NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1813

A STUDY OF FLOW CHANGES ASSOCIATED WITH AIRFOIL

SECTION DRAG RISE AT SUPERCRITICAL SPEEDS

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SUMMARY

A study of experimental pressure distributions and section charac-
teristics for several moderately thick airfoil sections was made. A
correlation appears to exist between the drag-divergence Mach number
and the free-stream Mach number for which sonic velocity occurs at the
airfoil crest, the chordwise station at which the airfoil surface is
tangent to the free-stream direction. It was found that, since the Mach
number for which sonic velocity occurs at the airfoil crest can be
estimated satisfactorily by means of the Prandtl-Glauert rule, a method
is provided whereby the drag-divergence Mach number of an airfoil
section at a given angle of attack can be estimated from the low-speed
pressure distribution and the airfoil profile. This method was used
to predict with a reasonable degree of accuracy the drag-divergence
Mach number of a considerable number of airfoil sections having diverse
shapes and a wide range of thickness-chord ratios.

The pressure distributions and section force characteristics of
several moderately thick airfoil sections at Mach numbers above the
drag-divergence Mach number were analyzed. Some of the characteristics
of the flow over these airfoils at supercritical Mach numbers are
discussed.

INTRODUCTION

The first intensive efforts to develop airfoil sections suitable
for high-subsonic-speed applications were directed toward obtaining
sections with the highest possible critical speeds. It was reasoned
that, if the first occurrence of local sonic velocity on the airfoil
surface could be delayed to higher free-stream Mach numbers, then, the

adverse lift and drag changes associated with supercritical speeds
could be delayed. However, experimental measurements show that the
onset of these adverse effects is not directly related to the
critical Mach number. For some types of airfoil sections the drag
rises rapidly as soon as the free-stream Mach number exceeds the
critical; whereas for other types no appreciable drag rise occurs
until the Mach number of the free stream is considerably above the
critical.

No adequate method has been presented for predicting the free-
stream Mach numbers at which the various supercritical flow changes
occur. The purpose of the present report is to investigate these
flow changes for two-dimensional transonic flow past conventional and
low-drag airfoil sections. The investigation is based primarily on
a systematic analysis of experimental data for airfoil sections of
15-percent-chord thickness. Although this thickness is greater than
is generally desirable for use at high speeds, it has the advantage
that airfoil-shape effects are less likely to be masked by boundary-
layer effects.

SYMBOLS

α airfoil section angle of attack

c airfoil chord

c_d airfoil section drag coefficient

c_l airfoil section lift coefficient

M ratio of local velocity to local velocity of sound

M_β free-stream Mach number at which sonic velocity is first
              reached at airfoil crest

M_{cr} critical Mach number of airfoil section (free-stream
              Mach number at which local sonic velocity is first
              reached on airfoil surface)

M_d drag-divergence Mach number (Mach number at which slope
              of curve of drag coefficient versus Mach number attains
              a value of 0.10)

M_o ratio of free-stream velocity to free-stream velocity
              of sound

M_s shock-stall Mach number (free-stream Mach number at which boundary-layer separation first occurs at rear of airfoil)

P pressure coefficient $\left(\dfrac{p-p_0}{q_0} \right)$

p local static pressure on the airfoil surface

p_0 static pressure of the free stream

q_0 dynamic pressure of the free stream

x chordwise distance from airfoil leading edge

$(x/c)_\beta$ airfoil crest (chordwise station at which the airfoil surface is tangent to the direction of the free stream)

DETERMINATION OF DRAG-DIVERGENCE MACH NUMBER

As the free-stream Mach number is increased through the critical value, the first abrupt change in airfoil section characteristics, for an airfoil section at a moderate angle of attack, is the rapid increase in drag coefficient. For some airfoil sections at certain angles of attack this drag rise begins at the critical Mach number; whereas for other cases it does not begin until the free-stream Mach number is considerably greater than the critical. Some high-speed pressure distributions and section characteristics (reference 1) for the NACA 65_2-215 (a = 0.5), 66,2-215 (a = 0.6), 0015, 23015, and 4415 airfoil sections have been studied in an attempt to determine the flow changes which are associated with the appearance of this more or less abrupt drag rise. In order to supplement these data, simultaneous pressure distributions and schlieren pictures of the flow around the NACA 23015 airfoil section were obtained in the Ames 1- by 3-1/2-foot high-speed wind tunnel. Before considering the specific problem of the flow changes associated with the appearance of the supercritical drag rise, the data for this airfoil section will be discussed in detail.

Measurements at Supercritical Speeds for
NACA 23015 Airfoil at 2° Incidence

Lift- and drag-coefficient variation with Mach number for the
NACA 23015 airfoil section at 2° angle of attack is presented in
figure 1. For this section at 2° angle of attack, the critical Mach
number, which is the free-stream Mach number at which sonic velocity
is first attained at some point on the airfoil surface, is 0.59.
The first abrupt change in the section characteristics occurs at a
somewhat higher free-stream Mach number, and consists of a drag-
coefficient increase from the low-speed value. That free-stream
Mach number at which the rate of increase of drag coefficient with
Mach number equals 0.1 is defined, as in reference 1, as the drag-
divergence Mach number. The free-stream Mach number at which the
experimental pressure distribution first indicates the presence of
marked boundary-layer thickness is termed the shock-stall Mach number.
More detailed information about the supercritical flow changes over
the NACA 23015 airfoil section at 2° angle of attack is obtained from
the schlieren pictures and corresponding pressure distributions pre-
sented in figure 2. At a Mach number of 0.60 there is a small region
of supersonic flow which contains alternate expansion and compression
regions, as can be seen from the schlieren picture. The pressure
distribution resembles that for subcritical speeds. When the free-
stream Mach number is increased to 0.65, the drag-divergence Mach
number, a strong shock wave appears near the airfoil crest (the
chordwise station at which the surface is tangent to the free-stream
direction) but no change in the boundary-layer thickness is apparent
in the schlieren photograph. From the pressure distribution, it is
seen that at the airfoil crest the local velocity is greater than
sonic. Thus, sonic velocity is reached at the airfoil crest at a Mach
number between 0.60 and 0.65. In figures 2(b), 2(c), and 2(d) the
terminal shock wave stands close to the chordwise station at which the
local pressure coefficient corresponds to sonic velocity. At 0.70 and
0.73 free-stream Mach numbers, there is a marked thickening of the
boundary layer ahead of the shock wave, which is located aft of the
airfoil crest. This rapid thickening of the boundary layer appears to
start at the airfoil crest. The pressure distribution over the rear
of the upper surface of the airfoil for free-stream Mach numbers of
0.70 and 0.73 resembles that for the smaller Mach numbers and thus
gives no indication of the presence of a thick boundary layer.
However, the schlieren photograph for a free-stream Mach number of 0.73
does indicate the presence of a thick boundary layer over the rear of
the airfoil upper surface. The drag coefficient at this Mach number is
about 0.04, which is comparable to the drag coefficient for low-speed
maximum lift at the same Reynolds number.

Figure 3(a) shows the variation of pressure coefficient with Mach

number at several chordwise stations on the upper surface of the NACA
23015 airfoil section at 2° angle of attack. Here the airfoil crest
is at the 22-percent-chord station on the upper surface. At points
well forward of the crest, that is, at the 5- and 10-percent-chord
stations, the local pressure coefficient ceases to fall with increas-
ing Mach number somewhat before the drag-divergence Mach number.
Closer to the crest, at the 15- and 20-percent-chord stations, the
smooth decrease in pressure coefficient with increasing Mach number
continues until the free-stream Mach number reaches the value for drag
divergence. After the drag-divergence Mach number is exceeded, the
pressure coefficients at points ahead of the airfoil crest rise in
such a manner as to maintain virtually constant local Mach numbers.
For points immediately behind the airfoil crest, the 25-, 30-, and
40-percent-chord stations on the upper surface, the local Mach
number does not rise smoothly from subsonic to supersonic values
since, at each chordwise position, as the shock wave moves past,
there is an abrupt rise from a local Mach number slightly less than
unity to a value greater than unity. However, at each of these
chordwise positions, the free-stream Mach number for which the local
velocity becomes sonic agrees quite well with the value obtained by
assuming the pressure-coefficient variation with free-stream Mach
number to follow the Prandtl-Glauert rule. For points far back on
the upper surface, the 60- and 85-percent-chord stations, at a free-
stream Mach number of about 0.73, there is a marked increase in the
rate of fall of local pressure coefficient with increasing free-
stream Mach number. The presence of a thick boundary layer over the
rear of the airfoil is thus indicated by the pressure distribution
and 0.73 is termed the shock-stall Mach number. A similar change in
the variation of local pressure coefficient with free-stream Mach
number occurs for the entire lower surface of the airfoil (fig. 3(b))
at a free-stream Mach number somewhat below 0.73. This marked
decrease of local pressure coefficient with increasing free-stream
Mach number resembles the pressure-distribution changes associated
with increasing boundary-layer separation as the lift coefficient of
an airfoil approaches maximum lift at low speeds. It is interesting
to note (fig. 1) that, for the NACA 23015 airfoil section at 2°
angle of attack, the marked decrease in lift coefficient with increas-
ing Mach number which begins at a Mach number of about 0.7 is
primarily a result of the rapid decrease of pressure coefficients on
the lower surface of the airfoil (fig. 3(b)).

<div align="center">

Measurements at Supercritical Speeds for NACA 23015
Airfoil at Various Angles of Attack

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In figure 4 are presented the variations of pressure coefficient

with Mach number at various chordwise stations for the NACA 23015
airfoil section at several angles of attack. The local pressure-
coefficient variation with Mach number at the crest location, for
each angle of attack shown in figure 4, is denoted by a solid
curve. It is seen that for each angle of attack the drag-divergence
Mach number differs only slightly from the free-stream Mach number
at which sonic velocity occurs at the airfoil crest. As the Mach
number is increased beyond the value for drag divergence there is a
rise in pressure coefficient with Mach number at points ahead of the
crest; and, at least up to a Mach number of 0.8, this pressure-
coefficient variation with free-stream Mach number is such as to
maintain practically constant local Mach numbers.

Origin of Drag Rise

The relation between the pressure-distribution changes noted for
the NACA 23015 airfoil section and the supercritical drag rise will
now be considered. At approximately the drag-divergence Mach number,
there is established over the forward portion of the upper surface of
the NACA 23015 airfoil a Mach number distribution which remains rela-
tively unchanged with some further increase of free-stream Mach
number. In figure 3, it is seen that, while pressure coefficients
ahead of the crest are becoming less negative, pressure coefficients
at points well aft of the airfoil crest become more negative as the
Mach number increases above that for drag divergence. This combina-
tion of pressure changes produces a rising pressure drag which appears
to be the primary factor in the supercritical drag rise. It should be
pointed out that the negatively increasing pressure coefficients aft
of the airfoil crest may result from two different causes: the
rearward growth of the supersonic region, and marked boundary-layer
thickening. Both of these causes may be present, as for example, on
the NACA 23015 airfoil section at 2° angle of attack.

Prediction of the Drag-Divergence Mach Number

In the preceding sections the flow changes over the NACA 23015
airfoil section at supercritical speeds were considered for various
angles of attack. It was noted that increasing the free-stream Mach
number produces no abrupt changes in the airfoil section character-
istics until the supersonic region extends behind the airfoil crest.
When this occurs, the drag begins to rise rapidly with increasing
Mach number. Figure 5 shows that for the NACA 23015 airfoil section
the calculated and experimental values of M_β, the Mach number at
which sonic velocity occurs at the airfoil crest, are in reasonable

agreement with measured values of the drag-divergence Mach number.
The experimental value of M_β at a given angle of attack was
obtained from a plot of the experimentally determined pressure
coefficient at the airfoil crest as a function of free-stream Mach
number. The free-stream Mach number at which this curve passed
through a value corresponding to a local Mach number of unity was
termed $M_{\beta_{exp}}$. The calculations consisted of determining the free-
stream Mach number at which sonic velocity was reached at the airfoil
crest by applying the Prandtl-Glauert rule to the low-speed experi-
mental pressure coefficient at the airfoil crest. For comparison,
there are also shown in figure 5 the critical Mach number M_{cr} and
the shock-stall Mach number M_S at which marked boundary-layer thick-
ening or separation is first indicated by negatively increasing
pressure coefficients over the tail of the airfoil. It is seen that
these latter two curves are not closely related to drag divergence.
Similar data for several other 15-percent-thick airfoil sections are
presented in figure 6. For all these cases, the Mach number for
which the velocity at the crest is calculated to be sonic is close to
the drag-divergence Mach number evaluated from the section drag
characteristics.

