NACA TN No. 1544

27 FEB 1948

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1544

STRENGTH OF THIN-WEB BEAMS WITH TRANSVERSE LOAD APPLIED AT AN INTERMEDIATE UPRIGHT

By L. Ross Levin

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

Washington
February 1948

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 1544

STRENGTH OF THIN-WEB BEAMS WITH TRANSVERSE LOAD APPLIED AT AN INTERMEDIATE UPRIGHT

By L. Ross Levin

SUMMARY

Results are presented of tests of several 24S-T aluminum-alloy thin-web beams with transverse load applied at the end of an intermediate upright. A method of computing stresses and predicting failures in these directly loaded uprights is presented. A comparison between the experimental and calculated results is given.

INTRODUCTION

Information on the design of uprights in diagonal-tension beams is limited at present to uprights which do not have any directly applied axial loads. In practice, however, part or all of the transverse load is frequently applied at one of the intermediate uprights and a method of designing the uprights seems desirable when this loading condition exists. Several beams were tested with loads applied at the intermediate uprights, and a method of designing the uprights was developed from these tests.

SYMBOLS

A_U	cross-sectional area of upright, square inches
A_{U_e}	effective cross-sectional area of upright, square inches (see reference 1)
A_{W_e}	cross-sectional area of web effective with uprights in compression, square inches (see appendix)
E	Young's modulus, ksi
I_F	moment of inertia of flange, inches^4
P	load, kips

P_U internal force in upright, kips

P_{pred} predicted ultimate load on beam (total load on beam, fig. 2), kips

P_{ult} ultimate load on beam (total load on beam, fig. 2), kips

S transverse shear force, kips

d spacing of uprights, inches

e distance from median plane of web to centroid of (single) upright, inches

h_e depth of beam measured between centroids of flanges, inches

h_U length of beam upright measured between centroids of upright-to-flange rivet patterns, inches

k diagonal-tension factor

t thickness of web, inches

t_U thickness of upright, inches

σ_U normal stress in upright, ksi

σ_o "basic" allowable stress for forced crippling of uprights (valid for stresses below proportional limit in compression of upright material), ksi

τ shear stress, ksi

τ_{cr} critical shear stress, ksi

τ_{ult} ultimate shear stress, ksi

ρ centroidal radius of gyration of cross section of upright about axis parallel to web (no sheet should be included), inches

Subscripts:

n number of station or bay

Superscripts:

DT diagonal tension

P direct load

TEST SPECIMENS AND TEST PROCEDURE

The test specimens were 24S-T aluminum-alloy diagonal-tension beams 80 inches long by 24 inches deep with 0.032-inch webs and 1- by 1- by $\frac{1}{16}$-inch uprights on one side of the web. Three beams had uprights 20 inches apart and three beams had uprights 10 inches apart. A sketch of the beams is given in figure 1; the properties of the beams are given in table 1.

The beams were tested as cantilevers with supports to prevent lateral buckling of the flanges. Figure 2 shows the manner in which each of the beams was loaded and gives the numbers of the bays and stations on each beam. The beams were loaded until failure occurred.

Strains were measured with resistance-type wire strain gages in the attached leg of the upright angle at the station 30 inches from the tip. In beam 6, which had 10-inch upright spacing and all of the load applied at the station 30 inches from the tip, strains were measured in the uprights at 20, 30, and 40 inches from the tip with resistance-type wire strain gages.

TEST RESULTS AND ANALYSIS

Stresses in Intermediate Uprights

Uprights without direct load.- The stress in an upright without any direct load was calculated by the methods given in reference 1. The average measured stresses and the average calculated stresses in the uprights (average over the length of the upright) of beams 1 and 4 are shown in figure 3. The trend of the measured stresses is approximately the same as the trend of the calculated stresses. At high loads the measured stresses and calculated stresses were very nearly the same.

