NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1255

THE EFFECT OF COMPRESSIBILITY ON THE GROWTH OF THE LAMINAR BOUNDARY LAYER ON LOW-DRAG WINGS AND BODIES

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SUMMARY

The development of the laminar boundary layer in a compressible fluid is considered. Formulas are given for determining the boundary-layer thickness and the boundary-layer Reynolds number, which is a measure of the boundary-layer stability, for airfoils and bodies of revolution.

It is shown that low drag coefficients can be maintained to considerably larger Reynolds numbers if these Reynolds numbers are associated with high Mach numbers rather than low Mach numbers. The primary cause of the increase in boundary-layer stability with increasing Mach number is viscosity changes resulting from aerodynamic heating.

INTRODUCTION

Experiments with a large number of low-drag airfoils have shown that as long as the transition from laminar to turbulent flow at the surface occurs between the minimum pressure position and the trailing edge of the airfoil, the low-drag characteristics of these airfoils are maintained, but that as the transition point moves forward of the minimum pressure position the drag coefficient increases more or less markedly depending on the airfoil pressure distribution.

It has been found that the boundary-layer Reynolds number R_δ, based on the boundary-layer thickness and the local velocity outside this layer, gives a fair measure of the stability of the boundary layer and, in consequence, may be used as a criterion for determining the point at which transition to turbulent flow takes place.

As noted in reference 1, the best estimates of the critical value of R_δ available at present were obtained from flight tests of an NACA 35-215 airfoil section which was tested as a glove on the B-18 airplane. Designating

$$R_\delta = \frac{V\delta}{\nu}$$

where V is the velocity outside the boundary layer; δ, the distance from the surface of the airfoil to a point in the boundary layer where the velocity has reached 0.707 V; and ν, the kinematic viscosity of the fluid, critical values of R_δ between 8000 and 9500 were observed in these flight tests.

From von Kármán's momentum relation (reference 2, p. 107) it is evident that, in an incompressible fluid, if (1) the boundary layer on a given body is laminar from the stagnation point to any given point on the body and (2) the boundary-layer velocity profile is at all points along the surface of the same form when considered nondimensionally in terms of δ and V; then at the given point on the surface, the boundary-layer Reynolds number R_δ is related to the body Reynolds number R (based on body dimensions and the stream speed) in the form

$$\frac{{R_\delta}^2}{R} = \text{constant}$$

If the constant is known for any body at the minimum pressure point it is possible to determine the body Reynolds number, which is the upper limit of the range of the low-drag coefficients, for a given value of ${R_\delta}_{crit}$. For nearly incompressible flow this constant may be evaluated by the method of reference 1. In those applications where the Mach number is not negligibly small, it is necessary to extend this method to take account of the compressibility effects. Such an extension of this method is the subject of this paper.

THEORY

The growth of the laminar boundary layer in a compressible fluid may be conveniently studied by von Kármán's momentum method. To this end, consider first the steady-state flow across the faces of an elemental parallelepiped at the surface of a two-dimensional body shown in figure 1. Let h, which is chosen so as to be independent of s, be the distance from the surface of the body to a point in the boundary layer where the fluid shear has become negligibly small.

The several contributions to the s component of the change in momentum across the parallelopiped will now be considered in turn.

The fluid entering the face normal to s per unit width introduces the momentum

$$\int_0^h \rho\, u^2\, dy$$

while that removed at the opposite face is

$$\int_0^h \rho\, u^2\, dy + ds \frac{d}{ds} \int_0^h \rho\, u^2\, dy$$

and hence the change in this contribution to the momentum is

$$ds\left(\frac{d}{ds} \int_0^h \rho\, u^2\, dy\right) \tag{1}$$

No contribution occurs at the surface of the airfoil but at the parallel face an amount $\rho_v\, vV\, ds$ is removed. Continuity requires that

$$\frac{\partial(\rho v)}{\partial y} = - \frac{\partial(\rho u)}{\partial s}$$

hence

$$\rho_v\, v = - \frac{d}{ds} \int_0^h \rho\, u\, dy$$

and so this momentum contribution becomes

$$- V\, ds\left(\frac{d}{ds} \int_0^h \rho\, u\, dy\right) \tag{2}$$

Since the flow is considered to be constant with time, the total change in the s component of momentum across the parallelepiped is given by the sum of equations (1) and (2).

The forces acting on the parallelepiped in the direction of s are the surface shear and the pressure difference between the surfaces normal to s. The shear force, using the established sign convention, is

$$-\tau\, ds \tag{3}$$

and, if the boundary layer is thin, it has been shown (reference 2, p. 83) that the pressure variation with y is negligible, so that the pressure force is

$$-h\frac{dp}{ds}\,ds \tag{4}$$

Now v is small compared to V, so that Bernoulli's equation for a compressible flow which is constant with respect to time may be written for the flow region outside the frictional influence of the boundary layer

$$\frac{dp}{ds} = -\rho_V V\frac{dV}{ds} = -\frac{1}{2}\frac{d\,(\rho_V V^2)}{ds} + \frac{1}{2}V^2\frac{d\rho_V}{ds}$$