The five airfoil sections, for which drag-divergence has been
related to the occurrence of sonic velocity at the airfoil crest, are
relatively thick sections. Since no rigorous causal relation between
these phenomena has been demonstrated, it is desirable to investigate
the applicability of the method for thinner airfoil sections. In
figure 7, calculated values of M_β and measured drag-coefficient
variation with Mach number are presented for a large number of
airfoil sections. The calculations consisted of applying the Prandtl-
Glauert compressibility correction to the low-speed pressure distribu-
tions in order to determine the Mach number at which sonic velocity
occurs at the airfoil crest. The drag data for the 6-percent-thick
symmetrical sections were obtained from reference 2 and the data for
the cambered 6-series airfoils were measured in the Ames 1- by 3-1/2-
foot high-speed wind tunnel on 6-inch-chord models which spanned the
1-foot dimension of the tunnel. It is seen that for each of these
airfoil sections a useful measure of the free-stream Mach number at
which drag divergence occurs is that Mach number at which the calcu-
lated velocity at the airfoil crest is sonic.

<center>DISCUSSION OF FLOW CHARACTERISTICS AT MACH NUMBERS</center>

<center>ABOVE THE DRAG-DIVERGENCE MACH NUMBER</center>

In the previous sections of the report some characteristics of
the supercritical flow past airfoil sections at subsonic free-stream

speeds have been pointed out. These characteristics will be con-
sidered further in this section of the report. Attention has already
been directed to the aft boundary of the supersonic region. In
figure 8, the variation of the chordwise extent of the region of
supersonic flow over the NACA 23015 airfoil section is shown as a
function of Mach number. It is seen that the fore and aft boundaries
shift until the drag-divergence Mach number is reached. The forward
sonic point remains relatively fixed for some further increase in
free-stream Mach number while the aft sonic point continues to move
rearward of the airfoil crest $(x/c)_\beta$. At Mach numbers above that
for drag divergence the rear sonic point lies close to the position
of the shock wave, as determined from the schlieren photographs.
Reference 3 shows that the chordwise location of the shock wave can
be altered by scale effects. The question therefore arises as to
whether these data, obtained at a test Reynolds number of about
2 million, apply at larger Reynolds numbers. Unpublished German
pressure-distribution data, for the NACA 23015 airfoil section at
0° incidence and a Reynolds number of about 5 million, are shown for
comparison in figure 8. The chordwise extent of the supersonic
region is nearly the same for both sets of experimental data.
However, as can be seen from figure 4, there is some small difference
between the variation of local pressure coefficients with Mach number
at 2 million and 5 million Reynolds number.

The appearance of a fixed chordwise location of the forward
sonic point at subsonic free-stream Mach numbers above that for drag-
divergence is to be expected since, as has been noted for the
NACA 23015 airfoil section, the pressure-coefficient variation at
these Mach numbers appears to be such as to maintain an approximately
constant local Mach number distribution over that portion of the
airfoil surface ahead of the crest. The location of the forward
sonic point for each of the airfoils of reference 1 is shown as a
function of Mach number in figure 9. The farthest forward station at
which pressures were measured for that report was the 2.5-percent-
chord station so that the angles of attack which could be considered
were limited to those for which the sonic point was near or behind
this station. It appears (fig. 9) that the forward sonic point is
fixed at Mach numbers above the drag-divergence Mach number only for
those cases for which the sonic point lies between the 2.5- and
15-percent-chord stations. When the sonic point is outside of this
region at the drag-divergence Mach number there is some movement of
the sonic point with further increase in Mach number. This movement
appears to be greatest for the two low-drag airfoil sections which
have, at a fixed Mach number, large variation of sonic point location
with angle of attack.

The experimental pressure distribution for each of the cases con-
sidered in figure 9 reveals that at Mach numbers above that of drag
divergence the maximum local Mach number occurs near the airfoil
crest. This phenomenon is probably characteristic of moderately thick
airfoil sections only; but, for these cases, it suggests the possibility
of predicting the peak pressure coefficient. In figure 10 the experi-
mental local Mach number distribution over the NACA 23015 airfoil
section at 2° angle of attack is shown for several free-stream Mach
numbers greater than that of drag divergence. The Prandtl-Meyer
theoretical Mach number distribution for this airfoil is shown for
comparison; although, of course, this theory is not valid for a super-
sonic region of limited extent normal to the airfoil chord. It is
seen that a close approximation to the experimental local Mach number
distribution can be obtained by taking one-half of the Prandtl-Meyer
Mach number increment per degree of surface turning. Figure 11 shows
that, for the airfoil sections considered in figure 9, a useful
approximation to the local Mach number at the airfoil crest for Mach
numbers above that for drag divergence is obtained by using one-half
the Mach number increment predicted by the Prandtl-Meyer theory. In
these calculations the sonic point was assumed to be that at the
drag-divergence Mach number.

CONCLUDING REMARKS

A good measure of the free-stream Mach number at which the abrupt
supercritical drag rise begins has been shown to be that Mach number
at which sonic local velocity occurs at the airfoil crest. This fact
provides a method for calculating the drag-divergence Mach number. The
method consists of determining the free-stream Mach number at which
sonic velocity occurs at the airfoil crest by applying the Prandtl-
Glauert compressibility factor to the low-speed pressure distribution.
The usefulness of this method is demonstrated by comparing the calcu-
lated drag-divergence Mach number with that determined experimentally
for a wide variety of airfoil shapes.

The mixed subsonic and supersonic flow fields which exist at
supercritical speeds are extremely difficult to treat mathematically.
The general considerations of this report are no substitute for such
an analysis, but they do present a simplified picture of this complex
type of flow which should be useful in planning further studies.

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        National Advisory Committee for Aeronautics,
                Moffett Field, Calif.

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   its Effect Upon Airfoil Pressure Distributions. NACA RM No.
   A7A02, 1947.
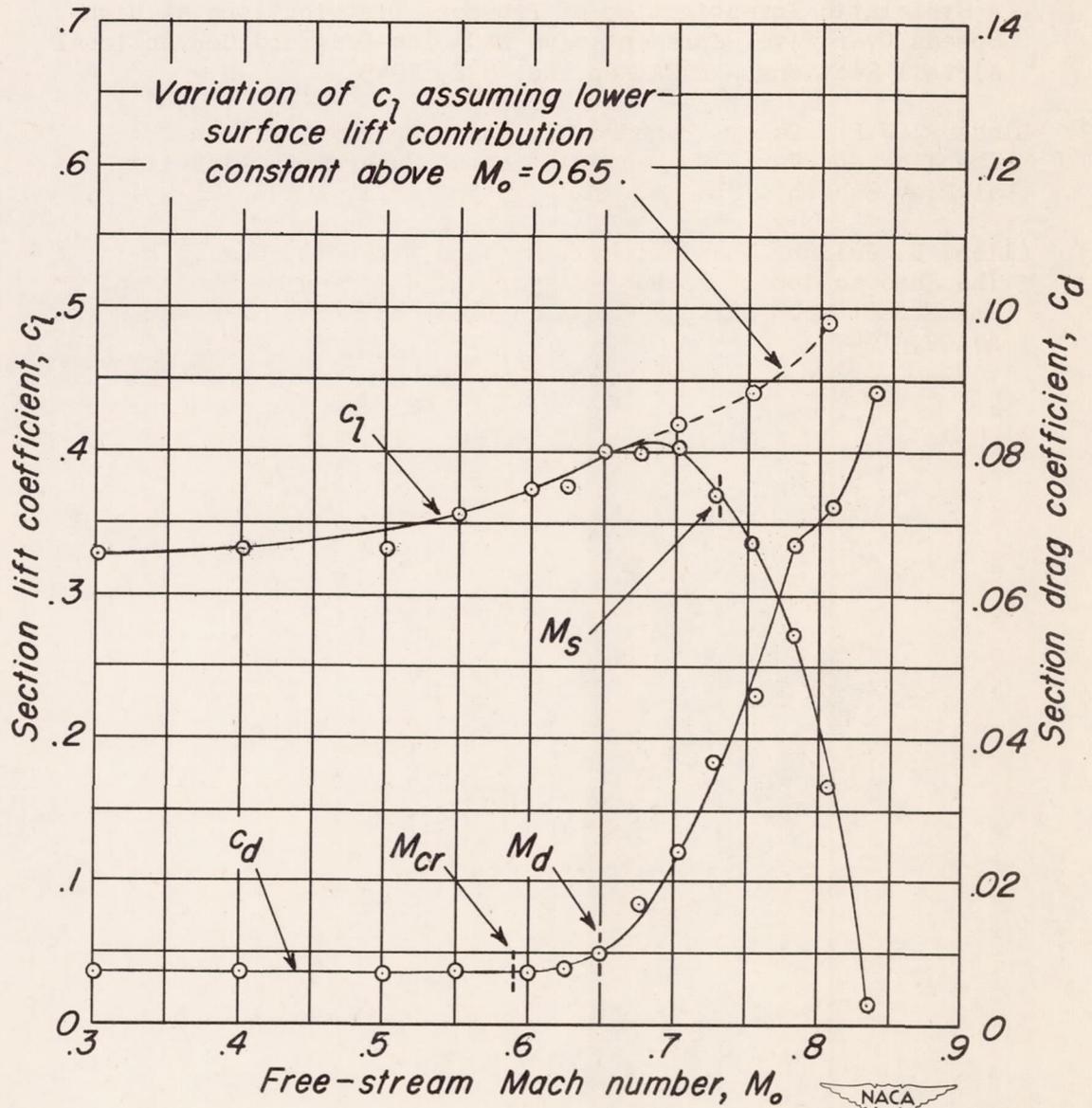
Figure 1.– Section lift and drag coefficients as functions of free–stream Mach number for NACA 23015 airfoil section at 2° angle of attack.
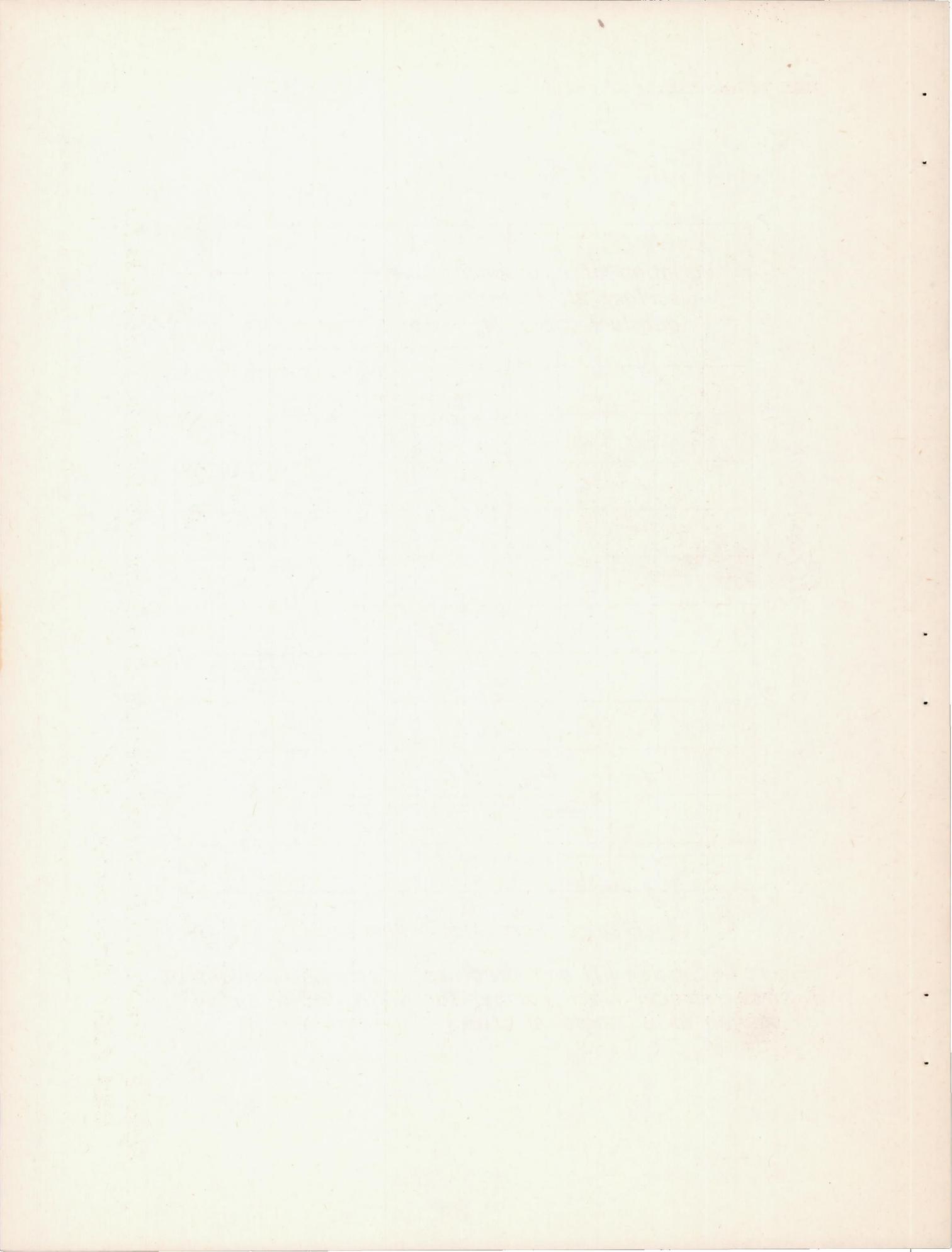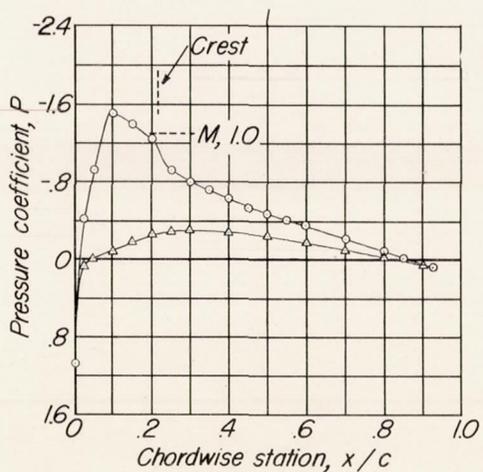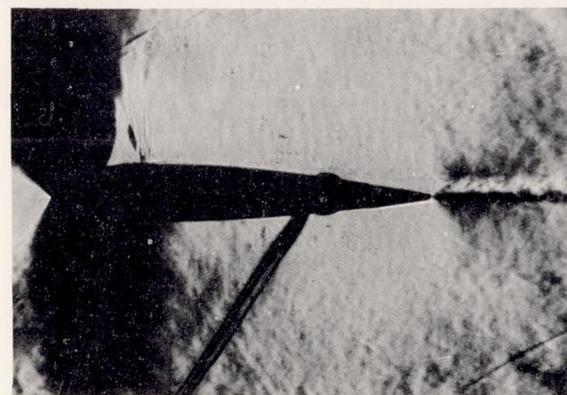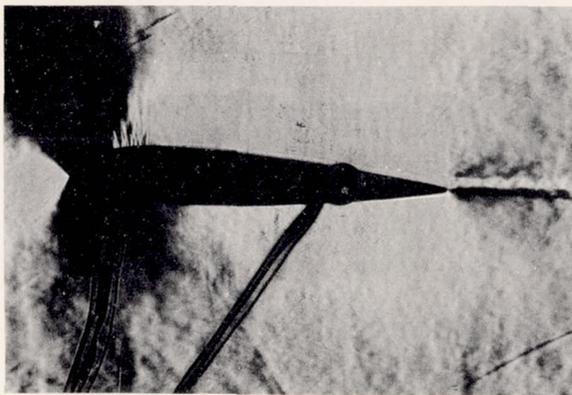
(a) M_o, 0.60
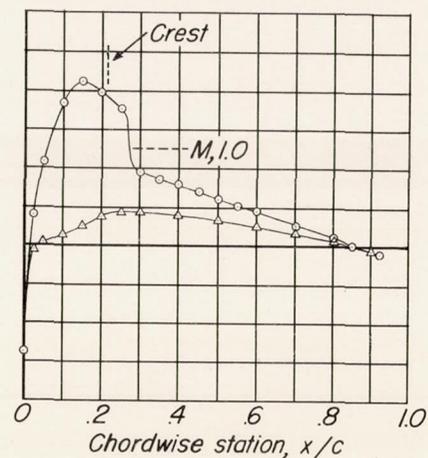
(b) M_o, 0.65