Uprights with direct load.- The theory presented in reference 1 is not applicable to the case of a load applied at one of the intermediate uprights; and in the tests which were made to verify this theory, the load was never applied at the upright on which measurements were taken. The only forces in the uprights of the beams discussed in reference 1 were the forces caused by the diagonal-tension action in the web. In beams which have some load applied at one of the intermediate uprights, there is a force P_U^{DT} in the uprights caused by diagonal-tension action in the web and a force P_U^{P} caused by some of the external load P being transmitted directly to the upright. The total force in the upright is then

$$P_U = P_U^{DT} + P_U^{P} \tag{1}$$

The tests, particularly those of beam 6 which had all the load applied at station 30, indicated that the load applied to the beam was distributed over several uprights. There were appreciable stresses outboard of the section where the load was applied, which would not have been present if all of the load applied at the end of an upright had been carried by that upright. In the estimation of the distribution of the external load to the uprights, the flange and uprights were assumed to deflect as shown in figure 4. The flange was assumed to have zero slope and zero deflection at the first upright on each side of the station where the load was applied. The force caused by the direct load acting on the uprights was assumed to vary linearly from a maximum of $\left(P_U{}^P\right)_n$ at the lower end to zero at the top. The force at the end of the upright directly over the load is then

$$\left(P_U{}^P\right)_n = P_n \frac{\left(A_{U_e} + A_{W_e}\right)_n d^3}{\left(A_{U_e} + A_{W_e}\right)_n d^3 + 12I_F h_U} \tag{2}$$

and the force on each adjacent upright is

$$\left(P_U{}^P\right)_{n+1} = \left(P_U{}^P\right)_{n-1} = \frac{P_n - \left(P_U{}^P\right)_n}{2} \tag{3}$$

The distribution of the load must be determined by successive approximation because the area of the web A_{W_e}, which is effective with the uprights in compression, is a function of the loading ratio τ/τ_{cr} and this ratio varies with the load distribution. For the beams discussed herein the fourth approximation was usually satisfactory. Some changes in the methods of analysis given in reference 1 were made when the upright forces and stresses caused by the diagonal-tension action in the web were calculated. These changes were made because the conditions in adjacent bays were not identical when the load was applied at one of the intermediate uprights; whereas the charts in reference 1 were based on the assumption that the conditions were identical in all bays. These details of the analysis are illustrated by the numerical example given in the appendix.

The upright stresses for beams 2, 3, 5, and 6 were calculated in the manner just presented. The differences between the average measured stresses and the average calculated stresses in the upright (average over the length of the upright) at station 30 were about the same on each of these beams where all or part of the load was applied at station 30 as on beams 1 and 4 which did not have any load applied at station 30. (See fig. 3.) Beams 1 and 4 were similar to the beams used in reference 1. The average measured and average calculated stresses in the uprights at stations 20, 30, and 40 of beam 6 are shown in figure 5. At stations 20 and 40 the average measured stresses were less than the average calculated stresses except for a few points at station 20 at the higher loads. The differences between the average measured and the average calculated stresses in these uprights at stations 20 and 40 were usually greater than the differences at station 30 where the load was applied.

Ultimate Load Tests

In beams 1 and 4 the ultimate loads were computed by the methods given in reference 1. The allowable stress was computed from

$$\sigma_o = 35k \sqrt{t_U/t} \tag{4}$$

which represents the average value of allowable upright stress for the beams discussed in reference 1.

The allowable upright stress at the neutral axis (longitudinal center line) of the beams loaded at an intermediate upright was obtained from formula (4). If k was not the same on both sides of the upright, the average value was used. The effective value given in reference 1 was used if k was less than 0.5. In reference 1, where all of the load on the uprights was caused by diagonal-tension action, σ_o was taken as the allowable value of the maximum stress in the upright which always occurred at the neutral axis of the beam. In beams 2, 3, 5, and 6, which had load applied at one of the intermediate uprights, the maximum computed stress occurred near the loaded end of the uprights; however, in the beams with load applied at an intermediate upright, the best method of predicting upright failures was to use σ_o as the allowable value of upright stress at the neutral axis (longitudinal center line) of the beam. It was necessary to check several uprights for the possibility of failure when the ultimate load on the beams with load at an upright was computed because neither the computed stresses nor the allowable stresses were the same in all uprights. Table 2 gives the ratio of actual ultimate load to predicted ultimate load P_{ult}/P_{pred} for each upright in which failure seemed likely. Both P_{ult} and P_{pred} are the total load on the beam. For the uprights which actually failed, P_{ult}/P_{pred} varied from 1.202 to 0.869. The range of P_{ult}/P_{pred} for the tests in reference 1 was from approximately 1.20 to 0.80 when P_{pred} was based on the allowable stresses given by formula (4).

