \\
C_{C\delta_L}\left(\frac{L^3}{EI}\right) &= -C_{F\delta_L}\left(\frac{L^3}{EI}\right)
\end{aligned}
\right\} \qquad \text{(A6)}
$$

and

$$j = \sqrt{EI/P}$$

For the right-hand span (fig. 6) the differential equation of equilibrium may be written as

$$EI \frac{d^2y}{dx^2} = -M_{F_R} - P\left(y - \delta\right) + S_{F_R}x \tag{A7}$$

and the boundary conditions to be satisfied at $x = 0$ are $y = \delta$ and $\frac{dy}{dx} = \theta$ and at $x = L$ are $y = 0$ and $\frac{dy}{dx} = 0$.

By solving the differential equation and satisfying the boundary conditions and the equations of statics for the span as a whole, the following expressions for the end shears and moments in the right-hand span are obtained:

$$S_{F_R} = C_{F\theta_R}\theta + C_{F\delta_R}\delta \tag{A8}$$

$$M_{F_R} = K_{F\theta_R}\theta + K_{F\delta_R}\delta \tag{A9}$$

$$S_{C_R} = C_{C\theta_R}\theta + C_{C\delta_R}\delta \tag{A10}$$

$$M_{C_R} = K_{C\theta_R}\theta + K_{C\delta_R}\delta \tag{A11}$$

where

$$\left.\begin{aligned}
K_{F\theta_R}\left(\frac{L}{EI}\right) &= \frac{-\frac{L}{j}\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{F\delta_R}\left(\frac{L^2}{EI}\right) &= \frac{-\left(\frac{L}{j}\right)^2\left(1 - \cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
C_{F\theta_R}\left(\frac{L^2}{EI}\right) &= K_{F\delta_R}\left(\frac{L^2}{EI}\right) \\
C_{F\delta_R}\left(\frac{L^3}{EI}\right) &= \frac{-\left(\frac{L}{j}\right)^3\sin\frac{L}{j}}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{C\theta_R}\left(\frac{L}{EI}\right) &= K_{F\delta_R}\left(\frac{L^2}{EI}\right) - K_{F\theta_R}\left(\frac{L}{EI}\right) \\
K_{C\delta_R}\left(\frac{L^2}{EI}\right) &= K_{F\delta_R}\left(\frac{L^2}{EI}\right) \\
C_{C\theta_R}\left(\frac{L^2}{EI}\right) &= -K_{F\delta_R}\left(\frac{L^2}{EI}\right) \\
C_{C\delta_R}\left(\frac{L^3}{EI}\right) &= -C_{F\delta_R}\left(\frac{L^3}{EI}\right)
\end{aligned}\right\} \qquad (A12)$$

and

$$j = \sqrt{EI/P}$$

Equations (A4), (A5), (A10), and (A11) are identical to equations (10), (12), (11), and (13), respectively. The terms in equations (A6) and (A12) are defined physically in the symbols.

<u>Derivation of balancing equations</u>.- The static equilibrium of the center joint of the beam-column in figure 4 requires that

$$F = C\delta + S_{F_L} + S_{F_R} \tag{A13}$$

and

$$M = K\theta + M_{F_L} + M_{F_R} \tag{A14}$$

Substitution of $S_{F_L}$ from equation (A2), $S_{F_R}$ from equation (A8), $M_{F_L}$ from equation (A3), and $M_{F_R}$ from equation (A9) in equations (A13) and (A14) results in

$$F = C_\theta \theta + C_\delta \delta \tag{A15}$$

$$M = K_\theta \theta + K_\delta \delta \tag{A16}$$

where

$$C_\theta = C_{F\theta_L} + C_{F\theta_R}$$

$$C_\delta = C + C_{F\delta_L} + C_{F\delta_R}$$

$$K_\theta = K + K_{F\theta_L} + K_{F\theta_R}$$

$$K_\delta = K_{F\delta_L} + K_{F\delta_R} = C_\theta$$

Equations (A15) and (A16) can be solved for $\delta$ and $\theta$ with the result that

$$\delta = -\frac{C_\theta}{D} M + \frac{K_\theta}{D} F \tag{A17}$$

$$\theta = \frac{C_\delta}{D} M - \frac{K_\delta}{D} F \tag{A18}$$

where

$$D = K_\theta C_\delta - C_\theta{}^2$$

For the special case in which the spring constant $K$ is infinite, $\theta = 0$, and, from equation (A15),

$$\delta = \frac{F}{C_\delta} \tag{A19}$$

If $C$ is infinite, $\delta = 0$, and, from equation (A16),

$$\theta = \frac{M}{K_\theta} \tag{A20}$$

Equations (A17), (A18), (A19), and (A20) are identical to equations (1), (2), (8), and (9), respectively.

## APPENDIX B

## DEFLECTION EQUATIONS

Graphs for calculating the five types of deflection discussed in the section entitled "Deflections and Bending Moments between Supports" are presented in figures 7 to 10 for a span having uniform EI and constant axial compression and the corresponding deflection equations will now be given. In these equations $x$ represents the distance of a point on the elastic curve from the left end of the span and $y$ represents the downward deflection of the point. Detailed derivations are omitted. The equations are obtained by the solution of the ordinary beam-flexure differential equation $M = -EI \dfrac{d^2y}{dx^2}$ and the satisfaction of simple boundary conditions.

Deflections due to lateral loading.- The following two types of loading will be considered:

(1) Uniform load: The deflections $y$ produced by a total uniformly distributed load $W$ (fig. 7) are defined by the equation

$$\frac{y}{\dfrac{WL^3}{EI}} = \frac{\sin\dfrac{L}{j}\dfrac{x}{L} + \sin\left[\dfrac{L}{j}\left(1 - \dfrac{x}{L}\right)\right] - \sin\dfrac{L}{j}}{2\left(\dfrac{L}{j}\right)^3\left(1 - \cos\dfrac{L}{j}\right)} - \frac{1}{2\left(\dfrac{L}{j}\right)^2}\frac{x}{L}\left(1 - \frac{x}{L}\right) \qquad \text{(B1)}$$

(2) Concentrated load: The deflections $y$ produced by a lateral load $Q$ acting a distance $c$ from the right end (figs. 8(a) to 8(e)) are given by the equations

$$\left.\begin{aligned}
\frac{y}{\frac{QL^3}{EI}} &= \frac{1}{\left(\frac{L}{j}\right)^2}\left\{\left[\frac{\sin\frac{L}{j}\frac{c}{L}\sin\frac{L}{j}\frac{x}{L}}{\frac{L}{j}\sin\frac{L}{j}} - \frac{c}{L}\frac{x}{L}\right.\right. \\
&\quad + H_1\left(\frac{\sin\frac{L}{j}\frac{x}{L}}{\sin\frac{L}{j}} - \frac{x}{L}\right) \\
&\quad \left.\left. + H_2\left[\frac{\sin\frac{L}{j}\left(1 - \frac{x}{L}\right)}{\sin\frac{L}{j}} - 1 + \frac{x}{L}\right]\right]\right\} \\
&\text{for } x \leqq L - c \text{ and} \\
\frac{y}{\frac{QL^3}{EI}} &= \frac{1}{\left(\frac{L}{j}\right)^2}\left\{\left[\frac{\sin\frac{L}{j}\left(1 - \frac{c}{L}\right)\sin\frac{L}{j}\left(1 - \frac{x}{L}\right)}{\frac{L}{j}\sin\frac{L}{j}} - \left(1 - \frac{c}{L}\right)\left(1 - \frac{x}{L}\right)\right.\right. \\
&\quad + H_1\left(\frac{\sin\frac{L}{j}\frac{x}{L}}{\sin\frac{L}{j}} - \frac{x}{L}\right) \\
&\quad \left.\left. + H_2\left[\frac{\sin\frac{L}{j}\left(1 - \frac{x}{L}\right)}{\sin\frac{L}{j}} - 1 + \frac{x}{L}\right]\right]\right\}
\end{aligned}\right\} \quad \text{(B2)}$$

for $x \geqq L - c$ where

$$H_1 = \frac{\left[\left(1 - \frac{c}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{c}{L}\right)\right]\left[\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right] - \left[\frac{c}{L}\sin\frac{L}{j} - \sin\frac{L}{j}\frac{c}{L}\right]\left[\frac{L}{j} - \sin\frac{L}{j}\right]}{\frac{L}{j}\left(1 - \cos\frac{L}{j}\right)\left(2\sin\frac{L}{j} - \frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}$$

$$H_2 = \frac{\left(\frac{c}{L}\sin\frac{L}{j} - \sin\frac{L}{j}\frac{c}{L}\right)\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right) - \left[\left(1 - \frac{c}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{c}{L}\right)\right]\left(\frac{L}{j} - \sin\frac{L}{j}\right)}{\frac{L}{j}\left(1 - \cos\frac{L}{j}\right)\left(2\sin\frac{L}{j} - \frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}$$



<u>Deflections due to displacement at left end</u>.- The deflections $y$ due to a displacement at the left end $\delta_L$ (fig. 9) are defined by the equation

$$\frac{y}{\delta_L} = \frac{\sin\frac{L}{j}\frac{x}{L}\sin\frac{L}{j} - \frac{x}{L}\frac{L}{j}\sin\frac{L}{j}\cos\frac{L}{j} + \left(\cos\frac{L}{j} - 1\right)\left[\cos\frac{L}{j}\left(1 - \frac{x}{L}\right) - \cos\frac{L}{j}\right]}{\sin^2\frac{L}{j} - \frac{L}{j}\sin\frac{L}{j}\cos\frac{L}{j} - \left(1 - \cos\frac{L}{j}\right)^2} \qquad \text{(B3)}$$

<u>Deflections due to rotation at left end</u>.- The deflections $y$ due to a rotation at the left end $\theta_L$ (fig. 10) are defined by the equation

$$\frac{y}{\theta_L L} = \frac{\left[\left(1 - \frac{x}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{x}{L}\right)\right]\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right) - \left(\frac{x}{L}\sin\frac{L}{j} - \sin\frac{L}{j}\frac{x}{L}\right)\left(\frac{L}{j} - \sin\frac{L}{j}\right)}{\left[-2\frac{L}{j}\left(1 - \cos\frac{L}{j}\right) + \left(\frac{L}{j}\right)^2\sin\frac{L}{j}\right]\sin\frac{L}{j}} \tag{B4}$$

Deflections due to displacement at right end.- The deflections $y$ due to a displacement at the right end $\delta_R$ (fig. 9) are defined by the equation

$$\frac{y}{\delta_R} = \frac{\sin\frac{L}{j}\left(1 - \frac{x}{L}\right)\sin\frac{L}{j} - \left(1 - \frac{x}{L}\right)\frac{L}{j}\sin\frac{L}{j}\cos\frac{L}{j} + \left(\cos\frac{L}{j} - 1\right)\left(\cos\frac{L}{j}\frac{x}{L} - \cos\frac{L}{j}\right)}{\sin^2\frac{L}{j} - \frac{L}{j}\sin\frac{L}{j}\cdot\cos\frac{L}{j} - \left(1 - \cos\frac{L}{j}\right)^2} \tag{B5}$$

Deflections due to rotation at right end.- The deflections $y$ due to a rotation at the right end $\theta_R$ (fig. 10) are defined by the equation

$$-\frac{y}{\theta_R L} = \frac{\left(\frac{x}{L}\sin\frac{L}{j} - \sin\frac{x}{L}\frac{L}{j}\right)\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right) - \left[\left(1 - \frac{x}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{x}{L}\right)\right]\left(\frac{L}{j} - \sin\frac{L}{j}\right)}{\left[-2\frac{L}{j}\left(1 - \cos\frac{L}{j}\right) + \left(\frac{L}{j}\right)^2\sin\frac{L}{j}\right]\sin\frac{L}{j}} \tag{B6}$$

# APPENDIX C

## PRELIMINARY COMPUTATIONS FOR ILLUSTRATIVE EXAMPLE

Computations at joint B.- Setting up the balancing equation (9) for joint B requires the preliminary evaluation of $K_\theta$ from equation (3), which in turn requires the calculation of $K_{F\theta_R}$. Setting up equations (11) and (13) for induced effects necessitates the evaluation of only $C_{C\theta_R}$ and $K_{C\theta_R}$ since $\delta$ at joint B equals zero. These quantities can be obtained by entering table 2 with the value of $\frac{L}{j}$ for span BC, namely $\frac{L}{j} = 3$, and substituting numerical values of $\frac{EI}{L}$, $\frac{EI}{L^2}$, and $\frac{EI}{L^3}$. These numerical values are

$$\left.\begin{aligned} \frac{EI}{L} &= \frac{29,000,000 \times 0.2}{80} = 72,500 \\ \frac{EI}{L^2} &= 906.25 \\ \frac{EI}{L^3} &= 11.328 \end{aligned}\right\} \tag{C1}$$

In this manner the following data are obtained:

$$K_{F\theta_R} = 2.62420 \frac{EI}{L} = 190,255 \tag{C2}$$

$$C_{C\theta_R} = -5.03565 \frac{EI}{L^2} = -4563.56 \tag{C3}$$

$$K_{C\theta_R} = 2.41145 \frac{EI}{L} = 174,830 \tag{C4}$$

From equation (3)

$$K_\theta = 0 + 0 + 190,255 = 190,255 \tag{C5}$$

The balancing equation (9) can now be written

$$\theta = \frac{M}{190,255} = 5.2561 \times 10^{-6} M \tag{C6}$$

Equation (11) for induced shear and equation (13) for induced moment can be written as

$$S_{C_R} = -4563.56\theta + 0 = -4563.56\theta \tag{C7}$$

$$M_{C_R} = 174,830\theta + 0 = 174,830\theta \tag{C8}$$

<u>Computations at joint C</u>.- The computations at joint C are similar to, but more extensive than, those at joint B because joint C can deflect as well as rotate and has two spans effective in resisting its motion instead of one.

The following data are obtained for joint C by entering table 1 with the value of $\frac{L}{j} = 3$ for span CB:

$$\left.\begin{aligned}
K_{F\theta_L} &= 2.62420\,\frac{EI}{L} = 190{,}255 \\
K_{F\delta_L} &= -5.03565\,\frac{EI}{L^2} = -4563.56 \\
C_{F\theta_L} &= -5.03565\,\frac{EI}{L^2} = -4563.56 \\
C_{F\delta_L} &= 1.07131\,\frac{EI}{L^3} = 12.1358 \\
K_{C\theta_L} &= 2.41145\,\frac{EI}{L} = 174{,}830 \\
K_{C\delta_L} &= -5.03565\,\frac{EI}{L^2} = -4563.56 \\
C_{C\theta_L} &= 5.03565\,\frac{EI}{L^2} = 4563.56 \\
C_{C\delta_L} &= -1.07131\,\frac{EI}{L^3} = -12.1358
\end{aligned}\right\} \qquad \text{(C9)}$$

The following information is obtained from table 2 by entering it with $\frac{L}{j} = 3$ for span CD:

$$
\left.
\begin{aligned}
K_{F\theta_R} &= 2.62420 \frac{EI}{L} = 190,255 \\
K_{F\delta_R} &= 5.03565 \frac{EI}{L^2} = 4563.56 \\
C_{F\theta_R} &= 5.03565 \frac{EI}{L^2} = 4563.56 \\
C_{F\delta_R} &= 1.07131 \frac{EI}{L^3} = 12.1358 \\
K_{C\theta_R} &= 2.41145 \frac{EI}{L} = 174,830 \\
K_{C\delta_R} &= 5.03565 \frac{EI}{L^2} = 4563.56 \\
C_{C\theta_R} &= -5.03565 \frac{EI}{L^2} = -4563.56 \\
C_{C\delta_R} &= -1.07131 \frac{EI}{L^3} = -12.1358
\end{aligned}
\right\} \qquad (C10)
$$

Expressions (3), (4), (5), and (7) can now be evaluated as

$$K_\theta = 0 + 190{,}255 + 190{,}255 = 380{,}510 \qquad \text{(C11)}$$

$$C_\theta = -4563.56 + 4563.56 = 0 \qquad \text{(C12)}$$

$$C_\delta = 686.7 + 12.1358 + 12.1358 = 710.972 \qquad \text{(C13)}$$

$$D = 380{,}510 \times 710.972 - 0 = 270{,}532{,}000 \qquad \text{(C14)}$$

The balancing equations (1) and (2) can now be set up as follows:

$$\delta = 0 + \frac{380{,}510}{270{,}532{,}000} F = 0.00140653\ F \qquad \text{(C15)}$$

$$\theta = \frac{710.972}{270{,}532{,}000} M - 0 = 2.62805 \times 10^{-6}\ M \qquad \text{(C16)}$$

Equations (10) to (13) for induced effects at neighboring joints can be written

$$S_{C_L} = 4563.56\theta - 12.1358\delta \qquad \text{(C17)}$$

$$S_{C_R} = -4563.56\theta - 12.1358\delta \qquad \text{(C18)}$$

$$M_{C_L} = 174{,}830\theta - 4563.56\delta \qquad \text{(C19)}$$

$$M_{C_R} = 174{,}830\theta + 4563.56\delta \qquad \text{(C20)}$$

<u>Calculation of exact results shown in table 3</u>.- From table 3 it is clear that all the $\delta$'s and $\theta$'s that were calculated after the second balancing of joint C are simply 2816.89/4888.95 times the corresponding preceding entries. The formula for the sum of an infinite geometric series being $S = \frac{a}{1 - r}$, where S is the sum, a the first term of the series, and r the ratio of any term to the preceding term, the total rotation of joint B can be calculated as

$$\theta_B = \frac{25.6968 \times 10^{-3}}{1 - \frac{2816.89}{4888.95}} = 60.6307 \times 10^{-3} \tag{C21}$$

Similarly, the deflection of joint C is

$$\delta_C = 0.410847 + \frac{0.164942}{1 - \frac{2816.89}{4888.95}} = 0.800022 \text{ inch} \tag{C22}$$

and the rotation of joint C is

$$\theta_C = -5.85845 \times 10^{-3} - \frac{11.8067 \times 10^{-3}}{1 - \frac{2816.89}{4888.95}}$$

$$= -33.7160 \times 10^{-3} \tag{C23}$$

## REFERENCES

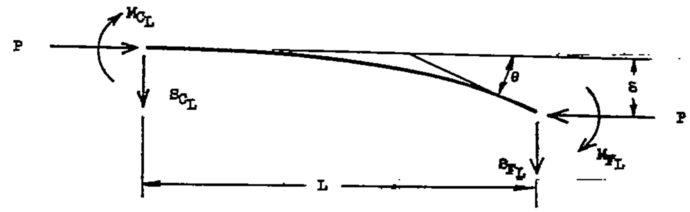
1. James, Benjamin Wylie: Principal Effects of Axial Load on Moment-Distribution Analysis of Rigid Structures. NACA TN No. 534, 1935.

2. Niles, Alfred S., and Newell, Joseph S.: Airplane Structures. Vol. II. Third ed., John Wiley & Sons, Inc., 1943, pp. 62-155.

3. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., 1936, pp. 6-8.

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED

$$\left[\begin{array}{ll} S_{C_L} = C_{C\theta_L}\theta + C_{C\delta_L}\delta & S_{F_L} = C_{F\theta_L}\theta + C_{F\delta_L}\delta \\ M_{C_L} = K_{C\theta_L}\theta + K_{C\delta_L}\delta & M_{F_L} = K_{F\theta_L}\theta + K_{F\delta_L}\delta \end{array}\right]$$




| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| 0 | 4.00000 | -6.00000 | 12.0000 | 2.00000 | -6.00000 | 6.00000 | -12.0000 |
| .1 | 3.99865 | -5.99901 | 11.9880 | 2.00036 | -5.99901 | 5.99901 | -11.9880 |
| .2 | 3.99465 | -5.99597 | 11.9519 | 2.00132 | -5.99597 | 5.99597 | -11.9519 |
| .3 | 3.98798 | -5.99099 | 11.8920 | 2.00300 | -5.99099 | 5.99099 | -11.8920 |
| .4 | 3.97862 | -5.98398 | 11.8080 | 2.00536 | -5.98398 | 5.98398 | -11.8080 |
| .5 | 3.96656 | -5.97495 | 11.6999 | 2.00840 | -5.97495 | 5.97495 | -11.6999 |
| .6 | 3.95177 | -5.96391 | 11.5678 | 2.01214 | -5.96391 | 5.96391 | -11.5678 |
| .7 | 3.93424 | -5.95083 | 11.4117 | 2.01658 | -5.95083 | 5.95083 | -11.4117 |
| .8 | 3.91394 | -5.93571 | 11.2314 | 2.02176 | -5.93571 | 5.93571 | -11.2314 |
| .9 | 3.89083 | -5.91853 | 11.0271 | 2.02769 | -5.91853 | 5.91853 | -11.0271 |
| 1.0 | 3.86488 | -5.89928 | 10.7986 | 2.03440 | -5.89928 | 5.89928 | -10.7986 |
| 1.01 | 3.86213 | -5.89724 | 10.7744 | 2.03511 | -5.89724 | 5.89724 | -10.7744 |
| 1.02 | 3.85935 | -5.89518 | 10.7500 | 2.03583 | -5.89518 | 5.89518 | -10.7500 |
| 1.03 | 3.85654 | -5.89310 | 10.7253 | 2.03656 | -5.89310 | 5.89310 | -10.7253 |
| 1.04 | 3.85370 | -5.89099 | 10.7004 | 2.03730 | -5.89099 | 5.89099 | -10.7004 |
| 1.05 | 3.85083 | -5.88887 | 10.6752 | 2.03804 | -5.88887 | 5.88887 | -10.6752 |
| 1.06 | 3.84793 | -5.88673 | 10.6499 | 2.03880 | -5.88673 | 5.88673 | -10.6499 |
| 1.07 | 3.84500 | -5.88456 | 10.6242 | 2.03956 | -5.88456 | 5.88456 | -10.6242 |
| 1.08 | 3.84204 | -5.88238 | 10.5984 | 2.04033 | -5.88238 | 5.88238 | -10.5984 |
| 1.09 | 3.83906 | -5.88017 | 10.5722 | 2.04111 | -5.88017 | 5.88017 | -10.5722 |
| 1.10 | 3.83604 | -5.87794 | 10.5459 | 2.04190 | -5.87794 | 5.87794 | -10.5459 |
| 1.11 | 3.83300 | -5.87569 | 10.5193 | 2.04269 | -5.87569 | 5.87569 | -10.5193 |
| 1.12 | 3.82992 | -5.87342 | 10.4924 | 2.04350 | -5.87342 | 5.87342 | -10.4924 |
| 1.13 | 3.82682 | -5.87113 | 10.4654 | 2.04431 | -5.87113 | 5.87113 | -10.4654 |
| 1.14 | 3.82369 | -5.86882 | 10.4380 | 2.04513 | -5.86882 | 5.86882 | -10.4380 |
| 1.15 | 3.82052 | -5.86648 | 10.4105 | 2.04596 | -5.86648 | 5.86648 | -10.4105 |
| 1.16 | 3.81733 | -5.86413 | 10.3827 | 2.04679 | -5.86413 | 5.86413 | -10.3827 |
| 1.17 | 3.81411 | -5.86175 | 10.3546 | 2.04764 | -5.86175 | 5.86175 | -10.3546 |
| 1.18 | 3.81086 | -5.85935 | 10.3263 | 2.04850 | -5.85935 | 5.85935 | -10.3263 |
| 1.19 | 3.80758 | -5.85693 | 10.2978 | 2.04936 | -5.85693 | 5.85693 | -10.2978 |
| 1.20 | 3.80426 | -5.85449 | 10.2690 | 2.05023 | -5.85449 | 5.85449 | -10.2690 |
| 1.21 | 3.80092 | -5.85203 | 10.2400 | 2.05111 | -5.85203 | 5.85203 | -10.2400 |
| 1.22 | 3.79755 | -5.84955 | 10.2107 | 2.05200 | -5.84955 | 5.84955 | -10.2107 |
| 1.23 | 3.79415 | -5.84705 | 10.1812 | 2.05290 | -5.84705 | 5.84705 | -10.1812 |
| 1.24 | 3.79072 | -5.84452 | 10.1514 | 2.05380 | -5.84452 | 5.84452 | -10.1514 |
| 1.25 | 3.78726 | -5.84197 | 10.1215 | 2.05472 | -5.84197 | 5.84197 | -10.1215 |
| 1.26 | 3.78377 | -5.83941 | 10.0912 | 2.05564 | -5.83941 | 5.83941 | -10.0912 |
| 1.27 | 3.78024 | -5.83682 | 10.0607 | 2.05658 | -5.83682 | 5.83682 | -10.0607 |
| 1.28 | 3.77669 | -5.83421 | 10.0300 | 2.05752 | -5.83421 | 5.83421 | -10.0300 |
| 1.29 | 3.77311 | -5.83157 | 9.99905 | 2.05847 | -5.83157 | 5.83157 | -9.99905 |
| 1.30 | 3.76949 | -5.82892 | 9.96784 | 2.05943 | -5.82892 | 5.82892 | -9.96784 |
| 1.31 | 3.76585 | -5.82625 | 9.93639 | 2.06040 | -5.82625 | 5.82625 | -9.93639 |
| 1.32 | 3.76217 | -5.82355 | 9.90470 | 2.06137 | -5.82355 | 5.82355 | -9.90470 |
| 1.33 | 3.75847 | -5.82083 | 9.87276 | 2.06236 | -5.82083 | 5.82083 | -9.87276 |
| 1.34 | 3.75473 | -5.81809 | 9.84058 | 2.06336 | -5.81809 | 5.81809 | -9.84058 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| 1.35 | 3.75097 | -5.81533 | 9.80816 | 2.06436 | -5.81533 | 5.81533 | -9.80816 |
| 1.36 | 3.74717 | -5.81255 | 9.77549 | 2.06538 | -5.81255 | 5.81255 | -9.77549 |
| 1.37 | 3.74334 | -5.80974 | 9.74258 | 2.06640 | -5.80974 | 5.80974 | -9.74258 |
| 1.38 | 3.73948 | -5.80691 | 9.70943 | 2.06743 | -5.80691 | 5.80691 | -9.70943 |
| 1.39 | 3.73559 | -5.80407 | 9.67603 | 2.06848 | -5.80407 | 5.80407 | -9.67603 |
| 1.40 | 3.73167 | -5.80119 | 9.64239 | 2.06953 | -5.80119 | 5.80119 | -9.64239 |
| 1.41 | 3.72771 | -5.79830 | 9.60851 | 2.07059 | -5.79830 | 5.79830 | -9.60851 |
| 1.42 | 3.72373 | -5.79539 | 9.57438 | 2.07166 | -5.79539 | 5.79539 | -9.57438 |
| 1.43 | 3.71971 | -5.79245 | 9.54001 | 2.07274 | -5.79245 | 5.79245 | -9.54001 |
| 1.44 | 3.71567 | -5.78950 | 9.50539 | 2.07383 | -5.78950 | 5.78950 | -9.50539 |
| 1.45 | 3.71159 | -5.78652 | 9.47053 | 2.07493 | -5.78652 | 5.78652 | -9.47053 |
| 1.46 | 3.70748 | -5.78352 | 9.43543 | 2.07604 | -5.78352 | 5.78352 | -9.43543 |
| 1.47 | 3.70333 | -5.78049 | 9.40009 | 2.07716 | -5.78049 | 5.78049 | -9.40009 |
| 1.48 | 3.69916 | -5.77745 | 9.36450 | 2.07829 | -5.77745 | 5.77745 | -9.36450 |
| 1.49 | 3.69495 | -5.77438 | 9.32866 | 2.07943 | -5.77438 | 5.77438 | -9.32866 |
| 1.50 | 3.69072 | -5.77129 | 9.29259 | 2.08058 | -5.77129 | 5.77129 | -9.29259 |
| 1.51 | 3.68645 | -5.76818 | 9.25626 | 2.08173 | -5.76818 | 5.76818 | -9.25626 |
| 1.52 | 3.68214 | -5.76505 | 9.21969 | 2.08290 | -5.76505 | 5.76505 | -9.21969 |
| 1.53 | 3.67781 | -5.76189 | 9.18288 | 2.08408 | -5.76189 | 5.76189 | -9.18288 |
| 1.54 | 3.67344 | -5.75871 | 9.14583 | 2.08527 | -5.75871 | 5.75871 | -9.14583 |
| 1.55 | 3.66904 | -5.75551 | 9.10853 | 2.08647 | -5.75551 | 5.75551 | -9.10853 |
| 1.56 | 3.66461 | -5.75229 | 9.07098 | 2.08768 | -5.75229 | 5.75229 | -9.07098 |
| 1.57 | 3.66015 | -5.74905 | 9.03319 | 2.08890 | -5.74905 | 5.74905 | -9.03319 |
| 1.58 | 3.65565 | -5.74578 | 8.99516 | 2.09013 | -5.74578 | 5.74578 | -8.99516 |
| 1.59 | 3.65112 | -5.74249 | 8.95689 | 2.0913 | -5.74249 | 5.74249 | -8.95689 |
| 1.60 | 3.64656 | -5.73918 | 8.91836 | 2.09262 | -5.73918 | 5.73918 | -8.91836 |
| 1.61 | 3.64197 | -5.73585 | 8.87960 | 2.09388 | -5.73585 | 5.73585 | -8.87960 |
| 1.62 | 3.63734 | -5.73249 | 8.84059 | 2.09515 | -5.73249 | 5.73249 | -8.84059 |
| 1.63 | 3.63268 | -5.72911 | 8.80133 | 2.09643 | -5.72911 | 5.72911 | -8.80133 |
| 1.64 | 3.62799 | -5.72571 | 8.76183 | 2.09773 | -5.72571 | 5.72571 | -8.76183 |
| 1.65 | 3.62326 | -5.72229 | 8.72208 | 2.09903 | -5.72229 | 5.72229 | -8.72208 |
| 1.66 | 3.61850 | -5.71884 | 8.68209 | 2.10035 | -5.71884 | 5.71884 | -8.68209 |
| 1.67 | 3.61371 | -5.71538 | 8.64185 | 2.10167 | -5.71538 | 5.71538 | -8.64185 |
| 1.68 | 3.60888 | -5.71189 | 8.60137 | 2.10301 | -5.71189 | 5.71189 | -8.60137 |
| 1.69 | 3.60402 | -5.70837 | 8.56064 | 2.10435 | -5.70837 | 5.70837 | -8.56064 |
| 1.70 | 3.59912 | -5.70484 | 8.51967 | 2.10571 | -5.70484 | 5.70484 | -8.51967 |
| 1.71 | 3.59420 | -5.70128 | 8.47845 | 2.10708 | -5.70128 | 5.70128 | -8.47845 |
| 1.72 | 3.58923 | -5.69770 | 8.43699 | 2.10846 | -5.69770 | 5.69770 | -8.43699 |
| 1.73 | 3.58424 | -5.69409 | 8.39528 | 2.10985 | -5.69409 | 5.69409 | -8.39528 |
| 1.74 | 3.57921 | -5.69046 | 8.35333 | 2.11126 | -5.69046 | 5.69046 | -8.35333 |
| 1.75 | 3.57414 | -5.68681 | 8.31113 | 2.11267 | -5.68681 | 5.68681 | -8.31113 |
| 1.76 | 3.56905 | -5.68314 | 8.26868 | 2.11410 | -5.68314 | 5.68314 | -8.26868 |
| 1.77 | 3.56391 | -5.67945 | 8.22599 | 2.11553 | -5.67945 | 5.67945 | -8.22599 |
| 1.78 | 3.55875 | -5.67573 | 8.18305 | 2.11698 | -5.67573 | 5.67573 | -8.18305 |
| 1.79 | 3.55354 | -5.67199 | 8.13987 | 2.11844 | -5.67199 | 5.67199 | -8.13987 |
| 1.80 | 3.54831 | -5.66822 | 8.09644 | 2.11991 | -5.66822 | 5.66822 | -8.09644 |
| 1.81 | 3.54304 | -5.66443 | 8.05277 | 2.12140 | -5.66443 | 5.66443 | -8.05277 |
| 1.82 | 3.53773 | -5.66062 | 8.00884 | 2.12289 | -5.66062 | 5.66062 | -8.00884 |
| 1.83 | 3.53239 | -5.65679 | 7.96468 | 2.12440 | -5.65679 | 5.65679 | -7.96468 |
| 1.84 | 3.52701 | -5.65293 | 7.92026 | 2.12592 | -5.65293 | 5.65293 | -7.92026 |
| 1.85 | 3.52160 | -5.64905 | 7.87560 | 2.12745 | -5.64905 | 5.64905 | -7.87560 |
| 1.86 | 3.51615 | -5.64515 | 7.83070 | 2.12900 | -5.64515 | 5.64515 | -7.83070 |
| 1.87 | 3.51067 | -5.64122 | 7.78554 | 2.13055 | -5.64122 | 5.64122 | -7.78554 |
| 1.88 | 3.50515 | -5.63727 | 7.74014 | 2.13212 | -5.63727 | 5.63727 | -7.74014 |
| 1.89 | 3.49960 | -5.63330 | 7.69450 | 2.13370 | -5.63330 | 5.63330 | -7.69450 |
| 1.90 | 3.49402 | -5.62930 | 7.64860 | 2.13529 | -5.62930 | 5.62930 | -7.64860 |
| 1.91 | 3.48838 | -5.62528 | 7.60246 | 2.13690 | -5.62528 | 5.62528 | -7.60246 |
| 1.92 | 3.48272 | -5.62124 | 7.55607 | 2.13852 | -5.62124 | 5.62124 | -7.55607 |
| 1.93 | 3.47702 | -5.61717 | 7.50944 | 2.14015 | -5.61717 | 5.61717 | -7.50944 |
| 1.94 | 3.47129 | -5.61308 | 7.46256 | 2.14179 | -5.61308 | 5.61308 | -7.46256 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| 1.95 | 3.46552 | -5.60896 | 7.41543 | 2.14345 | -5.60896 | 5.60896 | -7.41543 |
| 1.96 | 3.45971 | -5.60483 | 7.36805 | 2.14512 | -5.60483 | 5.60483 | -7.36805 |
| 1.97 | 3.45386 | -5.60067 | 7.32043 | 2.14680 | -5.60067 | 5.60067 | -7.32043 |
| 1.98 | 3.44796 | -5.59648 | 7.27256 | 2.14850 | -5.59648 | 5.59648 | -7.27256 |
| 1.99 | 3.44207 | -5.59227 | 7.22444 | 2.15020 | -5.59227 | 5.59227 | -7.22444 |
| 2.00 | 3.43611 | -5.58804 | 7.17608 | 2.15193 | -5.58804 | 5.58804 | -7.17608 |
| 2.01 | 3.43012 | -5.58378 | 7.12746 | 2.15366 | -5.58378 | 5.58378 | -7.12746 |
| 2.02 | 3.42409 | -5.57950 | 7.07860 | 2.15541 | -5.57950 | 5.57950 | -7.07860 |
| 2.03 | 3.41802 | -5.57520 | 7.02949 | 2.15717 | -5.57520 | 5.57520 | -7.02949 |
| 2.04 | 3.41192 | -5.57087 | 6.98013 | 2.15895 | -5.57087 | 5.57087 | -6.98013 |
| 2.05 | 3.40578 | -5.56651 | 6.93053 | 2.16074 | -5.56651 | 5.56651 | -6.93053 |
| 2.06 | 3.39960 | -5.56214 | 6.88068 | 2.16254 | -5.56214 | 5.56214 | -6.88068 |
| 2.07 | 3.39338 | -5.55774 | 6.83057 | 2.16436 | -5.55774 | 5.55774 | -6.83057 |
| 2.08 | 3.38712 | -5.55331 | 6.78022 | 2.16619 | -5.55331 | 5.55331 | -6.78022 |
| 2.09 | 3.38083 | -5.54886 | 6.72963 | 2.16803 | -5.54886 | 5.54886 | -6.72963 |
| 2.10 | 3.37450 | -5.54439 | 6.67878 | 2.16989 | -5.54439 | 5.54439 | -6.67878 |
| 2.11 | 3.36812 | -5.53989 | 6.62768 | 2.17177 | -5.53989 | 5.53989 | -6.62768 |
| 2.12 | 3.36171 | -5.53537 | 6.57634 | 2.17366 | -5.53537 | 5.53537 | -6.57634 |
| 2.13 | 3.35526 | -5.53082 | 6.52475 | 2.17556 | -5.53082 | 5.53082 | -6.52475 |
| 2.14 | 3.34878 | -5.52625 | 6.47291 | 2.17748 | -5.52625 | 5.52625 | -6.47291 |
| 2.15 | 3.34225 | -5.52166 | 6.42081 | 2.17941 | -5.52166 | 5.52166 | -6.42081 |
| 2.16 | 3.33568 | -5.51704 | 6.36848 | 2.18135 | -5.51704 | 5.51704 | -6.36848 |
| 2.17 | 3.32908 | -5.51239 | 6.31589 | 2.18332 | -5.51239 | 5.51239 | -6.31589 |
| 2.18 | 3.32243 | -5.50772 | 6.26305 | 2.18529 | -5.50772 | 5.50772 | -6.26305 |
| 2.19 | 3.31575 | -5.50303 | 6.20996 | 2.18728 | -5.50303 | 5.50303 | -6.20996 |
| 2.20 | 3.30902 | -5.49831 | 6.15662 | 2.18929 | -5.49831 | 5.49831 | -6.15662 |
| 2.21 | 3.30226 | -5.49357 | 6.10304 | 2.19131 | -5.49357 | 5.49357 | -6.10304 |
| 2.22 | 3.29545 | -5.48880 | 6.04920 | 2.19335 | -5.48880 | 5.48880 | -6.04920 |
| 2.23 | 3.28860 | -5.48401 | 5.99512 | 2.19540 | -5.48401 | 5.48401 | -5.99512 |
| 2.24 | 3.28172 | -5.47919 | 5.94078 | 2.19747 | -5.47919 | 5.47919 | -5.94078 |
| 2.25 | 3.27479 | -5.47435 | 5.88619 | 2.19956 | -5.47435 | 5.47435 | -5.88619 |
| 2.26 | 3.26782 | -5.46948 | 5.83136 | 2.20166 | -5.46948 | 5.46948 | -5.83136 |
| 2.27 | 3.26081 | -5.46459 | 5.77627 | 2.20377 | -5.46459 | 5.46459 | -5.77627 |
| 2.28 | 3.25376 | -5.45967 | 5.72094 | 2.20591 | -5.45967 | 5.45967 | -5.72094 |
| 2.29 | 3.24667 | -5.45473 | 5.66535 | 2.20806 | -5.45473 | 5.45473 | -5.66535 |
| 2.30 | 3.23954 | -5.44976 | 5.60951 | 2.21022 | -5.44976 | 5.44976 | -5.60951 |
| 2.31 | 3.23236 | -5.44476 | 5.55343 | 2.21240 | -5.44476 | 5.44476 | -5.55343 |
| 2.32 | 3.22514 | -5.43974 | 5.49709 | 2.21460 | -5.43974 | 5.43974 | -5.49709 |
| 2.33 | 3.21788 | -5.43470 | 5.44050 | 2.21682 | -5.43470 | 5.43470 | -5.44050 |
| 2.34 | 3.21058 | -5.42963 | 5.38366 | 2.21905 | -5.42963 | 5.42963 | -5.38366 |
| 2.35 | 3.20324 | -5.42453 | 5.32657 | 2.22130 | -5.42453 | 5.42453 | -5.32657 |
| 2.36 | 3.19585 | -5.41941 | 5.26923 | 2.22356 | -5.41941 | 5.41941 | -5.26923 |
| 2.37 | 3.18842 | -5.41427 | 5.31163 | 2.22585 | -5.41427 | 5.41427 | -5.31163 |
| 2.38 | 3.18095 | -5.40909 | 5.15379 | 2.22815 | -5.40909 | 5.40909 | -5.15379 |
| 2.39 | 3.17343 | -5.40390 | 5.09569 | 2.23047 | -5.40390 | 5.40390 | -5.09569 |
| 2.40 | 3.16587 | -5.39867 | 5.03734 | 2.23280 | -5.39867 | 5.39867 | -5.03734 |
| 2.41 | 3.15827 | -5.39342 | 4.97875 | 2.23515 | -5.39342 | 5.39342 | -4.97875 |
| 2.42 | 3.15062 | -5.38815 | 4.91989 | 2.23753 | -5.38815 | 5.38815 | -4.91989 |
| 2.43 | 3.14293 | -5.38285 | 4.86079 | 2.23992 | -5.38285 | 5.38285 | -4.86079 |
| 2.44 | 3.13519 | -5.37752 | 4.80144 | 2.24232 | -5.37752 | 5.37752 | -4.80144 |
| 2.45 | 3.12742 | -5.37216 | 4.74183 | 2.24475 | -5.37216 | 5.37216 | -4.74183 |
| 2.46 | 3.11959 | -5.36679 | 4.68197 | 2.24719 | -5.36679 | 5.36679 | -4.68197 |
| 2.47 | 3.11172 | -5.36138 | 4.62186 | 2.24966 | -5.36138 | 5.36138 | -4.62186 |
| 2.48 | 3.10381 | -5.35595 | 4.56149 | 2.25214 | -5.35595 | 5.35595 | -4.56149 |
| 2.49 | 3.09585 | -5.35049 | 4.50088 | 2.25464 | -5.35049 | 5.35049 | -4.50088 |
| 2.50 | 3.08784 | -5.34500 | 4.44001 | 2.25716 | -5.34500 | 5.34500 | -4.44001 |
| 2.51 | 3.07979 | -5.33949 | 4.37888 | 2.25970 | -5.33949 | 5.33949 | -4.37888 |
| 2.52 | 3.07170 | -5.33395 | 4.31751 | 2.26226 | -5.33395 | 5.33395 | -4.31751 |
| 2.53 | 3.06355 | -5.32839 | 4.25588 | 2.26484 | -5.32839 | 5.32839 | -4.25588 |
| 2.54 | 3.05536 | -5.32280 | 4.19400 | 2.26743 | -5.32280 | 5.32280 | -4.19400 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| 2.55 | 3.04713 | -5.31718 | 4.13186 | 2.27005 | -5.31718 | 5.31718 | -4.13186 |
| 2.56 | 3.03885 | -5.31154 | 4.06947 | 2.27269 | -5.31154 | 5.31154 | -4.06947 |
| 2.57 | 3.03052 | -5.30587 | 4.00683 | 2.27535 | -5.30587 | 5.30587 | -4.00683 |
| 2.58 | 3.02214 | -5.30017 | 3.94393 | 2.27803 | -5.30017 | 5.30017 | -3.94393 |
| 2.59 | 3.01372 | -5.29444 | 3.88079 | 2.28073 | -5.29444 | 5.29444 | -3.88079 |
| 2.60 | 3.00524 | -5.28869 | 3.81738 | 2.28344 | -5.28869 | 5.28869 | -3.81738 |
| 2.61 | 2.99673 | -5.28291 | 3.75372 | 2.28619 | -5.28291 | 5.28291 | -3.75372 |
| 2.62 | 2.98816 | -5.27711 | 3.68981 | 2.28895 | -5.27711 | 5.27711 | -3.68981 |
| 2.63 | 2.97954 | -5.27127 | 3.62564 | 2.29173 | -5.27127 | 5.27127 | -3.62564 |
| 2.64 | 2.97088 | -5.26541 | 3.56122 | 2.29453 | -5.26541 | 5.26541 | -3.56122 |
| 2.65 | 2.96216 | -5.25952 | 3.49654 | 2.29736 | -5.25952 | 5.25952 | -3.49654 |
| 2.66 | 2.95340 | -5.25361 | 3.43161 | 2.30021 | -5.25361 | 5.25361 | -3.43161 |
| 2.67 | 2.94459 | -5.24766 | 3.36643 | 2.30308 | -5.24766 | 5.24766 | -3.36643 |
| 2.68 | 2.93572 | -5.24169 | 3.30099 | 2.30597 | -5.24169 | 5.24169 | -3.30099 |
| 2.69 | 2.92681 | -5.23570 | 3.23529 | 2.30888 | -5.23570 | 5.23570 | -3.23529 |
| 2.70 | 2.91785 | -5.22967 | 3.16934 | 2.31182 | -5.22967 | 5.22967 | -3.16934 |
| 2.71 | 2.90884 | -5.22362 | 3.10313 | 2.31478 | -5.22362 | 5.22362 | -3.10313 |
| 2.72 | 2.89977 | -5.21753 | 3.03667 | 2.31776 | -5.21753 | 5.21753 | -3.03667 |
| 2.73 | 2.89066 | -5.21143 | 2.96995 | 2.32077 | -5.21143 | 5.21143 | -2.96995 |
| 2.74 | 2.88149 | -5.20529 | 2.90298 | 2.32380 | -5.20529 | 5.20529 | -2.90298 |
| 2.75 | 2.87228 | -5.19912 | 2.83575 | 2.32685 | -5.19912 | 5.19912 | -2.83575 |
| 2.76 | 2.86301 | -5.19293 | 2.76826 | 2.32992 | -5.19293 | 5.19293 | -2.76826 |
| 2.77 | 2.85368 | -5.18671 | 2.70052 | 2.33302 | -5.18671 | 5.18671 | -2.70052 |
| 2.78 | 2.84431 | -5.18046 | 2.63252 | 2.33615 | -5.18046 | 5.18046 | -2.63252 |
| 2.79 | 2.83488 | -5.17418 | 2.56426 | 2.33930 | -5.17418 | 5.17418 | -2.56426 |
| 2.80 | 2.82540 | -5.16787 | 2.49575 | 2.34247 | -5.16787 | 5.16787 | -2.49575 |
| 2.81 | 2.81587 | -5.16154 | 2.42698 | 2.34567 | -5.16154 | 5.16154 | -2.42698 |
| 2.82 | 2.80628 | -5.15517 | 2.35795 | 2.34889 | -5.15517 | 5.15517 | -2.35795 |
| 2.83 | 2.79664 | -5.14878 | 2.28866 | 2.35214 | -5.14878 | 5.14878 | -2.28866 |
| 2.84 | 2.78695 | -5.14236 | 2.21912 | 2.35541 | -5.14236 | 5.14236 | -2.21912 |
| 2.85 | 2.77720 | -5.13591 | 2.14932 | 2.35871 | -5.13591 | 5.13591 | -2.14932 |
| 2.86 | 2.76740 | -5.12943 | 2.07926 | 2.36204 | -5.12943 | 5.12943 | -2.07926 |
| 2.87 | 2.75754 | -5.12292 | 2.00895 | 2.36539 | -5.12292 | 5.12292 | -2.00895 |
| 2.88 | 2.74762 | -5.11639 | 1.93838 | 2.36877 | -5.11639 | 5.11639 | -1.93838 |
| 2.89 | 2.73765 | -5.10982 | 1.86754 | 2.37217 | -5.10982 | 5.10982 | -1.86754 |
| 2.90 | 2.72763 | -5.10323 | 1.79645 | 2.37560 | -5.10323 | 5.10323 | -1.79645 |
| 2.91 | 2.71754 | -5.09660 | 1.72511 | 2.37906 | -5.09660 | 5.09660 | -1.72511 |
| 2.92 | 2.70740 | -5.08995 | 1.65350 | 2.38255 | -5.08995 | 5.08995 | -1.65350 |
| 2.93 | 2.69721 | -5.08326 | 1.58163 | 2.38606 | -5.08326 | 5.08326 | -1.58163 |
| 2.94 | 2.68695 | -5.07656 | 1.50951 | 2.38960 | -5.07656 | 5.07656 | -1.50951 |
| 2.95 | 2.67664 | -5.06981 | 1.43712 | 2.39317 | -5.06981 | 5.06981 | -1.43712 |
| 2.96 | 2.66627 | -5.06304 | 1.36448 | 2.39677 | -5.06304 | 5.06304 | -1.36448 |
| 2.97 | 2.65584 | -5.05624 | 1.29158 | 2.40040 | -5.05624 | 5.05624 | -1.29158 |
| 2.98 | 2.64535 | -5.04941 | 1.21841 | 2.40406 | -5.04941 | 5.04941 | -1.21841 |
| 2.99 | 2.63481 | -5.04255 | 1.14499 | 2.40774 | -5.04255 | 5.04255 | -1.14499 |
| 3.00 | 2.62420 | -5.03565 | 1.07131 | 2.41145 | -5.03565 | 5.03565 | -1.07131 |
| 3.01 | 2.61353 | -5.02873 | .997365 | 2.41520 | -5.02873 | 5.02873 | -.997365 |
| 3.02 | 2.60280 | -5.02178 | .923161 | 2.41897 | -5.02178 | 5.02178 | -.923161 |
| 3.03 | 2.59202 | -5.01480 | .848697 | 2.42278 | -5.01480 | 5.01480 | -.848697 |
| 3.04 | 2.58117 | -5.00778 | .773971 | 2.42661 | -5.00778 | 5.00778 | -.773971 |
| 3.05 | 2.57026 | -5.00074 | .698986 | 2.43048 | -5.00074 | 5.00074 | -.698986 |
| 3.06 | 2.55929 | -4.99367 | .623740 | 2.43438 | -4.99367 | 4.99367 | -.623740 |
| 3.07 | 2.54825 | -4.98656 | .548231 | 2.43831 | -4.98656 | 4.98656 | -.548231 |
| 3.08 | 2.53715 | -4.97943 | .472461 | 2.44227 | -4.97943 | 4.97943 | -.472461 |
| 3.09 | 2.52600 | -4.97226 | .396430 | 2.44627 | -4.97226 | 4.97226 | -.396430 |
| 3.10 | 2.51478 | -4.96507 | .320138 | 2.45030 | -4.96507 | 4.96507 | -.320138 |
| 3.11 | 2.50349 | -4.95784 | .243582 | 2.45436 | -4.95784 | 4.95784 | -.243582 |
| 3.12 | 2.49214 | -4.95059 | .166765 | 2.45845 | -4.95059 | 4.95059 | -.166765 |
| 3.13 | 2.48073 | -4.94331 | .0896851 | 2.46259 | -4.94331 | 4.94331 | -.0896851 |
| 3.14 | 2.46897 | -4.93543 | .0123409 | 2.46647 | -4.93543 | 4.93543 | -.0123409 |