Figure 2.— Simultaneously obtained pressure distributions and schlieren photographs for NACA 23015 airfoil section; α, 2°.

(d) M_0, 0.73

(c) M_0, 0.70
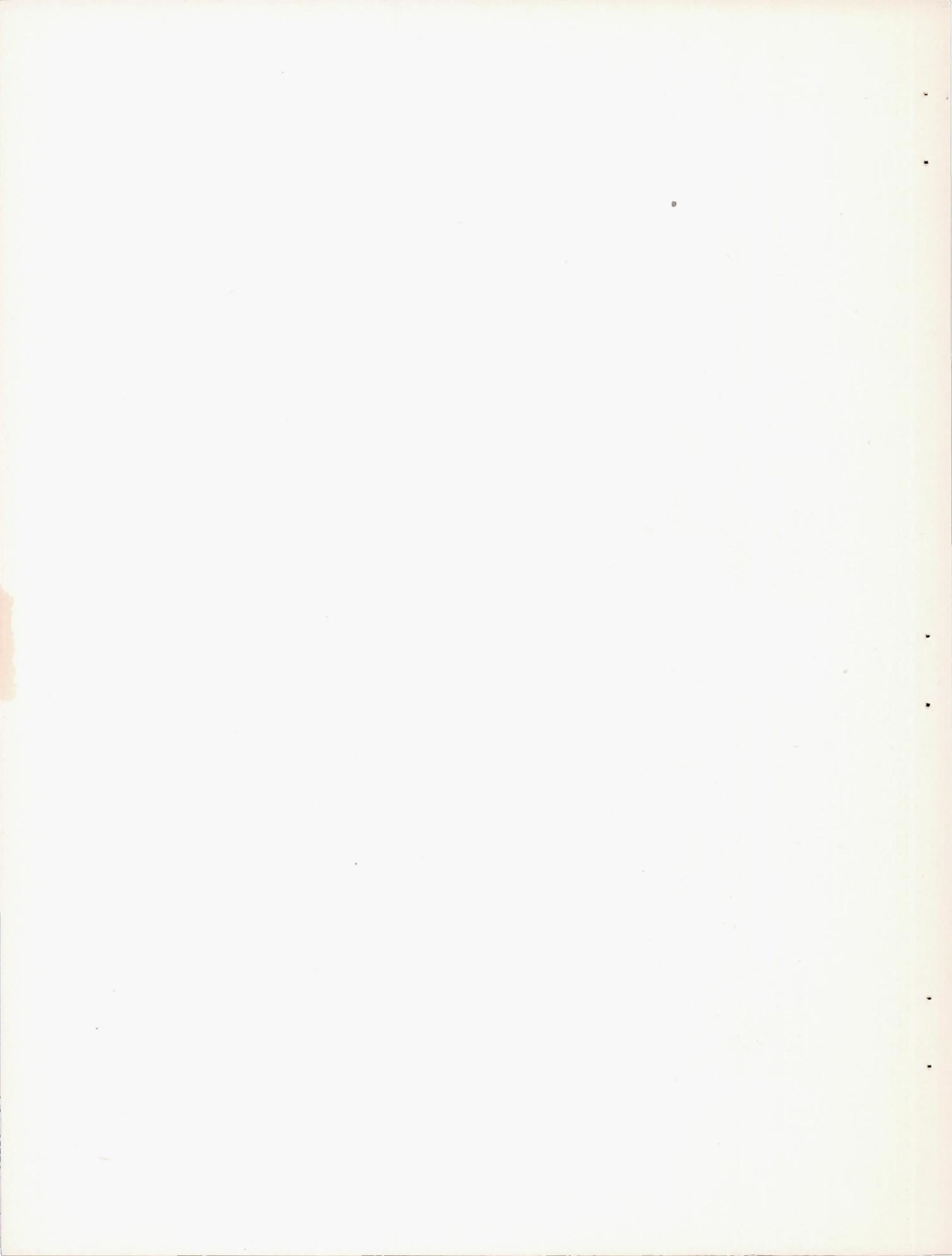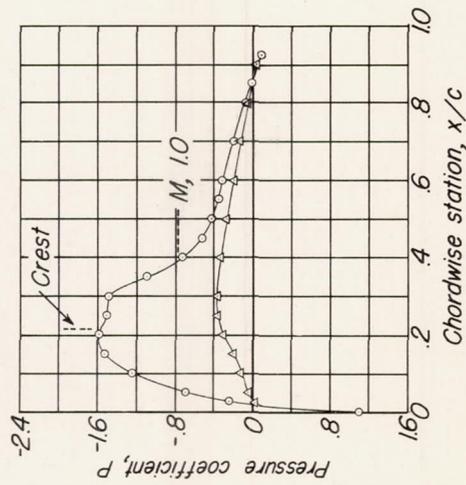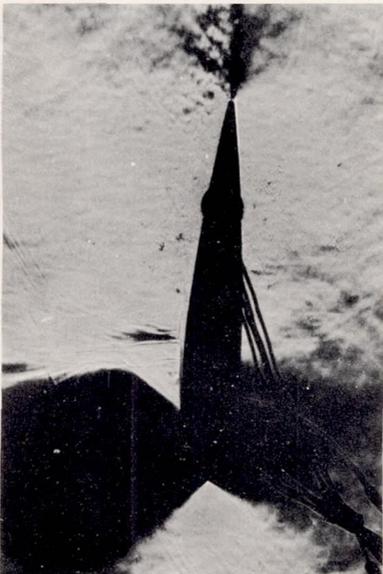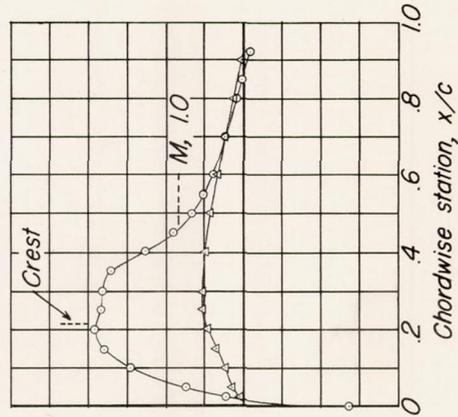
Figure 2. – Concluded.
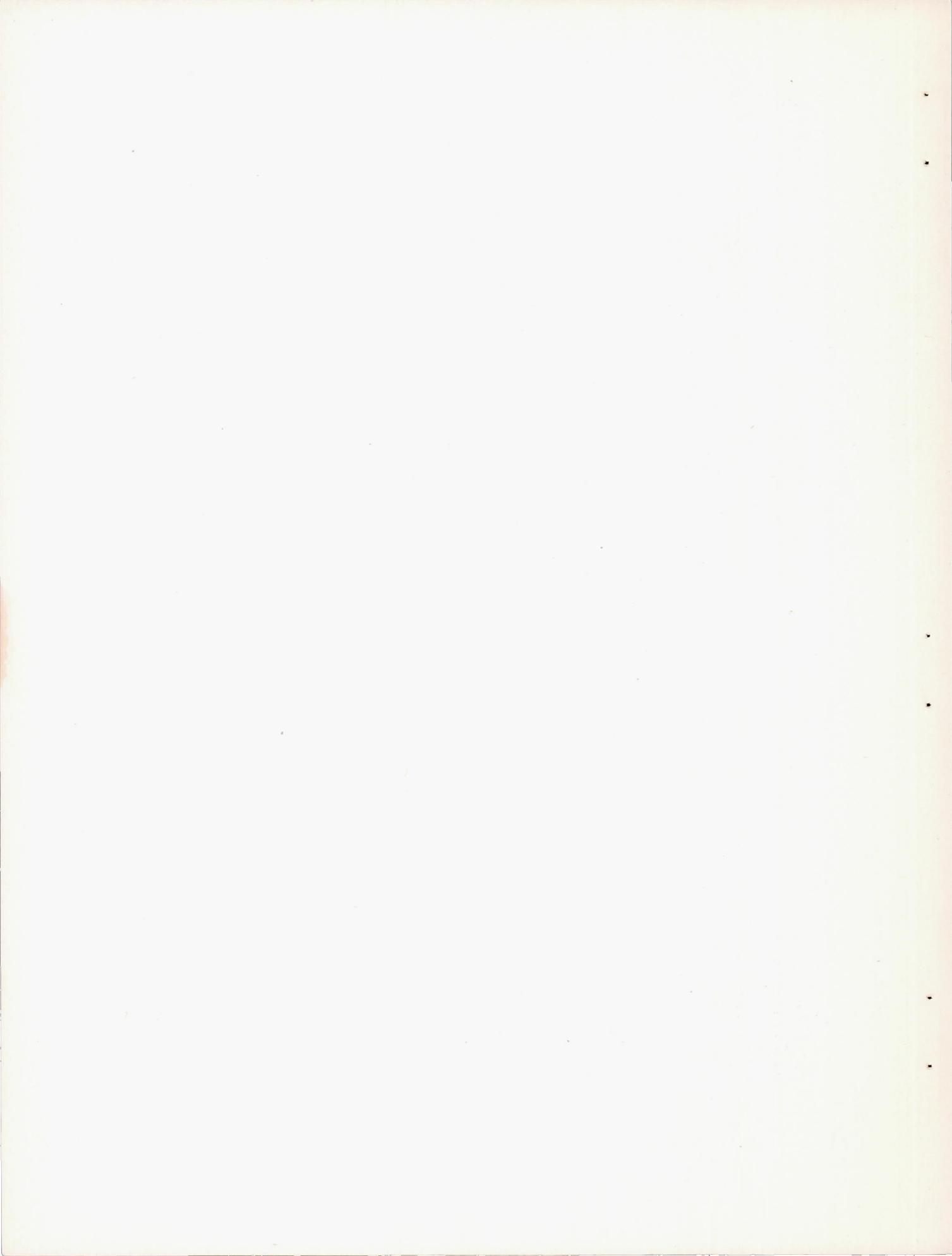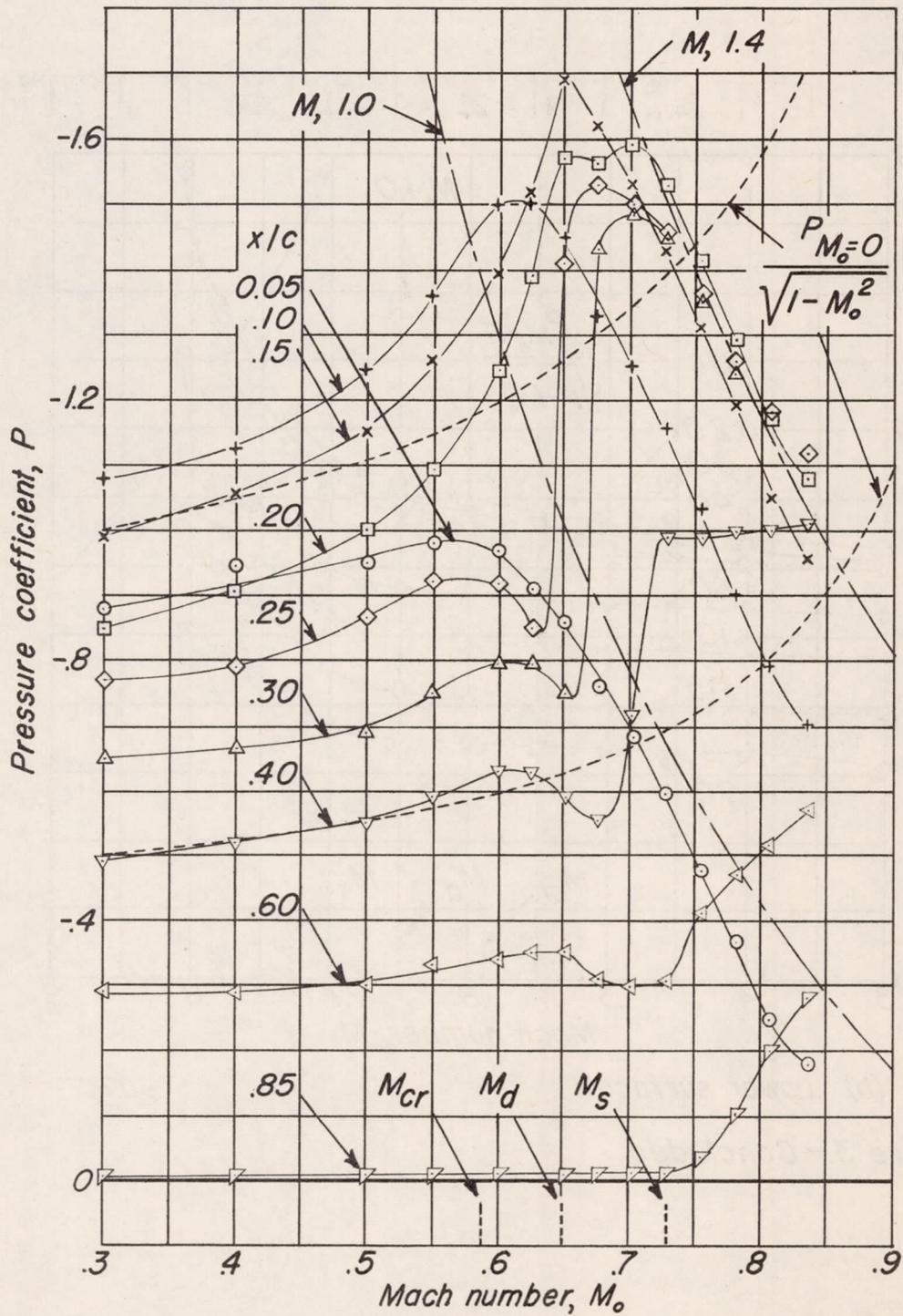
(a) Upper surface.
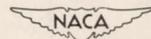
Figure 3.– Variation of pressure coefficient with Mach number at various chordwise stations of NACA 23015 airfoil section; α, 2°.
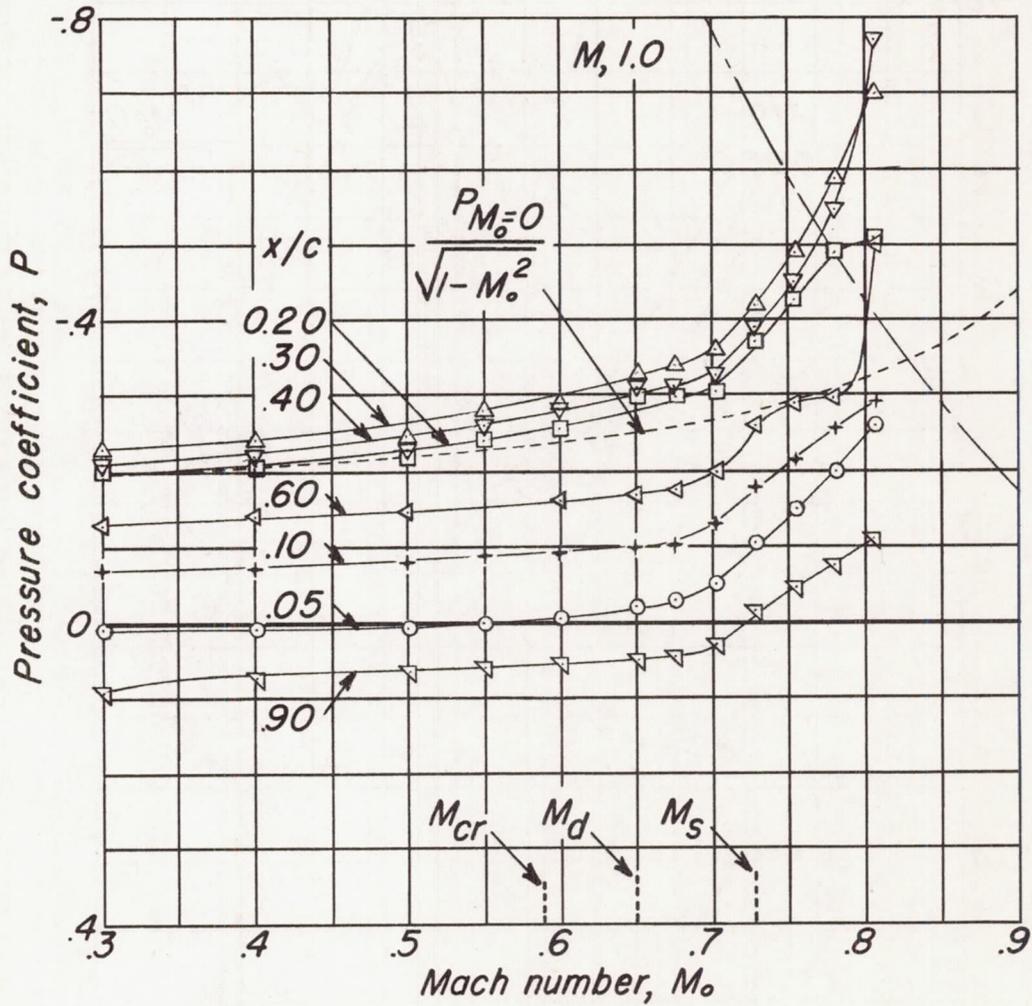
(b) Lower surface.

