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## COMPRESSIVE BUCKLING OF SIMPLY SUPPORTED PLATES WITH LONGITUDINAL STIFFENERS

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## SUMMARY

Charts are presented for the analysis of the stability under compression of simply supported rectangular plates with one, two, three, and an infinite number of identical equally spaced longitudinal stiffeners that have zero torsional stiffness.

## INTRODUCTION

The purpose of the present paper is to supply the aircraft structural designer with charts for the analysis of the stability under uniform compression of simply supported rectangular plates with identical equally spaced longitudinal stiffeners having no torsional stiffness (fig. 1). Although solutions of the problem have been previously presented in references 1 to 4, numerical results in the form of tables or charts which adequately cover the practical range of stiffener spacing and flexural stiffness are generally unavailable. Timoshenko (reference 1) presents tables giving the buckling stress of plates with one and two stiffeners for only a few values of stiffener spacing, area, and flexural stiffness, whereas Barbré (references 2 and 3) and Ratzersdorfer (reference 4) give only values of the minimum stiffener flexural stiffness required for the plate to buckle with no deflection of the stiffeners. Acknowledgment is made to Mr. Norman Grossman of the Republic Aviation Corporation for the discovery of an error in the results of reference 3.

Buckling charts are given in the present paper for plates with one, two, three, and an infinite number of longitudinal stiffeners. These charts indicate the relationship between the buckling stress coefficient and the plate bay aspect ratio for various values of the ratio of the stiffener flexural stiffness to the flexural stiffness of a plate bay and the ratio of the stiffener area to the area of a plate bay.

The stability equations from which the charts were computed are derived in the appendix by means of the Rayleigh-Ritz energy method. It is assumed in the derivation that the area and flexural stiffness of the stiffeners are concentrated along longitudinal lines at the middle surface of the plate and, as previously mentioned, that the stiffeners have zero torsional stiffness. The assumption of zero stiffener torsional stiffness usually applies with little error in the case of open-section stiffeners.

## SYMBOLS

| | |
|---|---|
| $x,y$ | coordinate axes (fig. 1) |
| $w$ | plate deflection |
| $E_p$ | Young's modulus for plate |
| $\mu$ | Poisson's ratio for plate |
| $t$ | plate thickness |
| $D$ | plate flexural stiffness per unit width $\left(\frac{E_p t^3}{12(1 - \mu^2)}\right)$ |
| $d$ | distance between stiffeners |
| $N$ | number of bays |
| $a$ | plate length |
| $\beta$ | aspect ratio of each bay $\left(\frac{a}{d}\right)$ |
| $EI$ | effective flexural stiffness of stiffener attached to plate |
| $\gamma = \frac{EI}{dD}$ | |
| $A$ | stiffener area |
| $\delta = \frac{A}{dt}$ | |
| $\sigma_{cr}$ | critical compressive stress |
| $k$ | buckling stress coefficient $\left(\frac{\sigma_{cr} d^2 t}{\pi^2 D}\right)$ |

| | |
|---|---|
| c | integer defining location of stiffener |
| q | integer defining number of buckles in y-direction $\left(1 \leqq q \leqq (N-1)\right)$ |
| m | integer defining number of buckles in x-direction |
| n,p,s | integers |

## RESULTS AND DISCUSSION

Charts are presented for the analysis of the stability under uniform compression of simply supported rectangular plates with one, two, three, and an infinite number of identical equally spaced longitudinal stiffeners having no torsional stiffness (fig. 1). These charts (figs. 2 to 5) show the relationship between the buckling stress coefficient $\frac{\sigma_{cr}d^2t}{\pi^2 D}$ and the plate bay aspect ratio $\frac{a}{d}$ for several values of $\frac{EI}{dD}$, the ratio of the stiffener flexural stiffness of a plate bay, and $\frac{A}{dt}$, the ratio of the stiffener area to the area of a plate bay. The equations from which these charts were computed are derived and discussed in the appendix.

The range of plate bay aspect ratios covered in the figures has been limited to values less than 8. The computations indicate that, for an aspect ratio greater than 8, the buckling stress coefficient is given very accurately by the smaller of the two values obtained from the approximate formulas

$$\frac{\sigma_{cr}d^2t}{\pi^2 D} = \frac{\left(\frac{m}{a/d} + \frac{1}{N^2}\frac{a/d}{m}\right)^2 + \left(\frac{m}{a/d}\right)^2 \frac{EI}{dD}}{1 + \frac{A}{dt}} \tag{1}$$

$$\frac{\sigma_{cr}d^2t}{\pi^2 D} = 4 \tag{2}$$

Consecutive integral values of m are substituted in equation (1) until a minimum value of the buckling coefficient is obtained for given values of $\frac{a}{d}$, $\frac{EI}{dD}$, and $\frac{A}{dt}$. Unlike the unstiffened plate, a stiffened plate with a bay aspect ratio of 8 may not be considered to be the equivalent of an infinitely long plate.

If, in figures 2 to 5, the plate bay aspect ratio is kept constant and the stiffener flexural stiffness is increased, a limiting value of the buckling stress which corresponds to the stress for buckling with no deflection of the stiffeners is reached. A further increase in stiffener flexural stiffness will not increase the buckling stress.

The dashed-line curves of figures 2 to 4 indicate a change in the number of half-waves in the longitudinal direction. As the stiffener flexural stiffness increases, the plate bay aspect ratio at which the change occurs also increases since the tendency of the stiffener to buckle as a simply supported column in one half-wave restricts the plate action more and more with increasing stiffener strength. In the case of the plate with an infinite number of stiffeners (fig. 5) the natural tendency of both the plate and the stiffeners is to buckle with one half-wave in the direction of loading so that only that type of buckling occurs.

Charts for the analysis of infinitely long plates with one, two, and three longitudinal stiffeners are given in figure 6. In these plates the stiffener flexural stiffness may be increased with a corresponding increase in buckling stress until a certain value of stiffness is reached. At this point the mode of buckling changes from buckling with deflection of the stiffeners to buckling with a node at each of the stiffeners. A further increase in stiffener flexural stiffness does not increase the buckling stress.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., January 18, 1949

# APPENDIX

## DERIVATION OF THE STABILITY CRITERIONS FOR UNIFORMLY COMPRESSED SIMPLY SUPPORTED RECTANGULAR PLATES WITH LONGITUDINAL STIFFENERS

The potential energy of a buckled rectangular plate with longitudinal stiffeners is equal to the difference of the bending energy $V_1 + V_2$ stored in the plate and stiffeners and the work $T_1 + T_2$ done by the compressive load in shortening the plate and stiffeners. The various components of the potential energy are given by the well-known equations (reference 1)

$$\left.\begin{aligned}
V_1 &= \frac{D}{2}\int_0^a\int_0^{Nd}\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\mu)\left[\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dx\,dy \\
V_2 &= \sum_{c=1}^{N-1}\frac{EI}{2}\int_0^a\left(\frac{\partial^2 w}{\partial x^2}\right)^2_{y=cd}dx \\
T_1 &= \frac{\sigma_{cr}t}{2}\int_0^a\int_0^{Nd}\left(\frac{\partial w}{\partial x}\right)^2 dx\,dy \\
T_2 &= \sum_{c=1}^{N-1}\frac{\sigma_{cr}A}{2}\int_0^a\left(\frac{\partial w}{\partial x}\right)^2_{y=cd}dx
\end{aligned}\right\} \tag{A1}$$

The plate is assumed to buckle in $m$ sinusoidal half-waves in the x-direction. The deflection function $w$ is therefore taken as the Fourier series

$$w = \sin \frac{m\pi x}{a} \sum_{n=1}^{\infty} a_n \sin \frac{n\pi y}{Nd} \tag{A2}$$

which satisfies the simple support conditions of zero deflection and zero moment at each edge. Substitution of equation (A2) in equation (A1) yields, after simplification, the potential energy

$$W = \frac{\pi^4 DNd}{8a^3} \left\{ \sum_{n=1}^{\infty} a_n^2 \left[ \left( m^2 + n^2 \frac{\beta^2}{N^2} \right)^2 - \beta^2 k m^2 \right] + \frac{2}{N} \sum_{c=1}^{N-1} m^2 (\gamma m^2 - \beta^2 k \delta) \left( \sum_{n=1}^{\infty} a_n \sin \frac{n\pi c}{N} \right)^2 \right\} \tag{A3}$$

