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STANDARD SYMBOLS FOR HELICOPTERS

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SUMMARY

A list of standard symbols for rotating-wing aircraft is presented. The list of symbols is subdivided for ready reference under appropriate headings, the symbols being restricted to those most generally used in studies of helicopter aerodynamics.

INTRODUCTION

Great interest has been shown by manufacturers of helicopters and by various government agencies charged with the design, evaluation, and procurement of helicopters in the standardization of symbols for use in their work. A survey was therefore conducted among members of the armed services, industry, certain educational institutions, and various government agencies in order to obtain an indication of their preferences. This survey was made by the Helicopter Committee of the Society of Automotive Engineers in cooperation with the NACA Subcommittee on Helicopters. The results of the survey indicated a definite preference for those symbols that had been most widely used in the past. Accordingly, a list of symbols that have appeared in many papers on rotating-wing aircraft has been prepared and correlated. This list, which is presented herein, was approved as standard by the members of the NACA Subcommittee on Helicopters at a meeting held on December 3, 1947. The membership of this Subcommittee is considered representative of the various groups contacted during the aforementioned survey.

The symbols listed have been limited for the present to those most generally used in helicopter aerodynamic studies. Specialized symbols necessary in vibration, stress, and stability work remain to be developed and standardized as these studies mature with time and use.

SYMBOLS

The units given in the definition of the symbols pertaining to velocity and angles were chosen to make the equations compatible and the coefficients of the correct magnitude. The units given are not intended to preclude the use of alternate units that might be more convenient for use in plots or tables, in which case the alternate unit should be specified.

An attempt was made to keep the symbols as general as possible in order not to restrict their use. For example, no specific formulas were given for the symbols representing induced velocities or performance parameters in order to allow the same symbols to represent these quantities, regardless of the exactness of the formulas used in evaluating them. For similar reasons, the choice of axes to go with the symbols for rotor rolling, pitching, and yawing moments was left open.

The list of symbols is subdivided for ready reference under appropriate headings.

Physical Quantities

The symbols representing the physical characteristics of the helicopter and of the medium in which it operates are as follows:

W gross weight of helicopter, pounds

b number of blades per rotor

R blade radius, feet

r radial distance to blade element, feet

x ratio of blade-element radius to rotor-blade radius (r/R)

c blade-section chord, feet

c_e equivalent blade chord (on thrust basis), feet $\left(\frac{\int_0^R cr^2\,dr}{\int_0^R r^2\,dr}\right)$

σ rotor solidity $\left(bc_e/\pi R\right)$

θ blade-section pitch angle; angle between line of zero lift of blade section and plane perpendicular to axis of no feathering, radians (See section "Air flow relative to rotor" for definition of axis of no feathering.)

θ_0 blade pitch angle at hub, radians

θ_1 difference between hub and tip pitch angles; positive when tip angle is larger, radians

e distance from drag hinge (vertical pin) to axis of rotation, feet m mass of blade per foot of radius, slugs per foot

I_1 mass moment of inertia of blade about flapping hinge, slug-feet^2

γ mass constant of rotor blade; expresses ratio of air forces to centrifugal forces $\left(c\rho aR^4/I_1\right)$ (See section "Blade-element aerodynamic characteristics" for definition of a)

ρ mass density of air, slugs per cubic foot

Air-Flow Parameters

Air flow relative to rotor.- Before the symbols associated with air flow relative to the rotor are listed, it is considered advisable to define an axis that is used as a reference for this system. This reference axis is called "the axis of no feathering" and is defined as the axis about which there is no first harmonic feathering or cyclic-pitch variation. The plane perpendicular to this axis has been termed the "rotor disk" in many papers on rotating-wing aircraft. Inasmuch as the axis of no feathering is basic to an understanding of many of the symbols included herein, its use and significance is discussed in detail in the appendix.

