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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1454

AN INVESTIGATION OF AIRCRAFT HEATERS

XXX - NOCTURNAL IRRADIATION AS A FUNCTION OF ALTITUDE AND ITS USE IN DETERMINATION OF HEAT REQUIREMENTS OF AIRCRAFT

By L. M. K. Boelter, H. Poppendiek, G. Young, and J. R. Andersen
University of California

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SUMMARY

Nocturnal radiation as used herein is atmospheric radiation excluding solar radiation. In the absence of radiation from the sun, only long-wavelength radiation exists in the atmosphere because of the range of temperatures present on the earth and in the atmosphere. Thus nocturnal irradiation is defined as irradiation coming from gases, clouds, and dust in the atmosphere and from the earth's surface.

This report contains generalized radiation charts for the calculation of the irradiation from water vapor, carbon dioxide, ozone, and the earth in terms of the appropriate parameters. Thus for a given set of meteorological data (temperature, pressure, and relative humidity as a function of altitude and an earth temperature), the nocturnal irradiation from above and below upon a horizontal area can be obtained as a function of altitude. These values of irradiation may then be used to make a heat-rate balance upon the area. One set of meteorological data is used as a typical example and is given in complete detail. The nocturnal irradiation as a function of altitude for three sets of meteorological data is presented to establish possible limiting cases of nocturnal irradiation.

Previous work, dealing with heat-rate balances on airplanes, personnel, and inanimate objects, simplified the radiation components as follows. The water vapor and carbon dioxide were postulated to be perfect transmitters, and the sky temperature was postulated to be at absolute zero. In the present report a heat-rate balance that both neglects and considers nocturnal radiation is made for a heat-transfer system. The errors involved in neglecting this radiation are discussed.

A discussion of the radiation charts available in the literature is given in an appendix.

INTRODUCTION

The radiant-energy exchange between an airplane and its surroundings (atmosphere and earth) at night as a function of altitude is important in order to complete a heat-rate balance upon an airplane. The radiant energy falling upon a horizontal differential area dA_1 at any elevation, is composed of three parts: (1) radiation from gases in the atmosphere above the horizontal area, (2) radiation from gases in the atmosphere below the horizontal area, and (3) the fraction of the radiation from the earth which has been transmitted through the atmosphere below.

Nocturnal radiation has been an important problem in the field of meteorology. Simpson (reference 1) in 1928 showed that the problem may be treated as follows. For clear nights the important radiating gas in the atmosphere is water vapor. By knowing the absorption properties of the water vapor, the amount of radiant energy incident on the ground originating from the atmosphere could be calculated. Möller and Mügge (reference 2) later developed a radiation chart based on this system which gives the nocturnal irradiation upon an area, from above and below, as a function of altitude. Elsasser, Andersen, and Ashburn (references 3 to 5) also presented similar radiation charts which are based on more recent absorption data of water vapor, and Elsasser and Ashburn accounted for the carbon-dioxide contribution to nocturnal radiation. The gaseous radiation obtained by each of these analyses consists in summing the radiation coming from each infinitesimal layer of gas and transmitted through the intermediate layers of gas. The absorptivity (or emissivity) of a gaseous layer is a function of concentration, temperature, pressure, and wavelength.

Andersen's method (reference 4) of determining nocturnal irradiation at the ground has been extended in this paper to determine gaseous radiation at various altitudes. Andersen's method, as shown in appendix B, applies to a system including radiation from water vapor only, excluding the effect of carbon dioxide and ozone. In order to calculate the nocturnal irradiation on an area, as a function of altitude, Andersen's procedure has been somewhat modified to account for carbon-dioxide radiation and absorption. A more direct method is presented for the determination of the radiation from the earth transmitted through the atmosphere. Ozone radiation at high altitudes has also been considered. (See section entitled "Carbon-dioxide and ozone radiation.") Generalized charts (see appendix A and figs. 1 to 5) are presented to facilitate the determination of gaseous radiation and radiation from the earth transmitted through the atmosphere. The procedure used to obtain these generalized charts is also presented. A typical example giving the nocturnal irradiation falling upon both sides of a horizontal area as a function of altitude for a given set of meteorological data and a given earth temperature is presented to illustrate the utility of the method.

The results of nocturnal irradiation upon a horizontal area from above and below for a very humid atmosphere and for a very dry atmosphere are given to establish possible limiting cases. For comparison, the nocturnal irradiation was computed by using Möller and Mügge's chart, Elsässer's chart, and Ashburn's chart for one of these atmospheric conditions. A discussion of these methods is also presented in appendix B.

The authors wish to express their gratitude to Dr. F. A. Brooks and Mr. E. H. Morrin for their many suggestions and to the Messrs. W. Elswick, V. Sanders, and M. Greenfield, who helped with the long and tedious calculations.

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IDEALIZED SYSTEM

The following ideal system has been chosen because it presents an approach to the actual system and because it is one which can be treated analytically without great difficulty. The actual system will be described first and then the idealized system, thus making evident the simplifications. Nocturnal radiation includes: (1) radiation from all radiating gases (water vapor, carbon dioxide, ozone, etc.) present in the atmosphere, (2) radiation upward from the earth which has been transmitted through the atmosphere, and (3) radiation from the dust of the atmosphere. The radiation from a gas in the atmosphere is a function of the gas temperature, gas pressure, gas density, and the gas absorption characteristics. The gas absorption characteristics may vary slightly depending upon the presence of other gas components. The emissive power of the earth's surface is a function of the earth's surface temperature and spectral emissivity. The spectral emissivity and temperature of the earth's surface will vary depending upon the locality.

Clouds exist in the atmosphere and act as radiation shields and ideal radiators. The temperature, pressure, and gas-density distributions of the atmosphere are transient. These distributions change principally as a result of the variation of the solar radiation during a 24-hour period.

The idealized system is a simplified version of the actual system. The radiating gases of the simplified system will be limited to water vapor, carbon dioxide, and ozone. A cloudless and dustless atmosphere will be postulated. Empirical expressions of the absorption coefficient of water vapor for values of ν from 0 to 584 centimeters^{-1} and from 752 to ∞ centimeters^{-1} ($\lambda = \infty$ to 17.1 microns and 13.3 to 0 microns)[1] as a function of temperature, pressure, moisture content, and wavelength are used. Carbon dioxide is considered to have a perfect absorption band from $\nu = 584$ to 752 centimeters^{-1} ($\lambda = 17.1$ to 13.3 microns). At high altitudes, ozone (see section entitled "Carbon-dioxide and ozone radiation") is considered to have a strong ($\alpha_\nu = 1$) absorption band from $\nu = 975$ to 1150 centimeters^{-1} ($\lambda = 10.25$ to 8.7 microns). Absorption characteristics of gases have been determined without the presence of other gases. It will be postulated that the absorption characteristic of a given gas will not change when other gases are present. (See table I.)

For a given locality an average earth temperature is used. The resulting generalized curves are expressed in a range of average earth-surface temperature. The earth will be postulated a gray body. (A gray body is defined as one which absorbs the same fraction of the incident radiant energy in all wavelengths, or one which has a constant spectral emissivity (absorptivity) in all wavelengths.) The meteorological data used in making nocturnal-radiation calculations are transient data. The idealized system is a steady-state system, and all the analytical expressions given are for a radiant system in the steady state.

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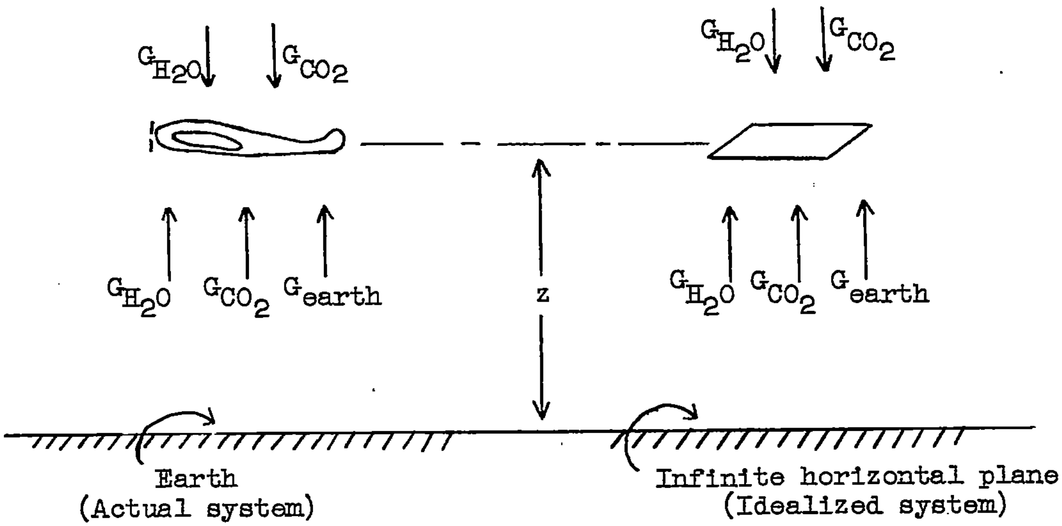
[1] From a search of the literature, it appears that many writers prefer to use wave number ν rather than wavelength λ

$$\nu = \frac{10^4}{\lambda} = \frac{f}{c}$$

where λ is wavelength in microns, ν is wave number in centimeters^{-1}, c is velocity of light in centimeters per second, and f is frequency of vibration in cycles per second. Table I gives the postulated absorption characteristics of the various gases as a function of both ν and λ.



As illustrated in the preceding diagram the irradiation from above and below upon a horizontal area may be used to obtain the irradiation upon an airplane flying in a horizontal position. The irradiation falling upon the area from above is composed of water-vapor and carbon-dioxide radiation. The irradiation on the area from below is composed of water vapor, carbon dioxide, and transmitted earth radiation. At altitudes of 40,000 feet, ozone radiation from above will be considered.

The results of calculations presented in this report are, of course, subject to experimental verification.

METHOD OF CALCULATION

In order to calculate the various components of irradiation upon the differential area indicated in the preceding section as a function of altitude, the following radiosonde data are required - pressure, temperature, and relative humidity as a function of altitude. With these data, values of h_0' are calculated as a function of altitude in the manner shown in the next paragraph. (For definitions of symbols, see appendix C.) The function h_0' is defined as the effective water-vapor content in the atmosphere up to any altitude z and is defined as follows:

$$h_0' = \frac{1}{\rho_s}\int_0^z \frac{P}{14.73}\sqrt{\frac{300}{T}}\rho\, dz = 1\int_0^z \frac{P}{14.73}\sqrt{\frac{300}{T}}\rho\, dz \qquad (1)$$

where

P	pressure at any altitude z from radiosonde data, pounds per square inch
T	temperature at any altitude z from radiosonde data, $^{\circ}K$
ρ	density of water vapor at any altitude z calculated from radiosonde data, grams per cubic centimeter
ρ_s	density of water at atmospheric pressure and temperature of 4° C, 1 gram per cubic centimeter
z	altitude, centimeters
h_0'	effective moisture content; subscript zero indicates reference altitude is at the ground, centimeters

When the values of P, T, and h_0' as a function of altitude are known, the five components of irradiation can readily be obtained by employing the generalized charts. (See appendix A and figs. 1 to 5.) Detailed procedures for obtaining these radiation components will be given in succeeding paragraphs.

Calculation of the function h_0'.- From the radiosonde data, temperature, and relative humidity, the partial vapor pressure is calculated. With the values of the partial pressure and the equation for an ideal gas,

$$\rho = 0.015\frac{p}{T} \qquad (2)$$

where

ρ	water-vapor density, grams per cubic centimeter
p	partial vapor pressure, pounds per square inch
T	absolute temperature, $^{\circ}K$

the moisture density is obtained as a function of altitude. Then by the nomograph (fig. 6) the quantity $\frac{P}{14.73}\sqrt{\frac{300}{T}}\rho$ is obtained as a function of altitude. Finally, values of h_0' are obtained by graphically integrating $\int_0^z \rho \frac{P}{14.73}\sqrt{\frac{300}{T}}\,dx$, or by using Simpson's rule (reference 6).

Water-vapor radiation.- The gaseous irradiation upon a horizontal differential area at any altitude has two water-vapor components, that from above $G_{H_2O,above}$ and that from below $G_{H_2O,below}$. Both $G_{H_2O,above}$ and $G_{H_2O,below}$ are obtained as follows: The water vapor is postulated to be an ideal radiator (see references 1 and 4) at wave numbers of 0 to 300 centimeters^{-1} and 1200 to ∞ centimeters^{-1}, and the irradiation due to water vapor, $G''_{H_2O,above}$ and $G''_{H_2O,below}$, at these wave numbers are thus a function of temperature only. The chart in figure 2 gives these values of irradiation as a function of temperature of the atmosphere adjacent to the differential area. The irradiation, $G'_{H_2O,above}$ and $G'_{H_2O,below}$, due to water vapor at wave numbers of 300 to 584 centimeters^{-1} and 752 to 1200 centimeters^{-1} (λ = 33.3 to 17.1 microns and 13.3 to 8.34 microns) upon a differential area at any altitude z_p are obtained by plotting the values of T and h'_{z_p} on the water-vapor radiation chart in figure 1. (See also appendix A.) The integrated area under the curve is the water-vapor radiation at these wave numbers. Since $G''_{H_2O,above}$ and $G''_{H_2O,below}$ are equal, the only difference between $G_{H_2O,above}$ and $G_{H_2O,below}$ is due to the difference of $G'_{H_2O,above}$ and $G'_{H_2O,below}$ because

$$G_{H_2O,above} = G'_{H_2O,above} + G''_{H_2O,above}$$

$$G_{H_2O,below} = G'_{H_2O,below} + G''_{H_2O;below}$$

The value of h'_{z_p} is defined by

$$h'_{z_p}(z) = \left| h'_0(z) - h'_0(z_p) \right| \tag{3}$$

where the vertical lines signify absolute values. The function $h'_{z_p}(z)$ is the effective moisture content measured either upward or downward from the differential area situated at the altitude z_p.

Carbon-dioxide and ozone radiation.- The two carbon-dioxide components, $G_{CO_2,above}$ and $G_{CO_2,below}$, are equal at any altitude, since carbon-dioxide radiation is postulated to be perfect in its absorption band. (See appendix A.) The carbon-dioxide radiation chart in figure 3 gives the irradiation due to carbon dioxide from either above or below $\left(G_{CO_2,above}\right.$ or $\left.G_{CO_2,below}\right)$, as a function of the air temperature immediately adjacent to the differential area. Thus, when the temperature of the air is known as a function of the altitude, this chart can be used to obtain $G_{CO_2,above}$ and $G_{CO_2,below}$ at any altitude.

The ozone component is neglected below altitudes of 40,000 feet from whence the ozone radiation chart (fig. 5) is used. This chart gives $G_{O_3,above}$ and $G_{O_3,below}$ as a function of temperature. Ozone radiation becomes important at high altitudes because relatively large quantities of the gas are found there (water-vapor and carbon-dioxide content are small). Ozone has two absorption bands; one extends from $\nu = 650$ to 750 centimeters^{-1} and the other from $\nu = 975$ to 1150 centimeters^{-1}. The ozone absorption band in the region $\nu = 650$ to 750 centimeters^{-1} coincides with one of the absorption bands of carbon dioxide which has been postulated an ideal radiator so that radiation occurring in the region has already been taken care of. The spectral emissivity of the absorption band in the region from $\nu = 975$ to 1150 centimeters^{-1} will be postulated to be $\epsilon_\nu = 1$. The method of obtaining ozone radiation upon the area dA at any altitude is the same as that for carbon dioxide.

Radiation from the earth.- The earth component reaching the horizontal area is obtained directly from the earth radiation chart in figure 4 when the temperature of the earth and $h'_0(z_p)$ are known.

Thus the irradiation upon a differential area at any altitude due to water vapor, carbon dioxide, ozone, and the earth is obtained from the following relations:

$$G_{above} = G_{H_2O,above} + G_{CO_2,above} + G_{O_3,above} \tag{4}$$

$$G_{below} = G_{H_2O,below} + G_{CO_2,below} + G_{earth} + G_{O_3,below} \tag{5}$$

The determination of the nocturnal irradiation may be simplified by a system of tabulation. See the example given in the following section.

APPLICATION

The nocturnal irradiation upon a horizontal area as a function of altitude is obtained by the previously mentioned method for three different sets of meteorological data. Radiosonde data, furnished by the Weather Bureau of the United States Department of Commerce, were obtained for (1) a clear mild night at Dallas, Texas; (2) a clear, hot, and moist night at Phoenix, Arizona; and (3) a clear, cold, and dry night at Barrow, Alaska.