CONCLUSIONS

Tests were made of 24S-T aluminum-alloy thin-web beams with a transverse load applied at the end of one of the intermediate uprights, and a method of computing stresses and predicting failures in these directly loaded uprights was developed.

Comparison of the test results with the method of analysis presented indicated the following conclusions:

1. The stresses in the uprights were predicted with about the same accuracy in the beams with a load applied at one of the intermediate uprights as in the beams which had all the load applied at the end upright.

2. The ultimate loads on the beams which had a load applied at an intermediate upright were predicted with about the same accuracy as the ultimate loads on the beams which had all the load applied on the end upright.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., November 5, 1947

APPENDIX

NUMERICAL EXAMPLE

The method of analysis presented in this paper is illustrated by calculating the stresses in the upright at station 30 of beam 6 when a load of 7 kips is applied at station 30. The location of the load and the numbers of the bays and stations are shown in figure 2. The necessary data for the analysis are as follows:

$$h_e = 24.28 \text{ in.}$$

$$h_U = 24.00 \text{ in.}$$

$$d = 10.0 \text{ in.}$$

$$t = 0.0332 \text{ in.}$$

$$t_U = 0.0650 \text{ in.}$$

$$A_U = 0.1288 \text{ sq in.}$$

$$A_{U_e} = 0.0732 \text{ sq in.}$$

$$\frac{A_{U_e}}{dt} = 0.2205$$

$$\rho = 0.311 \text{ in.}$$

$$e = 0.271 \text{ in.}$$

$$I_F = 0.82 \text{ in.}^4$$

$$\tau_{cr} = 0.850 \text{ ksi}$$

$$E = 10.6 \times 10^3 \text{ ksi}$$

The first step in calculating the stresses in the uprights is to determine the distribution of the external load to the uprights at stations 20, 30, and 40. This load distribution is given by formulas (2) and (3) which may be solved by successive approximations. For a first approximation it may be assumed that none of the web is effective with the uprights in compression; that is, $A_{W_e} = 0$.

First Approximation

The effective area of the web A_{W_e} is assumed to be zero and the load on the upright at station 30, which is computed from formula (2), is

$$\left(P_U P\right)_{30} = P_{30} \frac{\left(A_{U_e} + A_{W_e}\right)_{30} d^3}{\left(A_{U_e} + A_{W_e}\right)_{30} d^3 + 12 I_F h_U}$$

$$= P_{30} \frac{0.0732 \times 10.0^3}{(0.0732 \times 10.0^3) + 12(0.82 \times 24.00)}$$

$$= 0.236\ P_{30}$$

The load on the uprights at stations 20 and 40, which is calculated from formula (3), is

$$\left(P_U P\right)_{20} = \left(P_U P\right)_{40} = \frac{P_{30} - \left(P_U P\right)_{30}}{2}$$

$$= \frac{P_{30} - 0.236 P_{30}}{2}$$

$$= 0.382 P_{30}$$

Second Approximation

The distribution determined in the first approximation is used to determine an effective area of the web, which may be used to obtain a closer approximation of the load distribution. The first step is to calculate the shear loads and shear stresses in bays 3 and 4 from the distribution given by the first approximation. The web shear load at any section of the beam is the sum of all the loads $P_U P$ in the uprights outboard of that section. The shear stresses in bays 3 and 4 are

$$\tau_3 = \frac{S_3}{h_e t}$$

$$= \frac{\left(P_U P\right)_{20}}{h_e t}$$

$$= \frac{0.382 \times 7.00}{24.28 \times 0.0332}$$

$$= 3.31 \text{ ksi}$$

$$\tau_4 = \frac{S_4}{h_e t}$$

$$= \frac{\left(P_U P\right)_{20} + \left(P_U P\right)_{30}}{h_e t}$$

$$= \frac{(0.382 + 0.236)7.00}{24.28 \times 0.0332}$$

$$= 5.36 \text{ ksi}$$

The loading ratio τ/τ_{cr} for each bay is then

$$\left(\frac{\tau}{\tau_{cr}}\right)_3 = \frac{3.31}{0.850}$$

$$= 3.9$$

$$\left(\frac{\tau}{\tau_{cr}}\right)_4 = \frac{5.36}{0.850}$$

$$= 6.3$$

The next step is to determine the diagonal-tension factor k for each bay. By use of figure 7 of reference 1 which gives k as a function of τ/τ_{cr}, the values of k for bays 3 and 4 are as follows:

