A SIMPLE METHOD OF ESTIMATING THE SUBSONIC LIFT AND DAMPING IN ROLL OF SWEPTBACK WINGS

By Edward C. Polhamus

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SUMMARY

A method of modifying existing correction factors of lifting-surface theory to account approximately for the effects of sweep has been derived, and these factors have been applied to existing lifting-line theories for the lift and damping in roll of swept wings. Despite the simplicity of the resulting formulas the agreement with experimental data for low speeds is very good. The equation for lift is expressed entirely in terms of the geometric characteristics of the wing and the section-lift-curve slope; the necessity for any charts is thereby eliminated. The equation for the damping in roll, however, requires a chart for the determination of the effective lateral center of pressure for rolling moment due to rolling. If the Glauert-Prandtl transformation is used, the formulas obtained can be applied to swept wings at subsonic speeds below the critical speed.

INTRODUCTION

Lifting-line theory has been found to be inadequate for the prediction of the aerodynamic characteristics of low-aspect-ratio and sweptback wings and tedious lifting-surface methods have been resorted to. It would be useful, therefore, if some simple modifications could be applied to lifting-line theories which would permit their use in estimating the aerodynamic characteristics of wings having plan forms requiring the application of lifting-surface theories. Reference 1 presents relations for the lift and damping in roll of swept wings based on approximate lifting-line theory. However, it appears that the accuracy of these relations may be appreciably improved by modifying them to account for the edge-velocity correction factor for swept wings.

In the present paper, a simple although not rigorous edge-velocity correction factor for sweptback wings is utilized to obtain simple relationships for the lift and damping in roll for subsonic speeds. The resulting expressions are then compared with some experimental data obtained at low speeds.
SYMBOLS

The symbols used are defined as follows:

- $C_L$: lift coefficient (Lift/$qS$)
- $C_I$: rolling-moment coefficient (Rolling moment/$qS_b$)
- $\alpha$: angle of attack measured in plane of symmetry, degrees
- $\beta/2V$: wing-tip helix angle, radians
- $q$: dynamic pressure, pounds per square foot ($\frac{1}{2} \rho V^2$)
- $V$: free-stream velocity, feet per second
- $\rho$: mass density, slugs per cubic foot
- $S$: wing area, square feet
- $b$: wing span, feet
- $p$: rolling angular velocity, radians per second
- $a_0$: section lift-curve slope for section normal to quarter-chord line when placed in direction of free stream, per degree
- $A$: aspect ratio ($\frac{b^2}{S}$)
- $A_e$: equivalent aspect ratio
- $\Lambda$: sweep angle of wing quarter-chord line, degrees
- $\Lambda_e$: equivalent sweep angle, degrees
- $\lambda$: taper ratio ($\frac{\text{Tip chord}}{\text{Root chord}}$)
- $E_e$: effective edge-velocity correction factor for lift of unswept wings
- $E_{e\Lambda}$: effective edge-velocity correction factor for lift of swept wings
LIFT CHARACTERISTICS

Unswept wings. - Several theories have been developed for the prediction of the lift characteristics of finite-span unswept wings. The simplest of these is the well known lifting-line formula for elliptical wings which may be written as follows:

\[ C_{L_{\alpha}} = \frac{2\pi}{57.3} \left( \frac{A}{A + 2} \right) \]  

where the section lift-curve slope is assumed to be \( \frac{2\pi}{57.3} \). This formula, however, is inadequate to obtain accurate results for wings of low aspect ratio. Jones (reference 2) derived a correction factor \( E \) which is the ratio of the semiperimeter to the span. This correction is based on the fact that, for a given free-stream velocity, the local velocity around the trailing edge of a wing of elliptical plan form is less than that around the trailing edge of a two-dimensional wing at the same angle of attack by the factor \( 1/E \). The circulation required to satisfy the Kutta condition, therefore, is reduced by the same factor and a reduction in lift results. This correction factor, which is known as the Jones edge-velocity correction factor, has been modified by Swanson and Crandall who utilized the lifting-surface results obtained by the electromagnetic-analogy method. (See reference 3.) The surface load distribution
simulated in their experiments was represented by 50 vortex-filament wires and the results are considered to be reliable. Applying these corrections to the lifting-line equation (equation (1)) results in the following equation:

\[ CL_\alpha = \frac{2\pi}{57.3} \left( \frac{A}{AE_e + 2} \right) \]  