The calculations for the second set of meteorological data (for clear, hot, and moist weather) are presented here in detail to illustrate the method of calculation. For all three atmospheric conditions the radiosonde data and the results are tabulated and shown graphically. For the clear moist night of Phoenix, Arizona, results were also obtained by using Möller and Mügge's radiation chart, Elsasser's radiation chart, and Ashburn's radiation chart. The results and comparison are discussed in appendix B.

Following the method of calculation previously described, the nocturnal irradiation as a function of altitude was calculated as follows for a clear, hot, and moist night at Phoenix, Arizona.

(1) Tabulate the radiosonde data and then the corresponding values of z in feet, P, p_{sat} (see tables II and III), and p (all in lb/sq in. abs.); ρ in grams per cubic centimeter (note equation (2)); the function $\frac{P}{14.73}\sqrt{\frac{300}{T}}$ (see fig. 6); and, finally, $\rho\frac{P}{14.73}\sqrt{\frac{300}{T}}$ as shown in table IV.

(2) Plot the function $\rho\frac{P}{14.73}\sqrt{\frac{300}{T}}$ against z in centimeters (fig. 7).

(3) Using Simpson's rule, obtain $h_0'(z)$, defined by equation (1) as a function of altitude in the manner shown in table V.

(4) Plot h_0' (cm) and T (°C) as a function of z (ft). (See fig. 8.) When values of h_0' (cm) and T (°C) as a function of z (ft) are known, the irradiation upon a differential area at any altitude due to water vapor (in the spectral regions, $\nu = 0$ to 300 cm^{-1} and 1200 to ∞ cm^{-1}, or $\lambda = \infty$ to 33.4 microns and 8.34 to 0 microns), carbon dioxide, and the earth may be obtained directly from radiation charts II, III, and IV (figs. 2 to 4). For the irradiation at any altitude due to water vapor in the regions, $\nu = 300$ to 584 centimeters^{-1} and 752 to 1200 centimeters^{-1} ($\lambda = 33.3$ to 17.1 and 13.3 to 8.34 microns), the following procedure is convenient.

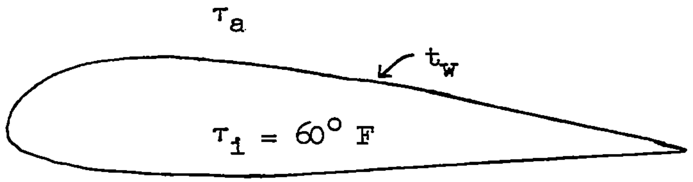
(5) Tabulate the function h_{z_p}', obtained by equation (3), as shown in table VI.

(6) Plot the values of h_{z_p}' and T (°C) of this table onto radiation chart I (fig. 1). The areas under the curves yield the last radiation component, G_{H_2O}', as a function of altitude; a typical example of radiation chart I is exhibited in figure 9 where $G_{H_2O, above}'$ and $G_{H_2O, below}'$ for $z_p = 5000$ feet are the areas under the two curves. The areas are easily obtained by Simpson's rule. The results of these calculations for the nocturnal irradiation as a function of altitude are shown in table VII and figures 10 and 11. Similar calculations were made for a clear, cold, and dry night at Barrow, Alaska, and for a clear and mild night at Dallas, Texas. The radiosonde data and the corresponding nocturnal irradiation for these cases as a function of altitude are shown in tables VIII to XI and in figures 12 and 13.

DISCUSSION

Figures 11 to 13 demonstrate that nocturnal irradiation from above and below upon a horizontal differential area varies greatly with altitude. For example, the nocturnal irradiation from the gases above varies from 113 Btu/(hr)(sq ft) at the ground to 25 Btu/(hr)(sq ft) at an altitude of 30,000 feet (meteorological data II). These values may be compared with the solar radiation when the sun is at the zenith which may be 200 Btu/(hr)(sq ft). If more accurate heat-rate balances can be made by considering the effect of nocturnal irradiation, a better estimate of the heating requirement of an airplane will result. The following illustration serves as an example only.

Consider an airplane flying in Alaska near the ground. Using the meteorological data III, a heat-rate balance can be written for the upper surface of the airplane wing, both excluding and considering radiation. It is desired to calculate the surface temperature and the heat loss per unit area when the temperature inside the wing is maintained at 60° F;



The following are the postulated conditions:

z	altitude taken near ground
τ_a	outside air temperature, -12° F
τ_i	air temperature inside wing, 60° F
t_w	upper-wing-surface temperature
f_{c_a}	unit thermal convective conductance between wing and outside air, 10 Btu/(hr)(sq ft)($^{\circ}$F)
f_{c_i}	unit thermal convective conductance between wing and inside air, 1 Btu/(hr)(sq ft)($^{\circ}$F)
G_{above}	nocturnal irradiation from above, 37 Btu/(hr)(sq ft)
ϵ_w	wing-surface gray-body emissivity, 0.5

The heat-rate balance excluding atmospheric radiation is (see reference 7)

$$\frac{q}{A} = f_{c_i}\left(\tau_i - t_w\right) = f_{c_a}\left(t_w - \tau_a\right)$$

$$1\left(60 - t_w\right) = 10\left(t_w + 12\right)$$

$$t_w = -5.45^\circ \text{ F}$$

$$\frac{q}{A} = 65.5 \text{ Btu/(hr)(sq ft)}$$

The heat-rate balance considering atmospheric radiation is

$$\frac{q}{A} = f_{c_i}\left(\tau_i - t_w\right) = f_{c_a}\left(t_w - \tau_a\right) + \epsilon_w \sigma \left(t_w + 460\right)^4 - \epsilon_w G_{above}$$

$$1\left(60 - t_w\right) = 10\left(t_w + 12\right) + 0.5 \times 17.3 \times 10^{-10}\left(t_w + 460\right)^4 - 0.5(37)$$

$$t_w = -7.5^\circ \text{ F}$$

$$\frac{q}{A} = 67.5 \text{ Btu/(hr)(sq ft)}$$

The difference in q/A amounts to 3 percent in this case.

The relative humidity for the preceding data was very high (90 percent). If the data had been taken during a dry day, G_{above} would have been much less. Also some areas of the airplane may have unit thermal convective conductances less than 10 Btu/(hr)(sq ft)(°F) depending upon the location of these areas and the air velocities over them. Thus for these two conditions the deviation between heat loss, considering and excluding nocturnal radiation, may exceed 3 percent.

Radiation may be a very important component in making a heat-rate balance upon an airplane standing on the ground. The limiting values of the skin temperature may be determined by the methods shown in this report for night and day conditions (when the sun is shining).

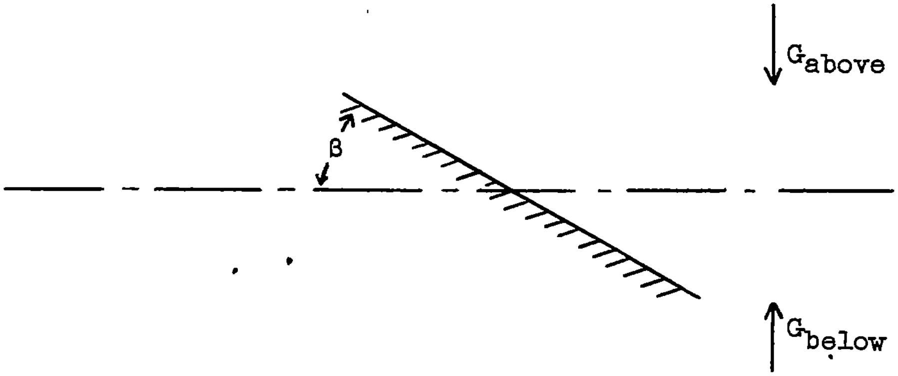
Figures 14 and 15 show a comparison of nocturnal irradiation as a function of altitude by the various radiation charts found in the literature. (See appendix B.) The results agree fairly well with those obtained in this report. The discrepancies may be due to the following considerations:

(1) Möller and Mügge did not have the latest water-vapor absorption data available and neglected carbon-dioxide radiation.

(2) Elsasser's water-vapor absorption data are essentially the same as those used in this report. However Elsasser, in obtaining the filtered earth radiation, used the artifice of postulating that the earth was an infinitely thick layer of water vapor at the temperature of the earth, as did Möller and Mügge and Ashburn.

(3) Ashburn's method differs from the one used in the present report in that Hottel and Mangelsdorf's water-vapor absorption data were used.

For areas other than horizontal the following values for nocturnal irradiation are suggested:



$$G_\beta = \left(\frac{1 + \cos\beta}{2}\right)G_{above} + \left(\frac{1 - \cos\beta}{2}\right)G_{below} \qquad (6)$$

Further considerations of irradiation on areas that are not horizontal can be obtained in reference 8.

CONCLUDING REMARKS

A study was made of nocturnal irradiation as a function of altitude with the following results:

1. The atmospheric-radiation charts obtained may be used to evaluate a more complete heat-rate balance on airplanes.

2. It was found that nocturnal irradiation from above and below decreases greatly as a function of altitude. The nocturnal-irradiation results for several sets of meteorological data are presented to establish possible limits of nocturnal irradiation.

Department of Engineering
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Berkeley, Calif., October 10, 1944

APPENDIX A

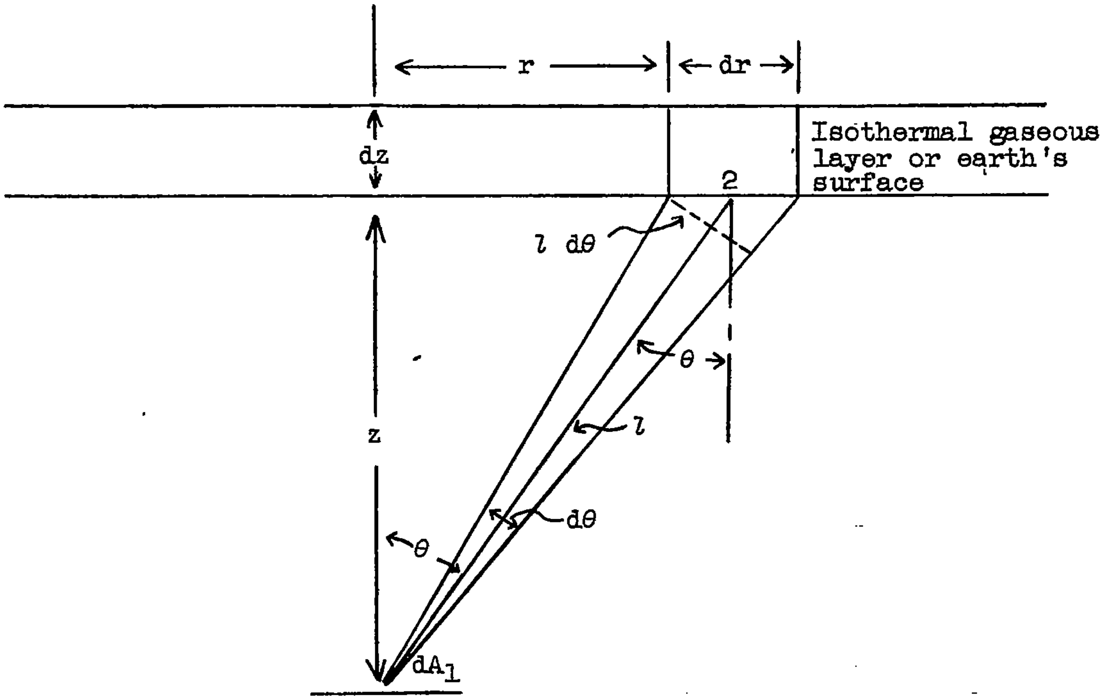
NOCTURNAL RADIATION CHARTS

For the determination of the atmospheric nocturnal irradiation as a function of altitude, five charts have been prepared. The principles upon which the charts are based are as follows: The atmosphere is postulated to be composed of uniform layers at constant temperature and of constant water-vapor and carbon-dioxide content. Ozone has been considered previously in the section entitled "Carbon-dioxide and ozone radiation." The emissive power of each isothermal layer is defined by the relation,

$$E = \int_0^\infty E_\nu \, d\nu = \int_0^\infty \epsilon_\nu B_\nu(T) \, d\nu \tag{A1}$$

Upon examining the absorption spectrum of the radiating gases in the atmosphere (water vapor and carbon dioxide), very strong absorption bands are seen to exist in various regions. In the regions $\nu = 0$ to 300 centimeters^{-1} and $\nu = 1200$ to ∞ centimeters^{-1}, water vapor is postulated to radiate as a perfect radiator; that is ϵ_ν the spectral emissivity, is unity. (See Andersen's report (reference 4) for the justification of this postulate.) Carbon dioxide has a very strong absorption band at the region $\nu = 584$ to 752 centimeters^{-1} and is also postulated to radiate as a perfect radiator. (See reference 3.) Thus ϵ_ν the spectral emissivity of the isothermal layers is also unity for $\nu = 584$ to 752 centimeters^{-1}. In the regions $\nu = 300$ to 584 centimeters^{-1} and $\nu = 752$ to 1200 centimeters^{-1}, water vapor is postulated to have the absorption characteristics in which $K_\nu = K_{0,\nu} \frac{P}{14.73}\sqrt{\frac{300}{T}}$ suggested by Elsasser (references 9 and 10) from the examination of experimental data. The spectral emissivity ϵ_ν in these regions is $K_{0,\nu}\, dh'$. (See the following paragraphs for the definitions of K_ν and $K_{0,\nu}$.) The spectral emissivity ϵ_ν of the earth's surface is also postulated to be unity.

The irradiation upon a differential area from either the earth's surface or a layer of gas with a nonabsorbing intervening medium may be found from the following considerations.



In the preceding diagram the monochromatic radiant energy incident upon dA_1 originating from surface 2 is, by the definition of $i_{2,\nu}$ (see reference 8)

$$d^3q = i_{2,\nu} \cos\theta \, dA_1 \, d\Omega \, d\nu$$

or

$$d^3q = \frac{i_{2,\nu} \cos\theta \cos\theta \, dA_1 \, dA_2 \, d\nu}{l^2} \tag{A2}$$

where

$$d\Omega = \frac{dA_2}{l^2} \cos\theta$$

Since

$$dA_2 = 2\pi r\ dr$$

$$r = l \sin \theta$$

$$\frac{l\ d\theta}{dr} = \cos \theta$$

and since the gaseous layers and the earth's surface are diffuse surfaces, or

$$i_{2,\nu} = \frac{E^{o}_{2,\nu}}{\pi}$$

equation (A2) becomes

$$d^3q_{2\longrightarrow 1} = 2E^{o}_{2,\nu} \cos \theta \sin \theta\ d\theta\ dA_1\ d\nu \tag{A3}$$

If the intervening space contains an absorbing medium, then by Beer's law,

$$E_{2,\nu} = E^{o}_{2,\nu}e^{-\int K(\nu,\rho,P,T)\ dl} \tag{A4}$$

where $K(\nu,\rho,P,T)$ is the absorption coefficient defined by equation (A4), the radiant energy originating from surface 2 incident upon the differential area dA_1 is

$$d^3q_{2\longrightarrow 1} = 2E_{2,\nu} \cos \theta \sin \theta\ d\theta\ dA_1\ d\nu$$

or

$$d^3q_{2\longrightarrow 1} = 2E^{\circ}_{2,\nu}\, e^{-\int K(\nu,\rho,P,T)\, dl} \cos\theta \sin\theta\, d\theta\, dA_1\, d\nu \qquad \text{(A5)}$$

Since the atmosphere is postulated to act as a black body except in the spectral regions of ν = 300 to 584 centimeters^{-1} and ν = 752 to 1200 centimeters^{-1}, information concerning the absorption coefficient $K(\nu,\rho,P,T)$ is needed only in these wave numbers. Data on the absorption coefficient $K(\nu,\rho,P,T)$ of water vapor in these spectral regions as a function of wavelength, concentrations, pressure, and temperature were analyzed by Elsasser (references 9 and 10). He proposed the following equations:

$$K(\nu,\rho,P,T)\, dl = K_{0,\nu} \frac{P}{14.73}\sqrt{\frac{300}{T}}\, \frac{dh}{\cos\theta} = K_{0,\nu} \frac{dh'}{\cos\theta} \qquad \text{(A6)}$$

where

$$K_{0,\nu} = \frac{75,000}{(\nu - 200)^2} \qquad \text{(A7)}$$

and

$$dh' = \frac{P}{14.73}\sqrt{\frac{300}{T}}\, dh = \frac{P}{14.73}\sqrt{\frac{300}{T}}\, \rho\, dz = \frac{P}{14.73}\sqrt{\frac{300}{T}}\, \rho \cos\theta\, dl \qquad \text{(A8)}$$

When these empirical equations for the absorption coefficient of water vapor in the spectral regions of ν = 300 to 584 centimeters^{-1} and ν = 752 to 1200 centimeters^{-1} are used and the postulate that the atmosphere acts as a black body in the remainder of the spectrum is employed, equation (A5) may be easily modified to give the nocturnal irradiation upon a differential area as a function of altitude with the radiant energy originating either from the gases in the atmosphere or from the earth's surface.