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| $\pi$ | 2.46740 | -4.93480 | 0 | 2.46740 | -4.93480 | 4.93480 | 0 |
| 3.15 | 2.45772 | -4.92867 | -.0652638 | 2.47096 | -4.92867 | 4.92867 | .0652638 |
| 3.16 | 2.44608 | -4.92125 | -.143132 | 2.47517 | -4.92125 | 4.92125 | .143132 |
| 3.17 | 2.43439 | -4.91381 | -.221263 | 2.47942 | -4.91381 | 4.91381 | .221263 |
| 3.18 | 2.42265 | -4.90638 | -.299658 | 2.48373 | -4.90638 | 4.90638 | .299658 |
| 3.19 | 2.41083 | -4.89889 | -.378316 | 2.48806 | -4.89889 | 4.89889 | .378316 |
| 3.20 | 2.39895 | -4.89138 | -.457239 | 2.49243 | -4.89138 | 4.89138 | .457239 |
| 3.21 | 2.38700 | -4.88384 | -.536424 | 2.49684 | -4.88384 | 4.88384 | .536424 |
| 3.22 | 2.37498 | -4.87626 | -.615875 | 2.50128 | -4.87626 | 4.87626 | .615875 |
| 3.23 | 2.36289 | -4.86866 | -.695590 | 2.50576 | -4.86866 | 4.86866 | .695590 |
| 3.24 | 2.35073 | -4.86101 | -.775568 | 2.51028 | -4.86101 | 4.86101 | .775568 |
| 3.25 | 2.33850 | -4.85334 | -.855811 | 2.51484 | -4.85334 | 4.85334 | .855811 |
| 3.26 | 2.32620 | -4.84564 | -.936321 | 2.51943 | -4.84564 | 4.84564 | .936321 |
| 3.27 | 2.31383 | -4.83790 | -1.01710 | 2.52407 | -4.83790 | 4.83790 | 1.01710 |
| 3.28 | 2.30139 | -4.83013 | -1.09814 | 2.52874 | -4.83013 | 4.83013 | 1.09814 |
| 3.29 | 2.28887 | -4.82233 | -1.17944 | 2.53345 | -4.82233 | 4.82233 | 1.17944 |
| 3.30 | 2.27629 | -4.81449 | -1.26101 | 2.53821 | -4.81449 | 4.81449 | 1.26101 |
| 3.31 | 2.26362 | -4.80662 | -1.34285 | 2.54300 | -4.80662 | 4.80662 | 1.34285 |
| 3.32 | 2.25089 | -4.79873 | -1.42495 | 2.54783 | -4.79873 | 4.79873 | 1.42495 |
| 3.33 | 2.23808 | -4.79079 | -1.50732 | 2.55271 | -4.79079 | 4.79079 | 1.50732 |
| 3.34 | 2.22520 | -4.78282 | -1.58996 | 2.55763 | -4.78282 | 4.78282 | 1.58996 |
| 3.35 | 2.21223 | -4.77482 | -1.67286 | 2.56258 | -4.77482 | 4.77482 | 1.67286 |
| 3.36 | 2.19920 | -4.76678 | -1.75603 | 2.56759 | -4.76678 | 4.76678 | 1.75603 |
| 3.37 | 2.18608 | -4.75871 | -1.83947 | 2.57263 | -4.75871 | 4.75871 | 1.83947 |
| 3.38 | 2.17289 | -4.75061 | -1.92318 | 2.57772 | -4.75061 | 4.75061 | 1.92318 |
| 3.39 | 2.15962 | -4.74247 | -2.00715 | 2.58285 | -4.74247 | 4.74247 | 2.00715 |
| 3.40 | 2.14627 | -4.73430 | -2.09139 | 2.58803 | -4.73430 | 4.73430 | 2.09139 |
| 3.41 | 2.13285 | -4.72610 | -2.17590 | 2.59325 | -4.72610 | 4.72610 | 2.17590 |
| 3.42 | 2.11934 | -4.71786 | -2.26068 | 2.59852 | -4.71786 | 4.71786 | 2.26068 |
| 3.43 | 2.10575 | -4.70958 | -2.34573 | 2.60383 | -4.70958 | 4.70958 | 2.34573 |
| 3.44 | 2.09209 | -4.70127 | -2.43105 | 2.60919 | -4.70127 | 4.70127 | 2.43105 |
| 3.45 | 2.07834 | -4.69293 | -2.51663 | 2.61460 | -4.69293 | 4.69293 | 2.51663 |
| 3.46 | 2.06450 | -4.68455 | -2.60249 | 2.62005 | -4.68455 | 4.68455 | 2.60249 |
| 3.47 | 2.05059 | -4.67614 | -2.68861 | 2.62555 | -4.67614 | 4.67614 | 2.68861 |
| 3.48 | 2.03659 | -4.66770 | -2.77501 | 2.63111 | -4.66770 | 4.66770 | 2.77501 |
| 3.49 | 2.02251 | -4.65921 | -2.86167 | 2.63671 | -4.65921 | 4.65921 | 2.86167 |
| 3.50 | 2.00834 | -4.65070 | -2.94861 | 2.64236 | -4.65070 | 4.65070 | 2.94861 |
| 3.51 | 1.99409 | -4.64214 | -3.03581 | 2.64806 | -4.64214 | 4.64214 | 3.03581 |
| 3.52 | 1.97975 | -4.63355 | -3.12329 | 2.65381 | -4.63355 | 4.63355 | 3.12329 |
| 3.53 | 1.96532 | -4.62493 | -3.21104 | 2.65961 | -4.62493 | 4.62493 | 3.21104 |
| 3.54 | 1.95081 | -4.61627 | -3.29906 | 2.66546 | -4.61627 | 4.61627 | 3.29906 |
| 3.55 | 1.93620 | -4.60758 | -3.38735 | 2.67137 | -4.60758 | 4.60758 | 3.38735 |
| 3.56 | 1.92151 | -4.59884 | -3.47591 | 2.67733 | -4.59884 | 4.59884 | 3.47591 |
| 3.57 | 1.90673 | -4.59007 | -3.56475 | 2.68335 | -4.59007 | 4.59007 | 3.56475 |
| 3.58 | 1.89185 | -4.58127 | -3.65386 | 2.68942 | -4.58127 | 4.58127 | 3.65386 |
| 3.59 | 1.87689 | -4.57243 | -3.74324 | 2.69554 | -4.57243 | 4.57243 | 3.74324 |
| 3.60 | 1.86183 | -4.56355 | -3.83289 | 2.70172 | -4.56355 | 4.56355 | 3.83289 |
| 3.61 | 1.84668 | -4.55464 | -3.92282 | 2.70796 | -4.55464 | 4.55464 | 3.92282 |
| 3.62 | 1.83143 | -4.54569 | -4.01302 | 2.71425 | -4.54569 | 4.54569 | 4.01302 |
| 3.63 | 1.81609 | -4.53670 | -4.10350 | 2.72061 | -4.53670 | 4.53670 | 4.10350 |
| 3.64 | 1.80066 | -4.52768 | -4.19425 | 2.72702 | -4.52768 | 4.52768 | 4.19425 |
| 3.65 | 1.78513 | -4.51862 | -4.28527 | 2.73349 | -4.51862 | 4.51862 | 4.28527 |
| 3.66 | 1.76950 | -4.50952 | -4.37657 | 2.74002 | -4.50952 | 4.50952 | 4.37657 |
| 3.67 | 1.75377 | -4.50038 | -4.46814 | 2.74661 | -4.50038 | 4.50038 | 4.46814 |
| 3.68 | 1.73794 | -4.49120 | -4.55999 | 2.75326 | -4.49120 | 4.49120 | 4.55999 |
| 3.69 | 1.72202 | -4.48199 | -4.65211 | 2.75997 | -4.48199 | 4.48199 | 4.65211 |
| 3.70 | 1.70599 | -4.47274 | -4.74451 | 2.76675 | -4.47274 | 4.47274 | 4.74451 |
| 3.71 | 1.68986 | -4.46345 | -4.83719 | 2.77359 | -4.46345 | 4.46345 | 4.83719 |
| 3.72 | 1.67363 | -4.45413 | -4.93014 | 2.78050 | -4.45413 | 4.45413 | 4.93014 |
| 3.73 | 1.65729 | -4.44764 | -5.02337 | 2.78747 | -4.44764 | 4.44764 | 5.02337 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| 3.74 | 1.64085 | -4.43536 | -5.11688 | 2.79451 | -4.43536 | 4.43536 | 5.11688 |
| 3.75 | 1.62431 | -4.42592 | -5.21066 | 2.80161 | -4.42592 | 4.42592 | 5.21066 |
| 3.76 | 1.60765 | -4.41644 | -5.30472 | 2.80879 | -4.41644 | 4.41644 | 5.30472 |
| 3.77 | 1.59089 | -4.40692 | -5.39906 | 2.81603 | -4.40692 | 4.40692 | 5.39906 |
| 3.78 | 1.57402 | -4.39736 | -5.49368 | 2.82334 | -4.39736 | 4.39736 | 5.49368 |
| 3.79 | 1.55705 | -4.38776 | -5.58857 | 2.83072 | -4.38776 | 4.38776 | 5.58857 |
| 3.80 | 1.53996 | -4.37813 | -5.68374 | 2.83817 | -4.37813 | 4.37813 | 5.68374 |
| 3.81 | 1.52275 | -4.36845 | -5.77920 | 2.84570 | -4.36845 | 4.36845 | 5.77920 |
| 3.82 | 1.50544 | -4.35873 | -5.87493 | 2.85329 | -4.35873 | 4.35873 | 5.87493 |
| 3.83 | 1.48802 | -4.34898 | -5.97095 | 2.86097 | -4.34898 | 4.34898 | 5.97095 |
| 3.84 | 1.47047 | -4.33918 | -6.06724 | 2.86871 | -4.33918 | 4.33918 | 6.06724 |
| 3.85 | 1.45281 | -4.32934 | -6.16381 | 2.87654 | -4.32934 | 4.32934 | 6.16381 |
| 3.86 | 1.43503 | -4.31947 | -6.26067 | 2.88444 | -4.31947 | 4.31947 | 6.26067 |
| 3.87 | 1.41713 | -4.30955 | -6.35780 | 2.89242 | -4.30955 | 4.30955 | 6.35780 |
| 3.88 | 1.39912 | -4.29959 | -6.45522 | 2.90047 | -4.29959 | 4.29959 | 6.45522 |
| 3.89 | 1.38098 | -4.28959 | -6.55292 | 2.90861 | -4.28959 | 4.28959 | 6.55292 |
| 3.90 | 1.36272 | -4.27955 | -6.65089 | 2.91683 | -4.27955 | 4.27955 | 6.65089 |
| 3.91 | 1.34434 | -4.26947 | -6.74916 | 2.92513 | -4.26947 | 4.26947 | 6.74916 |
| 3.92 | 1.32583 | -4.25935 | -6.84770 | 2.93352 | -4.25935 | 4.25935 | 6.84770 |
| 3.93 | 1.30720 | -4.24919 | -6.94653 | 2.94199 | -4.24919 | 4.24919 | 6.94653 |
| 3.94 | 1.28844 | -4.23898 | -7.04564 | 2.95054 | -4.23898 | 4.23898 | 7.04564 |
| 3.95 | 1.26955 | -4.22873 | -7.14504 | 2.95919 | -4.22873 | 4.22873 | 7.14504 |
| 3.96 | 1.25052 | -4.21844 | -7.24472 | 2.96792 | -4.21844 | 4.21844 | 7.24472 |
| 3.97 | 1.23137 | -4.20811 | -7.34468 | 2.97674 | -4.20811 | 4.20811 | 7.34468 |
| 3.98 | 1.21209 | -4.19774 | -7.44493 | 2.98565 | -4.19774 | 4.19774 | 7.44493 |
| 3.99 | 1.19267 | -4.18732 | -7.54546 | 2.99465 | -4.18732 | 4.18732 | 7.54546 |
| 4.00 | 1.17311 | -4.17686 | -7.64628 | 3.00374 | -4.17686 | 4.17686 | 7.64628 |
| 4.01 | 1.15342 | -4.16636 | -7.74739 | 3.01293 | -4.16636 | 4.16636 | 7.74739 |
| 4.02 | 1.13359 | -4.15581 | -7.84878 | 3.02222 | -4.15581 | 4.15581 | 7.84878 |
| 4.03 | 1.11362 | -4.14522 | -7.95046 | 3.03160 | -4.14522 | 4.14522 | 7.95046 |
| 4.04 | 1.09351 | -4.13459 | -8.05242 | 3.04108 | -4.13459 | 4.13459 | 8.05242 |
| 4.05 | 1.07325 | -4.12391 | -8.15468 | 3.05066 | -4.12391 | 4.12391 | 8.15468 |
| 4.06 | 1.05285 | -4.11319 | -8.25722 | 3.06034 | -4.11319 | 4.11319 | 8.25722 |
| 4.07 | 1.03230 | -4.10243 | -8.36005 | 3.07013 | -4.10243 | 4.10243 | 8.36005 |
| 4.08 | 1.01160 | -4.09162 | -8.46316 | 3.08002 | -4.09162 | 4.09162 | 8.46316 |
| 4.09 | .990754 | -4.08076 | -8.56657 | 3.09001 | -4.08076 | 4.08076 | 8.56657 |
| 4.10 | .969755 | -4.06987 | -8.67027 | 3.10011 | -4.06987 | 4.06987 | 8.67027 |
| 4.11 | .948603 | -4.05892 | -8.77425 | 3.11032 | -4.05892 | 4.05892 | 8.77425 |
| 4.12 | .927295 | -4.04794 | -8.87853 | 3.12064 | -4.04794 | 4.04794 | 8.87853 |
| 4.13 | .905830 | -4.03690 | -8.98309 | 3.13107 | -4.03690 | 4.03690 | 8.98309 |
| 4.14 | .884206 | -4.02582 | -9.08795 | 3.14162 | -4.02582 | 4.02582 | 9.08795 |
| 4.15 | .862420 | -4.01470 | -9.19310 | 3.15228 | -4.01470 | 4.01470 | 9.19310 |
| 4.16 | .840473 | -4.00353 | -9.29854 | 3.16306 | -4.00353 | 4.00353 | 9.29854 |
| 4.17 | .818360 | -3.99231 | -9.40427 | 3.17395 | -3.99231 | 3.99231 | 9.40427 |
| 4.18 | .796080 | -3.98105 | -9.51030 | 3.18497 | -3.98105 | 3.98105 | 9.51030 |
| 4.19 | .773632 | -3.96974 | -9.61661 | 3.19611 | -3.96974 | 3.96974 | 9.61661 |
| 4.20 | .751013 | -3.95839 | -9.72323 | 3.20737 | -3.95839 | 3.95839 | 9.72323 |
| 4.21 | .728221 | -3.94698 | -9.83013 | 3.21876 | -3.94698 | 3.94698 | 9.83013 |
| 4.22 | .705254 | -3.93553 | -9.93733 | 3.23028 | -3.93553 | 3.93553 | 9.93733 |
| 4.23 | .682109 | -3.92404 | -10.0448 | 3.24193 | -3.92404 | 3.92404 | 10.0448 |
| 4.24 | .658785 | -3.91249 | -10.1526 | 3.25371 | -3.91249 | 3.91249 | 10.1526 |
| 4.25 | .635279 | -3.90090 | -10.2607 | 3.26562 | -3.90090 | 3.90090 | 10.2607 |
| 4.26 | .611589 | -3.88926 | -10.3691 | 3.27767 | -3.88926 | 3.88926 | 10.3691 |
| 4.27 | .587712 | -3.87757 | -10.4778 | 3.28986 | -3.87757 | 3.87757 | 10.4778 |
| 4.28 | .563647 | -3.86583 | -10.5867 | 3.30219 | -3.86583 | 3.86583 | 10.5867 |
| 4.29 | .539390 | -3.85405 | -10.6960 | 3.31466 | -3.85405 | 3.85405 | 10.6960 |
| 4.30 | .514939 | -3.84221 | -10.8056 | 3.32727 | -3.84221 | 3.84221 | 10.8056 |
| 4.31 | .490292 | -3.83033 | -10.9154 | 3.34004 | -3.83033 | 3.83033 | 10.9154 |
| 4.32 | .465446 | -3.81839 | -11.0256 | 3.35295 | -3.81839 | 3.81839 | 11.0256 |
| 4.33 | .440399 | -3.80641 | -11.1361 | 3.36601 | -3.80641 | 3.80641 | 11.1361 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| 4.34 | 0.415147 | -3.79438 | -11.2468 | 3.37923 | -3.79438 | 3.79438 | 11.2468 |
| 4.35 | .389688 | -3.78229 | -11.3579 | 3.39261 | -3.78229 | 3.78229 | 11.3579 |
| 4.36 | .364019 | -3.77016 | -11.4693 | 3.40614 | -3.77016 | 3.77016 | 11.4693 |
| 4.37 | .338138 | -3.75797 | -11.5809 | 3.41984 | -3.75797 | 3.75797 | 11.5809 |
| 4.38 | .312040 | -3.74574 | -11.6929 | 3.43370 | -3.74574 | 3.74574 | 11.6929 |
| 4.39 | .285724 | -3.73345 | -11.8052 | 3.44773 | -3.73345 | 3.73345 | 11.8052 |
| 4.40 | .259187 | -3.72111 | -11.9178 | 3.46193 | -3.72111 | 3.72111 | 11.9178 |
| 4.41 | .232424 | -3.70873 | -12.0306 | 3.47630 | -3.70873 | 3.70873 | 12.0306 |
| 4.42 | .205434 | -3.69628 | -12.1438 | 3.49085 | -3.69628 | 3.69628 | 12.1438 |
| 4.43 | .178213 | -3.68379 | -12.2573 | 3.50558 | -3.68379 | 3.68379 | 12.2573 |
| 4.44 | .150756 | -3.67125 | -12.3711 | 3.52049 | -3.67125 | 3.67125 | 12.3711 |
| 4.45 | .123062 | -3.65865 | -12.4852 | 3.53558 | -3.65865 | 3.65865 | 12.4852 |
| 4.46 | .0951270 | -3.64599 | -12.5996 | 3.55087 | -3.64599 | 3.64599 | 12.5996 |
| 4.47 | .0669467 | -3.63329 | -12.7143 | 3.56634 | -3.63329 | 3.63329 | 12.7143 |
| 4.48 | .0385179 | -3.62053 | -12.8293 | 3.58201 | -3.62053 | 3.62053 | 12.8293 |
| 4.49 | .00983696 | -3.60772 | -12.9447 | 3.59788 | -3.60772 | 3.60772 | 12.9447 |
| 4.50 | -.0190999 | -3.59485 | -13.0603 | 3.61395 | -3.59485 | 3.59485 | 13.0603 |
| 4.51 | -.0482966 | -3.58193 | -13.1762 | 3.63023 | -3.58193 | 3.58193 | 13.1762 |
| 4.52 | -.0777570 | -3.56896 | -13.2925 | 3.64672 | -3.56896 | 3.56896 | 13.2925 |
| 4.53 | -.107485 | -3.55593 | -13.4090 | 3.66341 | -3.55593 | 3.55593 | 13.4090 |
| 4.54 | -.137485 | -3.54284 | -13.5259 | 3.68033 | -3.54284 | 3.54284 | 13.5259 |
| 4.55 | -.167761 | -3.52970 | -13.6431 | 3.69746 | -3.52970 | 3.52970 | 13.6431 |
| 4.56 | -.198317 | -3.51651 | -13.7606 | 3.71482 | -3.51651 | 3.51651 | 13.7606 |
| 4.57 | -.229158 | -3.50325 | -13.8784 | 3.73241 | -3.50325 | 3.50325 | 13.8784 |
| 4.58 | -.260288 | -3.48995 | -13.9965 | 3.75023 | -3.48995 | 3.48995 | 13.9965 |
| 4.59 | -.291712 | -3.47658 | -14.1149 | 3.76829 | -3.47658 | 3.47658 | 14.1149 |
| 4.60 | -.323435 | -3.46316 | -14.2337 | 3.78659 | -3.46316 | 3.46316 | 14.2337 |
| 4.61 | -.355461 | -3.44968 | -14.3527 | 3.80514 | -3.44968 | 3.44968 | 14.3527 |
| 4.62 | -.387795 | -3.43614 | -14.4721 | 3.82394 | -3.43614 | 3.43614 | 14.4721 |
| 4.63 | -.420442 | -3.42255 | -14.5918 | 3.84299 | -3.42255 | 3.42255 | 14.5918 |
| 4.64 | -.453408 | -3.40890 | -14.7118 | 3.86230 | -3.40890 | 3.40890 | 14.7118 |
| 4.65 | -.486697 | -3.39518 | -14.8321 | 3.88188 | -3.39518 | 3.39518 | 14.8321 |
| 4.66 | -.520316 | -3.38141 | -14.9528 | 3.90173 | -3.38141 | 3.38141 | 14.9528 |
| 4.67 | -.554269 | -3.36759 | -15.0737 | 3.92185 | -3.36759 | 3.36759 | 15.0737 |
| 4.68 | -.588563 | -3.35370 | -15.1950 | 3.94226 | -3.35370 | 3.35370 | 15.1950 |
| 4.69 | -.623202 | -3.33975 | -15.3166 | 3.96295 | -3.33975 | 3.33975 | 15.3166 |
| 4.70 | -.658194 | -3.32574 | -15.4385 | 3.98394 | -3.32574 | 3.32574 | 15.4385 |
| 4.71 | -.693545 | -3.31167 | -15.5608 | 4.00522 | -3.31167 | 3.31167 | 15.5608 |
| 4.72 | -.729259 | -3.29754 | -15.6833 | 4.02680 | -3.29754 | 3.29754 | 15.6833 |
| 4.73 | -.765345 | -3.28335 | -15.8062 | 4.04870 | -3.28335 | 3.28335 | 15.8062 |
| 4.74 | -.801808 | -3.26910 | -15.9294 | 4.07091 | -3.26910 | 3.26910 | 15.9294 |
| 4.75 | -.838656 | -3.25479 | -16.0529 | 4.09344 | -3.25479 | 3.25479 | 16.0529 |
| 4.76 | -.875895 | -3.24041 | -16.1768 | 4.11631 | -3.24041 | 3.24041 | 16.1768 |
| 4.77 | -.913532 | -3.22597 | -16.3010 | 4.13950 | -3.22597 | 3.22597 | 16.3010 |
| 4.78 | -.951574 | -3.21147 | -16.4255 | 4.16305 | -3.21147 | 3.21147 | 16.4255 |
| 4.79 | -.990030 | -3.19691 | -16.5503 | 4.18694 | -3.19691 | 3.19691 | 16.5503 |
| 4.80 | -1.02891 | -3.18228 | -16.6754 | 4.21119 | -3.18228 | 3.18228 | 16.6754 |
| 4.81 | -1.06821 | -3.16759 | -16.8009 | 4.23580 | -3.16759 | 3.16759 | 16.8009 |
| 4.82 | -1.10795 | -3.15283 | -16.9267 | 4.26079 | -3.15283 | 3.15283 | 16.9267 |
| 4.83 | -1.14814 | -3.13801 | -17.0529 | 4.28615 | -3.13801 | 3.13801 | 17.0529 |
| 4.84 | -1.18878 | -3.12313 | -17.1793 | 4.31191 | -3.12313 | 3.12313 | 17.1793 |
| 4.85 | -1.22989 | -3.10818 | -17.3061 | 4.33806 | -3.10818 | 3.10818 | 17.3061 |
| 4.86 | -1.27146 | -3.09316 | -17.4333 | 4.36462 | -3.09316 | 3.09316 | 17.4333 |
| 4.87 | -1.31352 | -3.07808 | -17.5607 | 4.39159 | -3.07808 | 3.07808 | 17.5607 |
| 4.88 | -1.35606 | -3.06293 | -17.6885 | 4.41899 | -3.06293 | 3.06293 | 17.6885 |
| 4.89 | -1.39911 | -3.04771 | -17.8167 | 4.44682 | -3.04771 | 3.04771 | 17.8167 |
| 4.90 | -1.44266 | -3.03243 | -17.9451 | 4.47509 | -3.03243 | 3.03243 | 17.9451 |
| 4.91 | -1.48674 | -3.01708 | -18.0740 | 4.50381 | -3.01708 | 3.01708 | 18.0740 |
| 4.92 | -1.53135 | -3.00166 | -18.2031 | 4.53300 | -3.00166 | 3.00166 | 18.2031 |
| 4.93 | -1.57650 | -2.98617 | -18.3326 | 4.56266 | -2.98617 | 2.98617 | 18.3326 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_L}\left(\frac{L}{EI}\right)$ | $K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_L}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_L}\left(\frac{L}{EI}\right)$ | $K_{C\delta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_L}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_L}\left(\frac{L^3}{EI}\right)$ |
| 4.94 | -1.62220 | -2.97061 | -18.4624 | 4.59281 | -2.97061 | 2.97061 | 18.4624 |
| 4.95 | -1.66847 | -2.95498 | -18.5925 | 4.62345 | -2.95498 | 2.95498 | 18.5925 |
| 4.96 | -1.71531 | -2.93929 | -18.7230 | 4.65460 | -2.93929 | 2.93929 | 18.7230 |
| 4.97 | -1.76275 | -2.92352 | -18.8539 | 4.68627 | -2.92352 | 2.92352 | 18.8539 |
| 4.98 | -1.81079 | -2.90769 | -18.9850 | 4.71847 | -2.90769 | 2.90769 | 18.9850 |
| 4.99 | -1.85944 | -2.89178 | -19.1165 | 4.75122 | -2.89178 | 2.89178 | 19.1165 |
| 5.00 | -1.90872 | -2.87580 | -19.2484 | 4.78452 | -2.87580 | 2.87580 | 19.2484 |
| 5.01 | -1.95865 | -2.85975 | -19.3806 | 4.81839 | -2.85975 | 2.85975 | 19.3806 |
| 5.02 | -2.00923 | -2.84362 | -19.5132 | 4.85265 | -2.84362 | 2.84362 | 19.5132 |
| 5.03 | -2.06048 | -2.82743 | -19.6461 | 4.88791 | -2.82743 | 2.82743 | 19.6461 |
| 5.04 | -2.11242 | -2.82116 | -19.7793 | 4.92358 | -2.81116 | 2.81116 | 19.7793 |
| 5.05 | -2.16507 | -2.79481 | -19.9129 | 4.95988 | -2.79481 | 2.79481 | 19.9129 |
| 5.06 | -2.21843 | -2.77839 | -20.0468 | 4.99682 | -2.77839 | 2.77839 | 20.0468 |
| 5.07 | -2.27253 | -2.76190 | -20.1811 | 5.03443 | -2.76190 | 2.76190 | 20.1811 |
| 5.08 | -2.32738 | -2.74533 | -20.3157 | 5.07272 | -2.74533 | 2.74533 | 20.3157 |
| 5.09 | -2.38301 | -2.72869 | -20.4507 | 5.11170 | -2.72869 | 2.72869 | 20.4507 |
| 5.10 | -2.43942 | -2.71197 | -20.5861 | 5.15139 | -2.71197 | 2.71197 | 20.5861 |
| 5.11 | -2.49665 | -2.69517 | -20.7218 | 5.19182 | -2.69517 | 2.69517 | 20.7218 |
| 5.12 | -2.55470 | -2.67830 | -20.8578 | 5.23300 | -2.67830 | 2.67830 | 20.8578 |
| 5.13 | -2.61360 | -2.66135 | -20.9942 | 5.27495 | -2.66135 | 2.66135 | 20.9942 |
| 5.14 | -2.67338 | -2.64432 | -21.1310 | 5.32336 | -2.64432 | 2.64432 | 21.1310 |
| 5.15 | -2.73404 | -2.62721 | -21.2681 | 5.36126 | -2.62721 | 2.62721 | 21.2681 |
| 5.16 | -2.79562 | -2.61003 | -21.4055 | 5.40565 | -2.61003 | 2.61003 | 21.4055 |
| 5.17 | -2.85815 | -2.59276 | -21.5434 | 5.45091 | -2.59276 | 2.59276 | 21.5434 |
| 5.18 | -2.92163 | -2.57541 | -21.6816 | 5.49704 | -2.57541 | 2.57541 | 21.6816 |
| 5.19 | -2.98611 | -2.55799 | -21.8201 | 5.54409 | -2.55799 | 2.55799 | 21.8201 |
| 5.20 | -3.05159 | -2.54048 | -21.9590 | 5.59207 | -2.54048 | 2.54048 | 21.9590 |
| 5.21 | -3.11812 | -2.52289 | -22.0983 | 5.64101 | -2.52289 | 2.52289 | 22.0983 |
| 5.22 | -3.18573 | -2.50522 | -22.2380 | 5.69094 | -2.50522 | 2.50522 | 22.2380 |
| 5.23 | -3.25443 | -2.48746 | -22.3780 | 5.74189 | -2.48746 | 2.48746 | 22.3780 |
| 5.24 | -3.32426 | -2.46962 | -22.5184 | 5.79388 | -2.46962 | 2.46962 | 22.5184 |
| 5.25 | -3.39525 | -2.45170 | -22.6591 | 5.84695 | -2.45170 | 2.45170 | 22.6591 |
| 5.26 | -3.46744 | -2.43369 | -22.8002 | 5.90113 | -2.43369 | 2.43369 | 22.8002 |
| 5.27 | -3.54086 | -2.41560 | -22.9417 | 5.95646 | -2.41560 | 2.41560 | 22.9417 |
| 5.28 | -3.61554 | -2.39742 | -23.0836 | 6.01296 | -2.39742 | 2.39742 | 23.0836 |
| 5.29 | -3.69152 | -2.37916 | -23.2258 | 6.07067 | -2.37916 | 2.37916 | 23.2258 |
| 5.30 | -3.76884 | -2.36081 | -23.3684 | 6.12964 | -2.36081 | 2.36081 | 23.3684 |
| 5.31 | -3.84754 | -2.34237 | -23.5114 | 6.18990 | -2.34237 | 2.34237 | 23.5114 |
| 5.32 | -3.92766 | -2.32384 | -23.6547 | 6.25150 | -2.32384 | 2.32384 | 23.6547 |
| 5.33 | -4.00924 | -2.30522 | -23.7984 | 6.31446 | -2.30522 | 2.30522 | 23.7984 |
| 5.34 | -4.09233 | -2.28652 | -23.9426 | 6.37885 | -2.28652 | 2.28652 | 23.9426 |
| 5.35 | -4.17698 | -2.26772 | -24.0870 | 6.44470 | -2.26772 | 2.26772 | 24.0870 |
| 5.36 | -4.26323 | -2.24884 | -24.2319 | 6.51207 | -2.24884 | 2.24884 | 24.2319 |
| 5.37 | -4.35114 | -2.22986 | -24.3772 | 6.58100 | -2.22986 | 2.22986 | 24.3772 |
| 5.38 | -4.44076 | -2.21079 | -24.5228 | 6.65155 | -2.21079 | 2.21079 | 24.5228 |
| 5.39 | -4.53214 | -2.19163 | -24.6688 | 6.72377 | -2.19163 | 2.19163 | 24.6688 |
| 5.40 | -4.62534 | -2.17238 | -24.8152 | 6.79773 | -2.17238 | 2.17238 | 24.8152 |
| 5.41 | -4.72043 | -2.15303 | -24.9620 | 6.87347 | -2.15303 | 2.15303 | 24.9620 |
| 5.42 | -4.81747 | -2.13359 | -25.1092 | 6.95106 | -2.13359 | 2.13359 | 25.1092 |
| 5.43 | -4.91653 | -2.11405 | -25.2568 | 7.03058 | -2.11405 | 2.11405 | 25.2568 |
| 5.44 | -5.01766 | -2.09442 | -25.4048 | 7.11208 | -2.09442 | 2.09442 | 25.4048 |
| 5.45 | -5.12096 | -2.07469 | -25.5531 | 7.19565 | -2.07469 | 2.07469 | 25.5531 |
| 5.46 | -5.22649 | -2.05486 | -25.7019 | 7.28135 | -2.05486 | 2.05486 | 25.7019 |
| 5.47 | -5.33434 | -2.03493 | -25.8510 | 7.36927 | -2.03493 | 2.03493 | 25.8510 |
| 5.48 | -5.44458 | -2.01491 | -26.0006 | 7.45950 | -2.01491 | 2.01491 | 26.0006 |
| 5.49 | -5.55732 | -1.99479 | -26.1505 | 7.55211 | -1.99479 | 1.99479 | 26.1505 |
| 5.50 | -5.67264 | -1.97456 | -26.3009 | 7.64720 | -1.97456 | 1.97456 | 26.3009 |
| 5.51 | -5.79063 | -1.95424 | -26.4516 | 7.74487 | -1.95424 | 1.95424 | 26.4516 |
| 5.52 | -5.91142 | -1.93381 | -26.6028 | 7.84523 | -1.93381 | 1.93381 | 26.6028 |
| 5.53 | -6.03509 | -1.91328 | -26.7543 | 7.94837 | -1.91328 | 1.91328 | 26.7543 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

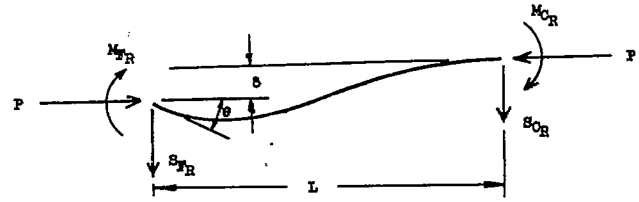
TABLE I - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Concluded