$$k_3 = 0.29$$

$$k_4 = 0.38$$

The area of the web which is effective with the upright in compression is given in reference 1 as

$$A_{W_e} = 0.5dt(1 - k)$$

and is applicable if k is constant. In beam 6, k is not the same in adjacent bays; therefore, the average value $\frac{k_3 + k_4}{2}$ is used to calculate A_{W_e} at station 30

$$A_{W_{e_{30}}} = 0.5dt \left(1 - \frac{k_3 + k_4}{2}\right)$$

$$= 0.5 \times 10.0 \times 0.0332 \left(1 - \frac{0.29 + 0.38}{2}\right)$$

$$= 0.1102 \text{ sq in.}$$

This effective web area is used to compute a new load distribution. The load on the upright at station 30 is

$$\left(P_U P\right)_{30} = P_{30} \frac{\left(A_{U_e} + A_{W_e}\right)_{30} d^3}{\left(A_{U_e} + A_{W_e}\right)_{30} d^3 + 12 I_F h_U}$$

$$= P_{30} \frac{(0.0732 + 0.1102)10.0^3}{(0.0732 + 0.1102)10.0^3 + 12(0.82 \times 24.00)}$$

$$= 0.438 P_{30}$$

The load on the uprights at station 20 and 40 is

$$\left(P_U P\right)_{20} \equiv \left(P_U P\right)_{40} = \frac{P_{30} - \left(P_U P\right)_{30}}{2}$$

$$= \frac{P_{30} - 0.438 P_{30}}{2}$$

$$= 0.281 P_{30}$$

This distribution is not the same as that obtained in the first approximation; therefore, a third approximation is made.

Third Approximation

The calculations required for the third approximation are similar to those which were made in the second approximation. The load distribution given by the third approximation is

$$\left(P_U{}^P\right)_{30} = 0.442P_{30}$$

$$\left(P_U{}^P\right)_{20} \equiv \left(P_U{}^P\right)_{40} = 0.279P_{30}$$

Fourth Approximation

The fourth approximation is obtained in the same manner as the previous approximations. The values for the diagonal-tension factors, the effective web area, and the load distribution are the same as those given by the third approximation. These values are

$$k_3 = 0.22$$

$$k_4 = 0.41$$

$$A_{We_{30}} = 0.1137 \text{ sq in.}$$

$$\left(P_U{}^P\right)_{30} = 0.442P_{30}$$

$$\left(P_U{}^P\right)_{20} \equiv \left(P_U{}^P\right)_{40} = 0.279P_{30}$$

The stresses in the uprights can now be computed from the quantities given by the fourth approximation. The general expression for the total stress in the upright is

$$\sigma_U = \sigma_U^{DT} + \sigma_U^{P}$$

The first step is to determine the average stress in the upright at station 30. The average upright stress (average over the length of the upright) caused by the direct load is

$$\left(\sigma_U^P\right)_{30av} = \frac{\left(P_U^P\right)_{30}}{2\left(A_{U_e} + A_{W_e}\right)_{30}}$$

$$= \frac{0.442 \times 7.00}{2(0.0732 + 0.1137)}$$

$$= 8.21 \text{ ksi}$$

The average upright stress caused by the diagonal-tension action in the web may be determined from figure 8 of reference 1 which gives σ_U/τ as a function of the loading ratio τ/τ_{cr} and the ratio A_{U_e}/dt. This chart was computed for beams in which the conditions in adjacent bays were identical; therefore, the effect of each bay on the stress in the upright was the same, and a single value of σ_U/τ could be used to indicate the stress in the upright. In beam 6 where the conditions in two adjacent bays are not the same, the upright stress caused by the diagonal-tension action in each bay is assumed to be half of that indicated by the value of σ_U/τ for that bay which is given by figure 8 of reference 1. By use of figure 8 of reference 1, the values of σ_U/τ for bays 3 and 4 are

$$\left(\frac{\sigma_U}{\tau}\right)_3 = 0.32$$

$$\left(\frac{\sigma_U}{\tau}\right)_4 = 0.68$$

At station 30 the average upright stress caused by the diagonal-tension action is