The symbols for the velocities, velocity parameters, and angles that are used in defining the air flow relative to the rotor follow:

V true airspeed of helicopter along flight path, feet per second

V_h horizontal component of true airspeed of helicopter, feet per second

V_v vertical component of true airspeed of helicopter, feet per second

Ω rotor angular velocity, radians per second

α rotor angle of attack; angle between projection in plane of symmetry of axis of no feathering and line perpendicular to flight path, positive when axis is pointing rearward, radians

v induced inflow velocity at rotor (always positive), feet per second

μ tip-speed ratio $\left(\frac{V \cos \alpha}{\Omega R}\right)$

λ inflow ratio $\left(\frac{V \sin \alpha - v}{\Omega R}\right)$

V' resultant velocity at rotor; vector sum of translational and induced velocities, feet per second

ψ blade azimuth angle measured from downwind position in direction of rotation, radians

Air flow relative to blade element.- The velocities and angles defining the air flow relative to the rotor-blade elements are illustrated on figure 1. The symbols for these quantities are listed as follows:

U_T component at blade element of resultant velocity perpendicular to blade-span axis and to axis of no feathering, feet per second

U_P component at blade element of resultant velocity perpendicular both to blade-span axis and U_T, feet per second

U_R component at blade element of resultant velocity parallel to blade-span axis and perpendicular to U_T, feet per second

U resultant velocity perpendicular to blade-span axis at blade element, feet per second

ϕ inflow angle at blade element in plane perpendicular to blade-span axis, radians $\left(\tan^{-1} \frac{U_P}{U_T}\right)$

α_r blade-element angle of attack, measured from line of zero lift, radians $(\theta + \phi)$

$\alpha_{(x)(\psi)}$ blade-element angle of attack at any radial position and at any blade azimuth angle, degrees; for example $\alpha_{(1.0)(270^\circ)}$ is blade-element angle of attack at tip of retreating blade at 270° azimuth position

Aerodynamic Characteristics

Blade-element aerodynamic characteristics.- The symbols for the two-dimensional aerodynamic characteristics of the airfoil sections comprising the rotor blade elements are listed as follows:

c_l section lift coefficient

c_{d_o} section profile-drag coefficient

$\delta_0, \delta_1, \delta_2$ coefficients in power series expressing c_{d_o} as a function of α_r $\left(c_{d_o} = \delta_0 + \delta_1\alpha_r + \delta_2{\alpha_r}^2\right)$

a slope of curve of section lift coefficient against section angle of attack (radian measure)

Rotor aerodynamic characteristics.- The symbols for the quantities that define the aerodynamic characteristics of the rotor are listed as follows:

L lift, pounds

D drag, pounds

T rotor thrust, pounds

Y lateral force, pounds

Q rotor-shaft torque, pound-feet

P rotor-shaft power, pound-feet per second

L' rolling moment, pound-feet

M pitching moment, pound-feet

N yawing moment, pound-feet

C_L lift coefficient $\left(\frac{L}{\frac{1}{2}\rho V^2 \pi R^2}\right)$

C_D drag coefficient $\left(\frac{D}{\frac{1}{2}\rho V^2 \pi R^2}\right)$

C_T thrust coefficient $\left(\frac{T}{\pi R^2 \rho (\Omega R)^2}\right)$

C_Y lateral-force coefficient $\left(\frac{Y}{\frac{1}{2}\rho V^2 \pi R^2}\right)$

C_Q rotor-shaft torque coefficient $\left(\frac{Q}{\pi R^2 \rho (\Omega R)^2 R}\right)$

C_P rotor-shaft power coefficient $\left(\dfrac{P}{\pi R^2 \rho (\Omega R)^3}\right)$

C_l rolling-moment coefficient $\left(\dfrac{L'}{\frac{1}{2}\rho V^2 \pi R^2 R}\right)$

C_m pitching-moment coefficient $\left(\dfrac{M}{\frac{1}{2}\rho V^2 \pi R^2 R}\right)$

C_n yawing-moment coefficient $\left(\dfrac{N}{\frac{1}{2}\rho V^2 \pi R^2 R}\right)$

B tip-loss factor; blade elements outboard of radius BR are assumed to have profile drag but no lift

M rotor figure of merit $\left(0.707 \dfrac{{C_T}^{3/2}}{C_Q}\right)$; the factor 0.707 is included to make the maximum (ideal) figure of merit equal to unity

Rotor-Blade Motion

Flapping motion.- Blade flapping motion may be described as the variation with azimuth angle of the blade flapping angle, the flapping angle being defined as the angle between the blade-span axis and the plane perpendicular to the axis of no feathering. This motion may be expressed as a function of the azimuth angle by the Fourier series

$$\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi - \cdot \cdot \cdot$$

where

β blade flapping angle at particular azimuth position

a_0 constant term in Fourier series that expresses β (radians); hence, the rotor coning angle

a_n coefficient of $\cos n\psi$ in expression for β

b_n coefficient of $\sin n\psi$ in expression for β

Feathering motion. - Feathering motion may be described as the variation with azimuth angle of the blade pitch angle at a representative radius, usually taken at 0.75 radius. This motion may be expressed as a function of the azimuth angle by the Fourier series

$$\theta = A_0 - A_1 \cos \psi - B_1 \sin \psi - A_2 \cos 2\psi - B_2 \sin 2\psi - . . .$$

where

θ blade pitch angle at particular azimuth position

A_0 constant term in Fourier series that expresses θ (radians); hence, the mean blade pitch angle at the representative radius