The unknown Fourier coefficients $a_n$ are determined from the condition that the potential energy be a minimum. This condition yields the following set of equations

$$\frac{\partial W}{\partial a_n} = 0 = a_n \left[ \left( m^2 + n^2 \frac{\beta^2}{N^2} \right)^2 - \beta^2 k m^2 \right] + \frac{2}{N} m^2 \left( \gamma m^2 - \beta^2 k \delta \right) \sum_{p=1}^{\infty} a_p \sum_{c=1}^{N-1} \sin \frac{p\pi c}{N} \sin \frac{n\pi c}{N} \tag{A4}$$

$$(n = 1, 2, 3, \ldots)$$

Equations (A4) are similar to equations encountered in the solution of the stability problem of reference 5 and may be solved by the method used in that paper. The stability criterions so obtained are

$$\gamma = \frac{-1/m^4}{\sum_{s=0}^{\infty} \frac{1}{\left[m^2 + \left(2s + \frac{q}{N}\right)^2\beta^2\right]^2 - \beta^2 k m^2} + \sum_{s=1}^{\infty} \frac{1}{\left[m^2 + \left(2s - \frac{q}{N}\right)^2\beta^2\right]^2 - \beta^2 k m^2}}$$

$$+ \frac{\beta^2 k \delta}{m^2} \tag{A5}$$

$$(q = 1, 2, \ldots N - 1)$$
$$(m = 1, 2, 3, \ldots)$$

which criterions correspond to buckling with deflection of the stiffeners and

$$k = \left(\frac{m}{\beta} + \frac{\beta}{m}\right)^2 \tag{A6}$$

which criterion corresponds to buckling with nodes at all of the stiffeners. Equations (A5) may be put into the closed form

$$\gamma = \frac{\frac{4}{\pi^2}\frac{\sqrt{k}\beta^3}{m^3}}{\dfrac{\frac{\sin\theta_1}{\theta_1}}{\cos\pi\frac{q}{N} - \cos\theta_1} - \dfrac{\frac{\sinh\theta_2}{\theta_2}}{\cos\pi\frac{q}{N} - \cosh\theta_2}} + \frac{\beta^2 k\delta}{m^2} \tag{A7}$$

where

$$\theta_1 = \pi\sqrt{\frac{m}{\beta}\left(\sqrt{k} - \frac{m}{\beta}\right)}$$

$$\theta_2 = \pi\sqrt{\frac{m}{\beta}\left(\sqrt{k} + \frac{m}{\beta}\right)}$$

and

$$m = 1, 2, 3 \ldots$$

$$q = 1, 2, \ldots N - 1$$

Computations made with equations (A6) and (A7) indicate that, except for a small range of values of $\beta (\beta < \sqrt{2})$ and $k$, the criterion corresponding to plate instability with one buckle in the y-direction ($q=1$) yields the highest value of the required stiffener flexural stiffness and hence the lowest buckling stress. For the ranges of $\beta$ and $k$ in which other buckling modes prevail, the difference between the highest stiffener flexural stiffness and the stiffener flexural stiffness computed from the criterion with $q = 1$ is negligible. These findings contradict results obtained by Barbré in reference 3 in which it is stated that, for a plate with two stiffeners, the criterion for the antisymmetrical buckling node $\left(\frac{q}{N} = \frac{2}{3}\right)$ gives buckling stresses appreciably lower than the buckling stresses computed from the criterion for the symmetrical buckling node $\left(\frac{q}{N} = \frac{1}{3}\right)$ for some values of the aspect ratio $\beta$. Mr. Norman Grossman of the Republic Aviation Corporation found that Barbré had made an error in comparing the critical stresses for the antisymmetrical buckling mode with the critical stresses for the symmetrical buckling mode. The results of the present paper agree with the corrected results of Barbre.

The computations indicate that, when the plate bay aspect ratio $\beta$ is greater than 8, accurate values of the stiffener flexural stiffness may be found by using only the first term in the first series of equation (A5). The stability criterion may then be written as

$$\gamma = -\left(1 + \frac{\beta^2}{N^2 m^2}\right)^2 + \frac{k\beta^2(1 + \delta)}{m^2} \tag{A8}$$

Equation (A8), which is equivalent to equation (1), is the stability criterion that would be obtained if the deflection function were approximated by

$$w = a_1 \sin \frac{m\pi x}{a} \sin \frac{\pi y}{Nd} \tag{A9}$$

## REFERENCES

1. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Company, Inc., 1936, pp. 372-378.

2. Barbré, Rudolf: Buckling Stresses of Rectangular Plates with Longitudinal Stiffeners under Uniform Compression. Translation 78, The David W. Taylor Model Basin, U. S. Navy, July 1943.

3. Barbré, R.: Stability of Rectangular Plates with Longitudinal or Transverse Stiffeners under Uniform Compression. NACA TM No. 904, 1939.

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5. Budiansky, Bernard, Seide, Paul, and Weinberger, Robert A.: The Buckling of a Column on Equally Spaced Deflectional and Rotational Springs. NACA TN No. 1519, 1948.

