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METHOD FOR DETERMINING PRESSURE DROP OF AIR FLOWING THROUGH CONSTANT-AREA PASSAGES FOR ARBITRARY HEAT-INPUT DISTRIBUTIONS

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SUMMARY

A method is presented that enables convenient determination of the pressure drop sustained by air flowing at high subsonic speeds in a constant-area passage under the simultaneous influence of friction and heat addition. The method is applicable for arbitrary heat-flux distribution along the flow-passage length. Air-pressure-drop working charts based on the method developed are presented.

INTRODUCTION

Many problems arise in aircraft heat-exchanger practice that involve the transfer of heat at high flux rates to a compressible fluid flowing at high speeds through constant-area flow passages. For example, in some recent heat-exchanger designs the temperature differential between heat-exchanger wall and fluid approached values of the order of 1200° R and the fluid velocities approached the choke condition (Mach number of 1).

Methods have been developed for determining the pressure drop experienced in the passage of air through the radiator tubes of reciprocating engines (references 1 and 2). These methods contain simplifying assumptions that, although adequate over the range of conditions encountered in aircraft radiators, cause appreciable error at the high airspeeds and heat-input rates considered herein.

In reference 3, the basic differential flow equation describing the pressure variations of a compressible fluid under the simultaneous action of friction and heat addition is numerically integrated for the specific case of air heated at constant passage-wall-temperature conditions. In reference 4, a closed-form solution of the basic differential flow equation is obtained for the special case of an exponential variation of fluid temperature with distance along the flow passage. Many cases arise, however, where the heat input to the fluid is a complicated function of distance along the flow passage.

In an investigation made at the NACA Lewis laboratory and 1359
reported herein, the basic differential flow equation is formally integrated by a method based on a simplification that is shown to introduce little error in the calculation of pressure drop over the Mach number and heat-input range encountered in high-flux-rate heat exchangers. The integration results are presented for air (constant ratio of the specific heats equal to 1.40) in convenient chart form. The method is applicable for arbitrary heat-flux distribution along the flow passage and for Mach numbers up to approximately 0.90.

Detailed examples are given that illustrate the methods presented herein.

ANALYSIS

Equations are derived that form the basis for the construction of charts describing the compressible air-flow process and hence the variation in pressure occurring in constant-area passages wherein simultaneous friction and heat-transfer effects are present. The charts are applicable to arbitrary heat-input distribution along the passage length.

When the heat-flux distribution along the flow passage is not explicitly given but instead the passage dimensions, wall temperature, and air-flow parameters are given, a simple method of determining the temperature rise of the air as a function of the distance along the passage is desirable. From this method, the heat-input distribution for application in the method of pressure-drop determination described in the preceding paragraph may be obtained. Equations are derived for the temperature of the air as a function of distance along the passage wall for two cases:

(a) Constant heat input along the passage

(b) Constant wall temperature

These equations, which are based on the recent heat-transfer correlation of references 5 and 6, are plotted to give convenient charts for determining these air-temperature distributions as a function of passage length.

Air-Pressure Drop with Arbitrary Heat-Flux Distribution along Passage Length

The differential momentum equation for the one-dimensional steady-state motion of a compressible fluid in a constant-area passage under the simultaneous influence of friction and heat-addition is

$$d(mV + pA) + dD_F = 0 \tag{1}$$

(All symbols are defined in appendix A.) In reference 7 it is shown, from energy and continuity considerations and the perfect gas law, that the total-momentum parameter $\frac{mV + pA}{m\sqrt{gRT}}$ (hereinafter denoted by the symbol M_p) is uniquely related to each of the flow parameters M, p/P, $pA/m\sqrt{gRT}$, t/T, and $V/\sqrt{gRT}$ for any value of ratio of specific heats γ corresponding to the total temperature T of the fluid. Hence, when the total-momentum parameter M_p and the total temperature T of the fluid are known, all the fluid-flow conditions (P, p, t, and V) are uniquely specified for a given mass flow per unit area. For convenience, the relations among the flow parameters are given in appendix B for the case of γ equal to a constant.

The variation of the total-momentum parameter M_p during the flow process involving friction and heat addition is due to the variations of both the total momentum $mV + pA$ and the total temperature T. Differentiation of M_p with respect to these two variables for constant mass flow gives

$$dM_p = \frac{d(mV + pA)}{m\sqrt{gRT}} - \frac{1}{2} M_p \frac{dT}{T} \tag{2}$$

The differential drag force due to friction is

$$dD_F = F \frac{1}{2} \rho V^2 A \frac{4\ dx}{D_e} \tag{3}$$

or, more conveniently,

$$dD_F = 2FmV \frac{dx}{D_e} \tag{4}$$

Combination of equations (1), (2), and (4) gives

$$\frac{dM_p}{M_p} = -\frac{2\frac{V}{\sqrt{gRT}}}{M_p}\frac{Fdx}{D_e} - \frac{1}{2}\frac{dT}{T} \tag{5}$$

The parameter φ is here introduced and defined by the equation

$$d\varphi = \frac{dM_p}{2\frac{V}{\sqrt{gRT}}} \tag{6}$$

The integration of equation (6) to give φ as a unique function of M_p and γ is made in appendix C (equation (C3)). A plot of φ against M_p for air ($\gamma = 1.40$) is presented and discussed later in the section PRESSURE-DROP CHARTS.

In terms of φ, equation (5) may be written

$$\left(1 + \frac{\frac{Fdx}{D_e}}{d\varphi}\right)\frac{dM_p}{M_p} = -\frac{1}{2}\frac{dT}{T} \tag{7}$$

Integration of equation (7) between any two stations in a flow passage gives

$$\int_{M_{p,1}}^{M_{p,2}}\left(1 + \frac{\frac{Fdx}{D_e}}{d\varphi}\right)\frac{dM_p}{M_p} = -\frac{1}{2}\log_e\frac{T_2}{T_1} \tag{8}$$

where subscript 1 denotes flow conditions at an upstream station in a flow passage and subscript 2, at a downstream station.

The simplification can be made that over short distances of tube length Δx, φ changes linearly with x, that is, $dx/d\varphi$ is closely given by $\frac{\Delta x}{\Delta\varphi} = \frac{\Delta x}{\varphi_2 - \varphi_1}$ where φ_2 corresponds to $M_{p,2}$ and φ_1 corresponds to $M_{p,1}$. With this simplification, the integration of equation (8) may be formally carried out to give

$$2\left(1 - \frac{\frac{F\,\Delta x}{D_e}}{\varphi_1 - \varphi_2}\right)\log_e \frac{M_{p,2}}{M_{p,1}} = -\log_e \frac{T_2}{T_1} \tag{9}$$

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Plots of $M_{p,2}$ from equation (9) have been prepared for air ($\gamma = 1.40$) for increment in friction-distance parameter $F\,\Delta x/D_e = 0.04$ and 0.01 over a wide range of values of $M_{p,1}$ and T_2/T_1. When the heat input per pound of air $\Delta H/mg$ in any increment of passage length Δx is known, the temperature T_2 may be obtained from air-enthalpy tables (reference 8); hence the plots of equation (9) permit calculation of $M_{p,2}$ from the known values of $M_{p,1}$, T_1, and $\Delta H/mg$. The use of these plots for calculating pressure drop in heat-exchanger passages and the validity of the assumption $\frac{dx}{d\varphi} = \frac{\Delta x}{\varphi_2 - \varphi_1}$ are discussed later in the sections PRESSURE-DROP CHARTS and Accuracy of Pressure-Drop Charts.

For the special case of adiabatic flow ($dT = 0$), equations (5) and (6) give $d\varphi = -F dx/D_e$. A brief analysis of the adiabatic process with friction is presented in appendix C.

Air-Temperature Variation along Flow Passage

for Two Special Modes of Heat Input

The air-temperature variation along the flow passage is herein related to the pertinent flow and heat-transfer parameters for (a) constant heat flux along the passage and (b) constant passage-wall temperature.

The differential relation equating the heat transferred from wall to fluid to the heat absorbed by the fluid is

$$mgc_p\,dT = hs(T_w - T)\,dx = dH \tag{10}$$

The heat-transfer coefficient h in equation (10) is given by

$$\frac{hD_e}{k_w} = 0.023\left(\frac{\rho V g D_e}{\mu_w}\right)^{0.8}\left(\frac{T}{T_w}\right)^{0.8}\left(\frac{c_{p,w}\mu_w}{k_w}\right)^{0.4} \tag{11}$$

This heat-transfer relation was obtained in the experimental investigations of references 5 and 6, which covered a wide range of air-flow and heating conditions, including Reynolds numbers from 10,000 to 500,000 and average wall temperatures up to 3000° R.

In the analysis, the variation of the specific heat of air at constant pressure with temperature must be known. The empirical relation used is

$$c_p = KT^{2/15} \tag{12}$$

which was found to closely approximate the actual variation of c_p with T over the range of temperature from 700° to 3000° R.

From equations (11) and (12) and the continuity relation, the following relation, of use in the analysis, is obtained:

$$\frac{Ah}{mgc_p} = 0.023 \left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \left(\frac{T}{T_w}\right)^{2/3} \left(\frac{k_w}{c_{p,w}\mu_w}\right)^{0.6} \tag{13}$$

Constant heat flux. - The case of constant heat flux is characterized by

$$\frac{dH}{dx} = \frac{H_t}{L} \tag{14}$$

From equations (10) and (14) and the definition of D_e,

$$\frac{H_t}{\frac{L}{D_e} mgc_p T} = 4 \frac{Ah}{mgc_p} \left(\frac{T_w}{T} - 1\right) \tag{15}$$

The total amount of heat added to the fluid in the over-all passage length L can be given as

$$H_t = mgc_{p,av}(T_{ex} - T_{en}) \tag{16}$$

or

$$\frac{H_t}{mgc_{p,ex}T_{ex}} = \frac{c_{p,av}}{c_{p,ex}} \left(1 - \frac{T_{en}}{T_{ex}}\right) \tag{17}$$

With the use of equation (12), the ratio $\frac{c_{p,av}}{c_{p,ex}}$ may be evaluated as follows:

$$\frac{c_{p,av}}{c_{p,ex}} = \frac{\int_{T_{en}}^{T_{ex}} c_p \, dT}{c_{p,ex}(T_{ex} - T_{en})} = \frac{15}{17}\left(\frac{1}{T_{ex}^{2/15}}\right)\left(\frac{T_{ex}^{17/15} - T_{en}^{17/15}}{T_{ex} - T_{en}}\right) \tag{18}$$

From equations (17) and (18),

$$\frac{H_t}{mgc_{p,ex}T_{ex}} = \frac{15}{17}\left[1 - \left(\frac{T_{en}}{T_{ex}}\right)^{17/15}\right] \tag{19}$$

Equations (13) and (15) are applicable at any cross section of the flow passage. When equations (13) and (15) are applied to the flow-passage exit, c_p and T are replaced by $c_{p,ex}$ and T_{ex}, respectively, and the subscript w is replaced by the subscript w,ex. Hence, from equations (13) and (15),

$$\frac{H_t}{\frac{L}{D_e} mgc_{p,ex}T_{ex}} = 0.092\left(\frac{A\mu_{w,ex}}{mgD_e}\right)^{0.2}\left(\frac{k_{w,ex}}{c_{p,w,ex}\mu_{w,ex}}\right)^{0.6}\left(\frac{T_{ex}}{T_{w,ex}}\right)^{2/3}\left(\frac{T_{w,ex}}{T_{ex}} - 1\right) \tag{20}$$

From equations (19) and (20),

$$\left(\frac{A\mu_{w,ex}}{mgD_e}\right)^{0.2}\frac{L}{D_e} = \frac{\frac{15}{17}\left[1 - \left(\frac{T_{en}}{T_{ex}}\right)^{17/15}\right]}{0.092\left(\frac{k_{w,ex}}{c_{pw,ex}\mu_{w,ex}}\right)^{0.6}\left(\frac{T_{ex}}{T_{en}}\right)^{2/3}\left(\frac{T_{en}}{T_{w,ex}}\right)^{2/3}\left[\left(\frac{T_{w,ex}}{T_{en}}\right)\left(\frac{T_{en}}{T_{ex}}\right) - 1\right]} \tag{21}$$

Equation (21) relates the over-all passage-distance parameter $\left(\frac{A\mu_{w,ex}}{mgD_e}\right)^{0.2}\frac{L}{D_e}$ to the ratio of exit- to entrance-air temperature T_{ex}/T_{en} and the ratio of passage-exit-wall temperature to entrance-air temperature $T_{w,ex}/T_{en}$. For heat addition at constant heat flux, the passage-exit-wall temperature is the maximum wall temperature. A plot of equation (21) is presented later for a constant value of $c_p\mu/k = 0.66$. The total air-temperature rise in the passage ΔT corresponding to any given value of T_{en} and $T_{W,ex}$ (maximum T_W for this mode of heat input) and the over-all passage-distance parameter can be obtained from this plot. The total heat input H_t corresponding to T_{en} and ΔT can be obtained from air-enthalpy tables (reference 8). Inasmuch as for this case the heat input is uniform along the passage, the heat added in any distance along the passage is $(x/L)H_t$; from this value the air temperature at any cross section of the flow passage (corresponding to distance x) can be obtained from the air-enthalpy tables.

Constant passage-wall temperature. - From equations (10) and (13) and the definition of D_e for $T_W = K$

$$\left(\frac{A\mu_W}{mgD_e}\right)^{0.2} \frac{dx}{D_e} = \frac{d\left(\frac{T}{T_W}\right)}{0.092\left(\frac{k_W}{c_{p,W}\mu_W}\right)^{0.6}\left(\frac{T}{T_W}\right)^{2/3}\left(1 - \frac{T}{T_W}\right)} \tag{22}$$

Integration of equation (22) gives

$$\left(\frac{A\mu_W}{mgD_e}\right)^{0.2} \frac{x}{D_e}\Bigg]_0^{x_0 + rL}$$

$$= \frac{1}{0.092\left(\frac{k_W}{c_{p,W}\mu_W}\right)^{0.6}} \left\{\sqrt{3}\tan^{-1}\left[\frac{2\left(\frac{T}{T_W}\right)^{1/3} + 1}{\sqrt{3}}\right] - \frac{1}{2}\log_e \frac{\left[1 - \left(\frac{T}{T_W}\right)^{1/3}\right]^3}{1 - \frac{T}{T_W}}\right\}_{0.20}^{T/T_W} \tag{23}$$

where, in the substitution of the integration limits, x is arbitrarily taken equal to zero at the passage cross section where T/T_W equals 0.20. This cross section may lie on a hypothetical forward extension of the given flow passage. The symbol x_0 is the distance from this arbitrary zero point (where $T/T_W = 0.20$) to the passage entrance, r

is the fraction of the actual passage length, and L is the actual passage length. When $r = 1$, equation (23) gives the value of T/T_w at the passage exit. A plot of equation (23) is presented later for $c_p\mu/k = 0.66$. From this plot of equation (23), the air-temperature variation, and hence the heat-input variation, along the passage length can be determined.

PRESSURE-DROP CHARTS

Plots of equation (9) that apply to flow-passage segments for which the friction-distance parameter $F\,\Delta x/D_e = 0.04$ and 0.01 are shown in figure 1. The total-momentum parameter at the exit of the flow-passage segment $M_{p,2}$ is plotted against the total-momentum parameter at the entrance to the flow-passage segment $M_{p,1}$ for a range of values of total-temperature ratio T_2/T_1 across the segment. In order to determine the variation in total-momentum parameter (and hence all other flow parameters) with distance along a flow passage, the flow passage is divided suitably into segments of such length that the friction-distance parameter $F\,\Delta x/D_e$ is 0.01 or 0.04 (or combinations thereof), and the total-momentum-parameter variation across each segment is obtained from figure 1. The dashed lines in figure 1 represent the condition where $M_{p,1} = M_{p,2}$ and are drawn to facilitate use of the figure. The details of using figure 1 are illustrated later by means of examples I and III.

When the FL/D_e of the entire flow passage is not a multiple of 0.01 (and hence not a multiple of 0.04), at least one segment of the flow passage cannot be characterized by either $F\,\Delta x/D_e = 0.04$ or 0.01; hence for this one segment figure 1 is not applicable. In this case, the division of the entire flow passage into segments is so made that all of the segments except the first upstream one (the entrance station of which corresponds to the flow-passage entrance) are characterized by $F\,\Delta x/D_e = 0.01$ or 0.04. The $F\,\Delta x/D_e$ of the first upstream segment will hence be less than 0.01. In this case, the approximation is made that the heat input in the first two upstream segments is added only in the second upstream segment and therefore the flow in the first upstream segment (of $F\,\Delta x/D_e$ less than 0.01) is adiabatic. The second upstream segment and all remaining segments are characterized by $F\,\Delta x/D_e = 0.04$ or 0.01 and hence figure 1 is applicable for these segments. The foregoing approximation as to location of heat addition near the entrance of a flow passage is justified on the basis that: (1) At the entrance of heat-exchanger passages the compressibility effects are relatively small, in which case the exact location along the passage of heat-addition is not critical; (2) the tube length corresponding to $F\,\Delta x/D_e$ less than 0.01 will be a small percentage of the over-all tube length in practical heat exchangers. The procedure just described is illustrated in detail by means of Example III.

In figure 2, the parameter φ is plotted against the total-momentum parameter M_p for air. It is shown in appendix C (equation (C2)) that for the case of adiabatic flow with friction $\varphi_1 - \varphi_2 = \frac{F\,\Delta x}{D_e}$. The use of figure 2 in conjunction with equation (C2) for determining the variation in total-momentum parameter along a flow passage for adiabatic flow with friction (no heat transfer) is illustrated by means of Example II.

The relations (for air) of the total-pressure parameter, the static-pressure parameter, and the Mach number to the total-momentum parameter are shown in figures 3, 4, and 5, respectively. The variation of M_p and T in a flow passage can therefore be translated into the variation of total and static pressure and Mach number in the flow passage.

AIR-TEMPERATURE-RISE CHARTS

The relation of the air-temperature rise to the maximum passage-wall temperature ($T_{w,ex}$ in this case) and the over-all passage-distance parameter for constant heat flux along the passage length is given in figure 6. The ratio of the fluid-exit to fluid-entrance temperature T_{ex}/T_{en} is the ordinate, the over-all passage-distance parameter $\left(\frac{A\mu_{w,ex}}{mgD_e}\right)^{0.2} \frac{L}{D_e}$ is the abscissa, and the ratio of the maximum passage-wall temperature to the entrance-air temperature $T_{w,ex}/T_{en}$ is the curve parameter.