In order to obtain the nocturnal irradiation due to the gases only (from above or below), equation (A5) is modified and integrated as follows:

$$G_{gases\rightarrow 1} = \frac{dq_{\epsilon}(2\rightarrow 1)}{dA_1} = \int_0^{300} B_\nu\left(T_1\right) d\nu$$

$$+ \int_{h'_{z_p}=0}^{h'_{z_p}} \int_{\nu=300}^{584} \int_{\theta=0}^{\pi/2} 2B_{2,\nu}(T)K_{0,\nu}e^{-\frac{1}{\cos\theta}\int K_{0,\nu}\, dh'_{z_p}} \sin\theta \, d\theta \, d\nu \, dh'_{z_p}$$

$$+ \int_{584}^{752} B_\nu\left(T_1\right) d\nu + \int_0^{h'_{z_p}} \int_{\nu=752}^{1200} \int_{\theta=0}^{\pi/2} 2B_{2,\nu}(T)K_{0,\nu}e^{-\frac{1}{\cos\theta}\int K_{0,\nu}\, dh_{z_p}} \sin\theta \, d\theta \, d\nu \, dh'_{z_p}$$

$$+ \int_{1200}^{\infty} B_\nu\left(T_1\right) d\nu \qquad \text{(A9)}$$

When

$$\frac{dG'_{H_2O}}{dh'_{z_p}} = \int_{\nu=300}^{584} \int_{\theta=0}^{\pi/2} 2B_\nu(T)K_{0,\nu} e^{-\frac{1}{\cos\theta}\int K_{0,\nu}\, dh'_{z_p}} \sin\theta \, d\theta \, d\nu$$

$$+ \int_{\nu=752}^{1200} \int_{\theta=0}^{\pi/2} 2B_\nu(T)K_{0,\nu} e^{-\frac{1}{\cos\theta}\int K_{0,\nu}\, dh'_{z_p}} \sin\theta \, d\theta \, d\nu \tag{A10}$$

equation (A9) becomes

$$G_{gases\rightarrow 1} = \frac{dq_{\epsilon(2\rightarrow 1)}}{dA_1} = \int_0^{300} B_\nu\left(T_1\right) d\nu + \int_0^{h'_{z_p}} \frac{dG'_{H_2O}}{dh'_{z_p}} dh'_{z_p} + \int_{584}^{752} B_\nu\left(T_1\right) d\nu$$

$$+ \int_{1200}^{\infty} B_\nu\left(T_1\right) d\nu \tag{A11}$$

In order to obtain the nocturnal irradiation upon dA_1 at altitudes above the ground due to the earth, equation (A5) is modified and integrated as follows:

$$G_{earth \rightarrow 1} = \frac{dq_{2 \rightarrow 1}}{dA_1} = \int_{\theta=0}^{\pi/2} \int_{\nu=300}^{584} E_{earth} 2 \sin \theta \cos \theta \; e^{-\frac{1}{\cos \theta} \int K_{0,\nu} \, dh'_{z_p}} \, d\theta \, d\nu$$

$$+ \int_{\theta=0}^{\pi/2} \int_{\nu=752}^{1200} E_{earth} 2 \sin \theta \cos \theta \; e^{-\frac{1}{\cos \theta} \int K_{0,\nu} \, dh'_{z_p}} \, d\theta \, d\nu \qquad \text{(A12)}$$

where $E_{earth} = \sigma T^4_{earth}$, and ϵ_{earth} is postulated as unity. The radiant energy leaving the earth at the other wave numbers is immediately absorbed by the water vapor and carbon dioxide, for these gases act as a black body at these wave numbers.

Of the five radiation charts mentioned, only four are used to obtain the nocturnal irradiation for altitudes up to 40,000 feet.

Radiation chart I.- The ordinates and abscissas of chart I (fig. 1) are $\frac{dG'_{H_2O}}{dh'_{z_p}}$ and h'_{z_p}, where $\frac{dG'_{H_2O}}{dh'_{z_p}}$ is defined by equation (A10). The curves shown are the relations between $\frac{dG'_{H_2O}}{dh'_{z_p}}$ and h'_{z_p} at constant temperature or $\left(\frac{dG'_{H_2O}}{dh'_{z_p}}\right)_T$ against h'_{z_p}. Thus by plotting the variation of T (°C) and h'_{z_p} with altitude of a particular atmosphere, the chart yields the relation $\frac{dG'_{H_2O}}{dh'_{z_p}}$ against h'_{z_p} for the system. The area under the curve G'_{H_2O} gives the nocturnal irradiation due to the water vapor in the atmosphere in the spectral regions of $\nu = 300$ to 584 centimeters^{-1} and $\nu = 752$ to 1200 centimeters^{-1}, which is the sum of the second and third integral of equation (A9) or the second integral of equation (A11).

Radiation chart II.- Chart II (fig. 2) gives the relation of

$$\int_0^{300} B_\nu(T)\, d\nu + \int_{1200}^{\infty} B_\nu(T)\, d\nu = G''_{H_2O} \text{ as a function of } T.$$

Therefore, by knowing the temperature adjacent to the differential area dA_1 the nocturnal irradiation from the atmosphere in these spectral regions is immediately obtained.

Radiation chart III.- Chart III (fig. 3) gives the carbon-dioxide contribution $G_{CO_2} = \int_{584}^{752} B_\nu(T)\, d\nu$, as a function of temperature. (See equations (A9) and (A11).)

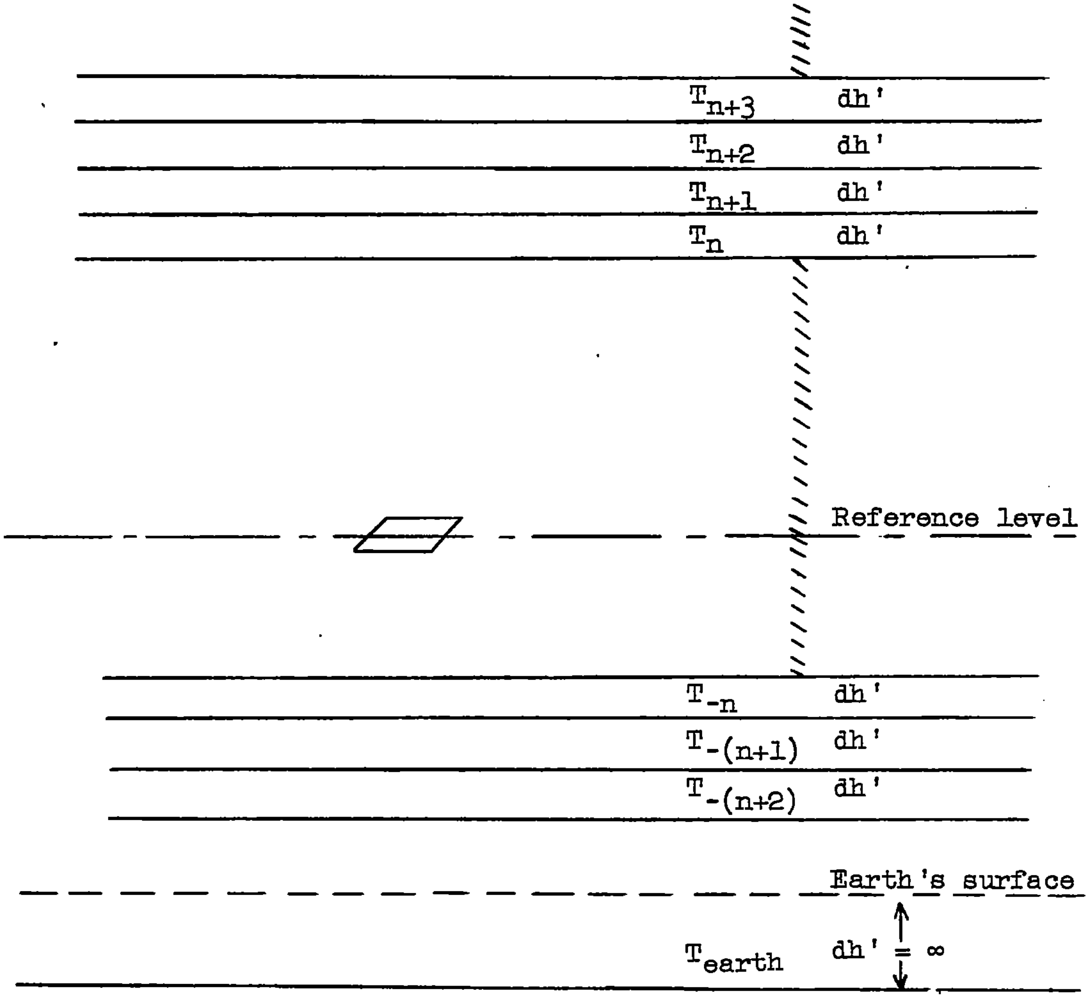
Radiation chart IV.- Chart IV (fig. 4) gives the relation of $G_{earth \longrightarrow 1}$ (see equation (A12)) as a function of T_{earth} and $h'_0(z_p)$. Thus by knowing the latter two variables, the earth's radiation contribution is easily obtained. This chart does not give the correct value of G_{earth} for $h'_0(z_p) = 0$ (equation (A12)). The value of G_{earth} for $h'_0(z_p) = 0$ is σT^4_{earth} (emissive power of the earth) and is equal to the sum of $G''_{H_2O}(T_{earth})$, $G_{CO_2}(T_{earth})$, and $G_{earth \longrightarrow 1}$ as obtained from radiation chart IV.

Radiation chart V.- This chart (fig. 5) is discussed in the section entitled "Carbon-dioxide and ozone radiation."

APPENDIX B

A DISCUSSION AND COMPARISON WITH PREVIOUS METHODS OF OBTAINING NOCTURNAL ATMOSPHERIC RADIATION

Möller and Mügge (reference 2) developed a radiation chart for the following ideal system. The radiating gas in the atmosphere is water vapor only. The atmosphere is a series of isothermal and constant-vapor-content differential layers, and the earth is replaced by a layer of gas of infinite thickness. The fundamental equations of Möller and Mügge for gaseous radiation in the atmosphere and their methods of integration are the same as those presented in appendix A



except for the absorptivity data. Although they recognized the equation for heat transfer between two flat surfaces with an absorbing medium, they developed a radiation chart for this system which considers the earth as an infinitely thick layer of water vapor.

Elsasser (reference 3) developed a similar radiation chart using more reliable recent water-vapor absorption data, and accounted for the carbon-dioxide radiation by means of the postulate that the carbon-dioxide radiation acts as a black body at certain wave numbers.

Andersen (reference 4) presented a radiation chart for the nocturnal atmospheric radiation incident upon the ground which considers water vapor only. This chart will also yield the water-vapor radiation upon a surface at levels above the ground, but it will not give the earth component filtering through the water vapor.

Ashburn's radiation chart (reference 5) is also similar to that of Möller and Mügge. Ashburn's method differs from Möller and Mügge's and Elsasser's methods in his choice of water-vapor-absorption data and his method of incorporating the carbon-dioxide contribution to the radiation. Ashburn used Hottel and Mangelsdorf's water-vapor-absorption data (reference 11).

The methods of Möller and Mügge, Elsasser, and Ashburn were used to calculate the nocturnal radiation for the aforementioned atmospheric condition for a clear, hot, and moist day in Arizona. These results are presented in table III and are compared with the method presented in this report in figures 14 and 15. In the calculations of nocturnal irradiation employing the charts of Möller and Mügge, Elsasser, and Ashburn, the function h_0' is obtained as suggested by Elsasser in reference 3 where the water-vapor-absorption correction $\sqrt{\frac{P}{1000}}$ is used instead of $\frac{P}{14.73}\sqrt{\frac{300}{T}}$. Thus

$$dh_0' = \rho\sqrt{\frac{P}{1000}}\,dz = \frac{w}{g}\sqrt{\frac{P}{1000}}\,dP$$

where

h_0' effective moisture content (see equation (1)), centimeters

g acceleration of gravity, ~ 1000 centimeters per second^2

w ratio of water vapor to air density from radiosonde data, grams per kilogram

A comparison of h_0' as a function of altitude calculated by Elsasser's and Andersen's methods is given in figure 16.

APPENDIX C

SYMBOLS

A	area of heat transfer perpendicular to direction of heat flow, sq ft
A_1	area of irradiated surface, sq ft
A_2	area of radiating surface, sq ft
$B_\nu(T)$	monochromatic black-body emissive power, Btu/(hr)(sq ft)(cm^{-1})
$B_{2,\nu}(T)$	unattenuated monochromatic black-body emissive power of area A_2, Btu/(hr)(sq ft)(cm^{-1})
c	speed of light, 2.998×10^{10} cm/sec
E	total emissive power, Btu/(hr)(sq ft)
E_{earth}	emissive power of earth, Btu/(hr)(sq ft)
E_ν	monochromatic emissive power, Btu/(hr)(sq ft)(cm^{-1})
$E^o_{2,\nu}$	monochromatic emissive power of area A_2, Btu/(hr)(sq ft)(cm^{-1})
$E'_{2,\nu}$	unattenuated monochromatic emissive power of area A_2, Btu/(hr)(sq ft)(cm^{-1})
f	frequency of vibration, cps
f_{c_a}	unit thermal convective conductance between wing and outside air, Btu/(hr)(sq ft)(°F)
f_{c_i}	unit thermal convective conductance between wing and inside air, Btu/(hr)(sq ft)(°F)
G	total nocturnal irradiation, Btu/(hr)(sq ft)
G_{CO_2}	irradiation from carbon dioxide in atmosphere, $\left(\int_{584}^{752} B_\nu(T)\,d\nu\right.$, Btu/(hr)(sq ft)$\left.\right)$
$G_{earth \longrightarrow 1} = G_{earth}$	irradiation from earth reaching area A_1, Btu/(hr)(sq ft)

$G_{gases \longrightarrow 1} = G_{gas}$	irradiation from atmospheric gases reaching area A_1, Btu/(hr)(sq ft)
G_{H_2O}	total irradiation from water vapor in atmosphere reaching area A_1, Btu/(hr)(sq ft)
G'_{H_2O}	irradiation from atmospheric water vapor in spectral regions of $\nu = 300$ to 584 cm^{-1} and 752 to 1200 cm^{-1}, Btu/(hr)(sq ft)
G''_{H_2O}	irradiation from atmospheric water vapor in spectral regions of $\nu = 0$ to 300 cm^{-1} and 1200 to $\infty \text{ cm}^{-1}$, Btu/(hr)(sq ft)
G_{O_3}	irradiation from ozone in atmosphere, Btu/(hr)(sq ft)

A subscript "below" or "above" is appended to all these G's to indicate whether the energy irradiated upon the horizontal area A_1 is coming from below or from above the area.

