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STRENGTH ANALYSIS OF STIFFENED BEAM WEBS

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SUMMARY

A previously published method for strength analysis of stiffened shear webs has been revised and extended. A set of formulas and graphs which cover all aspects of strength analysis is given, experimental data are presented, and the accuracy of the formulas as judged by comparison with these data is discussed. Revisions of some formulas have resulted in improved agreement with experimental stresses and with more rigorous theory, particularly for low ratios of applied shear to buckling shear. The scope of the experimental evidence has been greatly increased compared with the previous paper by incorporating the results of several investigations undertaken since then.

INTRODUCTION

Many of the shear webs used in aircraft structures are so thin that they buckle at a fraction of the ultimate load. A purely mathematical theory of basically simple form has been developed for the limiting case of webs so thin that their resistance to buckling is entirely negligible (reference 1). This theory of "pure diagonal tension" is too conservative for practical use because the resistance to buckling of practical webs - "incomplete-diagonal-tension webs" - is far from being negligible. A mathematical theory of incomplete diagonal tension has been developed (reference 2), but it requires such extensive calculations that its adaptability to stress analysis is questionable; moreover, no adequate check of its accuracy by comparing it with test results over a wide range has been published, and it is not sufficiently complete to explain upright failures, probably the most important item in the design of web systems.

In the face of such difficulties, practical stress-analysts have often resorted to entirely empirical formulas. There are two objections to such a procedure: Without the benefit of some guiding theory, a very large number of tests is required to insure the reliability of a given formula, and a formula established for the strength of one part of a beam is usually of little, if any, help in establishing formulas for other items.

The method of analysis given herein constitutes an attempt to avoid insofar as possible the objections to purely theoretical or purely empirical methods. The basis of the method is a semi-empirical engineering theory of incomplete diagonal tension, made as simple as possible by confining attention to over-all or average effects. The theory is formulated in such a way that the limiting conditions of fully developed diagonal tension and of zero diagonal tension (so-called "shear-resistant web") are included; it can therefore be regarded as an aid for interpolating between these limiting conditions.

The analysis is divided into two parts. The presentation of the theory and of the design formulas is given in part I and is kept very brief in order to approach as closely as possible the final form that it would take in a stress manual. Part II is devoted to a discussion and experimental verification of the formulas; it incorporates the results from a number of previous investigations. In order to keep the length of this part also to a minimum, the discussion has been confined to items of decided practical interest. Reading of part II is not necessary if interest is confined to routine application of the design formulas but is indispensable for anybody who wishes to interpret test results or to extend or modify the formulas in any respect.

The theory is basically the same as that previously published in references 3 and 4, but it has been modified in some respects and therefore supersedes the material given in these references.

SYMBOLS

A	cross-sectional area, square inches
E	Young's modulus, ksi
G	shear modulus, ksi
H	force in beam flange due to horizontal component of diagonal tension, kips
I	moment of inertia, inches^4

L	length of beam, inches
P	force, kips
Q	static moment about neutral axis of parts of cross section as specified by subscript, inches^3
R	coefficient of edge restraint (see formula (7))
S	transverse shear force, kips
d	spacing of uprights, inches
e	distance from median plane of web to centroid of (single) upright, inches
h	depth of beam, inches (see Special Combinations)
k	diagonal-tension factor
t	thickness, inches (used without subscript signifies thickness of web)
α	angle between neutral axis of beams and direction of diagonal tension, degrees
δ	deflection of beam, inches
ϵ	normal strain
ρ	centroidal radius of gyration of cross section of upright about axis parallel to web, inches (no sheet should be included)
σ	normal stress, ksi
τ	shear stress, ksi

Subscripts

DT	diagonal tension
F	flange
S	shear

U	upright
W	web
cr	critical
ult	ultimate
e	effective

Special Combinations

P_U	internal force in upright, kips
R''	shear force on rivets per inch run, kips per inch
R_{tot}	total shear strength (in single shear) of all rivets in one upright, kips
d_c	upright spacing measured as shown in figure 5(a)
h_c	depth of web measured as shown in figure 5(a)
h_e	depth of beam measured between centroids of flanges, inches
h_R	depth of beam measured between centroids of web-to-flange rivet patterns, inches
h_U	length of upright measured between centroids of upright-to-flange rivet patterns, inches
k_{ss}	theoretical buckling coefficient for plates with simply supported edges
σ_o	"basic" allowable stress for forced crippling of uprights (valid for stresses below proportional limit in compression of upright material), ksi
ωd	flange flexibility factor $\left(0.7d \sqrt[4]{\frac{t}{(I_C + I_T)h_e}}\right.$, where I_C and I_T are moments of inertia of compression flange and tension flange, respectively$\left.\right)$

I - THEORY AND FORMULAS

ENGINEERING THEORY OF INCOMPLETE DIAGONAL TENSION

In a plate girder subjected to a shear load less than the buckling load, the web plate is in a state of pure shear along the neutral axis as indicated by the inset diagram in figure 1(a). Above and below the neutral axis, normal stresses exist in a horizontal direction, but in the investigation of the web for the present purpose these stresses may be disregarded, and the stress diagram may be assumed valid over the entire depth of the web. The web stiffeners carry no stress.

If the web is thin, it will buckle at a certain critical shear load. If the load is increased beyond the critical value, the buckle pattern will gradually approach a configuration consisting of parallel folds (fig. 1(b)). In the theoretical limiting case of an infinitely thin sheet, the web carries pure tensile stresses in the direction of the folds as indicated by the inset diagram in figure 1(b). The angle α which these folds include with the horizontal axis of the beam is usually somewhat less than 45°. Simple statical considerations show that each upright carries a load

$$P_U = \tau td \tan \alpha \tag{1}$$

as reaction to the vertical component of the web tension, and each flange carries a compressive force

$$H = \frac{S}{2} \cot \alpha \tag{2}$$

as reaction to the horizontal component of the web tension. Formulas (1) and (2) can be evaluated once the angle α is known. The theory of pure diagonal tension (reference 1) shows that this angle is given by the formula

$$\tan^2\alpha = \frac{\epsilon - \epsilon_X}{\epsilon - \epsilon_y} \tag{3}$$

where ϵ is the strain in the web, ϵ_X is the strain in the flanges due to the force H, and ϵ_y is the strain in the upright. Elongation is considered as positive strain.

In a practical thin-web beam, the state of stress in the web is intermediate between pure shear and pure diagonal tension. An engineering theory of this intermediate state of incomplete diagonal tension may be based on the assumption that the total shear force S in the web can be divided into two parts, a part S_S carried by pure shear and a part S_{DT} carried by pure diagonal tension; thus

$$S = S_S + S_{DT}$$

This expression may be written in the form

$$\begin{aligned} S_{DT} &= kS \\ S_S &= (1 - k)S \end{aligned} \tag{4}$$

where k is the "diagonal-tension factor" which expresses the degree to which the diagonal tension is developed at a given load. With this factor, the state of pure shear is characterized by $k = 0$, and the state of pure diagonal tension, by $k = 1$. The stress condition of a web element is shown in figure 2 for the two limiting cases $k = 0$ and $k = 1$ and for an intermediate case.

The factor k has been established empirically by evaluating strain measurements on uprights because the stresses in the uprights constitute the most sensitive criterion for the degree to which the diagonal tension is developed. For loads less than five times the buckling loads, for which the accuracy of the experimental results is often poor, use was made of the calculations made by means of Levy's large-deflection theory of plates (references 5 and 6). From these experimental and theoretical data, it was found that k can be given by the expression

$$k = \tanh\left(0.5 \log_{10} \frac{\tau}{\tau_{cr}}\right) \tag{5}$$

As long as the web is resisting some compressive stress in a diagonal direction, it can also resist some compressive stress in the vertical direction and thus assist the uprights. If the distribution of these vertical compressive stresses is assumed to be sinusoidal immediately after buckling as indicated in figure 3, the total effective width of sheet cooperating with the upright is $0.5d$. The effective width will decrease as the diagonal tension develops and will become zero for fully developed diagonal tension ($k = 1$). If the effective width is assumed to decrease linearly with k, the effective area contributed by the web to the upright is

$$A_{W_e} = 0.5dt(1 - k) \tag{6}$$

A corresponding assumption could be made for the contribution of the web to the flange area as far as resistance to the force H given by formula (2) is concerned. This refinement, however, is probably unnecessary in the analysis of beam webs.

Formulas (4) to (6) are the fundamental formulas which generalize the theory of pure diagonal tension to cover the full range of incomplete diagonal tension from the limiting case of pure shear to the limiting case of fully developed diagonal tension. They enable the stress analyst to make a reasonably accurate estimate of the stresses in the uprights; the necessity of estimating these stresses with a much better accuracy than that afforded by the theory of pure diagonal tension has been the predominant reason for developing a theory of incomplete diagonal tension.

The theory expressed by formulas (4) to (6) defines only the "over-all" state of stress in the median plane of the web. It does not attempt to give an account of the detail distribution of these stresses, nor does it give any account of the bending stresses in the web sheet induced by the shear buckles. Consequently, all problems that involve the details of the web action require additional assumptions or empirical data for their solution. For a number of items (for instance, forces on the web attachment rivets), the magnitude is known for the limiting cases of pure shear and pure diagonal tension; for any intermediate case of incomplete diagonal tension, the magnitude can then be estimated by interpolating between the limiting cases with the factor k as argument. Straight-line interpolation is used unless empirical data or theoretical considerations indicate a different law of variation. For some quantities, straight-line interpolation is used for simplicity and conservativeness although the theory indicates a more complicated law.

A minor item from the theory of pure diagonal tension should be mentioned here. The vertical component of the diagonal tension in the web will bend the beam flanges as indicated in figure 4.

As a result, the tension will be relieved in some parts of the web and correspondingly increased in other parts of the web as indicated by the spacing of the diagonals in the figure. The redistribution of web tension, in turn, will decrease the secondary bending moments in the flanges. The theory of these effects is discussed in reference 1.

FORMULAS FOR STRESS ANALYSIS

Limitations of Formulas

The formulas given herein are believed to give reasonable assurance of conservative strength predictions provided that normal design practices and proportions are used. The most important points under this provision are that the uprights should not be too thin $\left(\text{say } \frac{t_U}{t} > 0.6\right)$ and that the upright spacing should not be too much outside the range $0.2 < \frac{d}{h} < 1.0$.

Very thin webs $\left(\frac{h}{t} > 1500\right)$ with single uprights, and very thick webs $\left(\frac{h}{t} < 200\right)$ have not been explored adequately. For web systems in these ranges, some possibility of unconservative predictions may exist.

The accuracy that may be expected of the strength predictions is discussed in part II, which presents the experimental evidence. The origin of the formulas and the references are also given in part II in order to keep part I free from details not necessary to the routine application of the formulas.

Critical Shear Stress

In the elastic range, the critical shear stress of the sheet between two uprights is calculated by the formula

$$\tau_{cr} = k_{ss}E\left(\frac{t}{d_c}\right)^2\left[R_h + \frac{1}{2}\left(R_d - R_h\right)\left(\frac{d_c}{h_c}\right)^3\right] \tag{7}$$

where

k_{ss} theoretical buckling coefficient (given in fig. 5(a)) for panel of length h_c and width d_c with simply supported edges

d_c width of sheet between uprights measured as shown in figure 5(a), inches

h_c depth of web measured as shown in figure 5(a), inches

R_h restraint coefficient for edges of sheet along upright (from fig. 5(b))

R_d restraint coefficient for edges of sheet along flanges (from fig. 5(b))

(If $d_c > h_c$, substitute h_c for d_c, d_c for h_c, R_d for R_h, and R_h for R_d.)

Curves of the critical shear stresses for plates of 24S-T aluminum alloy with simply supported edges are given in figure 6. To the right of the dashed line, these curves are plots of the theoretical equation

$$\tau_{cr} = k_{ss}E\left(\frac{t}{d_c}\right)^2$$

and may be used for most aluminum alloys. To the left of the dashed line, the curves represent straight-line tangents to the theoretical curves in a nonlogarithmic plot and are valid only for 24S-T alloy.

When the uprights are very thin, the value of τ_{cr} obtained by formula (7) may be less than that obtained by neglecting the presence of the uprights. Web systems of such abnormal proportions should not be designed by the formulas of the present paper.

Loading Ratio

The loading ratio is the ratio τ/τ_{cr} where τ is the depth-wise average of nominal web shear stress, that is, of the shear stress that would exist in the web if buckling were artificially prevented (by external restraints acting on the web).

When the depth of the flanges is small compared with the depth of the beam, and the flanges are angle sections, the stress τ may be computed by the formula

$$\tau = \frac{S_W}{h_e t} \tag{8}$$

In beams with other cross sections, the average nominal shear stress τ should be computed by the formula

$$\tau = \frac{S_W Q_F}{It}\left(1 + \frac{2Q_W}{3Q_F}\right) \tag{9}$$

where Q_F is the static moment about the neutral axis of the flange material and Q_W is the static moment about the neutral axis of the effective web material above the neutral axis. For the computations of I and Q, the effectiveness of the web must be estimated in first approximation. As second and final approximation, the effectiveness of the web may be taken as equal to $(1 - k)$, where k is the diagonal-tension factor determined in the next step; in other words, when I and Q are being computed, the effective thickness of the web is taken as $(1 - k)t$.

Diagonal-Tension Factor k

After the loading ratio τ/τ_{cr} has been computed, the diagonal-tension factor k can be computed by formula (5) or read from figure 7.

Average Stress in Upright

The average stress σ_U in a double upright (average over the length of the upright) is given by the formula

$$\sigma_U = \frac{k\tau \tan \alpha}{\frac{A_U}{dt} + 0.5(1 - k)}$$

which follows from formulas (1), (4), and (6). It can be evaluated with the help of figure 8 or figure 9 which give the ratio σ_U/τ as a function of the ratio A_U/dt and the loading ratio τ/τ_{cr}. The stress σ_U is uniformly distributed over the cross section of the upright until buckling of the upright begins.

The stress σ_U for a single upright is obtained in the same manner, except that the ratio A_U/dt is replaced by A_{U_e}/dt where

$$A_{U_e} = \frac{A_U}{1 + \left(\frac{e}{\rho}\right)^2} \tag{10}$$

For the single upright, σ_U is still an average over the length of the upright, but it applies only to the median plane of the web along the line of rivets connecting the upright to the web. In any given cross section of the upright, the compressive stress decreases with increasing distance from the web, because the upright is a column loaded eccentrically by the web tension. (For this reason, formulas for local crippling based on the assumption of a uniform distribution of stress over the cross section do not apply.)

Maximum Stress in Upright

The stress σ_U in an upright varies from a maximum at (or near) the neutral axis of the beam to a minimum at the ends of the upright ("gusset effect"). The ratio of the maximum stress to the average stress decreases as the upright spacing or the loading ratio increases. The empirical formula for the ratio is

$$\frac{\sigma_{U_{max}}}{\sigma_U} = 1 + \left[\left(\frac{\sigma_{U_{max}}}{\sigma_U}\right)_o - 1\right](1 - k) \tag{11}$$

where $\left(\frac{\sigma_{U_{max}}}{\sigma_U}\right)_o$ is the value of the ratio when the web has just buckled. The ratio is given graphically in figure 10.

Angle of Diagonal Tension

The angle α between the direction of the diagonal tension and the axis of the beam can be found with the aid of figure 11 after k and σ_U/τ have been determined.

Allowable Stresses in Uprights

The following four types of failure of uprights are conceivable:

(1) Column failure

(2) Forced crippling failure

(3) Natural crippling failure

(4) General elastic instability failure of web and stiffeners

Column failure.- Column failures in the usual meaning of the word (failure due to instability, without previous bowing) are possible only in double uprights. When column bowing begins, the uprights will force the web out of its original plane. The web tensile forces will then develop components normal to the plane of the web which tend to force the uprights back. This bracing action is taken into account by using a reduced "effective" column length of the upright L_e, which is given by the empirical formula

$$L_e = \frac{h_U}{\sqrt{1 + k^2\left(3 - 2\frac{d}{h}\right)}} \tag{12}$$

The stress σ_U at which column failure takes place can then be found by entering a standard column curve for the upright with the slenderness ratio L_e/ρ as argument.

The problem of "column" failures in single uprights has not been investigated to any extent, and test results are greatly at variance with theoretical results. The following two criterions are suggested for strength design:

(a) The stress σ_U should be no greater than the column yield stress for the upright material.

(b) The stress at the centroid of the upright (which is the average stress over the cross section) should be no greater than the allowable column stress for the slenderness ratio $h_U/2\rho$.

The first criterion accounts for the upright acting as an eccentrically loaded compression member; the second one is an attempt to take into account a two-wave type of buckling failure that has been observed in very slender uprights.

Forced crippling failure.- The shear buckles in the web will force buckling of the upright in the leg attached to the web, particularly if the upright is thinner than the web. These buckles give a lever arm to the compressive force acting in the leg and thereby produce a severe stress condition. The buckles in the attached leg will in turn induce buckling of the outstanding legs. In single uprights the outstanding legs are relieved to a considerable extent by virtue of the fact that the compressive stress decreases with distance from the web; the allowable stresses for single uprights are therefore somewhat higher than those for double uprights. Because the forced crippling is of a local nature, it is assumed to depend on the peak value $\sigma_{U_{max}}$ of the upright stress rather than on the average value.

The upright stress at which final collapse occurs is obtained by the following empirical method:

(1) Compute the allowable value of $\sigma_{U_{max}}$ for a perfectly elastic upright material by the formula

$$\sigma_o = 28k\sqrt{t_U/t} \qquad \text{(for single uprights)} \tag{13a}$$

$$\sigma_o = 25k\sqrt{t_U/t} \qquad \text{(for double uprights)} \tag{13b}$$

(2) If σ_o exceeds the proportional limit for the upright material, use as allowable value the stress corresponding to the compressive strain σ_o/E.

(3) If $k < 0.5$, use an effective value in formula (13a) or (13b) given by the expression

$$k_o = 0.15 + 0.7k \tag{13c}$$

Natural crippling failure.- The term "natural crippling failure" is used herein to denote a crippling failure resulting from a compressive stress uniformly distributed over the cross section of the upright. By this definition, it can occur only in double uprights. To avoid natural crippling failure, the peak stress $\sigma_{U_{max}}$ in the upright should be less than the crippling stress of the section for $\frac{L}{\rho} \rightarrow 0$. Natural crippling failure does not appear to be a controlling factor in actual designs.

General elastic instability of web and stiffeners.- Test experience so far has not indicated that general elastic instability need be considered in strength design. Apparently, the web system is safe against general elastic instability if the uprights are designed to fail by column action or by forced crippling at a shear load not much lower than the shear strength of the web. It should be borne in mind, however, that the test experience available at present is not adequate for very thin and for very thick webs. (See section of present paper entitled "Limitations of Formulas.")

Web Design

For design purposes, the peak value of the nominal web shear stress within a bay is taken as

$$\tau_{max} = \tau\left(1 + kC_1\right)\left(1 + kC_2\right) \tag{14}$$

where C_1 and C_2 are the factors given in figures 12 and 13, respectively. The factor C_1 constitutes a correction factor to allow for the angle α of the diagonal tension differing from 45°. The factor C_2 makes allowance for the stress concentration in the web brought about by flexibility of the flanges as discussed in connection with figure 4.