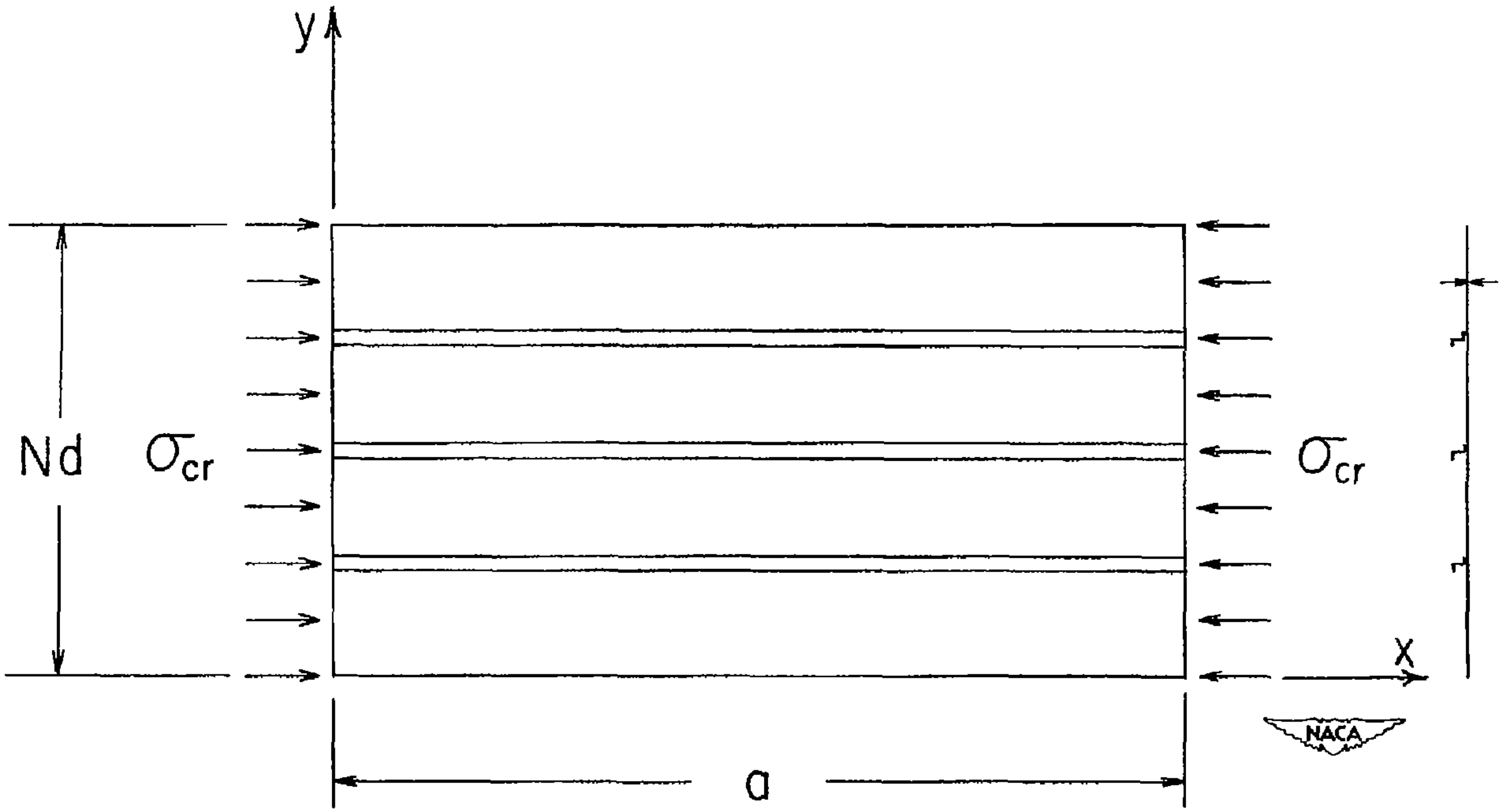
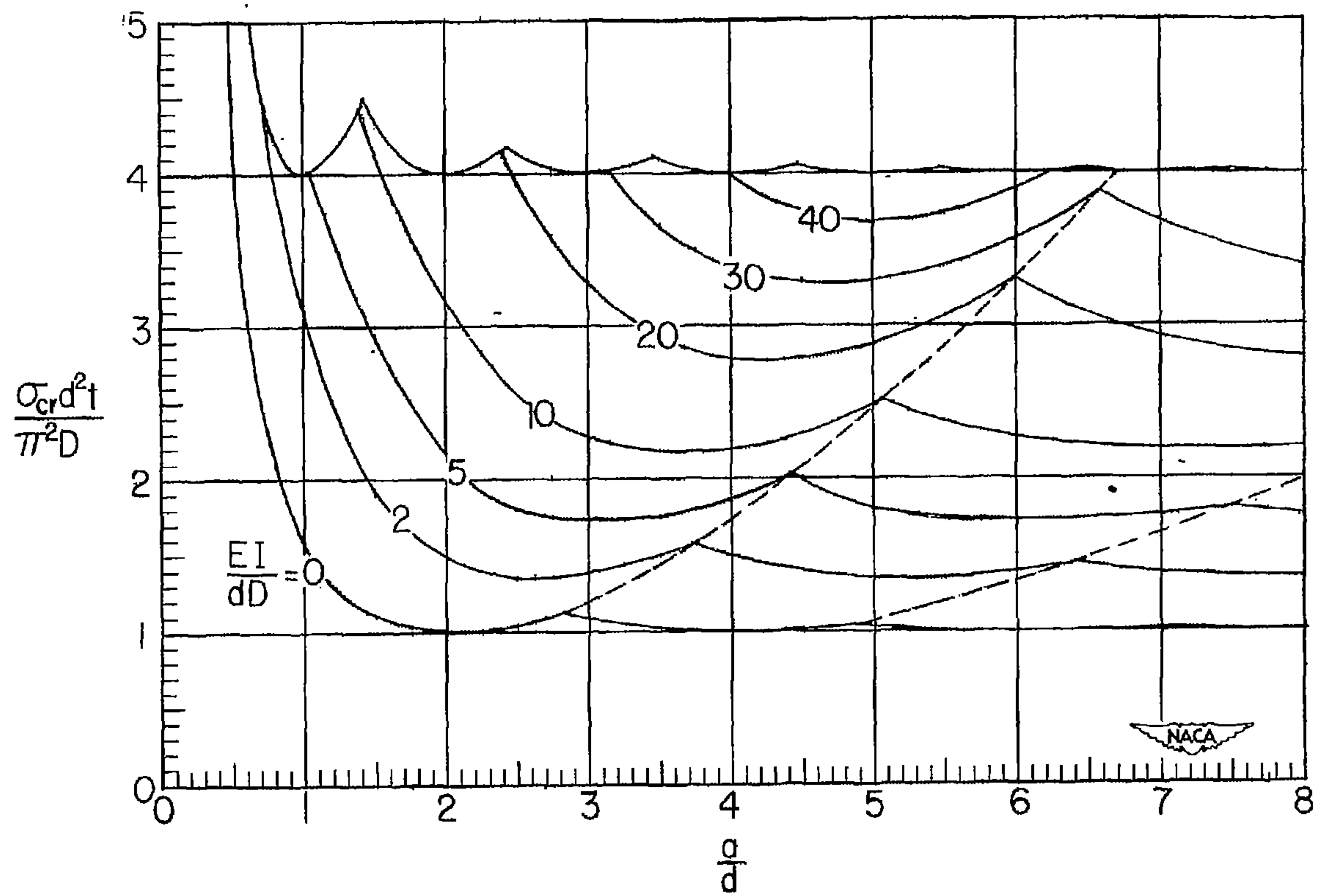
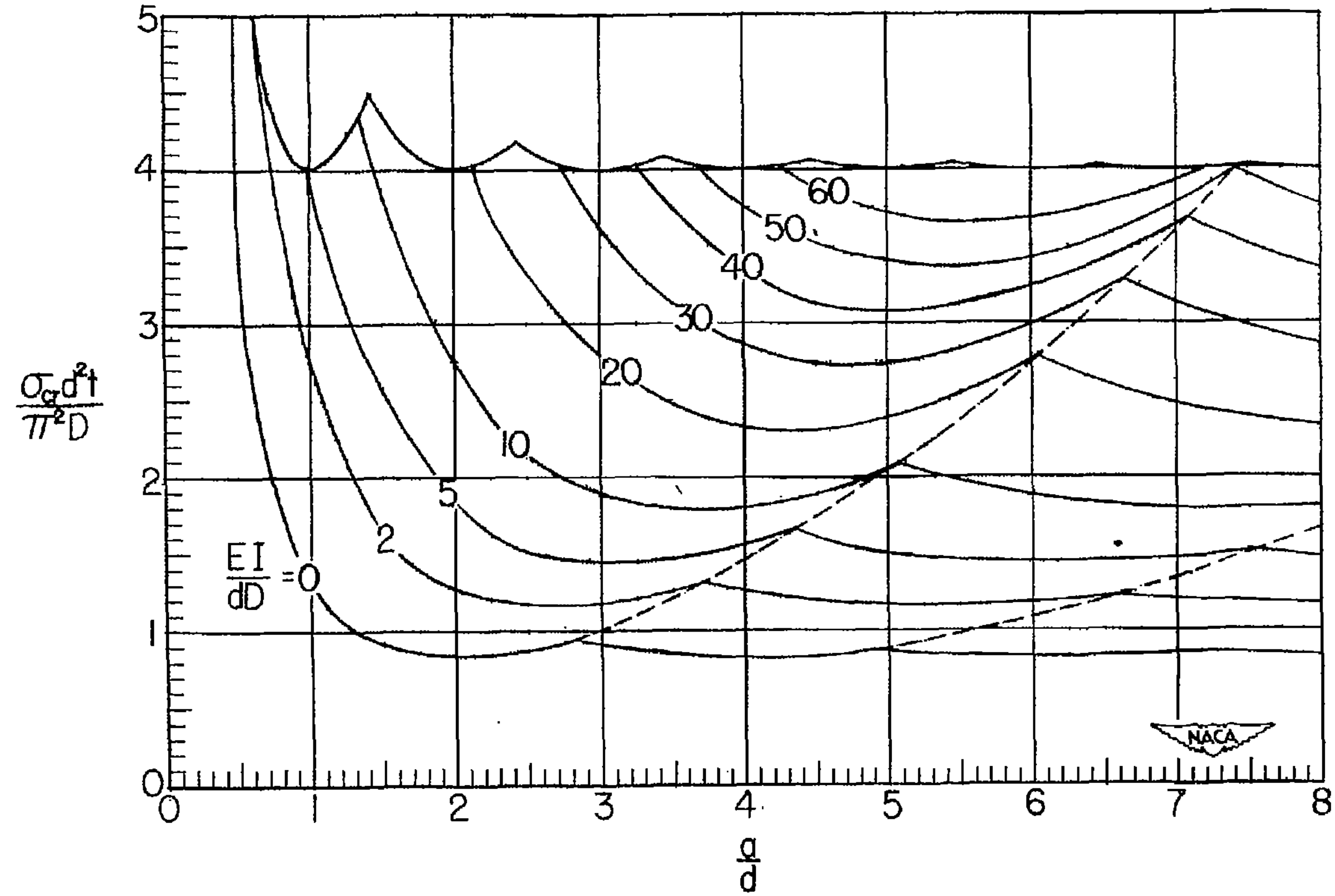



Figure 1.– Simply supported rectangular plate with longitudinal stiffeners (N = 4).



