NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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AMBIENT PRESSURE DETERMINATION AT HIGH ALTITUDES BY USE OF FREE-MOLECULE THEORY

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SUMMARY

A method based on free-molecule theory is presented for reducing pressure measurements obtained behind an orifice on a rapidly moving object to ambient air pressure. The method applies only in the region of the atmosphere where the mean free path of the air molecules is large with respect to a characteristic body dimension of the object. The method has been applied to a sample condition and the results obtained are compared with similar results computed from the gas-dynamic theory. The comparisons show the latter method to be in error at the altitudes which are attainable with modern rockets.

INTRODUCTION

Until recently the determination of atmospheric characteristics has been obtained from measurements made with balloon-borne instruments. With this method the altitude covered extended to approximately 25 miles. The advent of the V-2 and similar missiles, however, has offered the opportunity of measuring the characteristics of the atmosphere for altitudes of over 100 miles and several missiles have already been used for this purpose. In connection with these measurements it has been found desirable to determine the ambient pressure of the atmosphere by measuring the pressure at an orifice on the forward part of the missile. Several methods (references 1 and 2) based on gas-dynamic principles are available for reducing such measurements. The results obtained by these methods are subject to the limitation of the gas-dynamic theory which is that the mean free path of the molecules, that is, the average distance traveled by the molecules between impacts, is small with respect to the measuring device. The limiting altitude at which gas-dynamic theory becomes invalid and the theory of free-molecule behavior prevails cannot be accurately stated because of lack of experimental data. At altitudes above 75 miles the mean free path of the molecules is no longer small (reference 3), and the use of gas dynamics for interpreting data becomes increasingly in error. Inasmuch as no information on the subject is at present known to be available, the purpose of the present paper is to present a method based on the concept of free-molecule theory for use in connection with this problem.

SYMBOLS

a area of orifice, square centimeters

k Boltzmann's constant $\left(1.3805 \times 10^{-16} \text{ erg/}^{\circ}\text{C abs.}\right)$

m mass of one molecule, grams

M molecular weight

N number of molecules per cubic centimeter $\left(\frac{N_0 T_0}{p_0} \frac{p}{T} \frac{M_0}{M}\right)$

N_0 Loschmidt number; number of molecules per cubic centimeter at 0° C and normal pressure $\left(2.6870 \times 10^{19}/\text{atm/cm}^3\right)$

N_a total number of molecules crossing orifice area per second

p pressure, kilograms per square meter

T absolute temperature, °C absolute

V velocity of missile, centimeters per second

Z probability integral $\left(\frac{2}{\sqrt{\pi}} \int_0^{\frac{V}{\alpha} \sin \eta} e^{-V^2} \, dV\right)$

α most probable molecular velocity, centimeters per second $\left(\sqrt{\frac{2kT}{m}}\right)$

η angle between plane of orifice and flight-path direction, degrees

γ ratio of specific heats

Subscripts:

0 273° C absolute and sea-level atmospheric pressure

A conditions in ambient air

1 conditions within pressure chamber

ANALYSIS

At very high altitudes, the gas-dynamic theory is no longer applicable to the determination of ambient pressure because the mean free path of the molecules is relatively large. Figure 1, which has been reproduced from a plot of reference 3, shows the variation of molecular mean free path with altitude. In this region of relatively large mean free path, the pressure may be determined by the behavior of the molecules. The method of the analysis is based upon the principle that the pressure within a chamber vented to the ambient air by an orifice will be such that the number of molecules entering the chamber through the orifice is equal to the number of molecules leaving the chamber through the orifice. Such an orifice is shown in figure 2 to be a hole in a diaphragm the thickness of which is much smaller than the diameter of the orifice. Equilibrium of flow will be quickly established if the volume of the chamber is not too large. The analysis presented does not apply to pressures measured by means of a gage connected to the orifice by a long tube.