The allowable value of τ_{max} is determined by tests and depends on the value of the diagonal-tension factor k as well as on the details of the web-to-flange and web-to-upright fastenings. Figure 14 gives empirical curves for two aluminum alloys. It should be noted that these curves contain an allowance for the rivet factor; inclusion of this factor in these curves is possible because tests have shown that the ultimate shear stress based on the gross section (that is, without reduction for rivet holes) is almost constant within the normal range of rivet factor ($C_R > 0.6$). A separate check must be made, of course, to insure that the allowable bearing stresses between rivets and sheet are not exceeded.

Rivet Design

The load per inch run acting on the web-to-flange rivets is taken as

$$R'' = \frac{S_W}{h_R} (1 + 0.414k) \tag{15}$$

With double uprights, the web-to-upright rivets must provide sufficient longitudinal shear strength to make the two uprights act as an integral unit until column failure occurs. The total rivet shear strength (single shear strength of all rivets) required for an upright is

$$R_{tot} = \frac{2\sigma_{co}Q}{b} \frac{h_U}{L_e} \tag{16}$$

where

σ_{co} column yield strength of upright material (if σ_{co} is expressed in ksi, R_{tot} will be in kips)

Q static moment of cross section of one upright about an axis in the median plane of the web, inches^3

b width of outstanding leg of upright, inches

h_U/L_e ratio obtainable from formula (12)

The rivets must also have sufficient tensile strength to prevent the buckled sheet from lifting off the stiffener. The necessary strength is given by the tentative criterion

$$\text{Tensile strength (per inch) of rivets} > 0.15t\sigma_{ult} \tag{17}$$

where σ_{ult} is the tensile strength of the web.

For web-to-upright rivets on single uprights, the required tensile strength is given by the tentative criterion

$$\text{Tensile strength (per inch) of rivets} > 0.22t\sigma_{ult} \qquad (18)$$

(The tensile strength of a rivet is defined as the tensile load that causes any failure; if the sheet is thin, failure will consist in the pulling of the rivet through the sheet.)

No criterion for shear strength of the rivets on single uprights has been established; the criterion for tensile strength is probably adequate to insure a satisfactory design.

The pitch of the rivets on single uprights should be small enough to prevent inter-rivet buckling of the web (or the upright, if thinner than the web) at a compressive stress equal to $\sigma_{U_{max}}$. The pitch should also be less than $d/4$ in order to justify the assumption on edge support used in the determination of τ_{cr}. The two criterions for pitch are probably always fulfilled if the strength criterions are fulfilled and normal riveting practices are used.

The upright-to-flange rivets must carry the load existing in the upright into the flange.

$$P_U = \sigma_U A_U \qquad \text{(for double uprights)} \qquad (19)$$

$$P_U = \sigma_U A_{U_e} \qquad \text{(for single uprights)} \qquad (20)$$

These formulas neglect the gusset effect (decrease of σ_U towards the ends of the upright) in order to be conservative.

Secondary Bending Moments in Flanges

The secondary bending moment in a flange, caused by the vertical component of the diagonal tension (fig. 4), may be taken as

$$M = \frac{1}{12} k\tau t d^2 C_3 \qquad (21)$$

where C_3 is a factor given in figure 13. The moment given by formula (21) is the maximum moment in the bay and exists at the ends of the bay over the uprights. If C_3 and k are near unity, the moment in the middle of the bay is half as large as that given by formula (21) and of opposite sign. (See fig. 4.)

Shear Stiffness of Web

The theoretical effective shear modulus of a web G_e in partial diagonal tension is given by figure 15. This modulus is valid only in the elastic range.

II - DISCUSSION OF FORMULAS AND EXPERIMENTAL EVIDENCE

In the following part of the present paper, the formulas and the experimental evidence are discussed in the sequence in which the formulas appear in part I. The experimental evidence presented is based on results from manufacturers' tests and tests made in the Langley Structures Research Division of the NACA. All test evaluations are based on actual material properties insofar as possible.

TEST SPECIMENS

The analysis covers about 90 beams tested by four manufacturers and 32 beams tested by the NACA. Some of the manufacturers' tests could not be fully analyzed because the data were incomplete. The range of the tests can be defined as follows:

Ratios	Range
h/t	300 to 2500
d/h	0.18 to 0.91
A_U/dt	0.039 to 1.2
t_U/t	0.42 to 8.4

The NACA tests are discussed in greater detail than the manufacturers' tests because the strain measurements taken in these tests served as the main basis for establishing the diagonal-tension factor k. The essential data on all beams tested by the NACA are given herein in condensed form in order to obviate the necessity of referring to references 4, 7, and 8.

Each beam of the NACA series is given a code designation such as I-25-4D, with the following meaning:

I designates the test series of reference 4 (Series II is from reference 7, series III from reference 8, series IV from recent tests not previously published.)

25 is the approximate depth of the beam in inches

4 is the number of the beam within the series

D stands for double uprights (S for single uprights)

The basic data on the beams are given by figure 16 and table 1. The main results of the strength tests and of the calculations are given in table 2.

CRITICAL SHEAR STRESSES

Formulas for the critical shear stress of a flat plate with simply supported edges may be found in reference 9. Formula (7) for plates in which the edge conditions on one pair of edges differ from those on the other pair of edges was obtained by fitting an empirical curve to theoretical results for plates with one pair of edges simply supported ($R = 1$) and one pair of edges clamped ($R = 1.62$). The theoretical results were taken from references 10 to 13. Some of the results given in reference 11 were shown to be in error by Moheit, whose results are given in reference 13, but corrected values were not given for all cases that may be in error. A definite statement on the accuracy of formula (7) for the case of two edges clamped and two edges simply supported can therefore not be made, but it is believed that the formula has a maximum error of about 4 percent.

The restraint coefficients R given by figure 5(b) are based on incidental determinations of buckling stresses made in some beam tests. It should be realized, first of all, that representing R as a function of only t_U/t constitutes a rather extreme simplification of a very complex problem and, furthermore, the experimental determination of the critical stress in a beam web is a difficult problem. The curves given should, therefore, not be interpreted as means for a very exact determination of the critical stress, but as means for obtaining an approximate value of the critical stress adequate for the purpose of obtaining the diagonal-tension factor k. The upper curve of figure 5(b) is believed to be reasonably reliable because existing test data agree fairly well with it. Considerable doubt exists about the lower curve, particularly for $\frac{t_U}{t} < 1.2$, because the test data are not only very meager but also difficult to interpret.

The part of the curve near the origin is shown as a dashed line to indicate two facts. One is that no experimental evidence was available for this region. The other one is that the application of formula (7) in this region may give buckling stresses lower than those that would be obtained if the presence of the uprights were disregarded entirely and the web were considered as a plate framed by the beam flanges and the tip and root uprights. This obviously erroneous result is caused by the simplifying assumptions implied by formula (7); fortunately, it appears very improbable that the region in question will be approached in any actual web system designed to develop a strength somewhere near the shear strength of the web.

DIAGONAL-TENSION FACTOR k

Formula (5) for the factor k was obtained by fitting an empirical curve to values of k calculated from the strain measurements on the uprights of the NACA test beams. A direct comparison of the calculated values of k and the empirical expression is not given because it is of much less interest than the comparison of the experimental upright stresses with those predicted with the aid of the empirical k-curve.

ANALYSIS CHARTS

The analysis charts (figs. 8 and 9) were calculated from the formula for σ_U given in part I under the heading "Average Stress in Upright." This formula must be evaluated by successive approximations because $\tan \alpha$ is a function of σ_U according to formula (3). The flange area was assumed to be so large that ϵ_x could be neglected in formula (3).

The use of the analysis charts to determine the stresses in single uprights by means of the effective cross-section area A_{U_e} (formula (10)) implies several simplifying assumptions. Formula (10) is obtained from the familiar formula for eccentrically loaded compression members

$$\sigma = \frac{P}{A} + \frac{Pec}{I}$$

by setting $c = e$ and $I = \rho^2 A$. The implied assumptions are:

(a) The eccentricity e of the load is constant.

(b) The ratio e/ρ is not changed appreciably if the contribution of the web to the effective cross section of the upright is neglected.

Assumption (a) is plausible if the uprights are very closely spaced, because the web then moves with the uprights (reference 1). In general, however, both assumptions can be justified only by the fact that they have yielded good agreement with test results. Thick webs are likely to require more refined assumptions. If either assumption (a) or (b) is dropped, the analysis charts cannot be used for webs with single uprights.

VERIFICATION OF STRESS FORMULAS

In this section, results obtained by means of the engineering theory of incomplete diagonal tension will be compared with

(a) Experimental stresses deduced from NACA strain measurements on uprights

(b) Upright forces calculated by Levy's theory (references 5 and 6)

(c) Diagonal-tension factors k deduced from the tests of Lahde and Wagner (reference 14)

<u>Outline of procedure in NACA tests</u>.- In the NACA tests, the strains in the uprights were measured with electrical gages. In order to obtain a reasonably representative average, a fairly large number of gage stations was used on each upright (9 gage stations on the 25-inch beams of series II and III) and measurements were

taken on two or three uprights depending on the total number of uprights in the beam. After the strains had been converted to stresses with the help of stress-strain curves obtained from coupon tests, all the stresses for one beam were averaged to obtain the final value of σ_U shown in the plots.

<u>Comparison with experimental stresses in double uprights</u>.- On double uprights, a pair of gages was used at each strain station in order to average out bending stresses. The efficacy of this device depends on the degree to which the two stiffeners comprising the uprights act as an integral unit. A high degree of integral action was probably achieved in all the beams discussed herein, and will probably be achieved in any beam in which the uprights are not thinner than the web, provided that the rivet connection is adequate to prevent inter-rivet buckling.

Inspection of figure 17 shows that the calculated values of σ_U for double uprights are generally conservative or in close agreement with the experimental stresses; the only case of a decidedly unconservative prediction is beam IV-72-3D at high loads. A somewhat curious phenomenon is shown by beam IV-72-1D, for which σ_U begins to decrease with increasing load at a load well below the ultimate. The phenomenon appears to be linked to some extent with the effects of local buckling or forced crippling. The beam in question had next to the lowest ratio of t_U/t of all the double-upright beams tested, and the same phenomenon was exhibited to a much greater degree by the single-upright beam IV-72-4S, which had an even lower ratio of t_U/t. Some other double-upright beams with a somewhat higher ratio of t_U/t (beams III-25-4D and III-25-7D) appear to indicate a slight tendency toward the same phenomenon. It is, therefore, debatable whether σ_U really begins to decrease or whether the strain readings are falsified by local buckling stresses.

<u>Comparison with experimental stresses in single uprights</u>.- The predicted stresses σ_U for single uprights are valid only for the median plane of the web at the upright in question, whereas the strain measurements were taken on the exposed face of the attached leg of the upright. In order to permit a direct comparison, the predicted stresses were corrected to the plane of measurement, assuming linear stress variation in the uprights.

In single uprights it is not possible to average out bending stresses by using pairs of gages. Theoretically, they could be almost averaged out if gages were located on each crest and in each trough of the buckles, but the buckle patterns cannot be predicted with sufficient accuracy to achieve such a distribution of gages. In order to give some idea of the magnitude of the bending stresses, figure 17 shows for all beams with single uprights not only the average stress σ_U but also the lowest and the highest individual stress measured at any one gage in any of the uprights of the beam at each load. Inspection of the figure shows that the range from the lowest to the highest stress is quite large. In spite of this fact, however, the experimental average stress σ_U agrees quite well with the predicted stress in most cases, much closer in fact than for double uprights considering all tests, except that on the single-upright beams the stresses σ_U at the highest loads show a tendency to exceed the calculated stresses in a number of cases.

The phenomenon of a reversal in the curve of σ_U against load, noted for beam IV-72-1D, is exhibited very markedly by beam IV-72-4S. This beam had a ratio of t_U/t of unity, the lowest of all single-upright beams tested. The range of individual stresses is extremely large. Tensile stresses of large magnitude appear, whereas in all other beams the stress remained compressive; that is, the tensile stress due to local buckling was always less than the column compressive stress in the upright (with minor exceptions for beam IV-72-2S).

<u>Comparison with Levy's theory</u>.- On most of the beams tested the critical shear stress was too low to permit comparisons at low loading ratios τ/τ_{cr}. The only exceptions were the beams of series IV, and, as noted, the results on two of these were presumably partly invalidated by local bending effects. Fortunately the region of low τ/τ_{cr} is reasonably amenable to theoretical calculations. Levy has developed a suitable theory of plates with large deflections and has carried through calculations for several specific cases (references 5 and 6). Comparisons between the results obtained by the present semiempirical method and Levy's theoretical results are shown in figure 18.

The upright load $P_{U_{max}}$ rather than the upright stress is shown in figure 18 in order that the result for the theoretical limiting case of infinite upright area might be included. The agreement for $\frac{d}{h} = 0.4$ and $\frac{A_U}{dt} = 0.25$ is very close. For the two cases with $\frac{d}{h} = 1.0$, the agreement is not so close. For the finite-size upright $\left(\frac{A_U}{dt} = 0.25\right)$, the upright load predicted by the present theory is less than that predicted by Levy's theory, with a maximum deviation of about 30 percent at $\frac{\tau}{\tau_{cr}} = 1.6$; for the infinitely large upright, the deviation is of the same sign and somewhat larger. Although the agreement for $\frac{d}{h} = 1.0$ is not so good as might be desired, the agreement for this case as well as for $\frac{d}{h} = 0.40$ is very much better than that shown in references 5 and 6, which were based on the expression for k given in reference 3.

Knowledge of the strength of beams designed to fail (by simultaneous failure of the uprights and the web) at low ratios of τ/τ_{cr} is very incomplete at present. There are indications, however, that for efficient designs the spacing ratio d/h will probably be about 1/2. The semiempirical method of predicting σ_U appears to hold promise, therefore, of giving reasonable accuracy in the range most important for design even for webs that are commonly called "shear-resistant" webs rather than "incomplete-diagonal-tension" webs.

Comparison with tests of Lahde and Wagner.- The comparisons with experiments presented so far have been made for the stresses σ_U rather than for the diagonal-tension factor k because the stress is the directly measured quantity and is of greater direct interest. For the tests reported in reference 14, it is more convenient to compare values of the diagonal-tension factor k. These tests were not made on actual beams, but on plates in a special test jig. The test conditions were somewhat artificial and may not represent the conditions existing in a beam very well. There are also doubts concerning the evaluation and interpretation of the test results. As a matter of some interest, however, figure 19 shows the graph for k obtained by a comparison between formula (5) and experimental values of k deduced from these test results. In view of the factors of doubt mentioned, the agreement is perhaps as good as can be expected. The fact that the experimental points lie consistently above the curve may be explained in part by the fact that the critical stress has been taken at the theoretical value for clamped edges; it is well known that the fully clamped condition can be realized only imperfectly in tests, and lack of initial flatness would cause a further reduction of the critical stress.

Maximum stresses $\sigma_{U_{max}}$ in uprights.- Formula (11) for the ratio $\sigma_{U_{max}}/\sigma_U$ is based on Levy's theoretical results (references 5 and 6) for $\frac{d}{h} = 1.0$ and $\frac{d}{h} = 0.40$ and on the assumption of linear variation with d/h. Linear decrease of the ratio with k toward unity at $k = 1$ was also assumed.

On single uprights, experimental values of $\sigma_{U_{max}}/\sigma_U$ cannot be established with satisfactory accuracy because the local bending stresses cannot be eliminated from the measured total stresses. Figure 17 shows, therefore, calculated and experimental values of $\sigma_{U_{max}}$ only for double uprights. For a number of beams the calculated curves of $\sigma_{U_{max}}$ differ but little from the curves of σ_U; for these beams the experimental values of $\sigma_{U_{max}}$ were so close to the experimental values of σ_U that it was not feasible to show them on the plots. For the other beams the experimental ratios $\sigma_{U_{max}}/\sigma_U$ are generally of about the same order of magnitude as the calculated values. A close comparison is hardly warranted in view of the experimental scatter.

Distribution curves for σ_U are given in figure 20. The test points represent the average of corresponding stations on three uprights as well as the average of corresponding stations to either side of the neutral axis of the beam; they are, therefore, shown only in the lower half of the beam. The curve in the lower half is faired through the test points; the curve in the upper half is simply the mirror image of the faired curve in the lower half. The curves for the series IV beams show a very pronounced influence of the web splice plate along the neutral axis of the beams. The splice plate tends to act like a beam flange in producing gusset effect; in addition, the splice plate adds directly to the cross-sectional area of the upright. All these effects were neglected in the analysis, as was the increase in critical shear stress caused by the splice plate. For purposes of comparison, however, a second analysis was made for six beams with upright failures, based on the assumption that the web plate was simply supported along the splice line. This assumption is obviously optimistic and yielded predicted failing loads 7 percent greater (average for all six beams) than the analysis neglecting the presence of the splice plate. Of the six beams, three were from NACA series IV, and three were from manufacturers' tests on similar beams.

ANGLE OF DIAGONAL TENSION

In a fully developed diagonal-tension field, the direction of the diagonal tension coincides with the direction of the folds in the sheet (reference 1). When the diagonal-tension field is incompletely developed, as it is in any actual beam, the diagonal tension constitutes a component of stress which is separated out of the total stress purely mathematically, not physically. Moreover, the diagonal tension thus separated from the total actual stress is only an average for the entire bay and does not describe the details of the stress variation within the bay. The angle of diagonal tension consequently bears only a very loose relation to the physical folds in the sheet. A study of the contour map of a sheet just after buckling (reference 9) shows that the "folds" are curved to such an extent that an average direction cannot be defined to any degree of accuracy. No attempt has been made, therefore, to compare computed angles of diagonal tension with observed angles of folds.

ALLOWABLE STRESSES FOR UPRIGHTS

Column failure.- Out of a total of 19 beams with double-upright failures analyzed, nine were predicted to fail by column failure. In figure 21, the values of σ_U calculated for these beams at their respective failing loads are plotted against the effective slenderness ratio. The slenderness ratios were sufficiently high to make the Euler curve applicable in all cases; it was, therefore, possible to include uprights of 24S-T alloy as well as of 75S-T alloy.

The plot indicates that formula (12) for estimating the effective slenderness ratio is somewhat conservative considering all tests, but a sufficient number of points lie so close to the curve that a less conservative formula does not appear advisable.

Forced crippling failure.- Test observation has shown that the shear buckles in the web will force buckling or "crippling" of the upright in the leg attached to the web. The amount of the forced crippling will obviously depend primarily on the relative sturdiness of the upright and the web. The simplest parameter expressing relative sturdiness is the ratio t_U/t, and in reference 4 empirical formulas for allowable stress in the upright were given based on this parameter. Strength predictions based on these formulas, however, showed a rather large scatter, which indicated that additional parameters were necessary to define the failing stress more accurately. A considerable reduction in the scatter was effected by using the parameter $k\sqrt{t_U/t}$ instead of t_U/t, and at the same time using the maximum instead of the average stress in the uprights. (See formulas (13a) and (13b).)