It is convenient to rewrite this as

$$\frac{dp}{ds} = -\frac{d\,(\rho_V V^2)}{ds} + V^2\frac{d\rho_V}{ds} + \rho_V V\frac{dV}{ds}$$

$$= -\frac{d\,(\rho_V V^2)}{ds} + V\frac{d\,(\rho_V V)}{ds}$$

Moreover, since both ρ_V and V are independent of y, then, for reasons which will be evident later, rewrite the last equation as

$$\frac{dp}{ds} = -\frac{1}{h}\frac{d}{ds}\int_0^h \rho_V V^2 dy + \frac{V}{h}\frac{d}{ds}\int_0^h \rho_V V\, dy$$

so that the expression for the pressure force becomes

$$-h\frac{dp}{ds}\,ds = ds\left[\frac{d}{ds}\left(\int_0^h \rho_V V^2 dy\right) - V\frac{d}{ds}\int_0^h \rho_V V\, dy\right] \qquad (5)$$

Finally, equating the change in the s component of momentum across the parallelepiped to the s directed forces on the parallelepiped the "momentum" relation for the two-dimensional flow of a compressible fluid (i.e., with varying density) is

$$\tau = \frac{d}{ds}\int_0^h (\rho_V V^2 - \rho\, u^2)\, dy - V\frac{d}{ds}\int_0^h (\rho_V V - \rho\, u)\, dy \qquad (6)$$

It has been observed in a number of experiments with conventional low-drag and high critical compressibility speed airfoils that the Blasius-type profile is a good approximation to the actual boundary-layer profile over the forward region of the airfoil where the pressures are falling. An examination of the calculated boundary-layer profiles for a flat plate at a number of Mach numbers (reference 3) indicates that the form of the profile remains close to the Blasius type for subsonic flows. As a consequence it seems reasonable to assume, as is done in the analysis to follow, that the boundary layer over the surface of conventional low-drag and high critical speed airfoils will remain of the Blasius type throughout the subsonic speed range.

For adiabatic conditions, the local temperature and density outside the boundary layer are, respectively,

$$\left.\begin{aligned} T_V &= T_o\left\{1 - \frac{\gamma-1}{2}M^2\left[\left(\frac{V}{V_o}\right)^2 - 1\right]\right\} \\ \rho_V &= \rho_o\left\{1 - \frac{\gamma-1}{2}M^2\left[\left(\frac{V}{V_o}\right)^2 - 1\right]\right\}^{\frac{1}{\gamma-1}} \end{aligned}\right\} \qquad (7)$$

and to the order of M^2 the local density is

$$\rho_V = \rho_o\left\{1 - \frac{M^2}{2}\left[\left(\frac{V}{V_o}\right)^2 - 1\right]\right\}$$

where the subscript o denotes conditions in the free stream and the subscript v denotes conditions just outside the boundary layer at any point s along the airfoil, where the velocity is V; M is the free-stream Mach number, and $\gamma = c_p/c_v$, the ratio of specific heats.

Since the pressure is transmitted unchanged through the boundary layer, it follows from the law of Boyle and Charles that the density at any point y within the boundary layer is related to the local temperature by

$$\rho = \rho_v\left(\frac{T_v}{T}\right) \tag{8}$$

Further, it is shown in reference 3 that for a flat plate the temperature variation within the boundary layer, for the Prandtl number equal to unity, is given by

$$T = T_o + \left(T_{u=o} - T_o\right)\left[1 - \left(\frac{u}{V_o}\right)^2\right]$$

The Prandtl number is denoted by

$$Pr = \frac{c_p\mu}{k}$$

wherein, for the fluid,

μ the absolute viscosity coefficient

c_p the specific heat at constant pressure

and

k the thermal conductivity

For air, the Prandtl number is less than unity (at standard condition Pr for air is 0.733) but it is not expected that the form of the temperature variation as given in reference 3 will, for air, be seriously in error. For the airfoil it seems correct then to assume the temperature variation to be of the same form

$$T = T_V + \left(T_{u=o} - T_V\right)\left[1 - \left(\frac{u}{V}\right)^2\right] \qquad (9)$$

Moreover, the results of tests with a circular cylinder (reference 4) have indicated that the surface temperature may be given with reasonable accuracy by

$$T_{u=o} = \left[1 + \frac{\gamma-1}{2} M^2 \left(Pr^{\frac{1}{2}}\right)\left(\frac{V}{V_o}\right)^2\right] T_V \qquad (10)$$

Finally, from the relations of equations (7), (8), (9), and (10), to the order M^2, it may be found that

$$\rho = \rho_o \left\{1 - \frac{M^2}{2}\left[\left(\frac{V}{V_o}\right)^2 - 1 + (\gamma-1)\left(Pr^{\frac{1}{2}}\right)\left(\frac{V}{V_o}\right)^2\right]\left[1 - \left(\frac{u}{V}\right)^2\right]\right\} \qquad (11)$$

The surface unit shear is given by

$$\tau = \left(\mu \frac{du}{dy}\right)_{y=o}$$

Experiment has shown that μ varies as the absolute temperature to the 0.76 power and from equations (7) and (10) to the order of M^2

$$\frac{T_{u=o}}{T_o} = \left\{1 - \frac{M^2}{2}(\gamma-1)\left[\left(\frac{V}{V_o}\right)^2\left(1-Pr^{\frac{1}{2}}\right) - 1\right]\right\} \qquad (12)$$

and so, to the present order of approximation

$$\tau = \mu_o\left(\frac{du}{dy}\right)_{y=o}\left\{1 - \frac{0.76\ (\gamma-1)\ M^2}{2}\left[\left(\frac{V}{V_o}\right)^2\left(1-Pr^{\frac{1}{2}}\right) - 1\right]\right\} \qquad (13)$$