In connection with the development given by Tsien in reference 4 with regard to the mechanics of rarefied gases, an equation, based on the assumption that a Maxwellian velocity distribution exists, was developed to calculate the number of molecules crossing an orifice. This equation was used in the present analysis in which the pressure p, the temperature T, the free-stream velocity V, and the inclination of the orifice with respect to the free stream η were considered as variables. When the plane of the orifice is on the pressure side of a body, that is, on the nose of a missile (see fig. 2), the number of molecules crossing a unit area per second for any given values of p, T, V, and η may be calculated from the following equation:

$$\frac{N_a}{a} = \frac{N\alpha}{2\sqrt{\pi}} \left[e^{-\left(\frac{V}{\alpha} \sin \eta\right)^2} + \sqrt{\pi}\, \frac{V}{\alpha} \sin \eta \,(1 + Z) \right] \tag{1}$$

When the plane of the orifice is on the suction side of the body, for example, on the boattail portion of a missile, the number of molecules crossing the orifice per unit area per second is given by

$$\frac{N_a}{a} = \frac{N\alpha}{2\sqrt{\pi}} \left[e^{-\left(\frac{V}{\alpha} \sin \eta\right)^2} - \sqrt{\pi}\, \frac{V}{\alpha} \sin \eta \,(1 - Z) \right] \tag{2}$$

If the ratio of the molecular mean free path within the chamber to the orifice diameter is 10 or more, the number of molecules crossing the orifice per second in the outward direction, that is, from the inside to the outside, may be calculated from equation (1) where the area of the orifice is a.

For this calculation the velocity of the inside air is zero with respect to the orifice; therefore equations (1) and (2) become

$$\frac{N_a}{a} = \frac{N\alpha}{2\sqrt{\pi}} \tag{3}$$

The number of molecules passing outward through either a forward-facing or rearward-facing orifice therefore depends on the pressure and temperature within the measuring device. Establishing equilibrium gives

$$\left(\frac{N_a}{a}\right)_{inward} = \left(\frac{N_a}{a}\right)_{outward} \tag{4}$$

Substituting the expression for $\frac{N_a}{a}$ from equations (1) and (3), or (2) and (3), and for N and α into equation (4) and simplifying yields for a forward-facing orifice

$$\frac{p_1}{p_A} = \sqrt{\frac{T_1}{T_A}} \left[e^{-\left(\frac{V}{\alpha} \sin \eta\right)^2} + \sqrt{\pi}\,\frac{V}{\alpha} \sin \eta \,(1 + Z)\right] \tag{5}$$

and for a rearward-facing orifice

$$\frac{p_1}{p_A} = \sqrt{\frac{T_1}{T_A}} \left[e^{-\left(\frac{V}{\alpha} \sin \eta\right)^2} - \sqrt{\pi}\,\frac{V}{\alpha} \sin \eta \,(1 - Z)\right] \tag{6}$$

The quantity $\frac{p_1}{p_A}\sqrt{\frac{T_A}{T_1}}$ for forward-facing and rearward-facing orifices is shown in figures 3 and 4 as functions of $\frac{V}{\alpha}$ sin η.

When the plane of the orifice is parallel to the flight path ($\eta = 0^{\circ}$), equation (5) becomes

$$\frac{p_1}{p_A} = \sqrt{\frac{T_1}{T_A}} \tag{7}$$

This equation is in agreement with the results due to thermal transpiration as shown in reference 5.

For the pressure ratio at the stagnation point ($\eta = 90^\circ$), equation (5) becomes

$$\frac{p_1}{p_A} = \sqrt{\frac{T_1}{T_A}} \left[e^{-\left(\frac{V}{\alpha}\right)^2} + \sqrt{\pi}\, \frac{V}{\alpha} (1 + Z) \right] \tag{8}$$

APPLICATION AND COMPARISON

In order to calculate the ambient pressure by the method presented, it is necessary to determine by measurement or theory the following quantities: the pressure and temperature of the air within the chamber, the ambient temperature, the velocity of the projectile, and the composition of the air. The validity of the method will depend upon the accuracy with which these quantities can be determined.