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{FS_L}\left(\frac{L}{EI}\right)$ | $K_{FS_L}\left(\frac{L^2}{EI}\right)$ or $C_{FS_L}\left(\frac{L^2}{EI}\right)$ | $C_{FS_L}\left(\frac{L^3}{EI}\right)$ | $K_{CS_L}\left(\frac{L}{EI}\right)$ | $K_{CS_L}\left(\frac{L^2}{EI}\right)$ | $C_{CS_L}\left(\frac{L^2}{EI}\right)$ | $C_{CS_L}\left(\frac{L^3}{EI}\right)$ |
| 5.54 | -6.16177 | -1.89265 | -26.9063 | 8.05442 | -1.89265 | 1.89265 | 26.9063 |
| 5.55 | -6.29157 | -1.87191 | -27.0587 | 8.16349 | -1.87191 | 1.87191 | 27.0587 |
| 5.56 | -6.42464 | -1.85108 | -27.2115 | 8.27571 | -1.85108 | 1.85108 | 27.2115 |
| 5.57 | -6.56109 | -1.83013 | -27.3646 | 8.39122 | -1.83013 | 1.83013 | 27.3646 |
| 5.58 | -6.70107 | -1.80908 | -27.5183 | 8.51015 | -1.80908 | 1.80908 | 27.5183 |
| 5.59 | -6.84474 | -1.78792 | -27.6722 | 8.63266 | -1.78792 | 1.78792 | 27.6722 |
| 5.60 | -6.99225 | -1.76665 | -27.8267 | 8.75889 | -1.76664 | 1.76664 | 27.8267 |
| 5.61 | -7.14376 | -1.74527 | -27.9816 | 8.88903 | -1.74527 | 1.74527 | 27.9816 |
| 5.62 | -7.29947 | -1.72378 | -28.1368 | 9.02326 | -1.72378 | 1.72378 | 28.1368 |
| 5.63 | -7.45955 | -1.70219 | -28.2925 | 9.16174 | -1.70219 | 1.70219 | 28.2925 |
| 5.64 | -7.62422 | -1.68048 | -28.4486 | 9.30469 | -1.68048 | 1.68048 | 28.4486 |
| 5.65 | -7.79367 | -1.65866 | -28.6052 | 9.45233 | -1.65866 | 1.65866 | 28.6052 |
| 5.66 | -7.96815 | -1.63672 | -28.7622 | 9.60487 | -1.63672 | 1.63672 | 28.7622 |
| 5.67 | -8.14790 | -1.61467 | -28.9196 | 9.76257 | -1.61467 | 1.61467 | 28.9196 |
| 5.68 | -8.33166 | -1.59251 | -29.0774 | 9.92568 | -1.59251 | 1.59251 | 29.0774 |
| 5.69 | -8.52423 | -1.57023 | -29.2356 | 10.0945 | -1.57023 | 1.57023 | 29.2356 |
| 5.70 | -8.72139 | -1.54783 | -29.3943 | 10.2692 | -1.54783 | 1.54783 | 29.3943 |
| 5.71 | -8.92497 | -1.52532 | -29.5535 | 10.4503 | -1.52532 | 1.52532 | 29.5535 |
| 5.72 | -9.13528 | -1.50269 | -29.7130 | 10.6380 | -1.50269 | 1.50269 | 29.7130 |
| 5.73 | -9.35272 | -1.47993 | -29.8730 | 10.8327 | -1.47993 | 1.47993 | 29.8730 |
| 5.74 | -9.57765 | -1.45706 | -30.0335 | 11.0347 | -1.45706 | 1.45706 | 30.0335 |
| 5.75 | -9.81051 | -1.43407 | -30.1944 | 11.2446 | -1.43407 | 1.43407 | 30.1944 |
| 5.76 | -10.0517 | -1.41095 | -30.3557 | 11.4627 | -1.41095 | 1.41095 | 30.3557 |
| 5.77 | -10.3018 | -1.38771 | -30.5175 | 11.6895 | -1.38771 | 1.38771 | 30.5175 |
| 5.78 | -10.5613 | -1.36434 | -30.6797 | 11.9256 | -1.36434 | 1.36434 | 30.6797 |
| 5.79 | -10.8307 | -1.34085 | -30.8424 | 12.1715 | -1.34085 | 1.34085 | 30.8424 |
| 5.80 | -11.1106 | -1.31723 | -31.0055 | 12.4278 | -1.31723 | 1.31723 | 31.0055 |
| 5.81 | -11.4018 | -1.29348 | -31.1691 | 12.6953 | -1.29348 | 1.29348 | 31.1691 |
| 5.82 | -11.7049 | -1.26961 | -31.3332 | 12.9745 | -1.26961 | 1.26961 | 31.3332 |
| 5.83 | -12.0208 | -1.24560 | -31.4977 | 13.2664 | -1.24560 | 1.24560 | 31.4977 |
| 5.84 | -12.3502 | -1.22147 | -31.6627 | 13.5717 | -1.22147 | 1.22147 | 31.6627 |
| 5.85 | -12.6942 | -1.19720 | -31.8281 | 13.8914 | -1.19720 | 1.19720 | 31.8281 |
| 5.86 | -13.0537 | -1.17279 | -31.9940 | 14.2265 | -1.17279 | 1.17279 | 31.9940 |
| 5.87 | -13.4299 | -1.14826 | -32.1604 | 14.5781 | -1.14826 | 1.14826 | 32.1604 |
| 5.88 | -13.8240 | -1.12358 | -32.3272 | 14.9476 | -1.12358 | 1.12358 | 32.3272 |
| 5.89 | -14.2373 | -1.09877 | -32.4945 | 15.3361 | -1.09877 | 1.09877 | 32.4945 |
| 5.90 | -14.6719 | -1.07383 | -32.6623 | 15.7453 | -1.07383 | 1.07383 | 32.6623 |
| 5.91 | -15.1280 | -1.04874 | -32.8306 | 16.1768 | -1.04874 | 1.04874 | 32.8306 |
| 5.92 | -15.6089 | -1.02351 | -32.9994 | 16.6324 | -1.02351 | 1.02351 | 32.9994 |
| 5.93 | -16.1161 | -.998143 | -33.1686 | 17.1142 | -.998143 | .998143 | 33.1686 |
| 5.94 | -16.6519 | -.972631 | -33.3383 | 17.6845 | -.972631 | .972631 | 33.3383 |
| 5.95 | -17.2189 | -.946975 | -33.5086 | 18.1659 | -.946975 | .946975 | 33.5086 |
| 5.96 | -17.8201 | -.921174 | -33.6793 | 18.7412 | -.921174 | .921174 | 33.6793 |
| 5.97 | -18.4585 | -.895225 | -33.8505 | 19.3538 | -.895225 | .895225 | 33.8505 |
| 5.98 | -19.1380 | -.869126 | -34.0221 | 20.0071 | -.869126 | .869126 | 34.0221 |
| 5.99 | -19.8628 | -.842881 | -34.1944 | 20.7057 | -.842881 | .842881 | 34.1944 |
| 6.00 | -20.6375 | -.816483 | -34.3670 | 21.4540 | -.816483 | .816483 | 34.3670 |
| 6.01 | -21.4678 | -.789934 | -34.5402 | 22.2577 | -.789934 | .789934 | 34.5402 |
| 6.02 | -22.3598 | -.763230 | -34.7139 | 23.1230 | -.763230 | .763230 | 34.7139 |
| 6.03 | -23.3210 | -.736374 | -34.8882 | 24.0574 | -.736374 | .736374 | 34.8882 |
| 6.04 | -25.3598 | -.709359 | -35.0629 | 25.0692 | -.709359 | .709359 | 35.0629 |
| 6.05 | -25.4863 | -.682186 | -35.2381 | 26.1685 | -.682186 | .682186 | 35.2381 |
| 6.06 | -26.7121 | -.654853 | -35.4138 | 27.3669 | -.654853 | .654853 | 35.4138 |
| 6.07 | -28.0513 | -.627362 | -35.5902 | 28.6787 | -.627362 | .627362 | 35.5902 |
| 6.08 | -29.5205 | -.599706 | -35.7670 | 30.1202 | -.599706 | .599706 | 35.7670 |
| 6.09 | -31.1401 | -.571888 | -35.9443 | 31.7120 | -.571888 | .571888 | 35.9443 |
| 6.10 | -32.9343 | -.543900 | -36.1220 | 33.4782 | -.543900 | .543900 | 36.1220 |
| 6.11 | -34.9340 | -.515752 | -36.3006 | 35.4498 | -.515752 | .515752 | 36.3006 |
| 6.12 | -37.1765 | -.487433 | -36.4796 | 37.6639 | -.487433 | .487433 | 36.4796 |
| 6.13 | -39.7095 | -.458946 | -36.6593 | 40.1684 | -.458946 | .458946 | 36.6593 |
| 6.14 | -42.5931 | -.430281 | -36.8391 | 43.0234 | -.430281 | .430281 | 36.8391 |
| 6.15 | -45.9073 | -.401445 | -37.0196 | 46.3087 | -.401445 | .401445 | 37.0196 |
| 6.16 | -49.7565 | -.372434 | -37.2007 | 50.1289 | -.372424 | .372424 | 37.2007 |
| 6.17 | -54.2825 | -.343246 | -37.3824 | 54.6258 | -.343246 | .343246 | 37.3824 |
| 6.18 | -59.6822 | -.313880 | -37.5646 | 59.9960 | -.313880 | .313880 | 37.5646 |
| 6.19 | -66.2365 | -.284334 | -37.7474 | 66.5209 | -.284334 | .284334 | 37.7474 |
| 6.20 | -74.3621 | -.254605 | -37.9308 | 74.6167 | -.254605 | .254605 | 37.9308 |
| 6.21 | -84.7028 | -.224693 | -38.1147 | 84.9275 | -.224693 | .224693 | 38.1147 |
| 6.22 | -98.3097 | -.194593 | -38.2989 | 98.5043 | -.194593 | .194593 | 38.2989 |
| 6.23 | -117.028 | -.164309 | -38.4843 | 117.192 | -.164309 | .164309 | 38.4843 |
| 6.24 | -144.404 | -.133833 | -38.6700 | 144.538 | -.133833 | .133833 | 38.6700 |
| 6.25 | -188.268 | -.103166 | -38.8562 | 188.371 | -.103166 | .103166 | 38.8562 |
| 6.26 | -269.950 | -.0723062 | -39.0430 | 270.023 | -.0723062 | .0723062 | 39.0430 |
| 6.27 | -475.501 | -.0412474 | -39.2303 | 475.542 | -.0412474 | .0412474 | 39.2303 |
| 6.28 | -1971.76 | -.00999181 | -39.4227 | 1971.77 | -.00999181 | .00999181 | 39.4227 |
| $2\pi$ | $-\infty$ | 0 | -39.4784 | $\infty$ | 0 | 0 | 39.4784 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED

$$\begin{bmatrix} S_{F_R} = C_{F\theta_R}\theta + C_{F\delta_R}\delta & S_{C_R} = C_{C\theta_R}\theta + C_{C\delta_R}\delta \\ M_{F_R} = K_{F\theta_R}\theta + K_{F\delta_R}\delta & M_{C_R} = K_{C\theta_R}\theta + K_{C\delta_R}\delta \end{bmatrix}$$




| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\delta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_R}\left(\frac{L^3}{EI}\right)$ |
| 0 | 4.00000 | 6.00000 | 12.0000 | 2.00000 | 6.00000 | -6.00000 | -12.0000 |
| .1 | 3.99865 | 5.99901 | 11.9880 | 2.00036 | 5.99901 | -5.99901 | -11.9880 |
| .2 | 3.99465 | 5.99597 | 11.9519 | 2.00132 | 5.99597 | -5.99597 | -11.9519 |
| .3 | 3.98798 | 5.99099 | 11.8920 | 2.00300 | 5.99099 | -5.99099 | -11.8920 |
| .4 | 3.97862 | 5.98398 | 11.8080 | 2.00536 | 5.98398 | -5.98398 | -11.8080 |
| .5 | 3.96656 | 5.97495 | 11.6999 | 2.00840 | 5.97495 | -5.97495 | -11.6999 |
| .6 | 3.95177 | 5.96391 | 11.5678 | 2.01214 | 5.96391 | -5.96391 | -11.5678 |
| .7 | 3.93424 | 5.95083 | 11.4117 | 2.01658 | 5.95083 | -5.95083 | -11.4117 |
| .8 | 3.91394 | 5.93571 | 11.2314 | 2.02176 | 5.93571 | -5.93571 | -11.2314 |
| .9 | 3.89083 | 5.91853 | 11.0271 | 2.02769 | 5.91853 | -5.91853 | -11.0271 |
| 1.0 | 3.86488 | 5.89928 | 10.7986 | 2.03440 | 5.89928 | -5.89928 | -10.7986 |
| 1.01 | 3.86213 | 5.89724 | 10.7744 | 2.03511 | 5.89724 | -5.89724 | -10.7744 |
| 1.02 | 3.85935 | 5.89518 | 10.7500 | 2.03583 | 5.89518 | -5.89518 | -10.7500 |
| 1.03 | 3.85654 | 5.89310 | 10.7253 | 2.03656 | 5.89310 | -5.89310 | -10.7253 |
| 1.04 | 3.85370 | 5.89099 | 10.7004 | 2.03730 | 5.89099 | -5.89099 | -10.7004 |
| 1.05 | 3.85083 | 5.88887 | 10.6752 | 2.03804 | 5.88887 | -5.88887 | -10.6752 |
| 1.06 | 3.84793 | 5.88673 | 10.6499 | 2.03880 | 5.88673 | -5.88673 | -10.6499 |
| 1.07 | 3.84500 | 5.88456 | 10.6242 | 2.03956 | 5.88456 | -5.88456 | -10.6242 |
| 1.08 | 3.84204 | 5.88238 | 10.5984 | 2.04033 | 5.88238 | -5.88238 | -10.5984 |
| 1.09 | 3.83906 | 5.88017 | 10.5722 | 2.04111 | 5.88017 | -5.88017 | -10.5722 |
| 1.10 | 3.83604 | 5.87794 | 10.5459 | 2.04190 | 5.87794 | -5.87794 | -10.5459 |
| 1.11 | 3.83300 | 5.87569 | 10.5193 | 2.04269 | 5.87569 | -5.87569 | -10.5193 |
| 1.12 | 3.82992 | 5.87342 | 10.4924 | 2.04350 | 5.87342 | -5.87342 | -10.4924 |
| 1.13 | 3.82682 | 5.87113 | 10.4654 | 2.04431 | 5.87113 | -5.87113 | -10.4654 |
| 1.14 | 3.82369 | 5.86882 | 10.4380 | 2.04513 | 5.86882 | -5.86882 | -10.4380 |
| 1.15 | 3.82052 | 5.86648 | 10.4105 | 2.04596 | 5.86648 | -5.86648 | -10.4105 |
| 1.16 | 3.81733 | 5.86413 | 10.3827 | 2.04679 | 5.86413 | -5.86413 | -10.3827 |
| 1.17 | 3.81411 | 5.86175 | 10.3546 | 2.04764 | 5.86175 | -5.86175 | -10.3546 |
| 1.18 | 3.81086 | 5.85935 | 10.3263 | 2.04850 | 5.85935 | -5.85935 | -10.3263 |
| 1.19 | 3.80758 | 5.85693 | 10.2978 | 2.04936 | 5.85693 | -5.85693 | -10.2978 |
| 1.20 | 3.80426 | 5.85449 | 10.2690 | 2.05023 | 5.85449 | -5.85449 | -10.2690 |
| 1.21 | 3.80092 | 5.85203 | 10.2400 | 2.05111 | 5.85203 | -5.85203 | -10.2400 |
| 1.22 | 3.79755 | 5.84955 | 10.2107 | 2.05200 | 5.84955 | -5.84955 | -10.2107 |
| 1.23 | 3.79415 | 5.84705 | 10.1812 | 2.05290 | 5.84705 | -5.84705 | -10.1812 |
| 1.24 | 3.79072 | 5.84452 | 10.1514 | 2.05380 | 5.84452 | -5.84452 | -10.1514 |
| 1.25 | 3.78726 | 5.84197 | 10.1215 | 2.05472 | 5.84197 | -5.84197 | -10.1215 |
| 1.26 | 3.78377 | 5.83941 | 10.0912 | 2.05564 | 5.83941 | -5.83941 | -10.0912 |
| 1.27 | 3.78024 | 5.83682 | 10.0607 | 2.05658 | 5.83682 | -5.83682 | -10.0607 |
| 1.28 | 3.77669 | 5.83421 | 10.0300 | 2.05752 | 5.83421 | -5.83421 | -10.0300 |
| 1.29 | 3.77311 | 5.83157 | 9.99905 | 2.05847 | 5.83157 | -5.83157 | -9.99905 |
| 1.30 | 3.76949 | 5.82892 | 9.96784 | 2.05943 | 5.82892 | -5.82892 | -9.96784 |
| 1.31 | 3.76585 | 5.82625 | 9.93639 | 2.06040 | 5.82625 | -5.82625 | -9.93639 |
| 1.32 | 3.76217 | 5.82355 | 9.90470 | 2.06137 | 5.82355 | -5.82355 | -9.90470 |
| 1.33 | 3.75847 | 5.82083 | 9.87276 | 2.06236 | 5.82083 | -5.82083 | -9.87276 |
| 1.34 | 3.75473 | 5.81809 | 9.84058 | 2.06336 | 5.81809 | -5.81809 | -9.84058 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\delta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_R}\left(\frac{L^3}{EI}\right)$ |
| 1.35 | 3.75097 | 5.81533 | 9.80816 | 2.06436 | 5.81533 | -5.81533 | -9.80816 |
| 1.36 | 3.74717 | 5.81255 | 9.77549 | 2.06538 | 5.81255 | -5.81255 | -9.77549 |
| 1.37 | 3.74334 | 5.80974 | 9.74258 | 2.06640 | 5.80974 | -5.80974 | -9.74258 |
| 1.38 | 3.73948 | 5.80691 | 9.70943 | 2.06743 | 5.80691 | -5.80691 | -9.70943 |
| 1.39 | 3.73559 | 5.80407 | 9.67603 | 2.06848 | 5.80407 | -5.80407 | -9.67603 |
| 1.40 | 3.73167 | 5.80119 | 9.64239 | 2.06953 | 5.80119 | -5.80119 | -9.64239 |
| 1.41 | 3.72771 | 5.79830 | 9.60851 | 2.07059 | 5.79830 | -5.79830 | -9.60851 |
| 1.42 | 3.72373 | 5.79539 | 9.57438 | 2.07166 | 5.79539 | -5.79539 | -9.57438 |
| 1.43 | 3.71971 | 5.79245 | 9.54001 | 2.07274 | 5.79245 | -5.79245 | -9.54001 |
| 1.44 | 3.71567 | 5.78950 | 9.50539 | 2.07383 | 5.78950 | -5.78950 | -9.50539 |
| 1.45 | 3.71159 | 5.78652 | 9.47053 | 2.07493 | 5.78652 | -5.78652 | -9.47053 |
| 1.46 | 3.70748 | 5.78352 | 9.43543 | 2.07604 | 5.78352 | -5.78352 | -9.43543 |
| 1.47 | 3.70333 | 5.78049 | 9.40009 | 2.07716 | 5.78049 | -5.78049 | -9.40009 |
| 1.48 | 3.69916 | 5.77745 | 9.36450 | 2.07829 | 5.77745 | -5.77745 | -9.36450 |
| 1.49 | 3.69495 | 5.77438 | 9.32866 | 2.07943 | 5.77438 | -5.77438 | -9.32866 |
| 1.50 | 3.69072 | 5.77129 | 9.29259 | 2.08058 | 5.77129 | -5.77129 | -9.29259 |
| 1.51 | 3.68645 | 5.76818 | 9.25626 | 2.08173 | 5.76818 | -5.76818 | -9.25626 |
| 1.52 | 3.68214 | 5.76505 | 9.21969 | 2.08290 | 5.76505 | -5.76505 | -9.21969 |
| 1.53 | 3.67781 | 5.76189 | 9.18288 | 2.08408 | 5.76189 | -5.76189 | -9.18288 |
| 1.54 | 3.67344 | 5.75871 | 9.14583 | 2.08527 | 5.75871 | -5.75871 | -9.14583 |
| 1.55 | 3.66904 | 5.75551 | 9.10853 | 2.08647 | 5.75551 | -5.75551 | -9.10853 |
| 1.56 | 3.66461 | 5.75229 | 9.07098 | 2.08768 | 5.75229 | -5.75229 | -9.07098 |
| 1.57 | 3.66015 | 5.74905 | 9.03319 | 2.08890 | 5.74905 | -5.74905 | -9.03319 |
| 1.58 | 3.65565 | 5.74578 | 8.99516 | 2.09013 | 5.74578 | -5.74578 | -8.99516 |
| 1.59 | 3.65112 | 5.74249 | 8.95689 | 2.09137 | 5.74249 | -5.74249 | -8.95689 |
| 1.60 | 3.64656 | 5.73918 | 8.91836 | 2.09262 | 5.73918 | -5.73918 | -8.91836 |
| 1.61 | 3.64197 | 5.73585 | 8.87960 | 2.09388 | 5.73585 | -5.73585 | -8.87960 |
| 1.62 | 3.63734 | 5.73249 | 8.84059 | 2.09515 | 5.73249 | -5.73249 | -8.84059 |
| 1.63 | 3.63268 | 5.72911 | 8.80133 | 2.09643 | 5.72911 | -5.72911 | -8.80133 |
| 1.64 | 3.62799 | 5.72571 | 8.76183 | 2.09773 | 5.72571 | -5.72571 | -8.76183 |
| 1.65 | 3.62326 | 5.72229 | 8.72208 | 2.09903 | 5.72229 | -5.72229 | -8.72208 |
| 1.66 | 3.61850 | 5.71884 | 8.68209 | 2.10035 | 5.71884 | -5.71884 | -8.68209 |
| 1.67 | 3.61371 | 5.71538 | 8.64185 | 2.10167 | 5.71538 | -5.71538 | -8.64185 |
| 1.68 | 3.60888 | 5.71189 | 8.60137 | 2.10301 | 5.71189 | -5.71189 | -8.60137 |
| 1.69 | 3.60402 | 5.70837 | 8.56064 | 2.10435 | 5.70837 | -5.70837 | -8.56064 |
| 1.70 | 3.59912 | 5.70484 | 8.51967 | 2.10571 | 5.70484 | -5.70484 | -8.51967 |
| 1.71 | 3.59420 | 5.70128 | 8.47845 | 2.10708 | 5.70128 | -5.70128 | -8.47845 |
| 1.72 | 3.58923 | 5.69770 | 8.43699 | 2.10846 | 5.69770 | -5.69770 | -8.43699 |
| 1.73 | 3.58424 | 5.69409 | 8.39528 | 2.10985 | 5.69409 | -5.69409 | -8.39528 |
| 1.74 | 3.57921 | 5.69046 | 8.35333 | 2.11126 | 5.69046 | -5.69046 | -8.35333 |
| 1.75 | 3.57414 | 5.68681 | 8.31113 | 2.11267 | 5.68681 | -5.68681 | -8.31113 |
| 1.76 | 3.56905 | 5.68314 | 8.26868 | 2.11410 | 5.68314 | -5.68314 | -8.26868 |
| 1.77 | 3.56391 | 5.67945 | 8.22599 | 2.11553 | 5.67945 | -5.67945 | -8.22599 |
| 1.78 | 3.55875 | 5.67573 | 8.18305 | 2.11698 | 5.67573 | -5.67573 | -8.18305 |
| 1.79 | 3.55354 | 5.67199 | 8.13987 | 2.11844 | 5.67199 | -5.67199 | -8.13987 |
| 1.80 | 3.54831 | 5.66822 | 8.09644 | 2.11991 | 5.66822 | -5.66822 | -8.09644 |
| 1.81 | 3.54304 | 5.66443 | 8.05277 | 2.12140 | 5.66443 | -5.66443 | -8.05277 |
| 1.82 | 3.53773 | 5.66062 | 8.00884 | 2.12289 | 5.66062 | -5.66062 | -8.00884 |
| 1.83 | 3.53239 | 5.65679 | 7.96468 | 2.12440 | 5.65679 | -5.65679 | -7.96468 |
| 1.84 | 3.52701 | 5.65293 | 7.92026 | 2.12592 | 5.65293 | -5.65293 | -7.92026 |
| 1.85 | 3.52160 | 5.64905 | 7.87560 | 2.12745 | 5.64905 | -5.64905 | -7.87560 |
| 1.86 | 3.51615 | 5.64515 | 7.83070 | 2.12900 | 5.64515 | -5.64515 | -7.83070 |
| 1.87 | 3.51067 | 5.64122 | 7.78554 | 2.13055 | 5.64122 | -5.64122 | -7.78554 |
| 1.88 | 3.50515 | 5.63727 | 7.74014 | 2.13212 | 5.63727 | -5.63727 | -7.74014 |
| 1.89 | 3.49960 | 5.63330 | 7.69450 | 2.13370 | 5.63330 | -5.63330 | -7.69450 |
| 1.90 | 3.49401 | 5.62930 | 7.64860 | 2.13529 | 5.62930 | -5.62930 | -7.64860 |
| 1.91 | 3.48838 | 5.62528 | 7.60246 | 2.13690 | 5.62528 | -5.62528 | -7.60246 |
| 1.92 | 3.48272 | 5.62124 | 7.55607 | 2.13852 | 5.62124 | -5.62124 | -7.55607 |
| 1.93 | 3.47702 | 5.61717 | 7.50944 | 2.14015 | 5.61717 | -5.61717 | -7.50944 |
| 1.94 | 3.47129 | 5.61308 | 7.46256 | 2.14179 | 5.61308 | -5.61308 | -7.46256 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\delta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_R}\left(\frac{L^3}{EI}\right)$ |
| 1.95 | 3.46552 | 5.60896 | 7.41543 | 2.14345 | 5.60896 | -5.60896 | -7.41543 |
| 1.96 | 3.45971 | 5.60483 | 7.36805 | 2.14512 | 5.60483 | -5.60483 | -7.36805 |
| 1.97 | 3.45386 | 5.60067 | 7.32043 | 2.14680 | 5.60067 | -5.60067 | -7.32043 |
| 1.98 | 3.44798 | 5.59648 | 7.27256 | 2.14850 | 5.59648 | -5.59648 | -7.27256 |
| 1.99 | 3.44207 | 5.59227 | 7.22444 | 2.15020 | 5.59227 | -5.59227 | -7.22444 |
| 2.00 | 3.43611 | 5.58804 | 7.17608 | 2.15193 | 5.58804 | -5.58804 | -7.17608 |
| 2.01 | 3.43012 | 5.58378 | 7.12746 | 2.15366 | 5.58378 | -5.58378 | -7.12746 |
| 2.02 | 3.42409 | 5.57950 | 7.07860 | 2.15541 | 5.57950 | -5.57950 | -7.07860 |
| 2.03 | 3.41802 | 5.57520 | 7.02949 | 2.15717 | 5.57520 | -5.57520 | -7.02949 |
| 2.04 | 3.41192 | 5.57087 | 6.98013 | 2.15895 | 5.57087 | -5.57087 | -6.98013 |
| 2.05 | 3.40578 | 5.56651 | 6.93053 | 2.16074 | 5.56651 | -5.56651 | -6.93053 |
| 2.06 | 3.39960 | 5.56214 | 6.88068 | 2.16254 | 5.56214 | -5.56214 | -6.88068 |
| 2.07 | 3.39338 | 5.55774 | 6.83057 | 2.16436 | 5.55774 | -5.55774 | -6.83057 |
| 2.08 | 3.38712 | 5.55331 | 6.78022 | 2.16619 | 5.55331 | -5.55331 | -6.78022 |
| 2.09 | 3.38083 | 5.54886 | 6.72963 | 2.16803 | 5.54886 | -5.54886 | -6.72963 |
| 2.10 | 3.37450 | 5.54439 | 6.67878 | 2.16989 | 5.54439 | -5.54439 | -6.67878 |
| 2.11 | 3.36812 | 5.53989 | 6.62768 | 2.17177 | 5.53989 | -5.53989 | -6.62768 |
| 2.12 | 3.36171 | 5.53537 | 6.57634 | 2.17366 | 5.53537 | -5.53537 | -6.57634 |
| 2.13 | 3.35526 | 5.53082 | 6.52475 | 2.17556 | 5.53082 | -5.53082 | -6.52475 |
| 2.14 | 3.34878 | 5.52625 | 6.47291 | 2.17748 | 5.52625 | -5.52625 | -6.47291 |
| 2.15 | 3.34225 | 5.52166 | 6.42081 | 2.17941 | 5.52166 | -5.52166 | -6.42081 |
| 2.16 | 3.33568 | 5.51704 | 6.36848 | 2.18135 | 5.51704 | -5.51704 | -6.36848 |
| 2.17 | 3.32908 | 5.51239 | 6.31589 | 2.18332 | 5.51239 | -5.51239 | -6.31589 |
| 2.18 | 3.32243 | 5.50772 | 6.26305 | 2.18529 | 5.50772 | -5.50772 | -6.26305 |
| 2.19 | 3.31575 | 5.50303 | 6.20996 | 2.18728 | 5.50303 | -5.50303 | -6.20996 |
| 2.20 | 3.30902 | 5.49831 | 6.15662 | 2.18929 | 5.49831 | -5.49831 | -6.15662 |
| 2.21 | 3.30226 | 5.49357 | 6.10304 | 2.19131 | 5.49357 | -5.49357 | -6.10304 |
| 2.22 | 3.29545 | 5.48880 | 6.04920 | 2.19335 | 5.48880 | -5.48880 | -6.04920 |
| 2.23 | 3.28860 | 5.48401 | 5.99512 | 2.19540 | 5.48401 | -5.48401 | -5.99512 |
| 2.24 | 3.28172 | 5.47919 | 5.94078 | 2.19747 | 5.47919 | -5.47919 | -5.94078 |
| 2.25 | 3.27479 | 5.47435 | 5.88619 | 2.19956 | 5.47435 | -5.47435 | -5.88619 |
| 2.26 | 3.26782 | 5.46948 | 5.83136 | 2.20166 | 5.46948 | -5.46948 | -5.83136 |
| 2.27 | 3.26081 | 5.46459 | 5.77627 | 2.20377 | 5.46459 | -5.46459 | -5.77627 |
| 2.28 | 3.25376 | 5.45967 | 5.72094 | 2.20591 | 5.45967 | -5.45967 | -5.72094 |
| 2.29 | 3.24667 | 5.45473 | 5.66535 | 2.20806 | 5.45473 | -5.45473 | -5.66535 |
| 2.30 | 3.23954 | 5.44976 | 5.60951 | 2.21022 | 5.44976 | -5.44976 | -5.60951 |
| 2.31 | 3.23236 | 5.44476 | 5.55343 | 2.21240 | 5.44476 | -5.44476 | -5.55343 |
| 2.32 | 3.22514 | 5.43974 | 5.49709 | 2.21460 | 5.43974 | -5.43974 | -5.49709 |
| 2.33 | 3.21788 | 5.43470 | 5.44050 | 2.21682 | 5.43470 | -5.43470 | -5.44050 |
| 2.34 | 3.21058 | 5.42963 | 5.38366 | 2.21905 | 5.42963 | -5.42963 | -5.38366 |
| 2.35 | 3.20324 | 5.42453 | 5.32657 | 2.22130 | 5.42453 | -5.42453 | -5.32657 |
| 2.36 | 3.19585 | 5.41941 | 5.26923 | 2.22356 | 5.41941 | -5.41941 | -5.26923 |
| 2.37 | 3.18842 | 5.41427 | 5.31163 | 2.22585 | 5.41427 | -5.41427 | -5.31163 |
| 2.38 | 3.18095 | 5.40909 | 5.15379 | 2.22815 | 5.40909 | -5.40909 | -5.15379 |
| 2.39 | 3.17343 | 5.40390 | 5.09569 | 2.23047 | 5.40390 | -5.40390 | -5.09569 |
| 2.40 | 3.16587 | 5.39867 | 5.03734 | 2.23280 | 5.39867 | -5.39867 | -5.03734 |
| 2.41 | 3.15827 | 5.39342 | 4.97875 | 2.23515 | 5.39342 | -5.39342 | -4.97875 |
| 2.42 | 3.15062 | 5.38815 | 4.91989 | 2.23753 | 5.38815 | -5.38815 | -4.91989 |
| 2.43 | 3.14293 | 5.38285 | 4.86079 | 2.23992 | 5.38285 | -5.38285 | -4.86079 |
| 2.44 | 3.13519 | 5.37752 | 4.80144 | 2.24232 | 5.37752 | -5.37752 | -4.80144 |
| 2.45 | 3.12742 | 5.37216 | 4.74183 | 2.24475 | 5.37216 | -5.37216 | -4.74183 |
| 2.46 | 3.11959 | 5.36679 | 4.68197 | 2.24719 | 5.36679 | -5.36679 | -4.68197 |
| 2.47 | 3.11172 | 5.36138 | 4.62186 | 2.24966 | 5.36138 | -5.36138 | -4.62186 |
| 2.48 | 3.10381 | 5.35595 | 4.56149 | 2.25214 | 5.35595 | -5.35595 | -4.56149 |
| 2.49 | 3.09585 | 5.35049 | 4.50088 | 2.25464 | 5.35049 | -5.35049 | -4.50088 |
| 2.50 | 3.08784 | 5.34500 | 4.44001 | 2.25716 | 5.34500 | -5.34500 | -4.44001 |
| 2.51 | 3.07979 | 5.33949 | 4.37888 | 2.25970 | 5.33949 | -5.33949 | -4.37888 |
| 2.52 | 3.07170 | 5.33395 | 4.31751 | 2.26226 | 5.33395 | -5.33395 | -4.31751 |
| 2.53 | 3.06355 | 5.32839 | 4.25588 | 2.26484 | 5.32839 | -5.32839 | -4.25588 |
| 2.54 | 3.05536 | 5.32280 | 4.19400 | 2.26743 | 5.32280 | -5.32280 | -4.19400 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\theta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\theta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^3}{EI}\right)$ |
| 2.55 | 3.04713 | 5.31718 | 4.13186 | 2.27005 | 5.31718 | -5.31718 | -4.13186 |
| 2.56 | 3.03885 | 5.31154 | 4.06947 | 2.27269 | 5.31154 | -5.31154 | -4.06947 |
| 2.57 | 3.03052 | 5.30587 | 4.00683 | 2.27535 | 5.30587 | -5.30587 | -4.00683 |
| 2.58 | 3.02214 | 5.30017 | 3.94393 | 2.27803 | 5.30017 | -5.30017 | -3.94393 |
| 2.59 | 3.01372 | 5.29444 | 3.88079 | 2.28073 | 5.29444 | -5.29444 | -3.88079 |
| 2.60 | 3.00524 | 5.28869 | 3.81738 | 2.28344 | 5.28869 | -5.28869 | -3.81738 |
| 2.61 | 2.99673 | 5.28291 | 3.75372 | 2.28619 | 5.28291 | -5.28291 | -3.75372 |
| 2.62 | 2.98816 | 5.27711 | 3.68981 | 2.28895 | 5.27711 | -5.27711 | -3.68981 |
| 2.63 | 2.97954 | 5.27127 | 3.62564 | 2.29173 | 5.27127 | -5.27127 | -3.62564 |
| 2.64 | 2.97088 | 5.26541 | 3.56122 | 2.29453 | 5.26541 | -5.26541 | -3.56122 |
| 2.65 | 2.96216 | 5.25952 | 3.49654 | 2.29736 | 5.25952 | -5.25952 | -3.49654 |
| 2.66 | 2.95340 | 5.25361 | 3.43161 | 2.30021 | 5.25361 | -5.25361 | -3.43161 |
| 2.67 | 2.94459 | 5.24766 | 3.36643 | 2.30308 | 5.24766 | -5.24766 | -3.36643 |
| 2.68 | 2.93572 | 5.24169 | 3.30099 | 2.30597 | 5.24169 | -5.24169 | -3.30099 |
| 2.69 | 2.92681 | 5.23570 | 3.23529 | 2.30888 | 5.23570 | -5.23570 | -3.23529 |
| 2.70 | 2.91785 | 5.22967 | 3.16934 | 2.31182 | 5.22967 | -5.22967 | -3.16934 |
| 2.71 | 2.90884 | 5.22362 | 3.10313 | 2.31478 | 5.22362 | -5.22362 | -3.10313 |
| 2.72 | 2.89977 | 5.21753 | 3.03667 | 2.31776 | 5.21753 | -5.21753 | -3.03667 |
| 2.73 | 2.89066 | 5.21143 | 2.96995 | 2.32077 | 5.21143 | -5.21143 | -2.96995 |
| 2.74 | 2.88149 | 5.20529 | 2.90298 | 2.32380 | 5.20529 | -5.20529 | -2.90298 |
| 2.75 | 2.87228 | 5.19912 | 2.83575 | 2.32685 | 5.19912 | -5.19912 | -2.83575 |
| 2.76 | 2.86301 | 5.19293 | 2.76826 | 2.32992 | 5.19293 | -5.19293 | -2.76826 |
| 2.77 | 2.85368 | 5.18671 | 2.70052 | 2.33302 | 5.18671 | -5.18671 | -2.70052 |
| 2.78 | 2.84431 | 5.18046 | 2.63252 | 2.33615 | 5.18046 | -5.18046 | -2.63252 |
| 2.79 | 2.83488 | 5.17418 | 2.56426 | 2.33930 | 5.17418 | -5.17418 | -2.56426 |
| 2.80 | 2.82540 | 5.16787 | 2.49575 | 2.34247 | 5.16787 | -5.16787 | -2.49575 |
| 2.81 | 2.81587 | 5.16154 | 2.42698 | 2.34567 | 5.16154 | -5.16154 | -2.42698 |
| 2.82 | 2.80628 | 5.15517 | 2.35795 | 2.34889 | 5.15517 | -5.15517 | -2.35795 |
| 2.83 | 2.79664 | 5.14878 | 2.28866 | 2.35214 | 5.14878 | -5.14878 | -2.28866 |
| 2.84 | 2.78695 | 5.14236 | 2.21912 | 2.35541 | 5.14236 | -5.14236 | -2.21912 |
| 2.85 | 2.77720 | 5.13591 | 2.14932 | 2.35871 | 5.13591 | -5.13591 | -2.14932 |
| 2.86 | 2.76740 | 5.12943 | 2.07926 | 2.36204 | 5.12943 | -5.12943 | -2.07926 |
| 2.87 | 2.75754 | 5.12292 | 2.00895 | 2.36539 | 5.12292 | -5.12292 | -2.00895 |
| 2.88 | 2.74762 | 5.11639 | 1.93838 | 2.36877 | 5.11639 | -5.11639 | -1.93838 |
| 2.89 | 2.73765 | 5.10982 | 1.86754 | 2.37217 | 5.10982 | -5.10982 | -1.86754 |
| 2.90 | 2.72763 | 5.10323 | 1.79645 | 2.37560 | 5.10323 | -5.10323 | -1.79645 |
| 2.91 | 2.71754 | 5.09660 | 1.72511 | 2.37906 | 5.09660 | -5.09660 | -1.72511 |
| 2.92 | 2.70740 | 5.08995 | 1.65350 | 2.38255 | 5.08995 | -5.08995 | -1.65350 |
| 2.93 | 2.69721 | 5.08326 | 1.58163 | 2.38606 | 5.08326 | -5.08326 | -1.58163 |
| 2.94 | 2.68695 | 5.07656 | 1.50951 | 2.38960 | 5.07656 | -5.07656 | -1.50951 |
| 2.95 | 2.67664 | 5.06981 | 1.43712 | 2.39317 | 5.06981 | -5.06981 | -1.43712 |
| 2.96 | 2.66627 | 5.06304 | 1.36448 | 2.39677 | 5.06304 | -5.06304 | -1.36448 |
| 2.97 | 2.65584 | 5.05624 | 1.29158 | 2.40040 | 5.05624 | -5.05624 | -1.29158 |
| 2.98 | 2.64535 | 5.04941 | 1.21841 | 2.40406 | 5.04941 | -5.04941 | -1.21841 |
| 2.99 | 2.63481 | 5.04255 | 1.14499 | 2.40774 | 5.04255 | -5.04255 | -1.14499 |
| 3.00 | 2.62420 | 5.03565 | 1.07131 | 2.41145 | 5.03565 | -5.03565 | -1.07131 |
| 3.01 | 2.61353 | 5.02873 | .997365 | 2.41520 | 5.02873 | -5.02873 | -.997365 |
| 3.02 | 2.60280 | 5.02178 | .923161 | 2.41897 | 5.02178 | -5.02178 | -.923161 |
| 3.03 | 2.59202 | 5.01480 | .848697 | 2.42278 | 5.01480 | -5.01480 | -.848697 |
| 3.04 | 2.58117 | 5.00778 | .773971 | 2.42661 | 5.00778 | -5.00778 | -.773971 |
| 3.05 | 2.57026 | 5.00074 | .698986 | 2.43048 | 5.00074 | -5.00074 | -.698986 |
| 3.06 | 2.55929 | 4.99367 | .623740 | 2.43438 | 4.99367 | -4.99367 | -.623740 |
| 3.07 | 2.54825 | 4.98656 | .548231 | 2.43831 | 4.98656 | -4.98656 | -.548231 |
| 3.08 | 2.53715 | 4.97943 | .472461 | 2.44227 | 4.97943 | -4.97943 | -.472461 |
| 3.09 | 2.52600 | 4.97226 | .396430 | 2.44627 | 4.97226 | -4.97226 | -.396430 |
| 3.10 | 2.51478 | 4.96507 | .320138 | 2.45030 | 4.96507 | -4.96507 | -.320138 |
| 3.11 | 2.50349 | 4.95784 | .243582 | 2.45436 | 4.95784 | -4.95784 | -.243582 |
| 3.12 | 2.49214 | 4.95059 | .166765 | 2.45845 | 4.95059 | -4.95059 | -.166765 |
| 3.13 | 2.48073 | 4.94331 | .0896851 | 2.46259 | 4.94331 | -4.94331 | -.0896851 |
| 3.14 | 2.46897 | 4.93543 | .0123409 | 2.46647 | 4.93543 | -4.93543 | -.0123409 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED.- Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\delta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_R}\left(\frac{L^3}{EI}\right)$ |
| $\pi$ | 2.46740 | 4.93480 | 0 | 2.46740 | 4.93480 | -4.93480 | 0 |
| 3.15 | 2.45772 | 4.92867 | -.0652638 | 2.47096 | 4.92867 | -4.92867 | .0652638 |
| 3.16 | 2.44608 | 4.92125 | -.143132 | 2.47517 | 4.92125 | -4.92125 | .143132 |
| 3.17 | 2.43439 | 4.91381 | -.221263 | 2.47942 | 4.91381 | -4.91381 | .221263 |
| 3.18 | 2.42265 | 4.90638 | -.299658 | 2.48373 | 4.90638 | -4.90638 | .299658 |
| 3.19 | 2.41083 | 4.89889 | -.378316 | 2.48806 | 4.89889 | -4.89889 | .378316 |
| 3.20 | 2.39895 | 4.89138 | -.457239 | 2.49243 | 4.89138 | -4.89138 | .457239 |
| 3.21 | 2.38700 | 4.88384 | -.536424 | 2.49684 | 4.88384 | -4.88384 | .536424 |
| 3.22 | 2.37498 | 4.87626 | -.615875 | 2.50128 | 4.87626 | -4.87626 | .615875 |
| 3.23 | 2.36289 | 4.86866 | -.695590 | 2.50576 | 4.86866 | -4.86866 | .695590 |
| 3.24 | 2.35073 | 4.86101 | -.775568 | 2.51028 | 4.86101 | -4.86101 | .775568 |
| 3.25 | 2.33850 | 4.85334 | -.855811 | 2.51484 | 4.85334 | -4.85334 | .855811 |
| 3.26 | 2.32620 | 4.84564 | -.936321 | 2.51943 | 4.84564 | -4.84564 | .936321 |
| 3.27 | 2.31383 | 4.83790 | -1.01710 | 2.52407 | 4.83790 | -4.83790 | 1.01710 |
| 3.28 | 2.30139 | 4.83013 | -1.09814 | 2.52874 | 4.83013 | -4.83013 | 1.09814 |
| 3.29 | 2.28887 | 4.82233 | -1.17944 | 2.53345 | 4.82233 | -4.82233 | 1.17944 |
| 3.30 | 2.27629 | 4.81449 | -1.26101 | 2.53821 | 4.81449 | -4.81449 | 1.26101 |
| 3.31 | 2.26362 | 4.80662 | -1.34285 | 2.54300 | 4.80662 | -4.80662 | 1.34285 |
| 3.32 | 2.25089 | 4.79873 | -1.42495 | 2.54783 | 4.79873 | -4.79873 | 1.42495 |
| 3.33 | 2.23808 | 4.79079 | -1.50732 | 2.55271 | 4.79079 | -4.79079 | 1.50732 |
| 3.34 | 2.22520 | 4.78282 | -1.58996 | 2.55763 | 4.78282 | -4.78282 | 1.58996 |
| 3.35 | 2.21223 | 4.77482 | -1.67286 | 2.56258 | 4.77482 | -4.77482 | 1.67286 |
| 3.36 | 2.19920 | 4.76678 | -1.75603 | 2.56759 | 4.76678 | -4.76678 | 1.75603 |
| 3.37 | 2.18608 | 4.75871 | -1.83947 | 2.57263 | 4.75871 | -4.75871 | 1.83947 |
| 3.38 | 2.17289 | 4.75061 | -1.92318 | 2.57772 | 4.75061 | -4.75061 | 1.92318 |
| 3.39 | 2.15962 | 4.74247 | -2.00715 | 2.58285 | 4.74247 | -4.74247 | 2.00715 |
| 3.40 | 2.14627 | 4.73430 | -2.09139 | 2.58803 | 4.73430 | -4.73430 | 2.09139 |
| 3.41 | 2.13285 | 4.72610 | -2.17590 | 2.59325 | 4.72610 | -4.72610 | 2.17590 |
| 3.42 | 2.11934 | 4.71786 | -2.26068 | 2.59852 | 4.71786 | -4.71786 | 2.26068 |
| 3.43 | 2.10575 | 4.70958 | -2.34573 | 2.60383 | 4.70958 | -4.70958 | 2.34573 |
| 3.44 | 2.09209 | 4.70127 | -2.43105 | 2.60919 | 4.70127 | -4.70127 | 2.43105 |
| 3.45 | 2.07834 | 4.69293 | -2.51663 | 2.61460 | 4.69293 | -4.69293 | 2.51663 |
| 3.46 | 2.06450 | 4.68455 | -2.60249 | 2.62005 | 4.68455 | -4.68455 | 2.60249 |
| 3.47 | 2.05059 | 4.67614 | -2.68861 | 2.62555 | 4.67614 | -4.67614 | 2.68861 |
| 3.48 | 2.03659 | 4.66770 | -2.77501 | 2.63111 | 4.66770 | -4.66770 | 2.77501 |
| 3.49 | 2.02251 | 4.65921 | -2.86167 | 2.63671 | 4.65921 | -4.65921 | 2.86167 |
| 3.50 | 2.00834 | 4.65070 | -2.94861 | 2.64236 | 4.65070 | -4.65070 | 2.94861 |
| 3.51 | 1.99409 | 4.64214 | -3.03581 | 2.64806 | 4.64214 | -4.64214 | 3.03581 |
| 3.52 | 1.97975 | 4.63355 | -3.12329 | 2.65381 | 4.63355 | -4.63355 | 3.12329 |
| 3.53 | 1.96532 | 4.62493 | -3.21104 | 2.65961 | 4.62493 | -4.62493 | 3.21104 |
| 3.54 | 1.95081 | 4.61627 | -3.29906 | 2.66546 | 4.61627 | -4.61627 | 3.29906 |
| 3.55 | 1.93620 | 4.60758 | -3.38735 | 2.67137 | 4.60758 | -4.60758 | 3.38735 |
| 3.56 | 1.92151 | 4.59884 | -3.47591 | 2.67733 | 4.59884 | -4.59884 | 3.47591 |
| 3.57 | 1.90673 | 4.59007 | -3.56475 | 2.68335 | 4.59007 | -4.59007 | 3.56475 |
| 3.58 | 1.89185 | 4.58127 | -3.65386 | 2.68942 | 4.58127 | -4.58127 | 3.65386 |
| 3.59 | 1.87689 | 4.57243 | -3.74324 | 2.69554 | 4.57243 | -4.57243 | 3.74324 |
| 3.60 | 1.86183 | 4.56355 | -3.83289 | 2.70172 | 4.56355 | -4.56355 | 3.83289 |
| 3.61 | 1.84668 | 4.55464 | -3.92282 | 2.70796 | 4.55464 | -4.55464 | 3.92282 |
| 3.62 | 1.83143 | 4.54569 | -4.01302 | 2.71425 | 4.54569 | -4.54569 | 4.01302 |
| 3.63 | 1.81609 | 4.53670 | -4.10350 | 2.72061 | 4.53670 | -4.53670 | 4.10350 |
| 3.64 | 1.80066 | 4.52768 | -4.19425 | 2.72702 | 4.52768 | -4.52768 | 4.19425 |
| 3.65 | 1.78513 | 4.51862 | -4.28527 | 2.73349 | 4.51862 | -4.51862 | 4.28527 |
| 3.66 | 1.76950 | 4.50952 | -4.37657 | 2.74002 | 4.50952 | -4.50952 | 4.37657 |
| 3.67 | 1.75377 | 4.50038 | -4.46814 | 2.74661 | 4.50038 | -4.50038 | 4.46814 |
| 3.68 | 1.73794 | 4.49120 | -4.55999 | 2.75326 | 4.49120 | -4.49120 | 4.55999 |
| 3.69 | 1.72202 | 4.48199 | -4.65211 | 2.75997 | 4.48199 | -4.48199 | 4.65211 |
| 3.70 | 1.70599 | 4.47274 | -4.74451 | 2.76675 | 4.47274 | -4.47274 | 4.74451 |
| 3.71 | 1.68986 | 4.46345 | -4.83719 | 2.77359 | 4.46345 | -4.46345 | 4.83719 |
| 3.72 | 1.67363 | 4.45413 | -4.93014 | 2.78050 | 4.45413 | -4.45413 | 4.93014 |
| 3.73 | 1.65729 | 4.44764 | -5.02337 | 2.78747 | 4.44764 | -4.44764 | 5.02337 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\delta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_R}\left(\frac{L^3}{EI}\right)$ |
| 3.74 | 1.64085 | 4.43536 | -5.11688 | 2.79451 | 4.43536 | -4.43536 | 5.11688 |
| 3.75 | 1.62431 | 4.42592 | -5.21066 | 2.80161 | 4.42592 | -4.42592 | 5.21066 |
| 3.76 | 1.60765 | 4.41644 | -5.30472 | 2.80879 | 4.41644 | -4.41644 | 5.30472 |
| 3.77 | 1.59089 | 4.40692 | -5.39906 | 2.81603 | 4.40692 | -4.40692 | 5.39906 |
| 3.78 | 1.57402 | 4.39736 | -5.49368 | 2.82334 | 4.39736 | -4.39736 | 5.49368 |
| 3.79 | 1.55705 | 4.38776 | -5.58857 | 2.83072 | 4.38776 | -4.38776 | 5.58857 |
| 3.80 | 1.53996 | 4.37813 | -5.68374 | 2.83817 | 4.37813 | -4.37813 | 5.68374 |
| 3.81 | 1.52275 | 4.36845 | -5.77920 | 2.84570 | 4.36845 | -4.36845 | 5.77920 |
| 3.82 | 1.50544 | 4.35873 | -5.87493 | 2.85329 | 4.35873 | -4.35873 | 5.87493 |
| 3.83 | 1.48802 | 4.34898 | -5.97095 | 2.86097 | 4.34898 | -4.34898 | 5.97095 |
| 3.84 | 1.47047 | 4.33918 | -6.06724 | 2.86871 | 4.33918 | -4.33918 | 6.06724 |
| 3.85 | 1.45281 | 4.32934 | -6.16381 | 2.87654 | 4.32934 | -4.32934 | 6.16381 |
| 3.86 | 1.43503 | 4.31947 | -6.26067 | 2.88444 | 4.31947 | -4.31947 | 6.26067 |
| 3.87 | 1.41713 | 4.30955 | -6.35780 | 2.89242 | 4.30955 | -4.30955 | 6.35780 |
| 3.88 | 1.39912 | 4.29959 | -6.45522 | 2.90047 | 4.29959 | -4.29959 | 6.45522 |
| 3.89 | 1.38098 | 4.28959 | -6.55292 | 2.90861 | 4.28959 | -4.28959 | 6.55292 |
| 3.90 | 1.36272 | 4.27955 | -6.65089 | 2.91683 | 4.27955 | -4.27955 | 6.65089 |
| 3.91 | 1.34434 | 4.26947 | -6.74916 | 2.92513 | 4.26947 | -4.26947 | 6.74916 |
| 3.92 | 1.32583 | 4.25935 | -6.84770 | 2.93352 | 4.25935 | -4.25935 | 6.84770 |
| 3.93 | 1.30720 | 4.24919 | -6.94653 | 2.94199 | 4.24919 | -4.24919 | 6.94653 |
| 3.94 | 1.28844 | 4.23898 | -7.04564 | 2.95054 | 4.23898 | -4.23898 | 7.04564 |
| 3.95 | 1.26955 | 4.22873 | -7.14504 | 2.95919 | 4.22873 | -4.22873 | 7.14504 |
| 3.96 | 1.25052 | 4.21844 | -7.24472 | 2.96792 | 4.21844 | -4.21844 | 7.24472 |
| 3.97 | 1.23137 | 4.20811 | -7.34468 | 2.97674 | 4.20811 | -4.20811 | 7.34468 |
| 3.98 | 1.21209 | 4.19774 | -7.44493 | 2.98565 | 4.19774 | -4.19774 | 7.44493 |
| 3.99 | 1.19267 | 4.18732 | -7.54546 | 2.99465 | 4.18732 | -4.18732 | 7.54546 |
| 4.00 | 1.17311 | 4.17686 | -7.64628 | 3.00374 | 4.17686 | -4.17686 | 7.64628 |
| 4.01 | 1.15342 | 4.16636 | -7.74739 | 3.01293 | 4.16636 | -4.16636 | 7.74739 |
| 4.02 | 1.13359 | 4.15581 | -7.84878 | 3.02222 | 4.15581 | -4.15581 | 7.84878 |
| 4.03 | 1.11362 | 4.14522 | -7.95046 | 3.03160 | 4.14522 | -4.14522 | 7.95046 |
| 4.04 | 1.09351 | 4.13459 | -8.05242 | 3.04108 | 4.13459 | -4.13459 | 8.05242 |
| 4.05 | 1.07325 | 4.12391 | -8.15468 | 3.05066 | 4.12391 | -4.12391 | 8.15468 |
| 4.06 | 1.05285 | 4.11319 | -8.25722 | 3.06034 | 4.11319 | -4.11319 | 8.25722 |
| 4.07 | 1.03230 | 4.10243 | -8.36005 | 3.07013 | 4.10243 | -4.10243 | 8.36005 |
| 4.08 | 1.01160 | 4.09162 | -8.46316 | 3.08002 | 4.09162 | -4.09162 | 8.46316 |
| 4.09 | .990754 | 4.08076 | -8.56657 | 3.09001 | 4.08076 | -4.08076 | 8.56657 |
| 4.10 | .969755 | 4.06987 | -8.67027 | 3.10011 | 4.06987 | -4.06987 | 8.67027 |
| 4.11 | .948603 | 4.05892 | -8.77425 | 3.11032 | 4.05892 | -4.05892 | 8.77425 |
| 4.12 | .927295 | 4.04794 | -8.87853 | 3.12064 | 4.04794 | -4.04794 | 8.87853 |
| 4.13 | .905830 | 4.03690 | -8.98309 | 3.13107 | 4.03690 | -4.03690 | 8.98309 |
| 4.14 | .884206 | 4.02582 | -9.08795 | 3.14162 | 4.02582 | -4.02582 | 9.08795 |
| 4.15 | .862420 | 4.01470 | -9.19310 | 3.15228 | 4.01470 | -4.01470 | 9.19310 |
| 4.16 | .840473 | 4.00353 | -9.29854 | 3.16306 | 4.00353 | -4.00353 | 9.29854 |
| 4.17 | .818360 | 3.99231 | -9.40427 | 3.17395 | 3.99231 | -3.99231 | 9.40427 |
| 4.18 | .796080 | 3.98105 | -9.51030 | 3.18497 | 3.98105 | -3.98105 | 9.51030 |
| 4.19 | .773632 | 3.96974 | -9.61661 | 3.19611 | 3.96974 | -3.96974 | 9.61661 |
| 4.20 | .751013 | 3.95839 | -9.72323 | 3.20737 | 3.95839 | -3.95839 | 9.72323 |
| 4.21 | .728221 | 3.94698 | -9.83013 | 3.21876 | 3.94698 | -3.94698 | 9.83013 |
| 4.22 | .705254 | 3.93553 | -9.93733 | 3.23028 | 3.93553 | -3.93553 | 9.93733 |
| 4.23 | .682109 | 3.92404 | -10.0448 | 3.24193 | 3.92404 | -3.92404 | 10.0448 |
| 4.24 | .658785 | 3.91249 | -10.1526 | 3.25371 | 3.91249 | -3.91249 | 10.1526 |
| 4.25 | .635279 | 3.90090 | -10.2607 | 3.26562 | 3.90090 | -3.90090 | 10.2607 |
| 4.26 | .611589 | 3.88926 | -10.3691 | 3.27767 | 3.88926 | -3.88926 | 10.3691 |
| 4.27 | .587712 | 3.87757 | -10.4778 | 3.28986 | 3.87757 | -3.87757 | 10.4778 |
| 4.28 | .563647 | 3.86583 | -10.5867 | 3.30219 | 3.86583 | -3.86583 | 10.5867 |
| 4.29 | .539390 | 3.85405 | -10.6960 | 3.31466 | 3.85405 | -3.85405 | 10.6960 |
| 4.30 | .514939 | 3.84221 | -10.8056 | 3.32727 | 3.84221 | -3.84221 | 10.8056 |
| 4.31 | .490292 | 3.83033 | -10.9154 | 3.34004 | 3.83033 | -3.83033 | 10.9154 |
| 4.32 | .465446 | 3.81839 | -11.0256 | 3.35295 | 3.81839 | -3.81839 | 11.0256 |
| 4.33 | .440399 | 3.80641 | -11.1361 | 3.36601 | 3.80641 | -3.80641 | 11.1361 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\delta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_R}\left(\frac{L^3}{EI}\right)$ |
| 4.34 | .415147 | 3.79438 | -11.2468 | 3.37923 | 3.79438 | -3.79438 | 11.2468 |
| 4.35 | .389688 | 3.78229 | -11.3579 | 3.39261 | 3.78229 | -3.78229 | 11.3579 |
| 4.36 | .364019 | 3.77016 | -11.4693 | 3.40614 | 3.77016 | -3.77016 | 11.4693 |
| 4.37 | .338138 | 3.75797 | -11.5809 | 3.41984 | 3.75797 | -3.75797 | 11.5809 |
| 4.38 | .312040 | 3.74574 | -11.6929 | 3.43370 | 3.74574 | -3.74574 | 11.6929 |
| 4.39 | .285724 | 3.73345 | -11.8052 | 3.44773 | 3.73345 | -3.73345 | 11.8052 |
| 4.40 | .259187 | 3.72111 | -11.9178 | 3.46193 | 3.72111 | -3.72111 | 11.9178 |
| 4.41 | .232424 | 3.70873 | -12.0306 | 3.47630 | 3.70873 | -3.70873 | 12.0306 |
| 4.42 | .205434 | 3.69628 | -12.1438 | 3.49085 | 3.69628 | -3.69628 | 12.1438 |
| 4.43 | .178213 | 3.68379 | -12.2573 | 3.50558 | 3.68379 | -3.68379 | 12.2573 |
| 4.44 | .150756 | 3.67125 | -12.3711 | 3.52049 | 3.67125 | -3.67125 | 12.3711 |
| 4.45 | .123062 | 3.65865 | -12.4852 | 3.53558 | 3.65865 | -3.65865 | 12.4852 |
| 4.46 | .0951270 | 3.64599 | -12.5996 | 3.55087 | 3.64599 | -3.64599 | 12.5996 |
| 4.47 | .0669467 | 3.63329 | -12.7143 | 3.56634 | 3.63329 | -3.63329 | 12.7143 |
| 4.48 | .0385179 | 3.62053 | -12.8293 | 3.58201 | 3.62053 | -3.62053 | 12.8293 |
| 4.49 | .00983696 | 3.60772 | -12.9447 | 3.59788 | 3.60772 | -3.60772 | 12.9447 |
| 4.50 | -.0190999 | 3.59485 | -13.0603 | 3.61395 | 3.59485 | -3.59485 | 13.0603 |
| 4.51 | -.0482966 | 3.58193 | -13.1762 | 3.63023 | 3.58193 | -3.58193 | 13.1762 |
| 4.52 | -.0777570 | 3.56896 | -13.2925 | 3.64672 | 3.56896 | -3.56896 | 13.2925 |
| 4.53 | -.107485 | 3.55593 | -13.4090 | 3.66341 | 3.55593 | -3.55593 | 13.4090 |
| 4.54 | -.137485 | 3.54284 | -13.5259 | 3.68033 | 3.54284 | -3.54284 | 13.5259 |
| 4.55 | -.167761 | 3.52970 | -13.6431 | 3.69746 | 3.52970 | -3.52970 | 13.6431 |
| 4.56 | -.198317 | 3.51651 | -13.7606 | 3.71482 | 3.51651 | -3.51651 | 13.7606 |
| 4.57 | -.229158 | 3.50325 | -13.8784 | 3.73241 | 3.50325 | -3.50325 | 13.8784 |
| 4.58 | -.260288 | 3.48995 | -13.9965 | 3.75023 | 3.48995 | -3.48995 | 13.9965 |
| 4.59 | -.291712 | 3.47658 | -14.1149 | 3.76829 | 3.47658 | -3.47658 | 14.1149 |
| 4.60 | -.323435 | 3.46316 | -14.2337 | 3.78659 | 3.46316 | -3.46316 | 14.2337 |
| 4.61 | -.355461 | 3.44968 | -14.3527 | 3.80514 | 3.44968 | -3.44968 | 14.3527 |
| 4.62 | -.387795 | 3.43614 | -14.4721 | 3.82394 | 3.43614 | -3.43614 | 14.4721 |
| 4.63 | -.420442 | 3.42255 | -14.5918 | 3.84299 | 3.42255 | -3.42255 | 14.5918 |
| 4.64 | -.453408 | 3.40890 | -14.7118 | 3.86230 | 3.40890 | -3.40890 | 14.7118 |
| 4.65 | -.486697 | 3.39518 | -14.8321 | 3.88188 | 3.39518 | -3.39518 | 14.8321 |
| 4.66 | -.520316 | 3.38141 | -14.9528 | 3.90173 | 3.38141 | -3.38141 | 14.9528 |
| 4.67 | -.554269 | 3.36759 | -15.0737 | 3.92185 | 3.36759 | -3.36759 | 15.0737 |
| 4.68 | -.588563 | 3.35370 | -15.1950 | 3.94226 | 3.35370 | -3.35370 | 15.1950 |
| 4.69 | -.623202 | 3.33975 | -15.3166 | 3.96295 | 3.33975 | -3.33975 | 15.3166 |
| 4.70 | -.658194 | 3.32574 | -15.4385 | 3.98394 | 3.32574 | -3.32574 | 15.4385 |
| 4.71 | -.693545 | 3.31167 | -15.5608 | 4.00522 | 3.31167 | -3.31167 | 15.5608 |
| 4.72 | -.729259 | 3.29754 | -15.6833 | 4.02680 | 3.29754 | -3.29754 | 15.6833 |
| 4.73 | -.765345 | 3.28335 | -15.8062 | 4.04870 | 3.28335 | -3.28335 | 15.8062 |
| 4.74 | -.801808 | 3.26910 | -15.9294 | 4.07091 | 3.26910 | -3.26910 | 15.9294 |
| 4.75 | -.838656 | 3.25479 | -16.0529 | 4.09344 | 3.25479 | -3.25479 | 16.0529 |
| 4.76 | -.875895 | 3.24041 | -16.1768 | 4.11631 | 3.24041 | -3.24041 | 16.1768 |
| 4.77 | -.913532 | 3.22597 | -16.3010 | 4.13950 | 3.22597 | -3.22597 | 16.3010 |
| 4.78 | -.951574 | 3.21147 | -16.4255 | 4.16305 | 3.21147 | -3.21147 | 16.4255 |
| 4.79 | -.990030 | 3.19691 | -16.5503 | 4.18694 | 3.19691 | -3.19691 | 16.5503 |
| 4.80 | -1.02891 | 3.18228 | -16.6754 | 4.21119 | 3.18228 | -3.18228 | 16.6754 |
| 4.81 | -1.06821 | 3.16759 | -16.8009 | 4.23580 | 3.16759 | -3.16759 | 16.8009 |
| 4.82 | -1.10795 | 3.15283 | -16.9267 | 4.26079 | 3.15283 | -3.15283 | 16.9267 |
| 4.83 | -1.14814 | 3.13801 | -17.0529 | 4.28615 | 3.13801 | -3.13801 | 17.0529 |
| 4.84 | -1.18878 | 3.12313 | -17.1793 | 4.31191 | 3.12313 | -3.12313 | 17.1793 |
| 4.85 | -1.22989 | 3.10818 | -17.3061 | 4.33806 | 3.10818 | -3.10818 | 17.3061 |
| 4.86 | -1.27146 | 3.09316 | -17.4333 | 4.36462 | 3.09316 | -3.09316 | 17.4333 |
| 4.87 | -1.31352 | 3.07808 | -17.5607 | 4.39159 | 3.07808 | -3.07808 | 17.5607 |
| 4.88 | -1.35606 | 3.06293 | -17.6885 | 4.41899 | 3.06293 | -3.06293 | 17.6885 |
| 4.89 | -1.39911 | 3.04771 | -17.8167 | 4.44682 | 3.04771 | -3.04771 | 17.8167 |
| 4.90 | -1.44266 | 3.03243 | -17.9451 | 4.47509 | 3.03243 | -3.03243 | 17.9451 |
| 4.91 | -1.48674 | 3.01708 | -18.0740 | 4.50381 | 3.01708 | -3.01708 | 18.0740 |
| 4.92 | -1.53135 | 3.00166 | -18.2031 | 4.53300 | 3.00166 | -3.00166 | 18.2031 |
| 4.93 | -1.57650 | 2.98617 | -18.3326 | 4.56266 | 2.98617 | -2.98617 | 18.3326 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