Figure 22 is a plot of values of $\sigma_{U_{max}}$ computed from the failing loads with the aid of the analysis chart, for all beams of 24S-T alloy presumed to have failed by forced crippling. Omitted from the plot are three points for a series of three 10-inch beams which fell from 100 to 150 percent above the average curve. This discrepancy is so large that it justifies doubts as to the accuracy of the test data. It will be noted that all the points are fairly uniformly distributed about the average curve

$$\sigma_o = 35k\sqrt{t_U/t} \tag{13d}$$

The curve recommended for design (formula 13a), which is the lower edge of the scatter band, lies 20 percent below this curve. Only a single point lies appreciably below the design curve, and only five points lie distinctly above the upper edge of the band, which is 20 percent above the average curve. Two of these points are for beams having a value of $k < 0.5$; this range will be discussed presently.

In accordance with formula (13c), an effective value of k was used when the actual value was below 0.5. It will be noted in figure 22 that the point for one of the three 80-inch beams which fell in this range lies considerably above the upper edge of the scatter band. For the beam represented by this high point, the ratio of actual to predicted failing load is above 1.41; an exact value cannot be given because the actual failure was web failure, not upright failure. (It may be noted that the ratio 1.41 of actual to predicted failing load is appreciably less than the ratio 1.57 obtained by comparing the plotted point for $\sigma_{U_{max}}$ with the average curve for $\sigma_{U_{max}}$, because the relation between $\sigma_{U_{max}}$ and P is not linear.) Analysis of incomplete test data on a few 12-inch beams with τ/τ_{cr} from 1.5 to 2.5 (at failure) indicated very close agreement in some cases and nearly 50-percent conservativeness in other cases. These values indicate that strength predictions for beams designed to fail at $\frac{\tau}{\tau_{cr}} < 4$ may be considerably more conservative than indicated by the upper edge of the scatter band in figure 22.

Figure 23 shows that the available test data on beams of 75S-T alloy with single uprights agree with formula (13d) within the same scatter limits as those for 24S-T alloy. Figure 24 shows that none of the points for double-upright beams fall below the recommended design curve for such beams (formula 13b); the average curve is given by

$$\sigma_o = 30k \sqrt{t_U/t} \tag{13e}$$

which is about 14 percent lower than the corresponding value for single uprights given by formula (13d).

General elastic instability of web and stiffeners.- Experience with simple elements such as plates and stiffeners tested as individual columns has shown that elastic instability beginning at low stresses is not immediately followed by ultimate failure; the ultimate load may be several times the critical load. Only when elastic instability occurs at a fairly high stress does the ultimate failure follow soon after.

A similar condition appears to exist regarding the general elastic instability of a web with double stiffeners. Analysis of test data by means of existing theories of buckling of orthotropic plates has shown in a number of cases that the ultimate load was several times the calculated critical load. Because of experimental difficulties, very little effort has been made to determine experimentally the load at which instability begins. It should also be noted that existing theories have not dealt adequately with the problem of general elastic instability in web systems where the web has buckled between the stiffeners.

Analysis of preliminary data on thick-web beams (h/t about 100) has indicated that perhaps some correlation between ultimate load and theoretical critical load may be established if the critical stress is definitely above the proportional limit of the material. Because web systems with these proportions have been studied very little, it is not possible to state at present whether analysis for column failure and forced crippling failure automatically covers the possibility of general instability failure over the entire design range.

WEB DESIGN

Formula (14) for the peak web stress τ_{max} is simply a formula for linear interpolation between the limiting cases of pure shear ($k = 0$) and pure diagonal tension ($k = 1$). The factor C_1 is defined by

$$C_1 = \frac{1}{\sin 2\alpha} - 1 \tag{22}$$

According to the theory of pure diagonal tension (reference 1), the diagonal tension stress is

$$\sigma = \frac{2\tau}{\sin 2\alpha} \tag{23}$$

According to formula (3), $\alpha = 45^\circ$ if the flanges and the uprights are infinitely large $(\epsilon_x = \epsilon_y = 0)$; in this case, $\sin 2\alpha = 1$. The factor C_1 expresses, therefore, the excess stress caused by α differing from 45°, as it will in any actual structure with members of finite size. The factor C_2 is similarly equivalent to the theoretical factor C_2 given in reference 1.

The allowable values for τ_{max} given in figure 14 were based chiefly on tests of long webs subjected to loads approximating pure shear (reference 15). These tests showed that the ultimate value of τ_{max} was independent of the rivet factor in the practical range $(C_R > 0.6)$ as long as the bearing stresses did not exceed the allowable values. These tests yielded values of allowable stress at $k = 0$ and $k = 0.3$ as shown in figure 25. (The test points shown represent the average of all tests over the range of rivet factor covered.) The allowable stresses at $k = 1.0$ were estimated as follows: For fully developed diagonal tension, the tension stress in the web is given by formula (23); the ultimate nominal shear stress is therefore

$$\tau = \frac{\sigma}{2} \sin 2\alpha$$

or somewhat less than $\sigma/2$. This stress must be reduced to take into account the reduction of section caused by the presence of the rivet holes and the stress-concentration effects caused by these holes. The combined effect of these two factors was estimated as 0.75 for 24S-T alloy and as 0.81 for 75S-T alloy, the latter having a smaller factor of stress concentration.

Figure 25 shows also points obtained from tests on a square picture-frame jig (reference 16). The higher stresses developed in this jig may have been higher because friction between the sheet and the frame angles relieved the riveted joint. The effect of friction can be quite high, but it should probably not be relied on to operate under service conditions.

In six NACA beam tests in which web failure was predicted and observed, the ratio of actual to predicted failing load ranged from 0.95 to 1.06, with an average of 1.01. In six manufacturers' tests, the ratio was 1.14 ± 0.06. The allowable stresses obtained from figure 14 are therefore somewhat conservative on the average. All these comparisons are based on actual material properties. The two premature web failures shown in table 2 were due to damage caused by shop accidents and should be disregarded.

RIVET DESIGN

Web-to-flange rivets.- Failures of web-to-flange rivets were observed in five manufacturers' tests. When actual rivet strengths as determined by special tests were used as allowable values, the strengths developed in the beam tests ranged from 3 percent lower to 16 percent higher than predicted, with an average of 7 percent higher. When nominal rivet strengths were used as allowable strengths, the actual values ranged from 37 percent to 60 percent higher than predicted. These values reflect the well-known fact that nominal rivet strengths are usually quite conservative.

Formula (15) for the load (per inch run) on the flange rivets is a formula for straight-line interpolation between the limiting cases $k = 0$ and $k = 1.0$. If the engineering theory of incomplete diagonal tension were interpreted literally, separate rivet loads would be computed for the shear component and the diagonal-tension component of the shear load, and these loads would be added vectorially. The resulting formula would be from 7 percent to 9 percent less conservative than formula (15) in the range of the five tests under discussion. Consequently, if actual rivet strengths had been used as allowable strengths, the strength predictions based on this more rational formula would have been about 2 percent unconservative on the average and up to 12 percent unconservative in the extreme case. The more rational formula is therefore unconservative by a sufficient margin to give preference to formula (15). It should also be realized that the greater rationality is largely spurious; the engineering theory of incomplete diagonal tension claims only to represent the average stress condition in a bay, and these average conditions do not exist along the edges of the bay where the rivets are located.

Web-to-upright rivets.- Formula (16) for the rivet shear strength required in double uprights is a semiempirical formula and was taken from reference 17. The tests described in the reference showed that the column strength developed does not depend very critically on the rivet strength; figure 4 of the reference shows, for instance, that a reduction of the rivet strength to 50 percent of the required value reduces the column strength on the average to 92 percent of the value obtainable with adequate riveting. The average curve in this figure was used for evaluating the early NACA beam tests in which the rivet strengths were generally less than required by formula (16).

Formulas (17) and (18) for the required tensile strengths of the rivets represent an attempt to provide a criterion for safeguarding against a type of failure sometimes observed in tests. It is especially important for single uprights, because no criterion previously existed to determine the required rivet strength. Because no tests have been made to check specifically on this item, the available evidence is rather fragmentary and largely negative; that is, in most tests no failures were observed (or at least none were recorded). An additional difficulty is that rivet failures are often found after the failure of the beam, and it is then impossible to state whether the rivet failure was a primary one responsible for the beam failure or a secondary one that took place while the beam was failing for other reasons. In view of all these uncertainties, the coefficients given in formulas (17) and (18) should be considered only as tentative values.

For single uprights, the analysis was based on a total of 21 tests. Three failures were observed with the coefficient in formula (18) ranging from 0.10 to 0.13. No failures were observed in the remaining 18 beams, for which the coefficient ranged from 0.18 to 0.31. There were only two tests in the range from 0.18 to 0.22, however, the remaining 16 beams having coefficients above 0.22. The coefficient for the design formula was therefore taken as 0.22 in order to be conservative. If more tests had been available in the range from 0.13 to 0.18, a lower coefficient in the design formula might have been justified. Although the coefficient 0.22 may appear to be more conservative than necessary if the tests are taken at face value, it is not considered to be unduly severe. All the manufacturer's beams analyzed fulfilled this criterion, and presumably they represented acceptable riveting practices. The lower rivet strengths incorporated in a number of the NACA beams arose from the fact that these beams were intended primarily for strain-gage tests to determine the diagonal-tension factor k. In order to accomodate the strain gages, the rivet pitch was increased in some cases; in other cases countersunk rivets were used which have a relatively low tensile strength.

On beams with double uprights, two failures were observed with coefficients of 0.09 to 0.13. No failures were observed on four beams with coefficients above 0.12 and on two beams with coefficients of 0.07 and 0.08. The suggested design coefficient of 0.15 is therefore probably conservative. Failures were observed on several beams with coefficients ranging from 0.07 to 0.27, but the shear strengths of the rivets on these beams were from 25 to 75 percent below the strengths required by formula (16); these failures were therefore attributed to shear rather than to tensile loads.

Upright-to-flange rivets.- Although tests on a rather large number of beams were available for analysis, there were almost no records of failure in upright-to-flange rivets. This lack of failures can probably be attributed to the use of very conservative design formulas based on pure-diagonal-tension theory or slight modifications of this theory. In the NACA beams of series II and III, the excess strength arose from the fact that the beam flanges were used for a number of tests, and bolts instead of rivets were employed for all flange connections in order to facilitate disassembly after a test.

For beams with double uprights, one rivet failure was recorded. The existing nominal rivet strength was only about 5 percent below the required strength. The existing actual rivet strength was therefore probably well above the required strength, but the analysis was very uncertain because of a peculiar design feature (reinforced upright). In three beams, no failures were recorded, although the existing nominal rivet strengths ranged from 0.47 to 0.76 of the required strength; these values are so low that the existing actual rivet strengths were probably below the required strengths in at least two cases. The available evidence appears, therefore, to justify the conclusion that formula (19) for the rivet strength required on the ends of double uprights would generally be safe even if actual rivet strengths were used as allowable values.

For beams with single uprights, there were two records of failure although the rivet strengths were appreciably greater than required. The analyses were extremely uncertain, however, because several important dimensions of the beams were not given and had to be estimated or inferred. Against these two records of failure there are 13 records of successful joints in which the ratio of existing nominal rivet strength to required strength was less than unity, the two lowest ratios being 0.68 and 0.53. It appears therefore reasonably safe to draw the same conclusion as for double uprights, that is, that formula (20) would probably be safe, in general, even if actual rivet strengths were used as allowable values. An appreciable margin of safety should exist in practice because the allowable strength values for rivets are likely to be well below the actual strengths. It might be pointed out also that formulas (19) and (20) are inherently conservative because they neglect the gusset effect; however, this effect is small in many beams and may be overshadowed by unpredictable irregularities.

SECONDARY BENDING MOMENTS IN FLANGES

Experimental evidence on secondary bending stresses in the flanges is confined to a few measurements given in reference 3. Most of these measurements were made on beams with very flexible flanges beyond the range of efficient design and showed the predictions to be very conservative. In the range of normal flange flexibilities, the predictions were somewhat conservative. Formula (21) is probably always conservative because it neglects the fact that a web in incomplete diagonal tension contributes to the section modulus of the beam flanges.

SHEAR STIFFNESS OF WEB

The deflection of a cantilever beam is calculated by adding the so-called bending deflection and the shear deflection according to the formula

$$\delta = \frac{PL^3}{3EI} + \frac{PL}{h_e t G_e}$$

where G_e is the effective shear modulus given by figure 15. This effective shear modulus was calculated as follows: According to formula (4), the shear load S can be divided into a shear component and a diagonal-tension component. The total shear deflection is the sum of the deflections caused by these two components; the effective shear modulus for incomplete diagonal tension is therefore defined by the relation

$$\frac{1}{G_e} = \frac{1 - k}{G} + \frac{k}{G_{DT}} \tag{24}$$

where G_{DT} is the effective shear modulus of a web in pure diagonal tension. The value of G_{DT} can be calculated by the relations given in reference 1. A convenient formula which can be derived from these relations is

$$G_{DE} = \frac{E \sin^2 \alpha}{2} \tag{25}$$

on the simplifying assumption that the beam flanges are sufficiently large to permit neglecting the strain ϵ_x caused by the horizontal component of the diagonal-tension force.

When the web stresses exceed the proportional limit, a corrected value $\overline{G}_e$ must be used. A tentative curve for $\overline{G}_e/G_e$ was given in reference 4. This curve was based on a small number of unpublished tests and is rather uncertain. Since the shape of the stress-strain curve is known to be quite variable with the existing tolerances of composition of material and heat treatment, it is not likely that a high accuracy can be achieved in predicting deformations at stresses near or beyond the yield stress. Fortunately, there appears to be no practical need for such accuracy.

Figure 26 shows experimental and calculated deflections for the beams of series I. The agreement is very satisfactory.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., April 2, 1947

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TABLE I.- PROPERTIES OF TEST BEAMS

Beam	h_e (in.)	h_U (in.)	t (in.)	d (in.)	A_U (sq in.)	A_{U_e} (sq in.)	$\frac{A_U}{dt}$	$\frac{A_{U_e}}{dt}$	ρ (in.)	Flanges (2 ∠s) (in.) (a)	ωd	Material: Web	Material: Upright
I-40-1D	40.0	38.6	0.0425	10.0	0.338	0.338	0.795	0.795	0.256	2 × 2 × 1/4	0.88	24S-T	24S-T
I-40-2D	40.0	38.6	.0425	10.0	.384	.384	.903	.903	.490	2 × 2 × 1/4	.88	24S-T	24S-T
I-40-3D	41.4	38.6	.0392	20.0	.384	.384	.490	.490	.490	3 × 3 × 5/16	1.20	24S-T	24S-T
I-40-4Da	41.4	38.6	.0390	20.0	.353	.353	.454	.454	.351	3 × 3 × 5/16	1.20	24S-T	24S-T
I-40-4Db	41.4	38.6	.0390	20.0	.353	.353	.454	.454	.351	3 × 3 × 5/16	1.20	24S-T	24S-T
I-40-4Dc	41.4	38.6	.0390	20.0	.353	.353	.454	.454	.351	3 × 3 × 5/16	1.20	24S-T	24S-T
I-25-1D	25.0	23.9	.0102	10.0	.123	.123	1.206	1.206	.232	2 × 2 × 3/16	.98	24S-T	24S-T
I-25-2D	25.0	23.9	.0105	20.0	.123	.123	.586	.586	.232	2 × 2 × 3/16	1.97	24S-T	24S-T
I-25-3D	25.0	23.9	.0116	10.0	.110	.110	.952	.952	.167	2 × 2 × 3/16	1.02	17S-T	24S-T
I-25-4D	25.0	23.9	.0153	10.0	.114	.114	.747	.747	.182	2 × 2 × 3/16	1.09	24S-T	24S-T
I-25-5D	25.0	23.9	.0150	20.0	.269	.269	.897	.897	.247	2 × 2 × 3/16	2.14	24S-T	24S-T
I-25-6D	25.0	23.9	.0162	20.0	.206	.206	.635	.635	.241	2 × 2 × 3/16	2.19	24S-T	24S-T
I-25-7D	25.0	23.9	.0402	10.0	.101	.101	.252	.252	.291	2 × 2 × 3/16	1.38	24S-T	24S-T
II-25-1S	24.3	23.3	.0265	20.0	.212	.0972	.400	.183	.309	2 × 2 × 3/16	2.52	24S-T	24S-T
II-25-2S	24.3	23.3	.0265	20.0	.195	.0764	.368	.144	.407	2 × 2 × 1/4	2.36	24S-T	24S-T
II-25-3S	24.3	23.3	.0224	10.0	.153	.0790	.684	.352	.357	2 × 2 × 3/16	1.20	24S-T	24S-T
II-25-4S	24.3	23.3	.0257	10.0	.121	.0654	.471	.254	.314	2 × 2 × 1/4	1.17	24S-T	24S-T
II-25-5S	24.3	23.3	.0249	10.0	.194	.0752	.778	.302	.377	2 × 2 × 3/16	1.24	24S-T	24S-T
II-25-6S	24.3	23.3	.0248	20.0	.212	.0972	.427	.196	.309	2 × 2 × 3/16	2.46	24S-T	24S-T
II-25-7S	24.3	23.3	.0248	20.0	.108	.0412	.217	.0831	.582	2 × 2 × 1/4	2.32	24S-T	24S-T
II-25-8S	24.3	23.3	.0248	10.0	.109	.0431	.439	.174	.611	2 × 2 × 1/4	1.16	24S-T	24S-T
II-25-9S	24.3	23.3	.0245	10.0	.156	.0809	.637	.331	.358	2 × 2 × 3/16	1.23	24S-T	24S-T
III-25-1S	24.3	23.3	.0210	20.0	.105	.0354	.249	.0840	.541	2 × 2 × 1/4	2.18	Alclad 75S-T	Alclad 75S-T
III-25-2S	24.3	23.3	.0208	10.0	.108	.0376	.517	.180	.556	2 × 2 × 3/16	1.16	Alclad 75S-T	Alclad 75S-T
III-25-3S	24.3	23.3	.0395	10.0	.108	.0377	.273	.0955	.556	2 × 2 × 1/4	1.27	Alclad 75S-T	Alclad 75S-T
III-25-4D	24.3	23.3	.0206	10.0	.136	.136	.659	.659	.231	2 × 2 × 3/16	1.16	Alclad 75S-T	Alclad 75S-T
III-25-5S	24.3	23.3	.0204	10.0	.102	.0484	.496	.237	.318	2 × 2 × 1/4	1.08	Alclad 75S-T	Alclad 75S-T
III-25-6D	24.3	23.3	.0295	15.0	.107	.107	.242	.242	.283	2 × 2 × 3/16	1.91	Alclad 75S-T	Alclad 75S-T
III-25-7D	24.3	23.3	.0303	15.0	.100	.100	.220	.220	.221	2 × 2 × 3/16	1.92	Alclad 75S-T	Alclad 75S-T
III-25-8S	24.3	23.3	.0206	10.0	.152	.0754	.741	.367	.359	2 × 2 × 1/4	1.09	Alclad 75S-T	Alclad 75S-T
IV-72-1D	73.7	64.3	.1237	18.0	.762	.762	.342	.342	.594	(b)	.60	24S-T	24S-T
IV-72-2S	73.7	64.3	.1219	18.0	1.137	.383	.518	.175	1.110	(b)	.60	24S-T	24S-T
IV-72-3D	73.7	64.3	.1227	18.0	1.176	1.176	.533	.533	.939	(b)	.60	24S-T	24S-T
IV-72-4S	73.7	64.3	.1239	18.0	1.092	.425	.490	.191	1.618	(b)	.60	24S-T	24S-T

[a]Flanges of I-40 and IV-72 beams are steel; all others are 24S-T.
[b]See figure 16(c).