G_β	irradiation defined by equation (6), Btu/(hr)(sq ft)
g	acceleration of gravity, cm/sec^2
h	height expressed in terms of moisture content of atmosphere, cm precipitable water
h'	effective moisture content of atmosphere, cm
h'_0	effective moisture content; subscript zero indicates that the reference altitude is at the ground, cm
h'_{z_p}	effective moisture content; reference altitude is z_p, cm
$i_{2,\nu}$	monochromatic surface intensity of emission of area A_2, Btu/(hr)(sq ft)(steradian)(cm^{-1})
$K(\nu,\rho,T,P)$	absorption coefficient defined by equation (A4), cm^{-1}
$K_{0,\nu}$	absorption coefficient defined by equation (A7), cm^{-1}
l	distance, cm and ft
P	atmospheric pressure, mb and lb/sq in.
p	partial pressure of water vapor, lb/sq in.

p_{sat}	saturated pressure of water vapor, lb/sq in.
q	rate of heat transfer, Btu/hr
r	radius, cm and ft
T	temperature, °C and °K
T_{earth}	earth temperature, °C
t_w	temperature of wing surface, °F
w	ratio of water vapor to air density, grams/kg
z	altitude, cm, m, and ft
z_p	reference plane or altitude of horizontal area A_1, cm, m, and ft
α_ν	monochromatic absorptivity
β	angle, radians
ϵ_{earth}	mean emissivity of surface of earth
ϵ_w	wing-surface gray-body emissivity
ϵ_ν	monochromatic emissivity
θ	angle, radians
λ	wavelength, microns
ν	wave numbers, cm^{-1}
ρ	water-vapor density, $gram/cm^3$
ρ_s	density of water at atmospheric pressure and temperature of 4° C, 1 $gram/cm^3$
σ	Stefan-Boltzmann's radiation constant, Btu/(hr)(sq ft)($^\circ R^4$)
τ_a, τ_i	air temperature outside and inside wing, °F
Ω	solid angle, steradians

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TABLE I

POSTULATED ABSORPTION CHARACTERISTICS OF THE ATMOSPHERE

Wave number, ν (cm^{-1})	Wavelength, λ (microns)	$\alpha_\nu = \epsilon_\nu$		
		Water vapor	Carbon dioxide	Ozone
0 to 300	∞ to 33.4	1	0	0
300 to 584	33.4 to 17.1	$\epsilon_\nu =$ [a]$K_{0,\nu}\ dh'$	0	0
584 to 752	17.1 to 13.3	0	1	0
752 to 975	13.3 to 10.25	$\epsilon_\nu = K_{0,\nu}\ dh'$	0	0
975 to 1150	10.25 to 8.7	$\epsilon_\nu = K_{0,\nu}\ dh'$	0	[b]1
1150 to 1200	8.7 to 8.34	$\epsilon_\nu = K_{0,\nu}\ dh'$	0	0
1200 to ∞	8.34 to 0	1	0	0

[a] See appendix A for definition of $K_{0,\nu}\ dh'$.

[b] As indicated in the text, at high altitudes ozone becomes important. At high altitude G_{above} will be the sum of $G_{O_3,above}$, $G_{CO_2,above}$, and $G''_{H_2O,above}$.

TABLE II

SATURATION PRESSURE OF WATER VAPOR AS A FUNCTION OF TEMPERATURE

[See fig. 17 and references 12 and 13.]

Temperature (°F)	Temperature (°C)	Vapor pressure (in. Hg)	Vapor pressure (lb/sq in. abs.)
-60	-51.11	0.00101	0.0005076
-55	-48.33	.00143	.000718
-50	-45.56	.00200	.001005
-45	-42.78	.00277	.001393
-40	-40.00	.00380	.001912
-35	-37.22	.00520	.002615
-30	-34.44	.00701	.003525
-25	-31.67	.00946	.00476
-20	-28.89	.0126	.00633
-15	-26.11	.0168	.00845
-10	-23.33	.0221	.0112
-5	-20.56	.0290	.01458
0	-17.78	.0377	.01896
5	-15.00	.0489	.0246
10	-12.22	.0630	.0317
15	-9.44	.0807	.0406
20	-6.67	.1028	.0517
25	-3.89	.1304	.0656
30	-1.11	.1645	.0827
---	0	-------	.0886
---	5	-------	.1265
---	10	-------	.1780
---	15	-------	.2471
---	20	-------	.3386
---	25	-------	.4581
---	30	-------	.6132
---	35	-------	.8126
---	40	-------	1.0661

TABLE III

NOCTURNAL IRRADIATION FOR METEOROLOGICAL DATA II BY EXISTING METHODS

[See figs. 14 and 15.]

Altitude (ft)	G_{above} (Btu/(hr)(sq ft))	$G_{gas,below}$ (Btu/(hr)(sq ft))	G_{earth} (Btu/(hr)(sq ft))	G_{below} (Btu/(hr)(sq ft))
With Möller and Mügge's chart (reference 2)				
0	131.0	0	169	169
3,000	117.8	114	44.9	158.9
5,000	106.5	113.2	38.2	151.4
10,000	80.3	112.8	33.7	146.5
15,000	60.5	103.0	32.6	135.6
20,000	42.9	95.1	30.6	125.6
30,000	-----	85.3	27.7	113.0
With Elsasser's chart (reference 3)				
0	106.2	0	155.5	155.5
3,000	92.8	90.5	63.4	153.9
5,000	83.2	94.2	56.6	150.8
10,000	64.0	93.2	48.4	141.6
15,000	47.9	87.9	44.6	132.5
20,000	36.3	82.4	43.3	125.7
30,000	-----	73.2	42.4	115.6
With Ashburn's chart (reference 5)				
0	117.7	0	153.2	153.2
3,000	109.3	102.6	48.0	150.6
5,000	100.0	102.3	44.1	146.4
10,000	83.2	92.4	41.5	133.8
15,000	66.5	81.2	40.5	121.7
20,000	52.0	71.5	40.2	111.7
30,000	-----	56.5	39.2	95.7

TABLE IV

RADIOSONDE DATA AND CALCULATION OF $\rho \frac{P}{14.73}\sqrt{\frac{300}{T}}$ FOR METEOROLOGICAL DATA II (MOIST ATMOSPHERE)

[See fig. 7.]

Radiosonde data				Calculations of $\rho \frac{P}{14.73}\sqrt{\frac{300}{T}}$						
Altitude, z (m)	Temperature, T (°C)	Pressure, P (mb)	Relative humidity, (percent)	Altitude, z (ft)	Pressure, P (lb/sq in. abs.)	Saturated vapor pressure, p_{sat} (lb/sq in. abs.)	Partial vapor pressure, p (lb/sq in. abs.)	Vapor density (gram/cm^3)	$\frac{P}{14.73}\sqrt{\frac{300}{T}}$	$\rho \frac{P}{14.73}\sqrt{\frac{300}{T}}$ (gram/cm^3)
339	30.5	969	22	1,110	14.03	0.635	0.140	0.0690×10^{-4}	0.951	0.0657×10^{-4}
760	34.0	926	13	2,490	13.42	.767	.100	.0488	.906	.0443
1,670	26.1	835	13	5,480	12.10	.491	.0638	.0319	.827	.0264
2,700	16.2	741	16	8,860	10.73	.275	.0440	.0236	.743	.01756
3,390	12.8	682	15	11,100	9.89	.214	.0321	.0168	.688	.01158
4,320	5.0	610	17	14,160	8.84	.127	.0216	.0116	.625	.00725
4,910	3.5	567	18	16,100	8.22	.108	.0194	.0105	.581	.00610
5,650	-2.9	517	20	18,510	7.50	.0674	.0135	.00748	.537	.00402
6,400	-9.0	470	22	21,000	6.81	.0417	.00917	.00520	.492	.00256
7,200	-15.0	423	25	23,600	6.12	.0240	.00600	.00348	.450	.001567
7,620	-17.1	400	26	25,000	5.80	.0216	.00561	.00328	.429	.001406
9,240	-30.2	319	37	30,300	4.63	.00510	.00189	.00116	.351	.000407
11,000	-44.1	248	--	36,100	3.60	-------	-------	-------------	-----	-------------
12,830	-55.6	188	--	42,100	2.71	-------	-------	-------------	-----	-------------
15,110	-67.8	130	--	49,500	1.89	-------	-------	-------------	-----	-------------

TABLE V

THE FUNCTION h_0' AGAINST z FOR METEOROLOGICAL DATA II (MOIST ATMOSPHERE)

[See fig. 8.]

Altitude, z (cm)	$\rho \frac{P}{14.73}\sqrt{\frac{300}{T}}$	h_0' (cm) (a)	Altitude, z (ft)
0 × 10^3	[b]11.00 × 10^{-6}	0	0
40	5.80		
80	4.05	.51	2,625
120	3.24		
160	2.72	.774	5,249
200	2.32		
240	1.93	.96	7,874
280	1.75		
320	1.43	1.098	10,500
360	1.20		
400	1.00	1.193	13,130
440	.80		
480	.64	1.258	15,750
520	.50		
560	.42	1.300	18,370
600	.35		
640	.28	1.328	21,000
680	.23		
720	.18	1.345	23,620
760	.15		
800	.11	1.357	26,250
840	.08		
880	.06	1.363	28,870
920	.05		
960	.05	1.368	31,500

[a] Simpson's rule gives area for even intervals only.

[b] By extrapolation.

TABLE VI

THE FUNCTION h'_{z_p} AGAINST z FOR METEOROLOGICAL DATA II (MOIST ATMOSPHERE)

Altitude, z (ft)	Temperature, T (°C)	h'_0 (cm)	$h'_{3,000}$ (cm)	$h'_{5,000}$ (cm)	$h'_{10,000}$ (cm)	$h'_{15,000}$ (cm)	$h'_{20,000}$ (cm)	$h'_{25,000}$ (cm)	$h'_{30,000}$ (cm)
0	33.5	0.00	0.550	0.752	1.074	1.240	1.306	1.350	1.366
1,000	32.5	.266	.284	.468	.808	.974	1.040	1.084	1.100
2,000	30.2	.425	.125	.327	.649	.815	.881	.925	.941
3,000	29.9	.550	.00	.202	.524	.690	.756	.850	.816
4,000	28.8	.660	.110	.092	.414	.580	.646	.690	.706
5,000	27.0	.752	.202	.00	.322	.488	.554	.598	.614
6,000	24.8	.835	.285	.083	.239	.405	.471	.515	.531
7,000	22.5	.902	.352	.152	.172	.338	.404	.448	.464
8,000	20.0	.968	.418	.216	.106	.272	.338	.382	.398
9,000	18.0	1.023	.473	.271	.051	.217	.283	.327	.343
10,000	16.0	1.074	.524	.322	.00	.166	.232	.276	.292
11,000	14.0	1.118	.568	.366	.044	.122	.188	.232	.248
12,000	12.0	1.160	.610	.408	.086	.080	.146	.190	.206
13,000	8.8	1.190	.640	.438	.116	.050	.106	.160	.176
14,000	7.5	1.220	.670	.468	.146	.020	.086	.120	.146
15,000	5.0	1.240	.690	.488	.166	.00	.066	.110	.126
16,000	3.0	1.262	.712	.510	.188	.022	.034	.088	.104
17,000	1.0	1.278	.728	.526	.204	.038	.028	.072	.088
18,000	-1.5	1.292	.742	.540	.218	.052	.014	.058	.074
19,000	-4.0	1.306	.756	.554	.232	.066	.00	.044	.060
20,000	-5.5	1.318	.768	.566	.244	.078	.012	.032	.048
21,000	-9.0	1.328	.778	.576	.254	.088	.022	.022	.038
22,000	-11.1	1.335	.785	.583	.261	.095	.029	.015	.031
23,000	-13.5	1.340	.790	.588	.266	.100	.036	.010	.026
24,000	-16.0	1.346	.796	.594	.272	.106	.040	.004	.020
25,000	-18.0	1.350	.800	.598	.276	.110	.044	.00	.016
26,000	-20.5	1.355	.805	.603	.281	.115	.049	.005	.011
27,000	-22.7	1.358	.808	.606	.284	.118	.052	.008	.008
28,000	-25.0	1.360	.810	.608	.286	.120	.054	.010	.006
29,000	-27.5	1.363	.813	.711	.289	.123	.057	.013	.003
30,000	-30.0	1.366	.816	.714	.292	.126	.060	.016	.00

TABLE VII

NOCTURNAL IRRADIATION FOR METEOROLOGICAL DATA II (MOIST ATMOSPHERE)

[See figs. 10 and 11.]

z (ft)	T (°C)	h'_0 (cm)	$G'_{H_2O,above}$ (Btu/(hr)(sq ft))	$G''_{H_2O,above}$ (Btu/(hr)(sq ft))	$G_{CO_2,above}$ (Btu/(hr)(sq ft))	G_{above} (Btu/(hr)(sq ft))
0	33.5	0	47.8	38.3	26.8	112.9
3,000	29.9	.550	36.8	36.0	25.7	98.5
5,000	27.0	.752	30.1	34.5	25.0	89.6
10,000	16.0	1.074	18.1	28.6	22.1	68.8
15,000	5.0	1.240	9.48	23.5	19.3	52.28
20,000	-5.5	1.318	4.64	19.7	16.8	41.14
30,000	-30.0	1.366	0	13.0	11.6	24.6

z (ft)	T (°C)	h'_0 (cm)	$G'_{H_2O,below}$ (Btu/(hr)(sq ft))	$G''_{H_2O,below}$ (Btu/(hr)(sq ft))	$G_{CO_2,below}$ (Btu/(hr)(sq ft))	G_{earth} (Btu/(hr)(sq ft))	G_{below} (Btu/(hr)(sq.ft))
0	33.5	0	0	0	0	154.1	154.1
3,000	29.9	.550	34.4	36.0	25.7	55	151.1
5,000	27.0	.752	38.1	34.5	25.0	48	145.6
10,000	16.0	1.074	42.6	28.6	22.1	35	128.3
15,000	5.0	1.240	43.4	23.5	19.3	27.5	113.7
20,000	-5.5	1.318	42.6	19.7	16.8	22.5	101.6
30,000	-30.0	1.366	42.0	13.0	11.6	19.0	85.6

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TABLE VIII

RADIOSONDE DATA FOR METEOROLOGICAL DATA III (DRY ATMOSPHERE)

Altitude, z (m)	Temperature, T (°C)	Pressure, P (mb)	Relative humidity (percent)
8	-24.4	1024	94
230	-23.5	994	94
360	-19.9	978	84
510	-19.1	958	69
810	-20.1	919	62
1,540	-23.5	832	66
2,310	-27.0	747	92
2,520	-27.0	726	88
3,330	-31.6	648	88
4,160	-35.4	576	88
5,150	-41.5	499	88
6,610	-51.5	400	88
7,540	-56.3	346	88
7,800	-57.1	332	88
8,560	-61.3	294	88
8,970	-62.1	276	88
9,260	-61.6	263	88
9,800	-56.8	241	88
12,090	-50.3	169	88
13,510	-44.5	136	88
14,080	-43.9	125	88

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TABLE IX

NOCTURNAL IRRADIATION FOR METEOROLOGICAL DATA III (DRY ATMOSPHERE)

[See fig. 12.]

z (ft)	T (°C)	h'_0 (cm)	$G'_{H_2O,above}$ (Btu/(hr)(sq ft))	$G''_{H_2O,above}$ (Btu/(hr)(sq ft))	$G_{CO_2,above}$ (Btu/(hr)(sq ft))	G_{above} (Btu/(hr)(sq ft))
0	-24.4	0	9.45	14.3	12.7	36.45
3,000	-20.5	.060	6.94	15.3	13.5	35.74
5,000	-22.7	.087	5.52	14.5	13.0	33.02
10,000	-30.0	.140	2.46	13.0	11.6	27.06
15,000	-38.0	.163	.80	11.3	10.1	22.20
20,000	-48.0	.171	.30	9.6	8.4	18.30
30,000	-62.0	.175	0	7.5	6.3	13.8

z (ft)	T (°C)	h'_0 (cm)	$G'_{H_2O,below}$ (Btu/(hr)(sq ft))	$G''_{H_2O,below}$ (Btu/(hr)(sq ft))	$G_{CO_2,below}$ (Btu/(hr)(sq ft))	G_{earth} (Btu/(hr)(sq ft))	G_{below} (Btu/(hr)(sq ft))
0	-24.4	0	0	0	0	67.0	67.0
3,000	-20.5	.060	4.39	15.3	13.5	39.0	67.8
5,000	-22.7	.087	6.29	14.5	13.0	37.0	64.5
10,000	-30.0	.140	8.34	13.0	11.6	31.0	55.6
15,000	-38.0	.163	9.04	11.3	10.1	26.0	47.4
20,000	-48.0	.171	9.18	9.6	8.4	21.0	39.0
30,000	-62.0	.175	9.2	7.5	6.3	15.0	28.8

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TABLE X

RADIOSONDE DATA FOR METEOROLOGICAL DATA I

(MILD ATMOSPHERE)

Altitude, z (m)	Temperature, T (°C)	Pressure, P (mb)	Relative humidity (percent)
0	9	1013	80
1,000	9	900	60
2,000	8	794	50
3,000	---	700	42
4,000	-5	614	37
5,000	---	541	32
6,000	-15	474	30
8,000	-27	352	27
10,000	-40	262	24
12,000	-50	193	23

TABLE XI

NOCTURNAL IRRADIATION FOR METEOROLOGICAL DATA I (MILD ATMOSPHERE)

[See fig. 13.]