$$\left(\sigma_U^{DT}\right)_{30_{av}} = \frac{\left(\frac{\sigma_U}{\tau}\right)_3 \tau_3}{2} + \frac{\left(\frac{\sigma_U}{\tau}\right)_4 \tau_4}{2}$$

$$= \frac{0.32 \times 2.42}{2} + \frac{0.68 \times 6.26}{2}$$

$$= 2.53 \text{ ksi}$$

The average stress in the upright at station 30 when there is a load of 7.0 kips at station 30 is then

$$\sigma_{U_{30_{av}}} = \left(\sigma_U^{DT}\right)_{30_{av}} + \left(\sigma_U^{P}\right)_{30_{av}}$$

$$= 2.53 + 8.21$$

$$= 10.74 \text{ ksi}$$

When the strength of the directly loaded uprights is computed, the upright stress at the longitudinal center line of the beam must be computed. The stress at the center line caused by the direct load is the same as the average stress caused by the direct load; that is,

$$\left(\sigma_U^{P}\right)_{30_{CL}} = \left(\sigma_U^{P}\right)_{30_{av}}$$

$$= 8.21 \text{ ksi}$$

The upright stress, at the longitudinal center line of the beam, caused by the diagonal tension is also the maximum upright stress caused by the diagonal tension. This maximum upright stress may be determined with the aid of figure 10 in reference 1 which gives the ratio of the maximum stress to the average stress. By use of figure 10 of reference 1, the values of σ_{max}/σ_{av} for bays 3 and 4 are as follows:

$$\left(\frac{\sigma_{max}}{\sigma_{av}}\right)_3 = 1.39$$

$$\left(\frac{\sigma_{max}}{\sigma_{av}}\right)_4 = 1.30$$

The average value of σ_{max}/σ_{av} is used to compute the stress in the upright between bays 3 and 4. At station 30 the upright stress at the center line of the beam is

$$\left(\sigma_U{}^{DT}\right)_{30_{CL}} = \frac{1}{2}\left[\left(\frac{\sigma_{max}}{\sigma_{av}}\right)_3 + \left(\frac{\sigma_{max}}{\sigma_{av}}\right)_4\right]\left(\sigma_U{}^{DT}\right)_{30_{av}}$$

$$= \frac{1}{2}\ (1.39 + 1.30)\ 2.53$$

$$= 3.31 \text{ ksi}$$

The total upright stress at the center line of the beam at station 30 is then

$$\sigma_{U_{30_{CL}}} = \left(\sigma_U{}^{DT}\right)_{30_{CL}} + \left(\sigma_U{}^{P}\right)_{30_{CL}}$$

$$= 3.31 + 8.21$$

$$= 11.52 \text{ ksi}$$

REFERENCE

1. Kuhn, Paul, and Peterson, James P.: Strength Analysis of Stiffened Beam Webs. NACA TN No. 1364, 1947.

TABLE 1

PROPERTIES OF TEST BEAMS

Beam	h_e (in.)	h_U (in.)	t (in.)	t_U (in.)	d (in.)	A_U (sq in.)	A_{U_e} (sq in.)	$\frac{A_{U_e}}{dt}$	ρ (in.)	Flanges (2 ∠s) (in.)
1	24.28	24.00	0.0326	0.0663	20.0	0.1230	0.0698	0.1072	0.311	2 x 2 x 5/16
2	24.28	24.00	.0322	.0637	20.0	.1248	.0709	.1101	.311	2 x 2 x 5/16
3	24.28	24.00	.0319	.0631	20.0	.1248	.0710	.1112	.311	2 x 2 x 5/16
4	24.34	24.10	.0325	.0654	10.0	.1230	.0698	.2145	.311	2 x 2 x 5/16
5	24.28	24.00	.0328	.0650	10.0	.1241	.0706	.2150	.311	2 x 2 x 5/16
6	24.28	24.00	.0332	.0650	10.0	.1288	.0732	.2205	.311	2 x 2 x 5/16