Using the density relations of equations (7) and (11), the value of τ given by equation (13), and assuming the Blasius variation of u/V with y/h in the momentum equation (6), it was found that, to the order of M^2, the boundary-layer thickness δ is given by

$$\delta^2 = \frac{c\nu_o}{V_1}\left(\frac{V_o}{V_1}\right)^{8.17}\left[5.3\left\{1 + M^2\left[0.67\left(\frac{V_1}{V_o}\right)^2 - 0.35\right]\right\}\right.$$

$$\left.\int_0^{s_1/c}\left(\frac{V}{V_o}\right)^{8.17} d(s/c) - 0.44\, M^2 \int_0^{s_1/c}\left(\frac{V}{V_o}\right)^{10.17} d(s/c)\right] \quad (14)$$

where

c chord of the airfoil

ν_o the kinematic viscosity in the free stream

V_1 the velocity outside the boundary layer at the chordwise station, s_1/c for which the boundary layer is being computed

δ the boundary-layer thickness, which is considered in this analysis to be the distance from the surface of the airfoil to a point in the boundary layer where the ratio of the local velocity to the velocity outside the boundary layer is 0.707

To employ R_δ as a criterion for determining the stability of the laminar boundary layer, account must be taken of the fact that because of aerodynamic heating, the kinematic viscosity varies throughout the boundary layer. The value of ν used in calculating the boundary-layer Reynolds number should be that characteristic of the point in the boundary layer at which instability initiates. The theoretical analysis does not indicate the location of this point, but experiment indicates (as will be discussed later), that instability initiates near the inside of the boundary layer. The kinematic viscosity is given by

$$\nu_{u=o} = \left(\frac{\mu}{\rho}\right)_{u=o} = \frac{\mu_o\left(\frac{T_{u=o}}{T_o}\right)^{0.76}}{\rho_o\left(\frac{\rho_{u=o}}{\rho_o}\right)} \quad (15)$$

but equation (8) applies through the boundary layer so

$$\nu_{u=o} = \frac{\mu_o}{\rho_o}\,\frac{\left(\frac{T_{u=o}}{T_v}\right)^{1.76}\left(\frac{T_v}{T_o}\right)^{0.76}}{\left(\frac{\rho_v}{\rho_o}\right)} \quad (16)$$

and using the relations of equations (7) and (10)

$$\left(\frac{\nu_{u=o}}{\nu_o}\right) = \frac{\left[1 + \frac{\gamma-1}{2} M^2 \Pr^{\frac{1}{3}}\left(\frac{V}{V_o}\right)^2\right]^{1.76}}{\left\{1 - \frac{\gamma-1}{2} M^2\left[\left(\frac{V}{V_o}\right)^2 - 1\right]\right\}^{1.74}} = \frac{\left[1 + 0.17M^2\left(\frac{V}{V_o}\right)^2\right]^{1.76}}{\left\{1 - 0.20M^2\left[\left(\frac{V}{V_o}\right)^2 - 1\right]\right\}^{1.74}} \tag{17}$$

If

$$\frac{R_\delta}{R_c} = \frac{\frac{\delta V}{\nu_{u=o}}}{\frac{cV_o}{\nu_o}} = \left(\frac{\delta}{c}\right)\left(\frac{V}{V_o}\right)\left(\frac{\nu_o}{\nu_{u=o}}\right) \tag{18}$$

then with the value of δ given by equation (14)

$$\frac{{R_\delta}^2}{R_c} = \frac{\left\{1 - 0.20M^2\left[\left(\frac{V}{V_o}\right)^2 - 1\right]\right\}^{3.48}}{\left[1 + 0.17M^2\left(\frac{V}{V_o}\right)^2\right]^{3.52}\left(\frac{V_1}{V_o}\right)^{7.17}}\left[5.3\left\{1 + M^2\left[0.67\left(\frac{V_1}{V_o}\right)^2 - 0.35\right]\right\}\right.$$

$$\left.\int_o^{s_1/c}\left(\frac{V}{V_o}\right)^{8.17} d(s/c) - 0.44\, M^2\int_o^{s_1/c}\left(\frac{V}{V_o}\right)^{10.17} d(s/c)\right] \tag{19}$$

For moderate Mach numbers equation (19) may be approximated by

$$\frac{{R_\delta}^2}{R_c} = \left(\frac{V_o}{V_1}\right)^{7.17}\left[5.3\left\{1 - M^2\left[0.63\left(\frac{V_1}{V_o}\right)^2 - 0.34\right]\right\}\int_o^{s_1/c}\left(\frac{V}{V_o}\right)^{8.17} d(s/c)\right.$$

$$\left. - 0.44\, M^2\int_o^{s_1/c}\left(\frac{V}{V_o}\right)^{10.17} d(s/c)\right] \tag{20}$$