(2)

where the factor \( E_e \) is the effective edge-velocity correction factor. Because the expression for \( E_e \) is extremely complicated the values are presented in chart form in reference 3.

Utilizing an approximate treatment of the problem and considering the induced downwash angle at the three-quarter-chord point, Helmbold (reference 4) derived a simple relation for the lift-curve slope of unswept wings of elliptical plan form, which in the notation of the present paper may be expressed as

\[ CL_\alpha = \frac{2\pi}{57.3} \left( \frac{A}{\sqrt{A^2 + 4} + 2} \right) \]  

(3)

As the aspect ratio approaches zero, equation (3) approaches the following value:

\[ CL_\alpha = \frac{2\pi A}{2(57.3)} \]  

(4)

If equation (4) is converted to radians, this expression is the same as that given in reference 5 for low-aspect ratio wings.

The most exact mathematical treatment of the lift of finite-span wings is that of Krienes who calculated the lift-curve slope of four elliptical wings of different aspect ratios. (See reference 6.) In order to determine the accuracy of the various methods discussed previously they are compared with the results of reference 6 in figure 1. All of the values presented in figure 1 are based on a section lift-curve slope of \( 2\pi \) per radian. The comparison indicated that the methods of both references 3 and 4 are in good agreement with the results of reference 6. Although both methods are equally accurate the method of reference 4 has an advantage in that the expression for the lift-curve slope may be expressed entirely in terms of the aspect ratio, whereas a chart of the effective edge-velocity correction factor (which must be extrapolated for aspect ratios less than 3) is necessary for the method of reference 3.
Although the formula of reference 4 is applicable to unswept wings of any aspect ratio, it neglects to take account of the effects of sweep. A modification of this formula which is applicable to sweptback wings is developed in the following section.

**Swept wings.** - The following equation for the lift-curve slope of swept wings based on an approximate lifting-line theory has been developed in reference 1:

\[
C_{L\alpha} = \frac{2\pi}{57.3} \left( \frac{A \cos \Lambda}{A + 2 \cos \Lambda} \right)
\]  

(5)

Applying an effective edge-velocity correction factor for swept wings to equation (5) in a manner similar to that used for unswept wings (see equation (2)) results in the following expression:

\[
C_{L\alpha} = \frac{2\pi}{57.3} \left( \frac{A \cos \Lambda}{AE_{e\Lambda} + 2 \cos \Lambda} \right)
\]  

(6)

For swept wings the reduction of the trailing-edge velocity would probably depend on the reciprocal of the ratio of the semiperimeter of the equivalent ellipse to the span measured along the sweep line rather than normal to the plane of symmetry. Therefore, \( E_{e\Lambda} \) should be based on \( A / \cos^2 \Lambda \), the aspect ratio with the panels unswept rather than on \( A \), the aspect ratio with the panels swept. For this case, as well as for the case of the unswept wing, the effective edge-velocity factor must be obtained from the chart presented in reference 3. If, however, equations (2) and (3) are equated, the expression \( \frac{\sqrt{A^2 + 4}}{A} \) is obtained for the effective edge-velocity correction factor for unswept wings and the need for a chart is eliminated. Using the aspect ratio with the panels unswept results in the following expression for the effective edge-velocity correction factor for swept wings:

\[
E_{e\Lambda} = \sqrt{\frac{A^2}{\cos^4 \Lambda} + \frac{4}{\cos^2 \Lambda}}
\]

Substituting this expression for \( E_{e\Lambda} \) in equation (6) gives the following expression for the lift-curve slope of swept wings:
which reduces to

\[ C_{L\alpha} = \frac{2\pi}{57.3} \left( \frac{A}{\cos \Lambda} \sqrt{\frac{A^2}{\cos^4 \Lambda} + 4 + 2} \right) \]  

The variation of the lift-curve slope with aspect ratio for several sweep angles as calculated from equation (7) is presented in figure 2.