| $\frac{L}{j}$ | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\delta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\delta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\delta_R}\left(\frac{L^3}{EI}\right)$ |
| 4.94 | -1.62220 | 2.97061 | -18.4624 | 4.59281 | 2.97061 | -2.97061 | 18.4624 |
| 4.95 | -1.66847 | 2.95498 | -18.5925 | 4.62345 | 2.95498 | -2.95498 | 18.5925 |
| 4.96 | -1.71531 | 2.93929 | -18.7230 | 4.65460 | 2.93929 | -2.93929 | 18.7230 |
| 4.97 | -1.76275 | 2.92352 | -18.8539 | 4.68627 | 2.92352 | -2.92352 | 18.8539 |
| 4.98 | -1.81079 | 2.90769 | -18.9850 | 4.71847 | 2.90769 | -2.90769 | 18.9850 |
| 4.99 | -1.85944 | 2.89178 | -19.1165 | 4.75122 | 2.89178 | -2.89178 | 19.1165 |
| 5.00 | -1.90872 | 2.87580 | -19.2484 | 4.78452 | 2.87580 | -2.87580 | 19.2484 |
| 5.01 | -1.95865 | 2.85975 | -19.3806 | 4.81839 | 2.85975 | -2.85975 | 19.3806 |
| 5.02 | -2.00923 | 2.84362 | -19.5132 | 4.85285 | 2.84362 | -2.84362 | 19.5132 |
| 5.03 | -2.06048 | 2.82743 | -19.6461 | 4.88791 | 2.82743 | -2.82743 | 19.6461 |
| 5.04 | -2.11242 | 2.81116 | -19.7793 | 4.92358 | 2.81116 | -2.81116 | 19.7793 |
| 5.05 | -2.16507 | 2.79481 | -19.9129 | 4.95988 | 2.79481 | -2.79481 | 19.9129 |
| 5.06 | -2.21843 | 2.77839 | -20.0468 | 4.99682 | 2.77839 | -2.77839 | 20.0468 |
| 5.07 | -2.27253 | 2.76190 | -20.1811 | 5.03443 | 2.76190 | -2.76190 | 20.1811 |
| 5.08 | -2.32738 | 2.74533 | -20.3157 | 5.07272 | 2.74533 | -2.74533 | 20.3157 |
| 5.09 | -2.38301 | 2.72869 | -20.4507 | 5.11170 | 2.72869 | -2.72869 | 20.4507 |
| 5.10 | -2.43942 | 2.71197 | -20.5861 | 5.15139 | 2.71197 | -2.71197 | 20.5861 |
| 5.11 | -2.49665 | 2.69517 | -20.7218 | 5.19182 | 2.69517 | -2.69517 | 20.7218 |
| 5.12 | -2.55470 | 2.67830 | -20.8578 | 5.23300 | 2.67830 | -2.67830 | 20.8578 |
| 5.13 | -2.61360 | 2.66135 | -20.9942 | 5.27495 | 2.66135 | -2.66135 | 20.9942 |
| 5.14 | -2.67338 | 2.64432 | -21.1310 | 5.32336 | 2.64432 | -2.64432 | 21.1310 |
| 5.15 | -2.73404 | 2.62721 | -21.2681 | 5.36126 | 2.62721 | 2.62721 | 21.2681 |
| 5.16 | -2.79562 | 2.61003 | -21.4055 | 5.40565 | 2.61003 | -2.61003 | 21.4055 |
| 5.17 | -2.85815 | 2.59276 | -21.5434 | 5.45091 | 2.59276 | -2.59276 | 21.5434 |
| 5.18 | -2.92163 | 2.57541 | -21.6816 | 5.49704 | 2.57541 | -2.57541 | 21.6816 |
| 5.19 | -2.98611 | 2.55799 | -21.8201 | 5.54409 | 2.55799 | -2.55799 | 21.8201 |
| 5.20 | -3.05159 | 2.54048 | -21.9590 | 5.59207 | 2.54048 | -2.54048 | 21.9590 |
| 5.21 | -3.11812 | 2.52289 | -22.0983 | 5.64101 | 2.52289 | -2.52289 | 22.0983 |
| 5.22 | -3.18573 | 2.50522 | -22.2380 | 5.69094 | 2.50522 | -2.50522 | 22.2380 |
| 5.23 | -3.25443 | 2.48746 | -22.3780 | 5.74189 | 2.48746 | -2.48746 | 22.3780 |
| 5.24 | -3.32426 | 2.46962 | -22.5184 | 5.79388 | 2.46962 | -2.46962 | 22.5184 |
| 5.25 | -3.39525 | 2.45170 | -22.6591 | 5.84695 | 2.45170 | -2.45170 | 22.6591 |
| 5.26 | -3.46744 | 2.43369 | -22.8002 | 5.90113 | 2.43369 | -2.43369 | 22.8002 |
| 5.27 | -3.54086 | 2.41560 | -22.9417 | 5.95646 | 2.41560 | -2.41560 | 22.9417 |
| 5.28 | -3.61554 | 2.39742 | -23.0836 | 6.01296 | 2.39742 | -2.39742 | 23.0836 |
| 5.29 | -3.69152 | 2.37916 | -23.2258 | 6.07067 | 2.37916 | -2.37916 | 23.2258 |
| 5.30 | -3.76884 | 2.36081 | -23.3684 | 6.12964 | 2.36081 | -2.36081 | 23.3684 |
| 5.31 | -3.84754 | 2.34237 | -23.5114 | 6.18990 | 2.34237 | -2.34237 | 23.5114 |
| 5.32 | -3.92766 | 2.32384 | -23.6547 | 6.25150 | 2.32384 | -2.32384 | 23.6547 |
| 5.33 | -4.00924 | 2.30522 | -23.7984 | 6.31446 | 2.30522 | -2.30522 | 23.7984 |
| 5.34 | -4.09233 | 2.28652 | -23.9426 | 6.37885 | 2.28652 | -2.28652 | 23.9426 |
| 5.35 | -4.17698 | 2.26772 | -24.0870 | 6.44470 | 2.26772 | -2.26772 | 24.0870 |
| 5.36 | -4.26323 | 2.24884 | -24.2319 | 6.51207 | 2.24884 | -2.24884 | 24.2319 |
| 5.37 | -4.35114 | 2.22986 | -24.3772 | 6.58100 | 2.22986 | -2.22986 | 24.3772 |
| 5.38 | -4.44076 | 2.21079 | -24.5228 | 6.65155 | 2.21079 | -2.21079 | 24.5228 |
| 5.39 | -4.53214 | 2.19163 | -24.6688 | 6.72377 | 2.19163 | -2.19163 | 24.6688 |
| 5.40 | -4.62534 | 2.17238 | -24.8152 | 6.79773 | 2.17238 | -2.17238 | 24.8152 |
| 5.41 | -4.72043 | 2.15303 | -24.9620 | 6.87347 | 2.15303 | -2.15303 | 24.9620 |
| 5.42 | -4.81747 | 2.13359 | -25.1092 | 6.95106 | 2.13359 | -2.13359 | 25.1092 |
| 5.43 | -4.91653 | 2.11405 | -25.2568 | 7.03058 | 2.11405 | -2.11405 | 25.2568 |
| 5.44 | -5.01766 | 2.09442 | -25.4048 | 7.11208 | 2.09442 | -2.09442 | 25.4048 |
| 5.45 | -5.12096 | 2.07469 | -25.5531 | 7.19565 | 2.07469 | -2.07469 | 25.5531 |
| 5.46 | -5.22649 | 2.05486 | -25.7019 | 7.28135 | 2.05486 | -2.05486 | 25.7019 |
| 5.47 | -5.33434 | 2.03493 | -25.8510 | 7.36927 | 2.03493 | -2.03493 | 25.8510 |
| 5.48 | -5.44458 | 2.01491 | -26.0006 | 7.45950 | 2.01491 | -2.01491 | 26.0006 |
| 5.49 | -5.55732 | 1.99479 | -26.1505 | 7.55211 | 1.99479 | -1.99479 | 26.1505 |
| 5.50 | -5.67264 | 1.97456 | -26.3009 | 7.64720 | 1.97456 | -1.97456 | 26.3009 |
| 5.51 | -5.79063 | 1.95424 | -26.4516 | 7.74487 | 1.95424 | -1.95424 | 26.4516 |
| 5.52 | -5.91142 | 1.93381 | -26.6028 | 7.84523 | 1.93381 | -1.93381 | 26.6028 |
| 5.53 | -6.03509 | 1.91328 | -26.7543 | 7.94837 | 1.91328 | -1.91328 | 26.7543 |

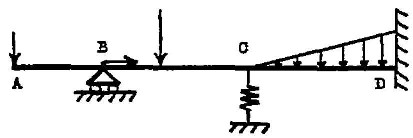
NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Concluded

| | Stiffness coefficients at free end | | | Coefficients for induced shear and moment at clamped end | | | |
|---|---|---|---|---|---|---|---|
| $\frac{L}{j}$ | $K_{F\theta_R}\left(\frac{L}{EI}\right)$ | $K_{F\theta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{F\theta_R}\left(\frac{L^3}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L}{EI}\right)$ | $K_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^2}{EI}\right)$ | $C_{C\theta_R}\left(\frac{L^3}{EI}\right)$ |
| 5.54 | -6.16177 | 1.89265 | -26.9063 | 8.05442 | 1.89265 | -1.89265 | 26.9063 |
| 5.55 | -6.29157 | 1.87191 | -27.0587 | 8.16349 | 1.87191 | -1.87191 | 27.0587 |
| 5.56 | -6.42464 | 1.85108 | -27.2115 | 8.27571 | 1.85108 | -1.85108 | 27.2115 |
| 5.57 | -6.56109 | 1.83013 | -27.3646 | 8.39122 | 1.83013 | -1.83013 | 27.3646 |
| 5.58 | -6.70107 | 1.80908 | -27.5183 | 8.51015 | 1.80908 | -1.80908 | 27.5183 |
| 5.59 | -6.84474 | 1.78792 | -27.6722 | 8.63266 | 1.78792 | -1.78792 | 27.6722 |
| 5.60 | -6.99225 | 1.76665 | -27.8267 | 8.75889 | 1.76664 | -1.76664 | 27.8267 |
| 5.61 | -7.14376 | 1.74527 | -27.9816 | 8.88903 | 1.74527 | -1.74527 | 27.9816 |
| 5.62 | -7.29947 | 1.72378 | -28.1368 | 9.02326 | 1.72378 | -1.72378 | 28.1368 |
| 5.63 | -7.45955 | 1.70219 | -28.2925 | 9.16174 | 1.70219 | -1.70219 | 28.2925 |
| 5.64 | -7.62422 | 1.68048 | -28.4486 | 9.30469 | 1.68048 | -1.68048 | 28.4486 |
| 5.65 | -7.79367 | 1.65866 | -28.6052 | 9.45233 | 1.65866 | -1.65866 | 28.6052 |
| 5.66 | -7.96815 | 1.63672 | -28.7622 | 9.60487 | 1.63672 | -1.63672 | 28.7622 |
| 5.67 | -8.14790 | 1.61467 | -28.9196 | 9.76257 | 1.61467 | -1.61467 | 28.9196 |
| 5.68 | -8.33166 | 1.59251 | -29.0774 | 9.92568 | 1.59251 | -1.59251 | 29.0774 |
| 5.69 | -8.52423 | 1.57023 | -29.2356 | 10.0945 | 1.57023 | -1.57023 | 29.2356 |
| 5.70 | -8.72139 | 1.54783 | -29.3943 | 10.2692 | 1.54783 | -1.54783 | 29.3943 |
| 5.71 | -8.92497 | 1.52532 | -29.5535 | 10.4503 | 1.52532 | -1.52532 | 29.5535 |
| 5.72 | -9.13528 | 1.50269 | -29.7130 | 10.6380 | 1.50269 | -1.50269 | 29.7130 |
| 5.73 | -9.35272 | 1.47993 | -29.8730 | 10.8327 | 1.47993 | -1.47993 | 29.8730 |
| 5.74 | -9.57765 | 1.45706 | -30.0335 | 11.0347 | 1.45706 | -1.45706 | 30.0335 |
| 5.75 | -9.81051 | 1.43407 | -30.1944 | 11.2446 | 1.43407 | -1.43407 | 30.1944 |
| 5.76 | -10.0517 | 1.41095 | -30.3557 | 11.4627 | 1.41095 | -1.41095 | 30.3557 |
| 5.77 | -10.3018 | 1.38771 | -30.5175 | 11.6895 | 1.38771 | -1.38771 | 30.5175 |
| 5.78 | -10.5613 | 1.36434 | -30.6797 | 11.9256 | 1.36434 | -1.36434 | 30.6797 |
| 5.79 | -10.8307 | 1.34085 | -30.8424 | 12.1715 | 1.34085 | -1.34085 | 30.8424 |
| 5.80 | -11.1106 | 1.31723 | -31.0055 | 12.4278 | 1.31723 | -1.31723 | 31.0055 |
| 5.81 | -11.4018 | 1.29348 | -31.1691 | 12.6953 | 1.29348 | -1.29348 | 31.1691 |
| 5.82 | -11.7049 | 1.26961 | -31.3332 | 12.9745 | 1.26961 | -1.26961 | 31.3332 |
| 5.83 | -12.0208 | 1.24560 | -31.4977 | 13.2664 | 1.24560 | -1.24560 | 31.4977 |
| 5.84 | -12.3502 | 1.22147 | -31.6627 | 13.5717 | 1.22147 | -1.22147 | 31.6627 |
| 5.85 | -12.6942 | 1.19720 | -31.8281 | 13.8914 | 1.19720 | -1.19720 | 31.8281 |
| 5.86 | -13.0537 | 1.17279 | -31.9940 | 14.2265 | 1.17279 | -1.17279 | 31.9940 |
| 5.87 | -13.4299 | 1.14826 | -32.1604 | 14.5781 | 1.14826 | -1.14826 | 32.1604 |
| 5.88 | -13.8240 | 1.12358 | -32.3272 | 14.9476 | 1.12358 | -1.12358 | 32.3272 |
| 5.89 | -14.2373 | 1.09877 | -32.4945 | 15.3361 | 1.09877 | -1.09877 | 32.4945 |
| 5.90 | -14.6715 | 1.07383 | -32.6623 | 15.7453 | 1.07383 | -1.07383 | 32.6623 |
| 5.91 | -15.1280 | 1.04874 | -32.8306 | 16.1768 | 1.04874 | -1.04874 | 32.8306 |
| 5.92 | -15.6089 | 1.02351 | -32.9994 | 16.6324 | 1.02351 | -1.02351 | 32.9994 |
| 5.93 | -16.1161 | .998143 | -33.1686 | 17.1142 | .998143 | -.998143 | 33.1686 |
| 5.94 | -16.6519 | .972631 | -33.3383 | 17.6245 | .972631 | -.972631 | 33.3383 |
| 5.95 | -17.2189 | .946975 | -33.5086 | 18.1659 | .946975 | -.946975 | 33.5086 |
| 5.96 | -17.8201 | .921174 | -33.6793 | 18.7412 | .921174 | -.921174 | 33.6793 |
| 5.97 | -18.4585 | .895225 | -33.8505 | 19.3538 | .895225 | -.895225 | 33.8505 |
| 5.98 | -19.1380 | .869126 | -34.0221 | 20.0071 | .869126 | -.869126 | 34.0221 |
| 5.99 | -19.8628 | .842881 | -34.1944 | 20.7057 | .842881 | -.842881 | 34.1944 |
| 6.00 | -20.6375 | .816483 | -34.3670 | 21.4540 | .816483 | -.816483 | 34.3670 |
| 6.01 | -21.4678 | .789934 | -34.5402 | 22.2577 | .789934 | -.789934 | 34.5402 |
| 6.02 | -22.3598 | .763230 | -34.7139 | 23.1230 | .763230 | -.763230 | 34.7139 |
| 6.03 | -23.3210 | .736374 | -34.8882 | 24.0574 | .736374 | -.736374 | 34.8882 |
| 6.04 | -25.3598 | .709359 | -35.0629 | 25.0692 | .709359 | -.709359 | 35.0629 |
| 6.05 | -25.4863 | .682186 | -35.2381 | 26.1685 | .682186 | -.682186 | 35.2381 |
| 6.06 | -26.7121 | .654853 | -35.4138 | 27.3669 | .654853 | -.654853 | 35.4138 |
| 6.07 | -28.0513 | .627362 | -35.5902 | 28.6787 | .627362 | -.627362 | 35.5902 |
| 6.08 | -29.5205 | .599706 | -35.7670 | 30.1202 | .599706 | -.599706 | 35.7670 |
| 6.09 | -31.1401 | .571888 | -35.9443 | 31.7120 | .571888 | -.571888 | 35.9443 |
| 6.10 | -32.9343 | .543900 | -36.1220 | 33.4782 | .543900 | -.543900 | 36.1220 |
| 6.11 | -34.9340 | .515752 | -36.3006 | 35.4498 | .515752 | -.515752 | 36.3006 |
| 6.12 | -37.1765 | .487433 | -36.4796 | 37.6639 | .487433 | -.487433 | 36.4796 |
| 6.13 | -39.7095 | .458946 | -36.6593 | 40.1684 | .458946 | -.458946 | 36.6593 |
| 6.14 | -42.5931 | .430281 | -36.8391 | 43.0234 | .430281 | -.430281 | 36.8391 |
| 6.15 | -45.9073 | .401445 | -37.0196 | 46.3087 | .401445 | -.401445 | 37.0196 |
| 6.16 | -49.7565 | .372434 | -37.2007 | 50.1289 | .372424 | -.372424 | 37.2007 |
| 6.17 | -54.2825 | .343246 | -37.3824 | 54.6258 | .343246 | -.343246 | 37.3824 |
| 6.18 | -59.6822 | .313880 | -37.5646 | 59.9960 | .313880 | -.313880 | 37.5646 |
| 6.19 | -66.2365 | .284334 | -37.7474 | 66.5209 | .284334 | -.284334 | 37.7474 |
| 6.20 | -74.3621 | .254605 | -37.9308 | 74.6167 | .254605 | -.254605 | 37.9308 |
| 6.21 | -84.7028 | .224693 | -38.1147 | 84.9275 | .224693 | -.224693 | 38.1147 |
| 6.22 | -98.3097 | .194593 | -38.2989 | 98.5043 | .194593 | -.194593 | 38.2989 |
| 6.23 | -117.028 | .164309 | -38.4843 | 117.192 | .164309 | -.164309 | 38.4843 |
| 6.24 | -144.404 | .133833 | -38.6700 | 144.538 | .133833 | -133833 | 38.6700 |
| 6.25 | -188.268 | .103166 | -38.8562 | 188.371 | .103166 | -.103166 | 38.8562 |
| 6.26 | -269.950 | .0723062 | -39.0430 | 270.023 | .0723062 | -.0723062 | 39.0430 |
| 6.27 | -475.501 | .0412474 | -39.2303 | 475.542 | .0412474 | -.0412474 | 39.2303 |
| 6.28 | -1971.76 | .00999181 | -39.4227 | 1971.77 | .00999181 | -.00999181 | 39.4227 |
| $2\pi$ | $-\infty$ | 0 | -39.4784 | $\infty$ | 0 | 0 | 39.4784 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 3 - FORCE AND MOMENT DISTRIBUTION COMPUTATIONS FOR ILLUSTRATIVE EXAMPLE




Joint C

$\delta = 0.00140653\ F$

$\theta = 2.62805 \times 10^{-6}\ M$

$M_{C_L} = 174,830\ \theta - 4563.56\ \delta$

Not used $\left\{ \begin{array}{l} S_{C_L} = 4563.56\ \theta - 12.1358\ \delta \\ S_{C_R} = -4563.56\ \theta - 12.1358\ \delta \\ M_{C_R} = 174,830\ \theta + 4563.56\ \delta \end{array} \right.$

Joint B

$\theta = 5.25610 \times 10^{-6}\ M$

$S_{C_R} = -4563.56\ \theta$

$M_{C_R} = 174,830\ \theta$

| | | B | | | | C | | | | D |
|---|---|---|---|---|---|---|---|---|---|---|
| | Shear | Moment | Moment | Shear | Shear | Moment | Moment | Shear | Shear | Moment |
| | -100 | 5000 | -6989.8 | -326.94 | -173.06<br>$F = 292.10$<br>$\delta = 0.410847$ in. | 4834.6 | -2605.4<br>$M = -2229.2$<br>$\theta = -5.85845 \times 10^{-3}$ | -119.04 | -280.96 | 3748.8 |
| | | | -2899.15<br>$M = 4888.95$<br>$\theta = 25.6968 \times 10^{-3}$ | | | | | | | |
| | | | | | -117.269<br>$F = 117.269$<br>$\delta = 0.164942$ in. | 4492.57 | $M = -4492.57$<br>$\theta = -11.8067 \times 10^{-3}$ | | | |
| | | | -2816.89<br>$M = 2816.89$<br>$\theta = 14.8059 \times 10^{-3}$ | | | | | | | |
| | | | | | -67.5676<br>$F = 67.5676$<br>$\delta = 0.0950359$ in. | 2588.52 | $M = -2588.52$<br>$\theta = -6.80276 \times 10^{-3}$ | | | |
| | | | -1623.03<br>$M = 1623.03$<br>$\theta = 8.53081 \times 10^{-3}$ | | | | | | | |
| | | | | | -38.9309<br>$F = 38.9309$<br>$\delta = 0.0547575$ in. | 1491.44 | $M = -1491.44$<br>$\theta = -3.91958 \times 10^{-3}$ | | | |
| | | | -935.149<br>$M = 935.149$<br>$\theta = 4.91524 \times 10^{-3}$ | | | | | | | |
| Total deflections and rotations after 10 cycles | | | $\theta_B = 60.3862 \times 10^{-3}$ | | $\delta_C = 0.797298$ in. | | $\theta_C = -33.5210 \times 10^{-3}$ | | | |
| After 20 cycles | | | $\theta_B = 60.6297 \times 10^{-3}$ | | $\delta_C = 0.800011$ in. | | $\theta_C = -33.7152 \times 10^{-3}$ | | | |
| Exact results | | | $\theta_B = 60.6307 \times 10^{-3}$ | | $\delta_C = 0.800022$ in. | | $\theta_C = -33.7160 \times 10^{-3}$ | | | |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 4 - COMPUTATIONS FOR THE DEFLECTION CURVE