z (ft)	T (°C)	h'_0 (cm)	$G'_{H_2O,\text{above}}$ (Btu/(hr)(sq ft))	$G''_{H_2O,\text{above}}$ (Btu/(hr)(sq ft))	$G_{CO_2,\text{above}}$ (Btu/(hr)(sq ft))	G_{above} (Btu/(hr)(sq ft))
0	15.6	0	40.3	28.3	22.0	90.6
3,000	9.0	.700	27.6	25.4	20.2	73.2
5,000	5.0	.945	20.65	23.5	19.3	63.45
10,000	-5.5	1.237	8.59	19.7	16.8	45.09
15,000	-15.0	1.326	2.84	16.7	14.7	34.24
20,000	-24.0	1.354	.82	14.4	13.8	29.02
30,000	-41.3	1.362	0	11.4	9.5	20.9

z (ft)	T (°C)	h'_0 (cm)	$G'_{H_2O,\text{below}}$ (Btu/(hr)(sq ft))	$G''_{H_2O,\text{below}}$ (Btu/(hr)(sq ft))	$G_{CO_2,\text{below}}$ (Btu/(hr)(sq ft))	G_{earth} (Btu/(hr)(sq ft))	G_{below} (Btu/(hr)(sq ft))
0	15.6	0	0	0	0	123.2	123.2
3,000	9.0	.700	31.0	25.4	20.2	37	113.6
5,000	5.0	.945	33.9	23.5	19.3	32	108.7
10,000	-5.5	1.237	36.45	19.7	16.8	23.3	96.25
15,000	-15.0	1.326	35.7	16.7	14.7	19.0	86.1
20,000	-24.0	1.354	36.0	14.4	13.8	16.0	80.2
30,000	-41.3	1.362	35.4	11.4	9.5	13.0	69.3

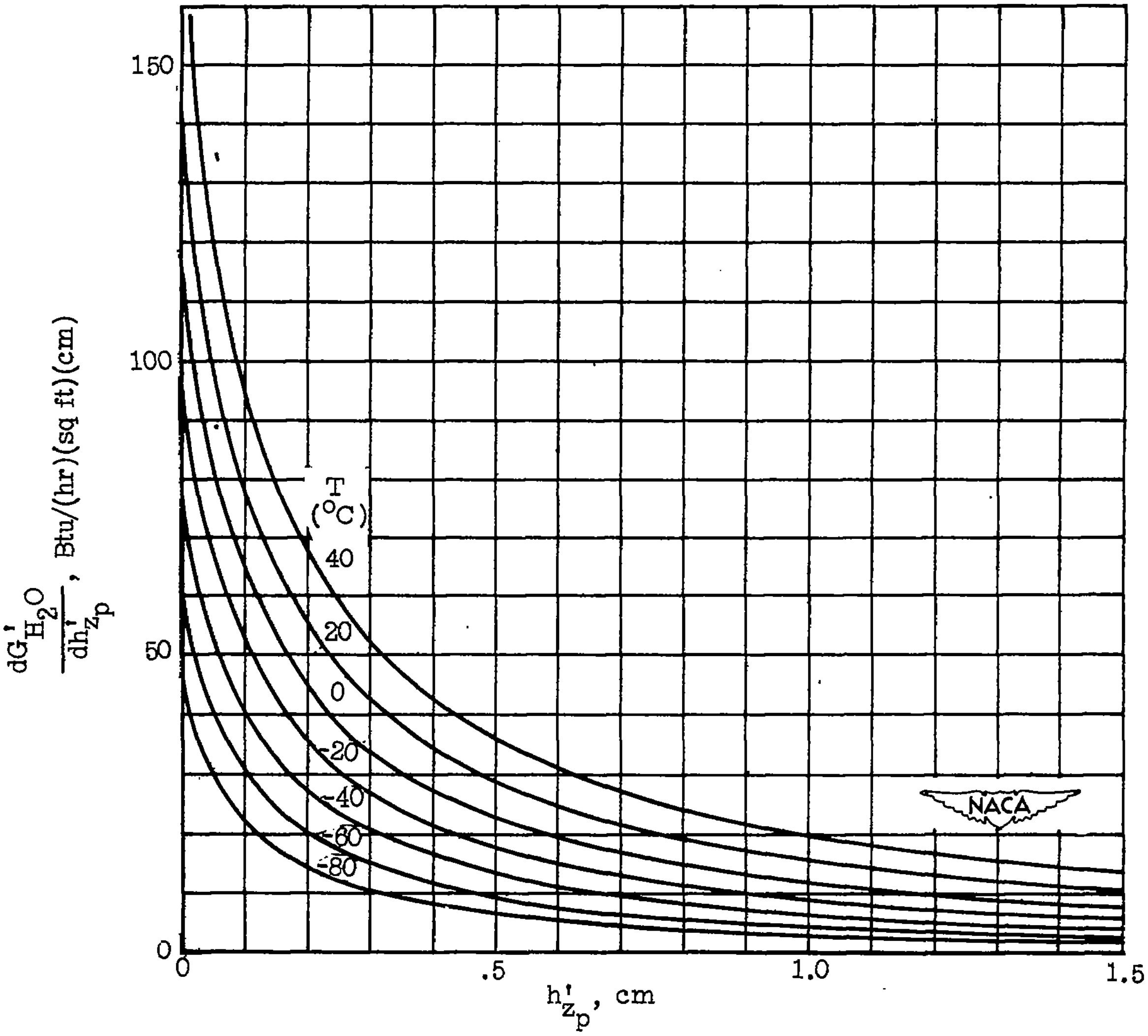

Figure 1.- Radiation chart I.

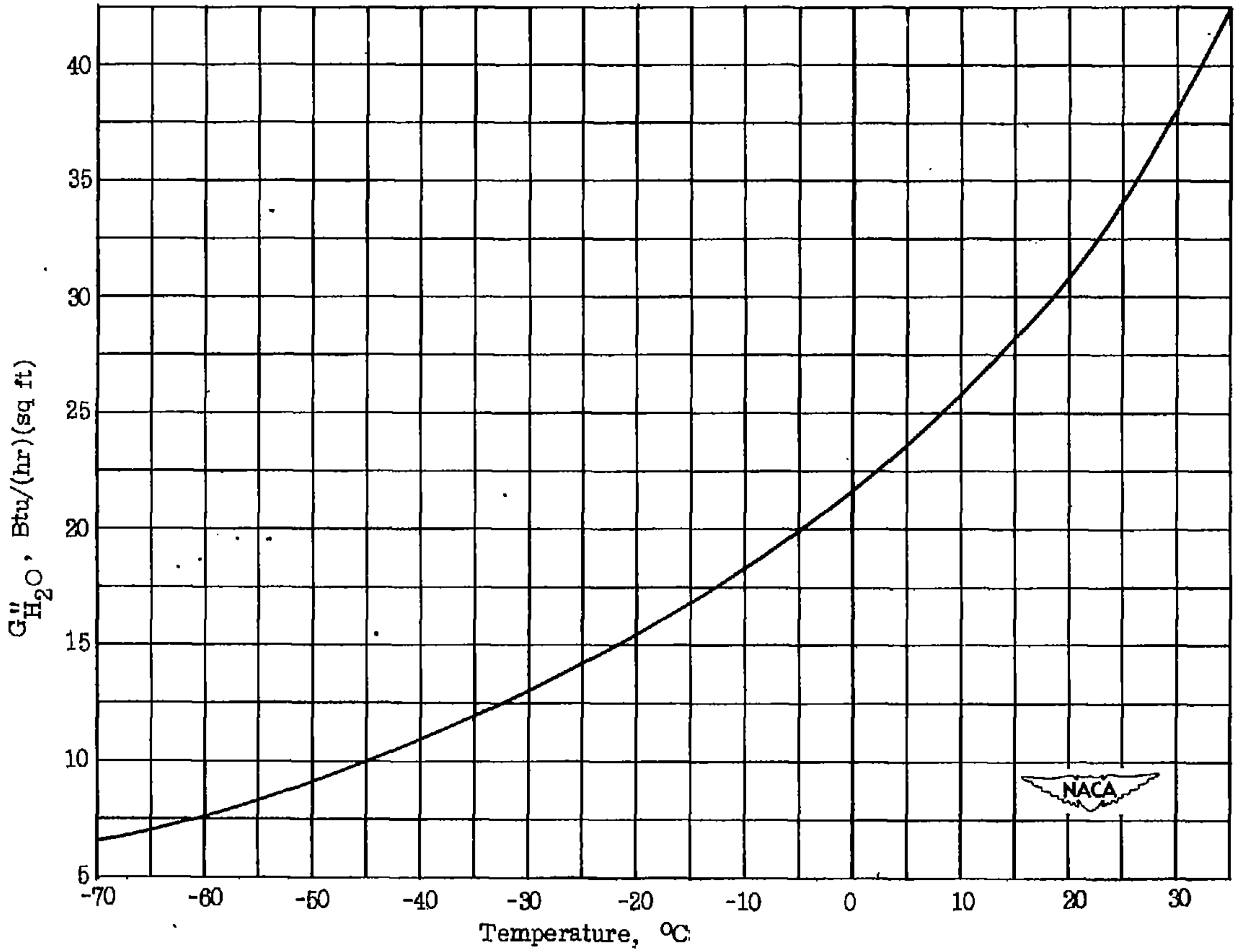

Figure 2.- Radiation chart II.

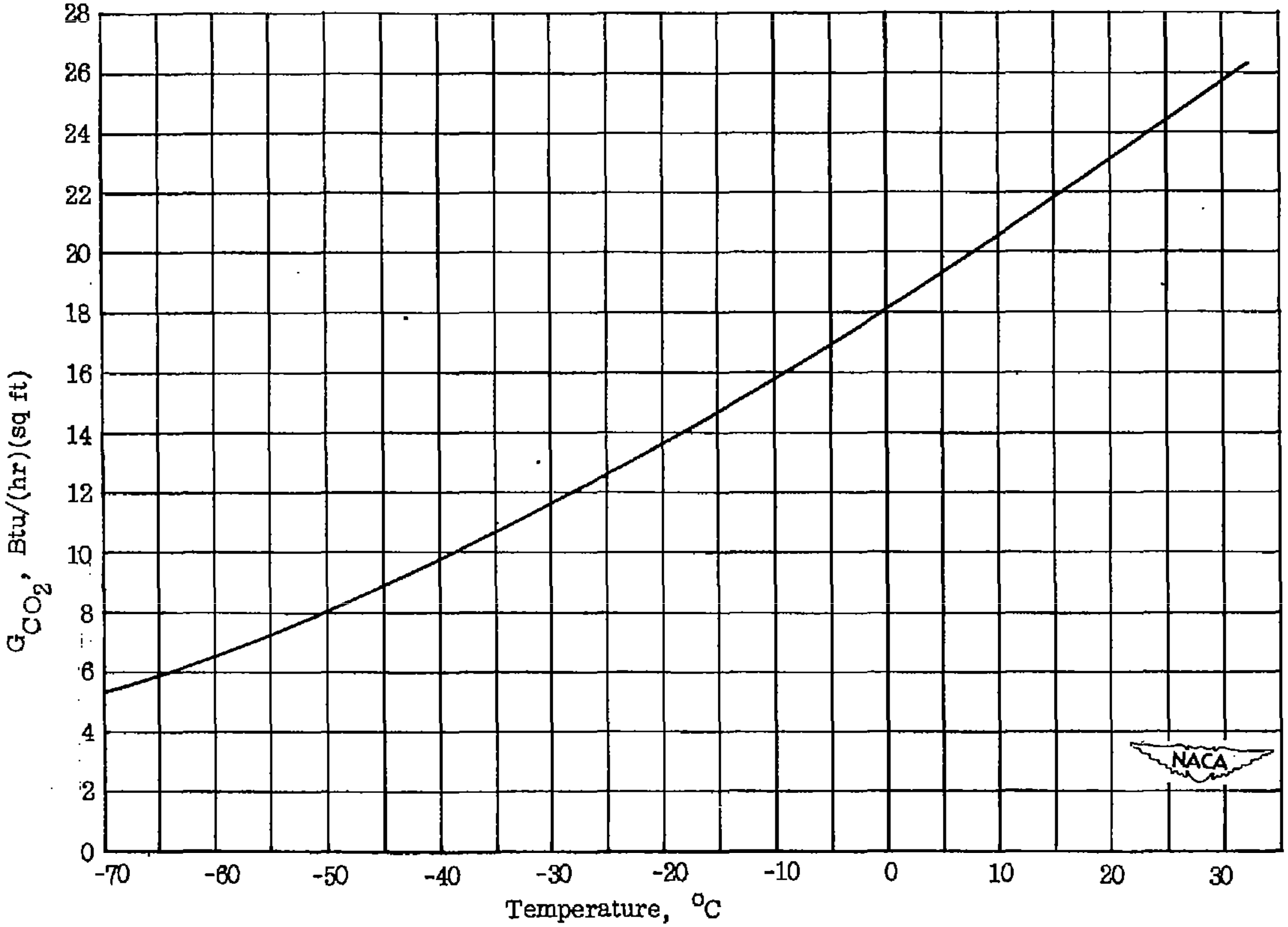

Figure 3.- Radiation chart III.

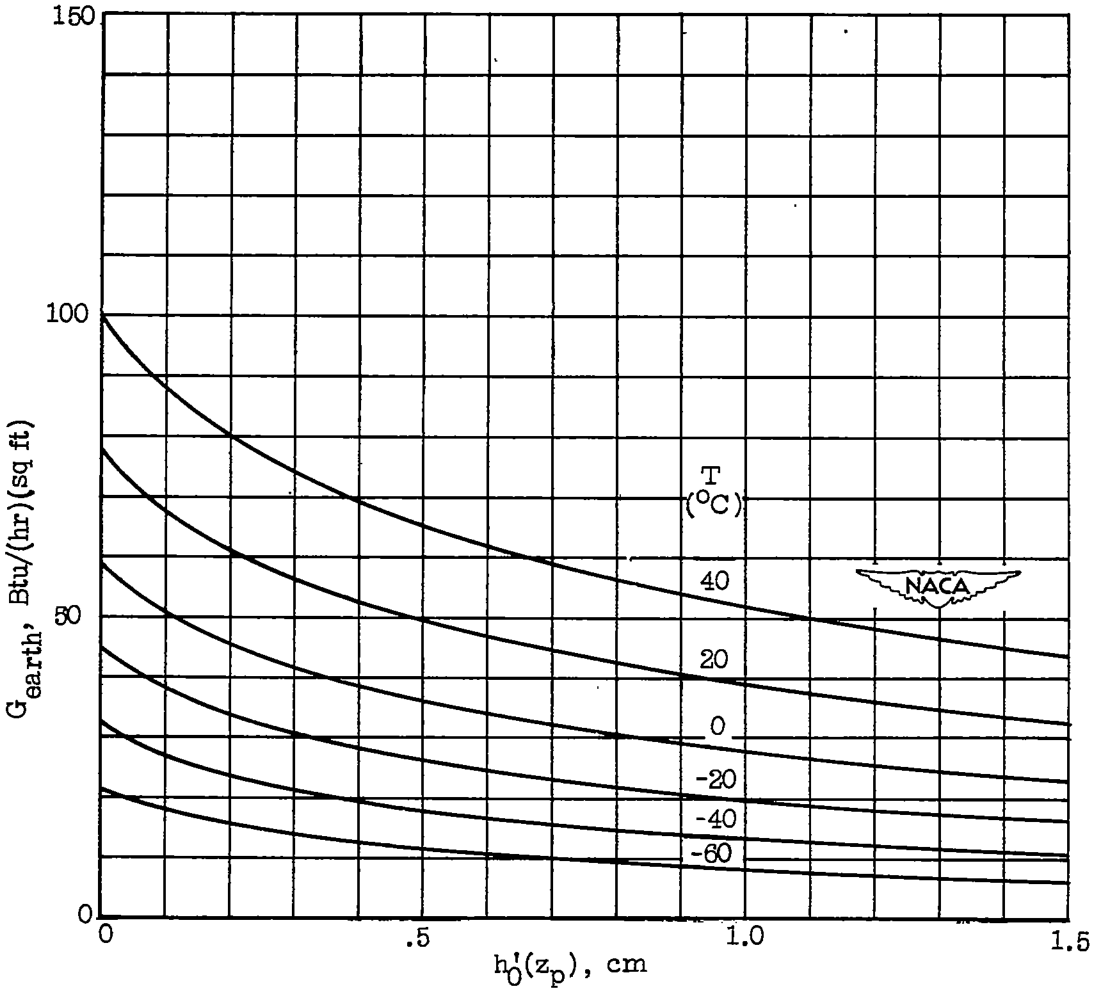

Figure 4.- Radiation chart IV.