A_n coefficient of $\cos n\psi$ in expression for θ

B_n coefficient of $\sin n\psi$ in expression for θ

In-plane motion. - The blade drag angle is defined as the angle between the blade-span axis and the line drawn through the rotor center of rotation and the drag hinge, the angle being positive in the direction of rotation. Changes in the blade drag angle are termed the "in-plane motion" of the rotor blades. This motion may be expressed as a function of the azimuth angle by the Fourier series

$$\zeta = E_0 + E_1 \cos \psi + F_1 \sin \psi + E_2 \cos 2\psi + F_2 \sin 2\psi + . . .$$

where

ζ blade drag angle at particular azimuth position

E_0 constant term in Fourier series that expresses ζ (radians); hence, the mean blade drag angle

E_n coefficient of $\cos n\psi$ in expression for ζ

F_n coefficient of $\sin n\psi$ in expression for ζ

Performance

Parameters useful for expressing helicopter performance are as follows:

$\left(\frac{D}{L}\right)_o$ rotor profile drag-lift ratio

$\left(\frac{D}{L}\right)_i$ rotor induced drag-lift ratio

$\left(\frac{D}{L}\right)_p$ parasite drag of helicopter components other than lifting rotors divided by rotor lift

$\left(\frac{D}{L}\right)_c$ drag-lift ratio representing angle of climb, positive in climb $\left(\tan^{-1}\frac{V_v}{V_h}\right)$

$\left(\frac{D}{L}\right)_r$ rotor drag-lift ratio; ratio of equivalent drag of rotor to rotor lift $\left(\left(\frac{D}{L}\right)_o + \left(\frac{D}{L}\right)_i\right)$

$\left(\frac{D}{L}\right)_u$ component of rotor resultant force along flight path (that is, useful component of rotor resultant force) divided by rotor lift $\left(\left(\frac{D}{L}\right)_p + \left(\frac{D}{L}\right)_c\right)$

$\frac{P}{L}$ shaft power parameter, where P is equal to rotor-shaft power divided by velocity along flight path and is therefore also equal to drag force that could be overcome by the shaft power at flight velocity $\left(\left(\frac{D}{L}\right)_r + \left(\frac{D}{L}\right)_u\right)$

f equivalent-flat-plate area representing parasite drag, based on unit drag coefficient, square feet $\left(\frac{\text{Helicopter parasite drag}}{\frac{1}{2}\rho V^2}\right)$

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX

EXPLANATION OF MEANING AND SIGNIFICANCE OF AXIS OF NO FEATHERING

At the time that the basic theoretical treatments of rotating-wing aircraft were made, the typical rotor arrangements involved the use of hinges to permit flapping but provided no mechanism by means of which both flapping and feathering could be introduced relative to the rotor shaft. The desired orientation of the rotor was achieved by tilting the rotor shaft. Since that time, the mechanical arrangement in most designs has been altered so that the rotor attitude is controlled by feathering. A controllable amount of first-harmonic blade-pitch change is thus introduced relative to the axis of the rotor shaft. The two systems are aerodynamically equal; the blades follow the same path relative to space axes, as regards both pitch angle and flapping angle, for any given flight condition regardless of the mechanical means used for achieving it. This fact may be confirmed by inspection and can also be demonstrated mathematically.

In practice, the present rotor systems involve both flapping and feathering, and, for comparison with theory based on either the assumption of no feathering or no flapping, a conversion would be necessary. Since most of the available treatments assume no feathering, it is expedient to convert to this condition.