The ratio of the fluid to wall temperature is related to the distance parameter in figure 7 for the case of constant passage-wall temperature. In the construction of figure 7, the distance x was arbitrarily taken equal to zero when the ratio of air to wall temperature equals 0.20; hence, the distance parameter in figure 7 must be taken as $\left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \left(\frac{x_0 + rL}{D_e}\right)$ where x_0 is the distance of the passage entrance from a hypothetical station in the forward extension of the passage characterized by $T/T_w = 0.20$, L is the over-all passage length, and r is the fraction of the passage length at which the value of T/T_w is to be determined. An illustration of the use of figure 7 in the solution of a pressure-drop problem is given in Example IV.

EXAMPLES ILLUSTRATING USE OF PRESSURE-DROP CHARTS

Example I

The use of figure 1 is illustrated for flow with friction and heat addition when FL/D_e is an exact multiple of 0.01.

Given:

(1) $FL/D_e = 0.270$

(2) $mg/A = 32.2$ (lb/(sec)(sq ft))

(3) $T_{en} = 600$ (°R)

(4) $P_{en} = 3000$ (lb/sq ft absolute)

(5) The heat-input $\Delta H/A$ variation along the flow passage in terms of the friction-distance parameter Fx/D_e is

Fx/D_e	$\Delta H/A$	[1]$\Delta H/mg$	[2]T/T_{en}
0	----	-----	1.000
.04	1018	31.62	1.218
.08	925	28.72	1.415
.12	848	26.34	1.594
.16	786	24.40	1.757
.20	713	22.15	1.904
.24	655	20.35	2.037
.25	157	4.87	2.069
.26	165	5.13	2.101
.27	157	4.89	2.133

[1]The value of $\Delta H/mg$ is obtained by dividing $\Delta H/A$ by mg/A (item 2).

[2]The value of T/T_{en} is obtained from the air-enthalpy tables (reference 8) for the given T_{en} (item 3) and enthalpy rise $\Delta H/mg$.

Determine: The total and static pressures at the passage exit.

The procedure is as follows:

(6) From item (5) the values of $F\ \Delta x/D_e$ and T_2/T_1 for the segments indicated in figure 8(a) are

Segment	$F\ \Delta x/D_e$	T_2/T_1
a	0.04	1.2180
b	.04	1.1617
c	.04	1.1265
d	.04	1.1023
e	.04	1.0837
f	.04	1.0699
g	.01	1.0157
h	.01	1.0155
i	.01	1.0152

(7) From items (2), (3), and (4),

$$\left(\frac{pA}{m\sqrt{gRT}}\right)_{en} = 2.955$$

(8) From item (7) and figure 3,

$$\left(\frac{mV+pA}{m\sqrt{gRT}}\right)_{en} = 3.126 = M_{p,1} \quad \text{(for segment a)}$$

(9) From items (6) and (8) and figure 1(b),

$$M_{p,2} \quad \text{(for segment a)} = 2.804$$

This step is traced out in figure 9 by the line 1-a.

Figure 9 is a reproduction of figure 1 and is inserted merely to aid in the explanation of the method of solution.

(10) $M_{p,1}$ (for segment b) = $M_{p,2}$ (for segment a).

Again from figure 1(a) and items (6) and (9)

$$M_{p,2} \quad \text{(for segment b)} = 2.568$$

The operation of obtaining item (10) can be performed as illustrated in figure 9. Proceed horizontally from point a to the 45° dashed line (point 2). The abscissa at point 2 is then $M_{p,1}$ for segment b. Proceed vertically downward to the appropriate T_2/T_1 value for segment b (point b). The ordinate for point b is then $M_{p,2}$ for segment b and also $M_{p,1}$ for segment c.

(11) The operations indicated are repeated in step-by-step fashion for each of the segments a through i. For segments g, h, and i, use is made of figure 1(d) applicable for $F\,\Delta x/D_e = 0.01$. These operations are traced in figure 9 where for each segment the point with ordinate and abscissa values that give $M_{p,2}$ and $M_{p,1}$ for the segment is marked by the letter designation for the segment. (For example, point d in fig. 9(a) gives $M_{p,2}$ and $M_{p,1}$ for segment d.) Although only the value for segment i is needed in this problem, a tabulation of the values of $M_{p,2}$ is presented:

Segment	$M_{p,2}$
a	2.804
b	2.568
c	2.381
d	2.224
e	2.087
f	1.962
g	1.9313
h	1.8992
i	1.8670

(12) For segment i, $M_{p,2}$ is M_p for the passage exit, as illustrated in figure 8(a). From figures (3) and (4) for $M_p = 1.8670$ given in item (11),

$$\left(\frac{PA}{m\sqrt{gRT}}\right)_{ex} = 1.4865$$

$$\left(\frac{pA}{m\sqrt{gRT}}\right)_{ex} = 0.917$$

(13) From item (5), $T_{ex}/T_{en} = 2.133$. From items (4), (7), and (12) therefore,

$$\frac{P_{ex}}{P_{en}} = \frac{1.4865}{2.955} \sqrt{2.133} = 0.7349$$

so that 1359

$$P_{ex} = (0.7349)(3000) = 2205 \text{ (lb/sq ft absolute)}$$

(14) From item (12),

$$\frac{p_{ex}}{P_{ex}} = \frac{0.917}{1.4865} = 0.6167$$

so that

$$p_{ex} = (0.6167)(2205) = 1360 \text{ (lb/sq ft absolute)}$$

(15) From item (12) and figure 5, the exit Mach number is $M = 0.86$.

Example II

The use of figure 2 is illustrated for adiabatic flow with friction.

Given:

(1) $M_p = 2.500$ at passage entrance $(M_{p,en})$

(2) $FL/D_e = 0.300$

Determine: M_p at passage exit $M_{p,ex}$

(3) From item (1) and figure (2), φ at the passage entrance is

$$\varphi_{en} = 0.528$$

(4) Equation (C2) of appendix C relates the change in φ between two stations in a constant-area flow passage to the friction-distance parameter between the two stations. Equation (C2) applied to the given problem can be written

$$\varphi_{en} - \varphi_{ex} = \frac{FL}{D_e}$$

(5) From items (2) and (3),

$$\varphi_{ex} = 0.528 - 0.300 = 0.228$$

(6) From item (5) and figure (2),

$$M_{p,ex} = 2.182$$

Example III

The use of figures 1 and 2 is illustrated for flow with friction and heat addition where FL/D_e is not an exact multiple of 0.01.

Given:

(1) $FL/D_e = 0.137$

(2) $M_{p,en} = 3.25$

(3) $T_{en} = 800$ (^{o}R)

The heat-input variation along the flow passage is specified and will be presented later in this example.

Determine: $M_{p,ex}$

The quantity FL/D_e is not an integral multiple of 0.01. Hence, as illustrated in the sketch of figure 8(b), the division of the passage into segments is so made that the one segment having a value of $F\,\Delta x/D_e$ other than 0.01 or 0.04 is the first upstream segment (segment a in fig. 8(b)). Segment a has $F\,\Delta x/D_e = 0.007$; segments b, c, and d have $F\,\Delta x/D_e = 0.04$; and segment e has $F\,\Delta x/D_e = 0.01$.

The assumption is then made that the heat added in segments a and b is added only in segment b; hence the flow in segment a is assumed to be adiabatic, in which case figure 2 and equation (C2) are used to obtain $M_{p,2}$ for segment a, as illustrated in Example II. For the remaining segments, figure 1 is used, as illustrated in Example I.

(4) The actual heat-input variation along the passage and the heat input modified as described in the preceding paragraph are given in the following table:

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Segment	$F\ \Delta x/D_e$	Actual $\Delta H/mg$	Modified $\Delta H/mg$	[1]Modified T_2/T_1
a	0.007	9.76	0	1.000
b	.04	51.93	61.69	1.313
c	.04	80.41	80.41	1.300
d	.04	101.60	101.60	1.280
e	.01	33.30	33.30	1.070

[1]The value of T_2/T_1 across each section corresponding to the modified heat input is obtained from the value of T_{en} (item (3)) and the modified enthalpy rise per pound of air $\Delta H/mg$ by means of the air-enthalpy tables of reference 8.

(5) From item (2) ($M_{p,en} = M_{p,1}$ for segment a), equation (C2), and figure 2 for $F\ \Delta x/D_e = 0.007$ (this step was illustrated in Example II),

$$M_{p,2}\ \text{(for segment a)} = 3.245$$

(6) From item (5) $\left[M_{p,2}\ \text{(for segment a)} = M_{p,1}\ \text{(for segment b)}\right]$ and figure 1(b) for $T_2/T_1 = 1.313$ (item (4)),

$$M_{p,2}\ \text{(for segment b)} = 2.803$$

(7) For the remaining segments, the procedure is the same as given in item (6) for segment b (and as illustrated in Example I); hence only a tabulation of the results is presented as follows:

Segment	$M_{p,2}$
a	3.245
b	2.803
c	2.425
d	2.100
e	2.017

(8) The value of $M_{p,ex}$ for the flow passage is then given by $M_{p,2}$ for segment e:

$$M_{p,ex} = 2.017$$

The procedure for obtaining the remaining flow parameters at the flow-passage exit from the calculated value of $M_{p,ex}$ was illustrated in Example I.

Example IV

The use of recent friction-data correlation to relate Fx/D_e and T/T_{en} for constant passage-wall temperature is illustrated.

Given:

(1) $T_{en}/T_w = 0.320$

(2) $\left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \frac{L}{D_e} = 15.0$

Determine: T/T_{en} as a function of Fx/D_e

Pressure-drop measurements were taken in the experimental heat-transfer investigations of reference 6 and the data reduced to determine the flow-friction factor F. The conventional correlation of F against bulk Reynolds number was unsatisfactory. Improved correlation was obtained, however, from a plot of $\frac{F\rho}{\rho_w}$ against $\frac{\rho_w VgD_e}{\mu_w}$. The best correlation line representing the data plotted in this manner is given by the equation

$$F = 0.046 \left(\frac{\mu_w}{\rho VgD_e}\right)^{0.2} \left(\frac{\rho_w}{\rho_{av}}\right)^{0.8}$$

This friction-factor equation is used in working out this example.

(3) By taking $\frac{\rho_w}{\rho_{av}} = \frac{T_{av}}{T_w}$, then from the friction-factor equation,

$$\frac{Fx}{D_e} = 0.046 \left(\frac{T_{av}}{T_w}\right)^{0.8} \left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \frac{x}{D_e}$$

(4) With reference to figure 7, at the flow-passage entrance $r = 0$ and $T/T_w = 0.32$ (item (1)). From figure 7,

$$\left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \frac{x_0}{D_e} = 3.67$$

hence 3.67 represents the value of the distance parameter from a hypothetical station (in the forward extension of the flow passage) at which T/T_w = the reference value of 0.20 to the entrance of the flow passage.

(5) For the flow-passage exit, $r = 1$ so that from items (2) and (4)

$$\left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \left(\frac{x_0 + rL}{D_e}\right) = 3.67 + 15.0 = 18.67$$

(6) From figure 7 and item (5),

$$\frac{T_{ex}}{T_w} = 0.777$$

(7) From items (1) and (6) with $T_{av} = \frac{T_{en} + T_{ex}}{2}$,

$$\frac{T_{av}}{T_w} = 0.549$$

(8) From items (3) and (7),

$$Fx/D_e = 0.0285 \left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \frac{x}{D_e}$$

(9) The value of T/T_w and Fx/D_e at a cross section, say, nine-fifteenths of the distance from entrance to exit of the flow passage, is obtained as follows:

$$r = 9/15$$

$$\left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \frac{rL}{D_e} = 9.0$$

$$\left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \frac{x_0 + rL}{D_e} = 3.67 + 9.0 = 12.67$$

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Hence, from figure 7,

$$\frac{T}{T_w} = 0.626$$

Inasmuch as $x = rL$, from item (8) the corresponding $Fx/D_e = 0.0285 \times 9.0 = 0.257$.

(10) In a manner similar to that outlined in item (9), T/T_w and Fx/D_e can be determined at any passage cross section. The value of T/T_{en} at the cross section is then given by $\dfrac{T/T_w}{T_{en}/T_w} = \dfrac{T/T_w}{0.320}$. A tabulation is made for various values of r as follows:

r	$\left(\frac{A\mu_w}{mgD_e}\right)^{0.2} \frac{x}{D_e}$	$\frac{T}{T_w}$	$\frac{Fx}{D_e}$	$\frac{T}{T_{en}}$
0	0	0.320	0	1.000
3/15	3	.426	.086	1.331
6/15	6	.530	.171	1.656
9/15	9	.626	.257	1.956
12/15	12	.708	.342	2.212
1	15	.777	.427	2.429

A plot of Fx/D_e against T/T_{en} can be made to obtain T/T_{en} for increments $F\,\Delta x/D_e$ of the passage equal to 0.04 or 0.01. The pressure drop in the passage is then obtained in the manner outlined in Example I or III.

Accuracy of Pressure-Drop Charts

In order to obtain an indication of the validity of the assumption used in obtaining equation (9) from equation (8) and hence of the accuracy of the pressure-drop charts, calculations were made for extreme conditions of heating and compressibility effects (1) by use of the charts and, (2) by numerical integration of the actual differential equation (equation (8)). The conditions investigated and the results obtained are tabulated as follows:

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M_{en}	M_{ex}	T_{ex}/T_{en}	Type of heat distribution	Δp(exact) (lb/sq ft)	Δp(chart) (lb/sq ft)
0.30	0.66	2.04	Constant wall temperature	1111	1107
0.25	0.46	2.00	Sine variation with distance	241	240
0.30	0.71	2.00	Sine variation with distance	1273	1274
0.40	0.85	1.557	Constant wall temperature	1323	1311

For the first two cases in the preceding table, the charts for $F\ \Delta x/D_e = 0.04$ were used to compute the chart pressure drop throughout the entire Mach number range from M_{en} to M_{ex}. In the third and fourth cases, the chart for $F\ \Delta x/D_e = 0.04$ was used up to Mach numbers of 0.67 and 0.65, respectively, and the chart for $F\ \Delta x/D_e = 0.01$ was used for the remaining flow process. The agreement between the exact and chart results for any of these cases is within 1 percent.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, March 30, 1950.

APPENDIX A

SYMBOLS

The following symbols are used in the calculations and the figures:

A cross-sectional area of flow passage, sq ft

c_p specific heat of fluid at constant pressure, Btu/(lb)($^{\circ}$R)

D_e equivalent diameter of flow passage, $4A/s$, ft

D_F drag force due to friction, lb

F friction factor, $\dfrac{-dp}{\frac{1}{2}\rho V^2 \dfrac{4dx}{D_e}}$

g mass conversion factor, 32.2 lb/slug

H heat added to fluid in distance x per unit time, Btu/sec

H_t heat added to fluid in total passage length L per unit time, Btu/sec

h heat-transfer coefficient between wall and fluid, Btu/(sec)(sq ft)($^{\circ}$R)

K constant

k thermal conductivity of fluid, Btu/(sec)(sq ft)($^{\circ}$R)(ft)

L total tube length, ft

M Mach number

m mass flow of fluid, slugs/sec

P total pressure, lb/sq ft absolute

p static pressure, lb/sq ft absolute

R gas constant for fluid (for air, 53.35), ft-lb/(lb)($^{\circ}$R)

r fraction of length of given flow passage, $0 \leq r \leq 1$

s wetted perimeter, ft 1359

T total temperature of fluid, $^{\circ}$R

T_w temperature of passage walls, $^{\circ}$R

t static temperature of fluid, $^{\circ}$R

V fluid velocity in flow passage, ft/sec

x distance along flow passage as measured from passage entrance, ft

x_0 distance from hypothetical station characterized by $T/T_w = 0.20$ to entrance of given flow passage, ft

γ ratio of specific heats

Δx length of flow-passage segment, ft

μ viscosity of fluid, lb/(ft)(sec)

ρ mass density of fluid, slugs/cu ft

Subscripts:

av fluid conditions evaluated at average fluid temperature in flow passage

en entrance of flow passage

ex exit of flow passage

w fluid conditions evaluated at average passage-wall temperature T_w

1 entrance station of flow-passage segment

2 exit station of flow-passage segment

Parameters:

M_p — total-momentum parameter $\dfrac{mV + pA}{m\sqrt{gRT}}$

$\dfrac{pA}{m\sqrt{gRT}}$ — static-pressure parameter

$\dfrac{PA}{m\sqrt{gRT}}$ — total-pressure parameter

$\dfrac{V}{\sqrt{gRT}}$ — velocity parameter

φ — parameter, defined by $d\varphi = \dfrac{dM_p}{2\dfrac{V}{\sqrt{gRT}}}$

$\dfrac{F\,\Delta x}{D_e}$ — increment in friction-distance parameter

$\dfrac{Fx}{D_e}$ — friction-distance parameter

$\dfrac{FL}{D_e}$ — over-all friction-distance parameter of given flow passage

$\left(\dfrac{A\mu_w}{mgD_e}\right)^{0.2} \dfrac{x}{D_e}$ — passage-distance parameter

$\left(\dfrac{A\mu_w}{mgD_e}\right)^{0.2} \dfrac{L}{D_e}$ — over-all passage-distance parameter of flow passage

APPENDIX B

RELATIONS AMONG FLOW PARAMETERS

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For convenience, the equations relating the parameters $V/\sqrt{gRT}$, t/T, M, $pA/m\sqrt{gRT}$, and p/P to the total-momentum parameter M_p are herein derived. The assumption is made that γ remains a constant.