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TABLE 2.- TEST DATA AND RESULTS

Beam	Calculated τ_{cr} (ksi)	P_{ult} (kips)	$\tau_{ult} = \frac{P_{ult}}{h_e t}$ (ksi)	$\frac{\tau_{ult}}{\tau_{cr}}$	k	Predicted P_{ult}			Observed failure	$\frac{P_{ult}}{P'}$ (e)	$\frac{P_{ult}}{P''}$ (f)
						P_1 (kips) (a)	P_2 (kips) (b)	P_3 (kips) (c)			
I-40-1D	2.00	27.4	16.1	8.05	0.43	45.6	27.5	80.7	Column	1.00	1.00
I-40-2D	1.98	39.3	23.1	11.7	.49	45.6	83.0	86.7	Flange	.86	----
I-40-3D	.424	37.0	22.8	53.7	.70	41.0	42.0	58.6	dWeb	.90	.90
I-40-4Da	.424	30.3	18.8	44.5	.68	40.6	26.5	52.2	Column	1.14	1.14
I-40-4Db	.424	32.1	19.9	47.0	.68	40.6	28.0	52.2	Column	1.14	1.14
I-40-4Dc	.424	35.7	22.1	52.2	.70	40.6	30.0	52.2	Column	1.19	1.19
I-25-1D	.117	6.8	26.6	228.0	.83	6.41	9.70	18.0	Web	1.06	1.06
I-25-2D	.038	6.3	24.0	624.0	.88	6.05	4.70	10.5	Column	1.34	1.34
I-25-3D	.151	7.6	26.2	173.0	.81	6.65	5.40	16.9	Web	1.41	1.14
I-25-4D	.262	7.8	20.4	78.0	.74	9.62	6.60	18.0	Column	1.18	1.18
I-25-5D	.080	10.9	29.1	366.0	.85	8.60	9.20	22.4	None	1.27	----
I-25-6D	.093	10.0	24.7	266.0	.84	9.20	8.60	17.7	Column	1.16	1.16
I-25-7D	1.15	12.7	12.6	10.9	.48	25.8	23.9	14.8	Forced crippling	.86	.86
II-25-1S	.208	13.2	20.5	98.5	.76	13.8	------	15.2	dWeb	.96	.96
II-25-2S	.208	11.4	17.7	85.0	.75	14.3	------	12.0	Forced crippling	.95	.95
II-25-3S	.416	13.9	25.4	61.1	.71	13.6	------	16.4	Web	1.02	1.02
II-25-4S	.543	13.5	21.6	39.8	.66	15.7	------	15.2	Forced crippling	.89	.89
II-25-5S	.512	15.5	25.6	50.0	.69	15.7	------	16.8	Web	.99	.99
II-25-6S	.189	13.6	22.6	119.0	.78	12.9	------	14.7	Web	1.05	1.05
II-25-7S	.170	6.7	11.1	65.4	.72	12.9	------	8.07	Forced crippling	.83	.83
II-25-8S	.469	10.2	16.9	36.0	.65	14.9	------	10.7	Forced crippling	.95	.95
II-25-9S	.505	15.4	25.8	51.1	.69	15.3	------	17.2	Web	1.01	1.01
III-25-1S	.124	6.7	13.0	105.0	.77	12.7	------	6.70	Forced crippling	1.00	1.00
III-25-2S	.336	9.5	18.7	55.6	.70	14.7	------	9.50	Forced crippling	1.00	1.00
III-25-3S	.981	14.8	15.4	15.7	.54	30.0	------	12.2	Forced crippling	1.21	1.21
III-25-4D	.442	14.3	28.5	64.6	.72	14.9	13.8	22.0	Column	1.04	1.04
III-25-5S	.338	13.5	27.2	80.5	.74	14.8	------	13.2	Forced crippling	1.02	1.02
III-25-6D	.390	11.4	15.8	40.5	.67	20.4	15.5	13.3	Forced crippling	.86	.86
III-25-7D	.412	10.5	14.3	34.7	.65	20.7	10.3	12.6	Column	1.02	1.02
III-25-8S	.344	14.2	28.5	83.0	.74	14.9	------	18.9	Web	.95	.95
IV-72-1D	4.02	178.5	19.6	4.87	.33	252	220	191	Forced crippling	.94	.94
IV-72-2S	3.33	212	23.6	7.08	.40	245	------	189	Web	1.12	.87
IV-72-3D	4.40	255	28.2	6.41	.38	251	488	255	Forced crippling	1.01	1.00
IV-72-4S	2.90	160	17.5	6.03	.37	248	------	156	Forced crippling	1.03	1.03

[a]For web failure.
[b]For column failure.
[c]For forced crippling failure.
[d]Premature web failure.
[e]P' is the lowest one of the predicted loads P_1, P_2, and P_3.
[f]P" is that predicted load (P_1, P_2, or P_3) which corresponds to the observed type of failure.

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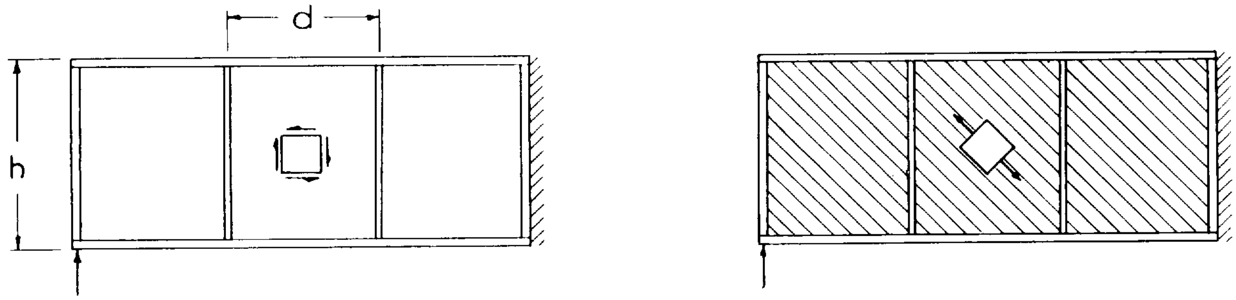
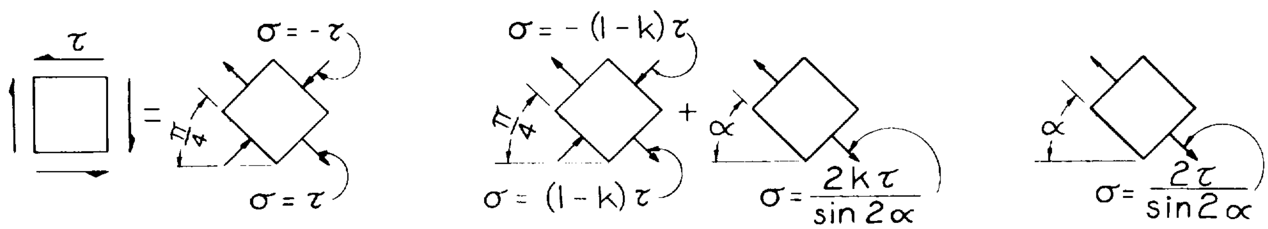


(a) Nonbuckled ("shear-resistant") web.

(b) Pure diagonal-tension web.

Figure 1.- State of stress in a beam web.



k = 0

0 < k < 1

k = 1

Figure 2.- Resolution of web stresses at different stages of diagonal tension.

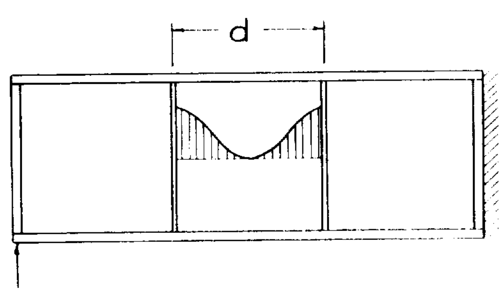


Figure 3.- Assumed distribution of (vertical) normal stress in web immediately after buckling.

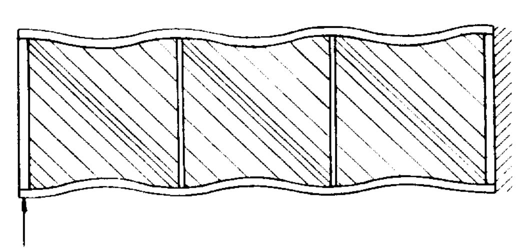
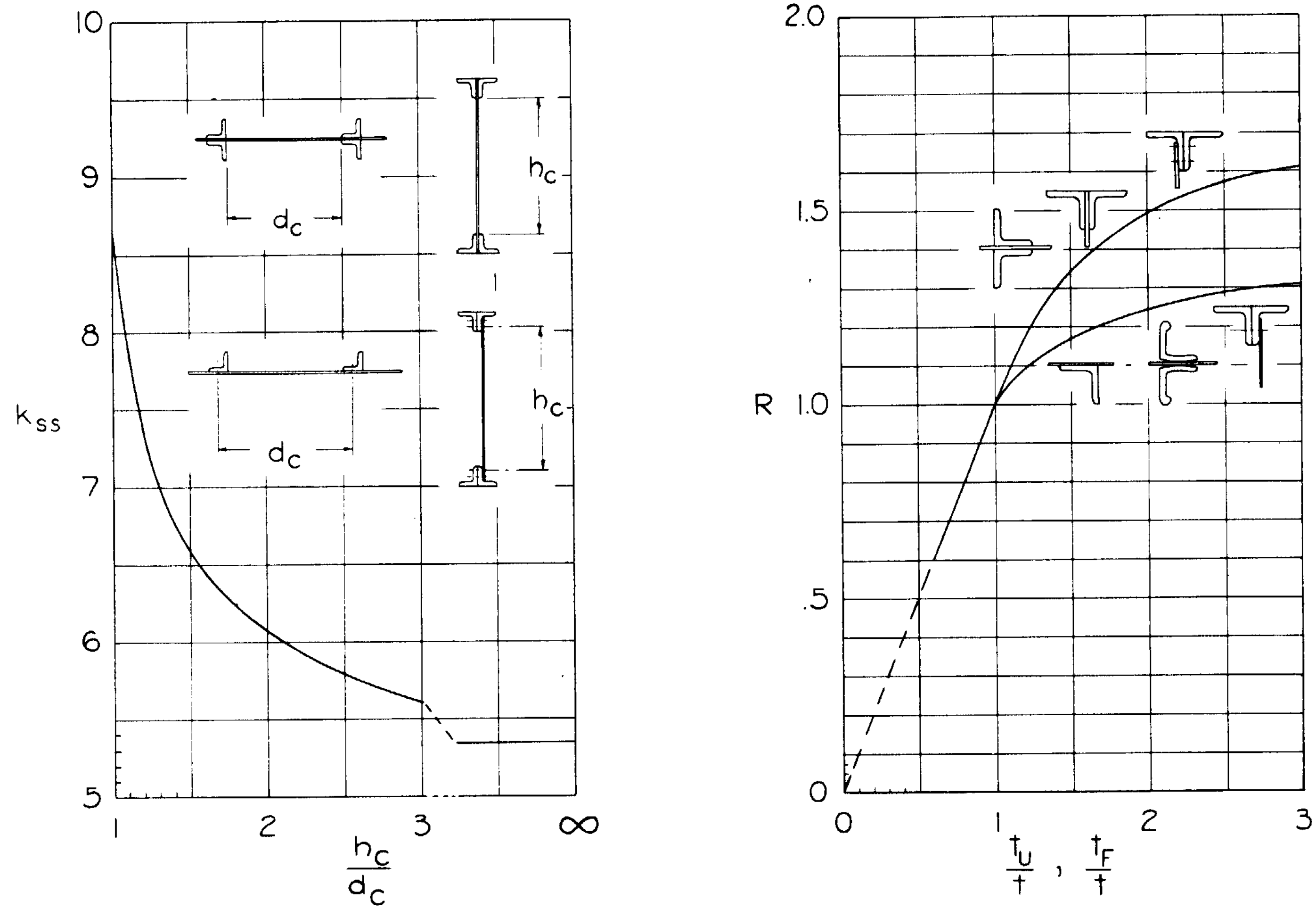


Figure 4.- Diagonal-tension beam with flexible flanges.



(a) Theoretical coefficients for plates with simply supported edges.

(b) Empirical restraint coefficients.

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Figure 5.- Coefficients for calculating web buckling stress.

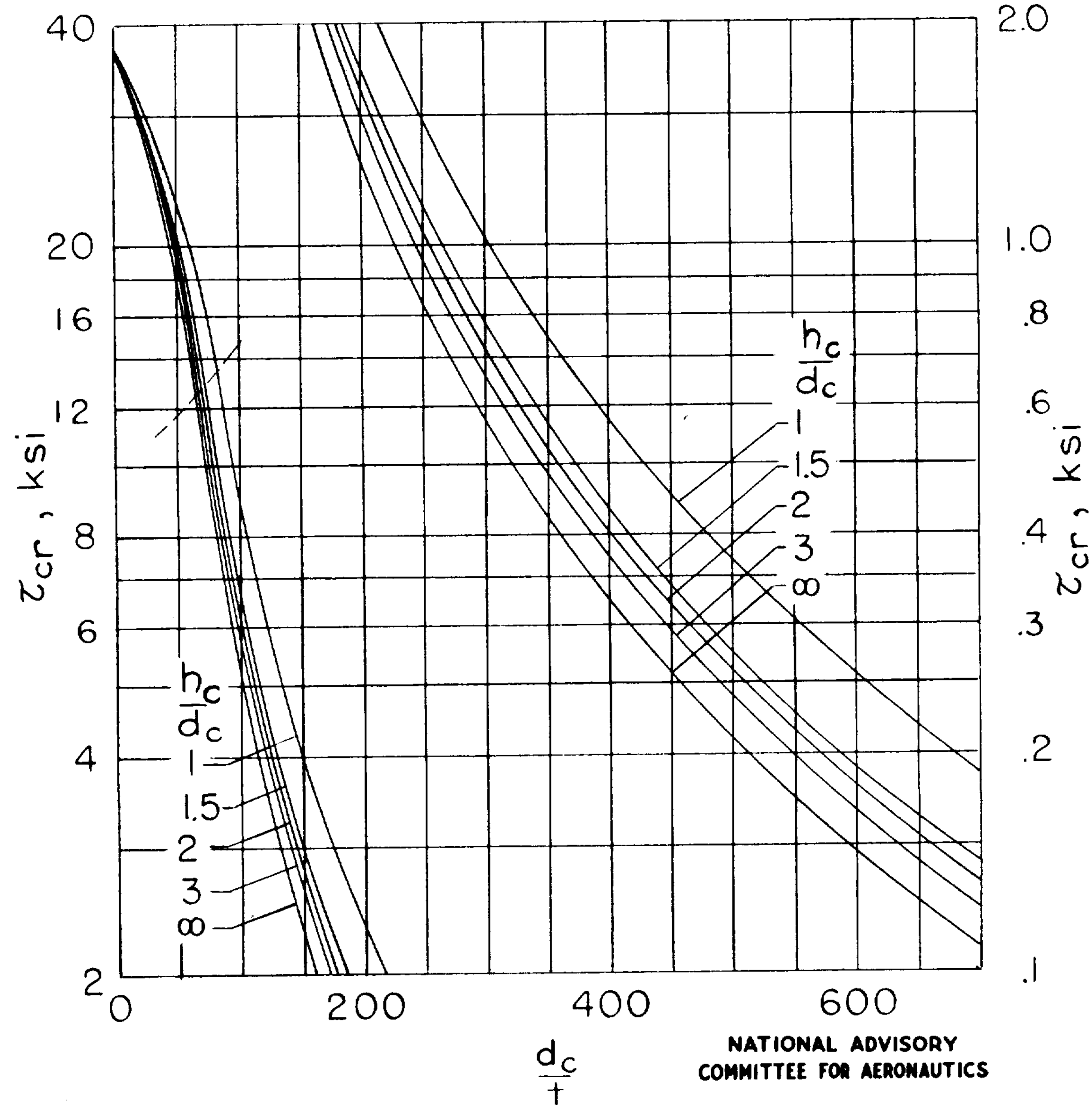

Figure 6.- Buckling stresses τ_{cr} for plates with simply supported edges. E = 10,600 ksi. (To left of dashed line, curves apply only to 24S-T aluminum alloy.)

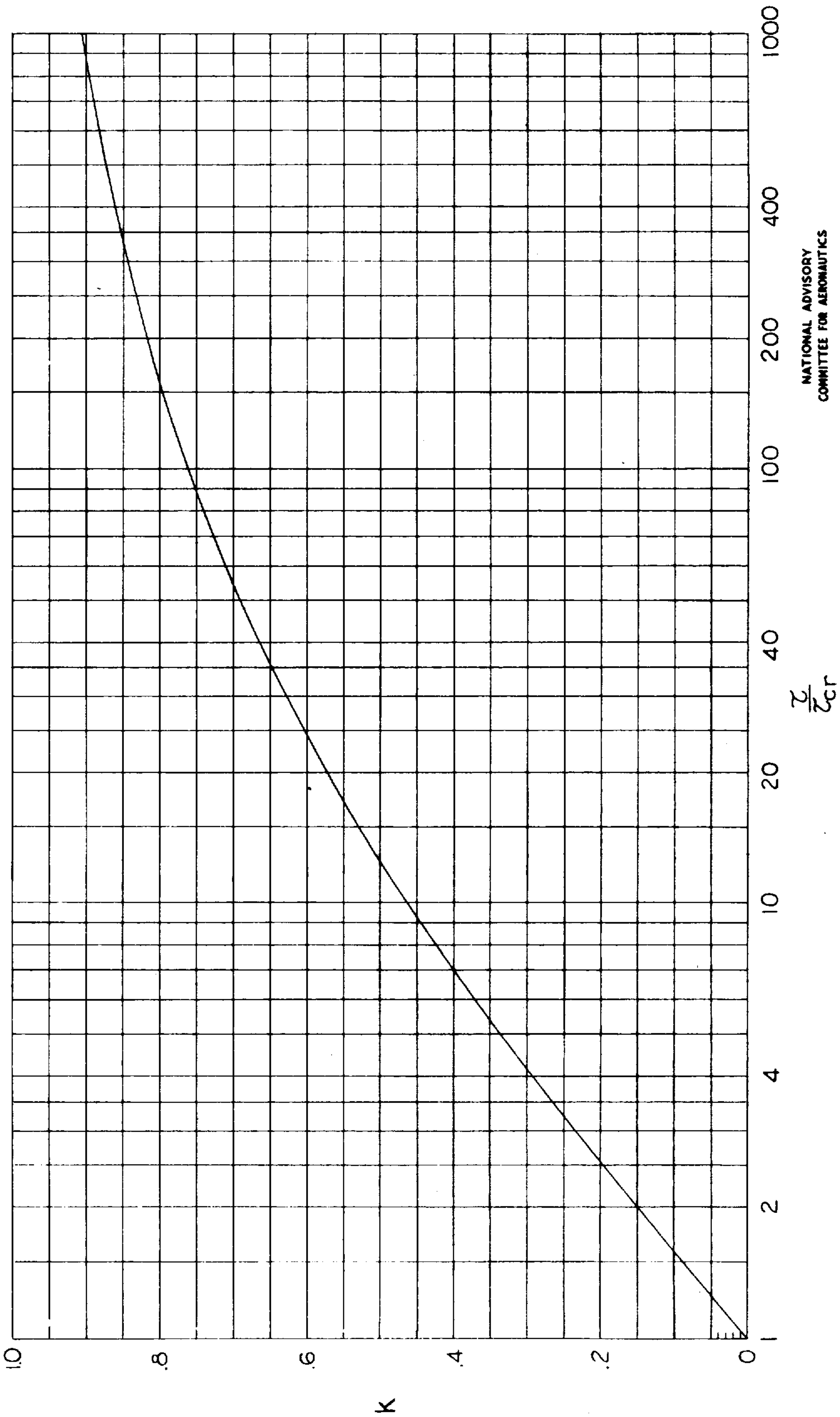

Figure 7.- Diagonal-tension factor k.