Equation (7), which is based on a section lift-curve slope of \( \frac{2\pi}{57.3} \) per degree, may be modified for any section slope provided that the slope is constant along the span, as follows:

\[ C_{L\alpha} = \frac{a_0 A}{\cos \Lambda \left( \frac{A^2}{\cos^4 \Lambda} + 4 + \frac{57.3 a_0}{\pi} \right)} \]  

where \( a_0 \) is the lift-curve slope of the airfoil section normal to the quarter-chord line.

For wings with airfoil sections placed parallel to the plane of symmetry or perpendicular to some line other than the quarter chord, it is suggested that the section lift-curve slope be determined for a section of the same series but with a thickness ratio equal to that in a plane normal to the quarter-chord line.

One of the main advantages of equation (8) is that the lift-curve slope is expressed entirely in terms of the wing geometry and section lift-curve slope.
Accuracy of method. - The effect of sweep as obtained by the use of equation (7) has been compared with some unpublished results by the electromagnetic-analogy method of reference 3 on a 60° sweptback elliptical wing of aspect ratio 3, and the agreement is excellent. The ratio of swept to unswept lift-curve slope obtained by the electromagnetic-analogy method is 0.697 and that obtained by the use of equation (7) is 0.698. The electromagnetic-analogy results are considered to be reliable, inasmuch as 40 wires were used and the downwash was measured at about 170 points on one wing panel.

In a further attempt to determine the accuracy of this method, a correlation has been made with available wind-tunnel data in the low lift range (fig. 3) and the agreement is very good. The experimental data shown in this figure were obtained from references 7 to 11 and unpublished data of the NACA laboratories. The wings (see table I) used in the correlation cover the following range of plan forms:

<table>
<thead>
<tr>
<th>Sweep</th>
<th>30° to 70°</th>
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</thead>
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<tr>
<td>Aspect ratio</td>
<td>1.07 to 5.20</td>
</tr>
<tr>
<td>Taper ratio</td>
<td>0 to 1.0</td>
</tr>
</tbody>
</table>

Equation (8) was used for the determination of the theoretical lift-curve slopes since most of the section lift-curve slopes were quite different from the thin-airfoil-theory value of $\frac{2\pi}{57.3}$ per degree. The section lift-curve slopes were obtained from references 12 and 13 and are presented in table I.

Although the theory is expected to be more reliable for wings with elliptical loading, the results of an investigation of the effect of taper on three sweptback wings (reference 7) indicates, as does the correlation, that the simplified expressions developed herein can probably be used for any plan form.

Although the present method gives good results for sweptback wings, this method is limited in regard to sweptforward wings in that it predicts the same values regardless of the direction of sweep, whereas experimental investigations have indicated that values obtained for sweptforward wings are different from those of sweptback wings. (See, for example, reference 8.)

DAMPING-IN-ROLL CHARACTERISTICS.

Unswept wings. - Lifting-line theory (reference 14) gives the following equation for the damping in roll of unswept-elliptical wings:

$$C_{p} = -\frac{1}{4} \left( \frac{\pi A}{A + 4} \right)$$  (9)
Swanson and Friddy have obtained, by the electromagnetic-analogy method (reference 15), an effective edge-velocity correction factor for roll $Ee'$ which when applied to equation (9) gives

$$C_{l_p} = - \frac{1}{4} \left( \frac{\pi A}{AEe' + 4} \right)$$

(10)

where $Ee'$ is obtained from the chart of reference 15.