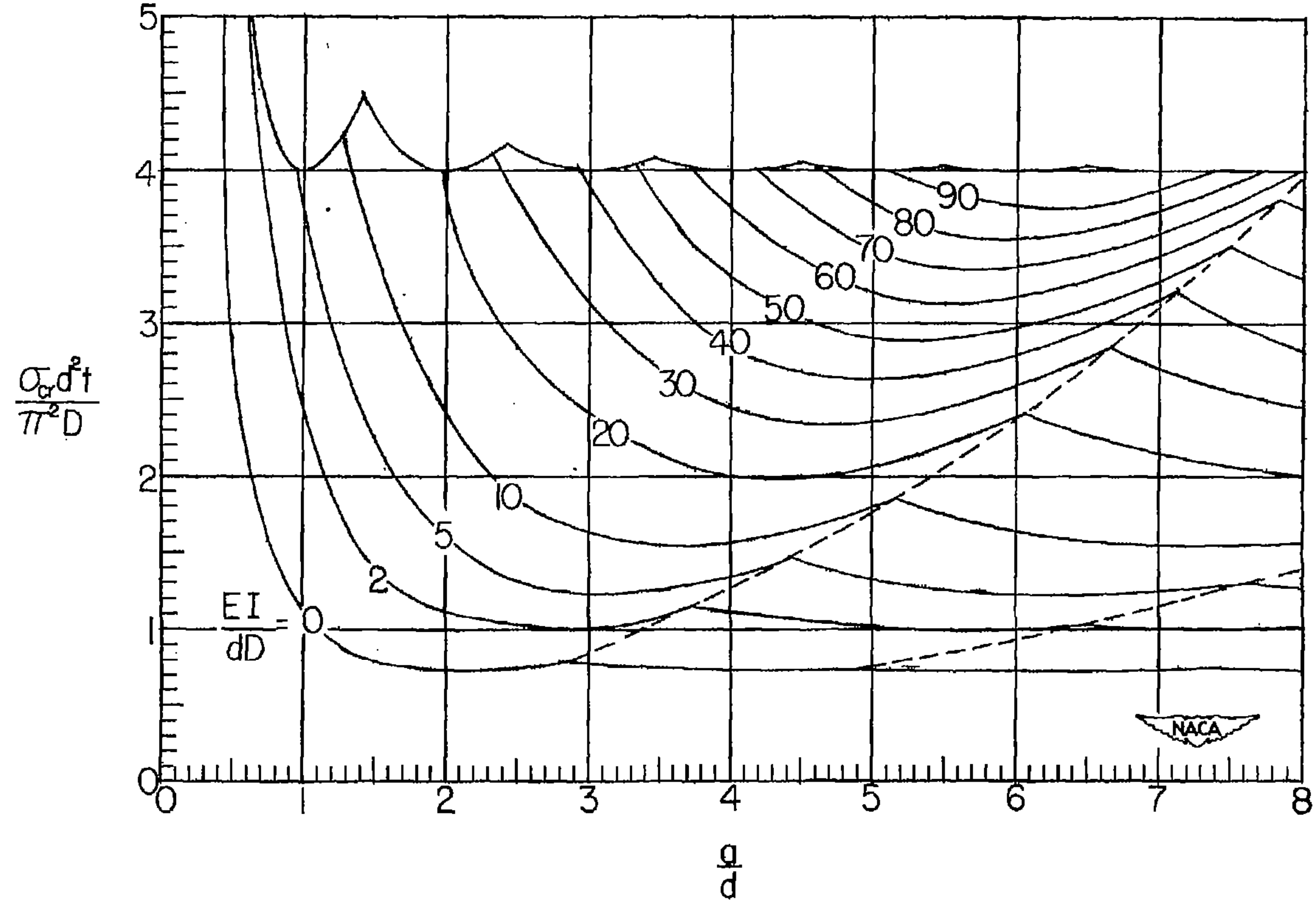

(a) $\frac{A}{dt} = 0.$

Figure 2.– Compressive buckling curves for plates with one longitudinal stiffener.



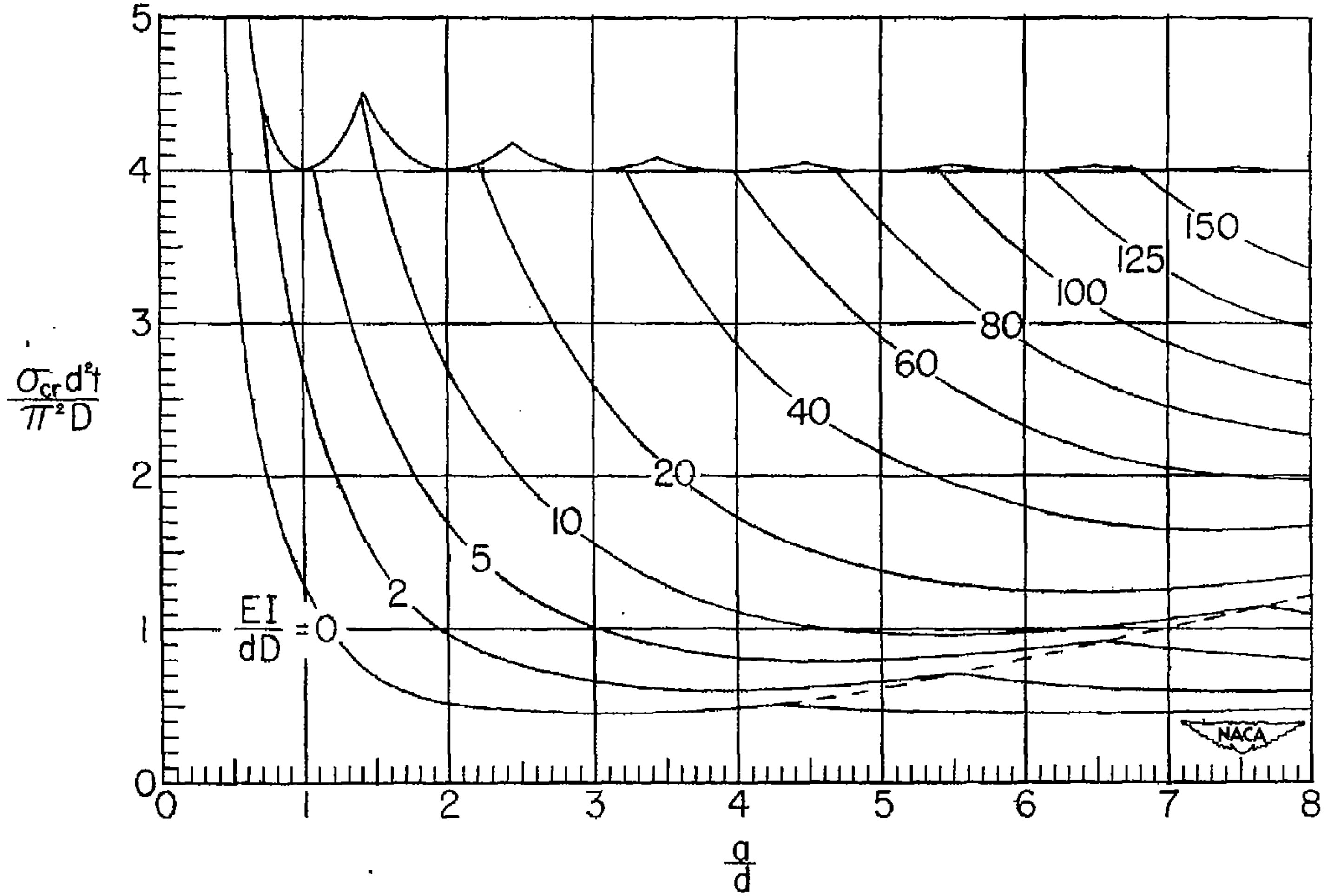

(b) $\frac{A}{dt} = 0.2.$

Figure 2.– Continued.