The lack of experimental data prevents an absolute comparison of results obtained by the methods based on free-molecule theory with the actual values. If values are assumed for the measured quantities, the ambient pressure can be calculated, and a comparison can be made of the results obtainable by free-molecule theory and gas-dynamic theory. The values assumed are as follows:

$p_1 = 2.087 \times 10^{-2}$ kilograms per square meter
$T_1 = 300^\circ$ C absolute
$V = 10^5$ centimeters per second
$M = 24$

The ambient temperature, calculable by methods such as reference 6, was assumed to be 375° C absolute. The value of the ratio of specific heats corresponding to a molecular weight of 24 was 1.461 as obtained from reference 7.

The ambient pressure was calculated by the method presented for pressure orifices on planes having angles of incidence from 0° to 90°. These results are shown in figure 5. Ambient pressure determined from gas-dynamic theory is also shown for values of γ of 1.405 and 1.461. The pressure determined for the orifice location at the stagnation point ($\eta = 90^\circ$) was calculated by using the method of reference 2. For orifice locations on planes having η other than 0° and 90° the pressure was calculated from the theory of conical flow presented in reference 1 and expanded in reference 8.

Figure 5 shows a comparison of the ambient pressures obtained by free-molecule theory and by gas-dynamic theory and illustrates the differences obtained if a nonapplicable theory is used. The difference at the stagnation point ($\eta = 90^{\circ}$) due to the use of gas-dynamic theory is 30 percent when $\gamma = 1.405$ is assumed. By accounting for the change in the value of γ at high altitudes by using $\gamma = 1.461$ in gas-dynamic theory, the difference is reduced to 16.4 percent.

CONCLUDING REMARKS

The comparison of the ambient pressure determined by free-molecule theory with the ambient pressure determined by gas-dynamic methods showed a percentage difference large enough to indicate that the correctness of the results depends upon the method chosen. The results obtainable by the method presented should be checked by experiment when the means becomes available.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., December 28, 1948

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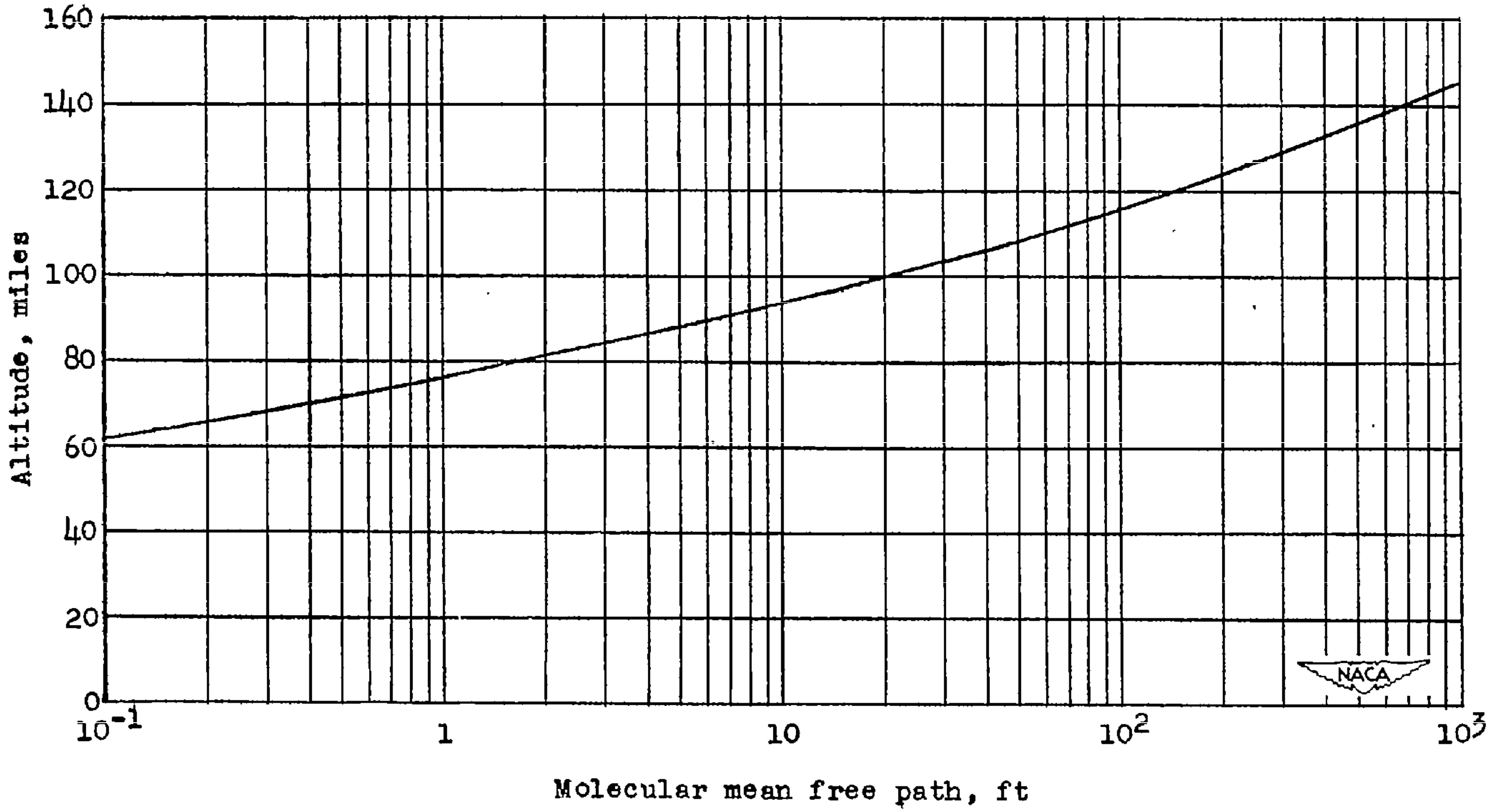