NACA

TABLE 2

TEST DATA AND RESULTS

Beam	Calculated τ_{cr} (ksi)	P_{ult} (kips) (a)	τ_{ult} (ksi)		$\frac{\tau_{ult}}{\tau_{cr}}$		P_{pred} (kips) (a)			$\frac{P_{ult}}{P_{pred}}$			Observed location of failure (station)	Location of load (station)
			Station -30	Station +30	Station -30	Station +30	Station 30	Station 40	Station 50	Station 30	Station 40	Station 50		
1	0.303	11.70	14.78	14.78	48.7	48.7	13.45	----------	13.45	0.869	----------	0.869	30,50	0
2	.282	11.80	8.07	14.53	26.6	48.0	10.80	----------	13.16	1.093	----------	.898	50	0,30
3	.270	7.80	.50	9.55	1.85	35.4	7.80	----------	12.15	1.000	----------	.642	30	30
4	.820	15.50	19.72	19.72	24.1	24.1	17.15	17.15	17.15	.904	0.904	.904	40,50	0
5	.832	16.60	13.78	17.54	16.5	21.1	15.00	13.80	----------	1.105	1.202	----------	40,50	0,30
6	.850	11.50	4.17	10.08	4.91	11.8	11.95	12.15	----------	.962	.947	----------	30	30

[a]Total load on beam (fig. 2).

NACA

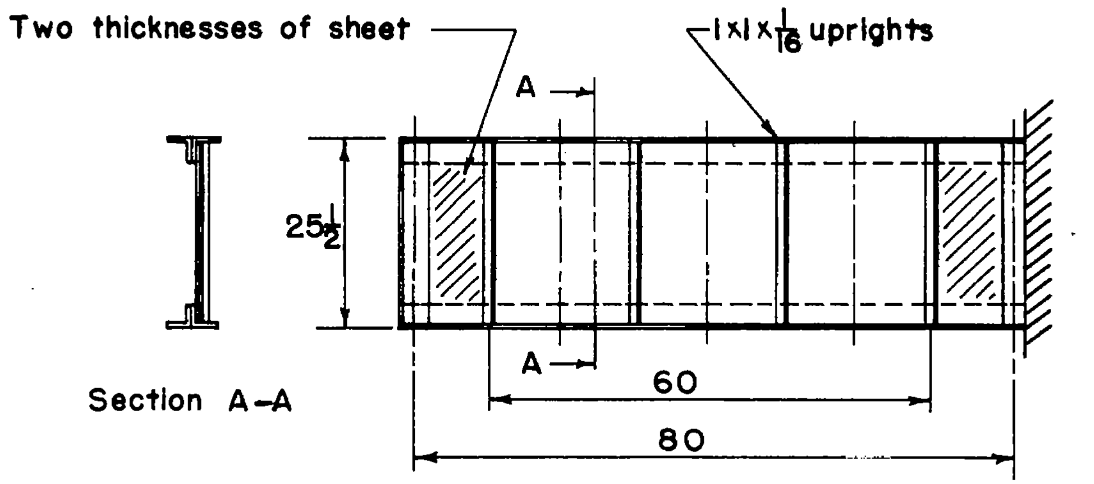

Figure 1.— Nominal dimensions of test beams.
(All dimensions are in inches.)