Figure 3.- Concluded.
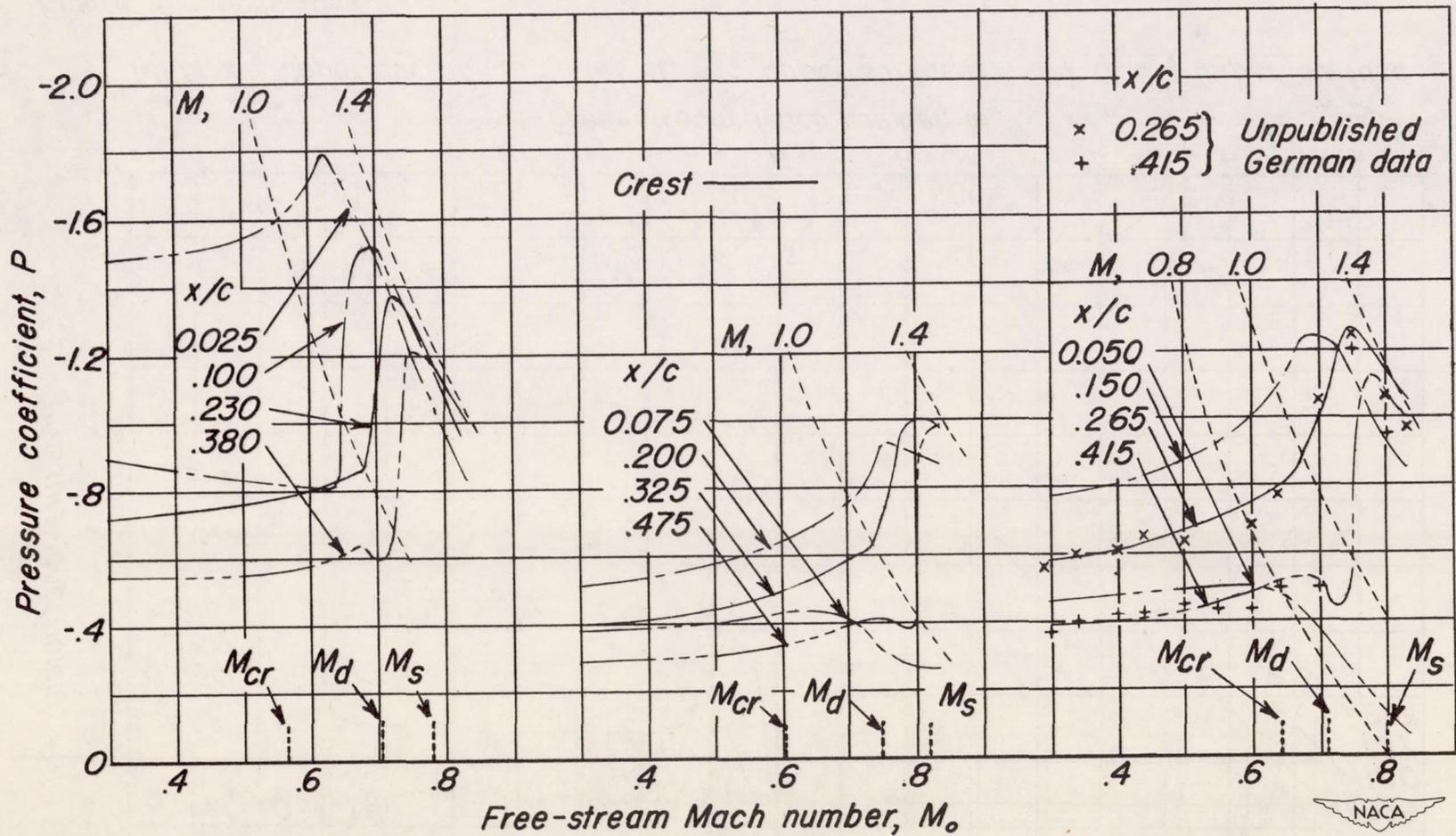
(a) α, $-4°$; lower surface.   (b) α, $-2°$; upper surface.   (c) α, $0°$; upper surface.