Figure 1.– Variation of molecular mean free path of air with altitude from reference 3.

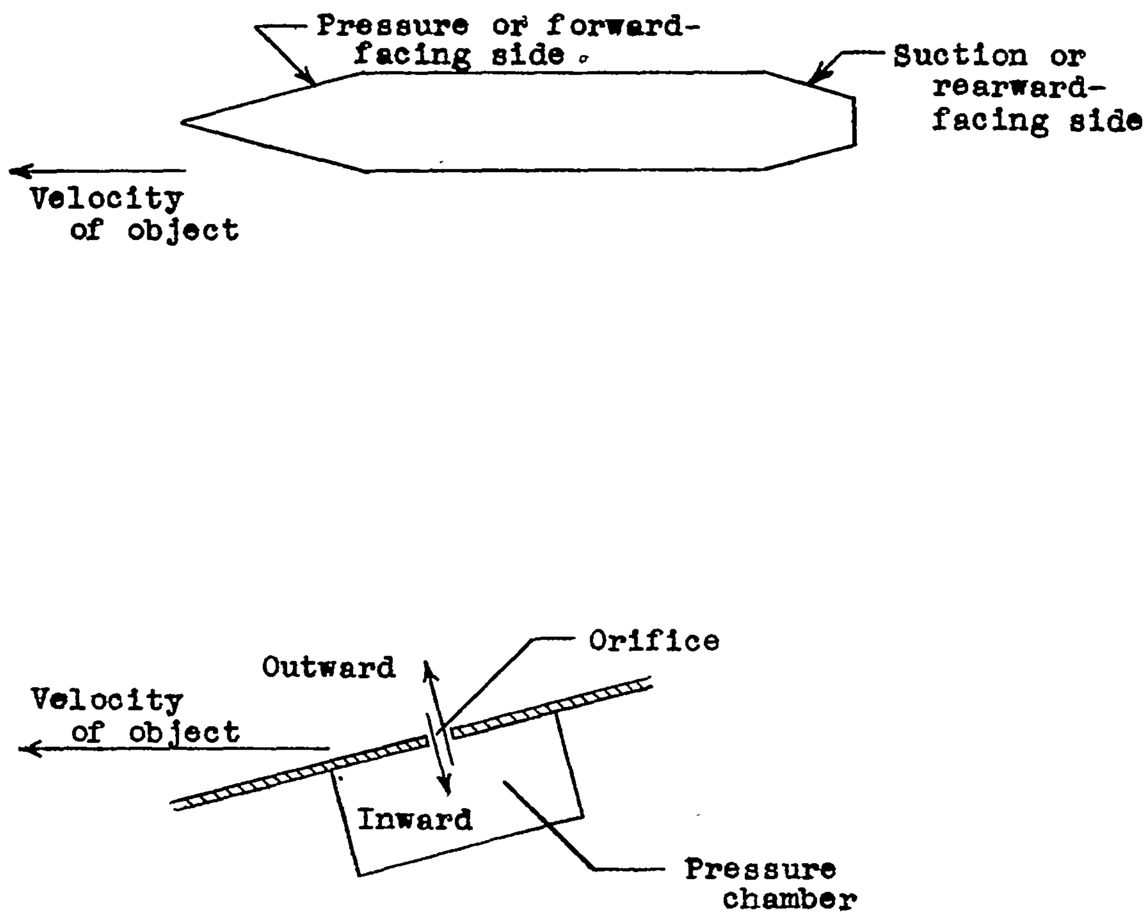

Figure 2.– Explanation of terms and directions.