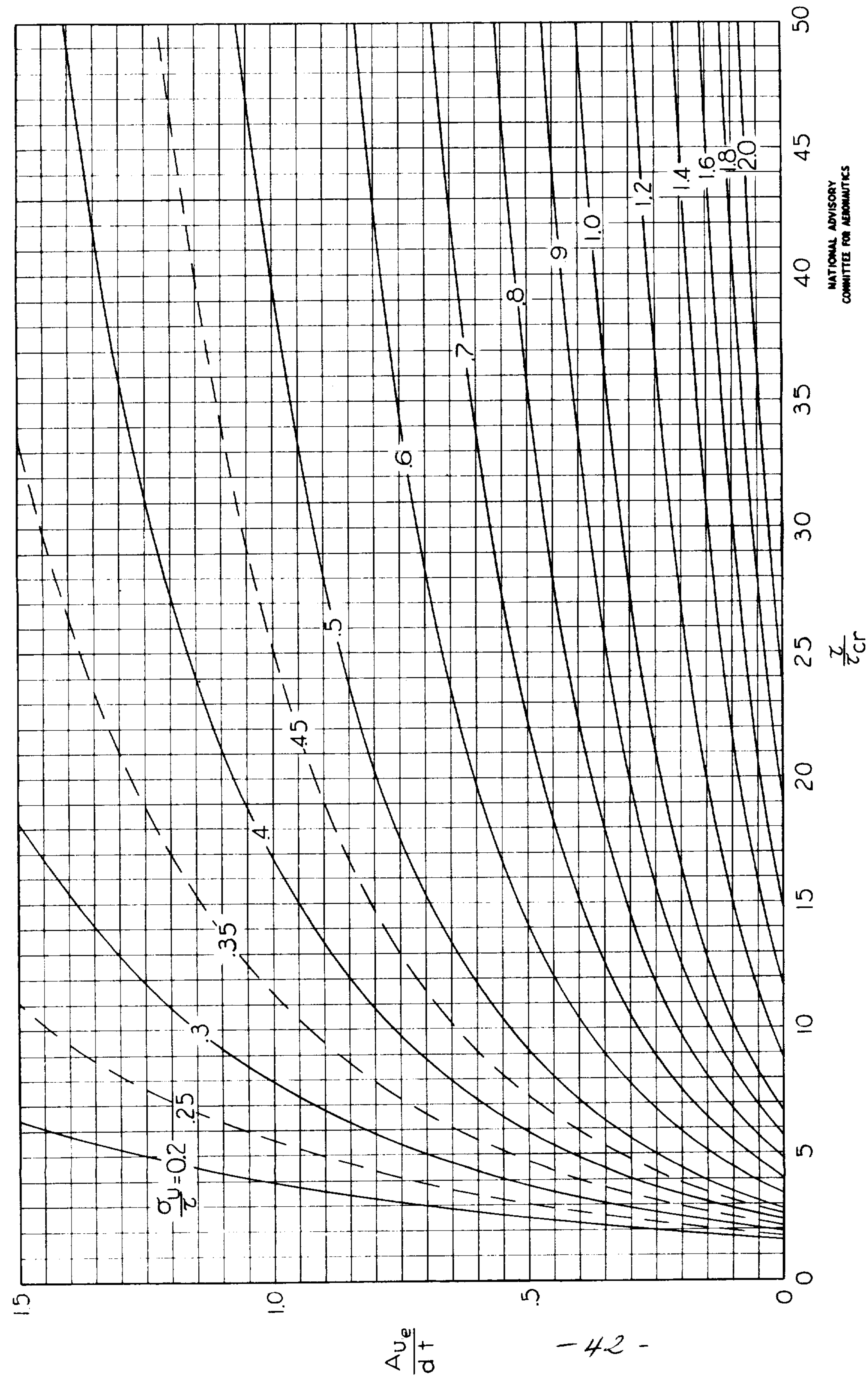

Figure 8.- Diagonal-tension analysis chart (straight scale).

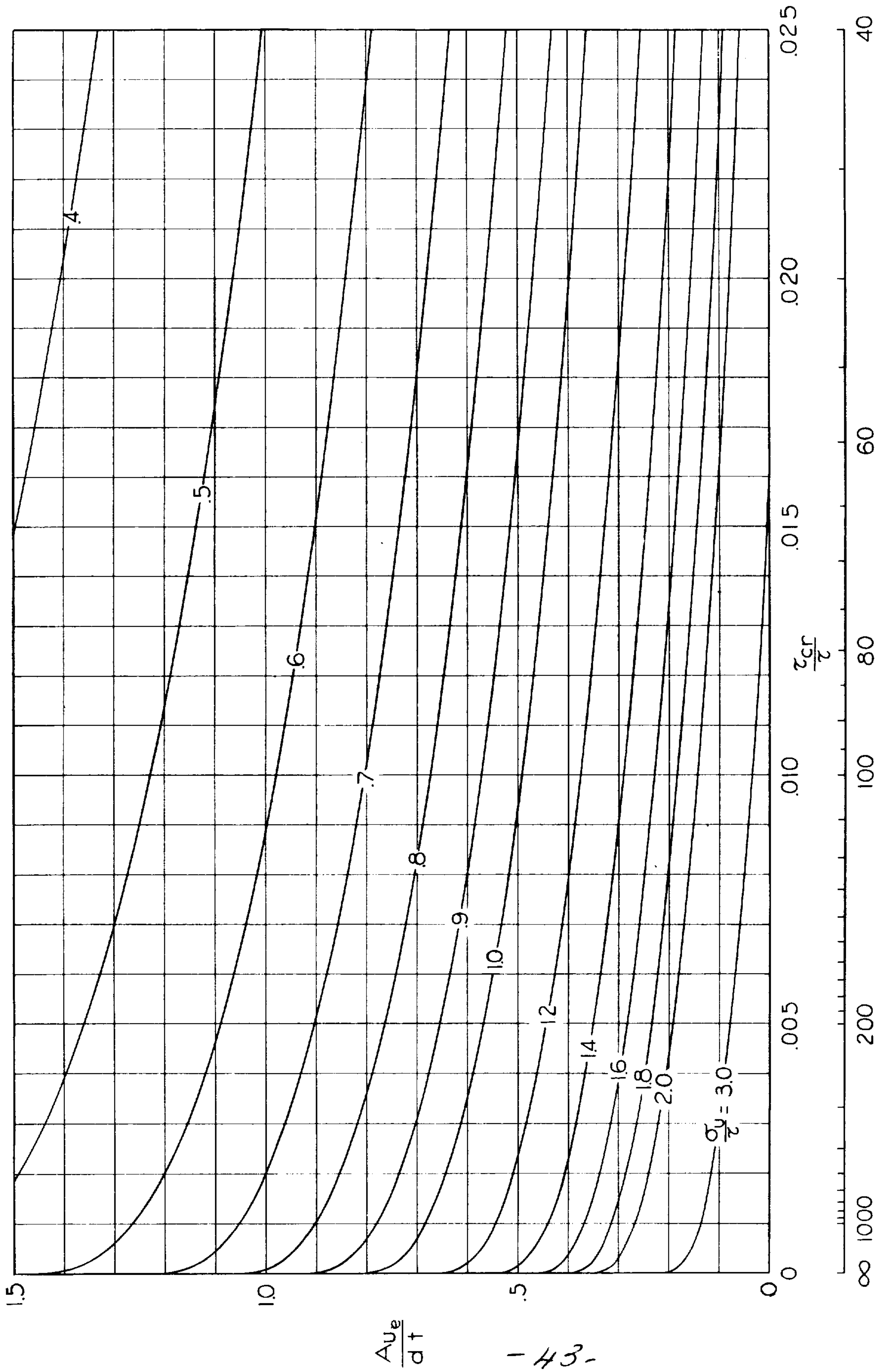

Figure 9.- Diagonal-tension analysis chart (semilogarithmic scale).

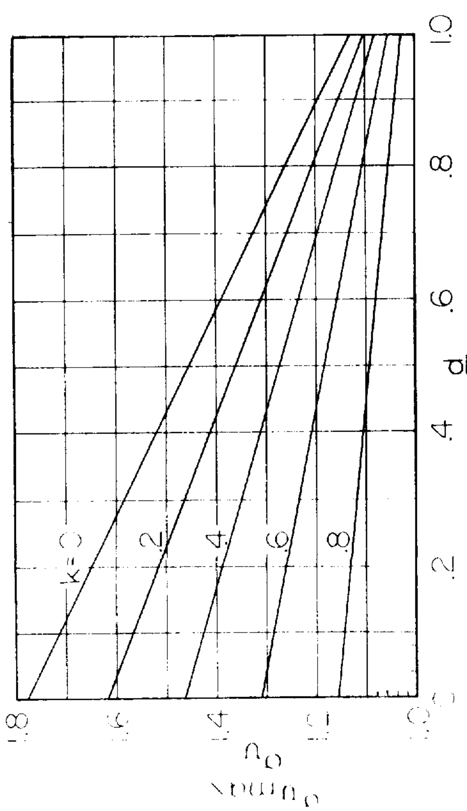

Figure 10.- Ratio of maximum stress to average stress in upright.

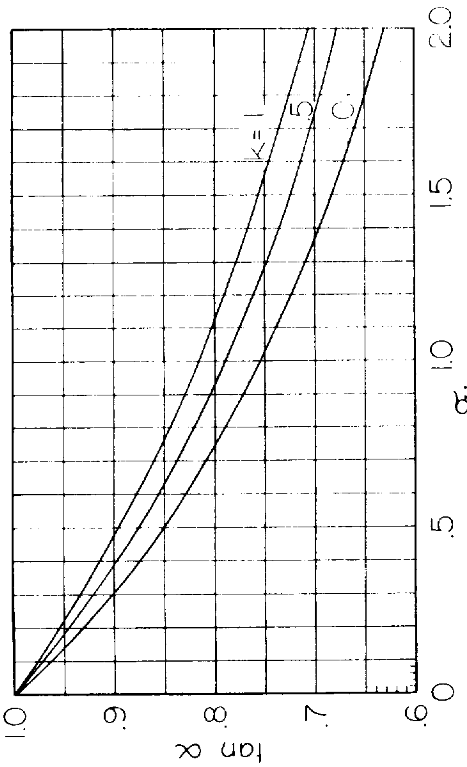

Figure 11.- Graph for determining angle α.

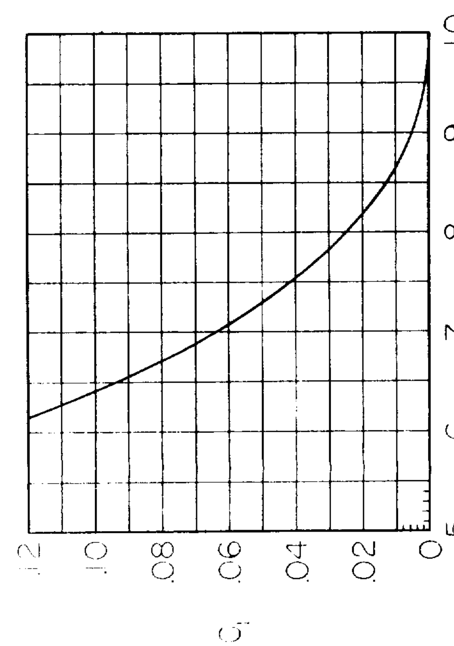

Figure 12.- Stress-concentration factor C_1.

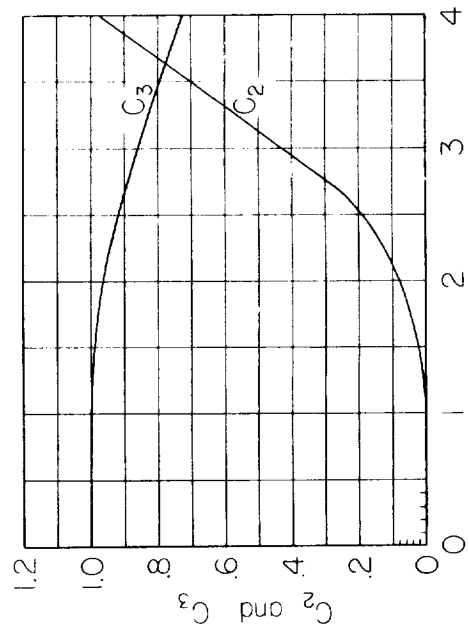

Figure 13.- Stress-concentration factors C_2 and C_3; $\left(\omega d = 0.7d \sqrt[4]{\frac{t}{(I_C + I_T) h_e}}\right)$

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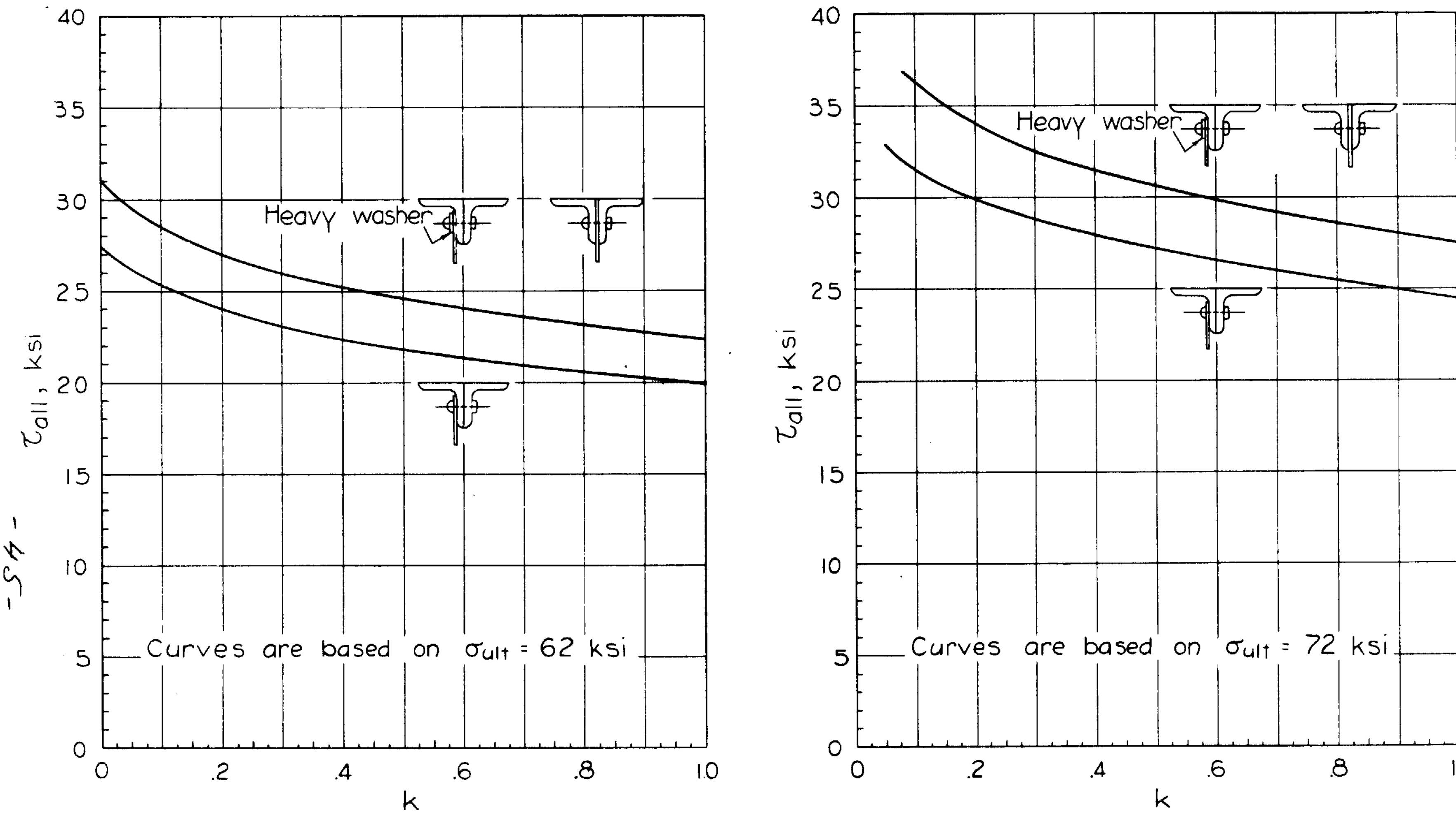

(a) 24S-T aluminum alloy.

(b) Alclad 75S-T aluminum alloy.

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Figure 14.- Allowable values of nominal web shear stress.