For the compressible fluid boundary layer of a body of revolution, the momentum relation may be found to be

$$\tau_o r = \frac{d}{ds}\int_o^h \left(\rho_V V^2 - \rho\ u^2\right) r\ dy - V\frac{d}{ds}\int_o^h \left(\rho_V V - \rho\ u\right) r\ dy \quad (21)$$

and under the previous assumptions as to temperature and density variation and the shape of the boundary-layer velocity profile, it may be shown that to the order of M^2

$$\delta^2 = \frac{L\nu_o}{V_1}\left(\frac{V_o}{V_1}\right)^{8.17}\left(\frac{L}{r_1}\right)^2\left[5.3\left\{1 + M^2\left[0.67\left(\frac{V_1}{V_o}\right)^2 - 0.35\right]\right\}\right.$$

$$\left.\int_o^{s_1/L}\left(\frac{r}{L}\right)^2\left(\frac{V}{V_o}\right)^{8.17} d(s/L) - 0.44\ M^2\int_o^{s_1/L}\left(\frac{r}{L}\right)^2\left(\frac{V}{V_o}\right)^{10.17} d(s/L)\right] \quad (22)$$

and

$$\frac{R_\delta^2}{R_L} = \left(\frac{L}{r_1}\right)^2\left(\frac{V_o}{V_1}\right)^{7.17}\left[5.3\left\{1 - M^2\left[0.63\left(\frac{V_1}{V_o}\right)^2 - 0.34\right]\right\}\right.$$

$$\left.\int_o^{s_1/L}\left(\frac{r}{L}\right)^2\left(\frac{V}{V_o}\right)^{8.17} d(s/L) - 0.44\ M^2\int_o^{s_1/L}\left(\frac{r}{L}\right)^2\left(\frac{V}{V_o}\right)^{10.17} d(s/L)\right] \quad (23)$$

where

r_1 radius of the body at s_1

r radius of the body at s where the velocity is V

L length of the body

$R_L = V_o L/\nu_o$

and the remaining symbols are as previously designated.

To apply the equations, the velocity distribution at the Mach number M must be ascertained. When the experimental pressure coefficient P distribution is known at the desired Mach number, the distribution of V/V_o may be found using Bernoulli's equation for a compressible fluid. For air this equation is

$$\left(\frac{V}{V_o}\right)^2 = 1 + \frac{1 - [1+0.7025\ M^2P]^{0.2888}}{0.2025\ M^2} \tag{24}$$

Values obtained from this equation are given in table I.

For two-dimensional flow, where the pressure coefficient distribution is known for $M = 0$, that for the desired Mach number may be calculated using the von Kármán-Tsien equation (reference 5)

$$P = \frac{P_{M=0}}{\sqrt{1-M^2} + \dfrac{M^2\ P_{M=0}}{2\ (1+\sqrt{1-M^2})}} \tag{25}$$

Values obtained from this equation are given in table II.

DISCUSSION AND CONCLUSIONS

An investigation of the boundary-layer thickness at a point 55 percent of the chord behind the leading edge on the upper surface of an NACA 66, 2-420 airfoil at several Mach numbers was conducted in the 16-foot wind tunnel at the Ames Aeronautical Laboratory. Using the measured pressure distributions at the same Mach numbers, the boundary-layer thickness was calculated by equation (14) which considers effects of compressibility and aerodynamic heating, and by the corresponding equation of reference 1 which neglects these effects. The calculated variation of boundary-layer thickness with Mach number as determined from these equations and the several experimentally measured values are shown on figure 2. That the theoretical variation of δ is valid is indicated by the close agreement between the calculated values obtained from equation (14) and the experimental results.

As noted previously, in order that R_δ may be used as a criterion for the stability of the boundary layer, it is essential to determine where, within the boundary layer, transition to turbulent flow initiates. In experimental investigations with flat plates (reference 6), it was found that slow fluctuations of flow occur within the boundary layer though they are not apparent near the outside of the layer. Jones (reference 7) obtained experimental results substantiating these data and suggested that the phenomena of transition to turbulent flow may be the direct result of intermittent instability due to transient separation of the flow from the surface. If this is true, then transition must initiate near the inside of the boundary layer.

The experimental data available showing the effects of compressibility and aerodynamic heating indicate that transition does arise near the surface. These data were obtained with four NACA 27-212 airfoil models of different chords by measuring the maximum Reynolds number for which low drag was maintained. It was found that the values of this critical Reynolds number for the smaller chord airfoils, which required higher Mach numbers than the larger chord airfoils to reach a given Reynolds number, were much higher than those for the larger chord airfoils. The variation of critical Reynolds number with Mach number computed from equation (19) and experimental measurements for the NACA 27-212 airfoils are shown in figure 3. It is seen that when the boundary-layer Reynolds number is based on the inside viscosity and the low Mach number experimental points are fitted to the theoretical curve, the calculated effect of compressibility is in agreement with experiment. In this connection, a calculation was made to determine the ratio $R^*/R^*_{M=0}$ using the viscosity corresponding to conditions outside the boundary. The calculation predicted a decreasing value of $R^*/R^*_{M=0}$ with Mach number, which is contrary to the observed fact.

Recently an experimental investigation was made of the effect of heating the surface of a low-drag airfoil by heating elements placed within the wing. It was found, when heat was so applied as to maintain the entire surface of the airfoil over which the laminar flow occurred at a constant temperature increment above the temperature of the ambient stream, that the boundary layer was destabilized so that the critical Reynolds number was decreased. With the same temperature increment but with only the surface in the immediate vicinity of the transition region heated, the destabilizing effect on the boundary layer was even more marked. These results, in contrast to the results of the experiments with the NACA 27-212 airfoils previously alluded to, would indicate that viscosity considerations alone are not sufficient to explain the effect of heat on the stability of the boundary layer unless, as in the cases of aerodynamic heating and no heating, the temperature gradient in the boundary layer at the surface is zero. In the heated airfoil investigation, heating only the leading-edge section was expected to produce a temperature variation in the boundary layer at transition similar to that obtained in aerodynamic heating. This was attempted but the heat so transferred was insufficient to materially influence the temperature variation over and above that occurring naturally at the Mach numbers of the tests so that no further conclusions could be drawn.