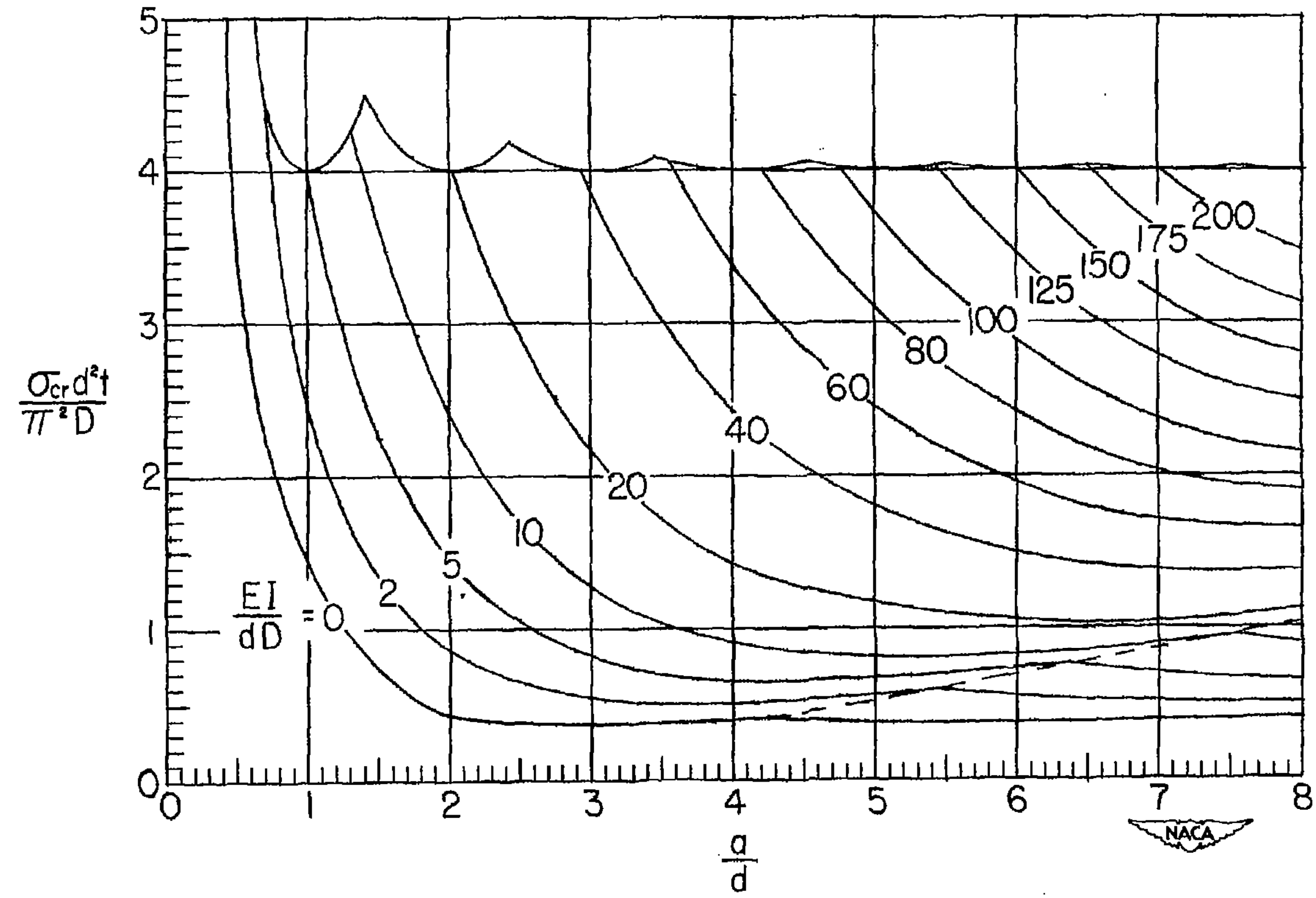

(c) $\frac{A}{dt} = 0.4.$

Figure 2.— Concluded.



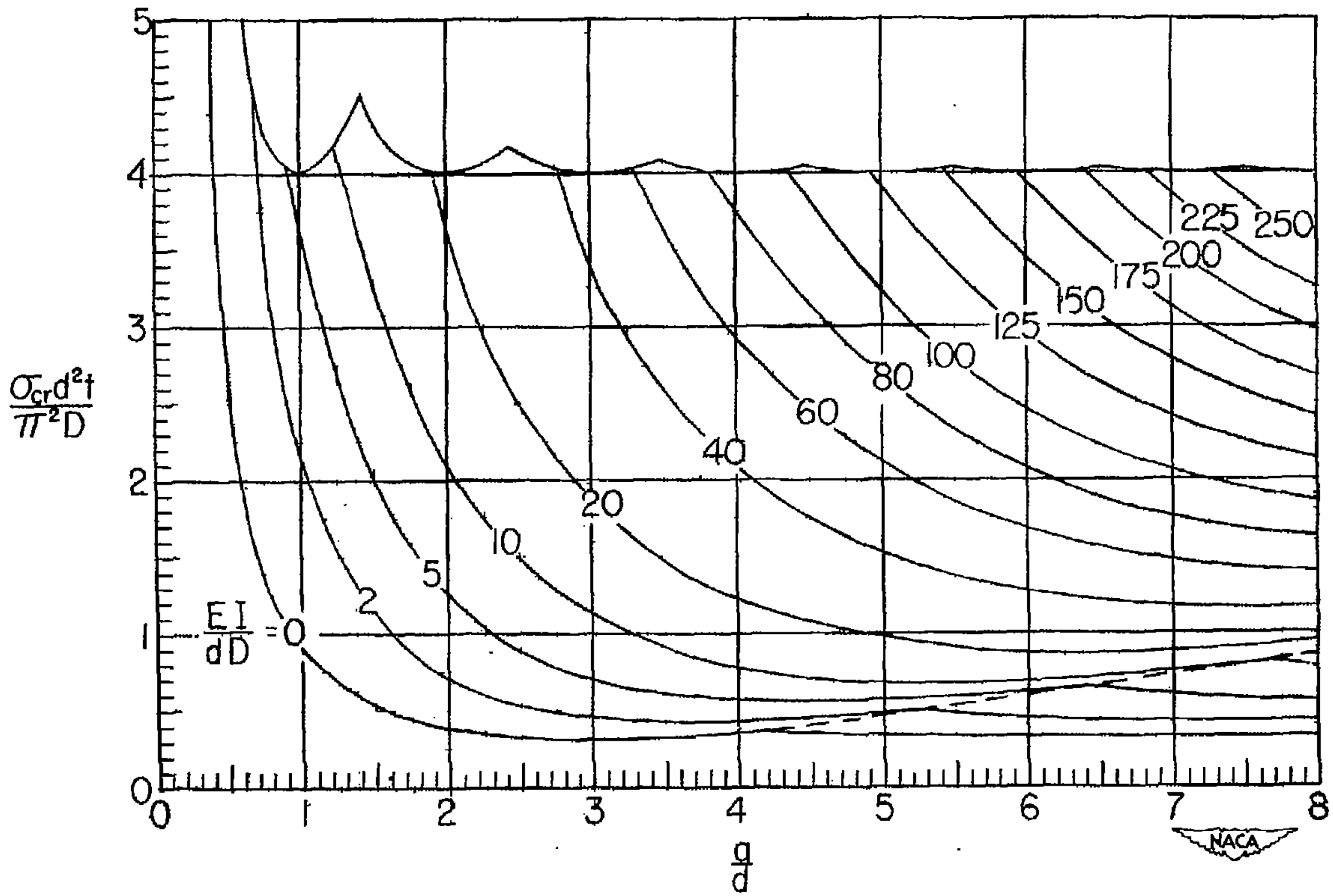

(a) $\frac{A}{dt} = 0$.

Figure 3.– Compressive buckling curves for plates with two longitudinal stiffeners.