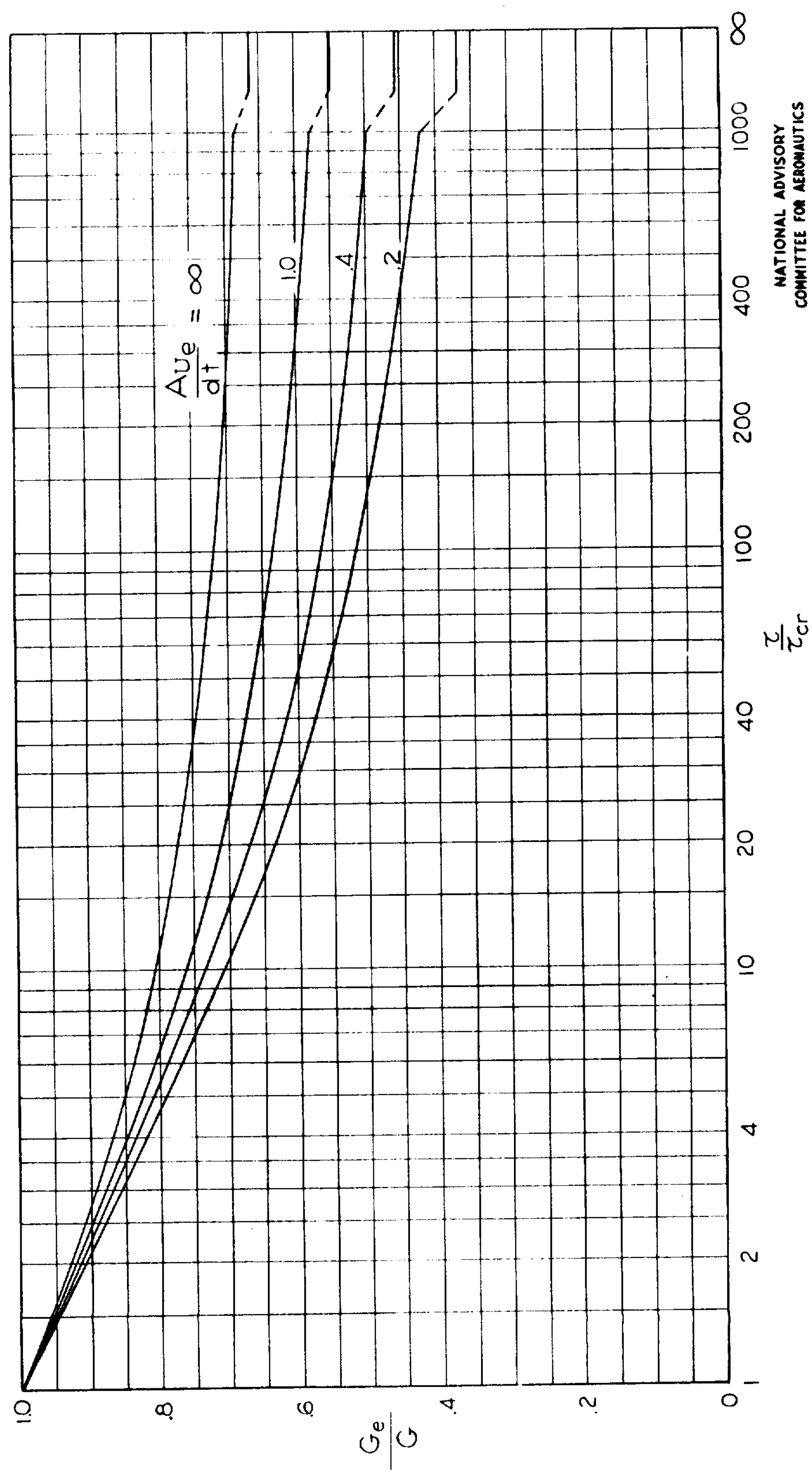
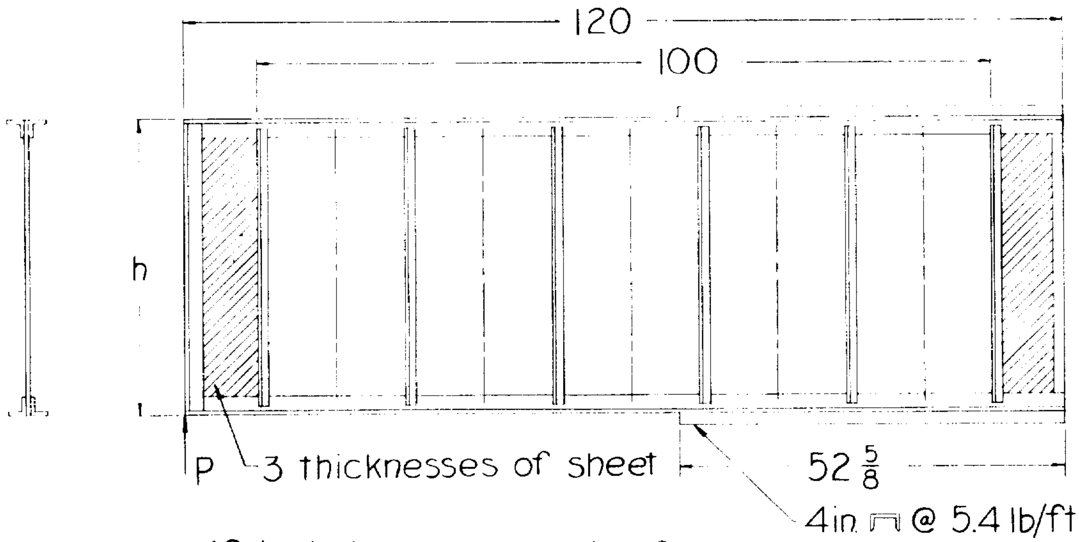
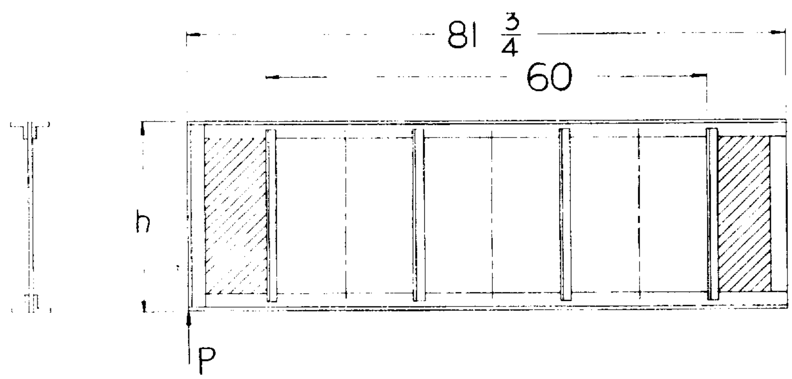


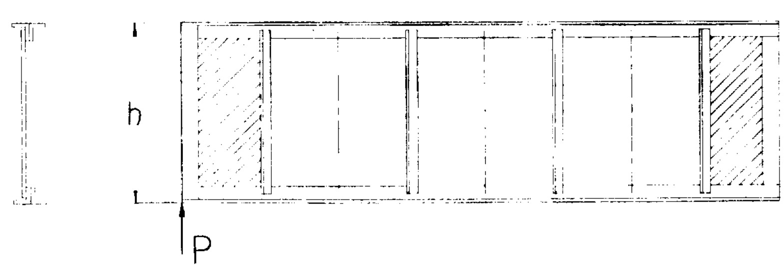
Figure 15.- Effective shear modulus of diagonal-tension webs.



40-inch beams - series I



25-inch beams - series I, II-25-6S and II-25-9S



25-inch beams series II and III

(a) General dimensions of test beams of series I, II, and III.

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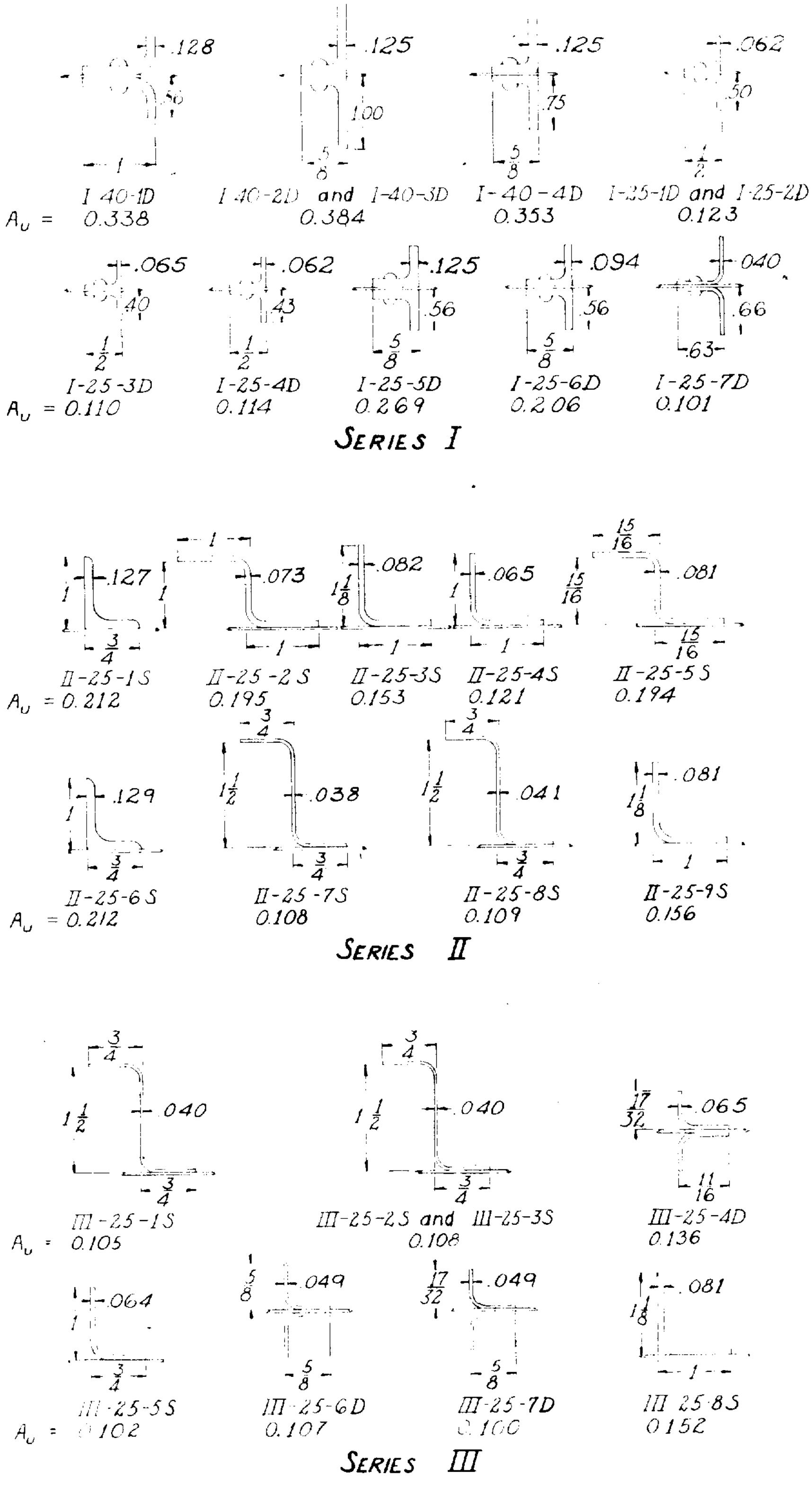
Figure 16.- Dimensions of test beams.



(b) Dimensions of uprights of test beams of series I, II, and III.

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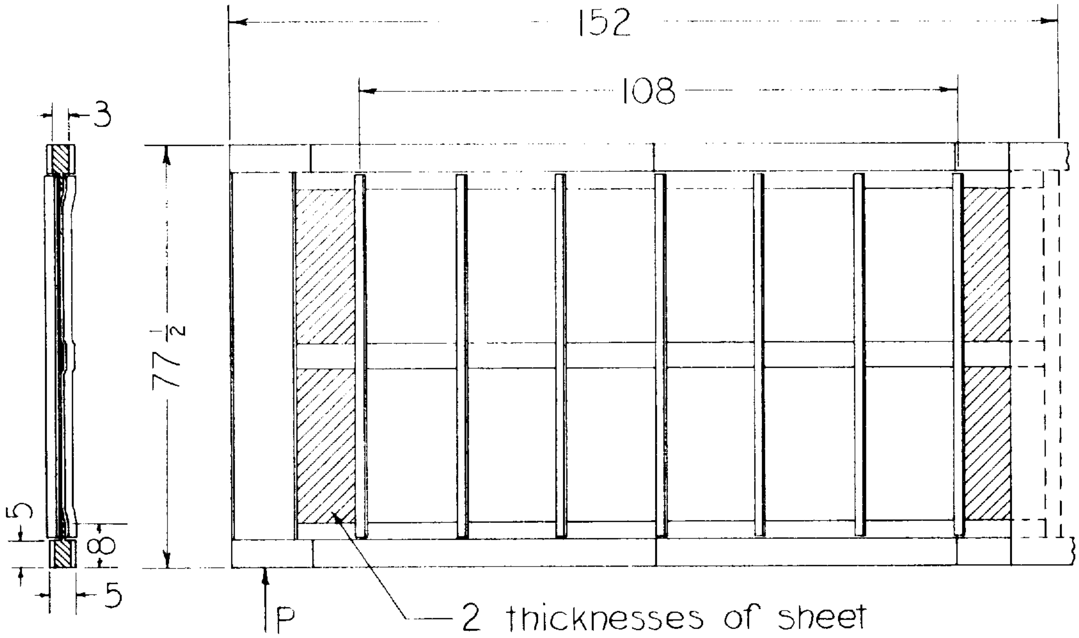
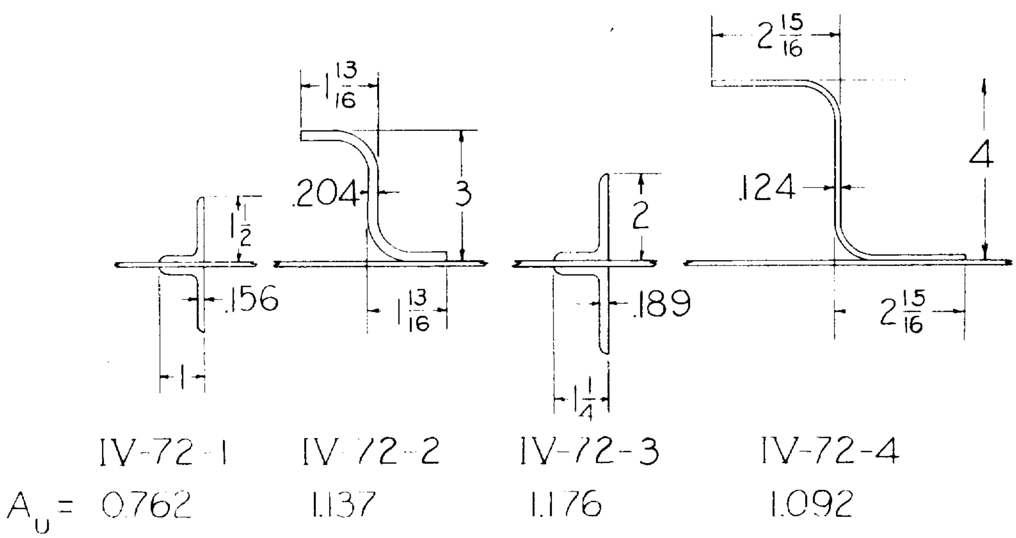
Figure 16.- Continued.



General dimensions of test beams



Dimensions of uprights

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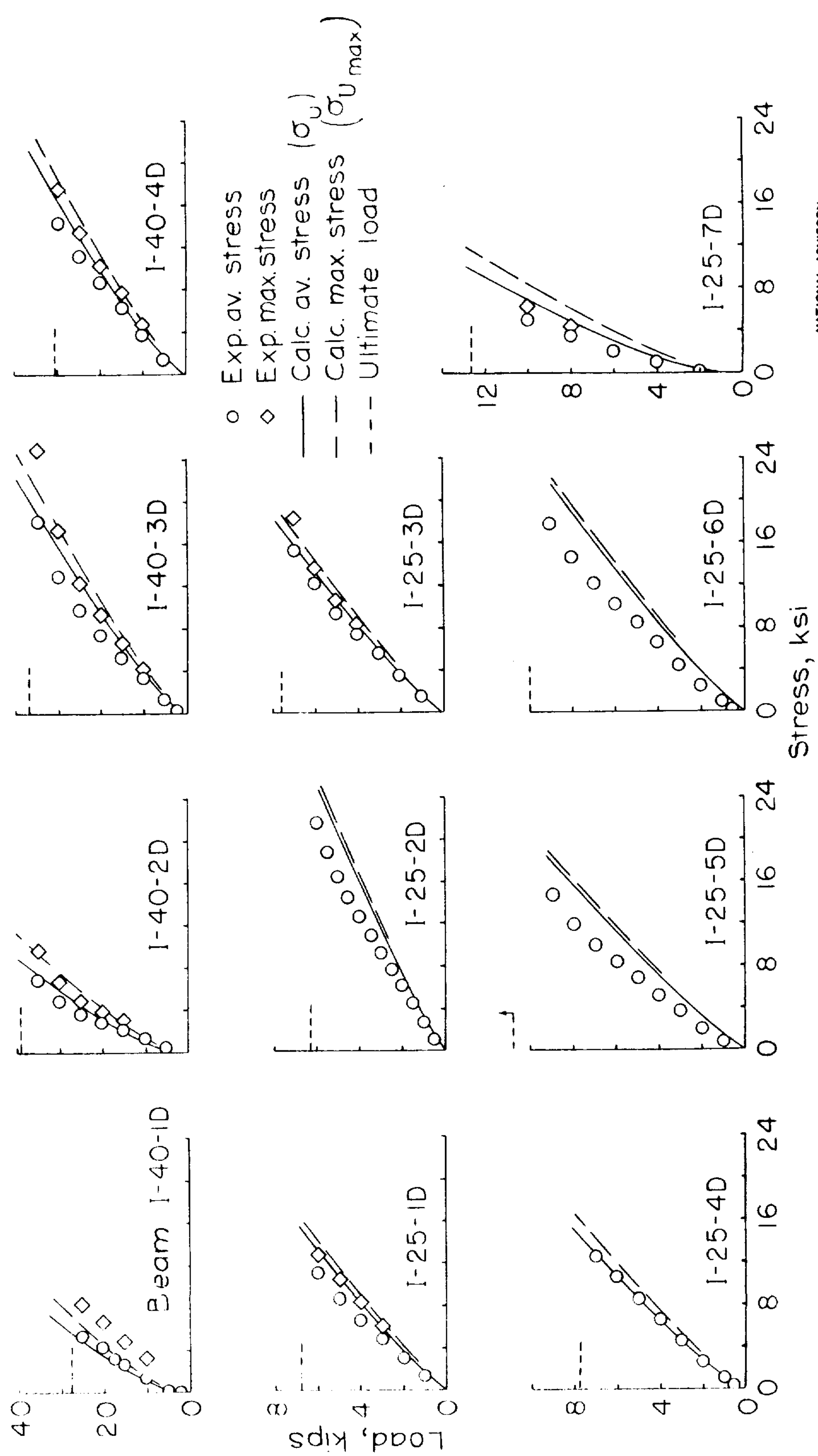
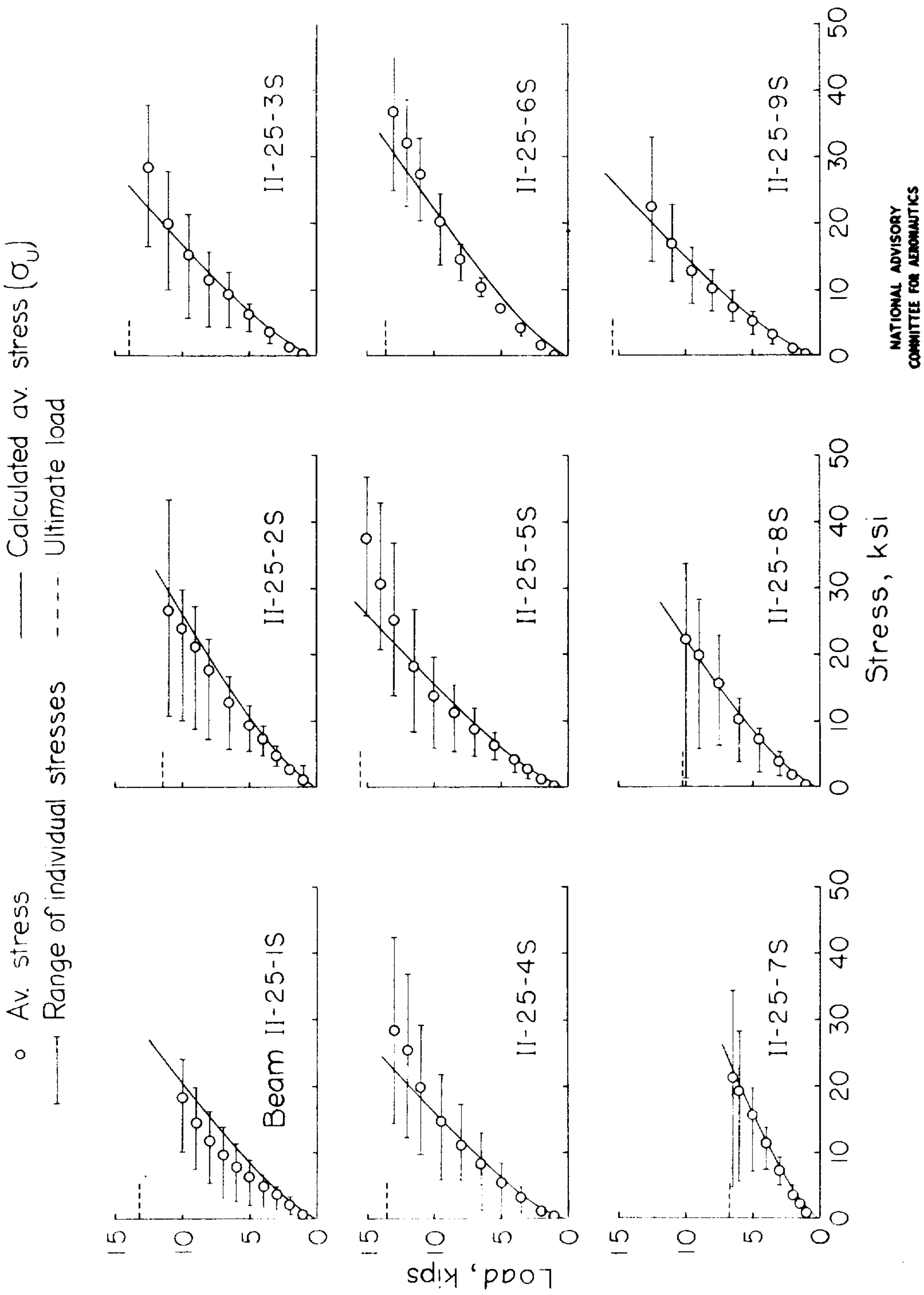
(c) Dimensions of test beams and uprights of series IV.