From the continuity equation and the perfect gas law,

$$m = \rho AV = \frac{pAV}{gRt} \tag{B1}$$

which can be written as

$$\frac{t}{T} = \left(\frac{pA}{m\sqrt{gRT}}\right)\left(\frac{V}{\sqrt{gRT}}\right) = \left(M_p - \frac{V}{\sqrt{gRT}}\right)\left(\frac{V}{\sqrt{gRT}}\right) \tag{B2}$$

One form of the conservation of energy equation is

$$\frac{t}{T} = 1 - \frac{\gamma - 1}{2\gamma}\left(\frac{V}{\sqrt{gRT}}\right)^2 \tag{B3}$$

From equations (B2) and (B3),

$$\frac{1}{2}\left(\frac{V}{\sqrt{gRT}}\right)^2 - \frac{\gamma}{\gamma + 1} M_p \left(\frac{V}{\sqrt{gRT}}\right) + \frac{\gamma}{\gamma + 1} = 0 \tag{B4}$$

Thus $\frac{V}{\sqrt{gRT}}$ is obtained as an explicit function of M_p

$$\frac{V}{\sqrt{gRT}} = \frac{\gamma}{\gamma + 1} M_p \pm \sqrt{\left(\frac{\gamma}{\gamma + 1} M_p\right)^2 - \frac{2\gamma}{\gamma + 1}} \tag{B5}$$

From equations (B3), (B5), and the isentropic relation,

$$\left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}} = \frac{t}{T} = \frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{\gamma}\left(\frac{\gamma}{\gamma+1}M_p\right)\left[\left(\frac{\gamma}{\gamma+1}M_p\right) \pm \sqrt{\left(\frac{\gamma}{\gamma+1}M_p\right)^2 - \frac{2\gamma}{\gamma+1}}\right] \tag{B6}$$

Because

$$M = \frac{V}{\sqrt{\gamma gRt}} = \frac{1}{\sqrt{\gamma}}\left(\frac{V}{\sqrt{gRT}}\right)\sqrt{\frac{T}{t}} \tag{B7}$$

Then from equations (B5) to (B7),

$$M = \frac{\frac{\gamma}{\gamma+1}M_p \pm \sqrt{\left(\frac{\gamma}{\gamma+1}M_p\right)^2 - \frac{2\gamma}{\gamma+1}}}{\sqrt{\frac{2\gamma^2}{\gamma+1} - (\gamma-1)\left(\frac{\gamma}{\gamma+1}M_p\right)\left[\left(\frac{\gamma}{\gamma+1}M_p\right) \pm \sqrt{\left(\frac{\gamma}{\gamma+1}M_p\right)^2 - \frac{2\gamma}{\gamma+1}}\right]}} \tag{B8}$$

The parameter $pA/m\sqrt{gRT}$ may be obtained by direct substitution of equation (B5) into equation (B2). In the application of these equations to a particular flow, the sign used in front of the radical should be plus if the flow is supersonic and minus if the flow is subsonic.

APPENDIX C

METHOD OF INTEGRATION OF EQUATION (6)

A brief analysis of the adiabatic compressible-flow process involving friction only is herein presented together with the method of integration of equation (6).

For the adiabatic flow of a compressible fluid with friction present, $dT = 0$ and hence from equations (5) and (6) 1359

$$d\varphi = -d\left(\frac{Fx}{D_e}\right) \tag{C1}$$

so that

$$\varphi_1 - \varphi_2 = \left(\frac{Fx}{D_e}\right)_2 - \left(\frac{Fx}{D_e}\right)_1 \tag{C2}$$

The variation of φ with M_p for either adiabatic flow or non-adiabatic flow may be obtained by substituting equation (B5), with the subsonic sign prefixing the radical, into equation (6) and integrating. The results following this procedure are

$$\varphi = \frac{1}{8}\left\{M_p\left[M_p + \sqrt{M_p^2 - 2\left(\frac{\gamma+1}{\gamma}\right)}\right] - M_{p,0}\left[M_{p,0} + \sqrt{M_{p,0}^2 - 2\left(\frac{\gamma+1}{\gamma}\right)}\right] - 2\left(\frac{\gamma+1}{\gamma}\right)\log_e\left[\frac{M_p + \sqrt{M_p^2 - 2\left(\frac{\gamma+1}{\gamma}\right)}}{M_{p,0} + \sqrt{M_{p,0}^2 - 2\left(\frac{\gamma+1}{\gamma}\right)}}\right]\right\} \tag{C3}$$

where M_p and $M_{p,0}$ are the upper and lower limits of integration on M_p; φ and φ_0 are the upper and lower limits of integration on φ, and φ_0 has been taken equal to zero when $M_p = M_{p,0}$.

A large-scale plot of equation (C3) for $\gamma = 1.40$ is presented in figure 2 for $M_{p,0} = 1.8516$ (the value of the momentum parameter at a Mach number of 1 with $\gamma = 1.40$).

REFERENCES

1. Nielsen, Jack N.: High-Altitude Cooling. III - Radiators. NACA ARR L4I11b, 1944.

2. Becker, John V., and Baals, Donald D.: Simple Curves for Determining the Effects of Compressibility on Pressure Drop through Radiators. NACA ACR L4I23, 1944.

3. Valerino, Michael F.: Generalized Charts for Determination of Pressure Drop of a High-Speed Compressible Fluid in Heat-Exchanger Passages. I - Air Heated in Smooth Passages of Constant Area with Constant Wall Temperature. NACA RM E8G23, 1948.

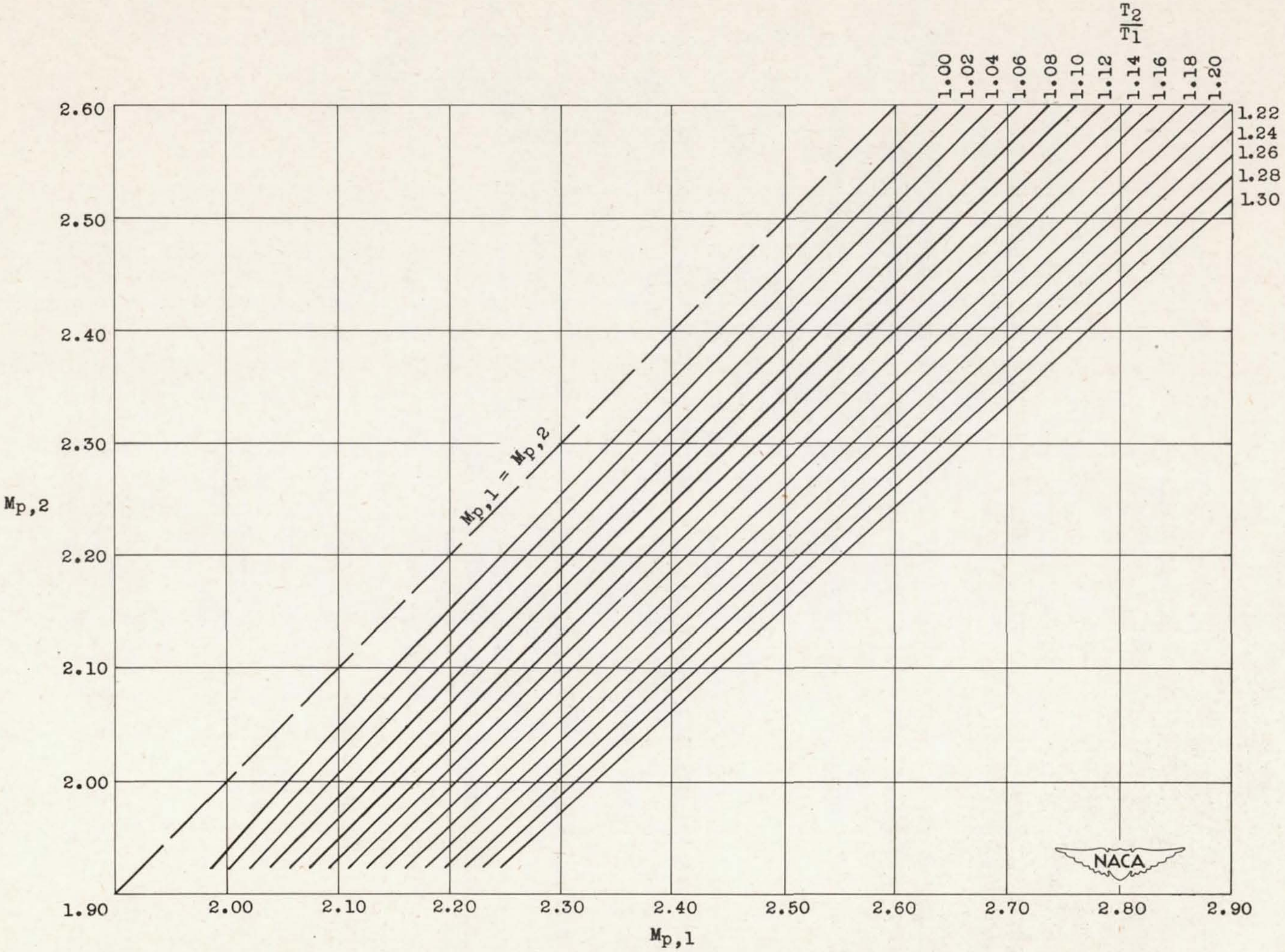
4. Sibulkin, Merwin, and Koffel, William K.: Chart for Simplifying Calculations of Pressure Drop of a High-Speed Compressible Fluid under Simultaneous Action of Friction and Heat Transfer - Application to Combustion-Chamber Cooling Passages. NACA TN 2067, 1950.

5. Humble, Leroy V., Lowdermilk, Warren H., and Grele, Milton: Heat Transfer from High-Temperature Surfaces to Fluids. I - Preliminary Investigation with Air in Inconel Tube with Rounded Entrance, Inside Diameter of 0.4 Inch, and Length of 24 Inches. NACA RM E7L31, 1948.

6. Lowdermilk, Warren H., and Grele, Milton D.: Heat Transfer from High-Temperature Surfaces to Fluids. II - Correlation of Heat-Transfer and Friction Data for Air Flowing in Inconel Tube with Rounded Entrance. NACA RM E8L03, 1949.

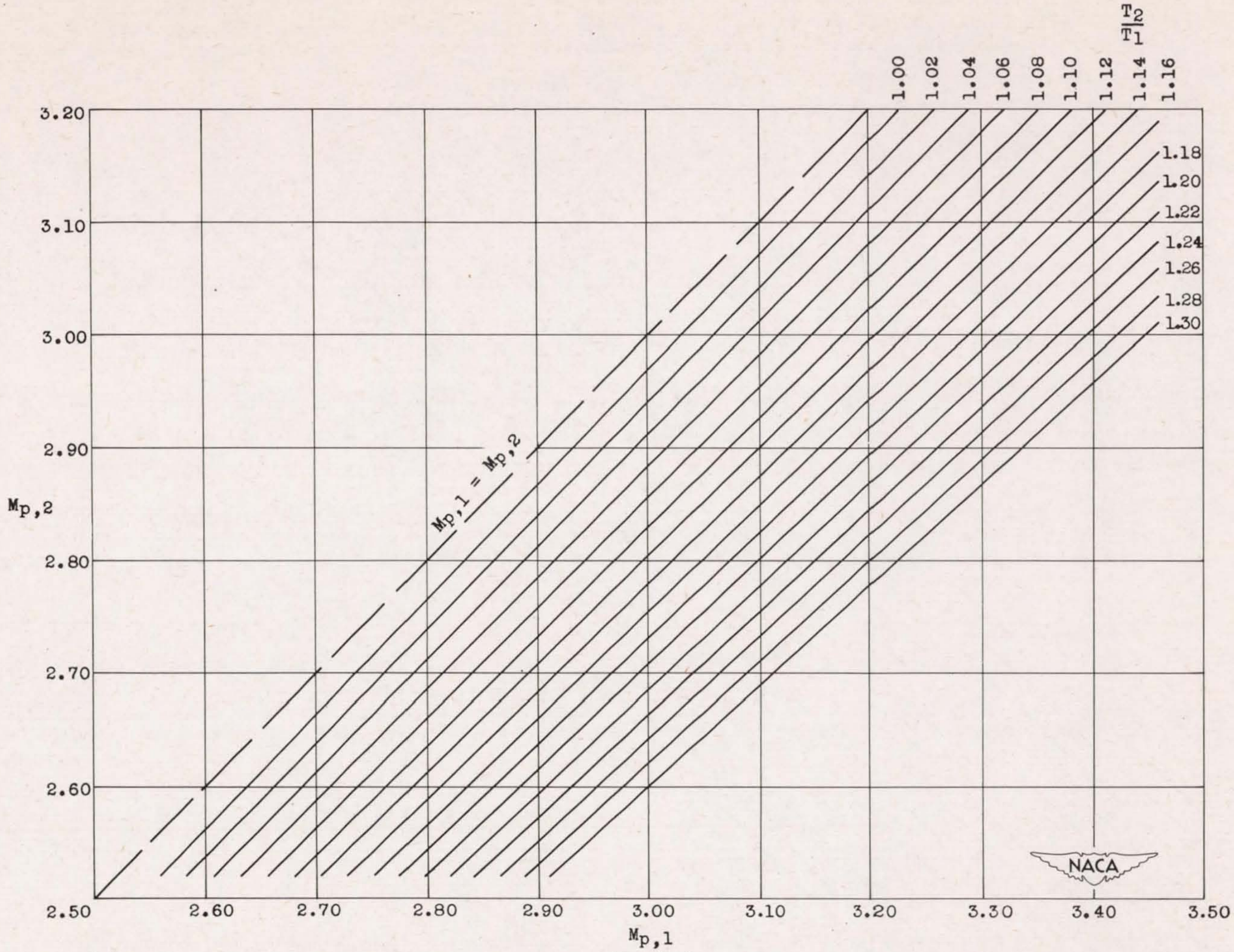
7. Turner, L. Richard, Addie, Albert N., and Zimmerman, Richard H.: Charts for the Analysis of One-Dimensional Steady Compressible Flow. NACA TN 1419, 1948.

8. Keenan, Joseph H., and Kaye, Joseph: Gas Tables. John Wiley & Sons, Inc., 1948.



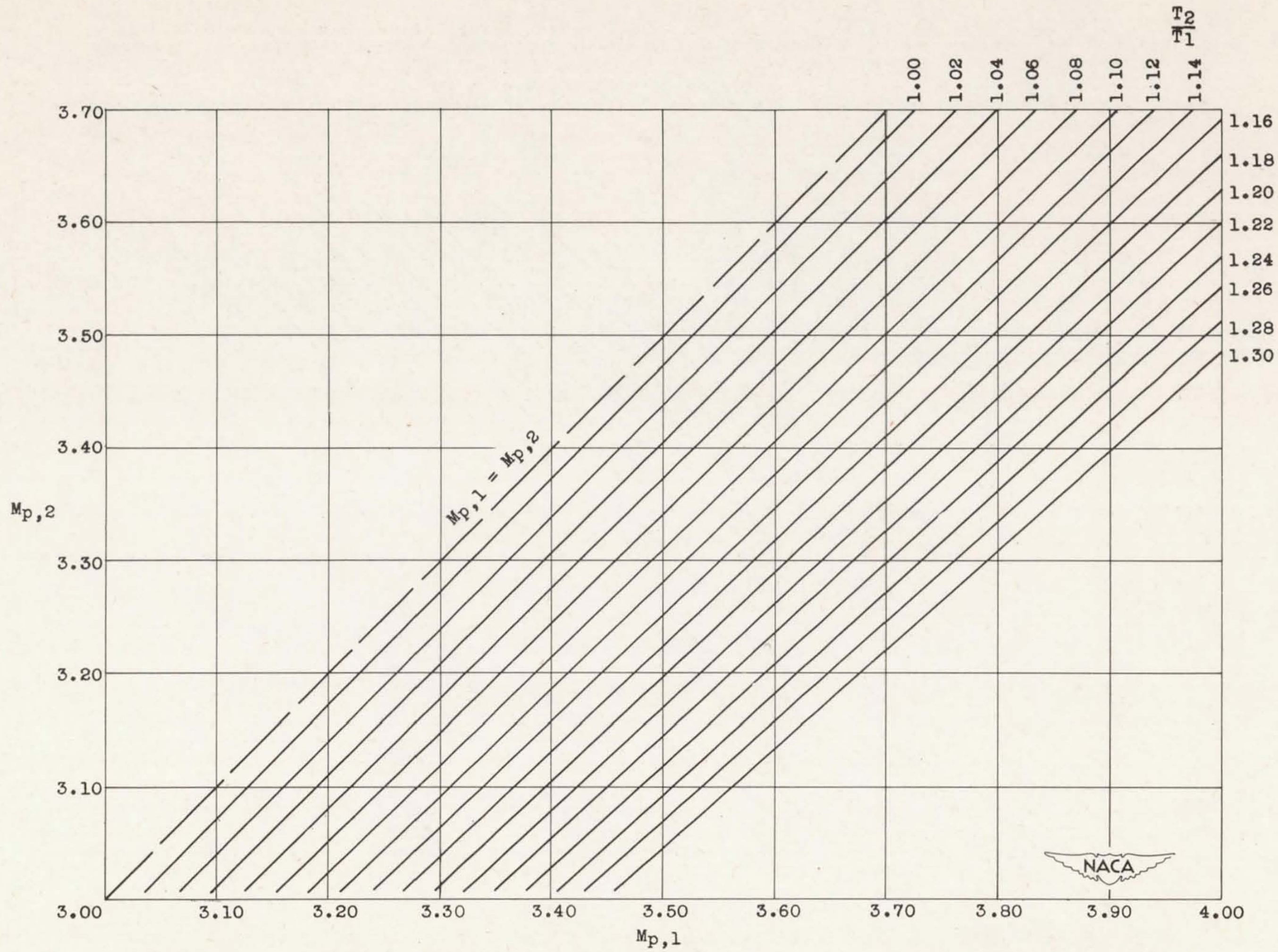
(a) Increment in friction-distance parameter $(F\Delta x/D_e)$, 0.04; range of total-momentum parameter at station 2, 1.92 to 2.52.

Figure 1. - Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition (γ = 1.40). (A 17- by 22-in. print of this fig. is attached.)