OF SPAN BC (ILLUSTRATIVE EXAMPLE)

$\theta_B = 0.06063$
$\delta_C = 0.8000$ inch
$\theta_C = -0.033716$
} from table 3

$Q = 500$ pounds

$L = 80$ inches

$\frac{EI}{L^3} = 11.328$ pounds per inch

$\frac{c}{L} = 0.6$

$\frac{L}{j} = 3$

| $\frac{x}{L}$ | $\frac{y_1}{\left(\frac{QL^3}{EI}\right)}$ | $\frac{y_2}{\theta_B L}$ | $\frac{y_3}{\theta_C L}$ | $\frac{y_4}{\delta_C}$ | $y_1$ (in.) | $y_2$ (in.) | $y_3$ (in.) | $y_4$ (in.) | Total deflection, y (in.) (a) |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| .1 | .00078 | .086 | -.011 | .025 | .035 | .419 | .030 | .020 | .504 |
| .2 | .00258 | .144 | -.040 | .096 | .114 | .698 | .108 | .077 | .997 |
| .3 | .00458 | .173 | -.080 | .207 | .202 | .837 | .215 | .166 | 1.420 |
| .4 | .00596 | .175 | -.121 | .346 | .263 | .847 | .326 | .277 | 1.713 |
| .5 | .00612 | .155 | -.155 | .500 | .270 | .753 | .419 | .400 | 1.842 |
| .6 | .00520 | .121 | -.175 | .654 | .230 | .586 | .471 | .523 | 1.810 |
| .7 | .00366 | .080 | -.173 | .793 | .161 | .386 | .465 | .634 | 1.646 |
| .8 | .00194 | .040 | -.144 | .904 | .086 | .195 | .388 | .723 | 1.392 |
| .9 | .00057 | .011 | -.086 | .975 | .025 | .054 | .233 | .780 | 1.092 |
| 1.0 | 0 | 0 | 0 | 1.000 | 0 | 0 | 0 | .800 | .800 |

a

$y = y_1 + y_2 + y_3 + y_4$

$y_1$ = deflection due to lateral load from figure 8(d)

$y_2$ = deflection due to rotation of joint B from figure 10

$y_3$ = deflection due to rotation of joint C from figure 10

$y_4$ = deflection due to deflection of joint C from figure 9

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE 5 - COMPUTATIONS FOR BENDING-MOMENT DIAGRAM

FOR SPAN BC (ILLUSTRATIVE EXAMPLE)

$$P = 8156.25 \text{ pounds}$$

$$S_{BC} = -213.82 \text{ pounds}$$

$$M_{BC} = -5000 \text{ inch-pounds}$$

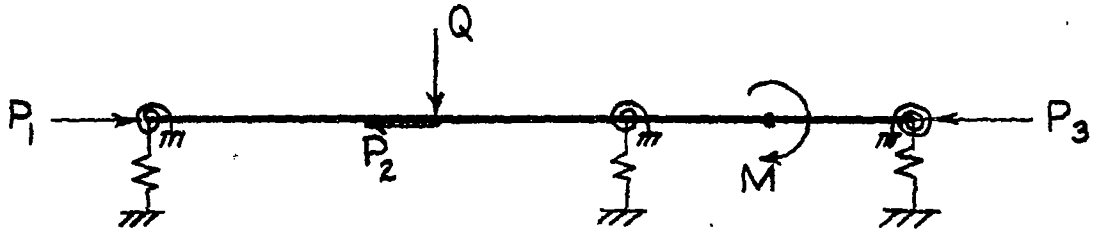
$$\Delta = y - (y)_{\frac{x}{L}=0}$$

Bending moment at $x \leq 32$ in. $= M_{BC} + P\Delta - S_{BC}x$

Bending moment at $x \geq 32$ in. $= M_{BC} + P\Delta - S_{BC}x - 500(x - 32)$

| $\frac{x}{L}$ | $x$ (in.) | $y$ from table 4 (in.) | $\Delta$ (in.) | $P\Delta$ (in.-lb) | $-S_{BC}x$ (in.-lb) | $-500(x - 32)$ for $x \geq 32$ (in.-lb) | Bending moment (in.-lb) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | ------------ | -5000 |
| .1 | 8 | .504 | .504 | 4100 | 1710 | ------------ | 810 |
| .2 | 16 | .997 | .997 | 8130 | 3420 | ------------ | 6550 |
| .3 | 24 | 1.420 | 1.420 | 11580 | 5130 | ------------ | 11710 |
| .4 | 32 | 1.713 | 1.713 | 13970 | 6840 | 0 | 15810 |
| .5 | 40 | 1.842 | 1.842 | 15020 | 8550 | -4000 | 14570 |
| .6 | 48 | 1.810 | 1.810 | 14760 | 10260 | -8000 | 12020 |
| .7 | 56 | 1.646 | 1.646 | 13430 | 11970 | -12000 | 8400 |
| .8 | 64 | 1.392 | 1.392 | 11350 | 13680 | -16000 | 4030 |
| .9 | 72 | 1.092 | 1.092 | 8910 | 15400 | -20000 | -690 |
| 1.0 | 80 | .800 | .800 | 6530 | 17110 | -24000 | -5360 |

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 1.- Continuous beam-column on elastic supports.

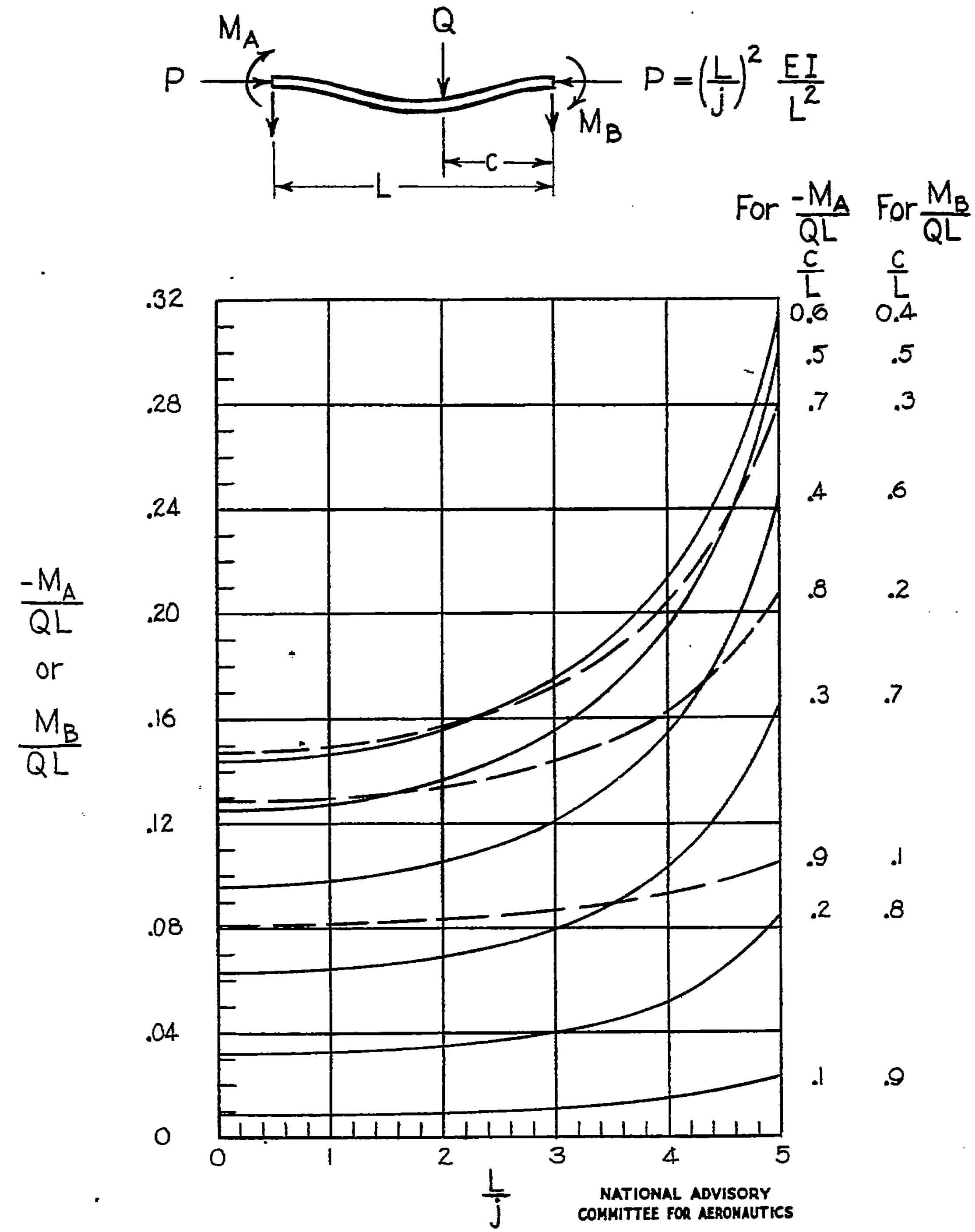



Figure 2.- Fixed-end moment coefficients for a beam-column subjected to a concentrated lateral load.

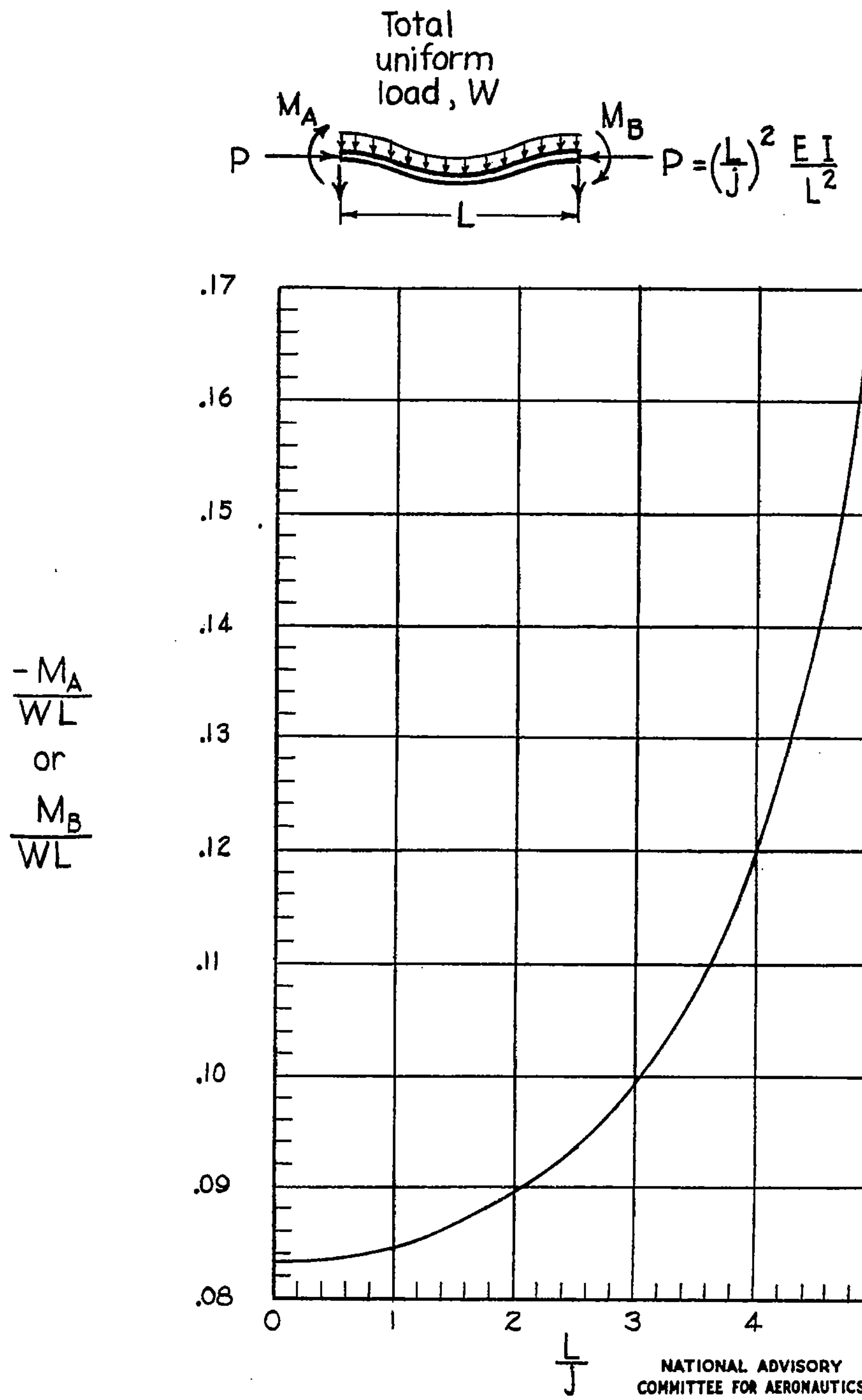
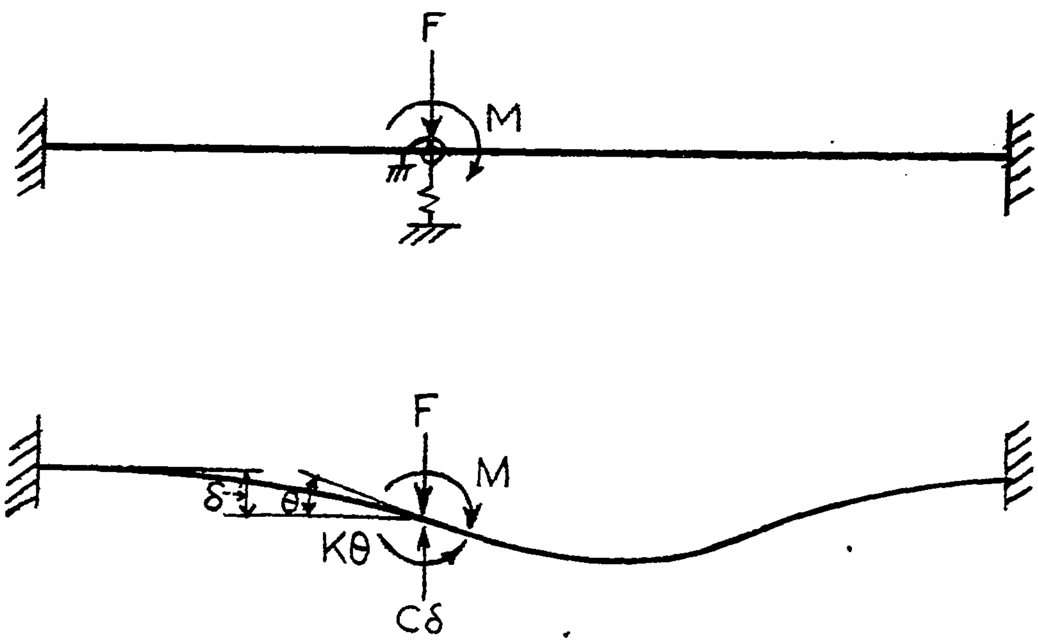
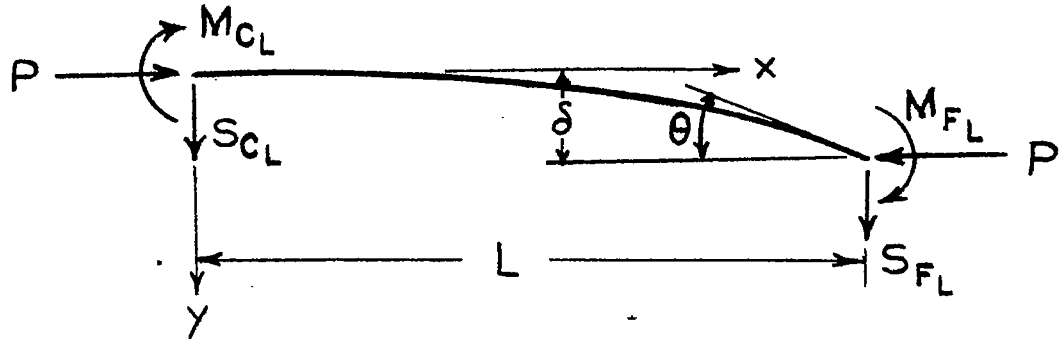



Figure 3.- Fixed-end moment coefficients for a beam-column subjected to a uniformly distributed lateral load.




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 4.- Two-span fundamental structural unit in force and moment distribution.




NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 5.- Free-body diagram of left-hand span.

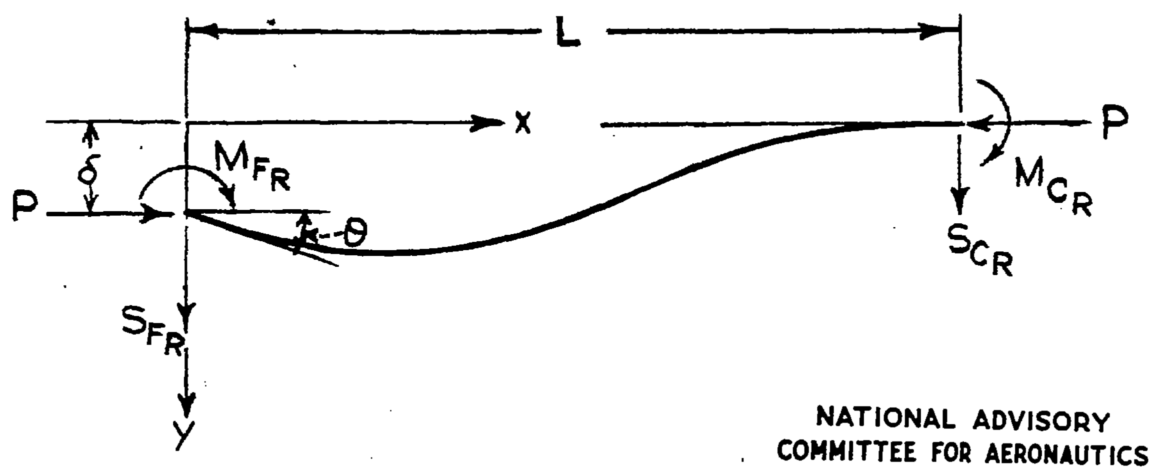



Figure 6.- Free-body diagram of right-hand span.

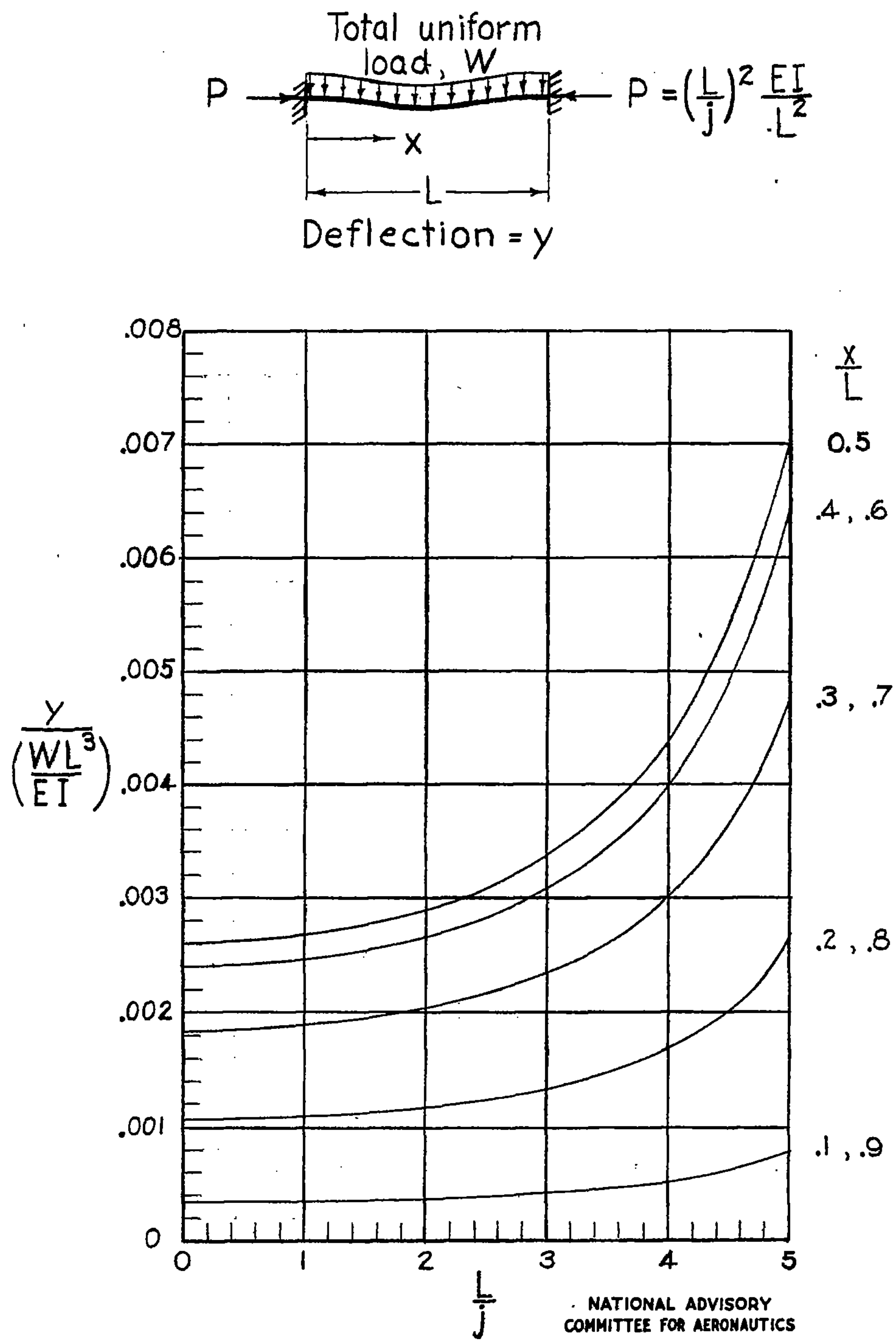
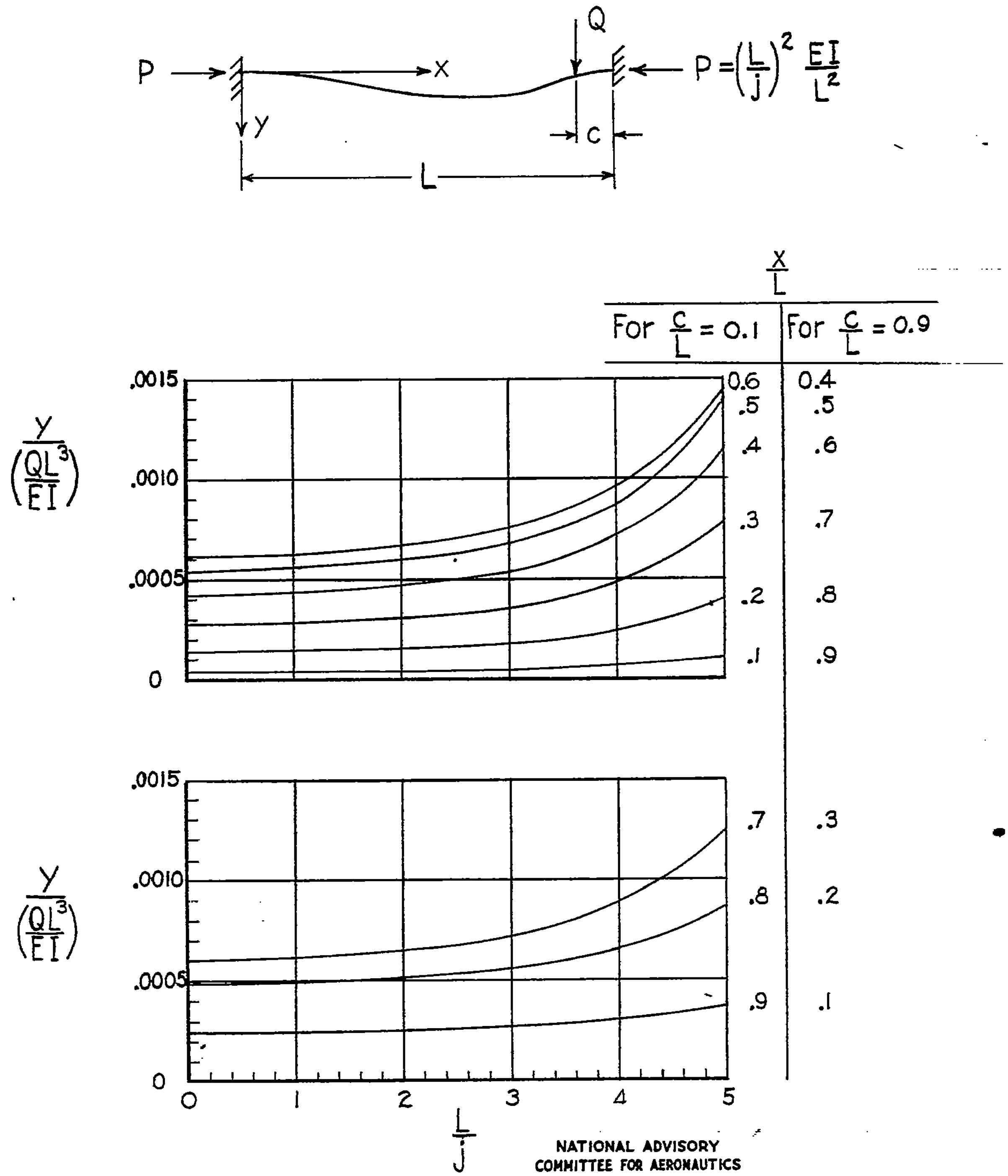
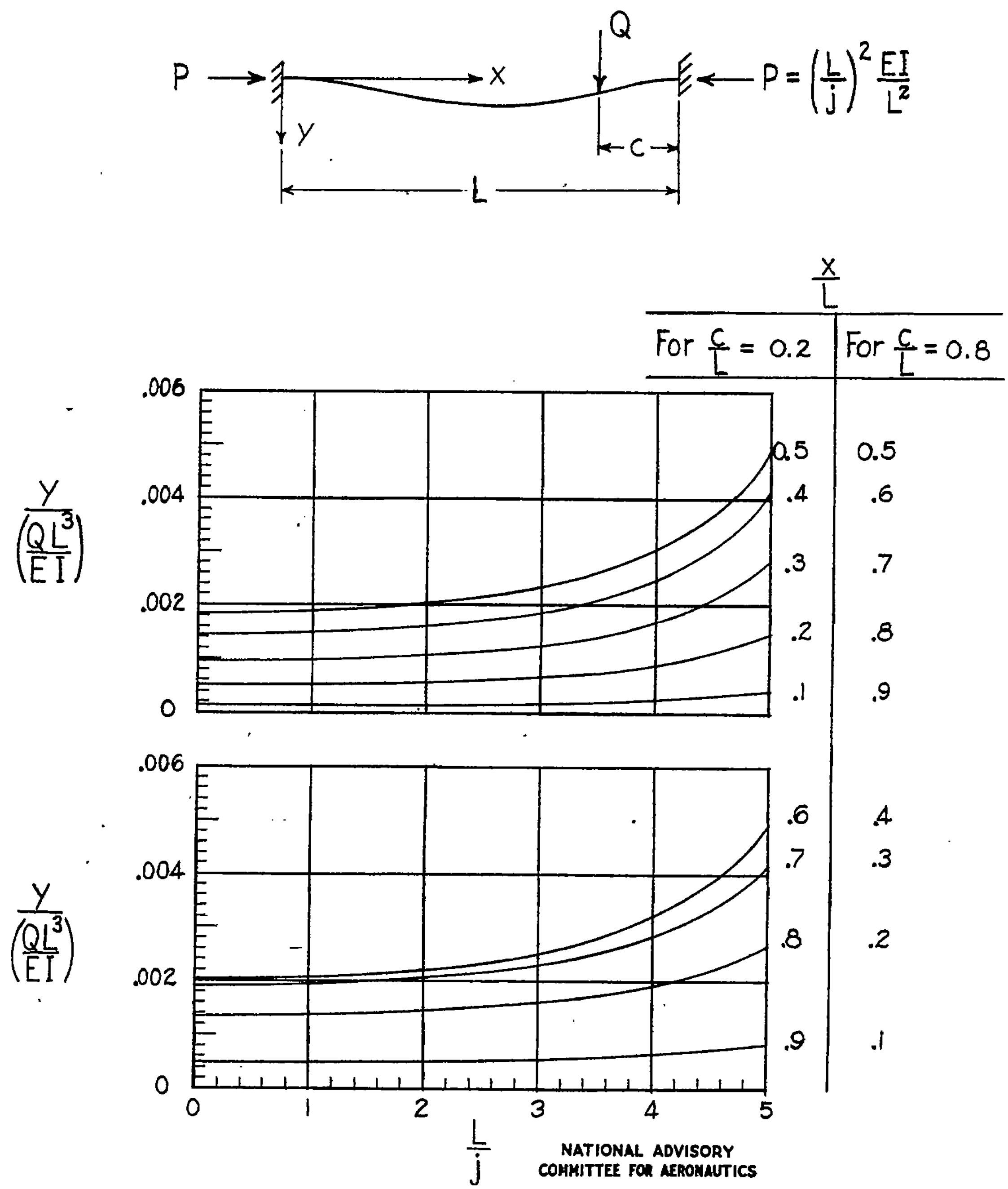



Figure 7.- Deflections of a clamped-end beam-column due to a uniformly distributed lateral load.