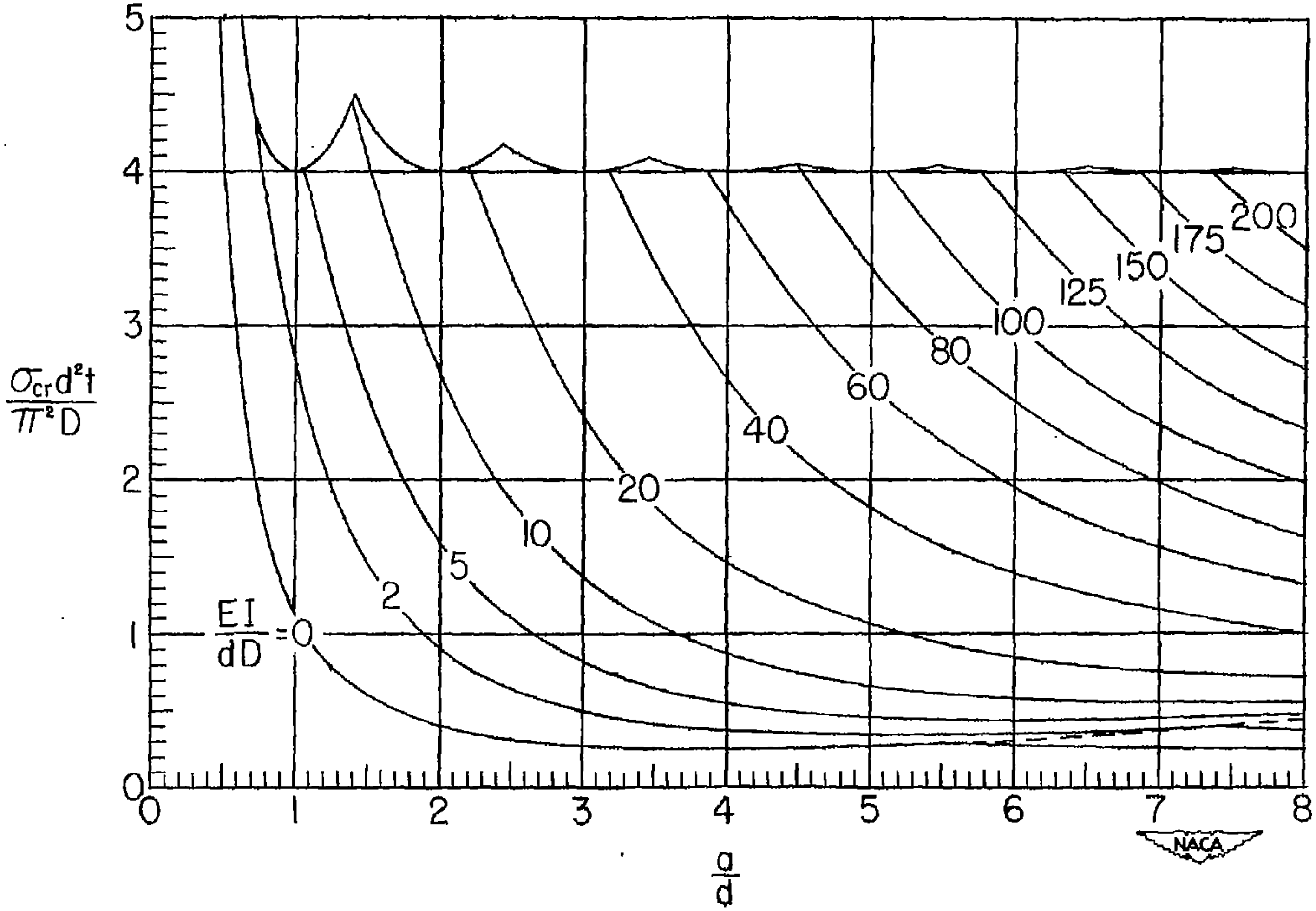

(b) $\frac{A}{dt} = 0.2$.

Figure 3.– Continued.



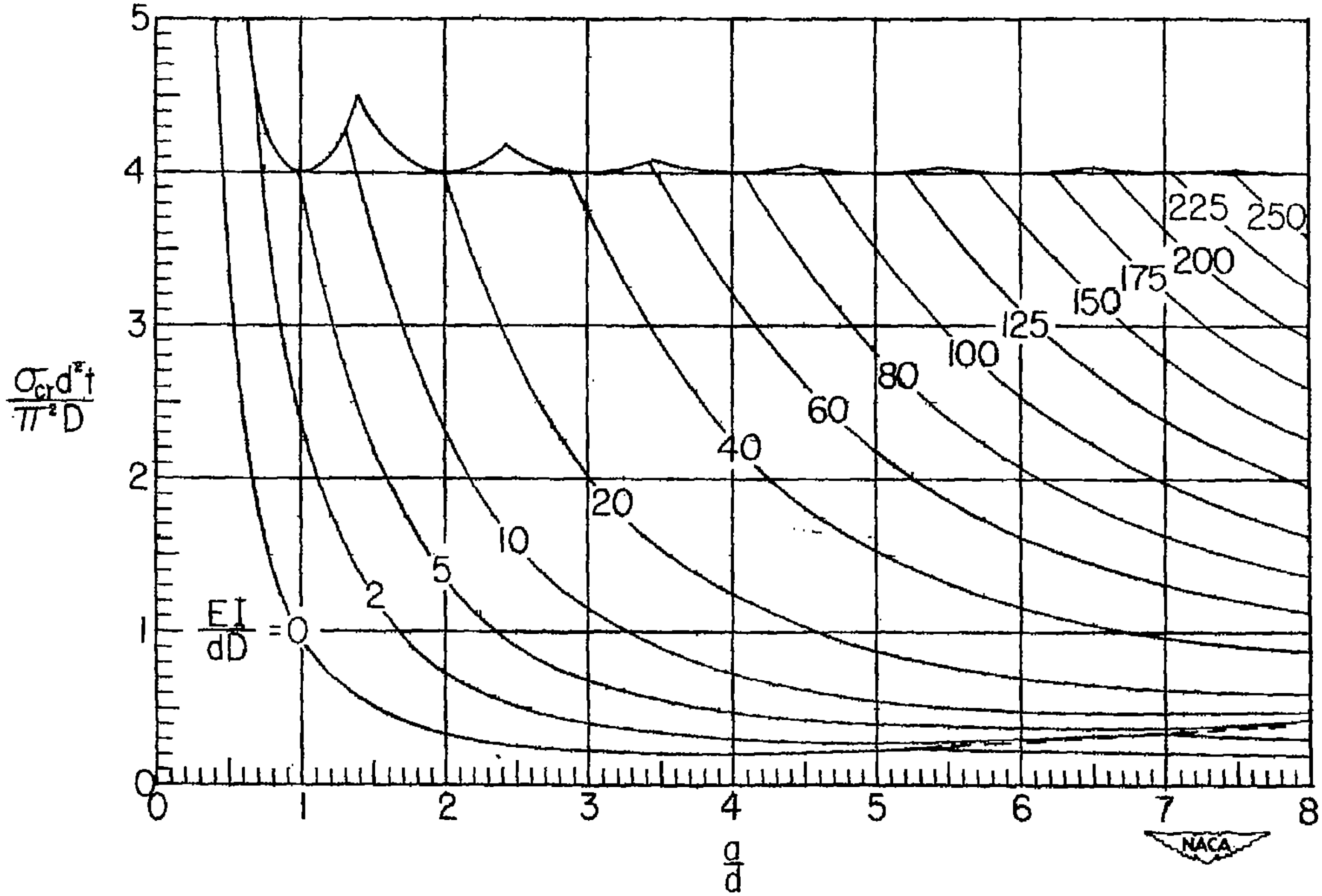

(c) $\frac{A}{dt} = 0.4$.

Figure 3.– Concluded.



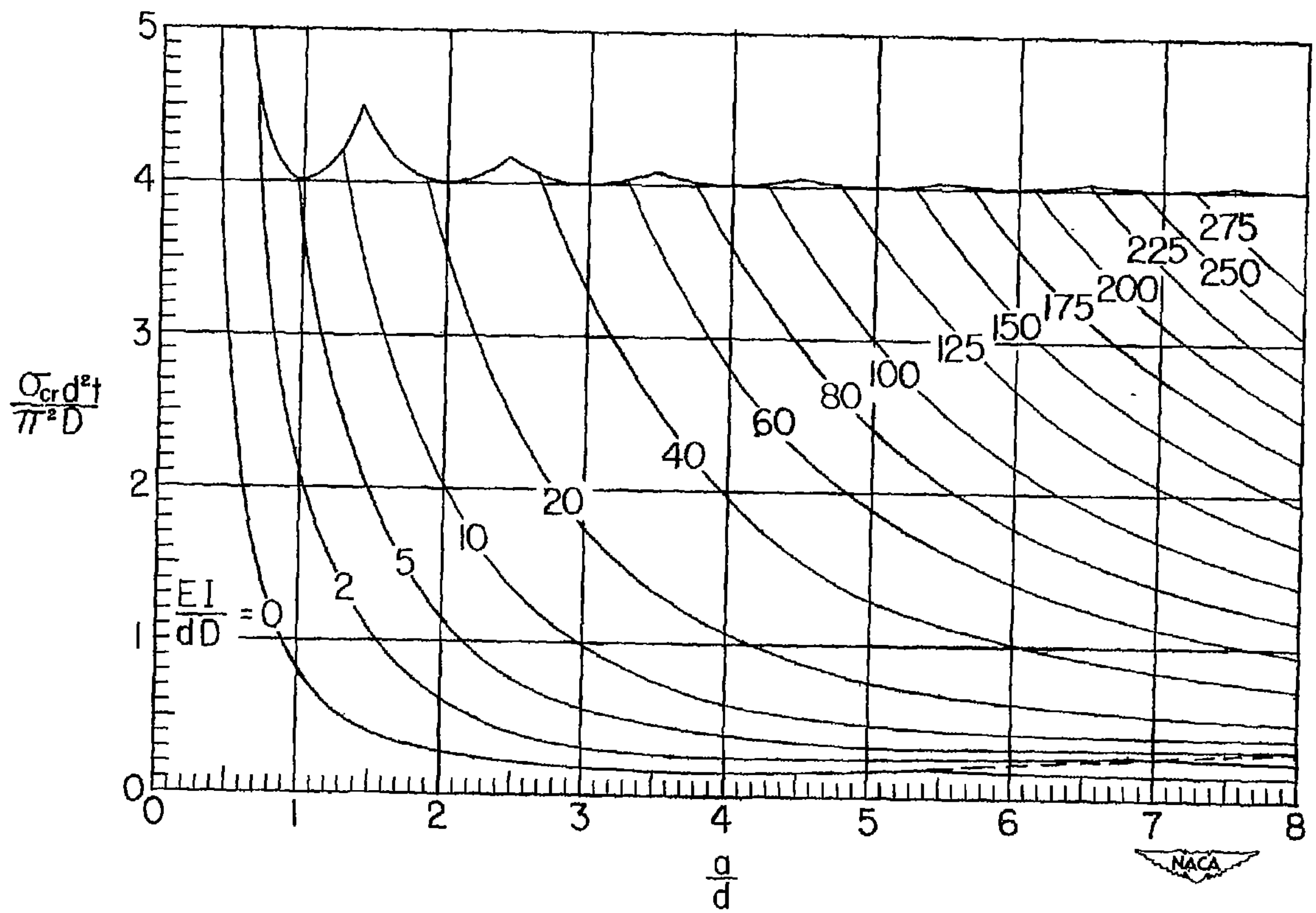

(a) $\frac{A}{dt} = 0.$

Figure 4.– Compressive buckling curves for plates with three longitudinal stiffeners.