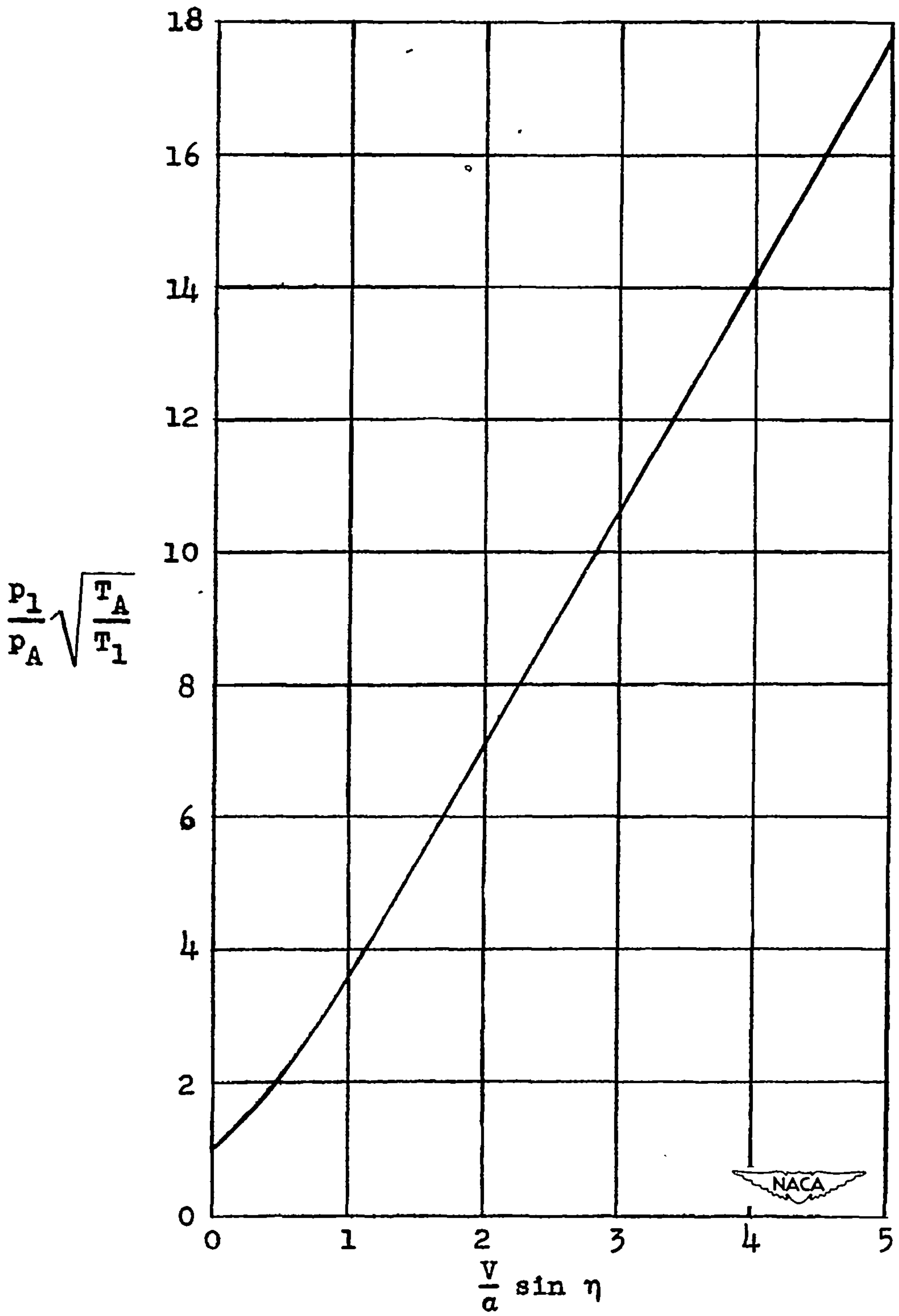

Figure 3.– Plot of $\frac{p_1}{p_A}\sqrt{\frac{T_A}{T_1}}$ as a function of $\frac{V}{a}\sin\eta$ for forward-facing orifice.