Figure 4.— Pressure coefficient at various chordwise locations as a function of free-stream Mach number for NACA 23015 airfoil section.
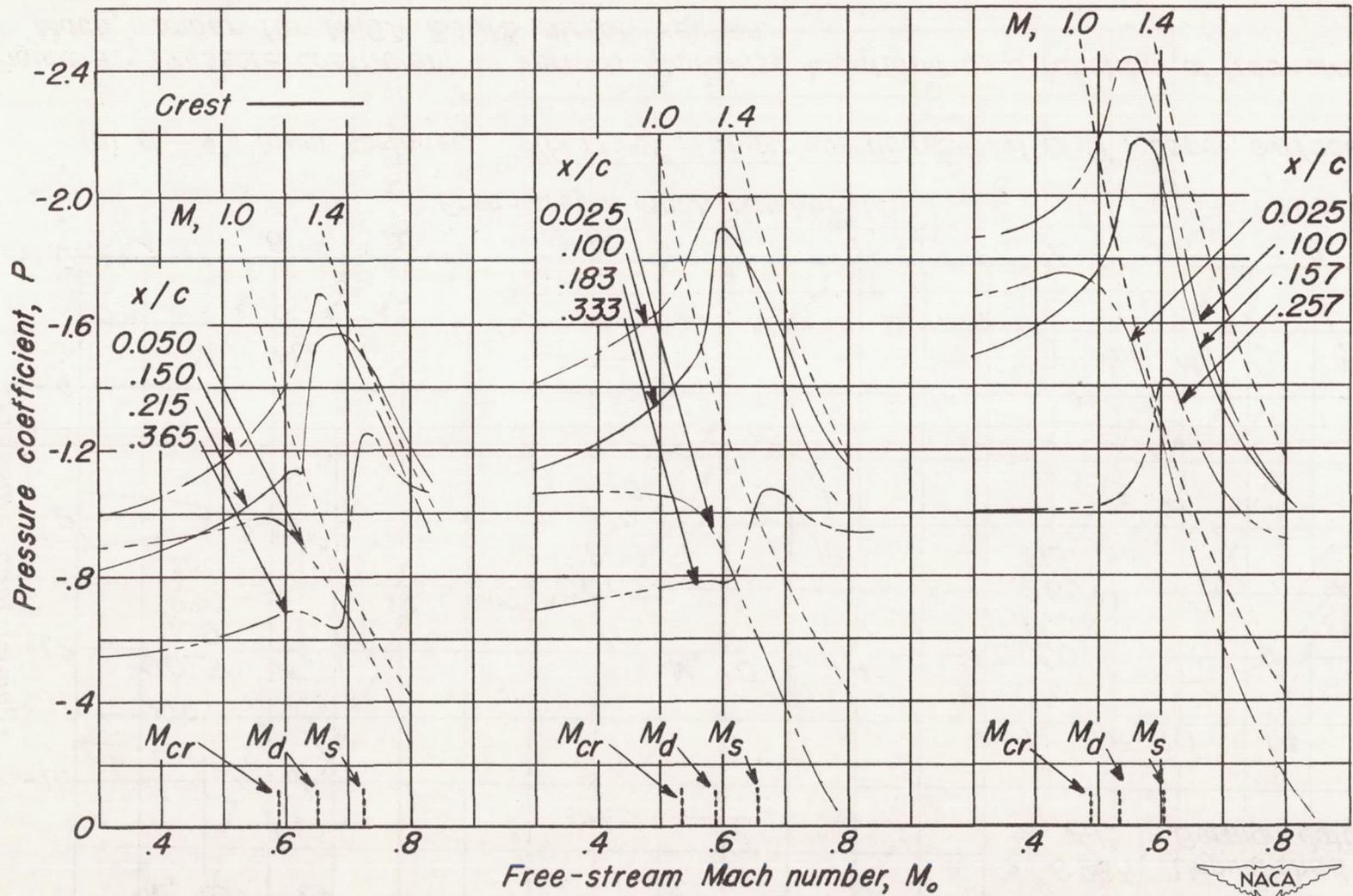
(d) α, $2°$; upper surface. (e) α, $4°$; upper surface. (f) α, $6°$; upper surface.

Figure 4.- Concluded.
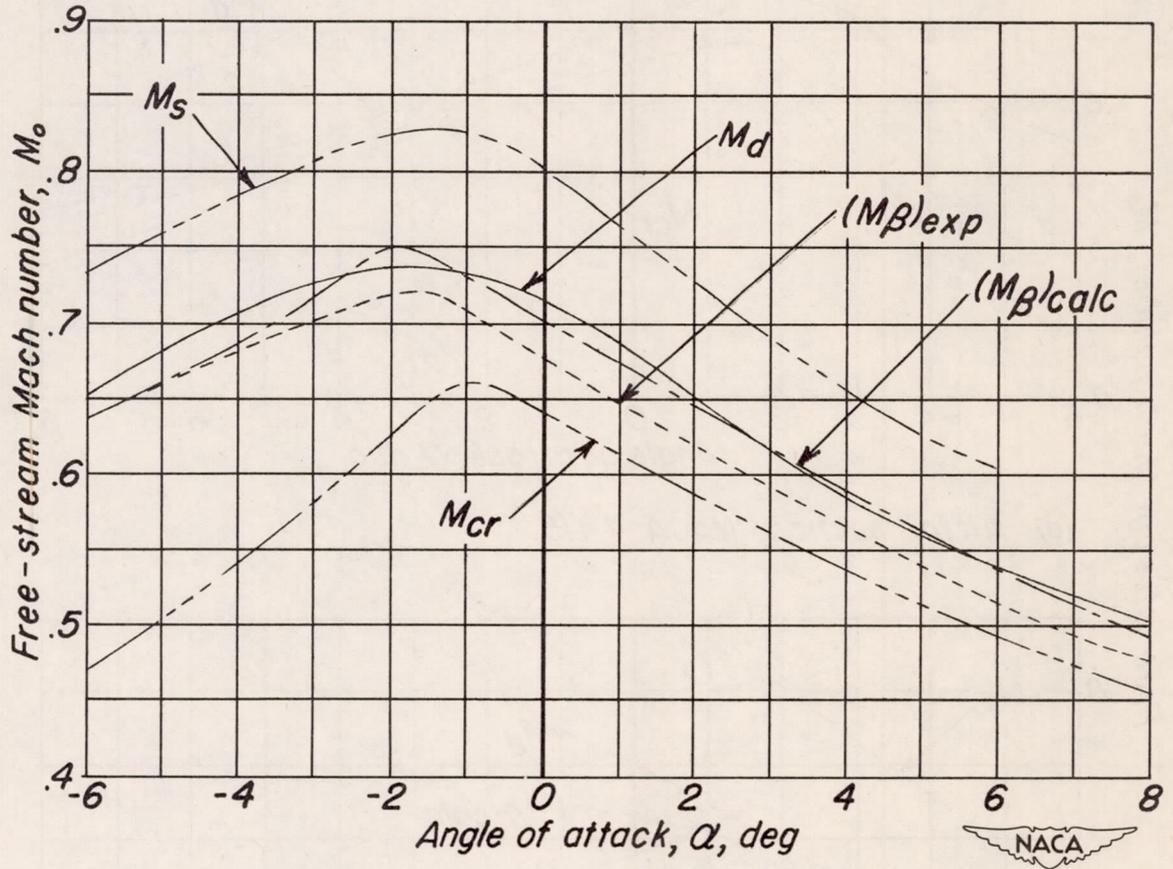
Figure 5.—Variation of critical, drag-divergence, and shock-stall Mach numbers with angle of attack for the NACA 23015 airfoil section.
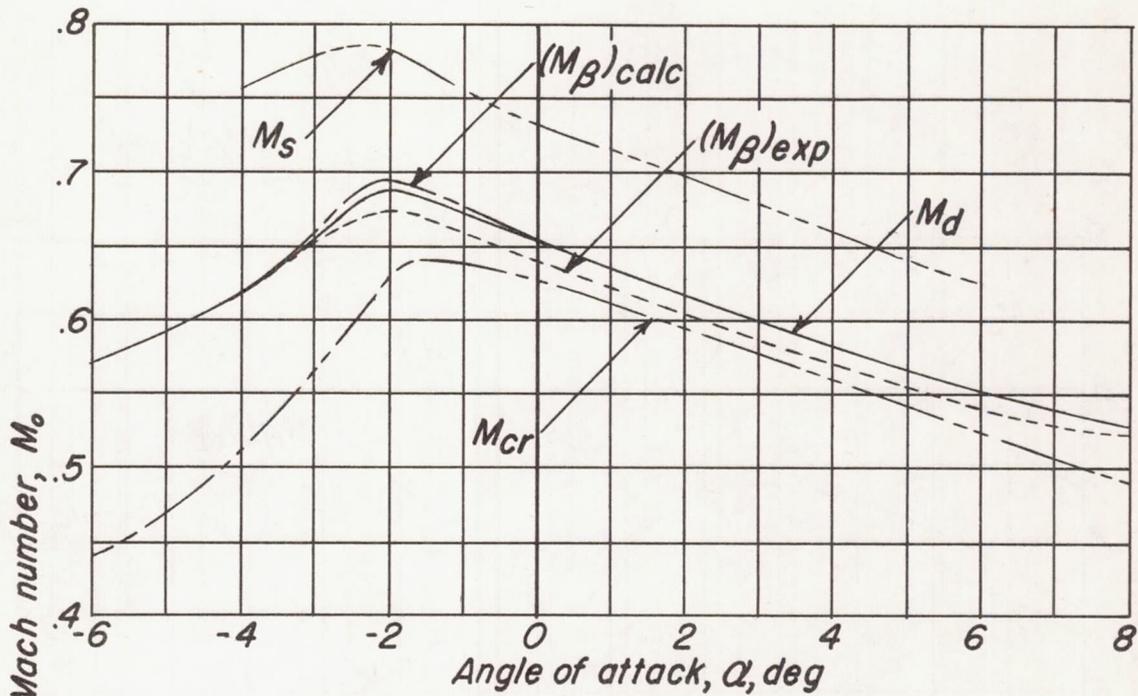
(a) Airfoil section, NACA 4415.
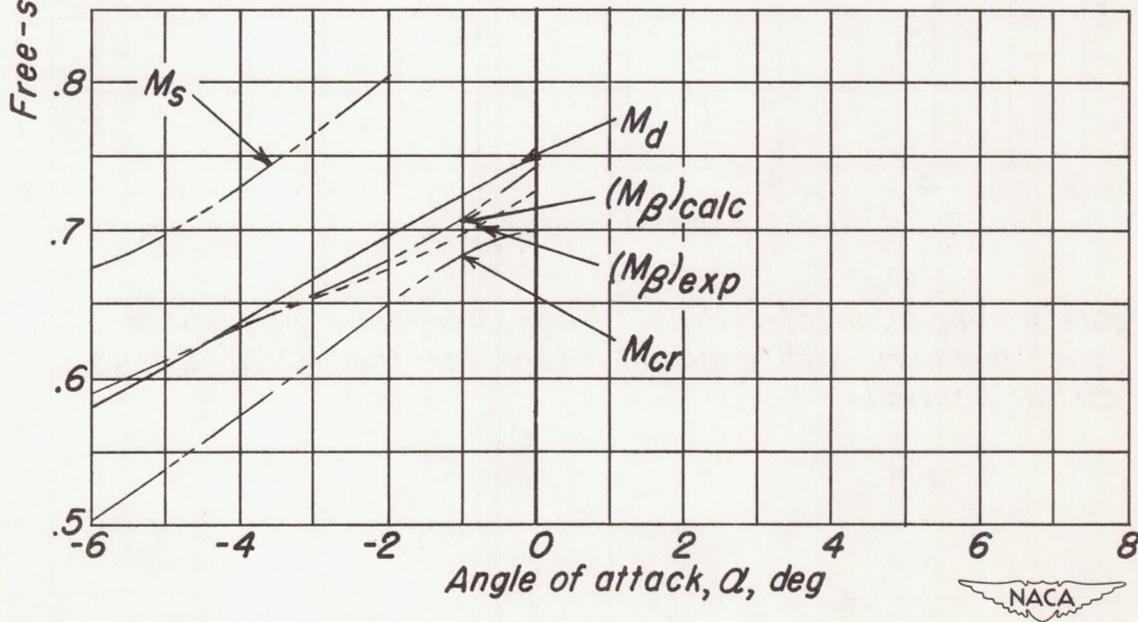
(b) Airfoil section, NACA 0015.