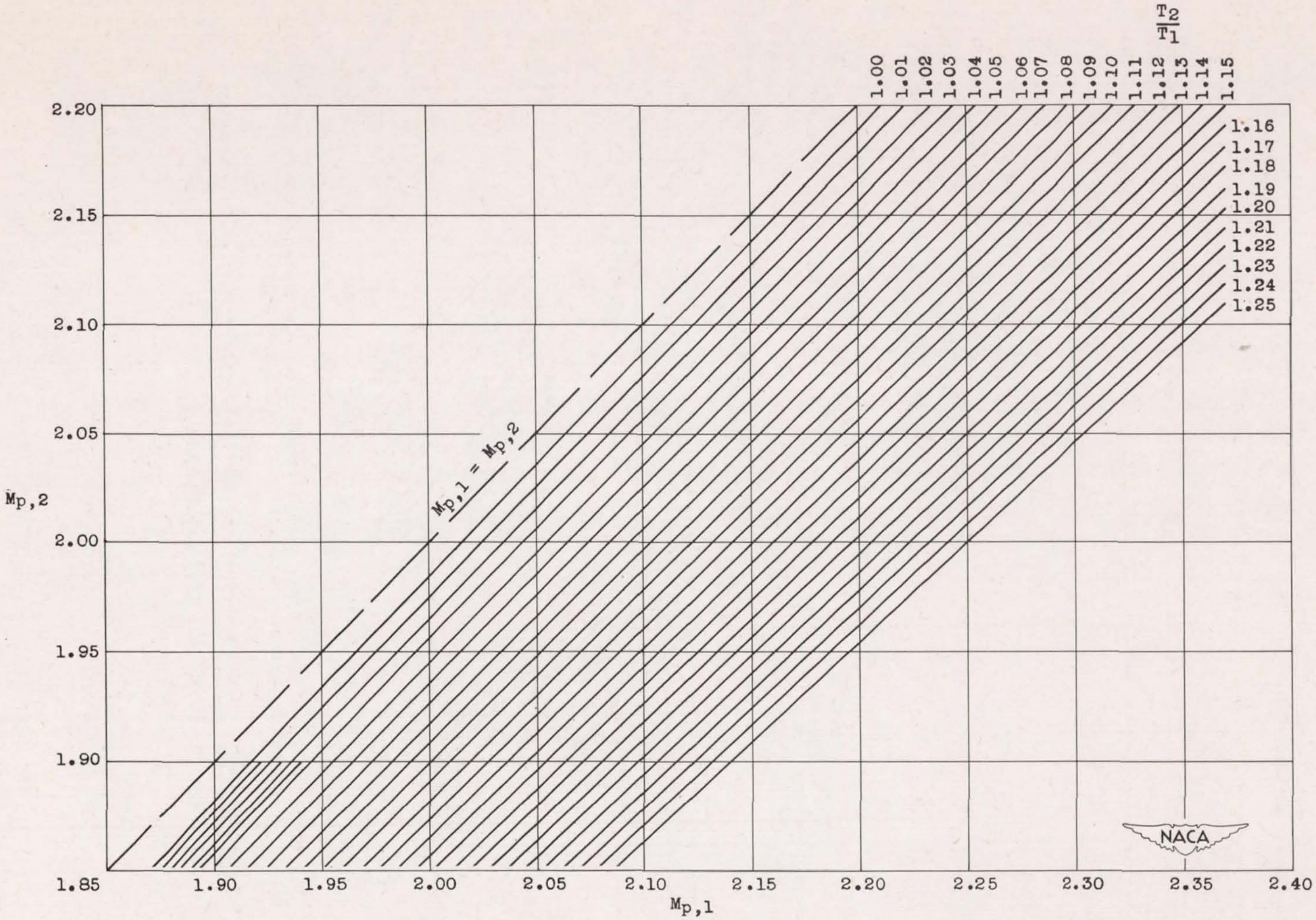
(b) Increment in friction-distance parameter ($F\Delta x/D_e$), 0.04; range of total-momentum parameter at station 2, 2.52 to 3.01.

Figure 1. - Continued. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition (γ = 1.40). (A 17- by 22-in. print of this fig. is attached.)



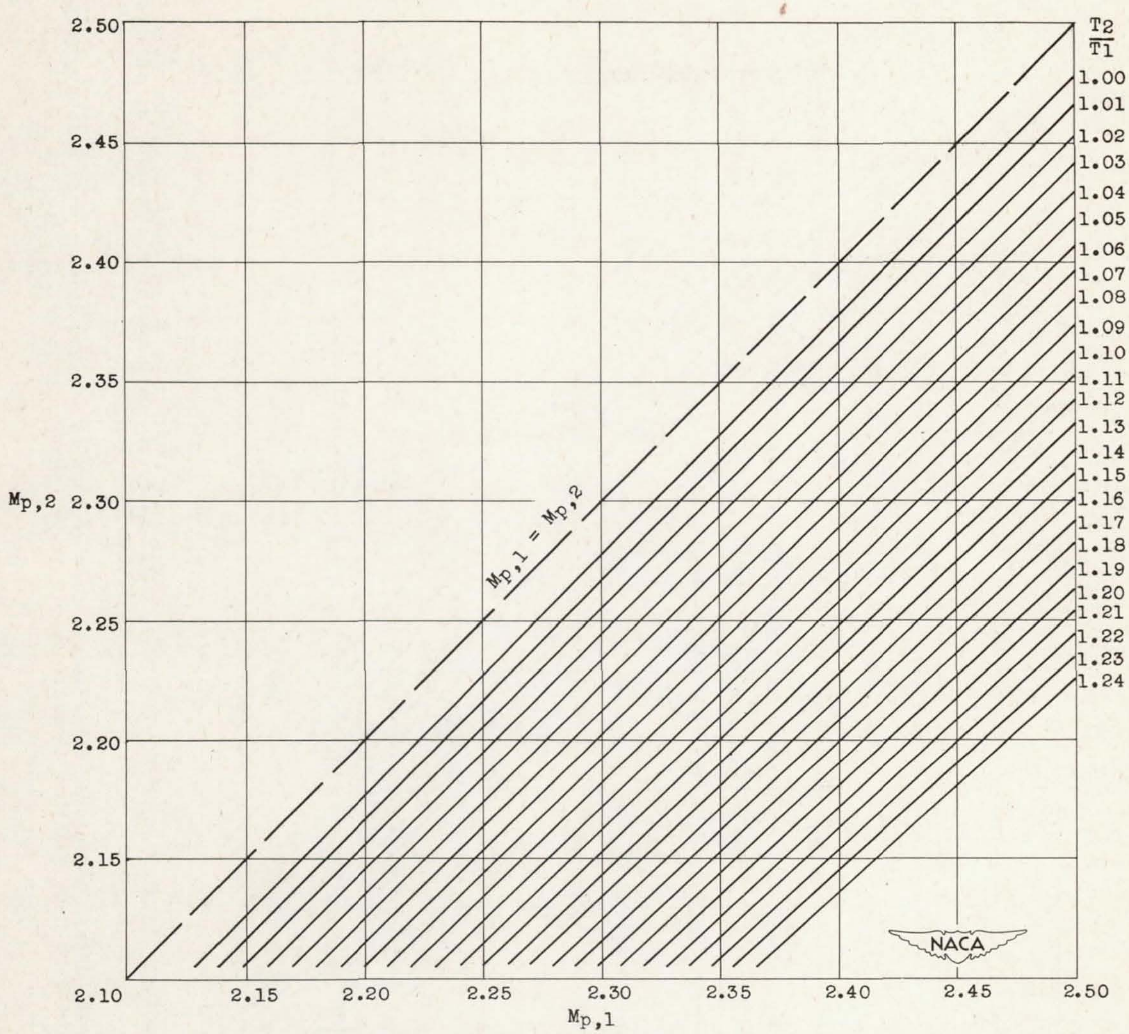
(c) Increment in friction-distance parameter ($F\Delta x/D_e$), 0.04; range of total-momentum parameter at station 2, 3.01 to 3.48.

Figure 1. - Continued. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition (γ = 1.40). (A 17- by 22-in. print of this fig. is attached.)



(d) Increment in friction-distance parameter $(F\Delta x/D_e)$, 0.01; range of total-momentum parameter at station 2, 1.852 to 2.109.

Figure 1. - Continued. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition (γ = 1.40). (A 17- by 22-in. print of this fig. is attached.)



(e) Increment in friction-distance parameter ($F\Delta x/D_e$), 0.01; range of total-momentum parameter at station 2, 2.105 to 2.226.

Figure 1. - Concluded. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition (γ = 1.40). (A 17- by 22-in. print of this fig. is attached.)

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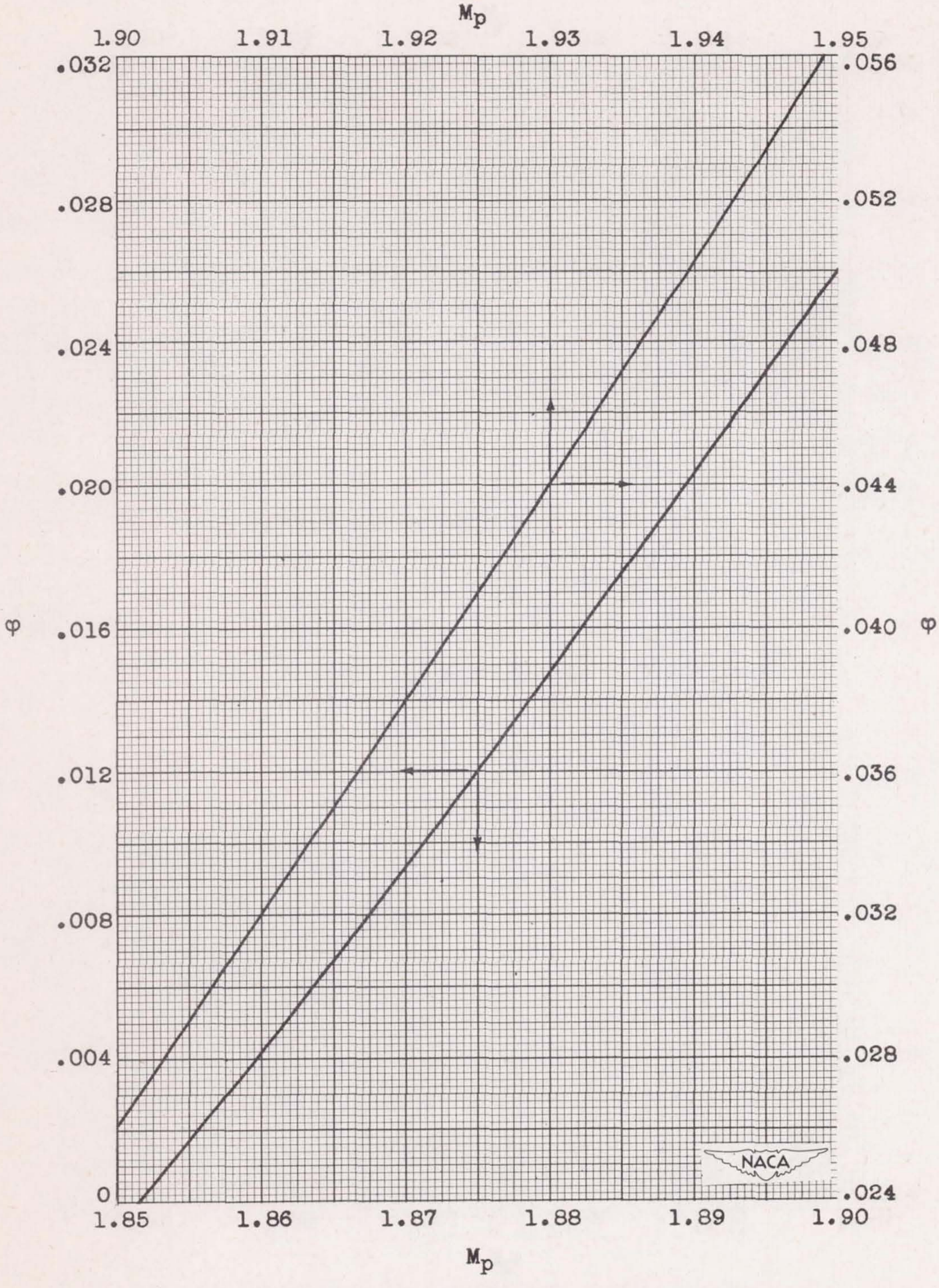
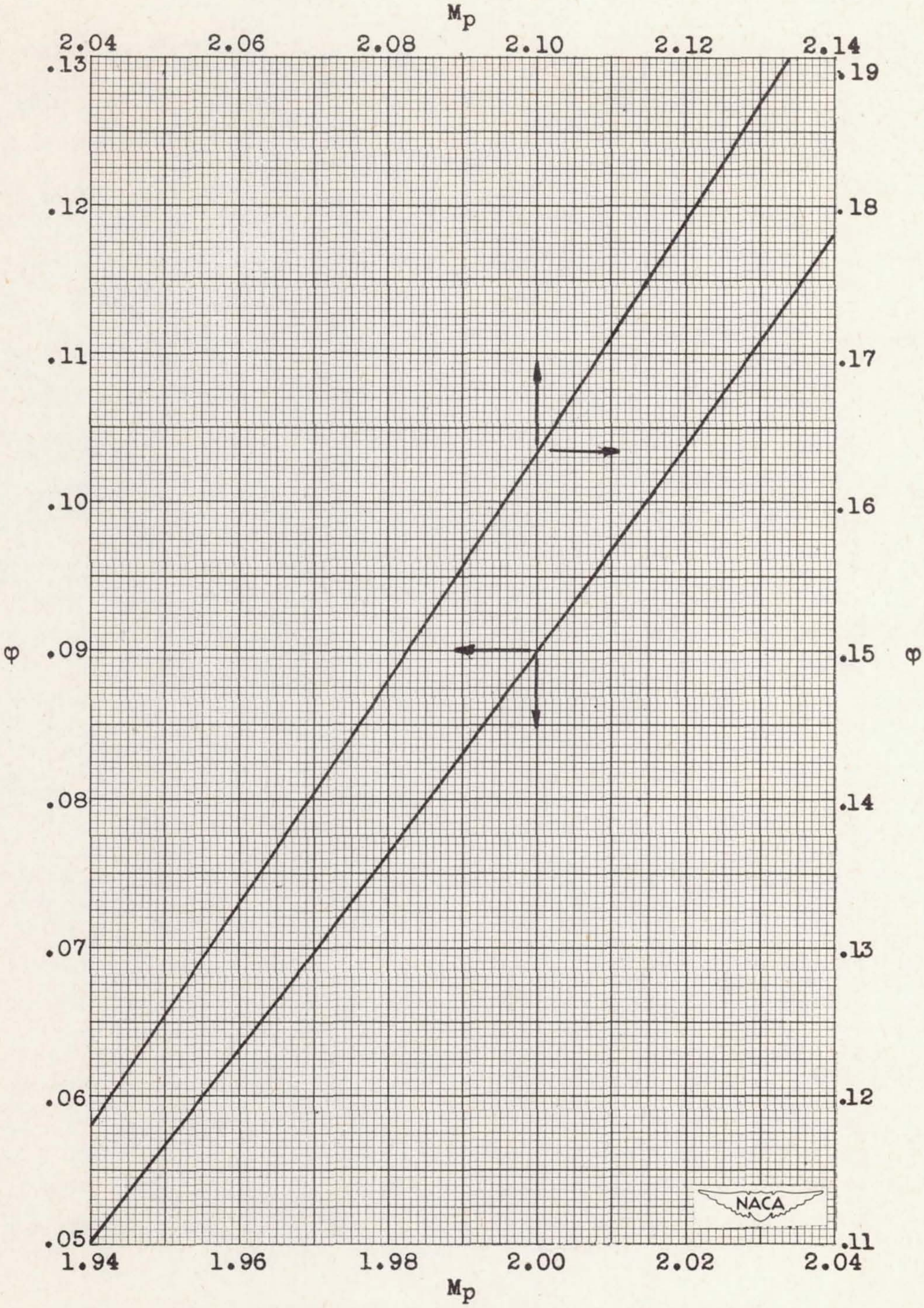


(a) Range of total-momentum parameter, 1.8516 (choke) to 1.9488.

Figure 2. - Relation between parameter φ and total-momentum parameter for air (γ = 1.40).



(b) Range of total-momentum parameter, 1.940 to 2.134.

Figure 2. - Continued. Relation between parameter φ and total-momentum parameter for air (γ = 1.40).

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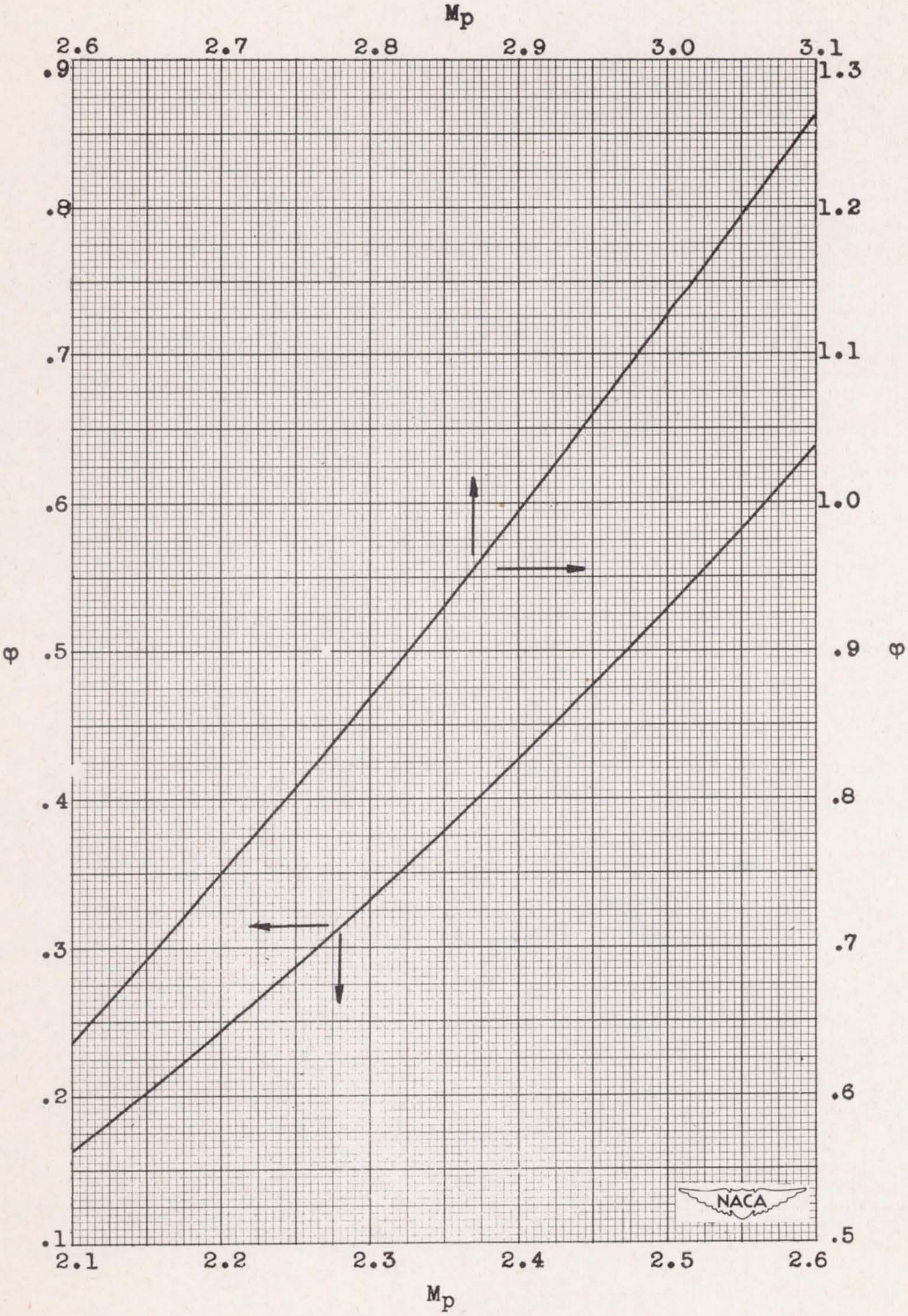
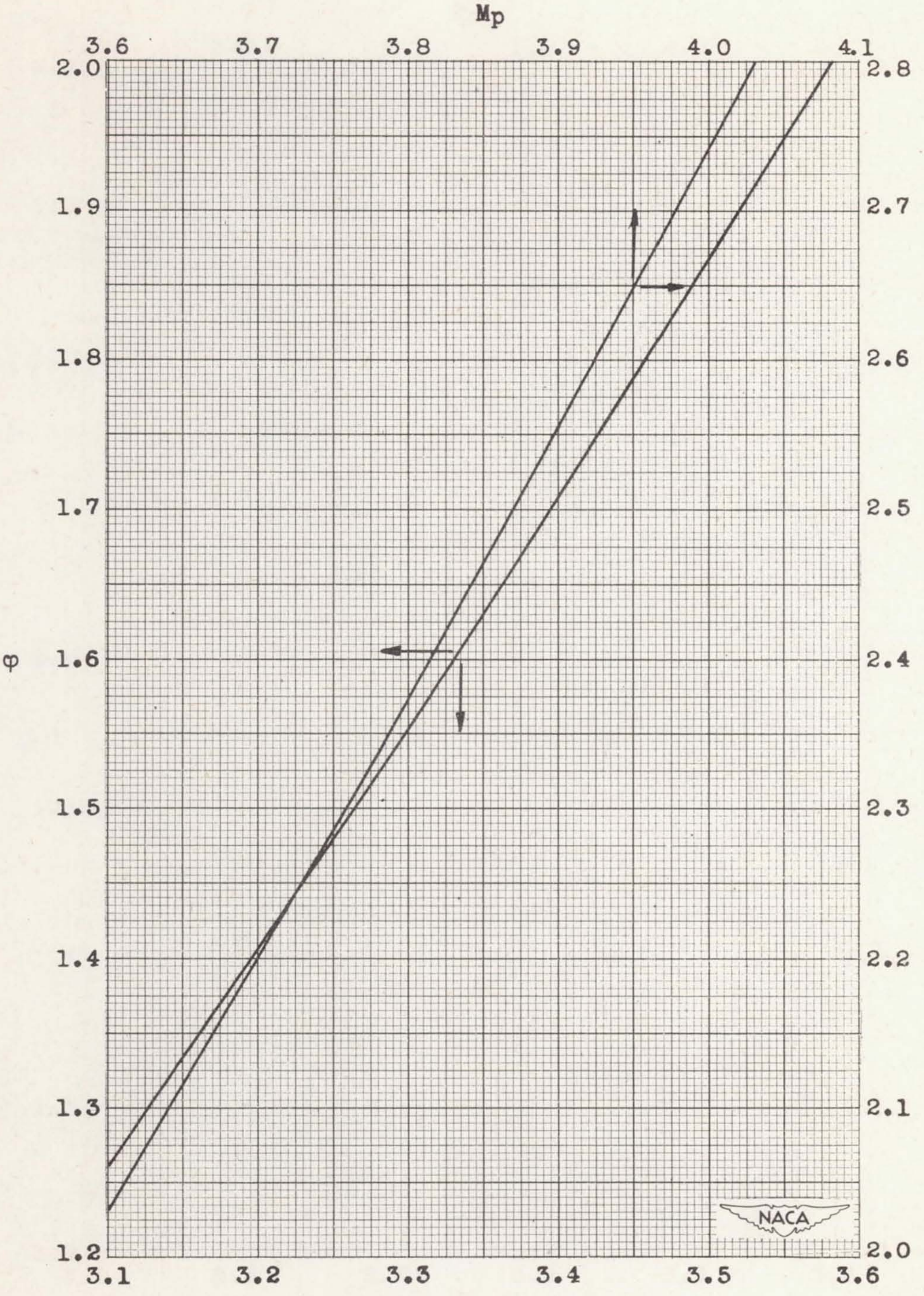