The conversion involved is simply a change of reference axes, which can be more easily understood by reference to figure 2. Figure 2(a) shows a longitudinal cross-section of the rotor cone for a system without feathering (that is, a pure flapping system) and the longitudinal tilt of the rotor shaft required to orient the rotor to correspond to some particular flight condition. In figure 2(b) the flight condition is assumed to be the same and, hence, the tilt of the rotor cone with respect to the flight path is the same, but the tilt is achieved by a combination of shaft tilt and blade feathering relative to this shaft. For clarity, the feathering is assumed to be achieved by a "swash" (or control) plate linked in such a manner that a 1° longitudinal tilt of the plate produces 1° of feathering in the blades when the blades are in the lateral position and no feathering when the blades are alined longitudinally. The blades do not change pitch with reference to the plane of the swash plate, as they revolve, no matter how the swash plate is tilted. In figure 2(a) the blades do not change pitch as referred to the shaft axis while revolving. The axis of the swash plate in figure 2(b) is thus equivalent to the shaft axis in figure 2(a) insofar as periodic pitch-angle variation is concerned. Since the swash plate need not be rigged as assumed in this figure and since the true significance of its axis is that no feathering is introduced relative to it, this axis is termed the axis of no feathering rather than the swash-plate axis. It corresponds to the shaft axis for an equivalent (or pure flapping) system incorporating no means for feathering.

A lateral cross-section of the rotor cone similar to figure 2 may be drawn which would show the relationships between flapping and feathering in that plane.

Flapping and feathering angles referred to the rotor shaft can be converted to flapping about the axis of no feathering by means of simple relationships. In the expressions that follow, the subscript "s" indicates that the quantity is referenced to the rotor shaft. The measured flapping relative to the shaft is expressed as

$$\beta_s = a_{0_s} - a_{1_s} \cos \psi - b_{1_s} \sin \psi - a_{2_s} \cos 2\psi - b_{2_s} \sin 2\psi \ldots$$

The measured pitch angle relative to the shaft is expressed as

$$\theta_s = A_{0_s} - A_{1_s} \cos \psi - B_{1_s} \sin \psi - A_{2_s} \cos 2\psi - B_{2_s} \sin 2\psi \ldots$$

Flapping with respect to the axis of no feathering is expressed as

$$\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi \ldots$$

Pitch angle with respect to the axis of no feathering is expressed as

$$\theta = A_0 - A_2 \cos 2\psi - B_2 \sin 2\psi \ldots$$

The first harmonic terms become zero by definition. Then, the equations for transferring from the shaft axis to the axis of no feathering are:

$$a_0 = a_{0_s}$$

$$A_0 = A_{0_s}$$

$$a_1 = a_{1_s} + B_{1_s}$$

$$b_1 = b_{1_s} - A_{1_s}$$

All harmonics greater than the first are unaffected by the transfer of axes.

If the angle between the perpendicular to the rotor shaft and the air stream is defined as α_s, the rotor angle of attack α is given by