Figure 6.— Variation of critical, drag-divergence, and shock-stall
Mach numbers with angle of attack for several NACA airfoil
sections.
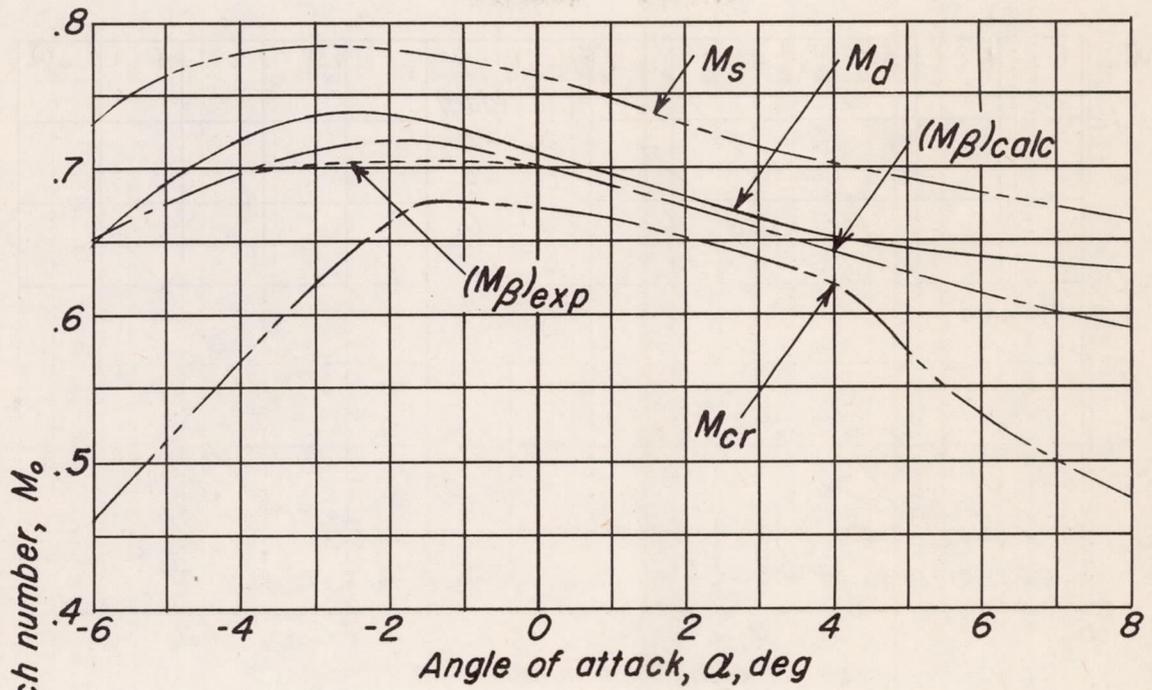
(c) Airfoil section, NACA 65_2-215, a=0.5.
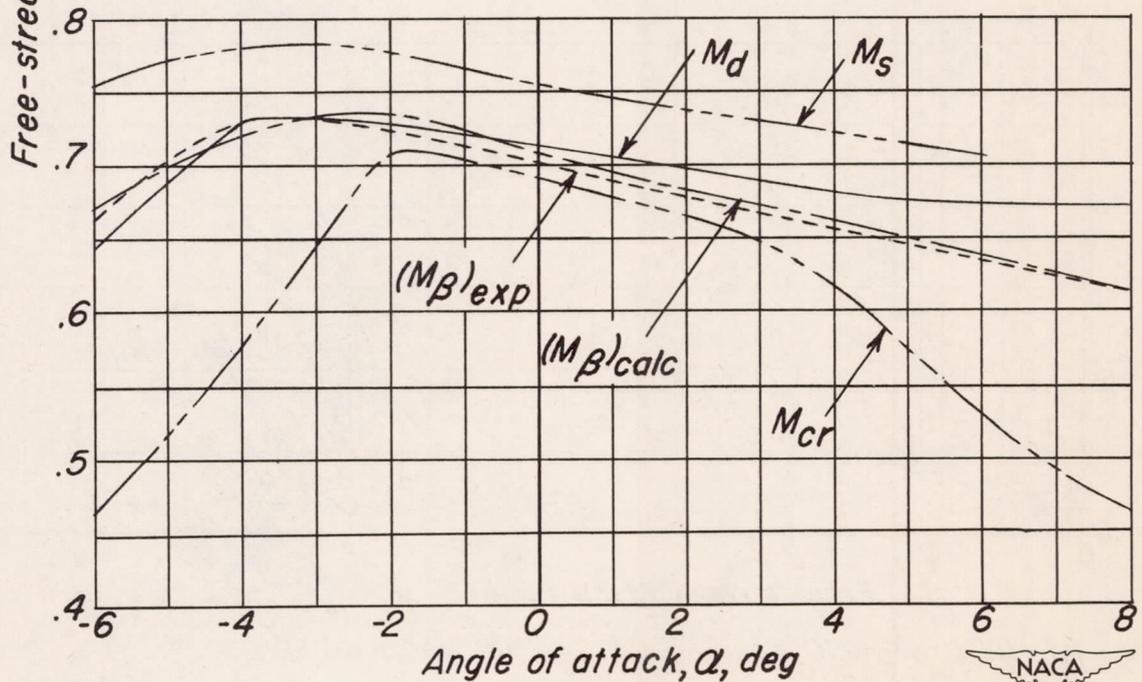
(d) Airfoil section, NACA 66,2-215, a=0.6.

Figure 6.—Concluded.

(a) NACA 64₁-206.   (b) NACA 64₁-208.
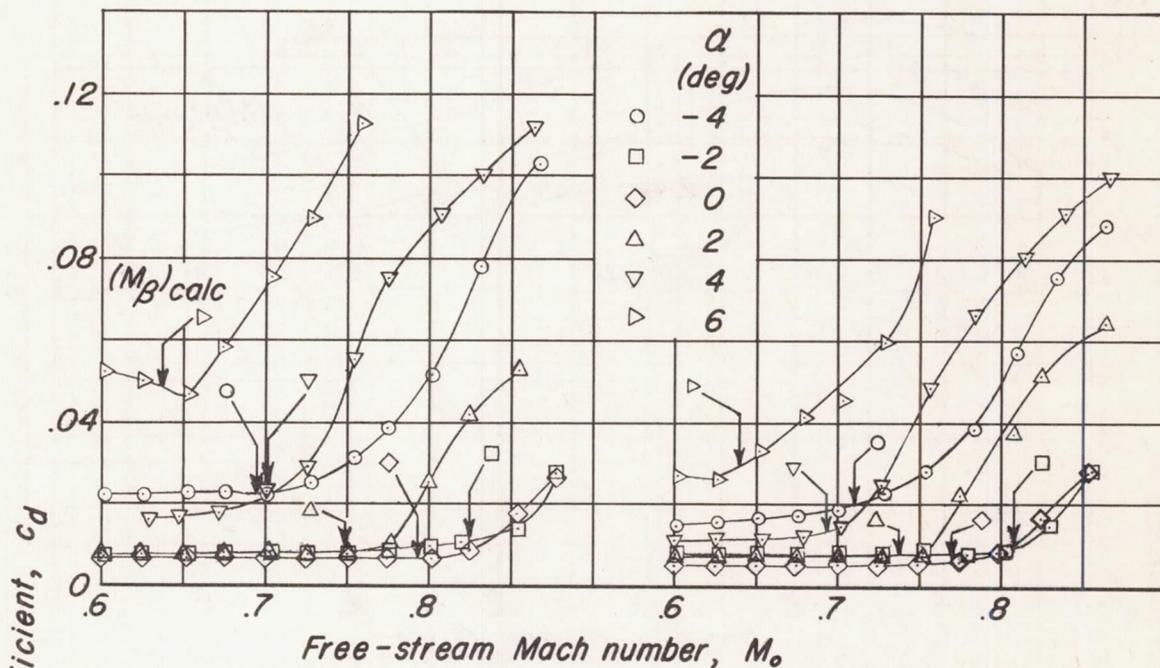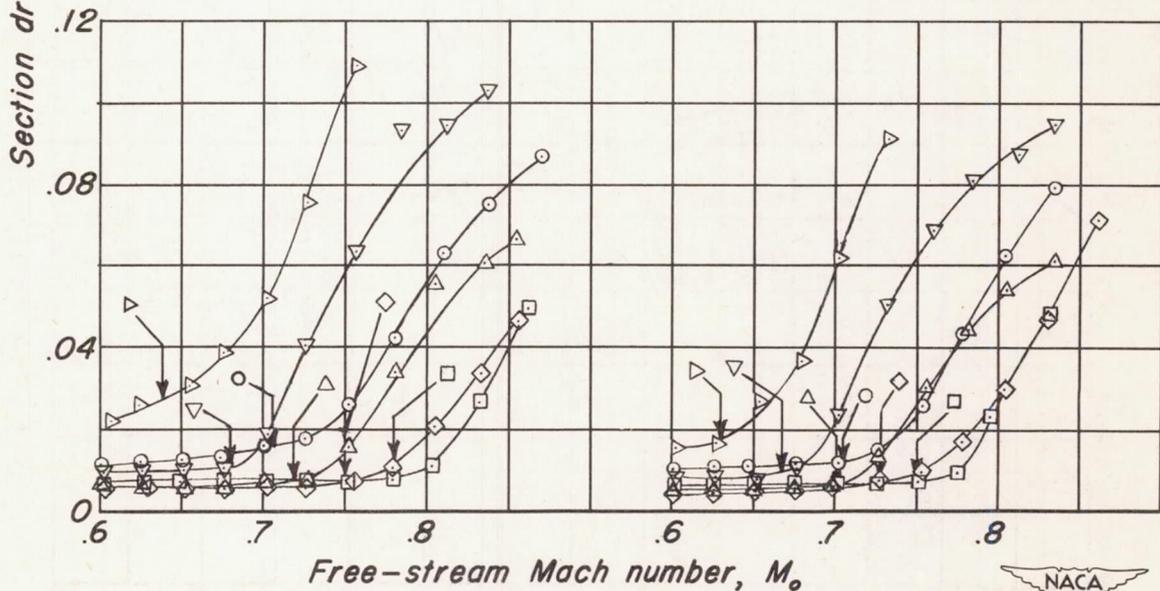
(c) NACA 64₁-210.   (d) NACA 64₁-212.
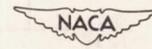
Figure 7.- Calculated Mach number for occurence of sonic velocity at
   airfoil crest and variation of section drag coefficient with Mach
   number for a variety of airfoil sections.

(e) NACA 65₁-206.

(f) NACA 65₁-208.