(c) Range of total-momentum parameter, 2.1 to 3.1.

Figure 2. - Continued. Relation between parameter φ and total-momentum parameter for air (γ = 1.40).



(d) Range of total-momentum parameter, 3.10 to 4.03.

Figure 2. - Concluded. Relation between parameter φ and total-momentum parameter for air (γ = 1.40).

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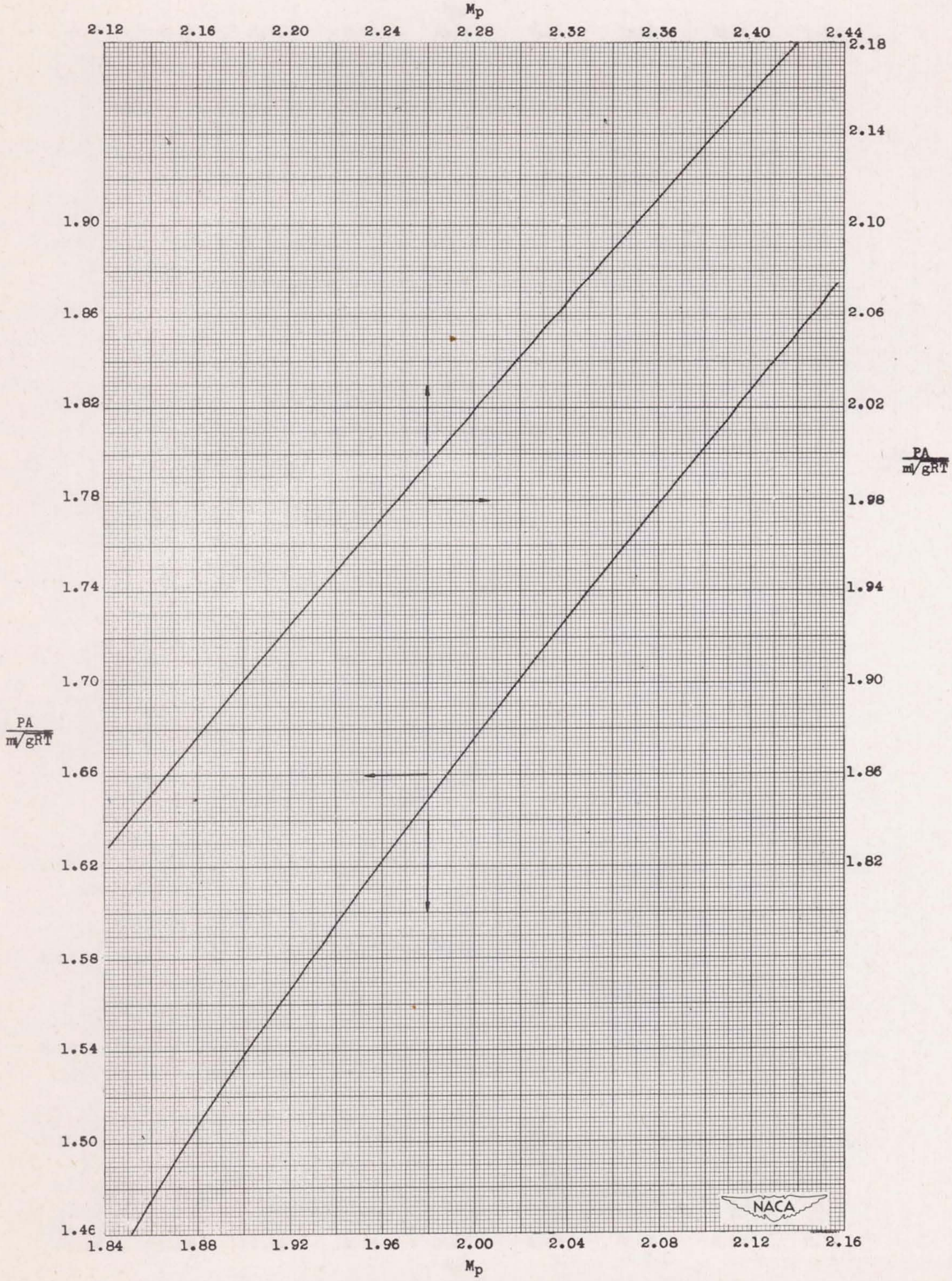
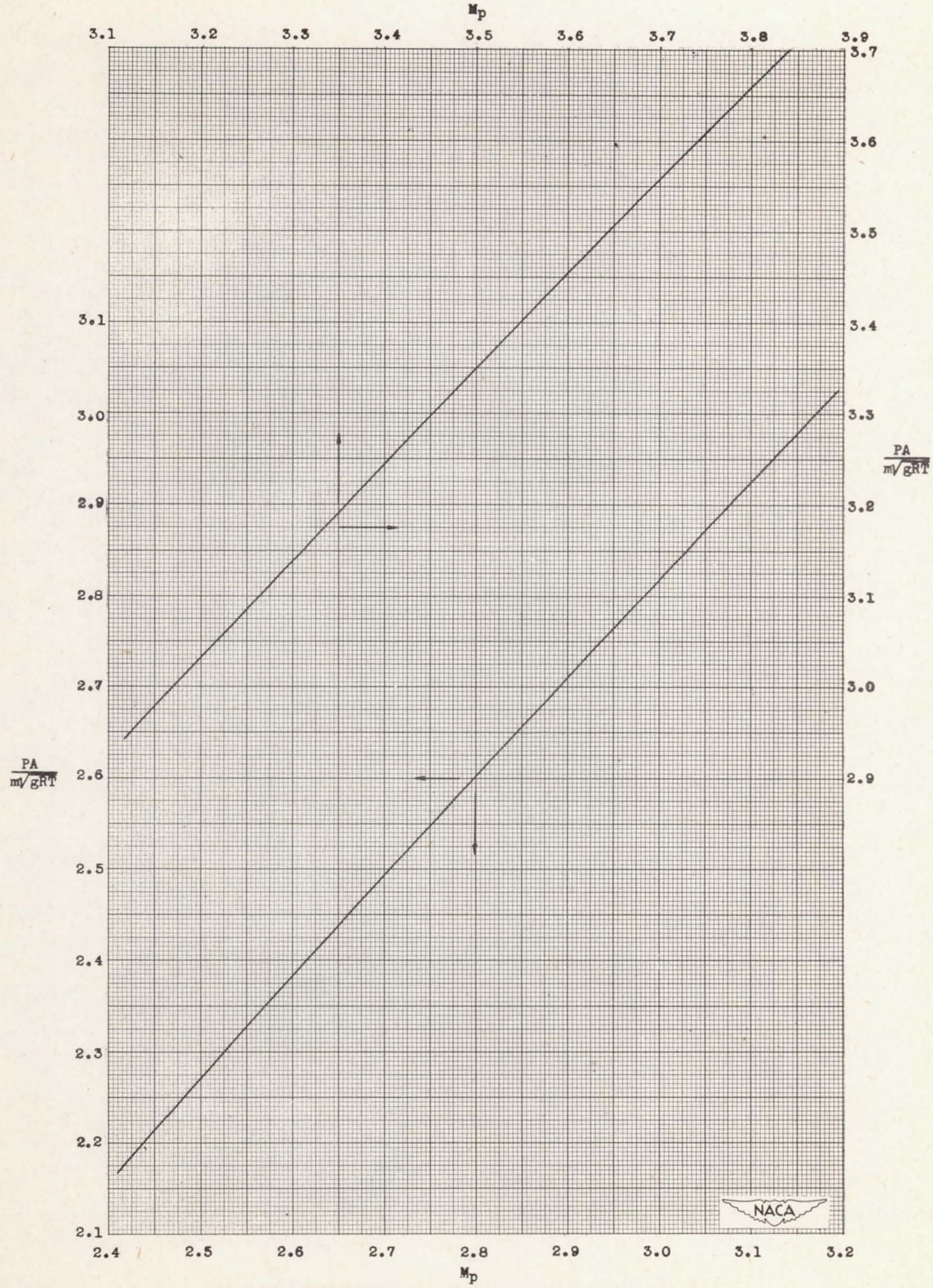


(a) Range of total-momentum parameter, 1.852 (choke) to 2.418.

Figure 3. - Relation between total-pressure parameter and total-momentum parameter for air (γ = 1.40).



(b) Range of total-momentum parameter, 2.41 to 3.83.

Figure 3. - Continued. Relation between total-pressure parameter and total-momentum parameter for air (γ = 1.40).

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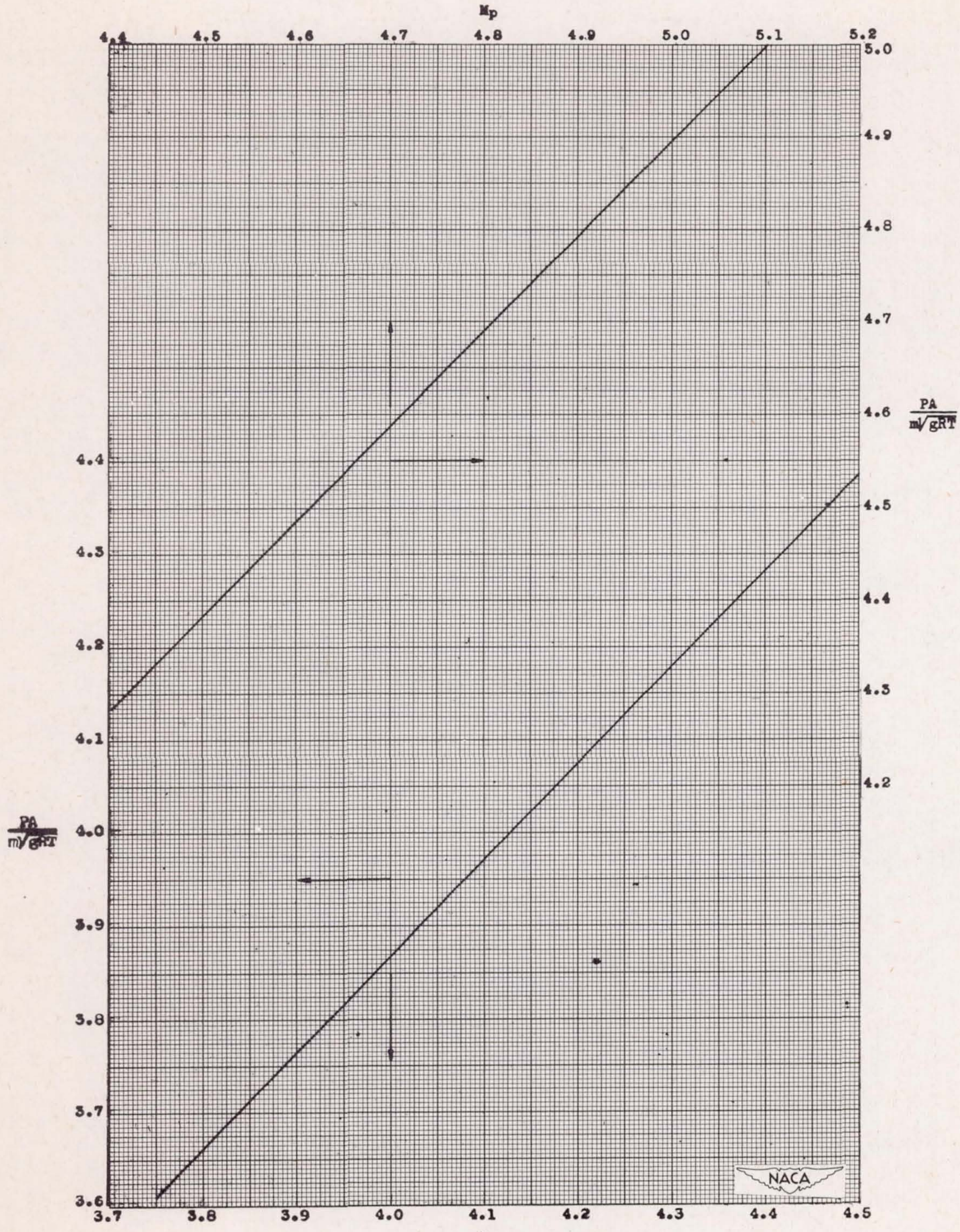
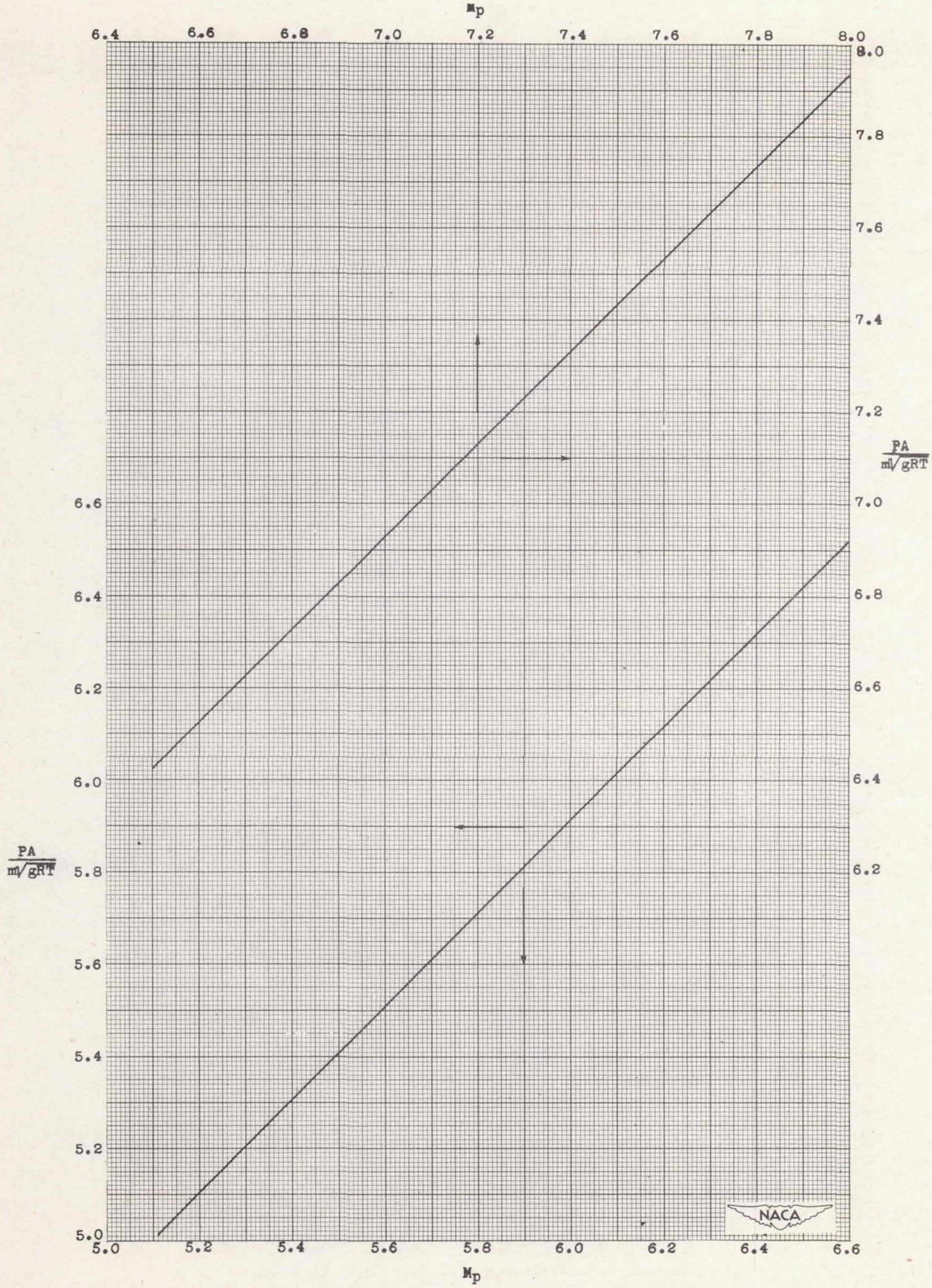


(c) Range of total-momentum parameter, 3.75 to 5.10.