An exact solution of the boundary-layer momentum equation can be obtained for compressible flow over a flat plate. In figure 4, the exact theoretical variation of the flat-plate critical Reynolds number is compared with values obtained from equations (19) and (20). It is seen from this figure that, even at supersonic Mach numbers, equation (19) is in good agreement with the exact solution. Considerable caution should be used in applying the analysis of the present report at such large Mach numbers. The boundary-layer thickness computed by equation (14) should be reasonably accurate even at somewhat supersonic Mach numbers; but there is only scant experimental basis for the assumption that the same value of R_δ determines the limit of stability of the laminar boundary layer at sonic velocities and at low speeds. In consequence, it is considered that the equations for stability developed in this report should only be used over the range of Mach numbers for which equations (19) and (20) are essentially in agreement.

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REFERENCES

1. Jacobs, E. N., and von Doenhoff, A. E.: Formulas for Use in Boundary-Layer Calculations on Low-Drag Wings. NACA ACR, Aug. 1941.

2. Durand, William F. (Ed.): Aerodynamic Theory. Vol. III, Julius Springer, (Berlin) 1935.

3. Emmons, H. W., and Brainerd, J. G.: Temperature Effects in a Laminar Compressible-Fluid Boundary Layer Along a Flat Plate. Jour. Appl. Mech., vol. 8, no. 3, Sept. 1941, pp. A-105-110.

4. Eckert, E.: Temperature Recording in High-Speed Gases. NACA TM No. 983, 1941.

5. von Kármán, Th.: Compressibility Effects in Aerodynamics. Jour. Aero. Sci., vol. 8, no. 9, July 1941, pp. 337-355.

6. Dryden, Hugh L.: Air Flow in the Boundary Layer Near a Plate. NACA Rep. No. 562, 1936.

7. Jones, B. Melvill: Flight Experiments on the Boundary Layer. Jour. Aero. Sci., vol. 5, no. 3, Jan. 1938, pp. 81-94.

Values of $\left(\frac{V}{V_c} - \sqrt{1 - P}\right)$

P \ M	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.7	0.0011	0.0043	0.0097	0.0168	0.0255	0.0358	0.0454	0.0579
.6	.0007	.0028	.0063	.0109	.0164	.0232	.0298	.0381
.5	.0004	.0017	.0039	.0069	.0105	.0145	.0194	.0248
.4	.0002	.0010	.0023	.0041	.0062	.0087	.0117	.0149
.3	.0001	.0006	.0012	.0020	.0031	.0047	.0061	.0077
.2	.0000	.0003	.0006	.0009	.0013	.0020	.0027	.0033
.1	0	.0001	.0002	.0004	.0006	.0008	.0010	.0012
0	0	0	0	0	0	0	0	0
−0.1	0	0.0000	0.0002	0.0003	0.0004	0.0006	0.0008	0.0010
−.2	0.0000	.0001	.0004	.0009	.0011	.0016	.0023	.0031
−.3	.0001	.0003	.0009	.0016	.0025	.0037	.0052	.0069
−.4	.0002	.0006	.0016	.0027	.0044	.0065	.0089	.0121
−.5	.0003	.0009	.0023	.0042	.0069	.0100	.0137	– – –
−.6	.0004	.0013	.0032	.0059	.0096	.0141	.0195	– – –
−.7	.0005	.0018	.0043	.0078	.0127	.0187	– – –	– – –
−.8	.0007	.0023	.0055	.0100	.0161	.0242	– – –	– – –
−.9	.0008	.0029	.0068	.0124	.0200	.0304	– – –	– – –
−1.0	.0009	.0035	.0083	.0151	.0245	– – –	– – –	– – –
−1.1	.0011	.0042	.0099	.0179	.0292	– – –	– – –	– – –
−1.2	.0013	.0050	.0115	.0210	.0340	– – –	– – –	– – –

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TABLE II

Values of $(P - P_{M=0})$

$P_{M=0}$ \ M	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.7	0.0022	0.0094	0.0220	0.0428	0.0660	0.1032	0.1550	0.234
.6	.0021	.0086	.0201	.0392	.0619	.0974	.1475	.226
.5	.0019	.0077	.0179	.0345	.0556	.0882	.1365	.214
.4	.0016	.0064	.0150	.0297	.0479	.0758	.1203	.184
.3	.0012	.0052	.0119	.0231	.0383	.0598	.0989	.150
.2	.0008	.0037	.0080	.0162	.0273	.0420	.0727	.110
.1	.0004	.0020	.0040	.0082	.0142	.0221	.0399	.0613
0	0	0	0	0	0	0	0	0
−0.1	−0.0005	−0.0022	−0.0049	−0.0098	−0.0160	−0.0271	−0.0450	−0.0724
−.2	−.0011	−.0046	−.0104	−.0201	−.0346	−.0562	−.0970	−.157
−.3	−.0017	−.0078	−.0164	−.0312	−.0542	−.0895	−.1540	−.2555
−.4	−.0024	−.0102	−.0230	−.0438	−.0772	−.1264	−.2145	— — —
−.5	−.0032	−.0130	−.0305	−.0571	−.1006	−.1667	−.2776	— — —
−.6	−.0040	−.0161	−.0378	−.0706	−.1273	−.2080	−.3440	— — —
−.7	−.0049	−.0194	−.0461	−.0859	−.1559	−.2570	— — —	— — —
−.8	−.0058	−.0229	−.0546	−.1021	−.1868	−.3086	— — —	— — —
−.9	−.0067	−.0269	−.0642	−.1199	−.2186	−.3645	— — —	— — —
−1.0	−.0076	−.0312	−.0742	−.1382	−.2515	— — —	— — —	— — —
−1.1	−.0086	−.0349	−.0855	−.1579	−.2865	— — —	— — —	— — —
−1.2	−.0098	−.0396	−.0971	−.1783	−.3245	— — —	— — —	— — —