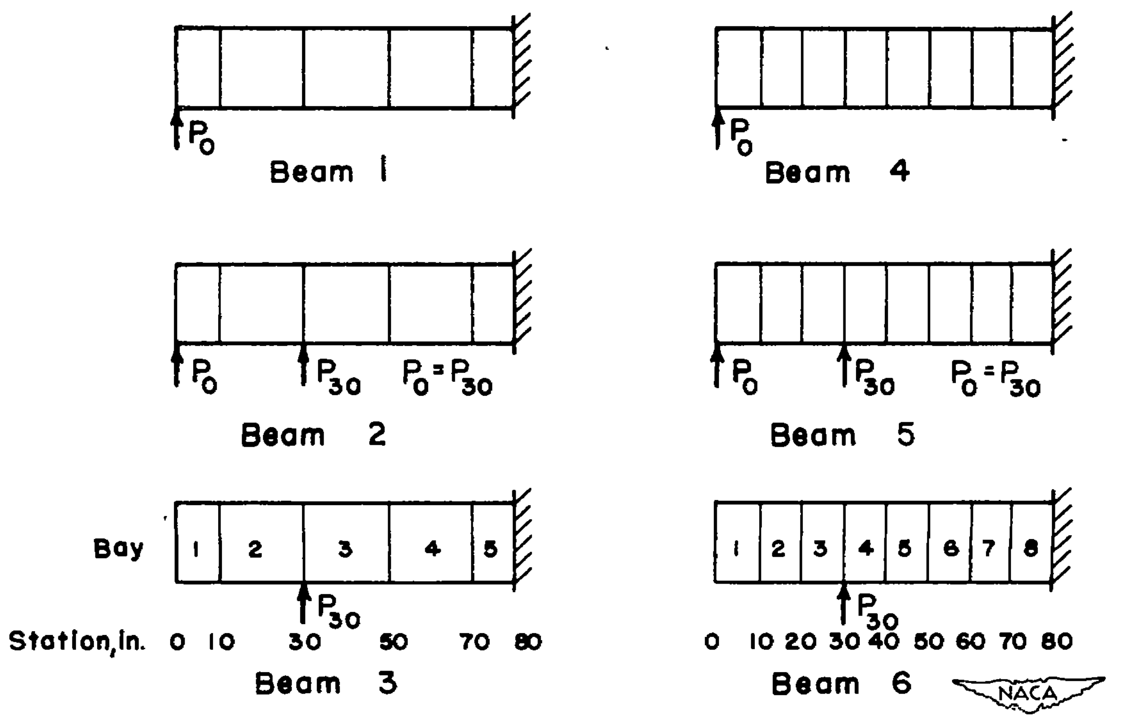

Figure 2.— Location of loads and numbers of stations and bays on each beam.