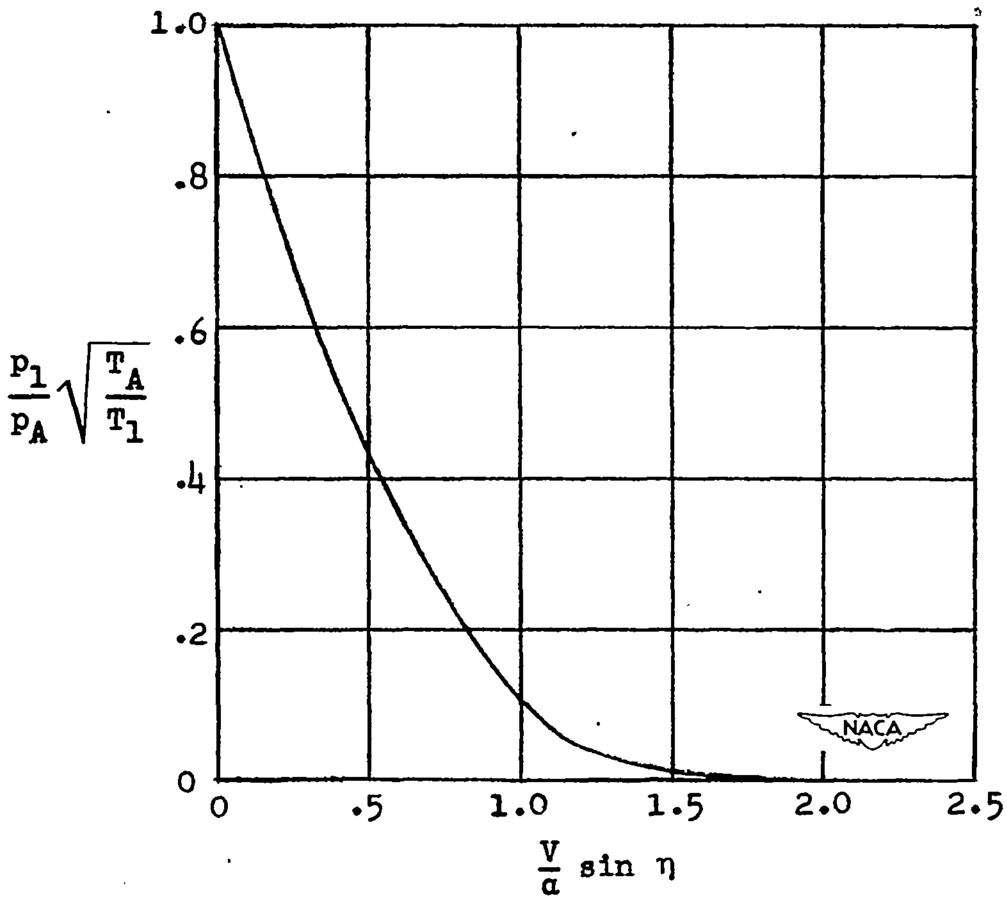

Figure 4.– Plot of $\frac{p_1}{p_A}\sqrt{\frac{T_A}{T_1}}$ as a function of $\frac{V}{a} \sin \eta$ for rearward-facing orifice.