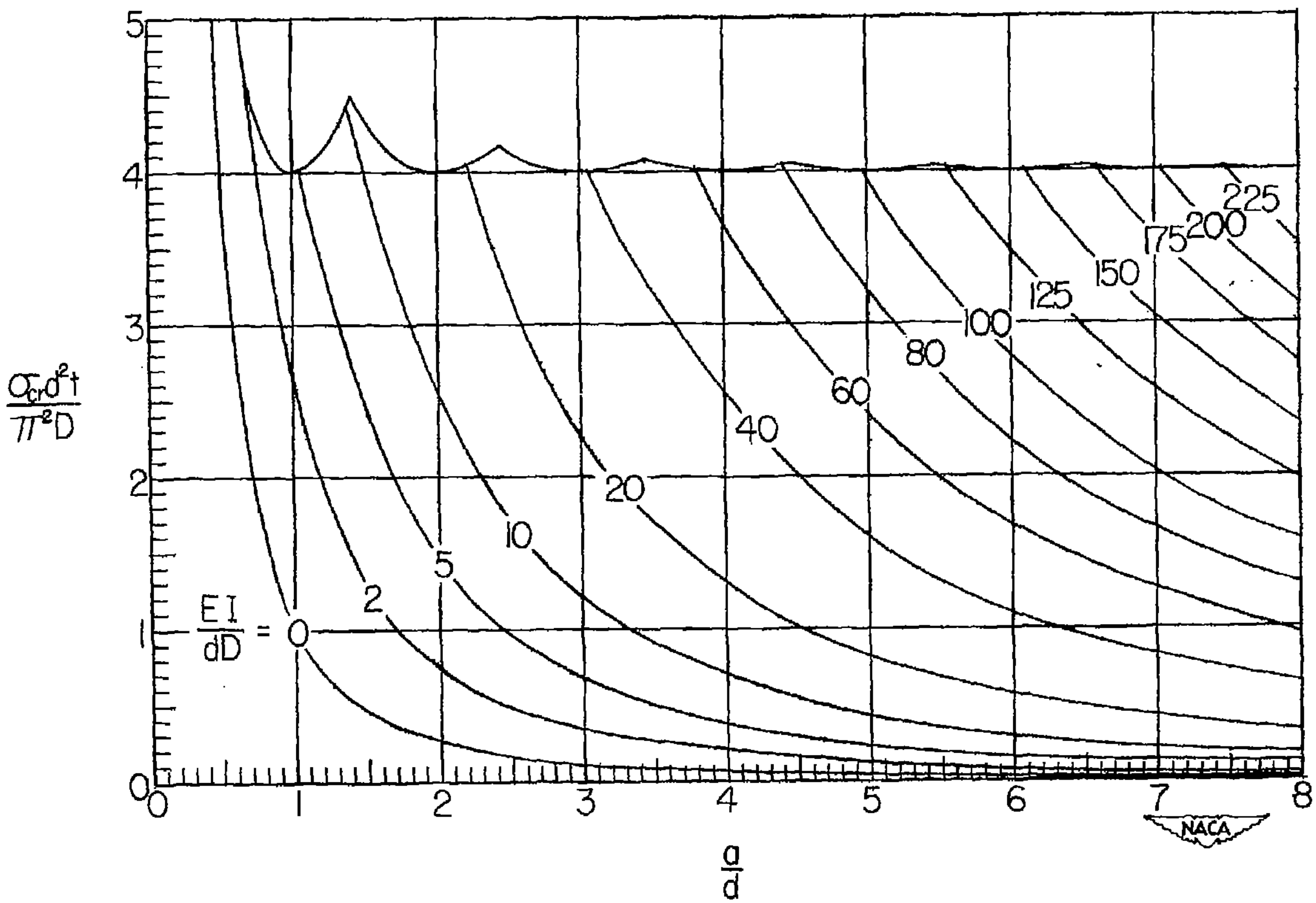

(b) $\frac{A}{dt} = 0.2.$

Figure 4.– Continued.



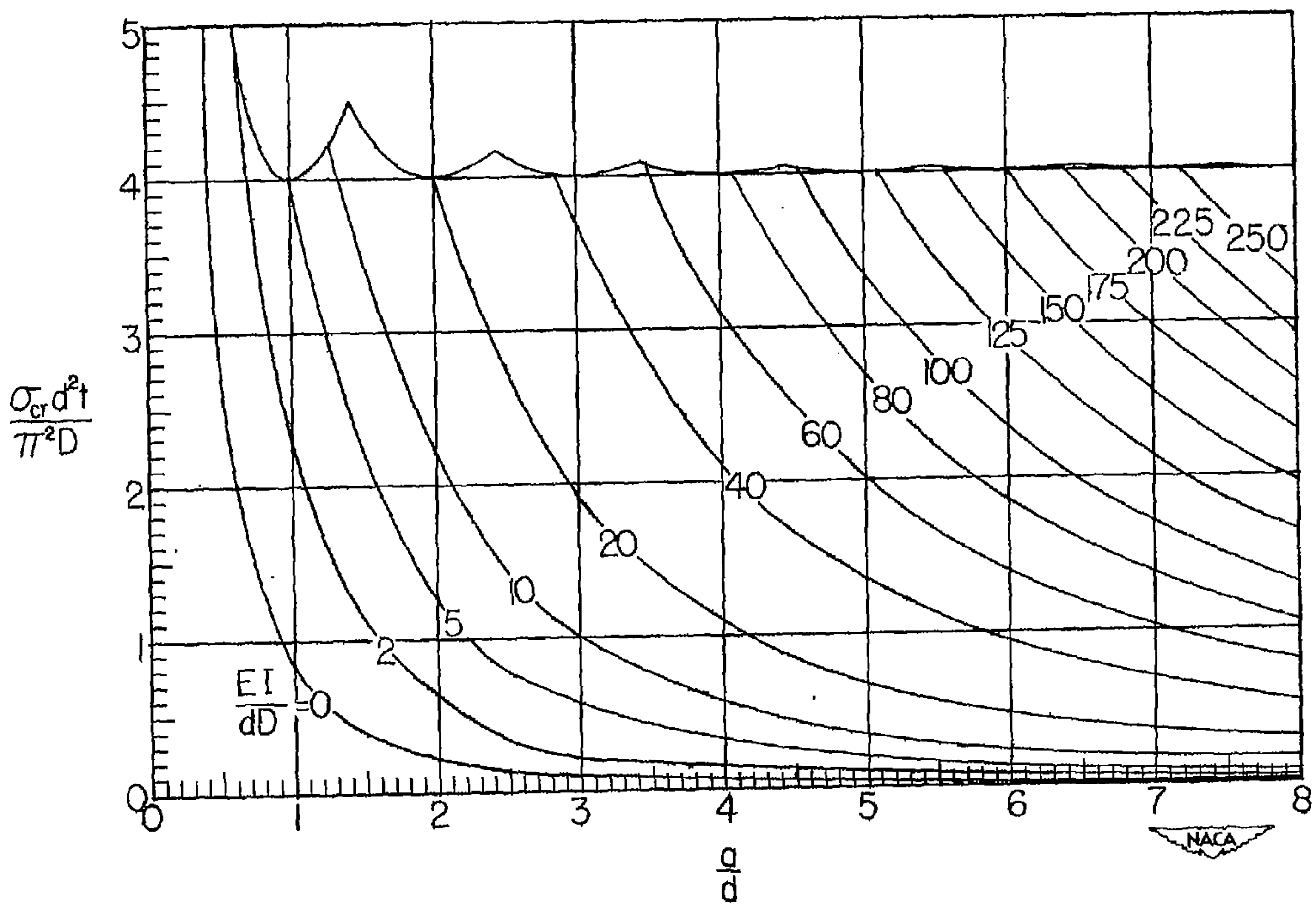

(c) $\frac{A}{dt} = 0.4.$

Figure 4.– Concluded.