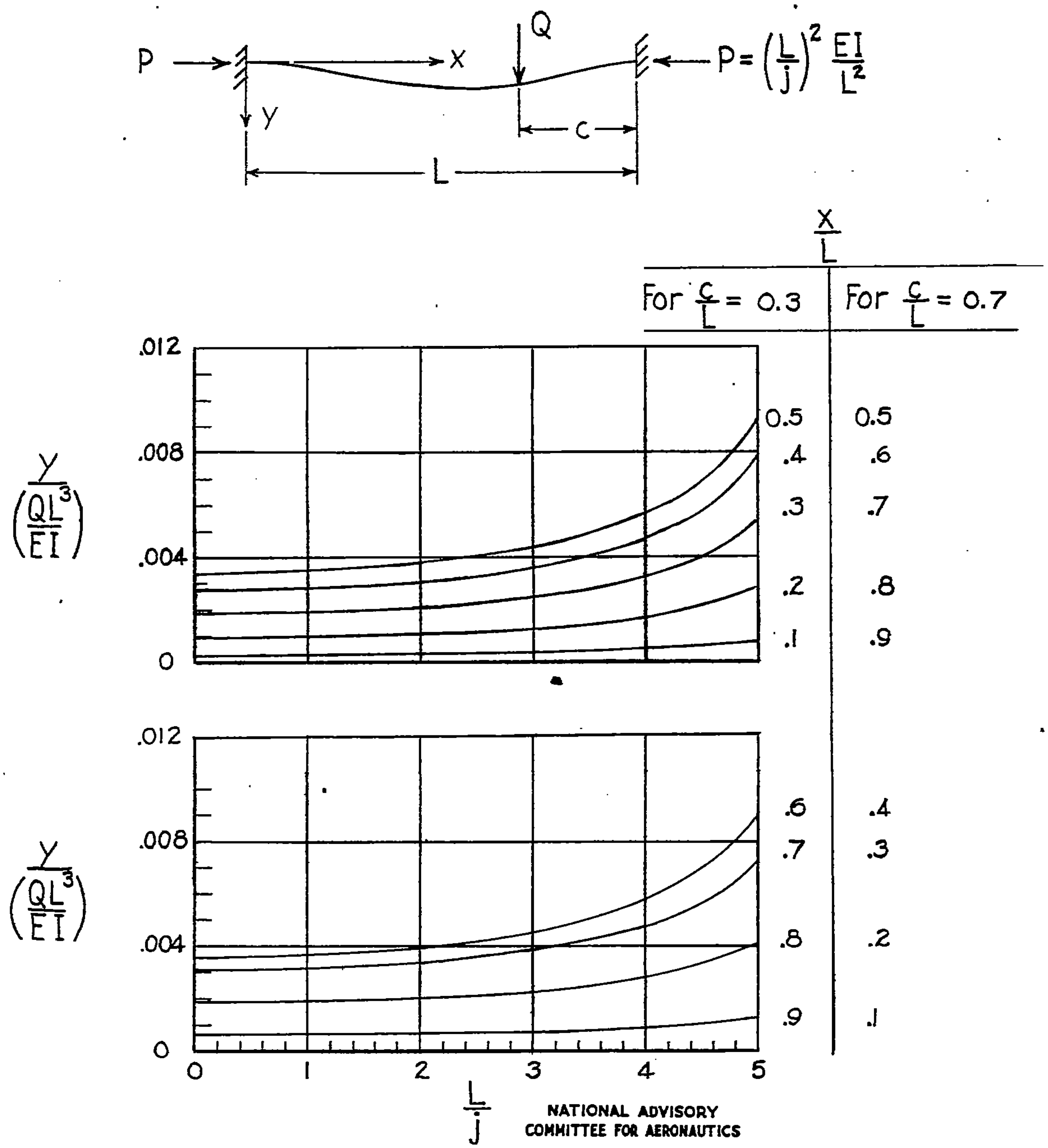

(a) $\frac{c}{L} = 0.1$ or $0.9$.

Figure 8.- Deflections of a clamped-end beam-column due to a concentrated lateral load.



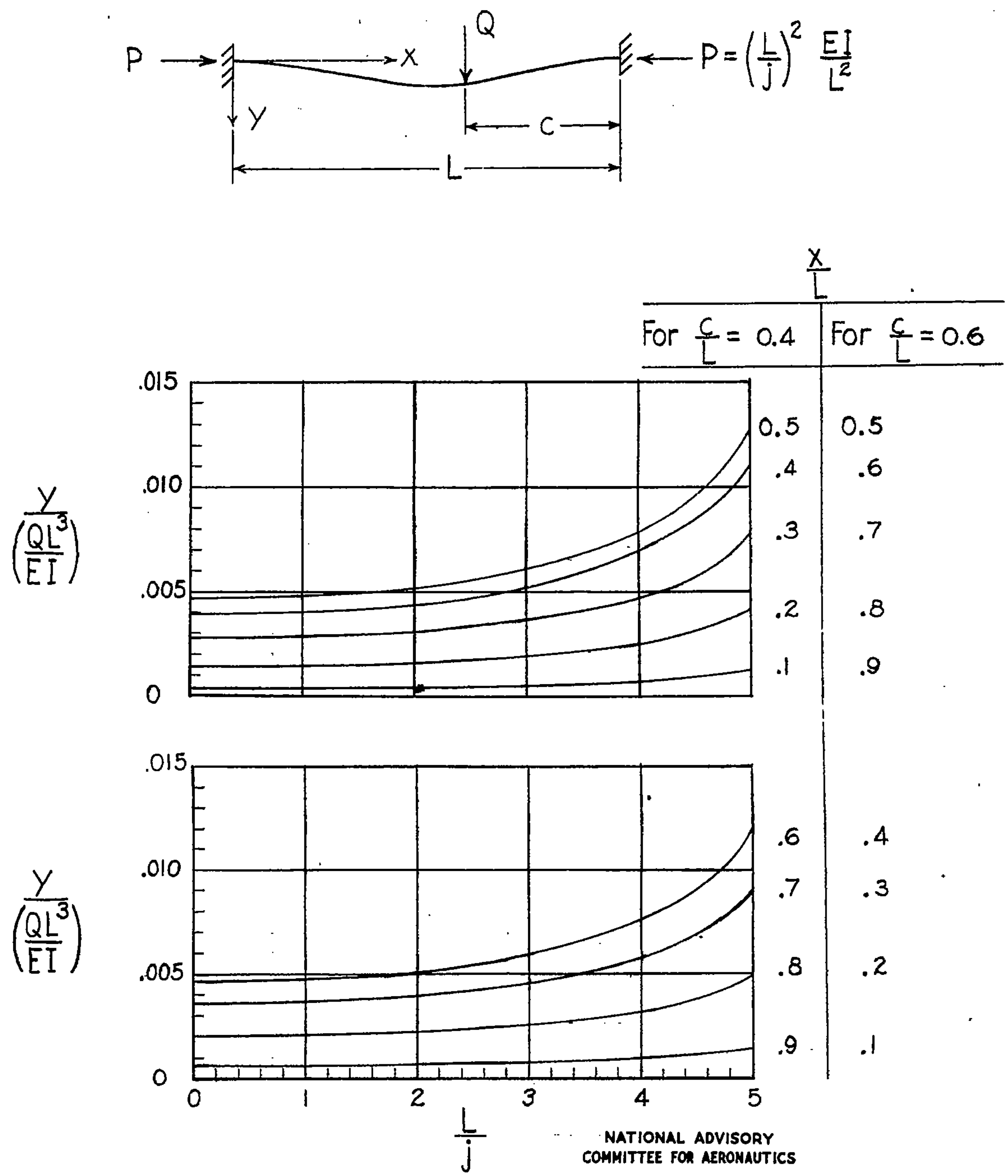

(b) $\frac{c}{L} = 0.2$ or 0.8.

Figure 8.- Continued.




(c) $\frac{c}{L}$ = 0.3 or 0.7.

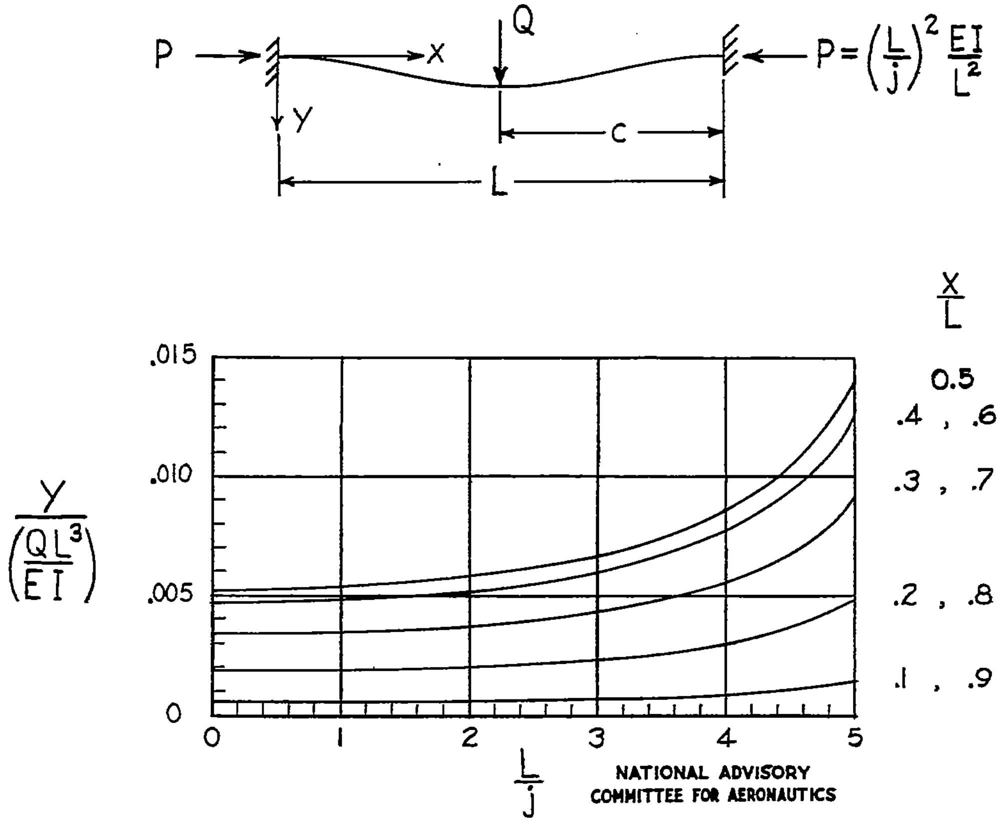
Figure 8.- Continued.




NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

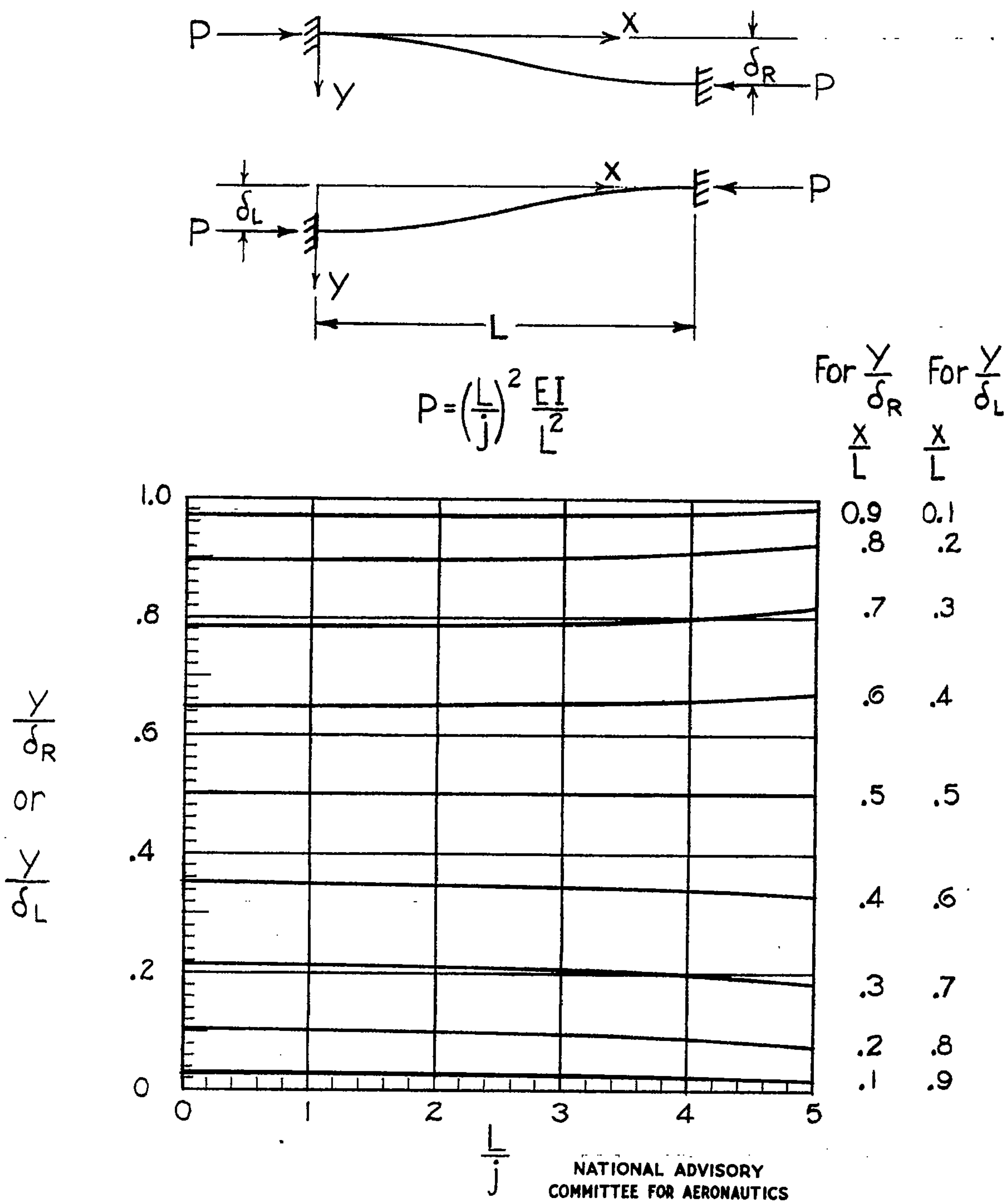
(d) $\frac{c}{L} = 0.4$ or 0.6.

Figure 8.- Continued.




(e) $\frac{c}{L} = 0.5$.

Figure 8.- Concluded.




NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Figure 9.- Deflections of a clamped-end column due to lateral movement of one end.

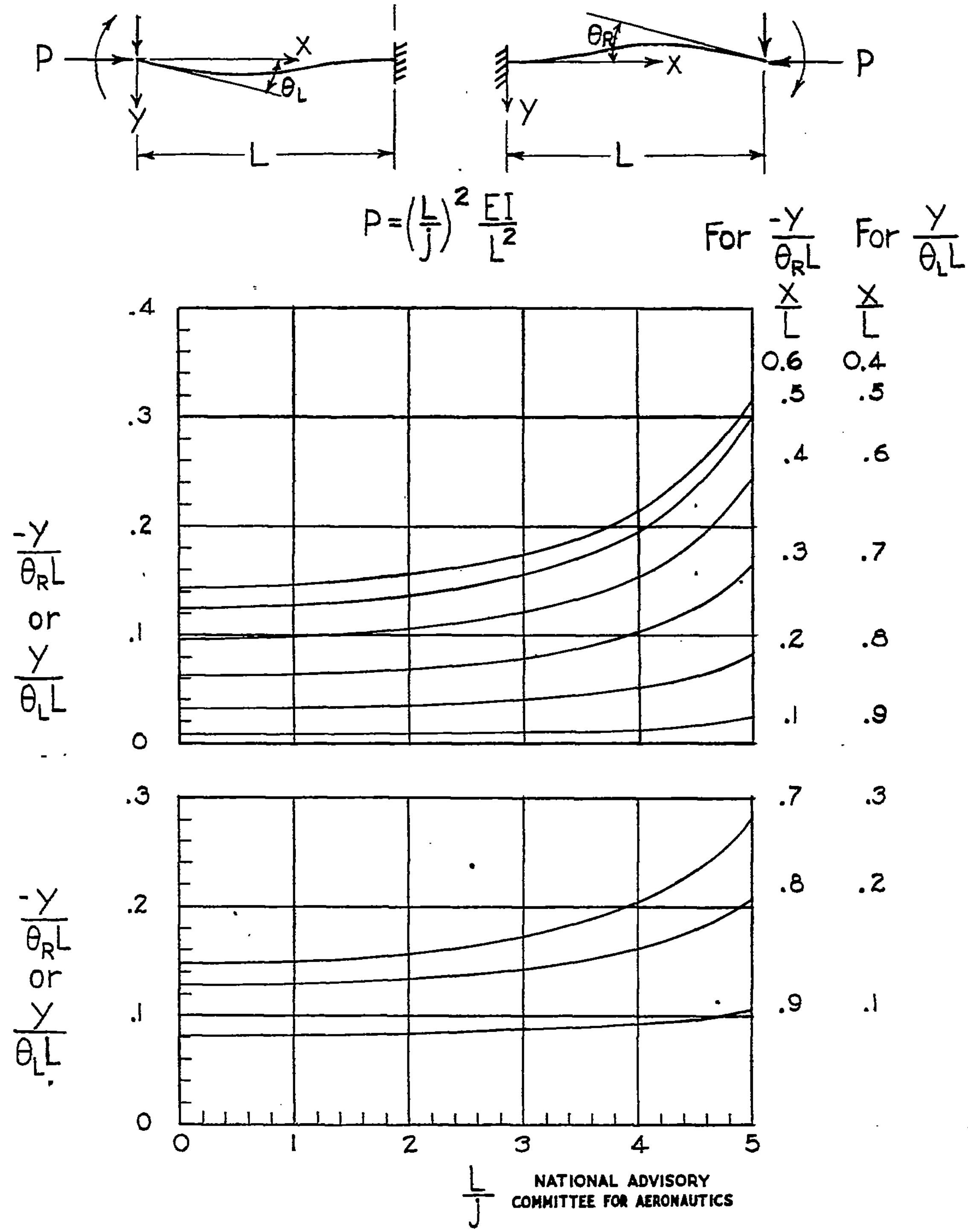
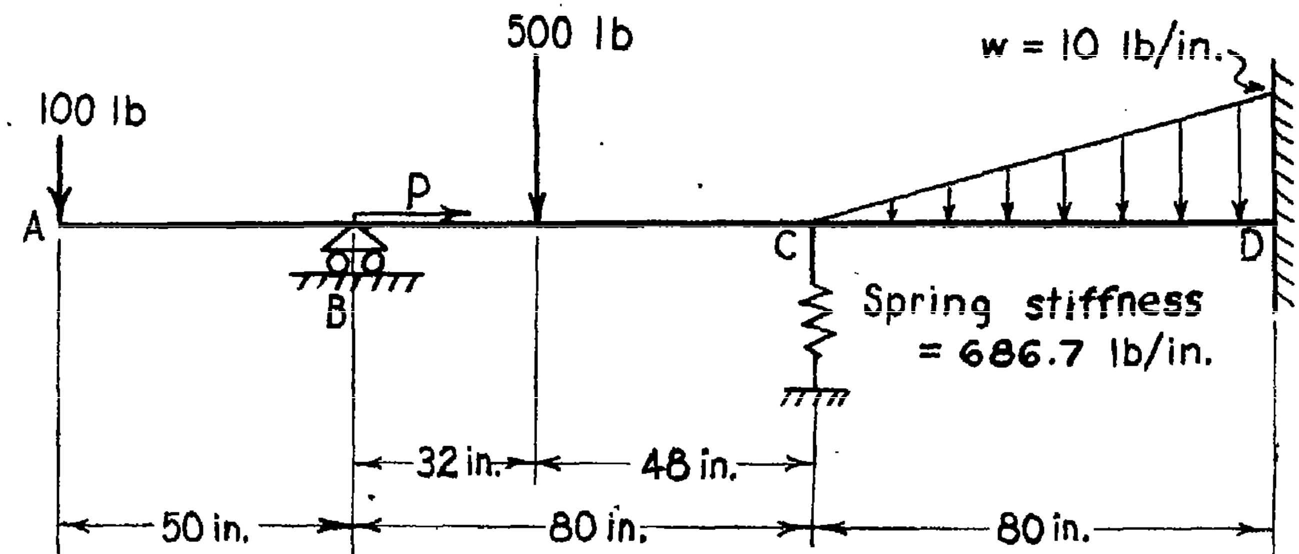



Figure 10.- Deflections of a clamped-end column due to rotation of one end.




$E = 29 \times 10^6$ psi

$I = 0.2$ inch$^4$

$\frac{L}{j} = 3$ for spans BC and CD

Span AB has no axial load.



Figure 11. - Structure analyzed for illustrative example.

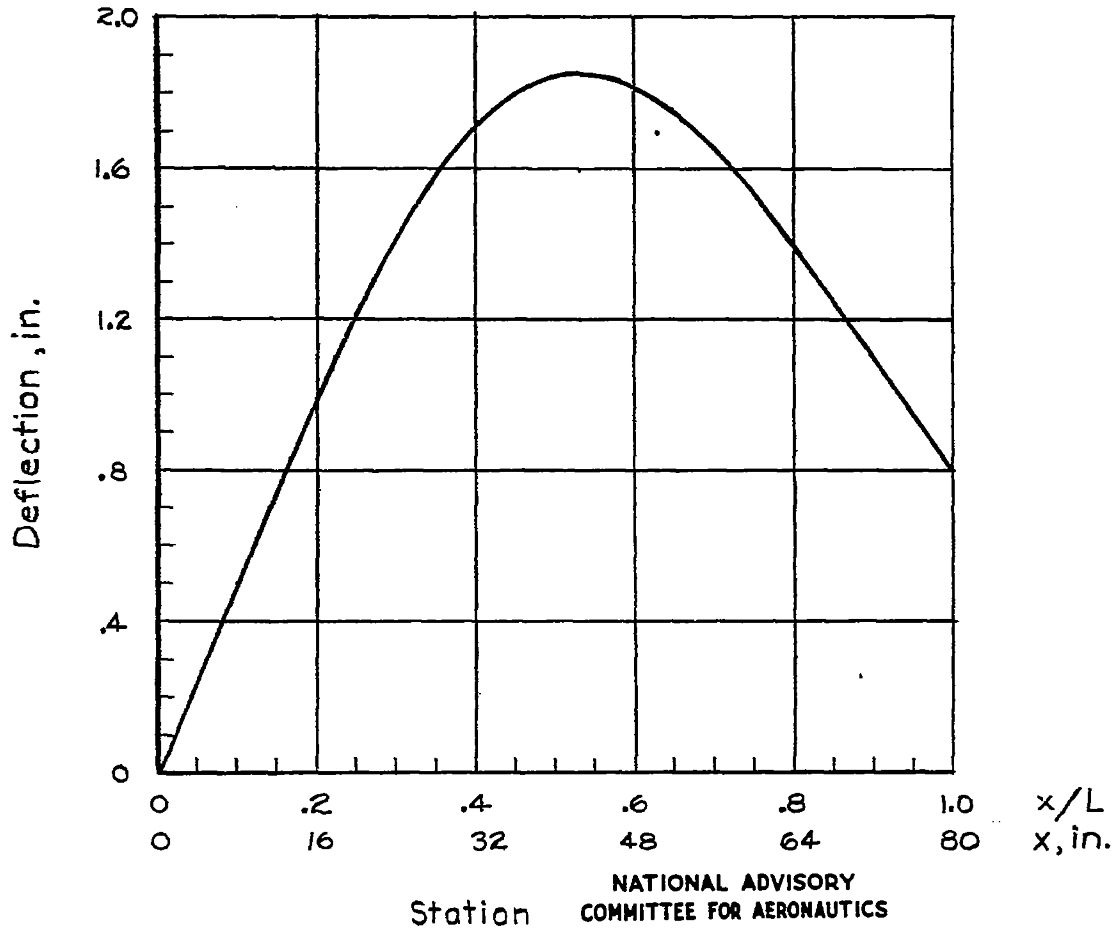



Figure 12.- Deflections of span BC in illustrative example.

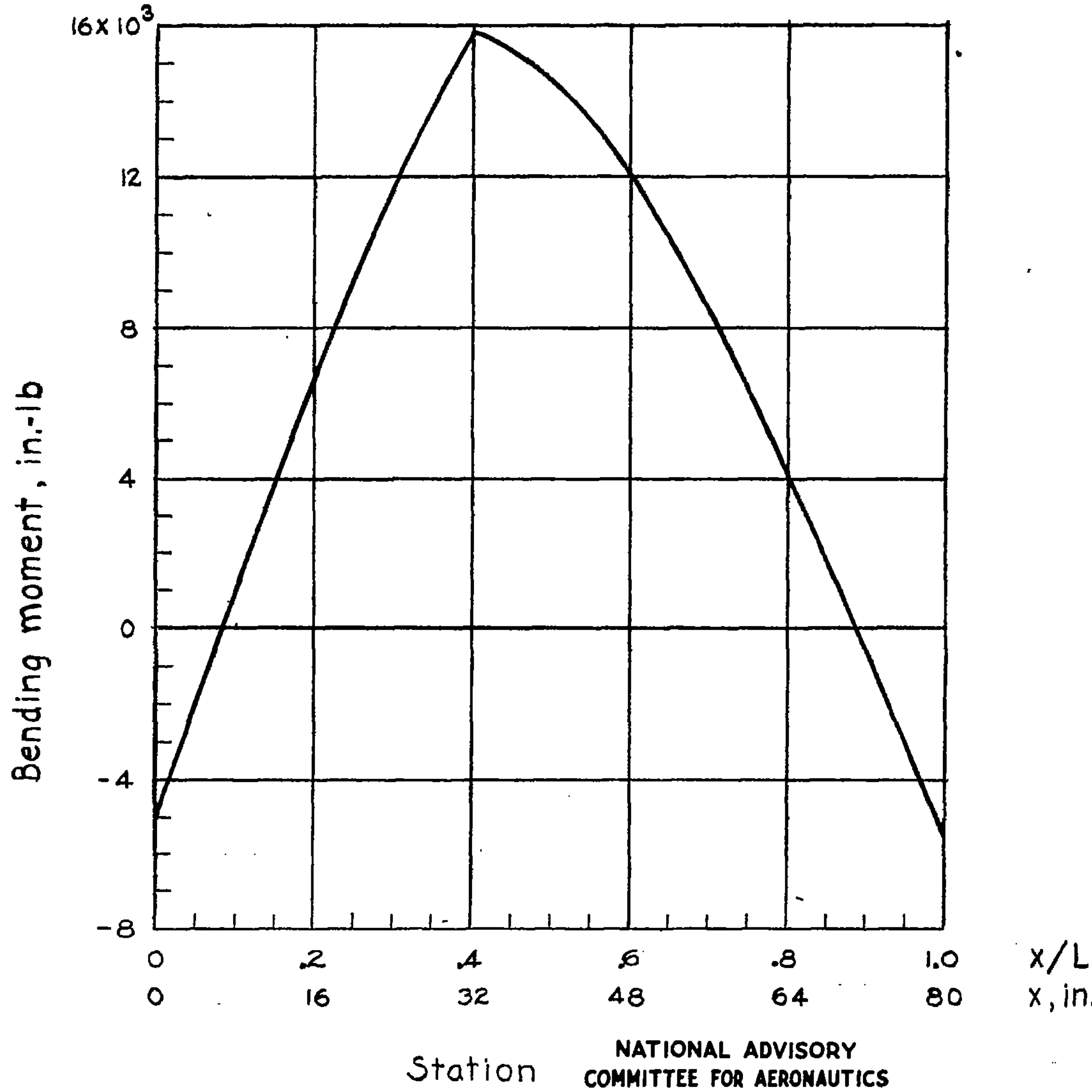



Figure 13.- Bending-moment diagram for span BC in illustrative example.