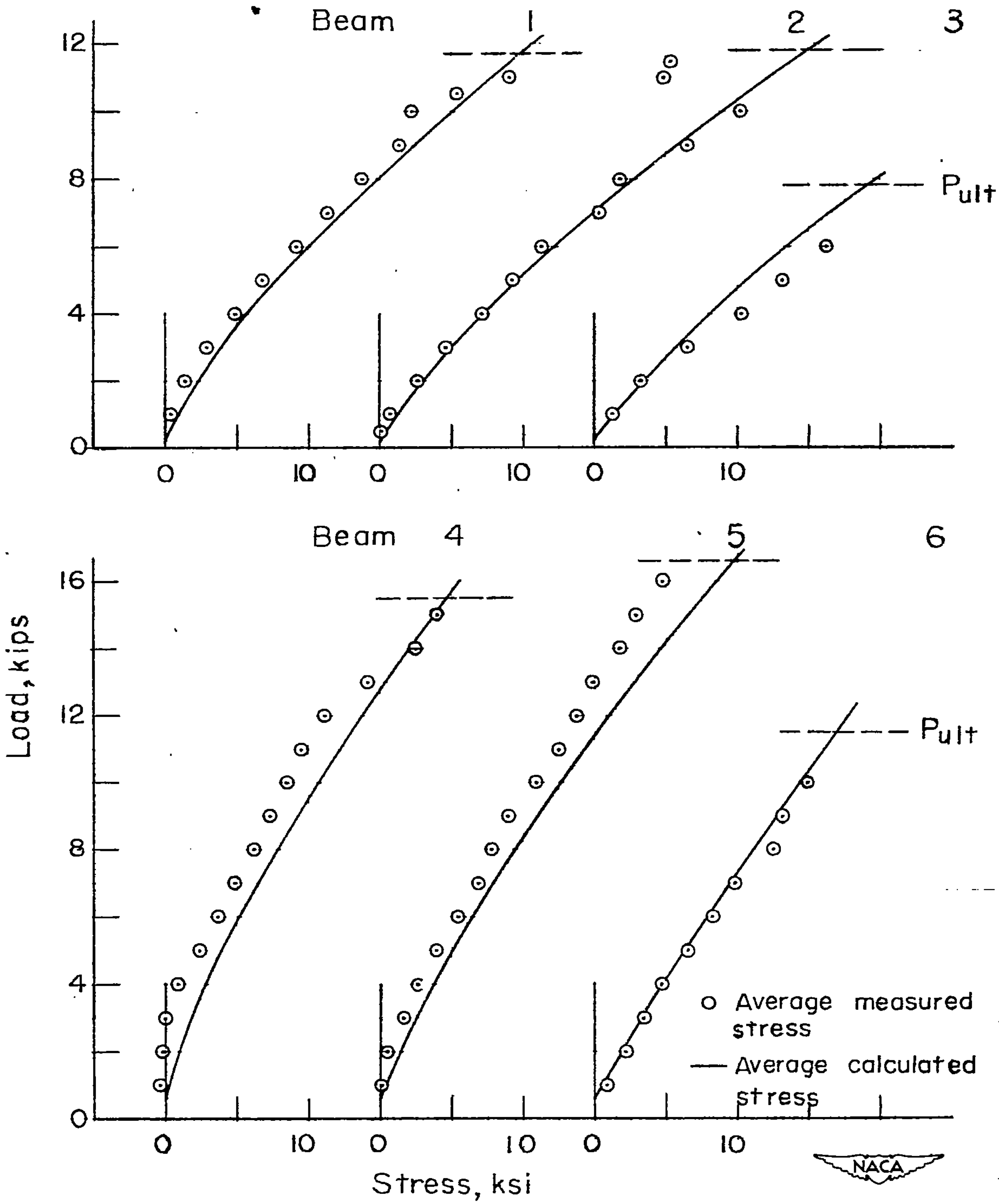

Figure 3.— Stresses in uprights at station 30.

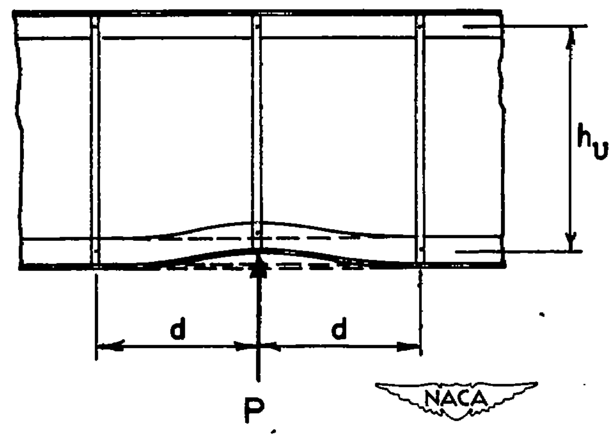

Figure 4.– Assumed deflection of flange and upright.

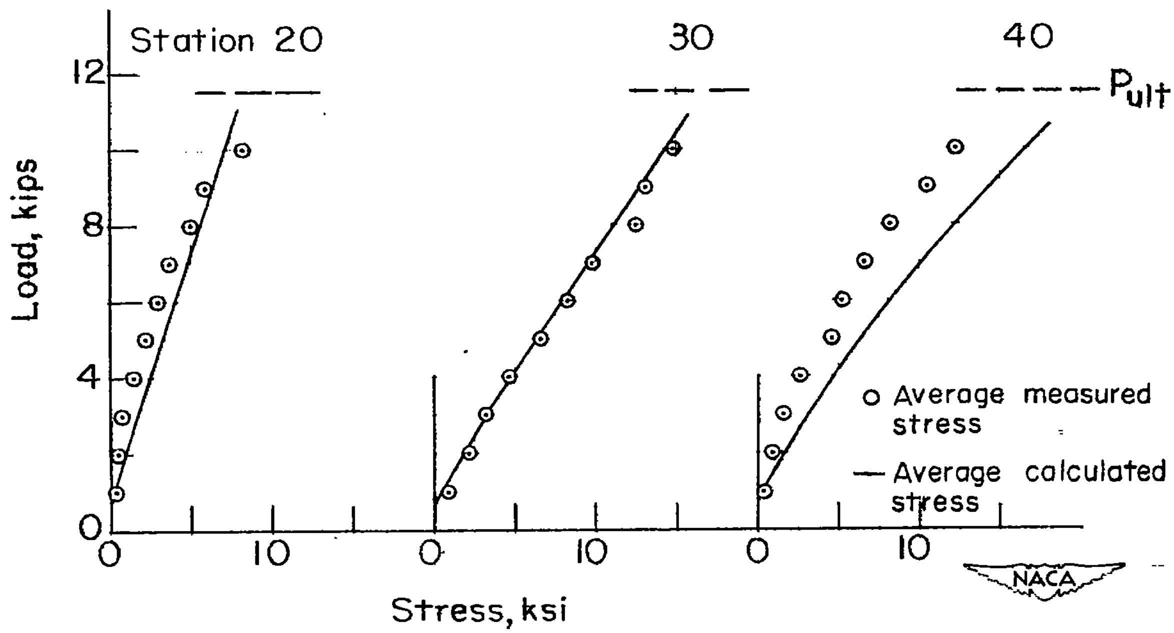

Figure 5.– Stresses in uprights of beam 6.