Figure 3. - Continued. Relation between total-pressure parameter and total-momentum parameter for air (γ = 1.40).



(d) Range of total-momentum parameter, 5.11 to 8.00.

Figure 3. - Concluded. Relation between total-pressure parameter and total-momentum parameter for air (γ = 1.40).

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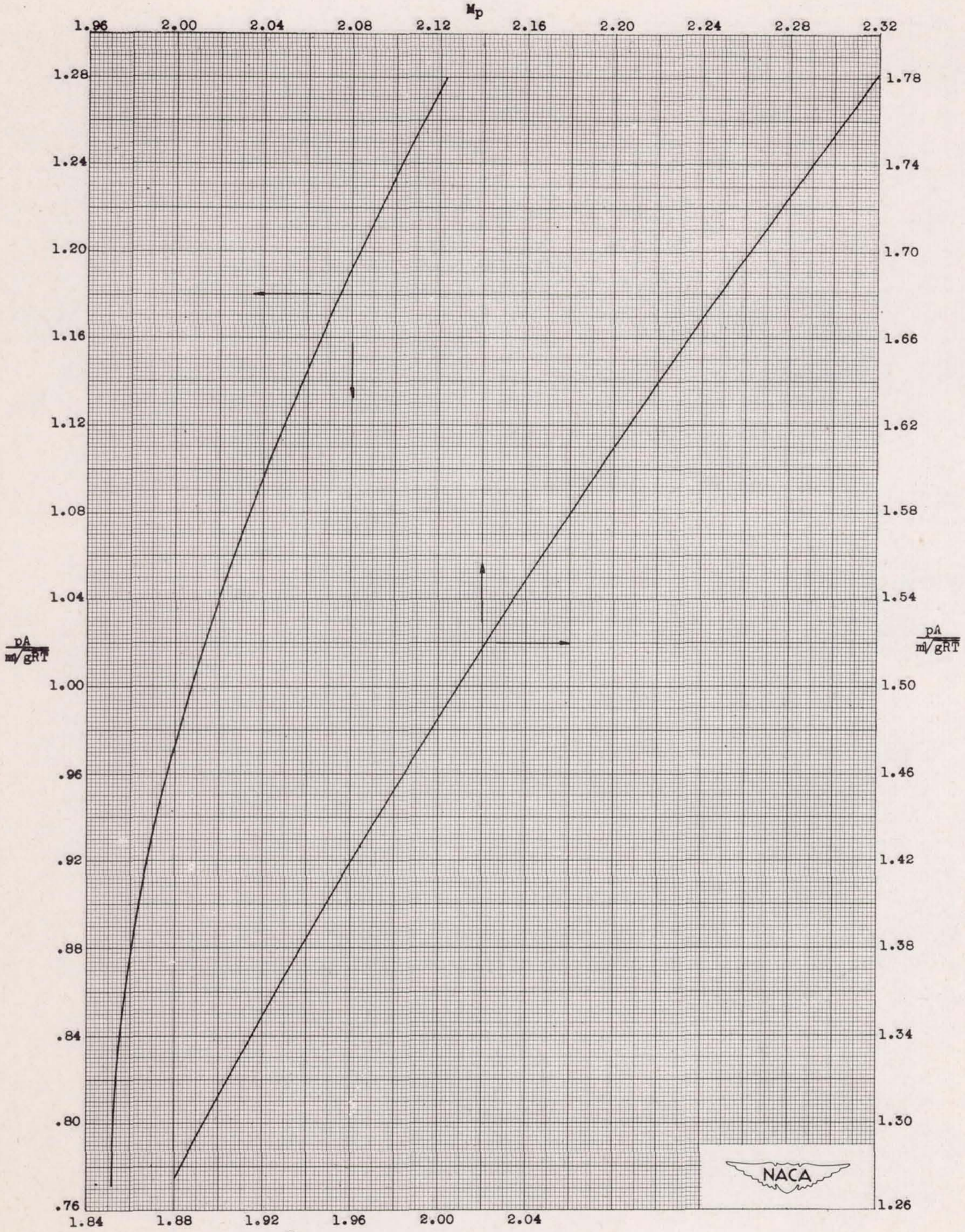
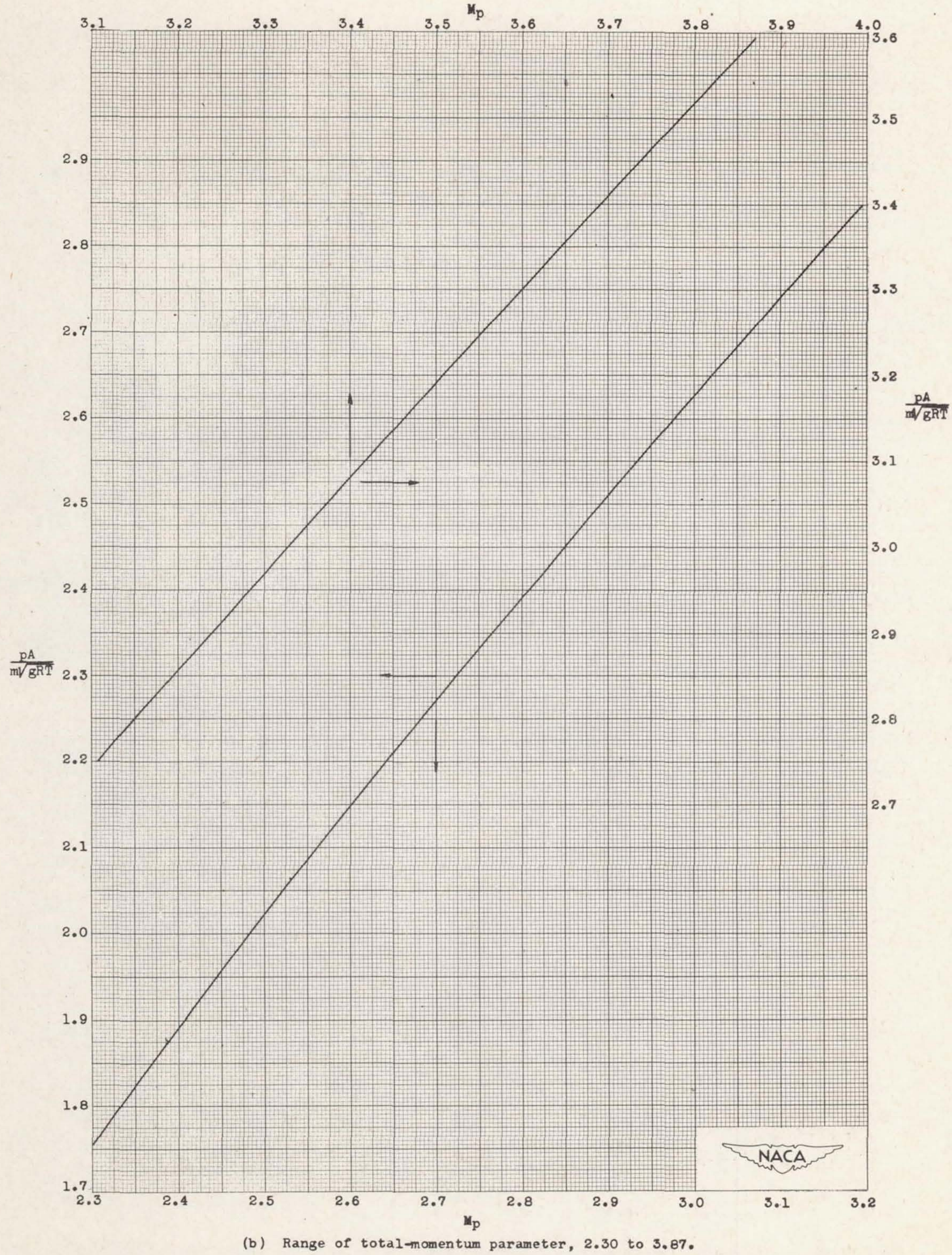


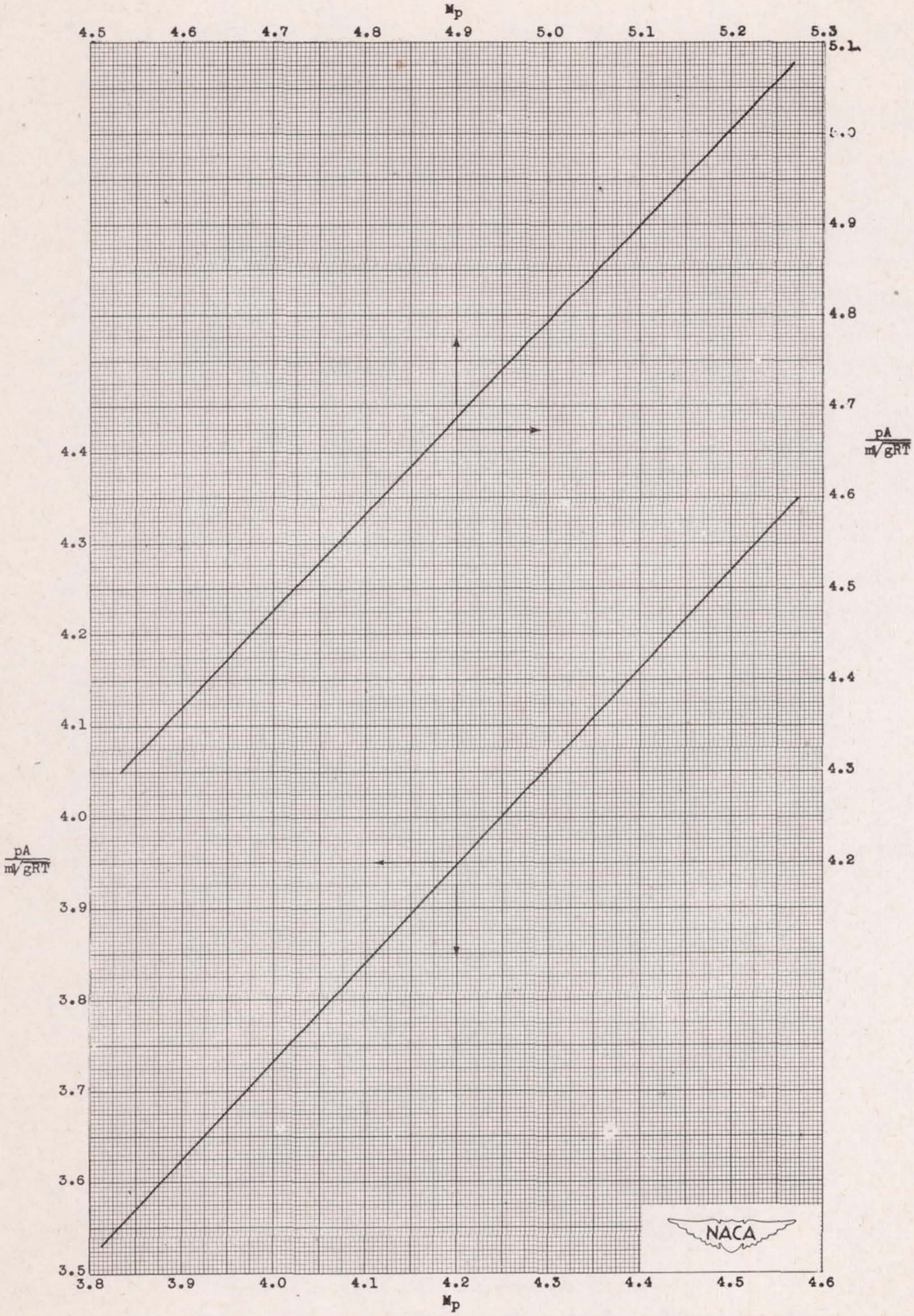
(a) Range of total-momentum parameter, 1.852 to 2.320.

Figure 4. - Relation between static-pressure parameter and total-momentum parameter for air (γ = 1.40).



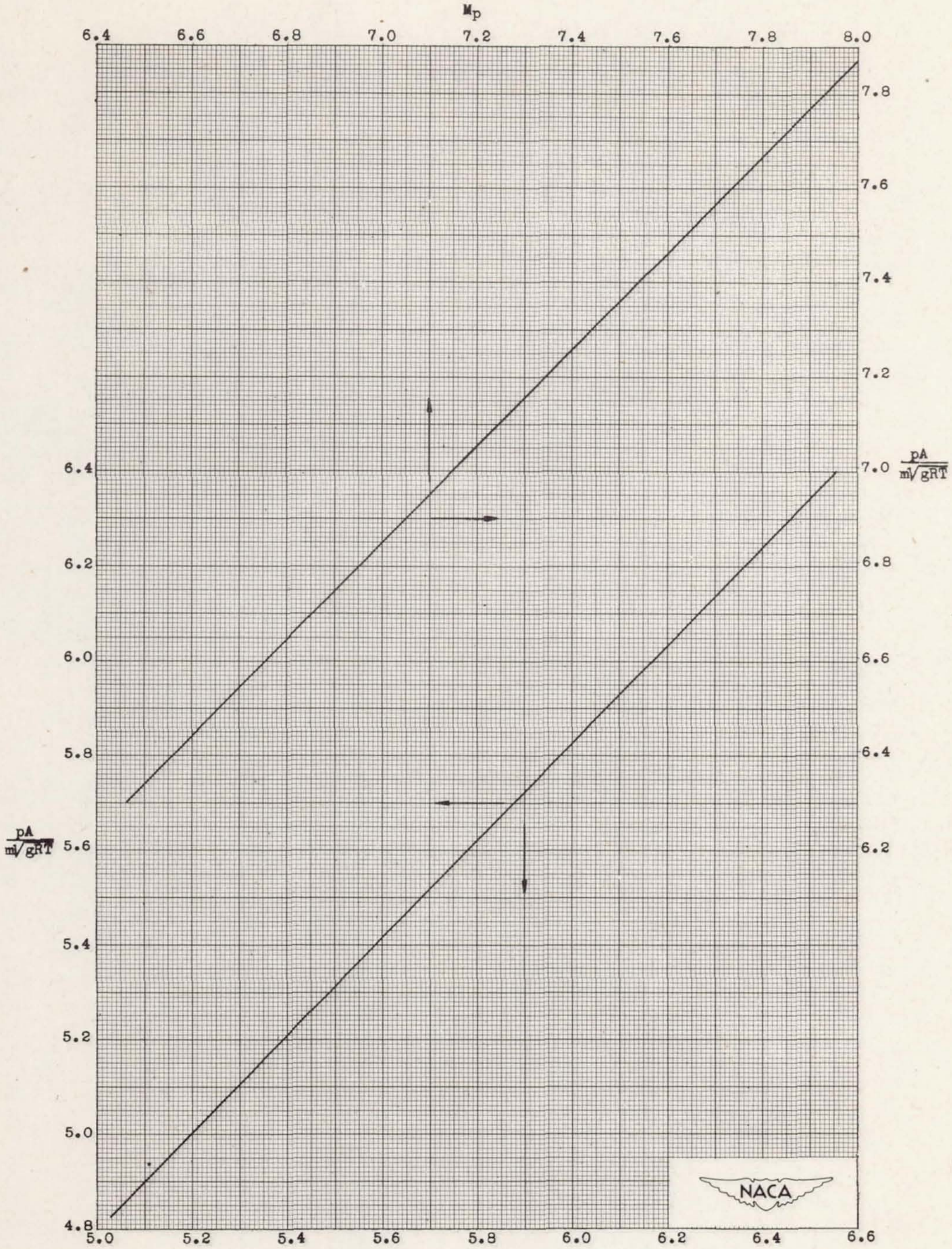
(b) Range of total-momentum parameter, 2.30 to 3.87.

Figure 4. - Continued. Relation between static-pressure parameter and total-momentum parameter for air (γ = 1.40).



(c) Range of total-momentum parameter, 3.81 to 5.27.

Figure 4. - Continued. Relation between static-pressure parameter and total-momentum parameter for air (γ = 1.40).



(d) Range of total-momentum parameter, 5.03 to 8.00.

Figure 4. - Concluded. Relation between static-pressure parameter and total-momentum parameter for air (γ = 1.40).