Figure 16.- Concluded.



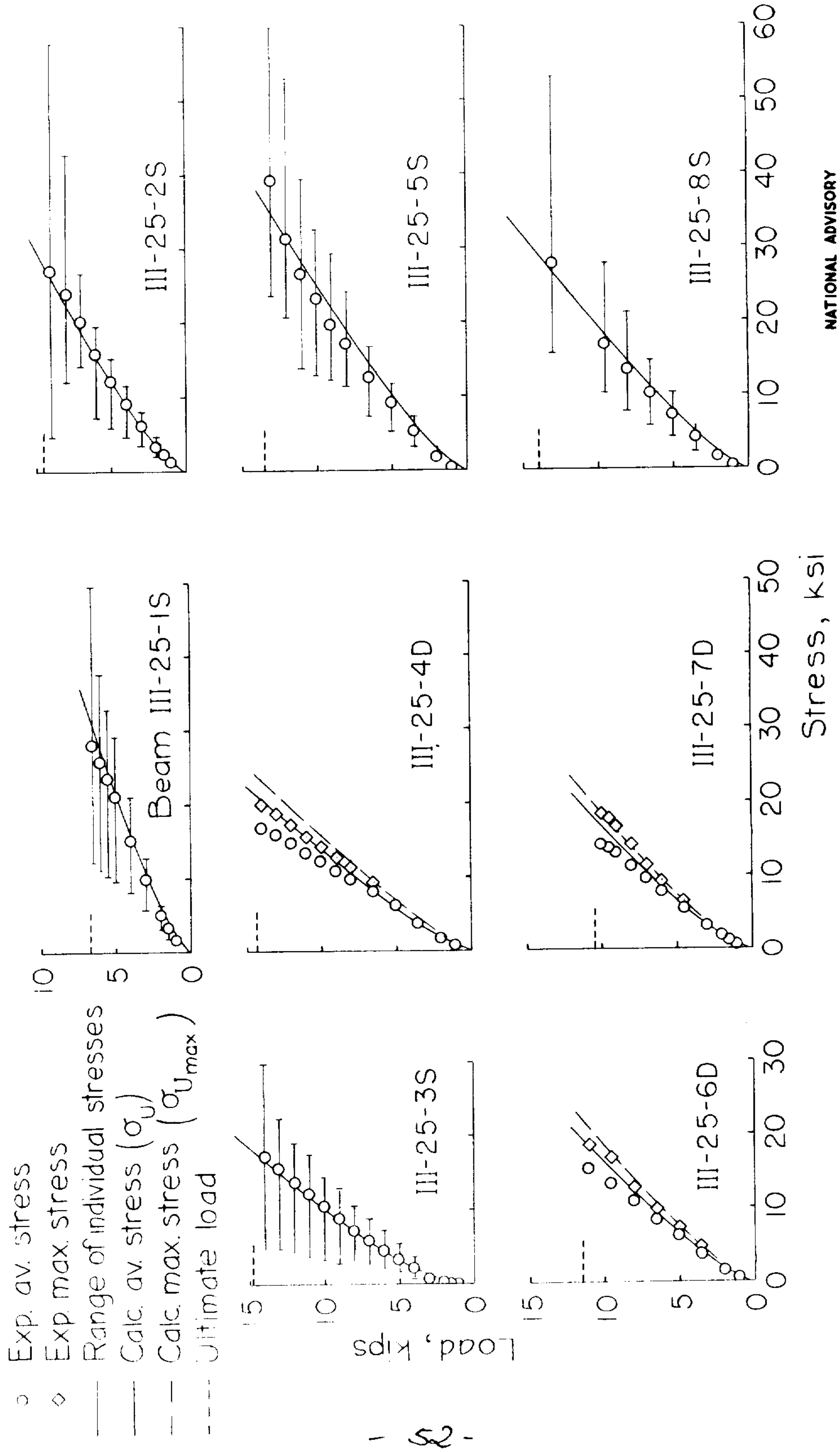
(a) Series I.

Figure 17.- Stresses in uprights of test beams.

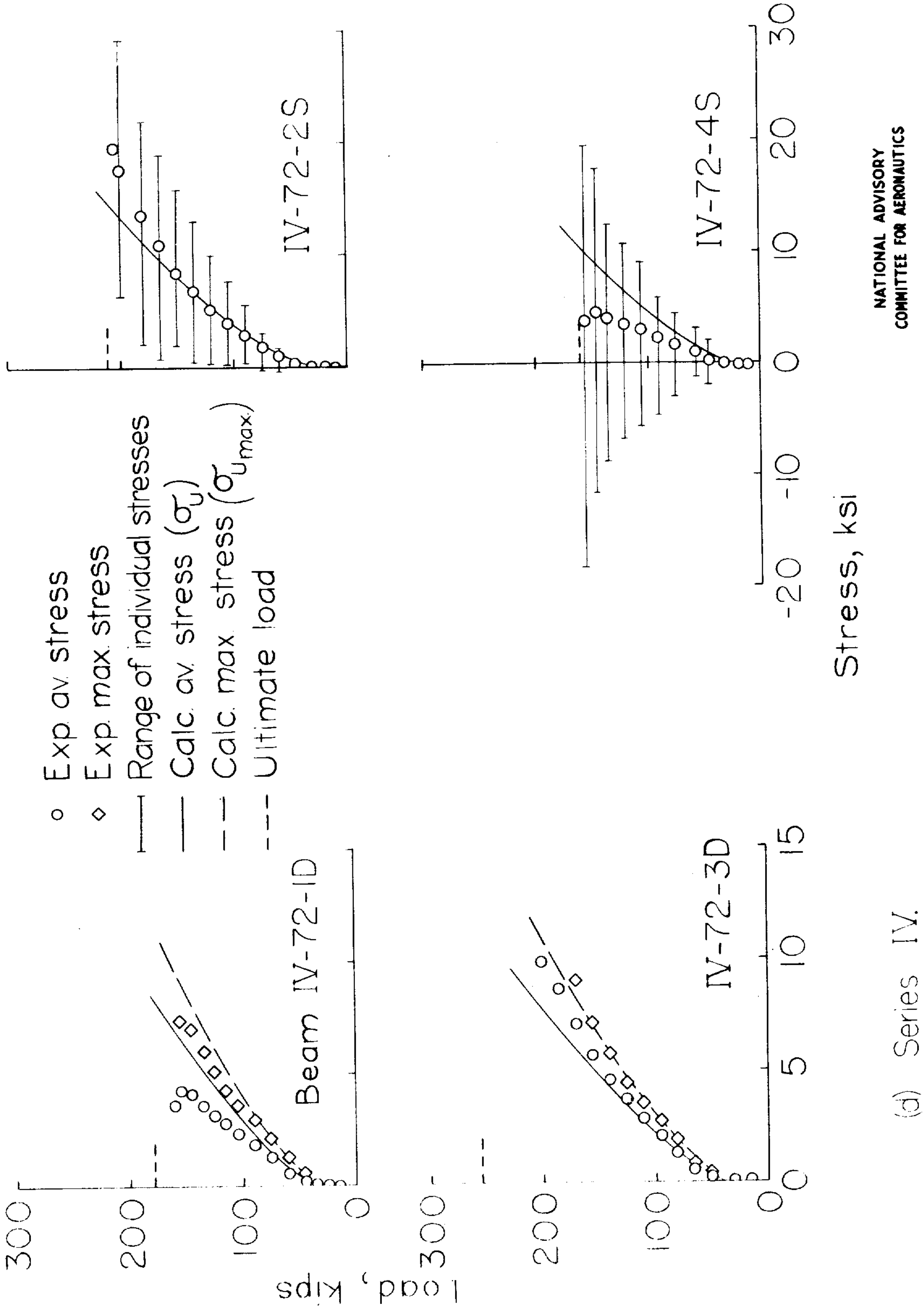


(b) Series II.

Figure 17.- Continued.



(c) Series III.
Figure 17.- Continued.



(d) Series IV.

Figure 17.- Concluded.

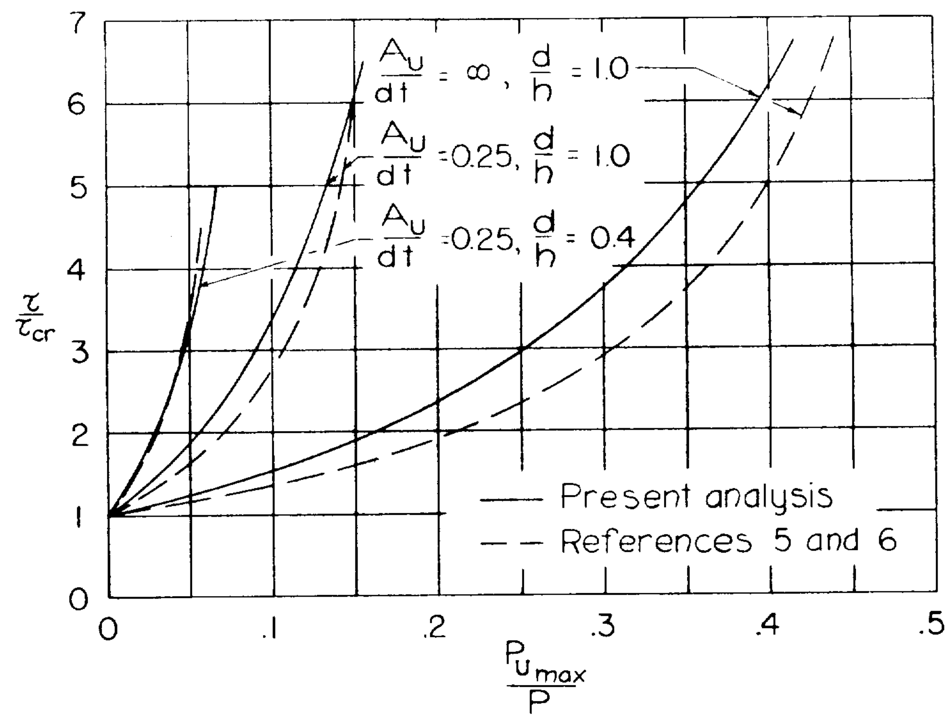


Figure 18.- Forces in uprights computed by present method and by Levy's theory (references 5 and 6).

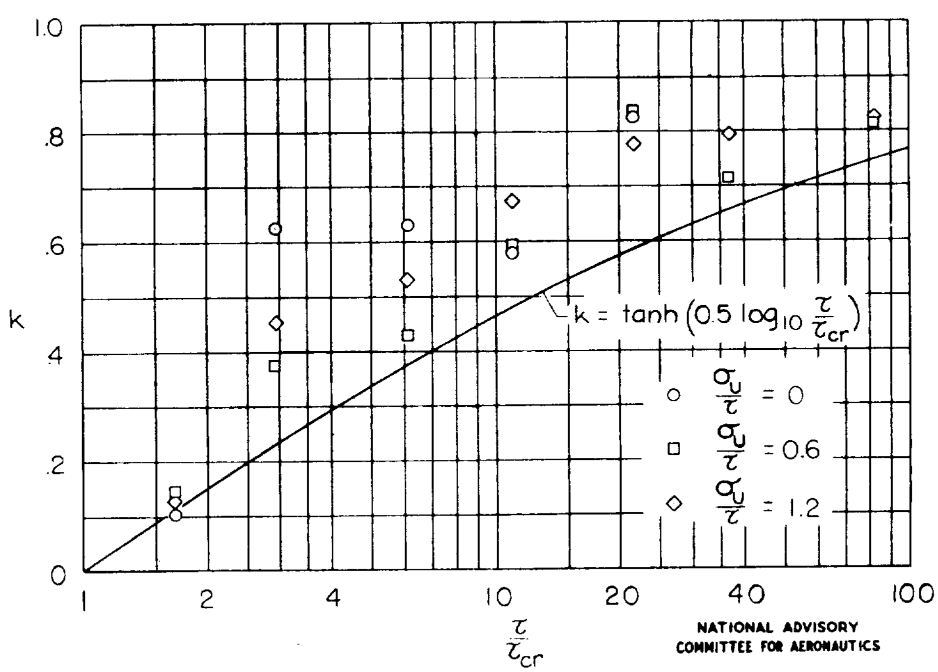


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Figure 19.- Diagonal-tension factors k obtained by present method and by Lahde-Wagner experimental data (reference 14).

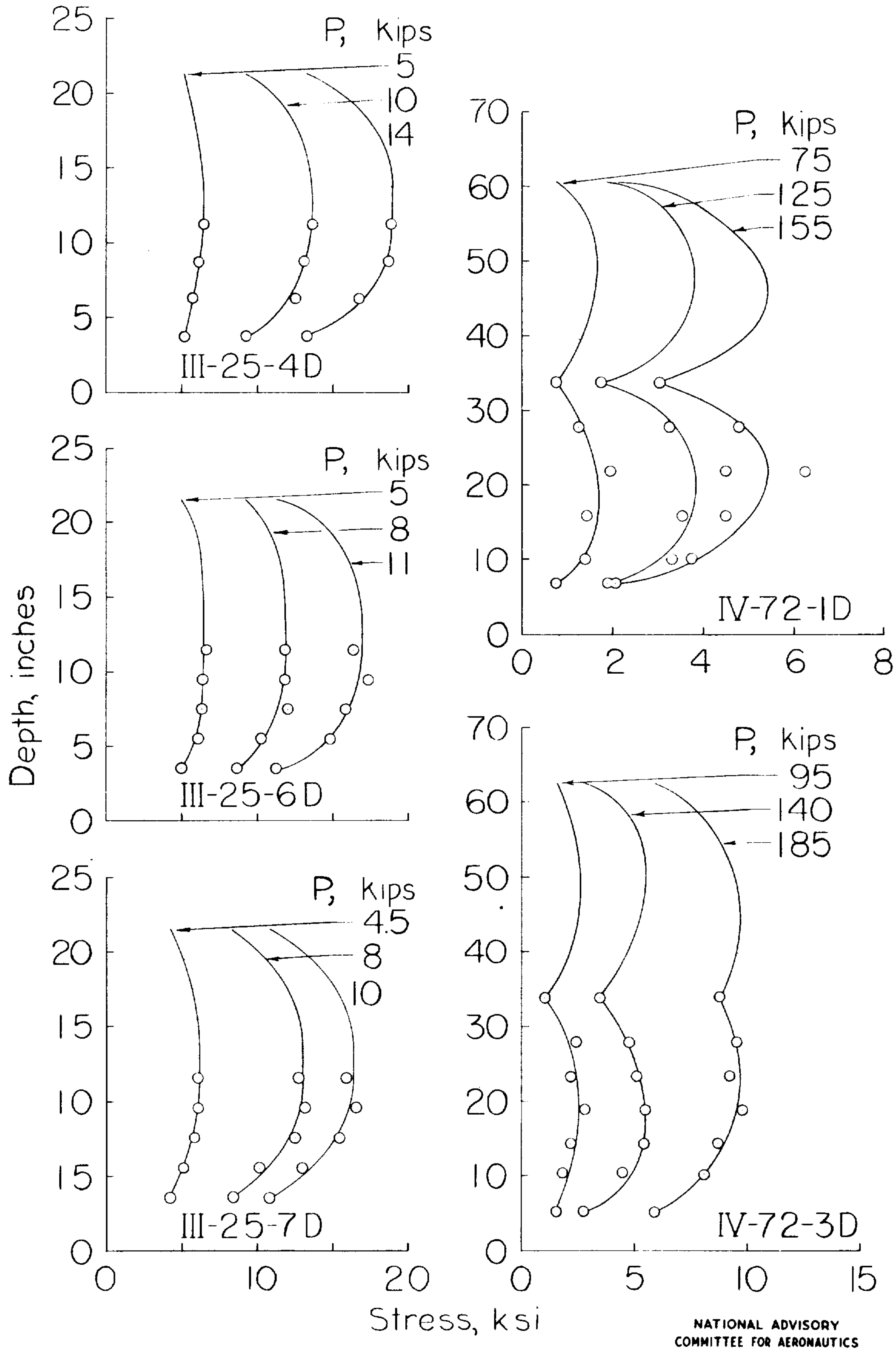

Figure 20.- Transverse distribution of stresses in uprights.