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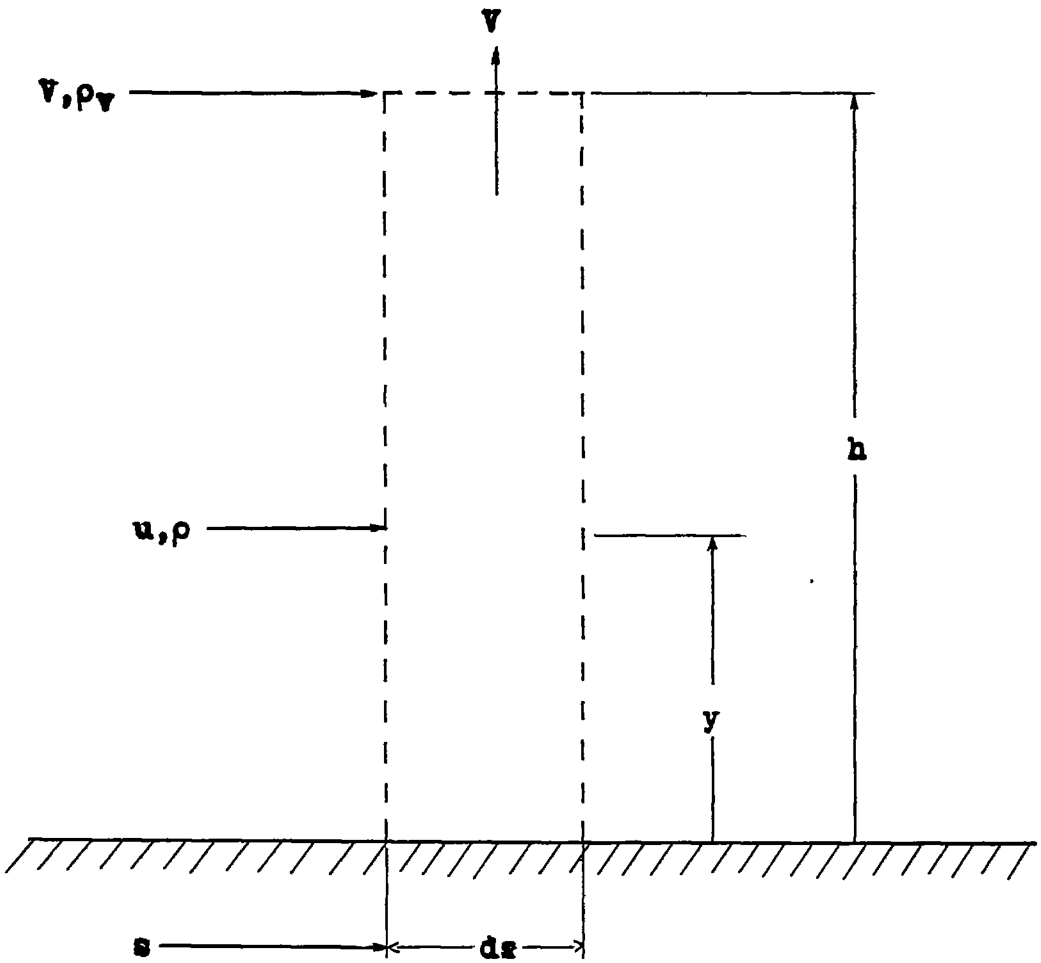

Figure 1.- Boundary layer coordinates.