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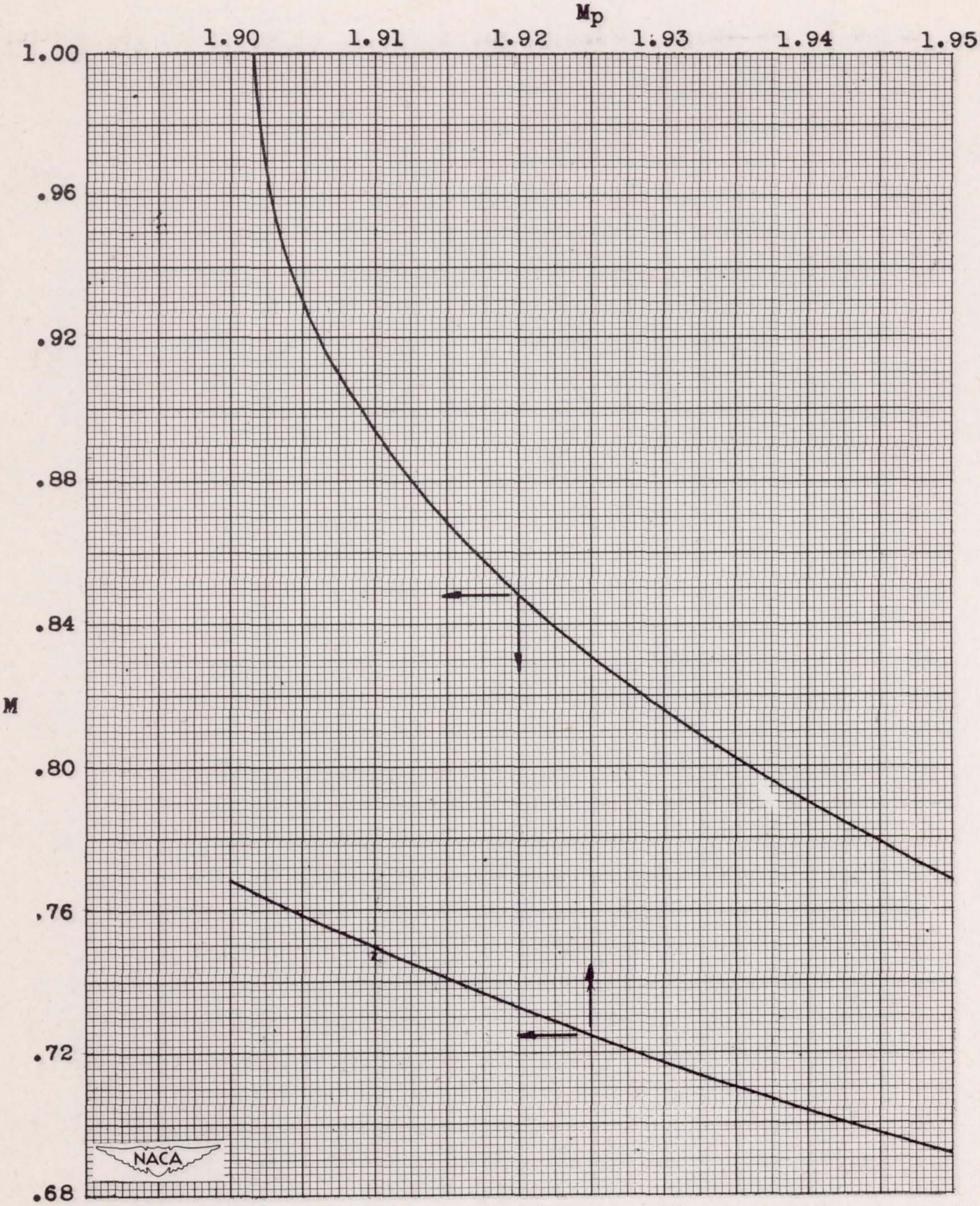
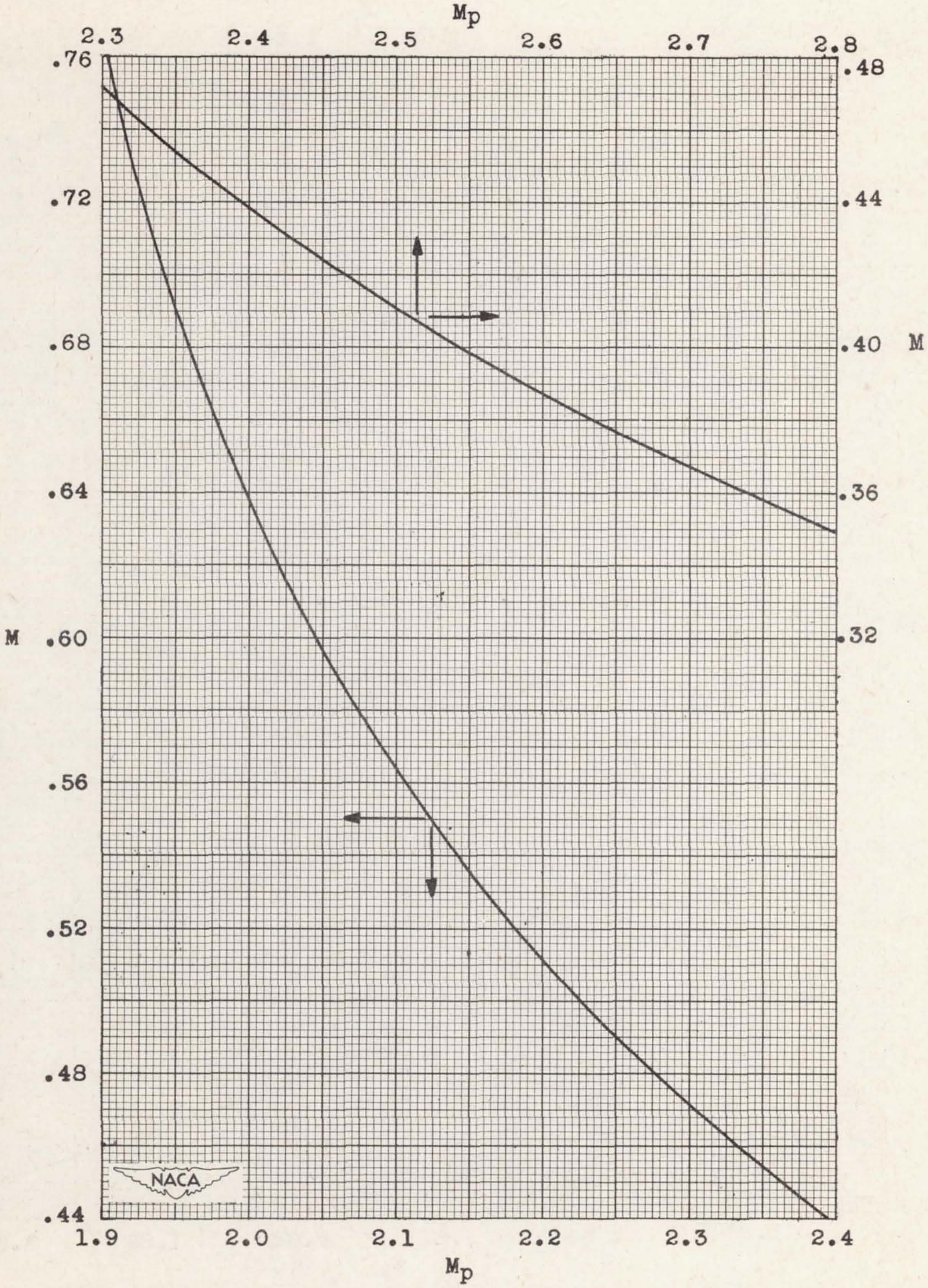


(a) Range of total-momentum parameter, 1.8516 (choke) to 1.9500.

Figure 5. - Relation between Mach number and total-momentum parameter for air (γ = 1.40).



(b) Range of total-momentum parameter, 1.905 to 2.800.

Figure 5. - Continued. Relation between Mach number and total-momentum parameter for air (γ = 1.40).

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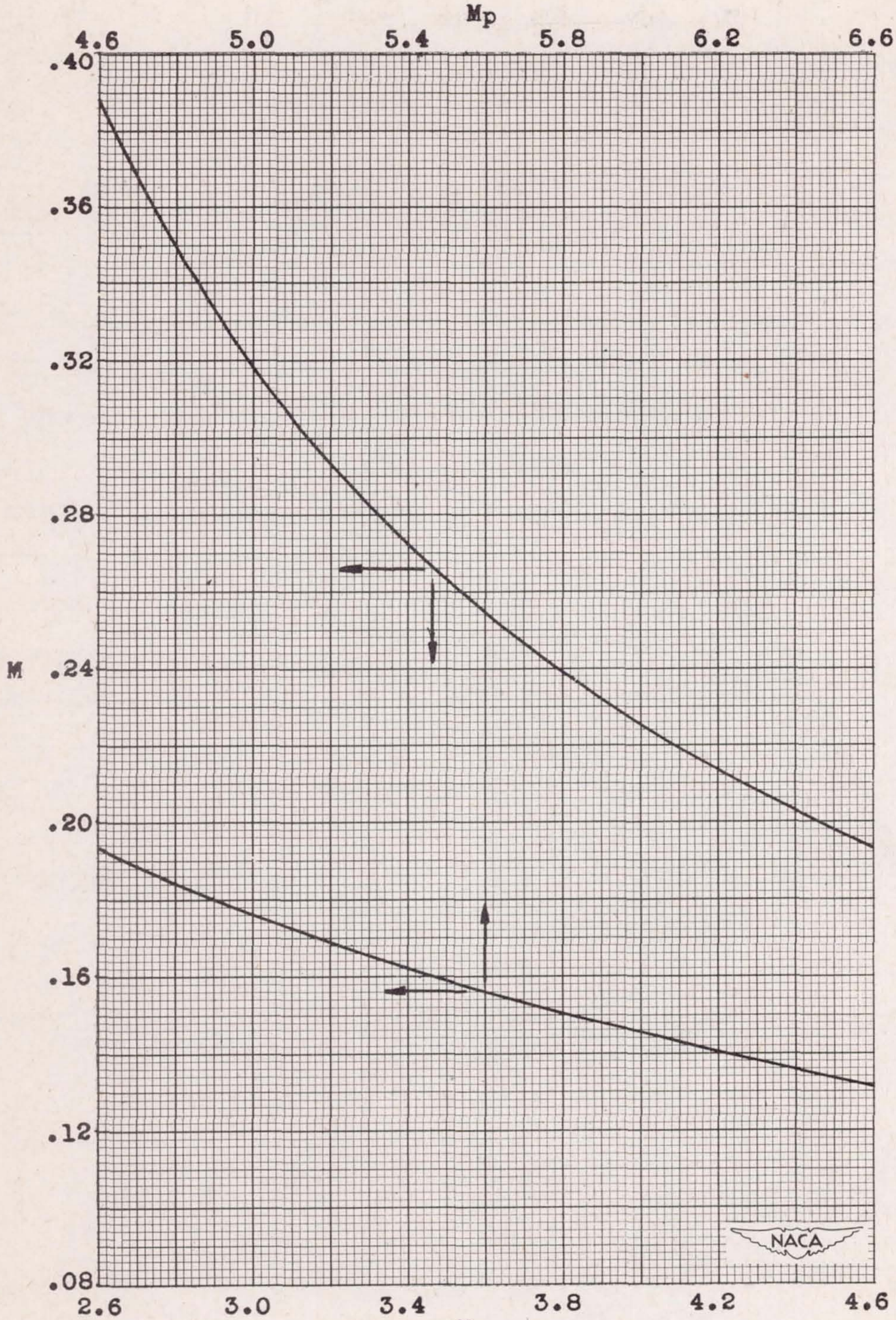

(c) Range of total-momentum parameter, 2.600 to 6.600.

Figure 5. - Concluded. Relation between Mach number and total-momentum parameter for air (γ = 1.40).

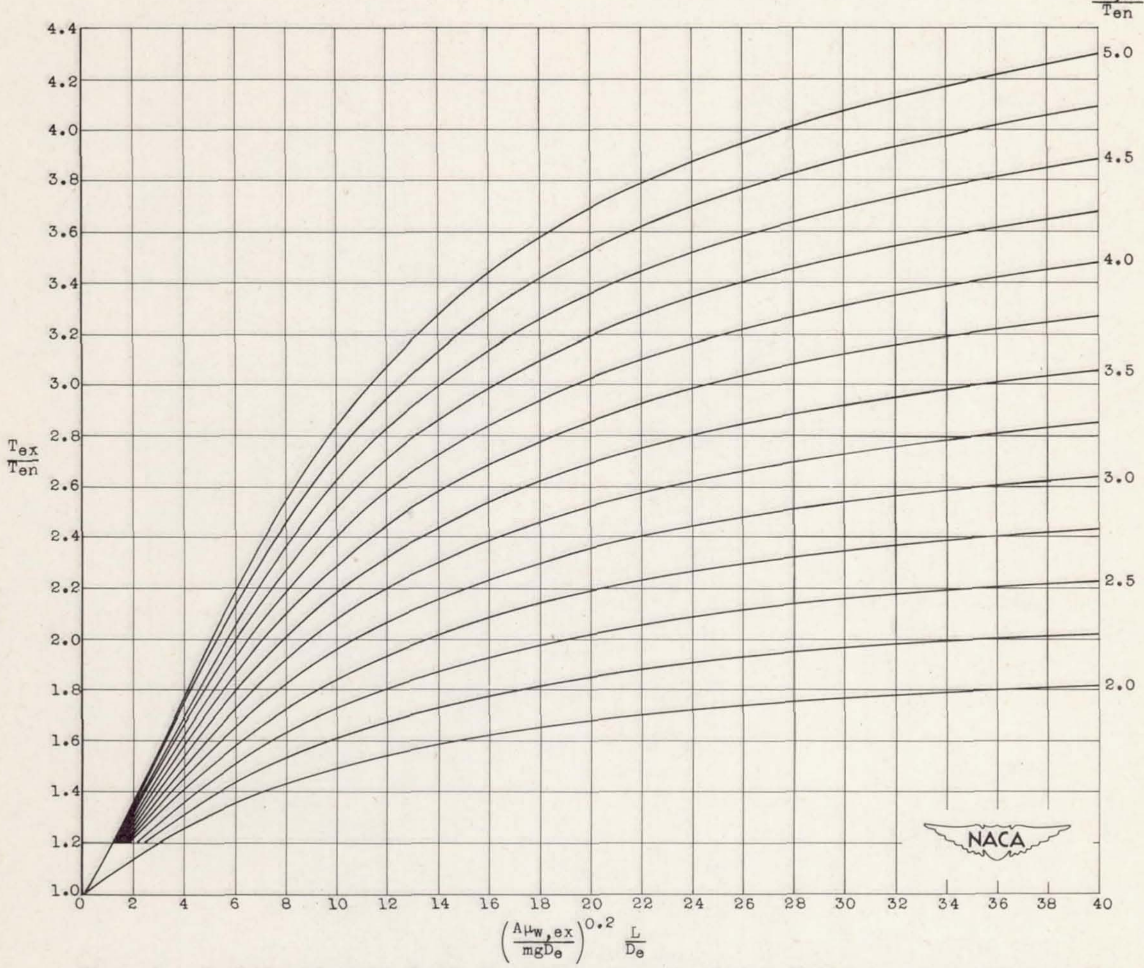

Figure 6. - Variation of ratio of exit fluid temperature to entrance fluid temperature with over-all passage-distance parameter for various ratios of maximum (exit, in this case) wall temperature to entrance fluid temperature. Turbulent flow of air for constant rate of heat-input conditions. (A 17- by 22-in. print of this fig. is attached.)