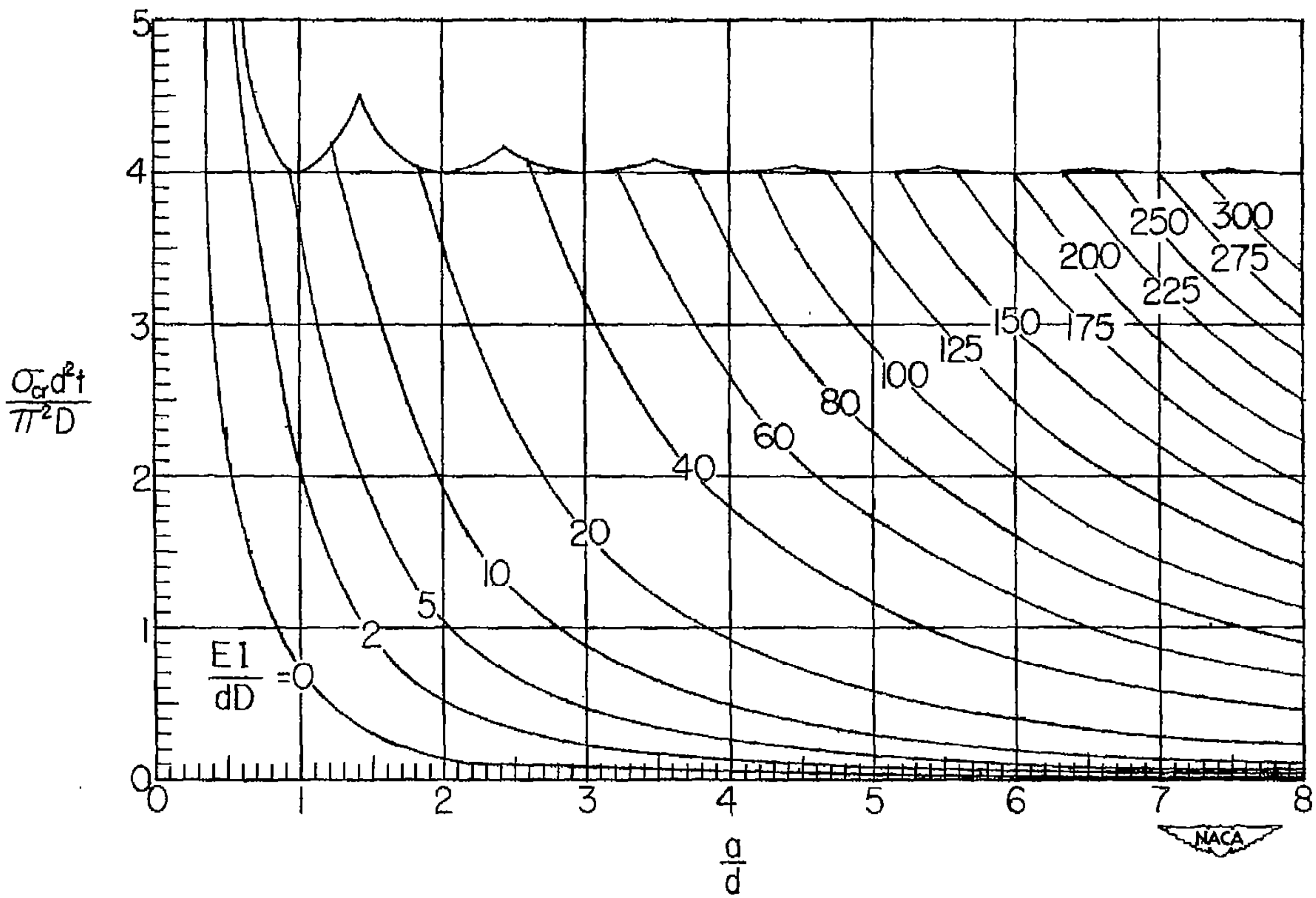

(a) $\frac{A}{dt} = 0.$

Figure 5.– Compressive buckling curves for plates with an infinite number of longitudinal stiffeners.




(b) $\frac{A}{dt} = 0.2.$

Figure 5.- Continued.




(c) $\frac{A}{dt} = 0.4.$

Figure 5.– Concluded.

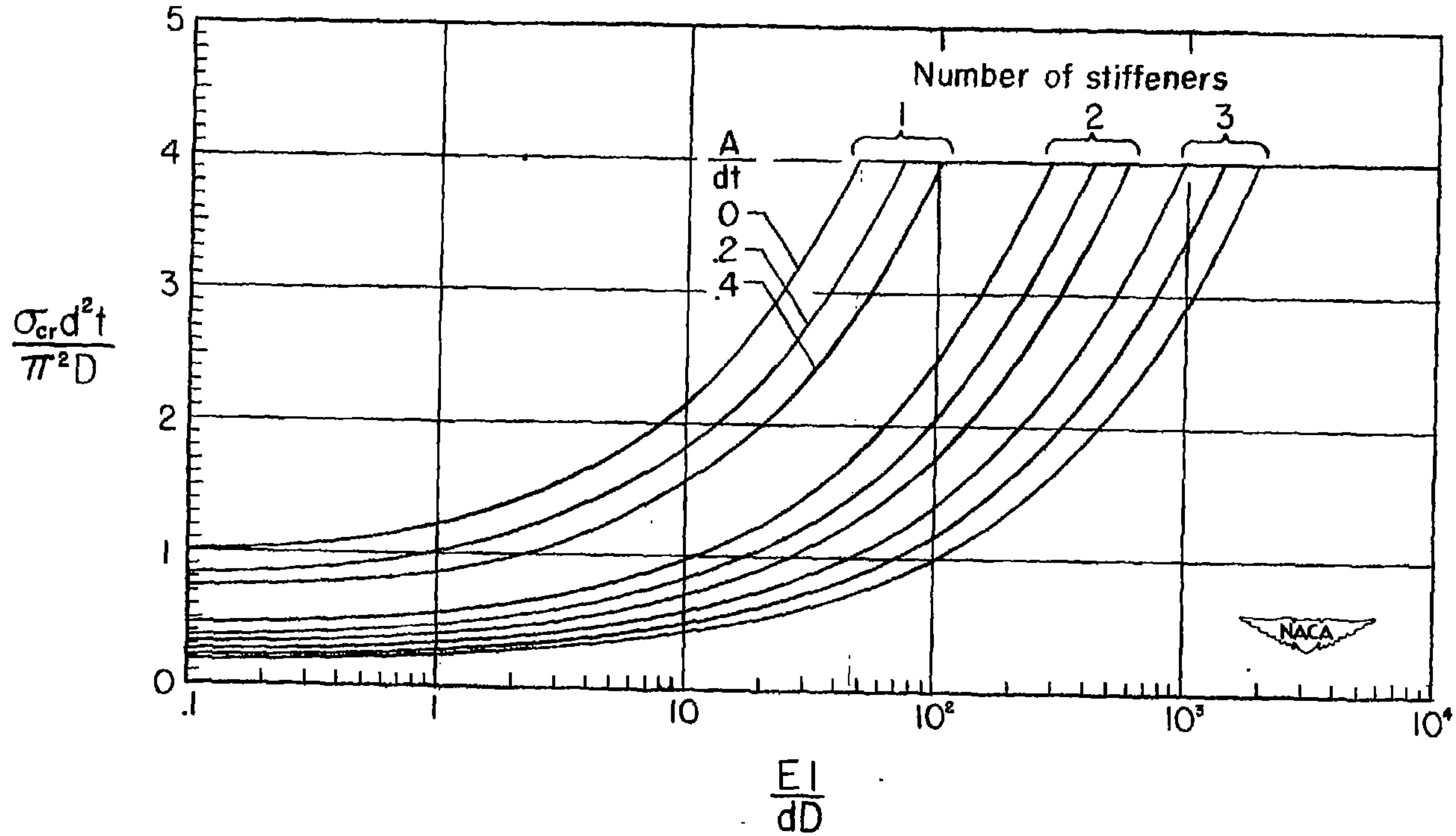



Figure 6.– Compressive buckling curves for infinitely long plates with longitudinal stiffeners.