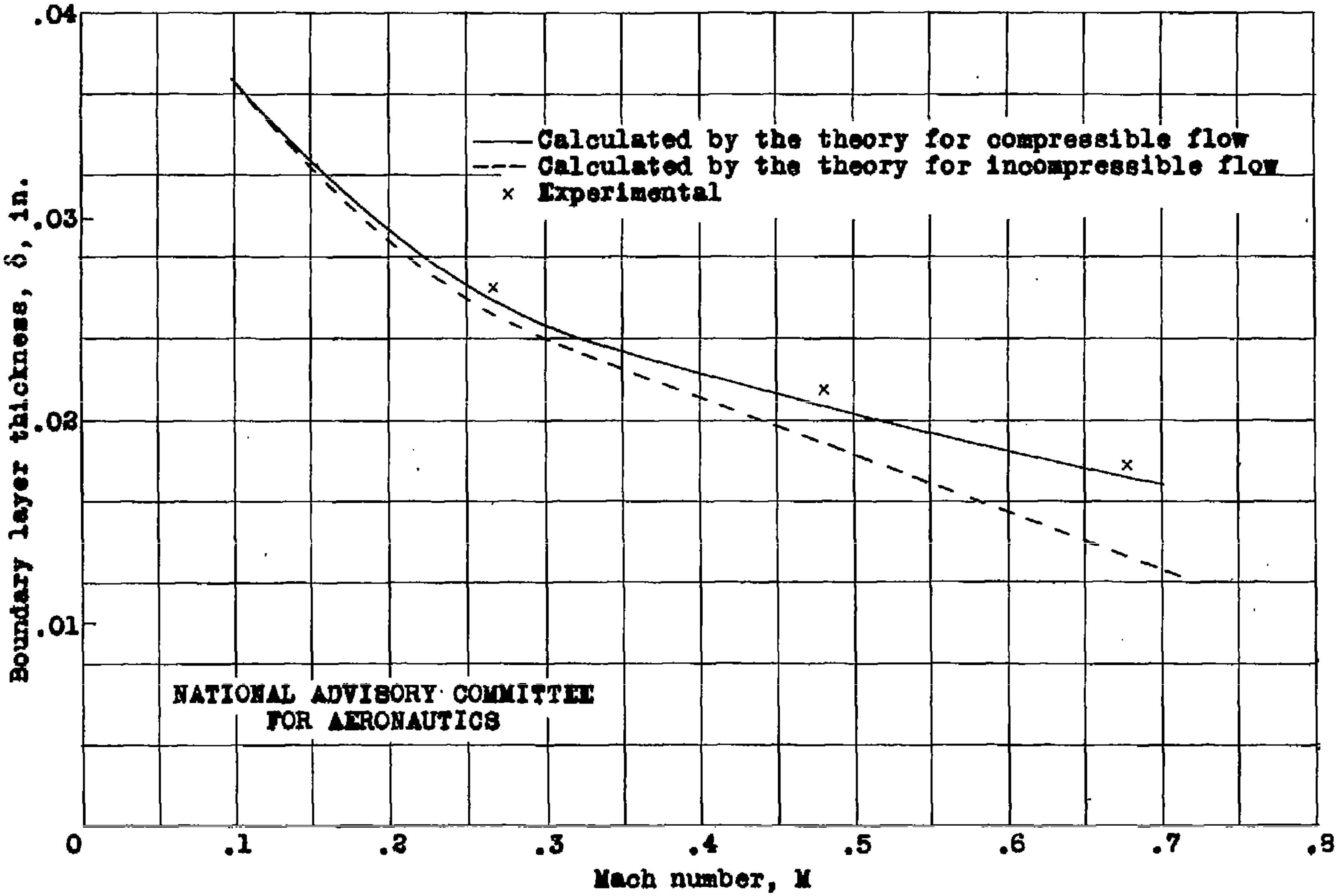

Figure 2.- The effect of compressibility on the boundary-layer thickness, δ. NACA 66, 2-420 airfoil; $s_1/c = 0.55$.