In the section on the lift-curve slope it was shown that, based on the theory of reference 4 (equation (3)), the effective edge-velocity correction factor for lift can be expressed as $\sqrt{\frac{A^2}{4} + 4}$. The results of reference 15 indicate that the effective edge-velocity correction factor for roll for any aspect ratio is very nearly equal to that for lift at one half the aspect ratio. The effective edge-velocity correction factor for roll can therefore be written as $\sqrt{\frac{A^2}{4} + 4}$, which eliminates the necessity for the chart of reference 15 and when applied to equation (10) results in the following equation for the damping in roll of unswept elliptical wings:

$$C_{l_p} = - \frac{1}{4} \left( \frac{\pi A}{2 \sqrt{\frac{A^2}{4} + 4} + 4} \right)$$

(11)

As the aspect ratio approaches zero, equation (11) approaches

$$C_{l_p} = - \frac{\pi A}{32}$$

(12)

the value presented in reference 16 for low aspect-ratio-triangular wings. A comparison of the various methods of estimating the damping in roll of elliptical wings is presented as figure 4. This comparison indicates that equation (11) is in good agreement with the lifting-surface-theory results (equation (10)).

**Swept wings.** If the correction factor for swept wings obtained in reference 1 by means of approximate lifting-line theory is utilized, equation (9) for the damping in roll of elliptical wings may be modified to include the effects of sweep as follows:
\[ C_l_p = -\frac{1}{4} \left( \frac{\pi A \cos \Lambda}{A + 4 \cos \Lambda} \right) \]  

(13)

Assuming that the expression for the effective edge-velocity correction factor for roll (see section entitled "Unswept wings") can be corrected for sweep effects by the same method employed for the lift results in the expression

\[
\sqrt{\frac{A^2}{4 \cos^4 \Lambda} + 4} \quad \frac{A}{2 \cos^2 \Lambda}
\]

which, when applied to equation (13), gives the following expression:

\[
C_l_p = -\frac{1}{4} \left( \frac{\pi A \cos \Lambda}{\sqrt{\frac{A^2}{4 \cos^4 \Lambda} + 4}} \right) + 4 \cos \Lambda
\]

(14)

which reduces to

\[
C_l_p = -\frac{1}{4} \left( \frac{\pi A}{2 \cos \Lambda \sqrt{\frac{A^2}{4 \cos^4 \Lambda} + 4} + 4} \right)
\]

(15)

Results obtained by the use of equation (15) are presented in figure 5. Equation (15) does not show any taper effects and is based on a section lift-curve slope of \(2\pi\) per radian. Inasmuch as taper ratio and section lift-curve slope are important parameters affecting the damping in roll of wings, equation (15) is modified in a manner similar to that presented in reference 1 to give

\[
C_l_p = -\frac{1}{2} \frac{57.3 a_o A \left( \frac{f_{L_p}}{b} \right)^2}{2 \cos \Lambda \sqrt{\frac{A^2}{4 \cos^4 \Lambda} + 4} + \frac{2a_o}{\pi}}
\]

(16)
where \( \frac{\bar{y}_L}{b} \) is an effective lateral center of pressure for rolling moment due to rolling. Values of \( \frac{\bar{y}_L}{b} \) for various taper ratios and aspect ratios must be obtained from charts and are replotted from reference 1 in figure 6 for convenience. These values are presented in a different form from those in reference 1 and are extended to a taper ratio of 0.25 and extrapolated to zero taper. This method of estimating the taper effect of swept wings can be considered only approximate inasmuch as it neglects any sweep effect on the spanwise position of the center of the rolling load.