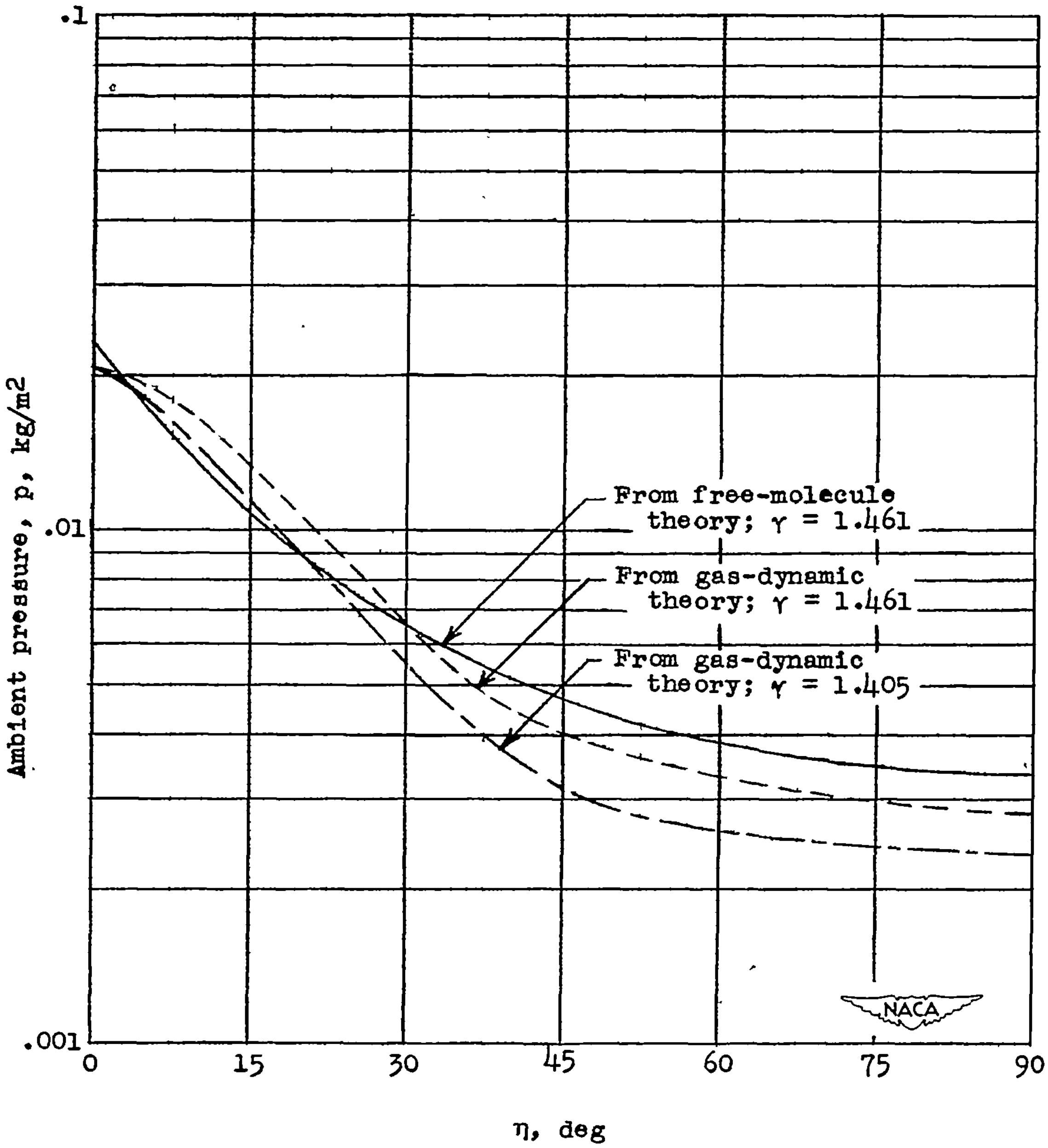

Figure 5.- Variation of ambient pressure determination with method of calculation. $p_1 = 2.087 \times 10^{-2}$ kilograms per square meter; $T_1 = 300^{\circ}$ C absolute; $V = 10^{5}$ centimeters per second; $T_A = 375^{\circ}$ C absolute.