(g) NACA 65₁-210
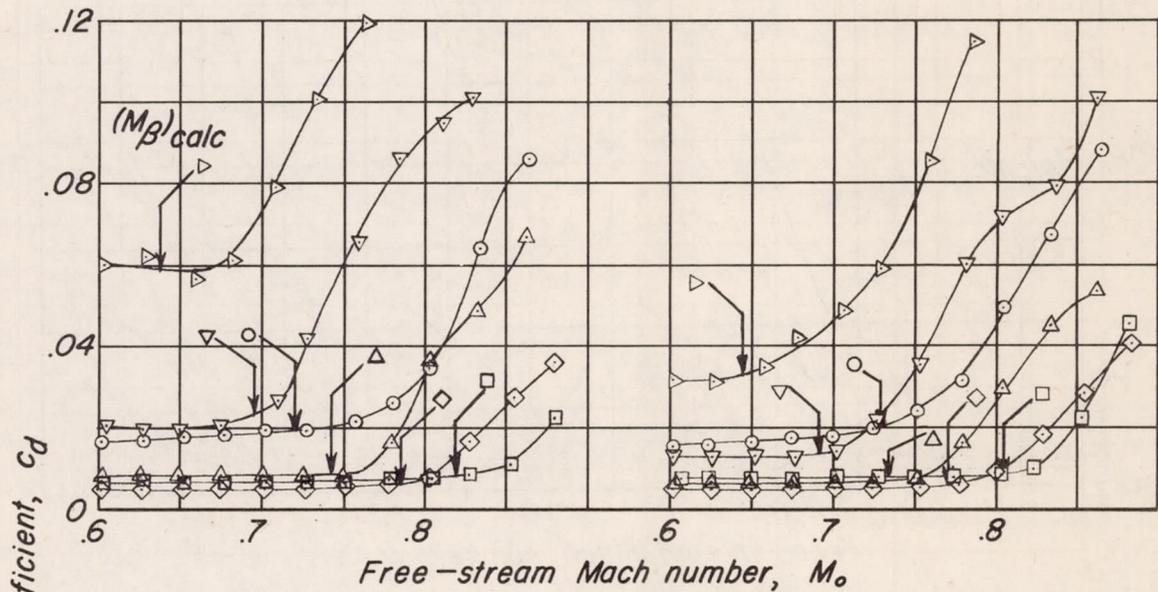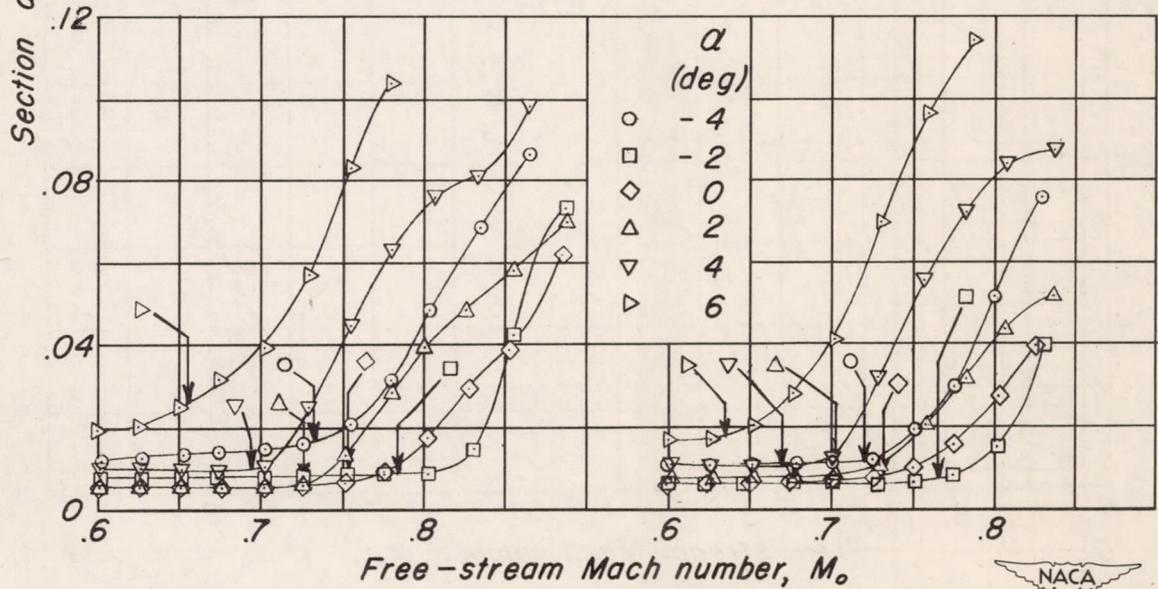
(h) NACA 65₁-212.
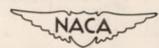
Figure 7.- Continued.
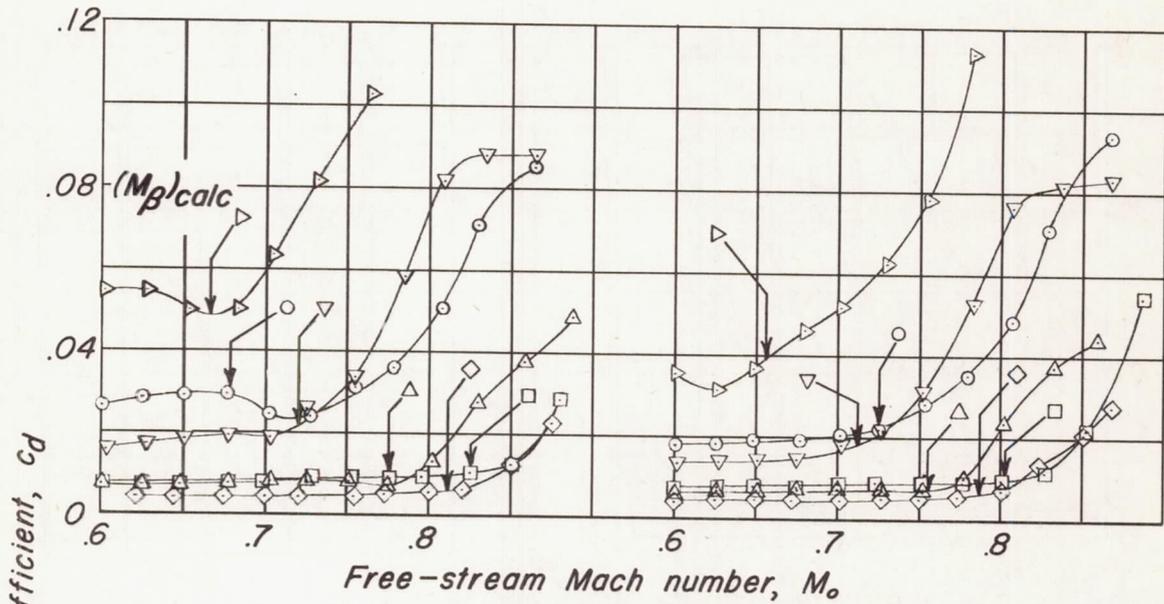
(i) NACA 66,-206.   (j) NACA 66,-208.
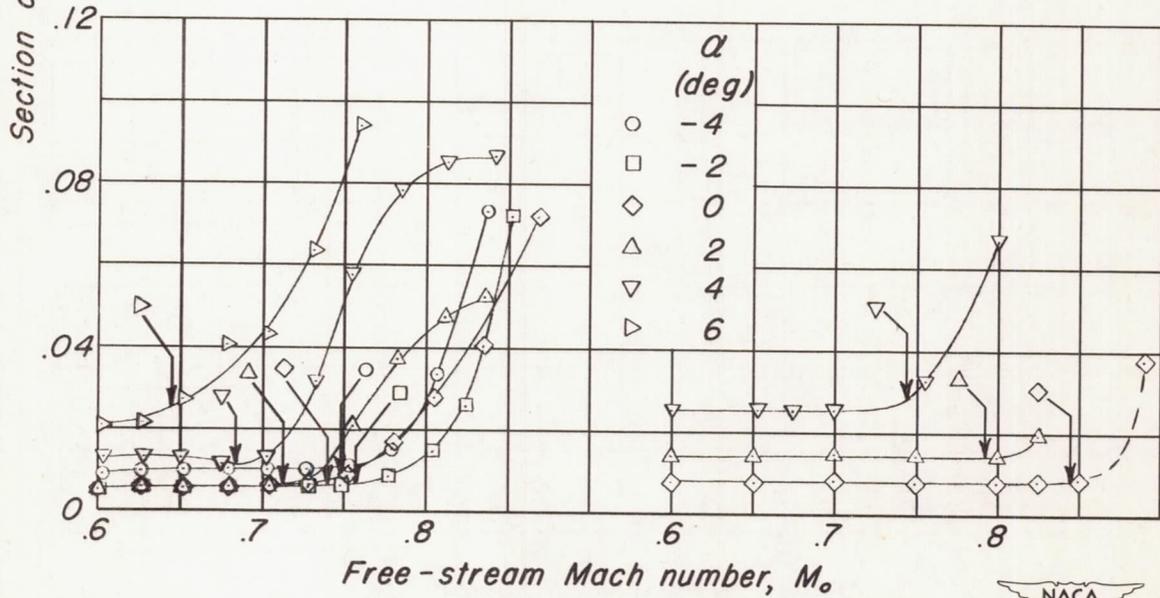
(k) NACA 66,-212.   (l) NACA 66-006.
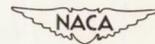
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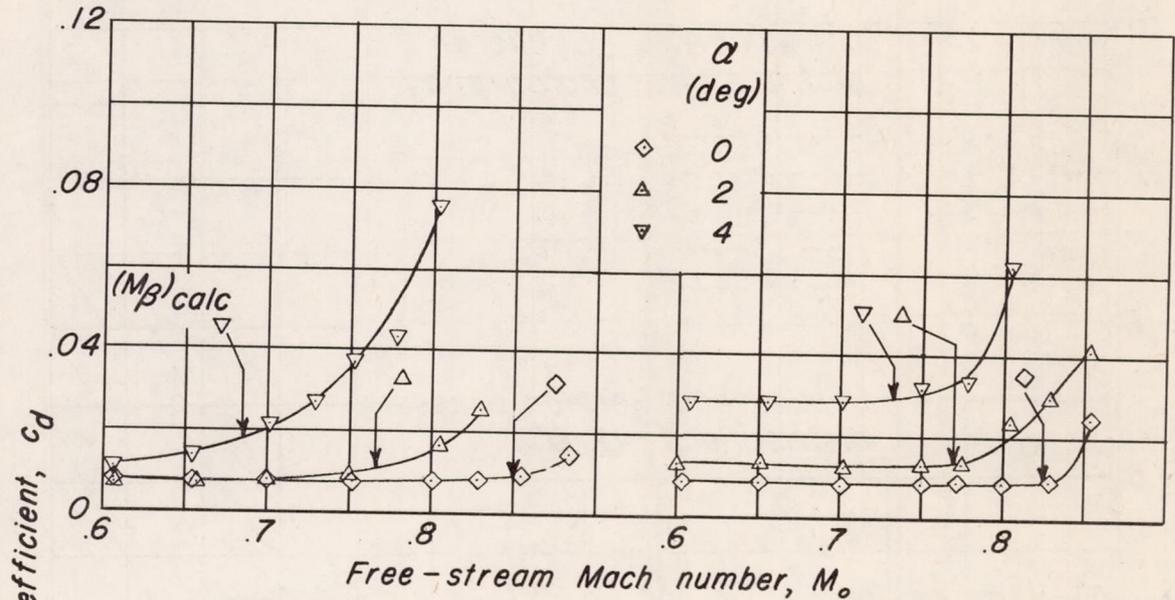
(m) NACA 0006.

(n) 2S-(30)(03)-(30)(03).
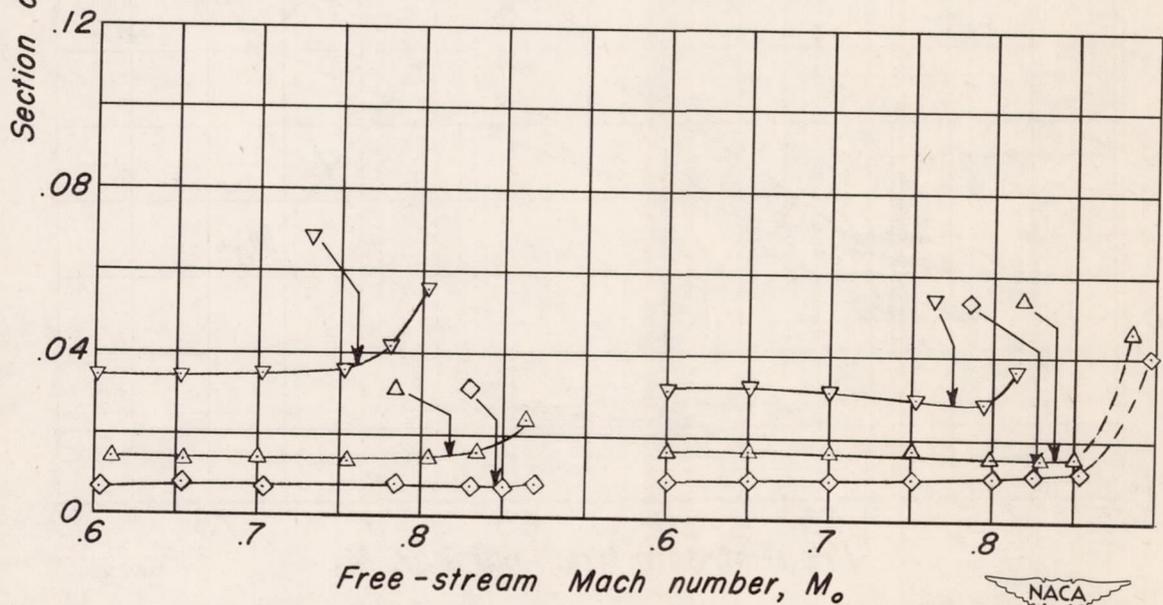
(o) 2S-(50)(03)-(50)(03).

(p) 2S-(70)(03)-(70)(03).