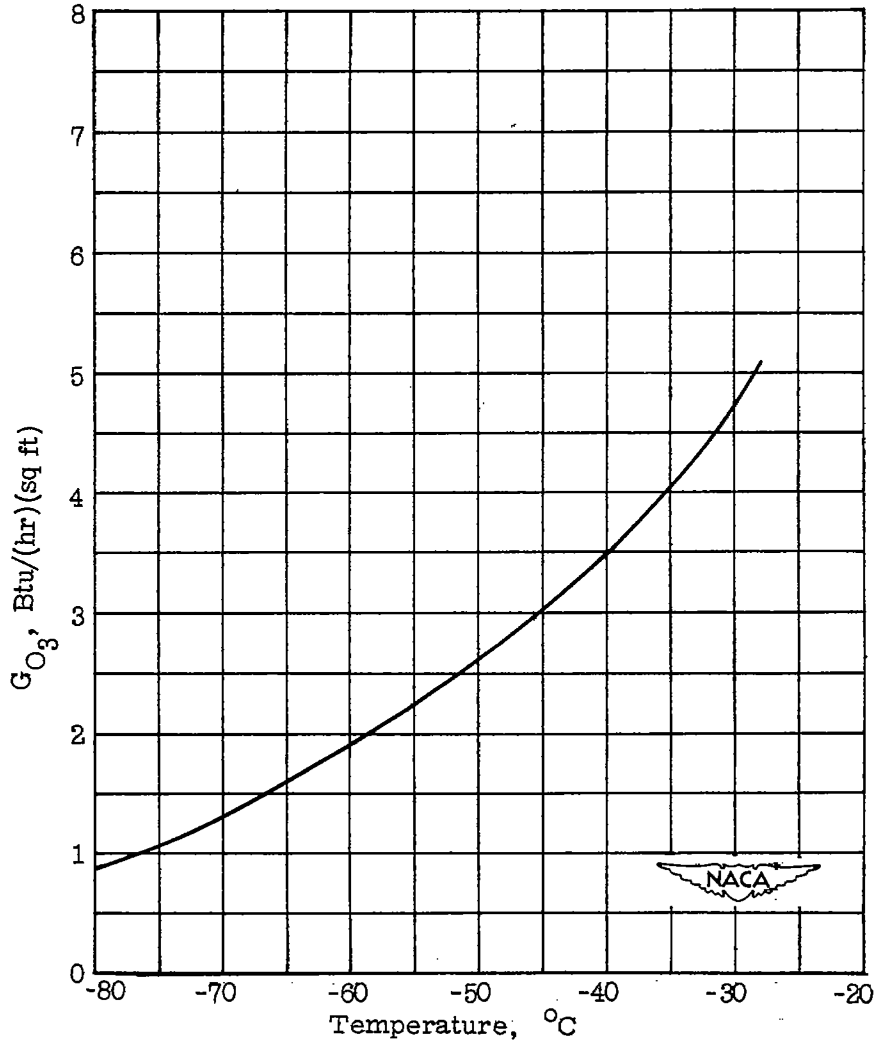

Figure 5.- Radiation chart V.

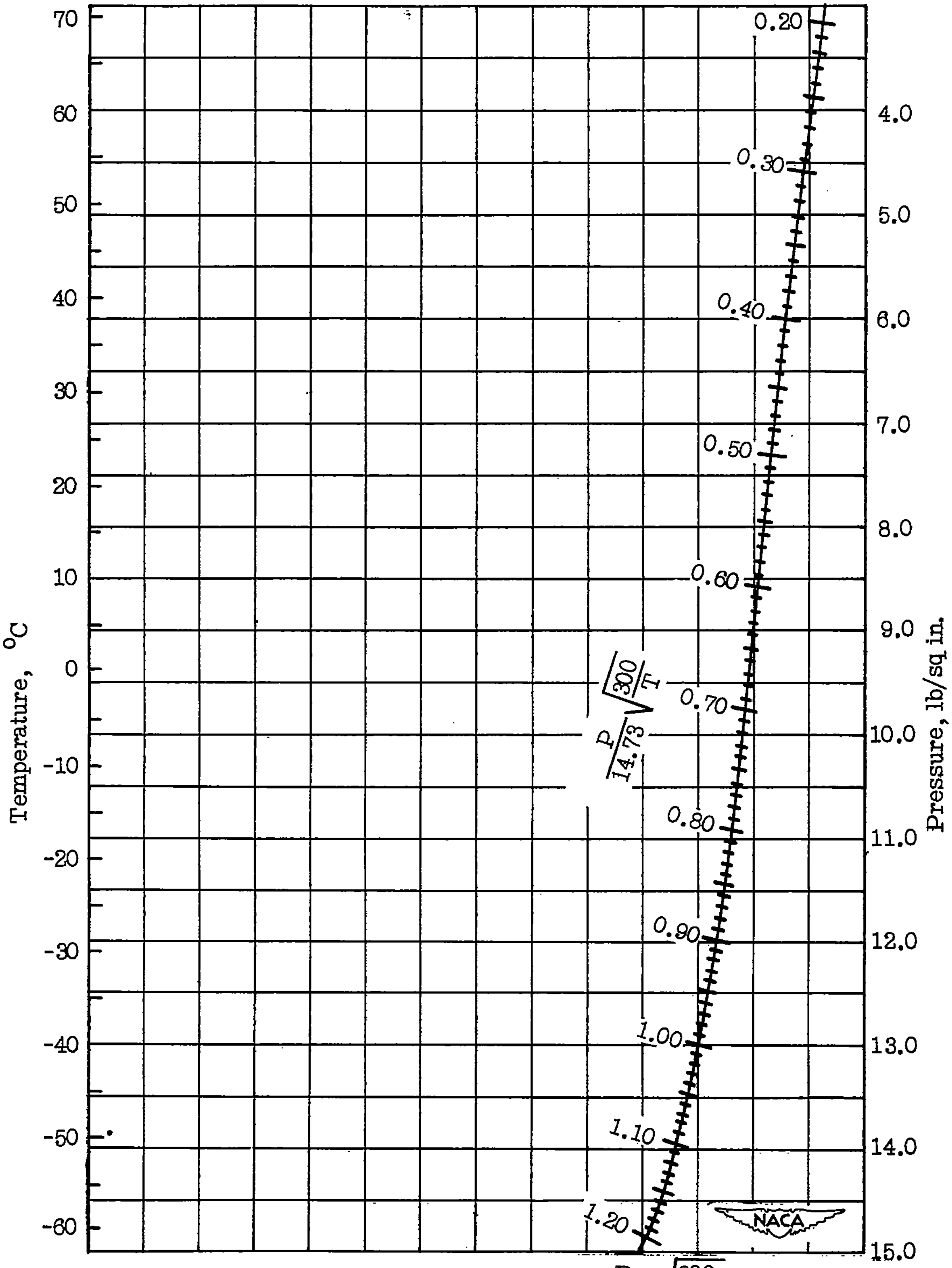

Figure 6.- Nomograph for $\frac{P}{14.73}\sqrt{\frac{300}{T}}$.

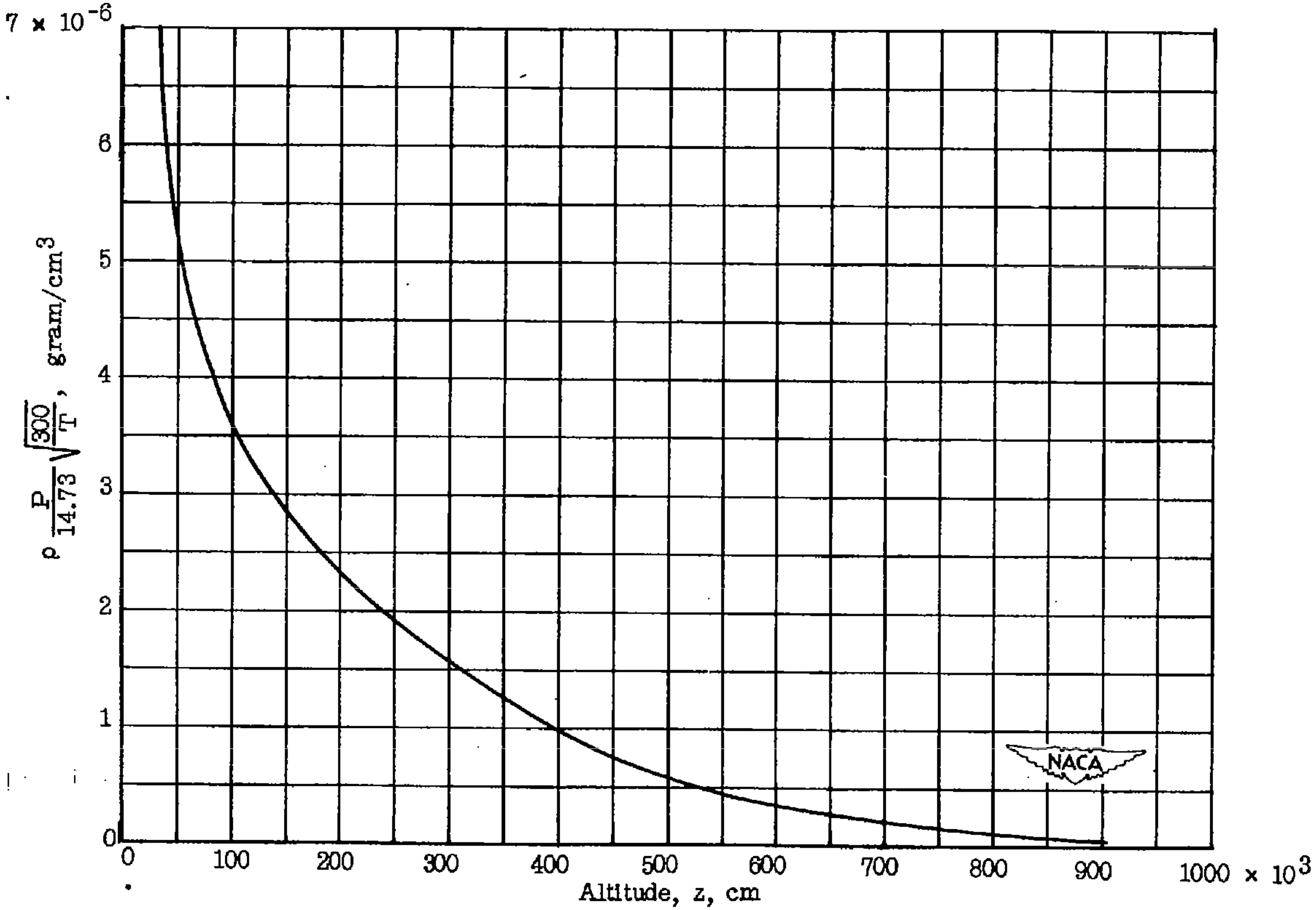

Figure 7.- The function $\rho \frac{P}{14.73}\sqrt{\frac{300}{T}}$ against altitude z. (For meteorological data II; see table IV.)

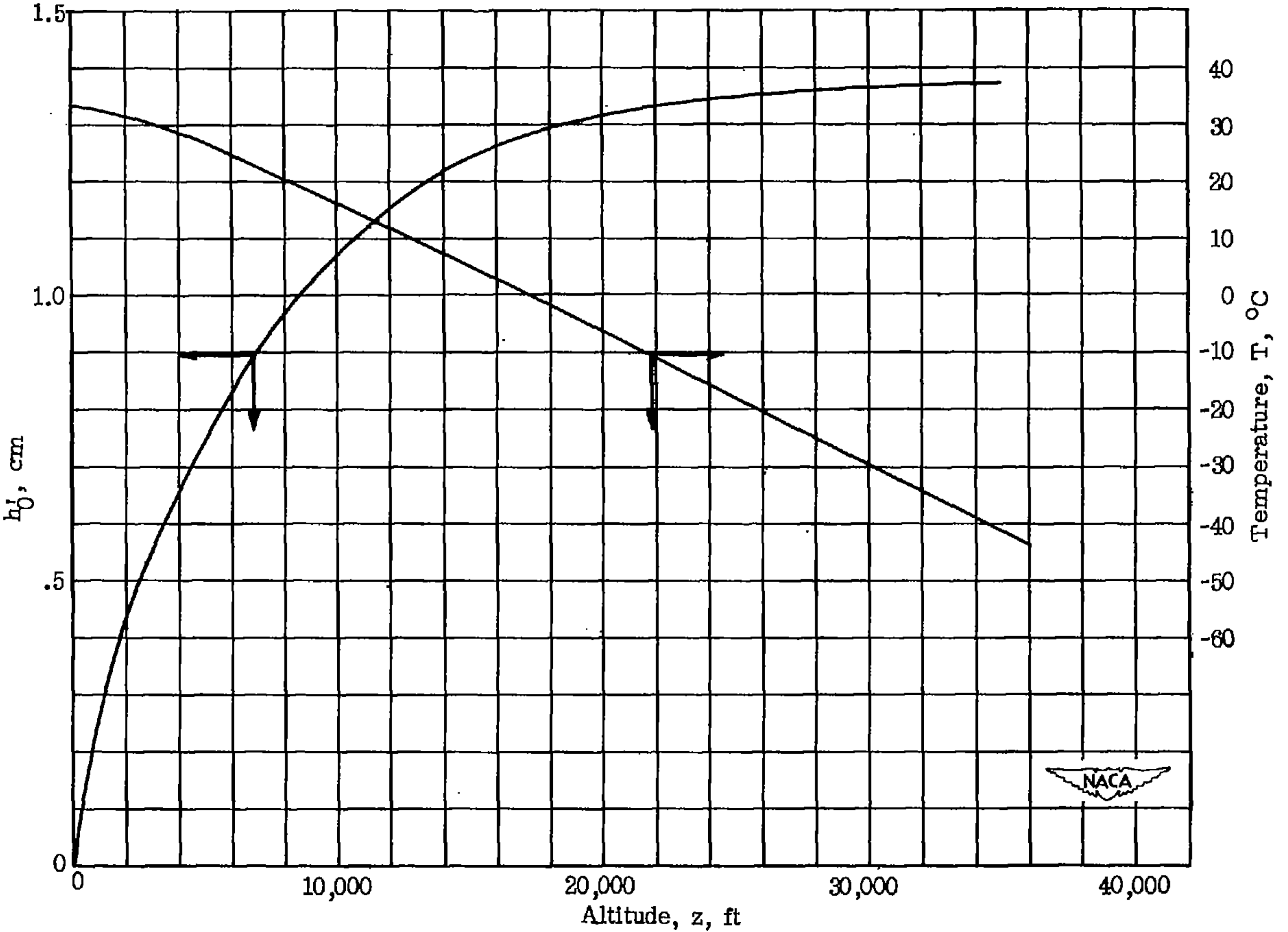

Figure 8.- The function h_0' and temperature T against altitude z. (For meteorological data II; see tables IV and V.)

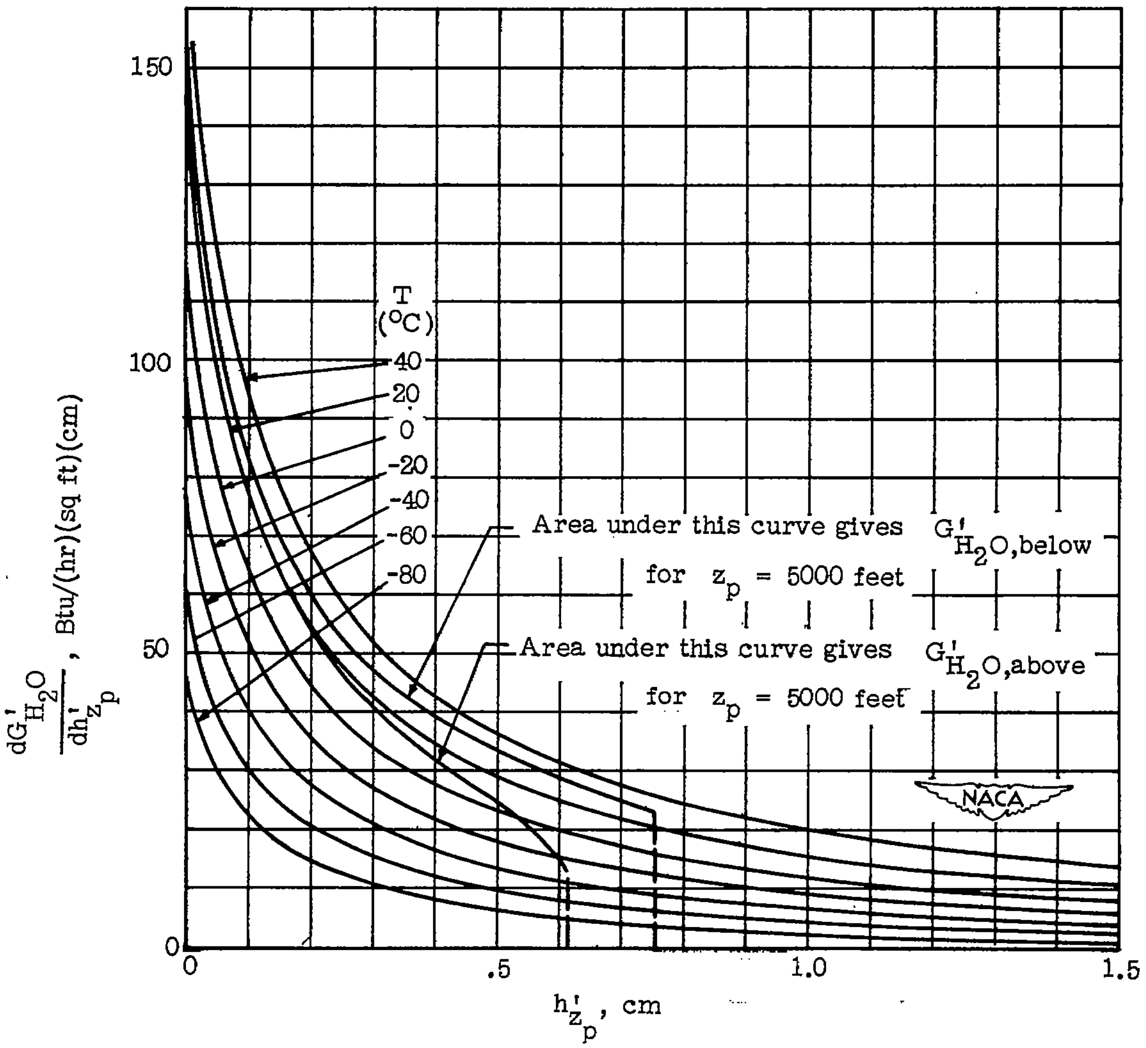

Figure 9.- Application of radiation chart I. (For meteorological data II; see table VI.)