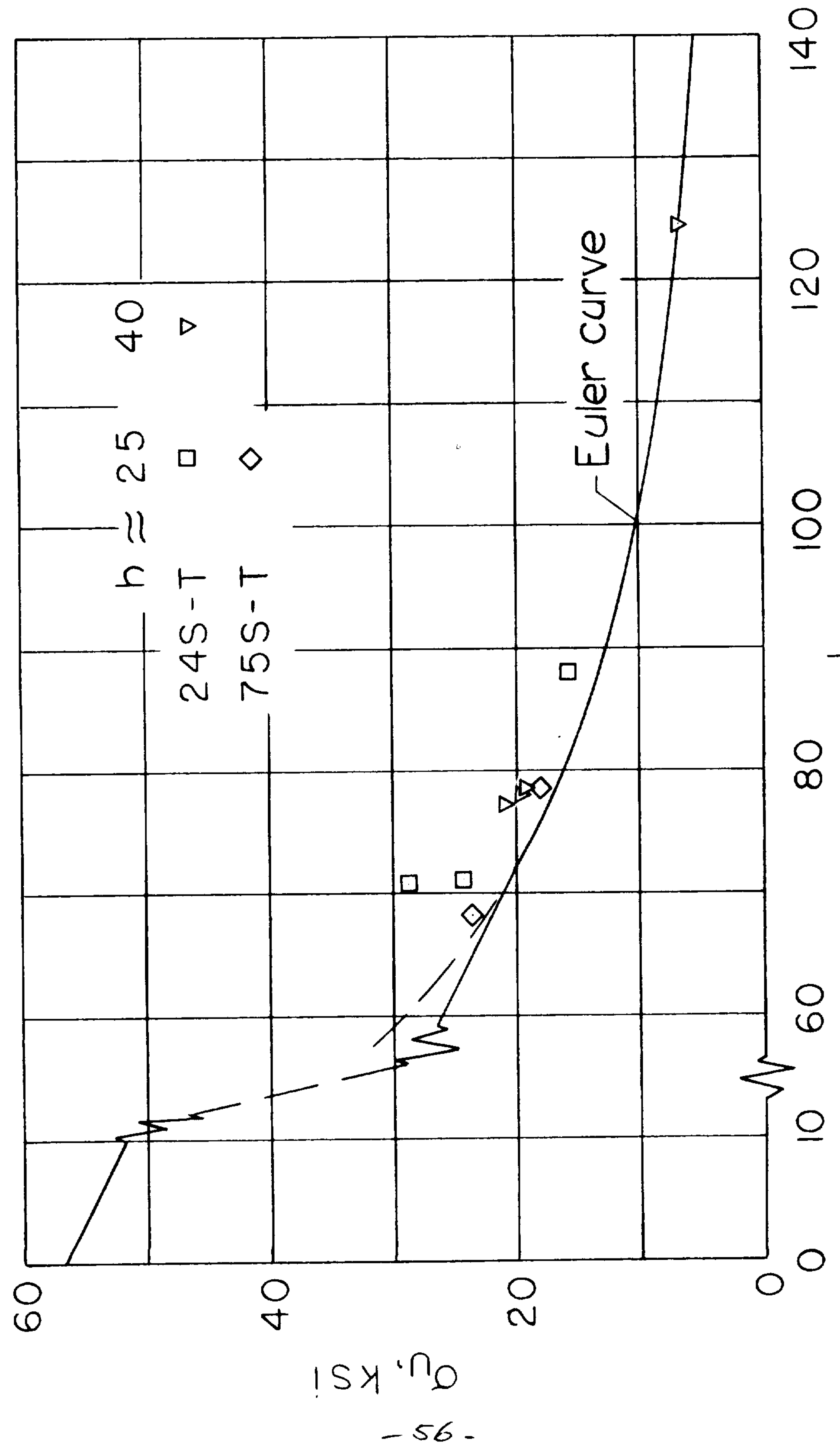

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Figure 21.- Stresses in double uprights failing by column action.

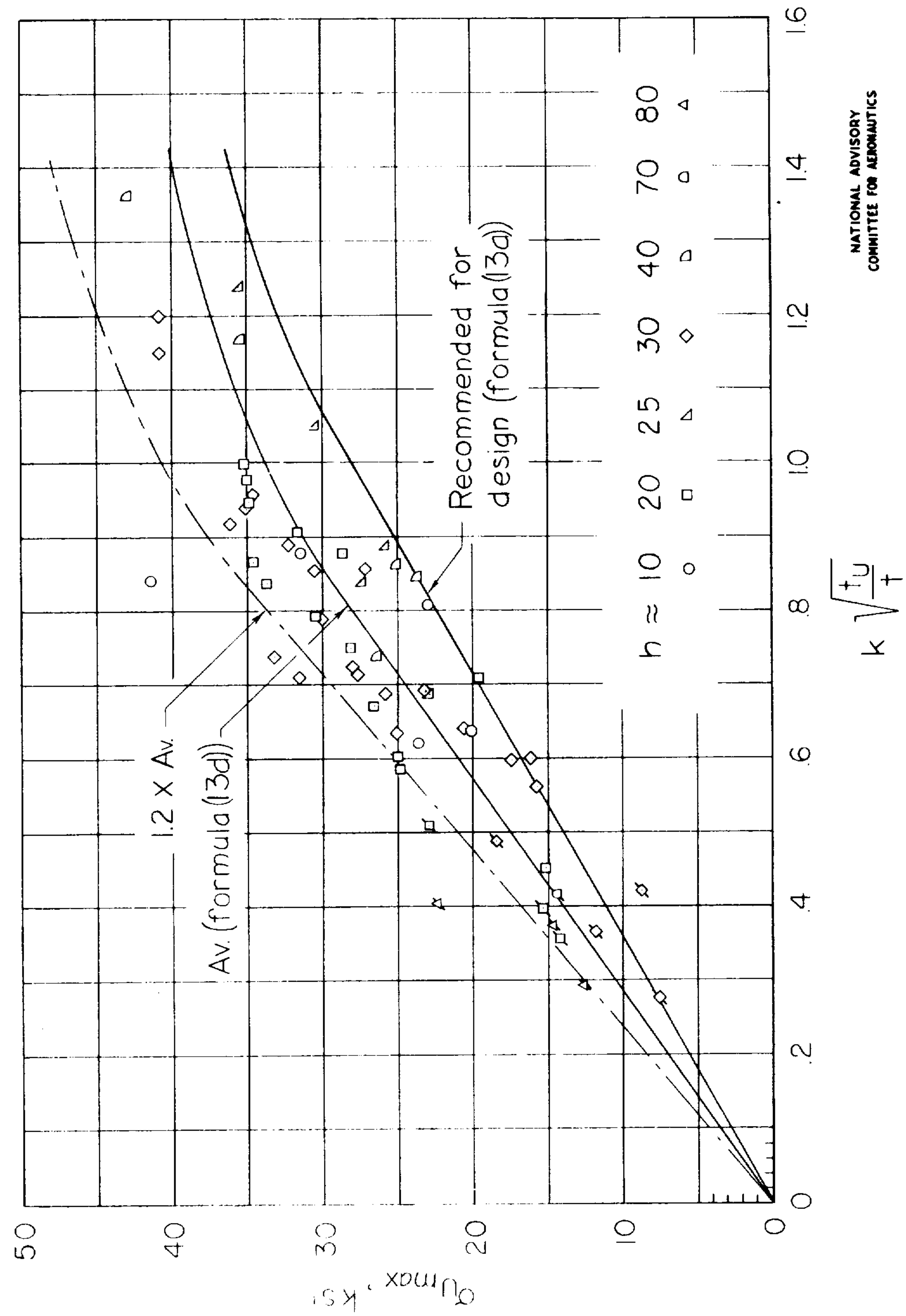

Figure 22.- Stresses in single uprights of 24S-T aluminum alloy at failure caused by forced crippling. Symbols with tails denote webs with k<0.5.

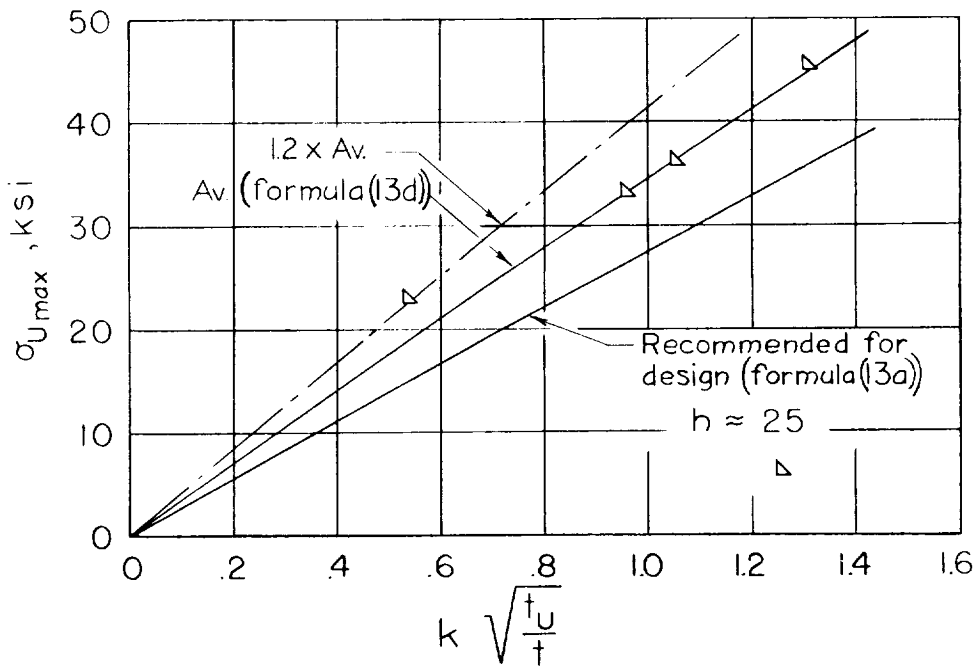

Figure 23.- Stresses in single uprights of 75S-T aluminum alloy at failure caused by forced crippling.

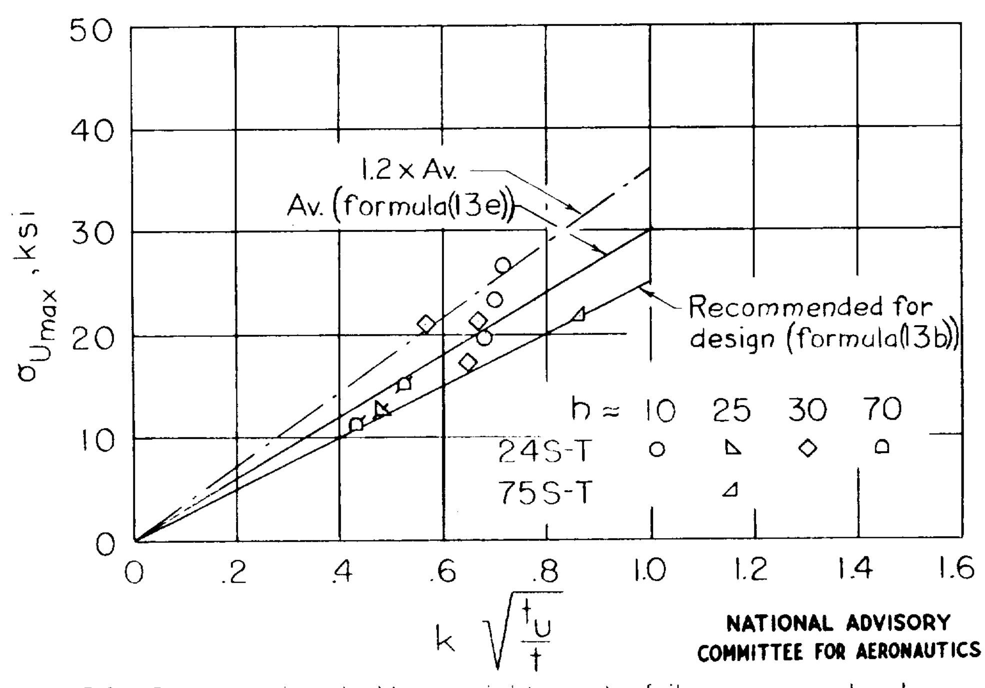

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Figure 24.- Stresses in double uprights at failure caused by forced crippling. Symbols with tails denote webs with $k < 0.5$.

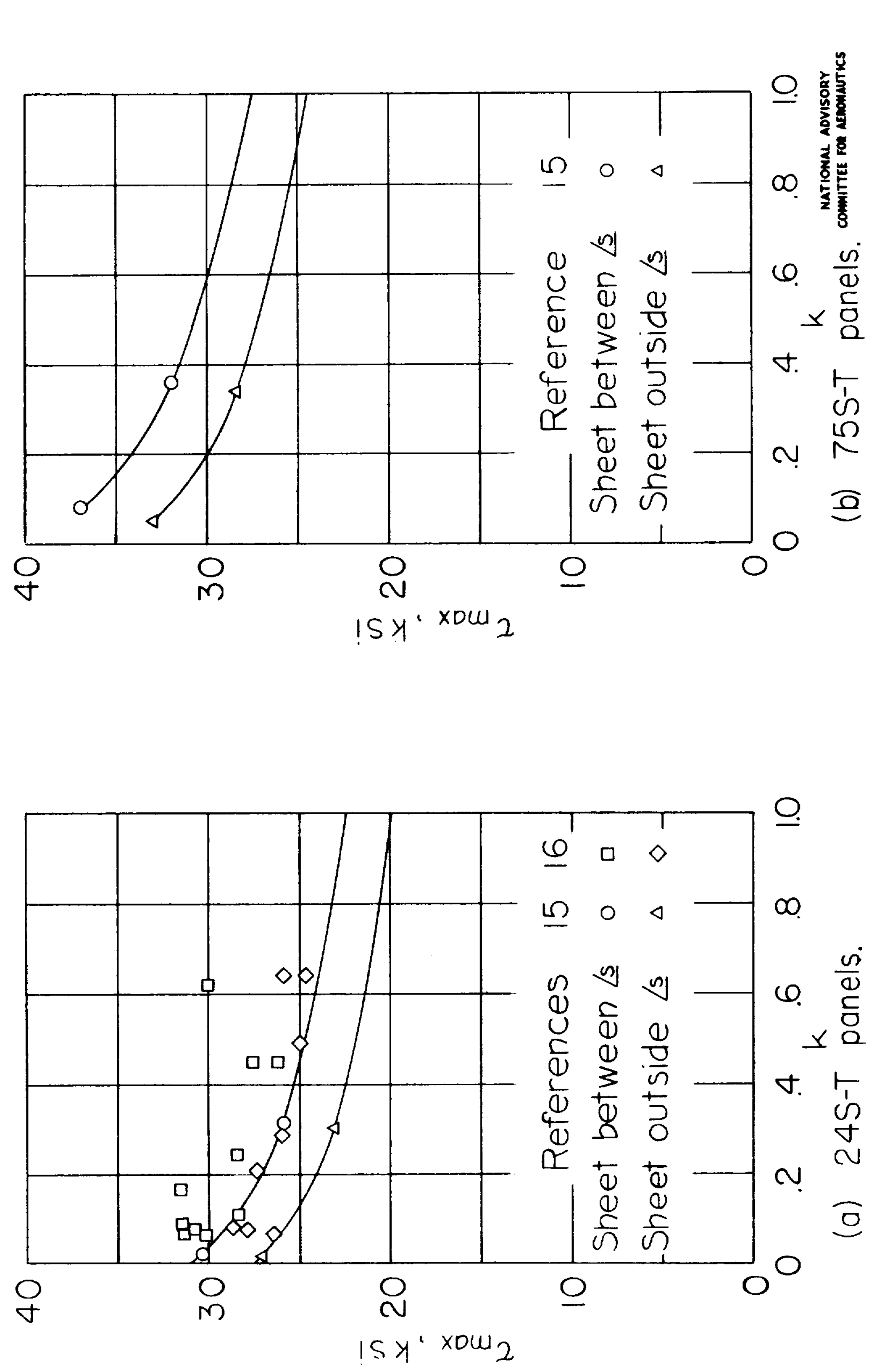

Figure 25.- Stresses in shear webs at failure. (Stresses at k=1.0 were estimated from tension tests.)

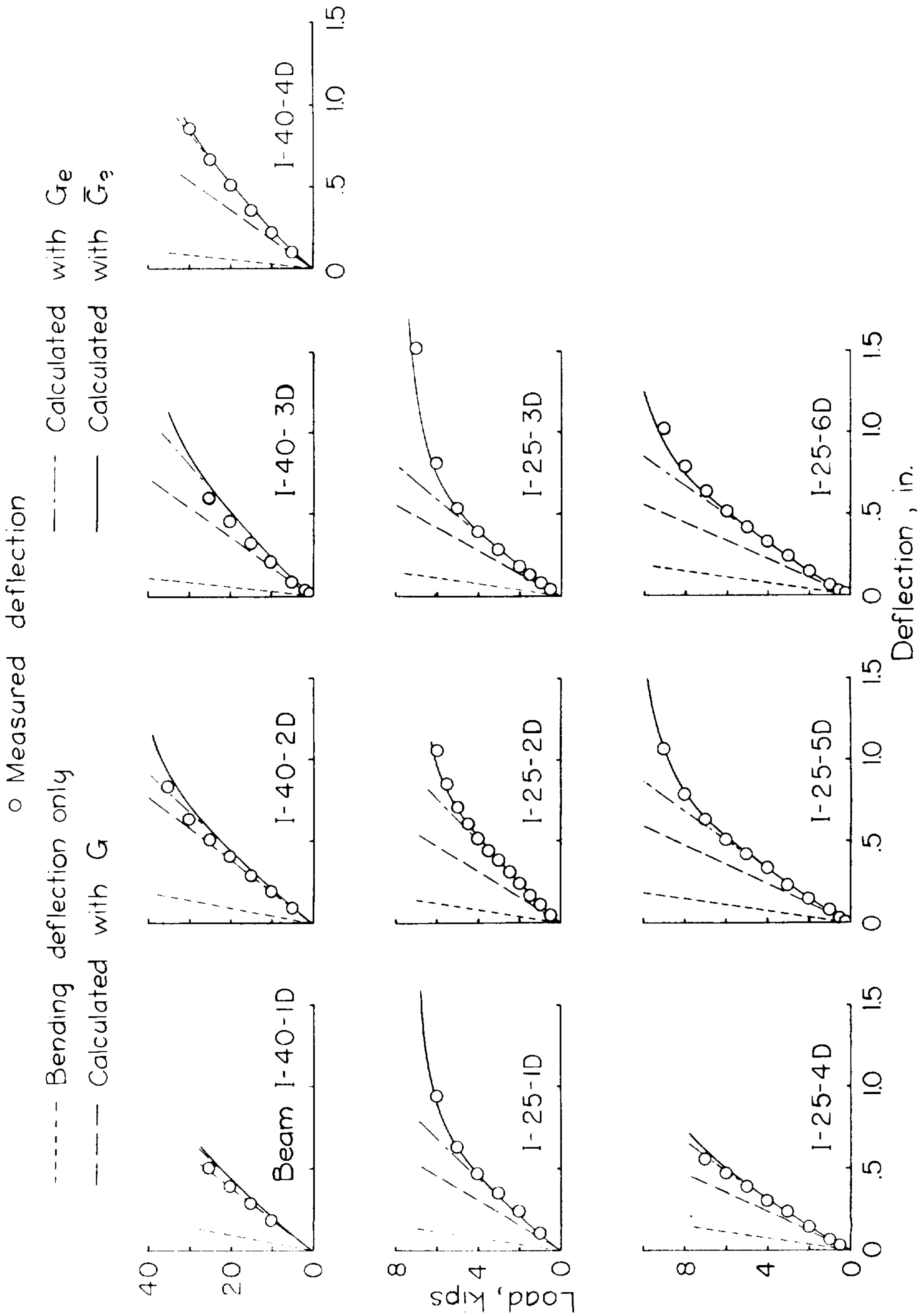

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Figure 26.- Deflection of series I beams.