Figure 7.- Concluded.
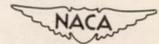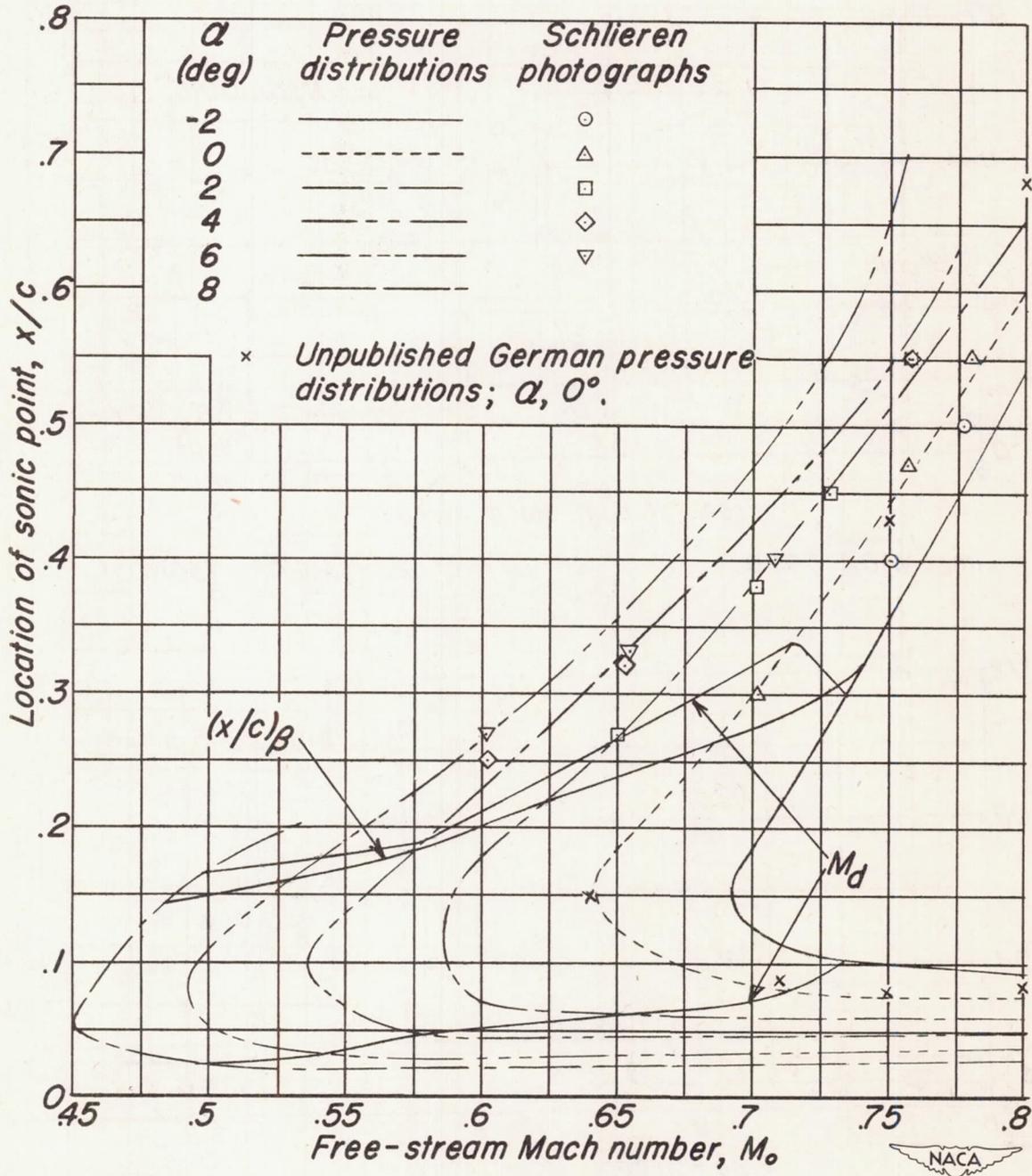
Figure 8.— Boundaries of supersonic region on upper surface of
NACA 23015 airfoil section as a function of free-stream
Mach number for various angles of attack.
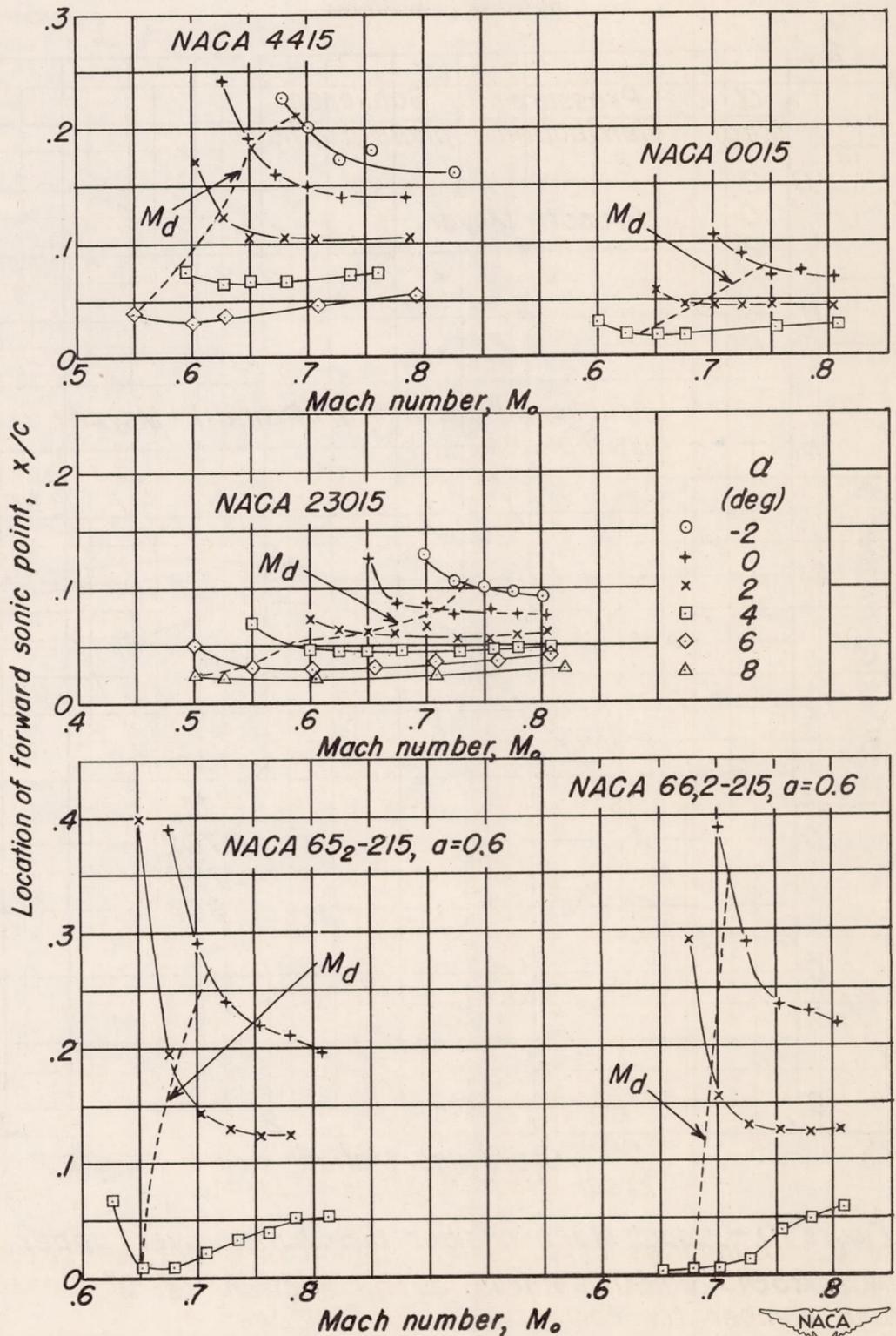
Figure 9.— Location of forward sonic point as a function of free-stream Mach number for several NACA airfoil sections.
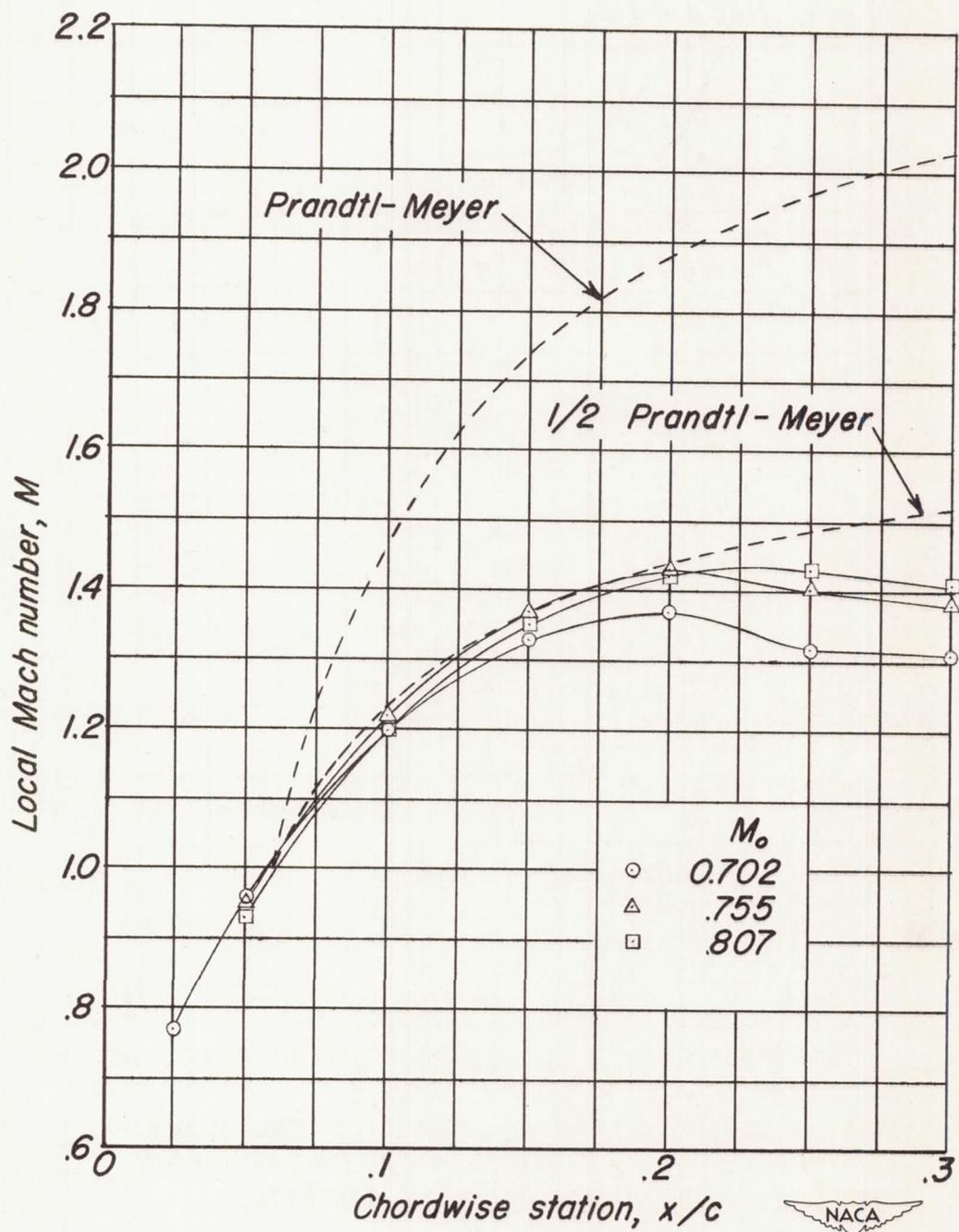
Figure 10.— Local Mach number distribution over upper surface of NACA 23015 airfoil section; α, 2°.
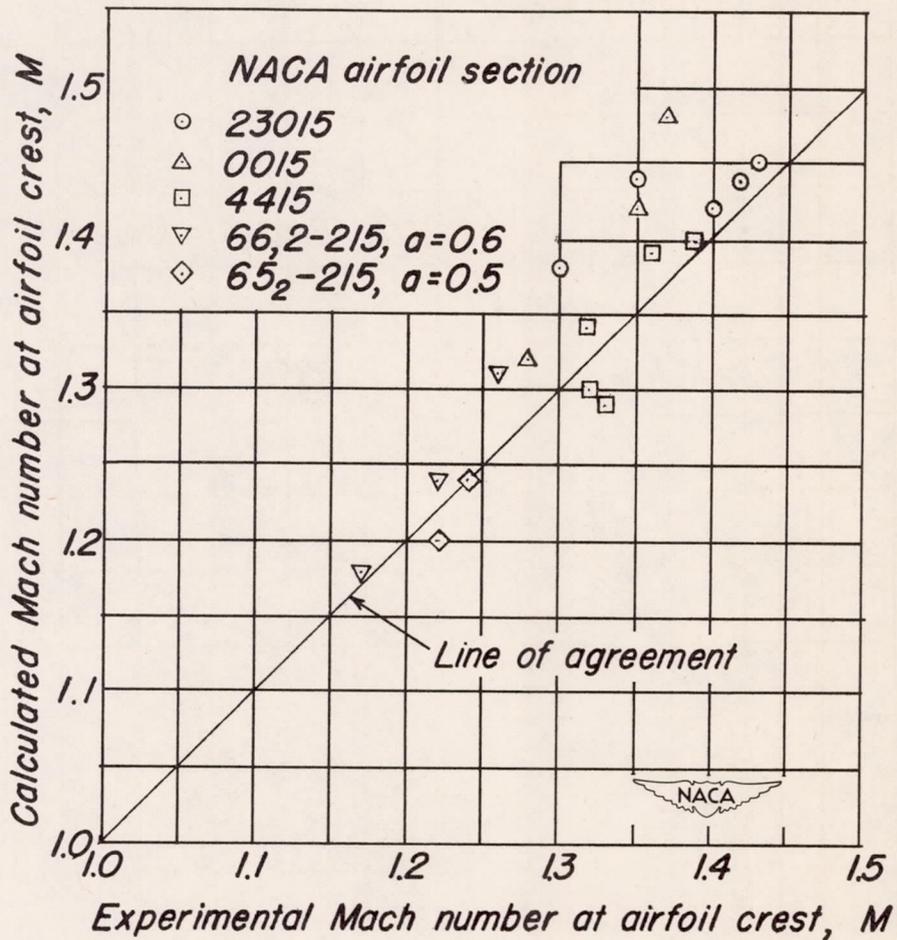
Figure 11.— Comparison of calculated and experimental values of local Mach number at airfoil crest for several NACA airfoil sections.