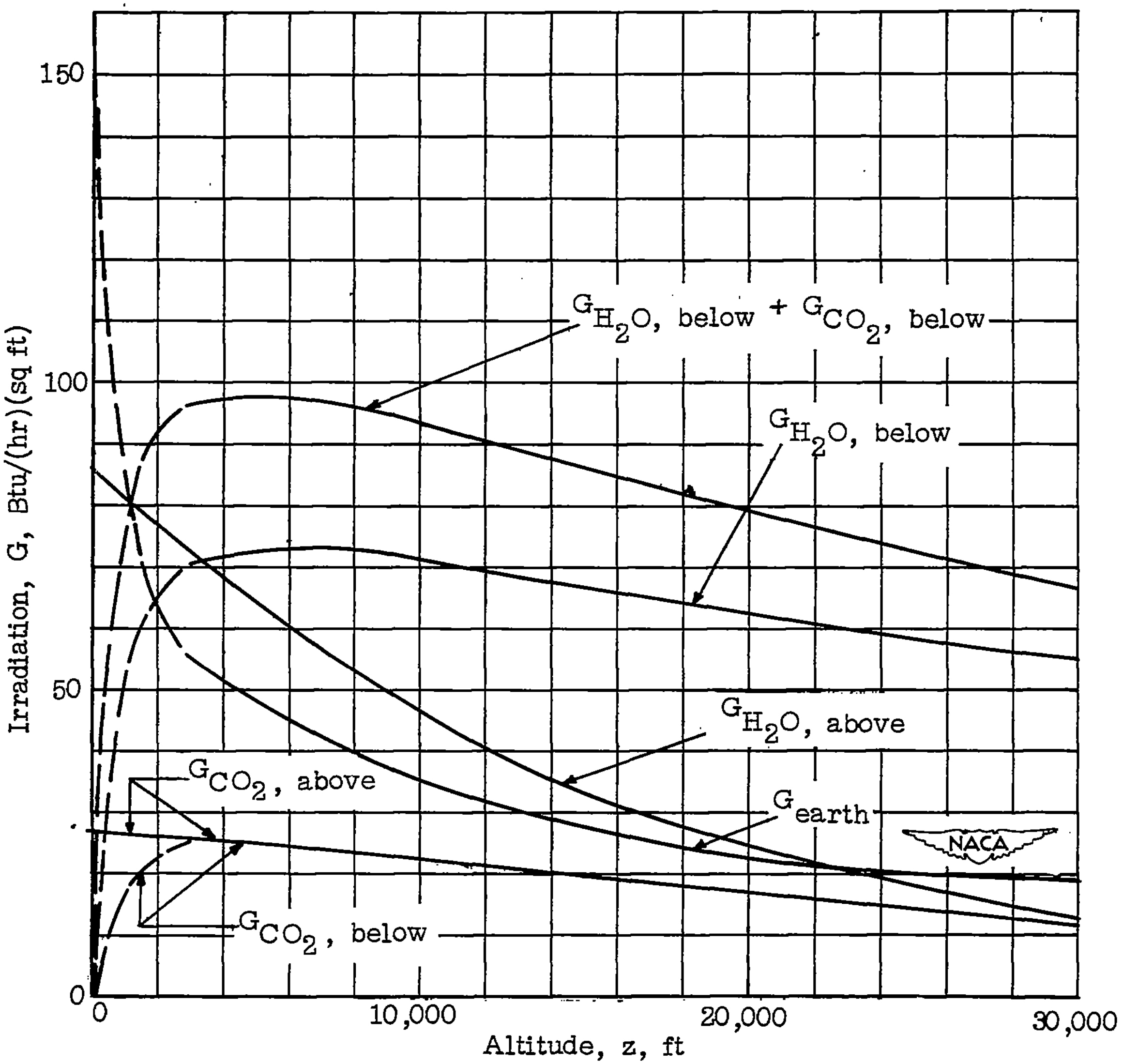

Figure 10.- Components of nocturnal irradiation G for meteorological data II. (See table VII.)

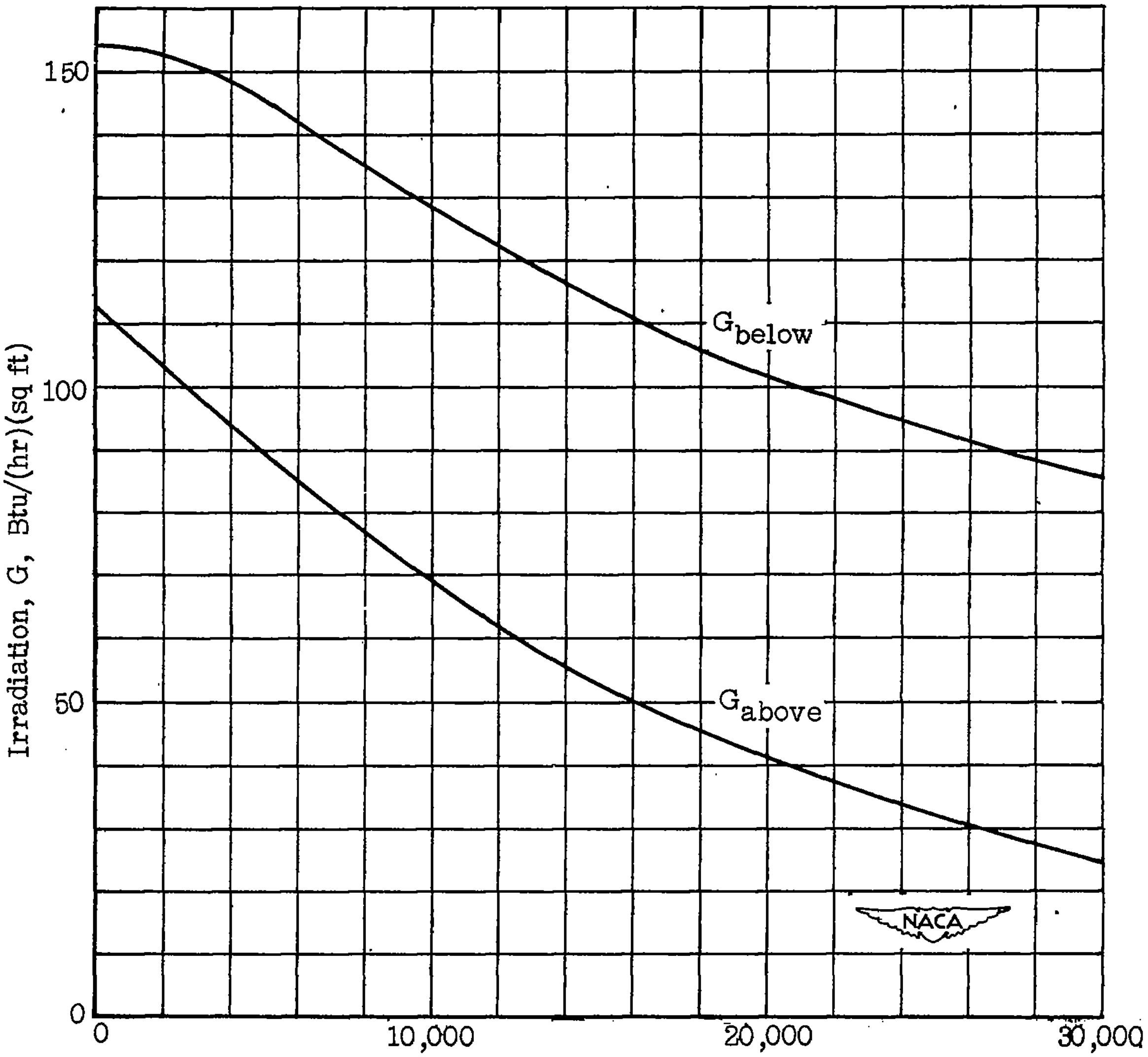

Figure 11.- Nocturnal irradiation G as a function of altitude z for meteorological data II. (See table VII.)

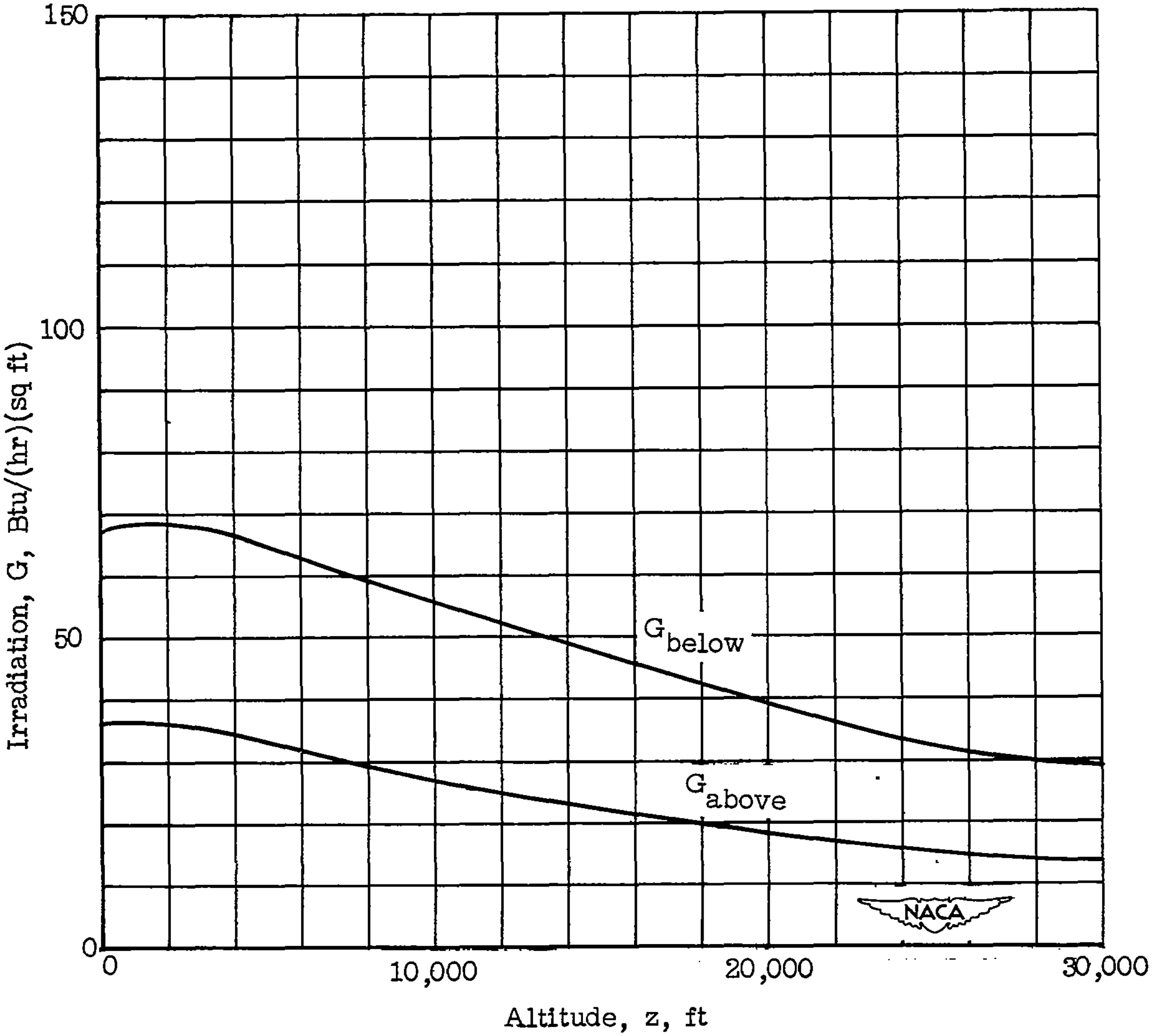

Figure 12.- Nocturnal irradiation G as a function of altitude z for meteorological data III. (See table IX.)

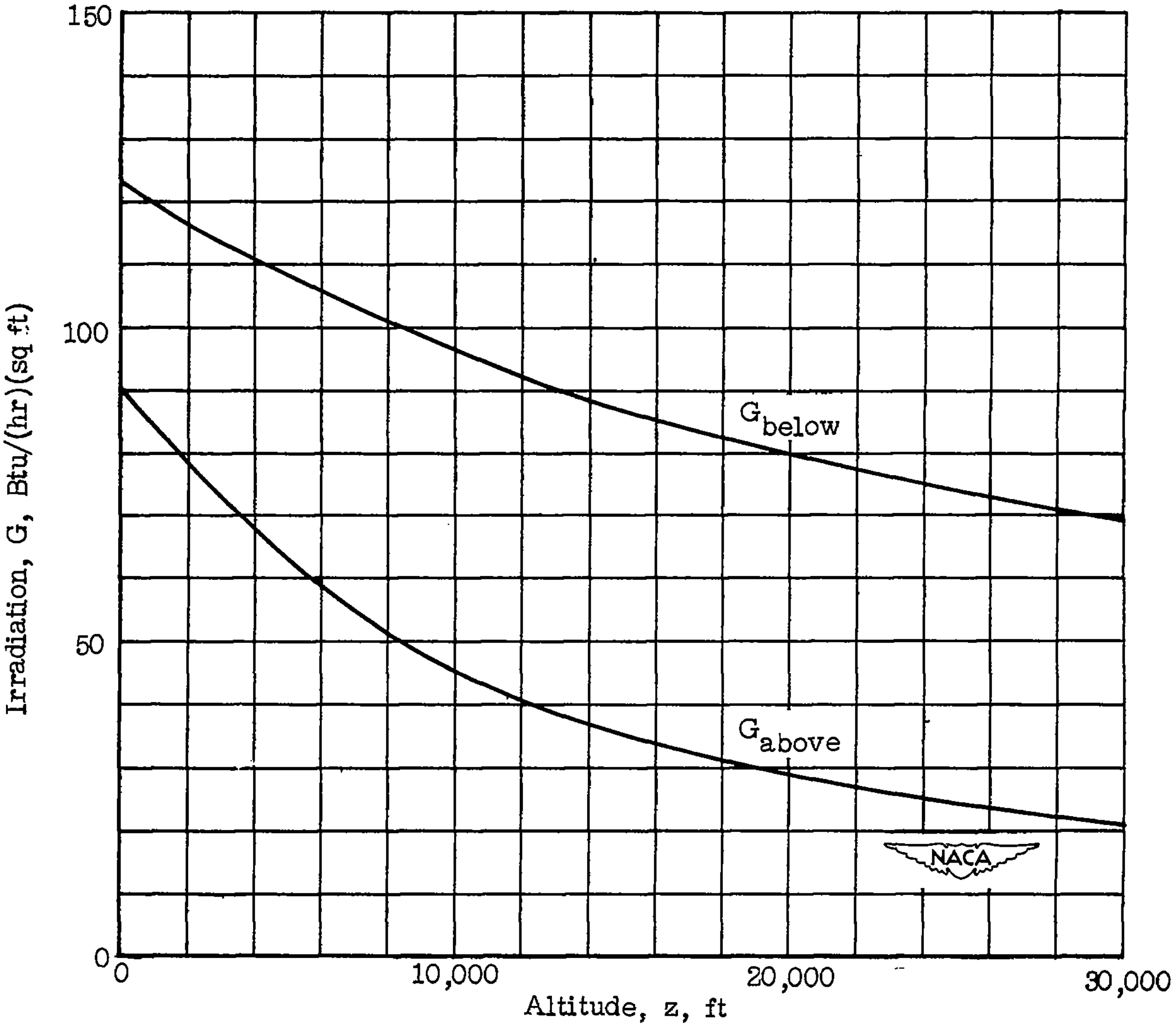

Figure 13.- Nocturnal irradiation G as a function of altitude z for meteorological data I. (See table XI.)