**Accuracy of method.** - A correlation of the results obtained by equation (16) with some low-speed experimental results obtained in the low lift range in the Langley stability tunnel is presented in figure 7. The geometric characteristics of the wings are presented in table II. The agreement is not so good as that obtained for the lift-curve slope and the theory seems to overestimate the damping in roll. On the basis of this correlation it is suggested that values obtained from equation (16) be multiplied by the factor 0.94. No data are available to test the accuracy of the method as applied to swept-forward wings in roll; however, consideration of the results discussed in the section on lift indicate that less accuracy might be obtained for swept-forward wings than for swept-back wings.

**COMPRESSIBILITY EFFECTS**

The lift and damping in roll of a wing in compressible flow below the critical Mach number may be determined by utilizing the Glauert-Prandtl transformation, that is, by calculating these parameters for an equivalent wing in incompressible flow and increasing these values by \( \frac{1}{\sqrt{1 - M^2}} \). (See reference 17.) The equivalent wing in compressible flow has an aspect ratio that is reduced by the factor \( \sqrt{1 - M^2} \) and a sweep angle the tangent of which is increased by the factor \( \frac{1}{\sqrt{1 - M^2}} \). The geometric characteristics of the equivalent wing become

\[
A_e = A \sqrt{1 - M^2}
\]

\[
\tan \Lambda_e = \frac{\tan \Lambda}{\sqrt{1 - M^2}}
\]
A method of modifying existing correction factors of lifting-surface theory to account approximately for the effects of sweep has been derived, and these factors have been applied to existing lifting-line theories for the lift and damping in roll of swept wings. The resulting expressions are very simple. The lift-curve slope is expressed entirely in terms of the wing geometry and section lift-curve slope; the necessity for any charts is thereby eliminated. The equation for the damping in roll, however, requires a chart for the determination of the effective lateral center of pressure for rolling moment due to rolling. Despite the simplicity of the expressions, the agreement with low-speed experimental data on sweptback wings is very good. If the Glauert-Prandtl transformation is used, the formulas obtained can be applied to swept wings at subsonic speeds below the critical speed.

Although the present method gives good results for sweptback wings, this method is limited in regard to sweptforward wings in that it predicts the same values regardless of the direction of sweep, whereas experimental investigations have indicated that values obtained for sweptforward wings are different from those of sweptback wings.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., February 23, 1949
REFERENCES


**TABLE I**

**WINGS USED IN LIFT-CURVE SLOPE CORRELATION**

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<tr>
<th>Wing</th>
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*Unpublished data from NACA Laboratories.*
### TABLE II

WINGS USED IN DAMPING-IN-ROLL CORRELATION

[Experimental data obtained from unpublished tests made in the Langley stability tunnel]

<table>
<thead>
<tr>
<th>Wing</th>
<th>Λ (deg)</th>
<th>Λ</th>
<th>λ</th>
<th>Airfoil section</th>
<th>$a_0$ from reference 12 or 13</th>
<th>Test-point symbol (fig. 7)</th>
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Figure 1.— Variation of $C_{L_\alpha}$ with aspect ratio for unswept wings of elliptical plan form as predicted by several methods. $a_0 = \frac{2\pi}{57.3}$. 
Figure 2.— Variation of $C_{L\alpha}$ with sweep and aspect ratio (equation 7).

$$a_0 = \frac{2\pi}{57.3}.$$
Figure 3.—Comparison of experimental lift-curve slopes with values obtained by equation (8). (Test-point symbols identified in table I.)
Figure 4.— Variation of $C_{lp}$ with aspect ratio for unswept wings of elliptical plan form as predicted by several methods. $a_0 = \frac{2\pi}{57.3}$.

Figure 5.— Variation of $C_{lp}$ with sweep and aspect ratio (equation 15).

$$ a_0 = \frac{2\pi}{57.3} $$
Figure 6.— Variation of the effective lateral center of pressure for rolling moment due to rolling with aspect ratio and taper ratio. (Data from reference 1.)

Figure 7.— Comparison of experimental values of $C_{lp}$ with values obtained by equation (16). (Test-point symbols identified in table II.)