$$\alpha = \alpha_s - B_{1_s}$$

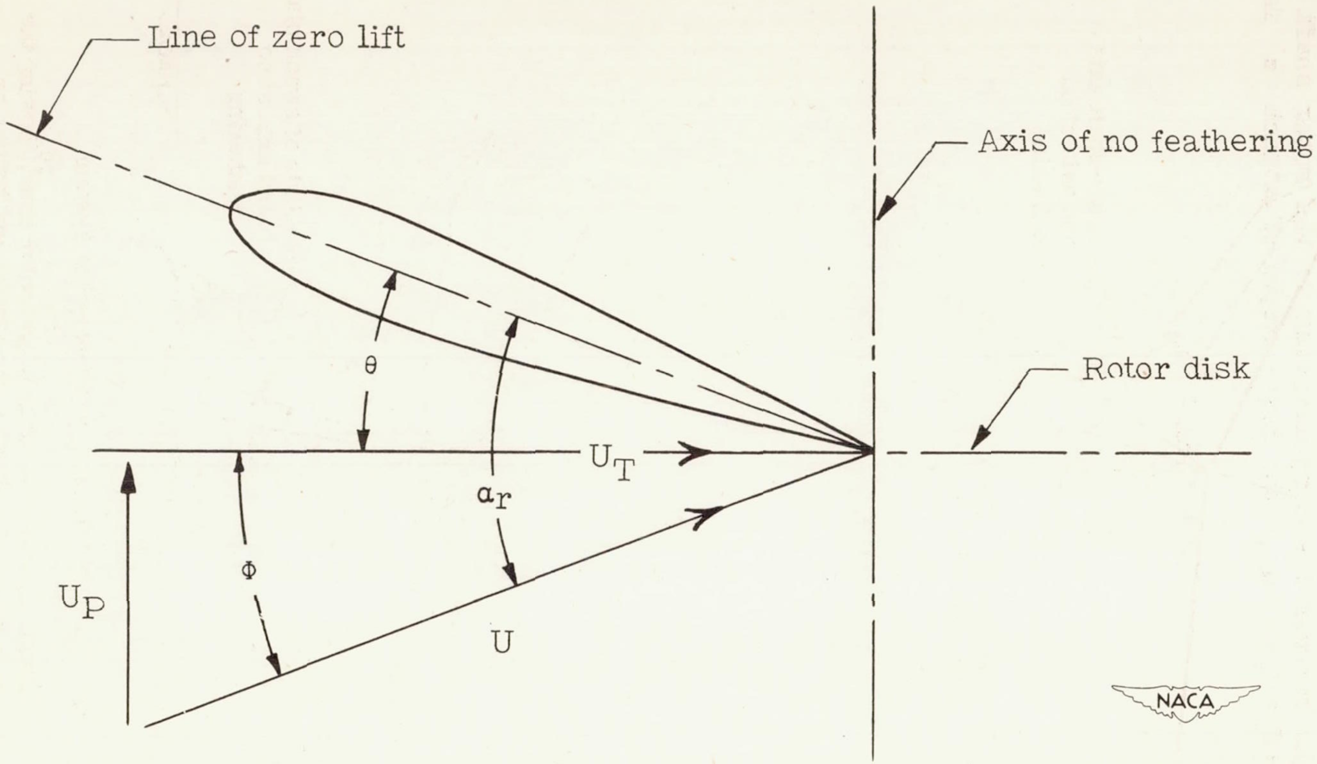
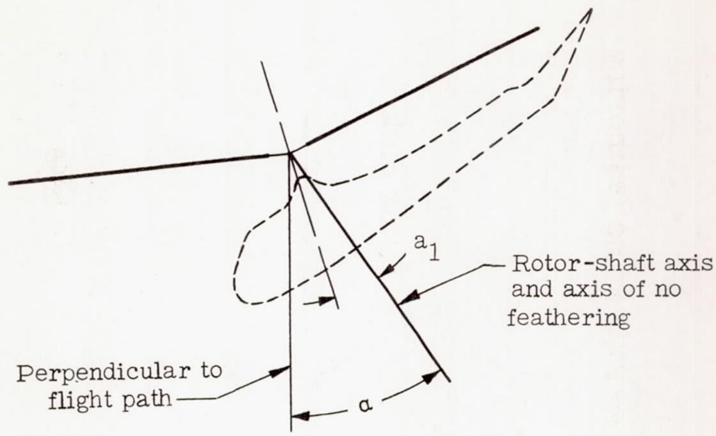
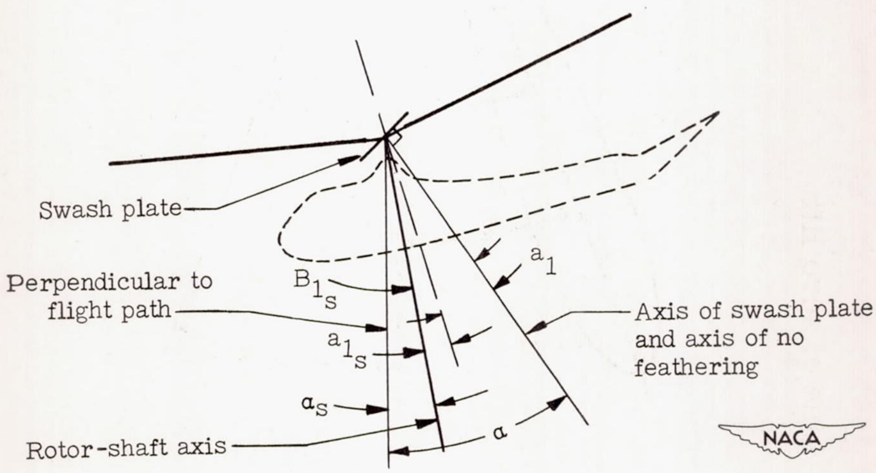


Figure 1.- Air flow relative to a blade element.



(a) Pure flapping system.



(b) Combined flapping and feathering system.

Figure 2.- Longitudinal cross sections of the rotor cone, showing the equivalence of a system involving both flapping and feathering (referred to the shaft axis) to a pure flapping system (referred to the axis of no feathering).