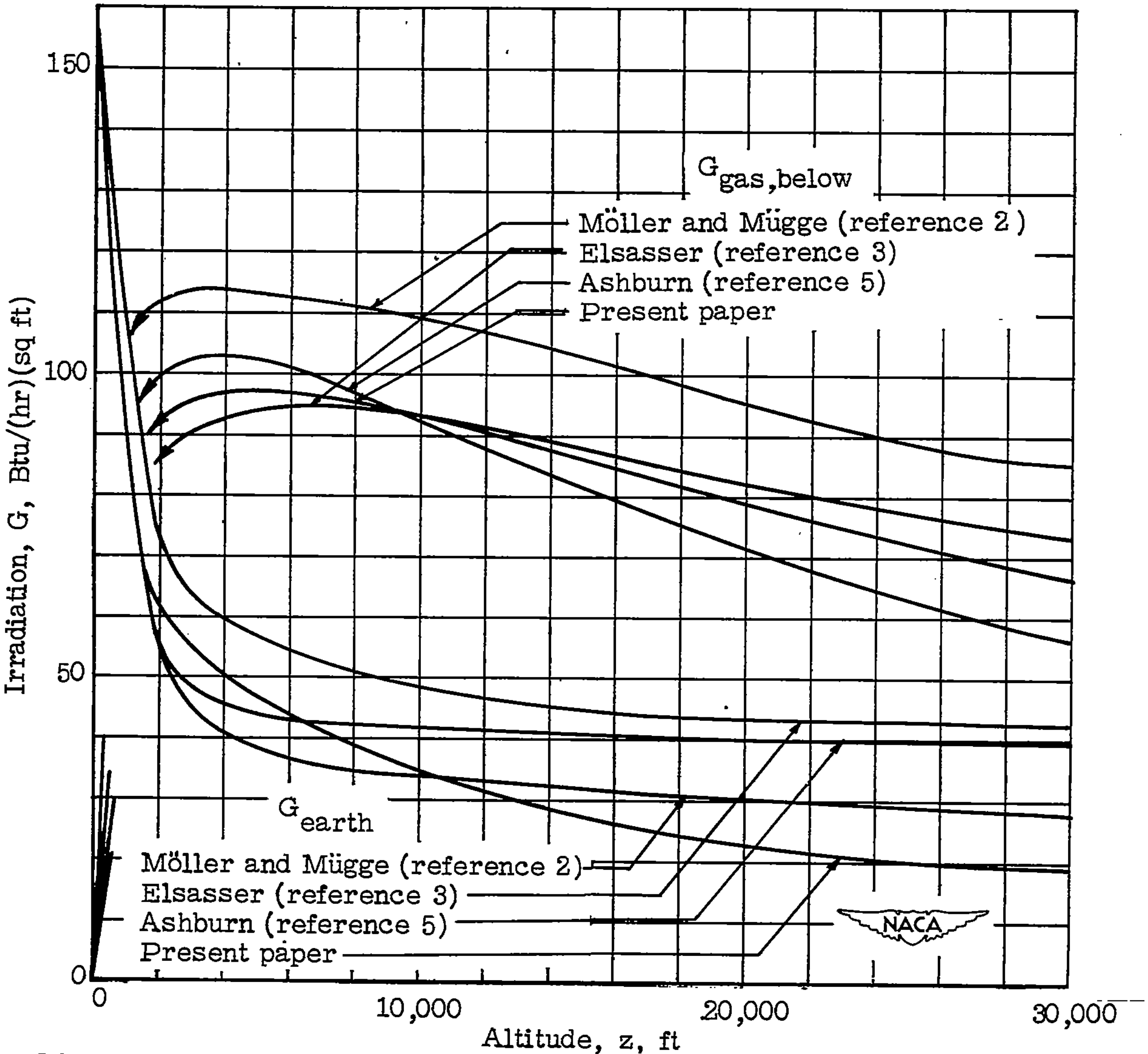

Figure 14.- Comparison of $G_{gas, below}$ and G_{earth} for meteorological data II. (See table III.)

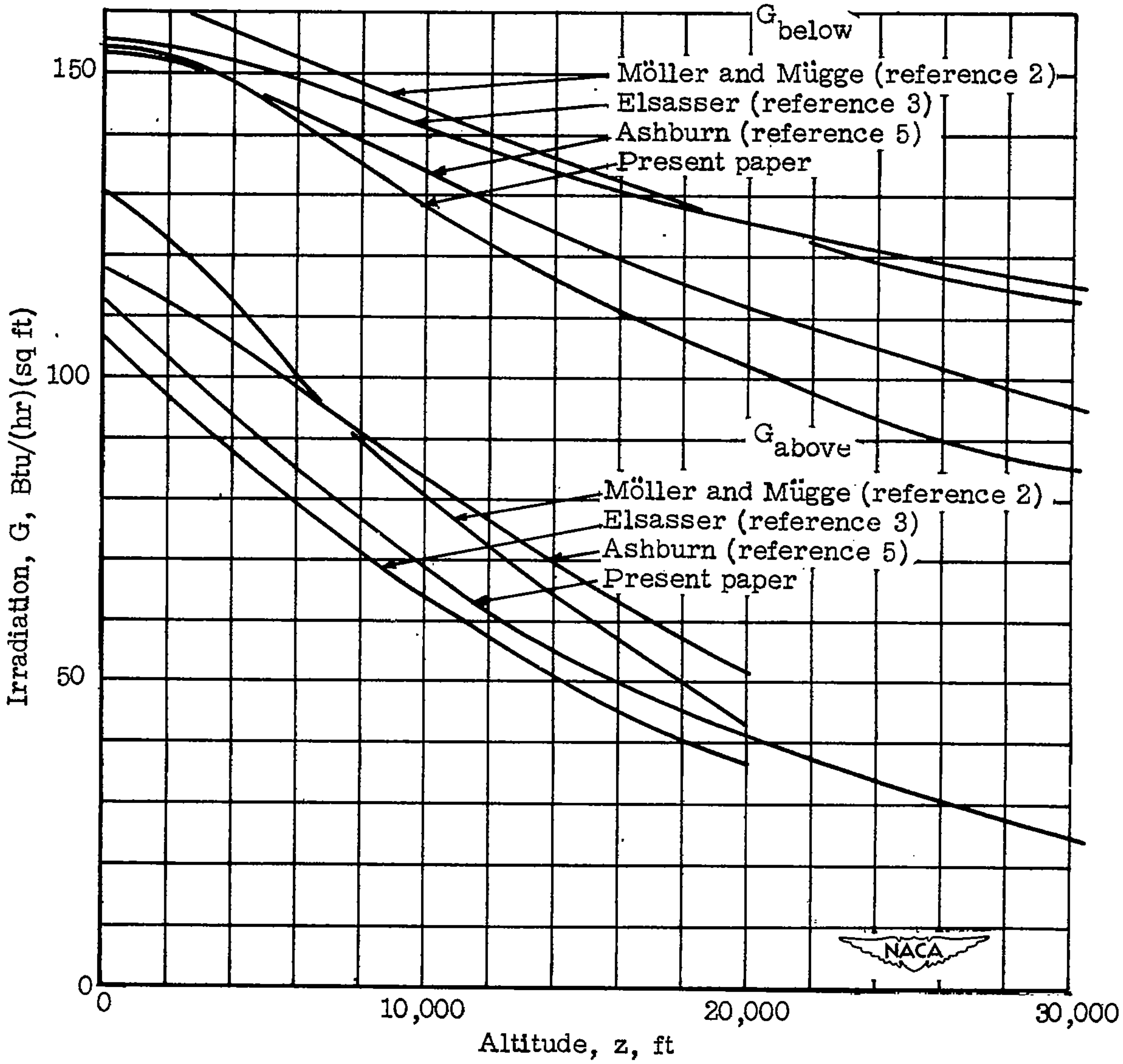

Figure 15.- Comparison of G_{below} and G_{above} for meteorological data II. (See table III.)

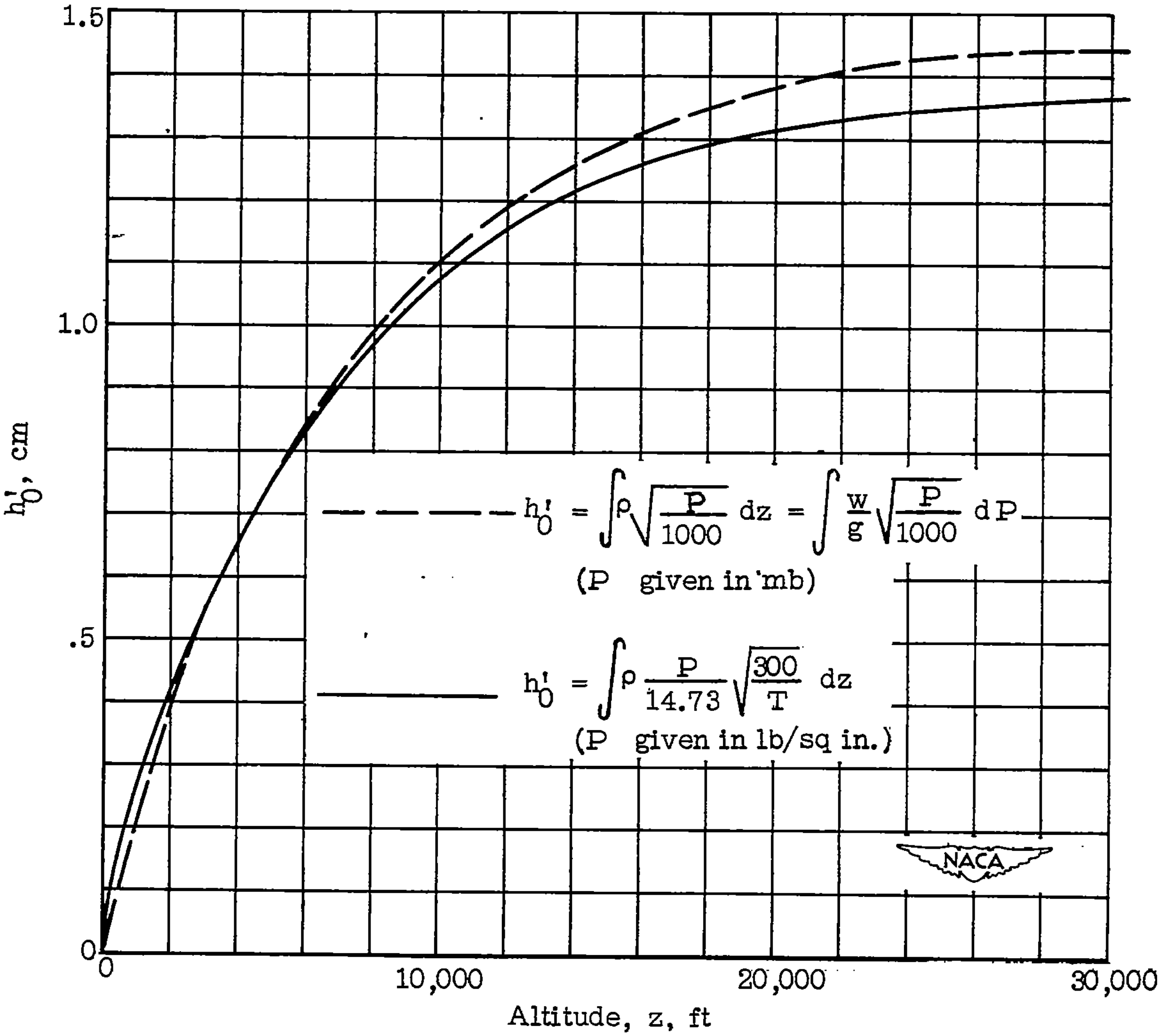

Figure 16.- Comparison of the function h_0'. (For meteorological data II.)

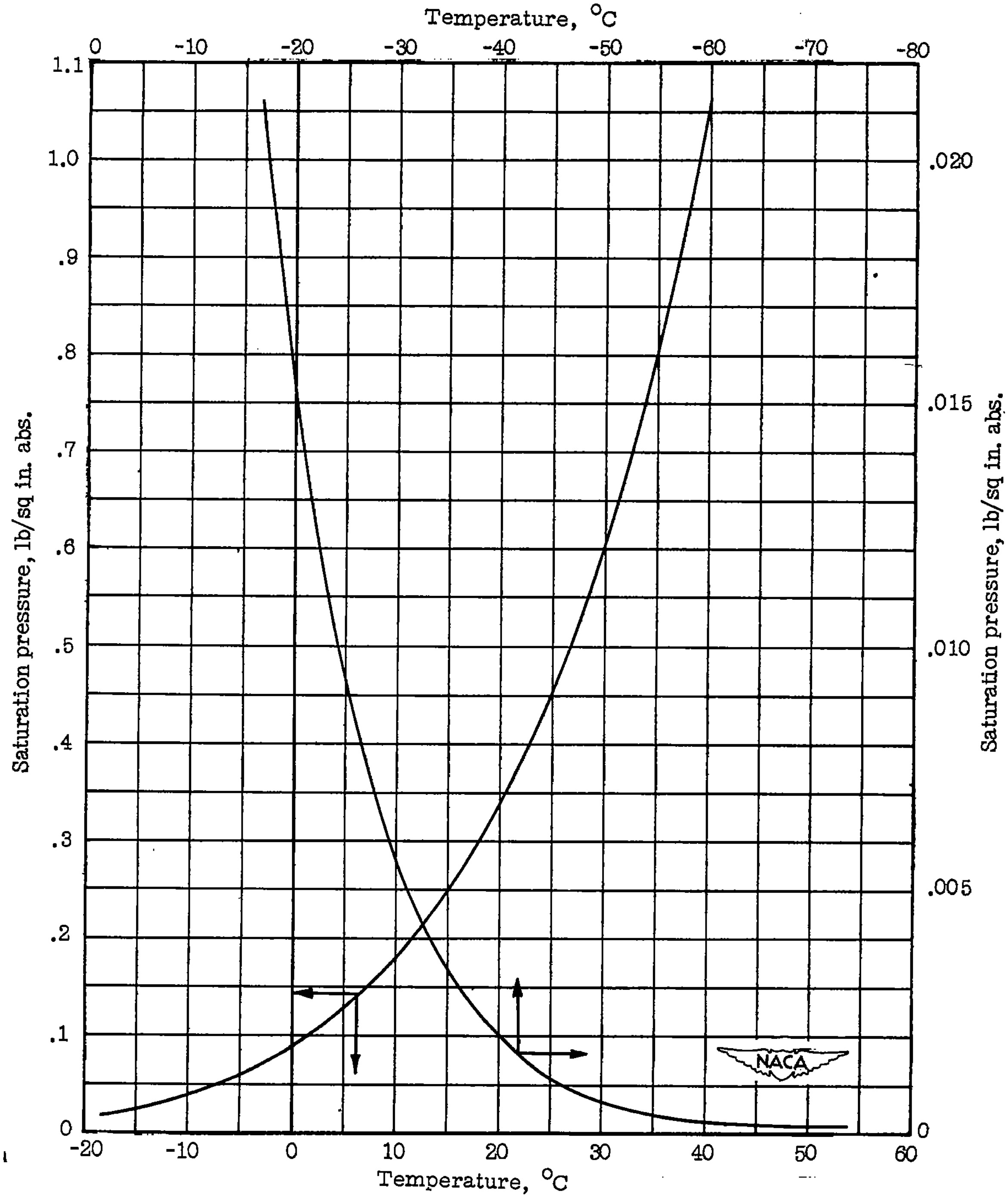

Figure 17.- Saturated vapor pressure against temperature. (See table II.)