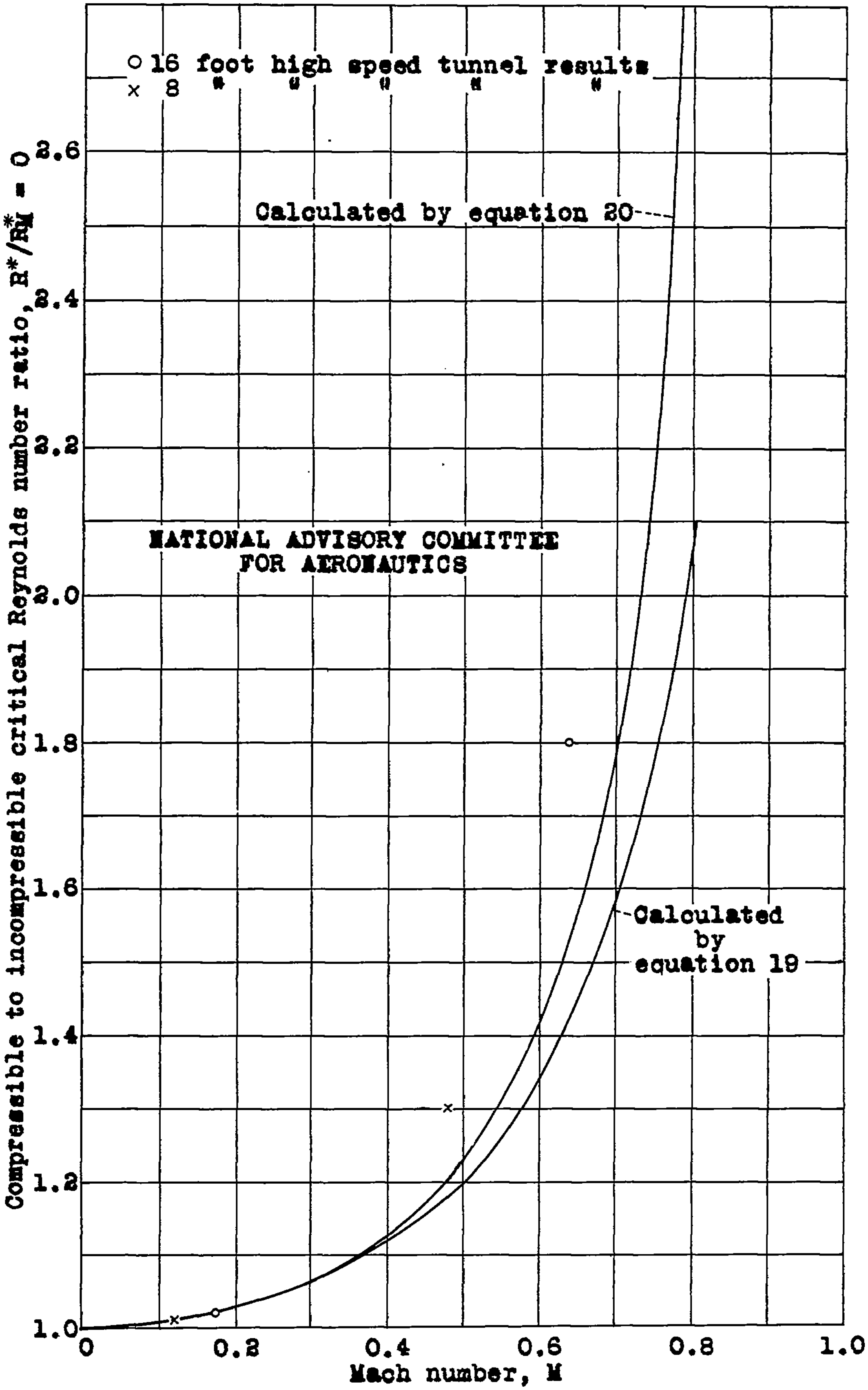

Figure 3.- The effect of compressibility on the critical Reynolds number ratio of the NACA 27-212 airfoil.

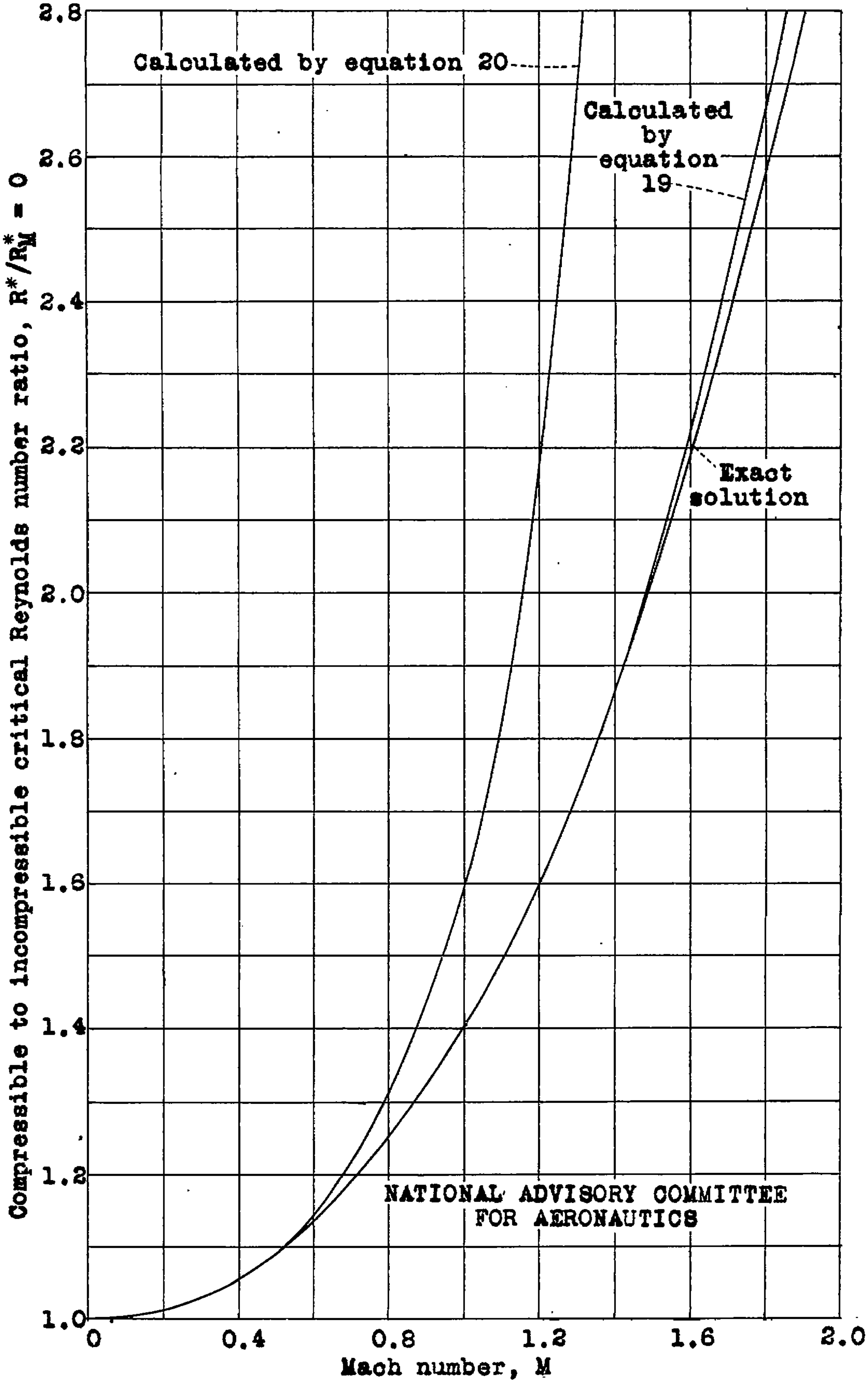

Figure 4.- The effect of compressibility on the critical Reynolds number ratio of a flat plate.