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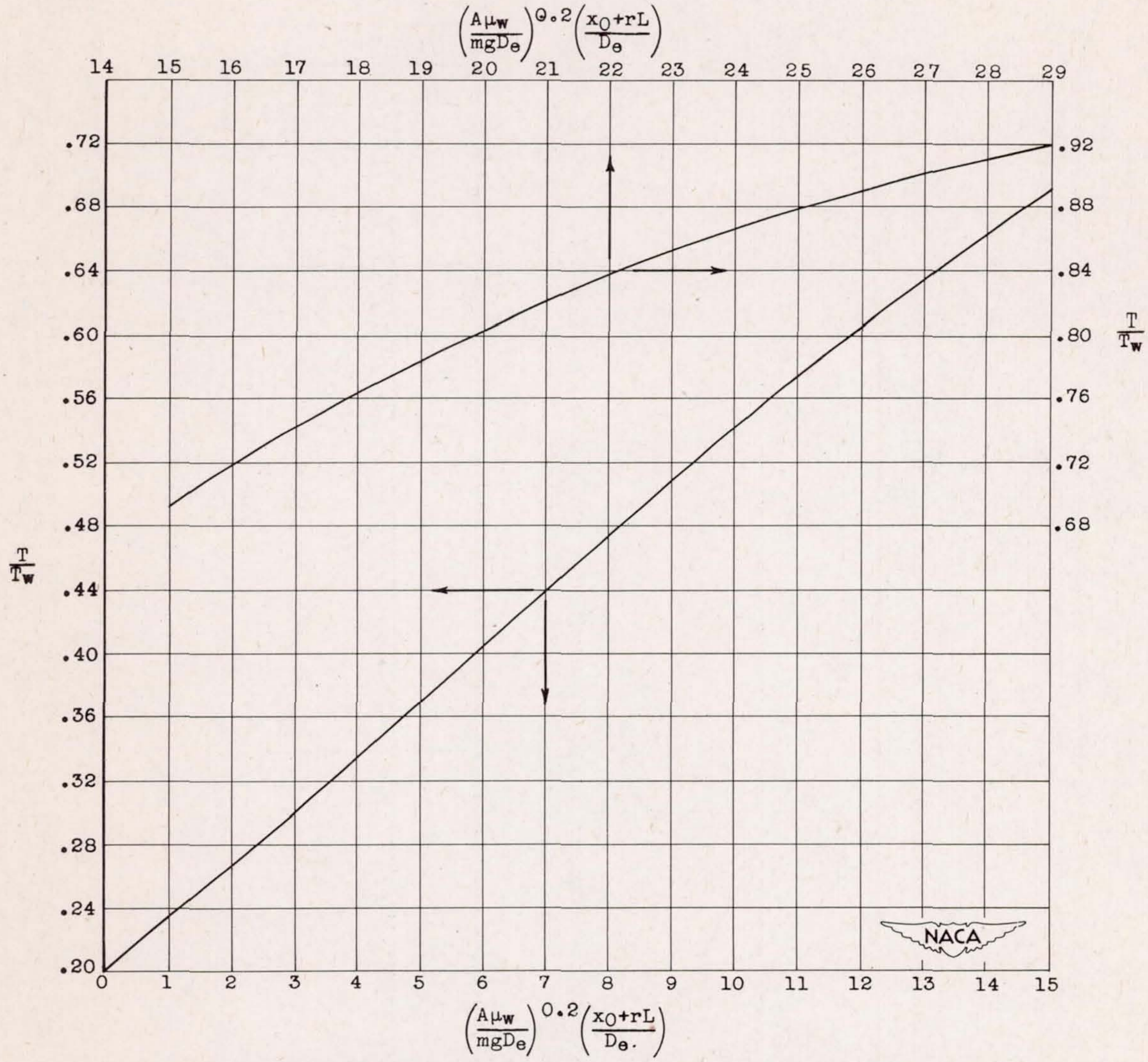
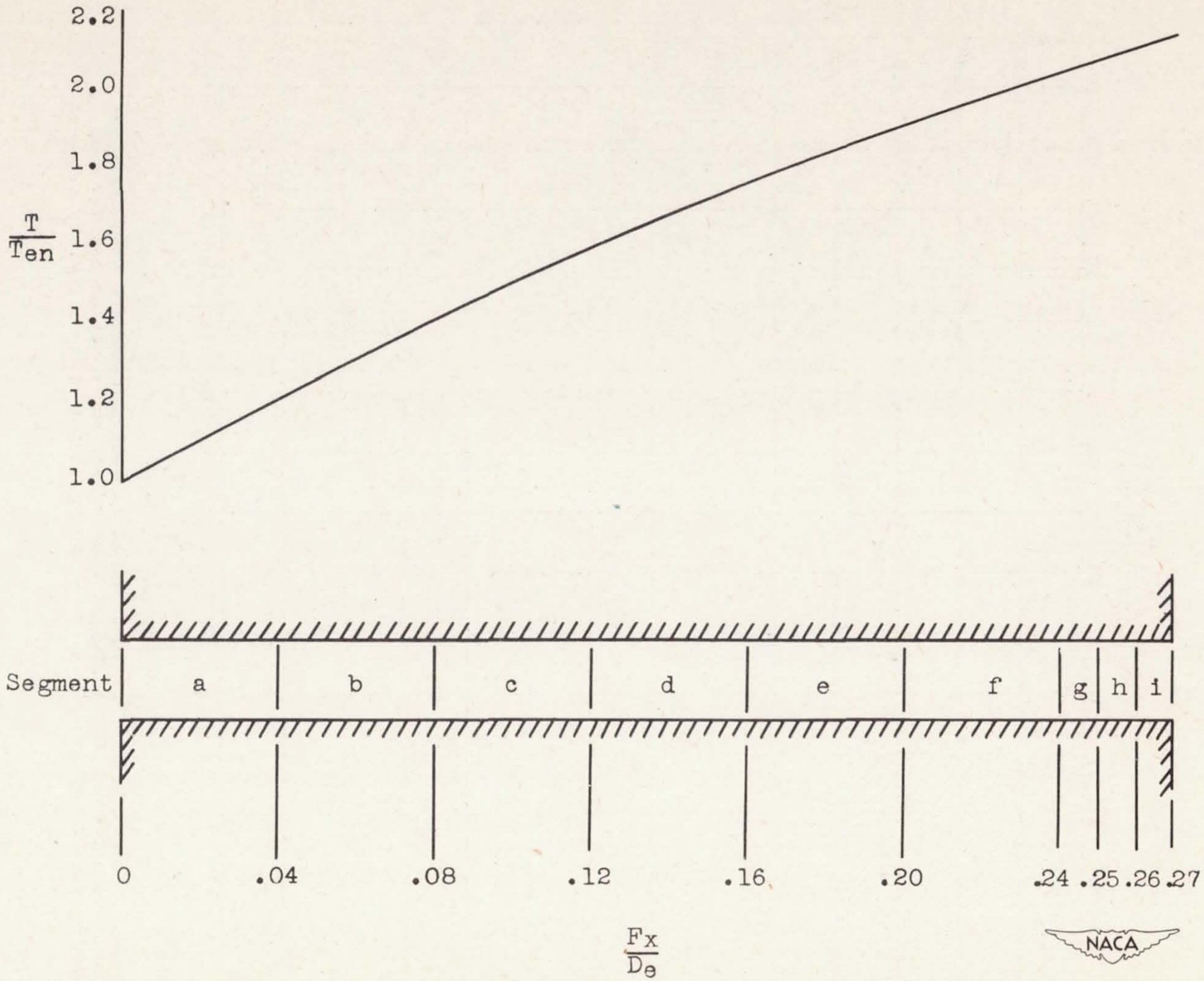
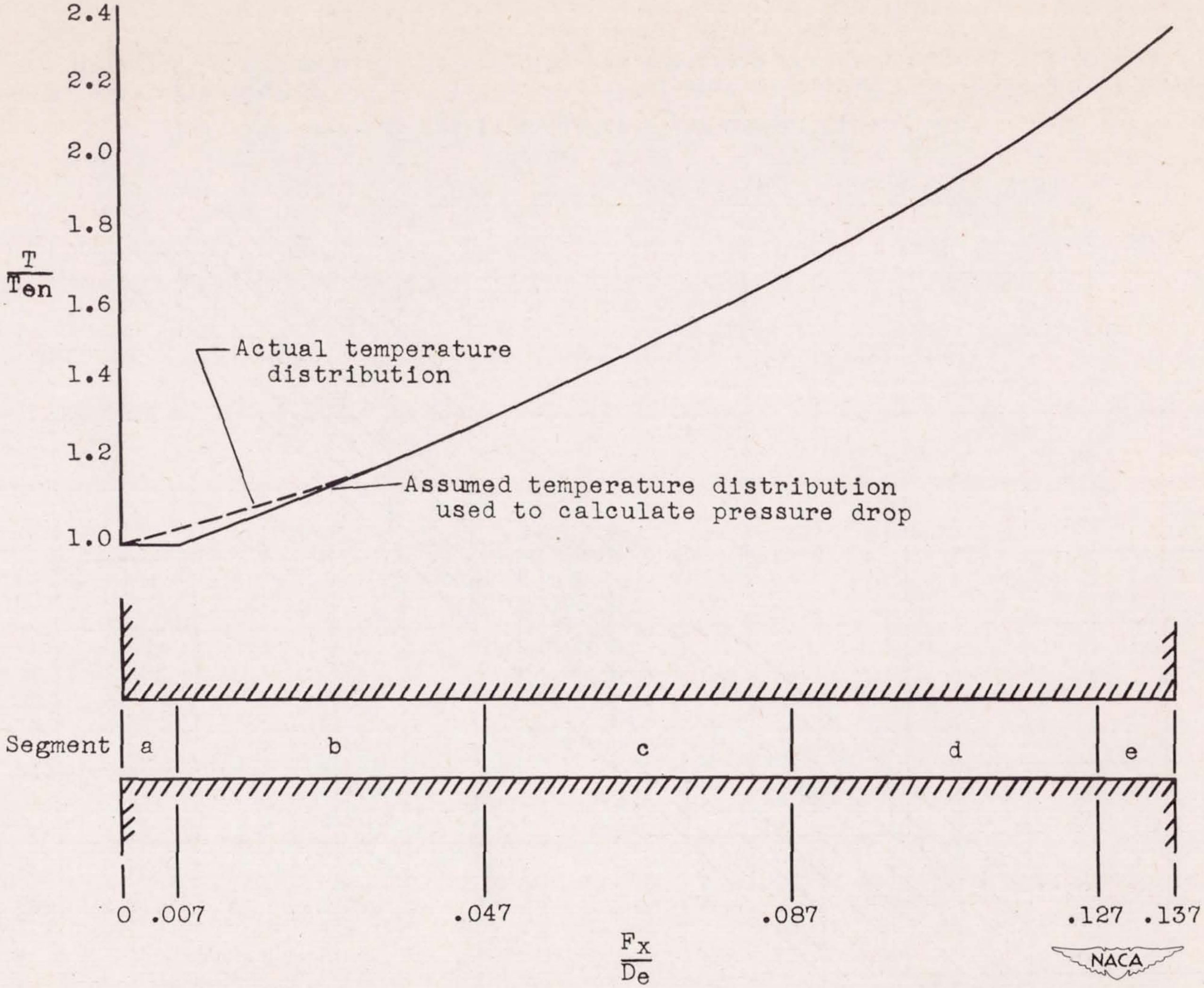


Figure 7. - Variation of ratio of fluid to wall temperature with passage-distance parameter for turbulent flow of air and constant passage-wall temperature conditions. (A 17- by 22-in. print of this fig. is attached.)



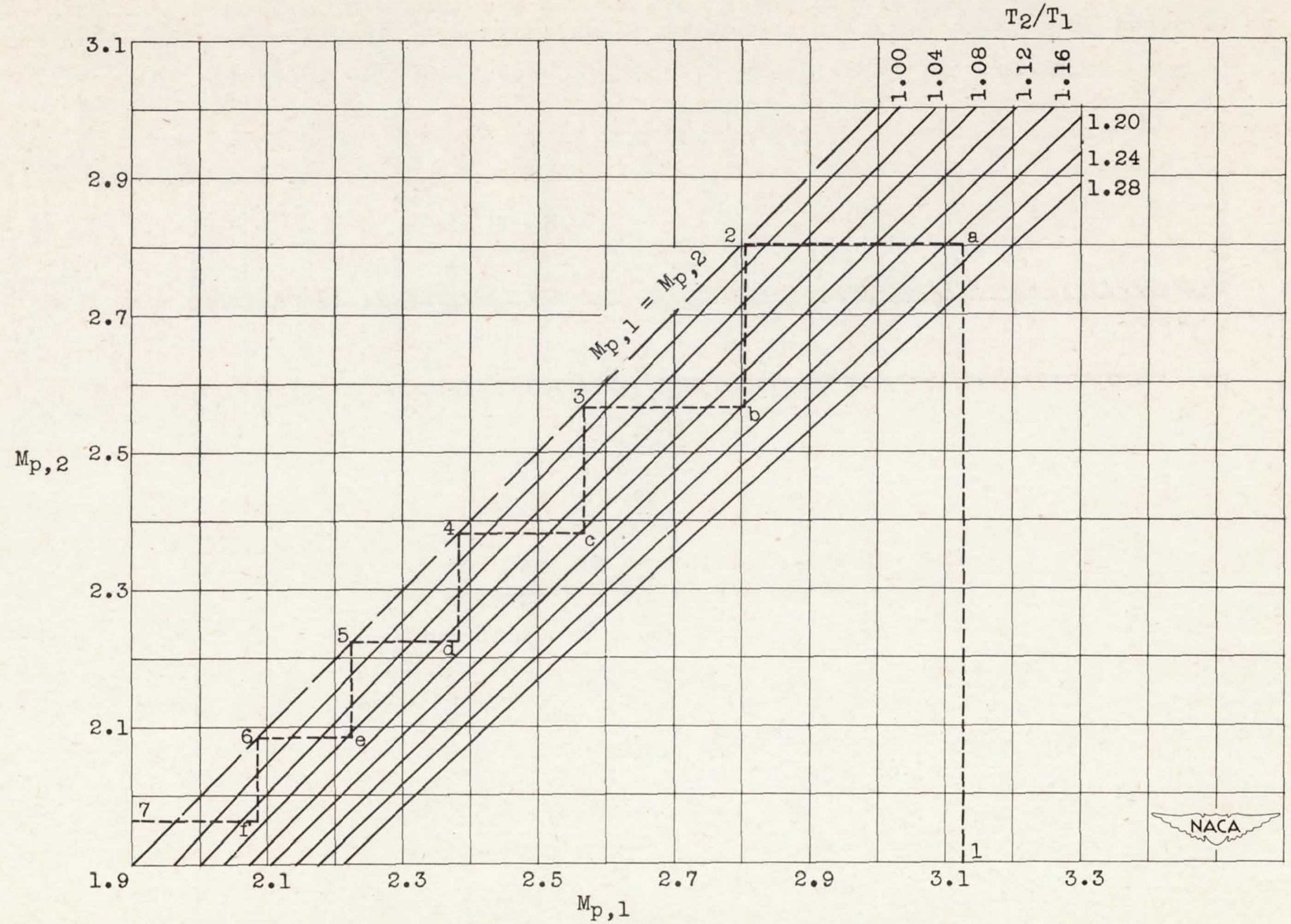
(a) Division of flow-passage length in Example I into segments a to i.

Figure 8. - Illustration of method of dividing total flow-passage length into segments for use in pressure-drop charts.



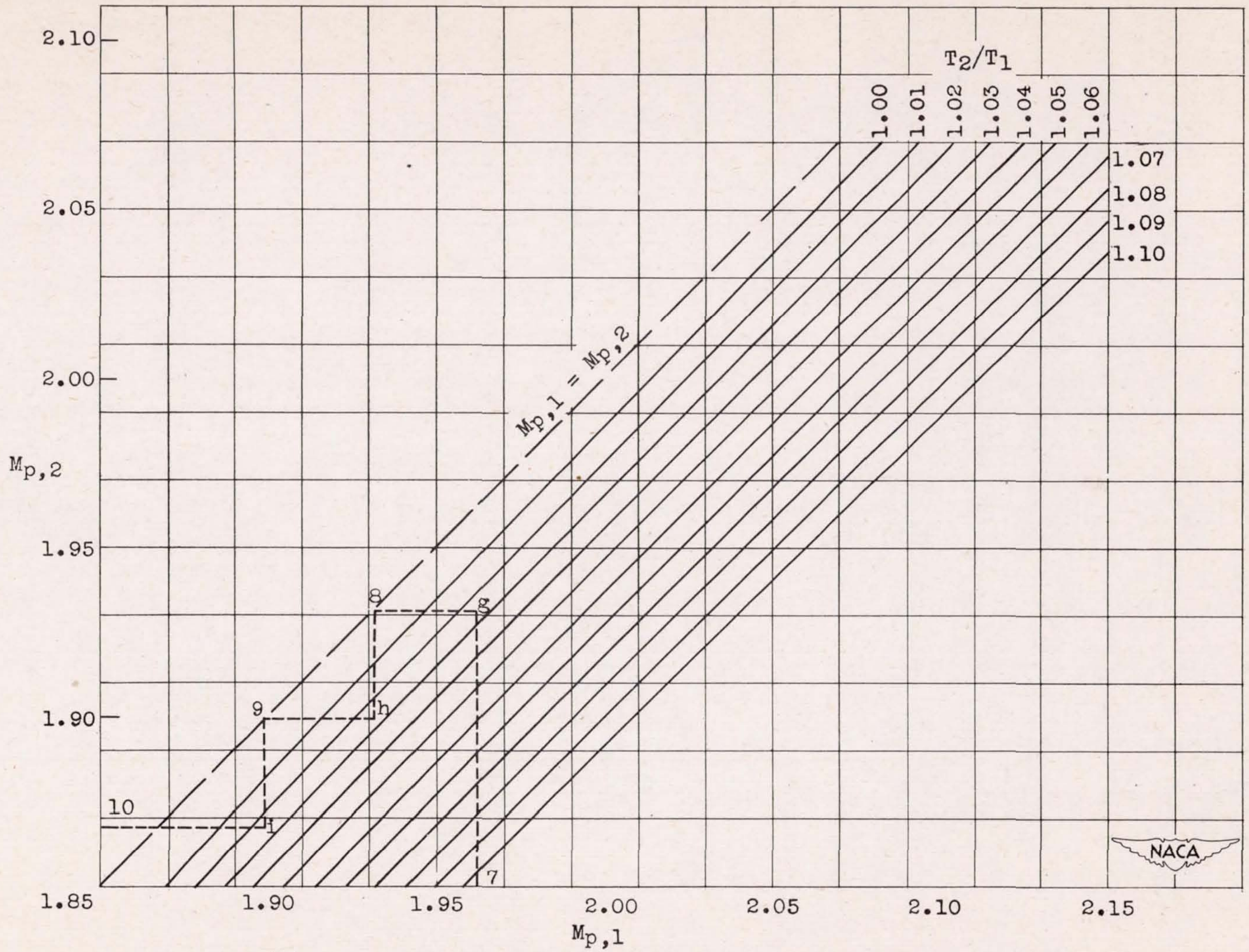
(b) Division of flow-passage length in Example III into segments a to e.

Figure 8. - Concluded. Illustration of method of dividing total flow-passage length into segments for use in pressure-drop charts.



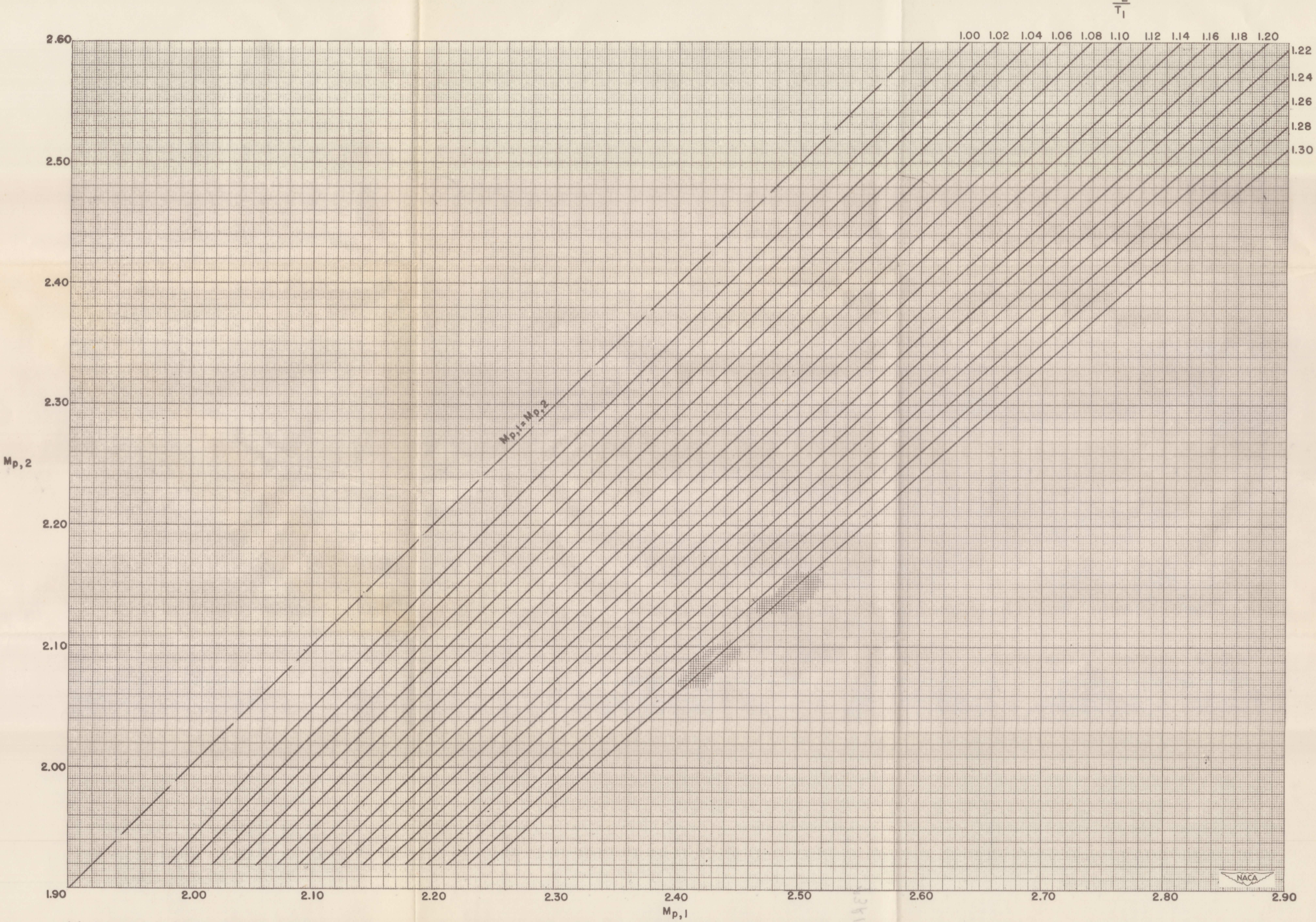
(a) Increment in friction-distance parameter, 0.04.

Figure 9. - Illustration of use of figure 1 describing compressible-flow process in constant-area duct with simultaneous friction and arbitrary heat addition ($\gamma = 1.40$).



(b) Increment in friction-distance parameter, 0.01.

Figure 9. - Concluded. Illustration of use of figure 1 describing compressible-flow process in constant-area duct with simultaneous friction and arbitrary heat addition ($\gamma = 1.40$).



(a) Increment in friction-distance parameter $(F\Delta x/D_e)$, 0.04; range of total-momentum parameter at station 2, 1.92 to 2.52.

Figure 1.- Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition ($\gamma = 1.40$).

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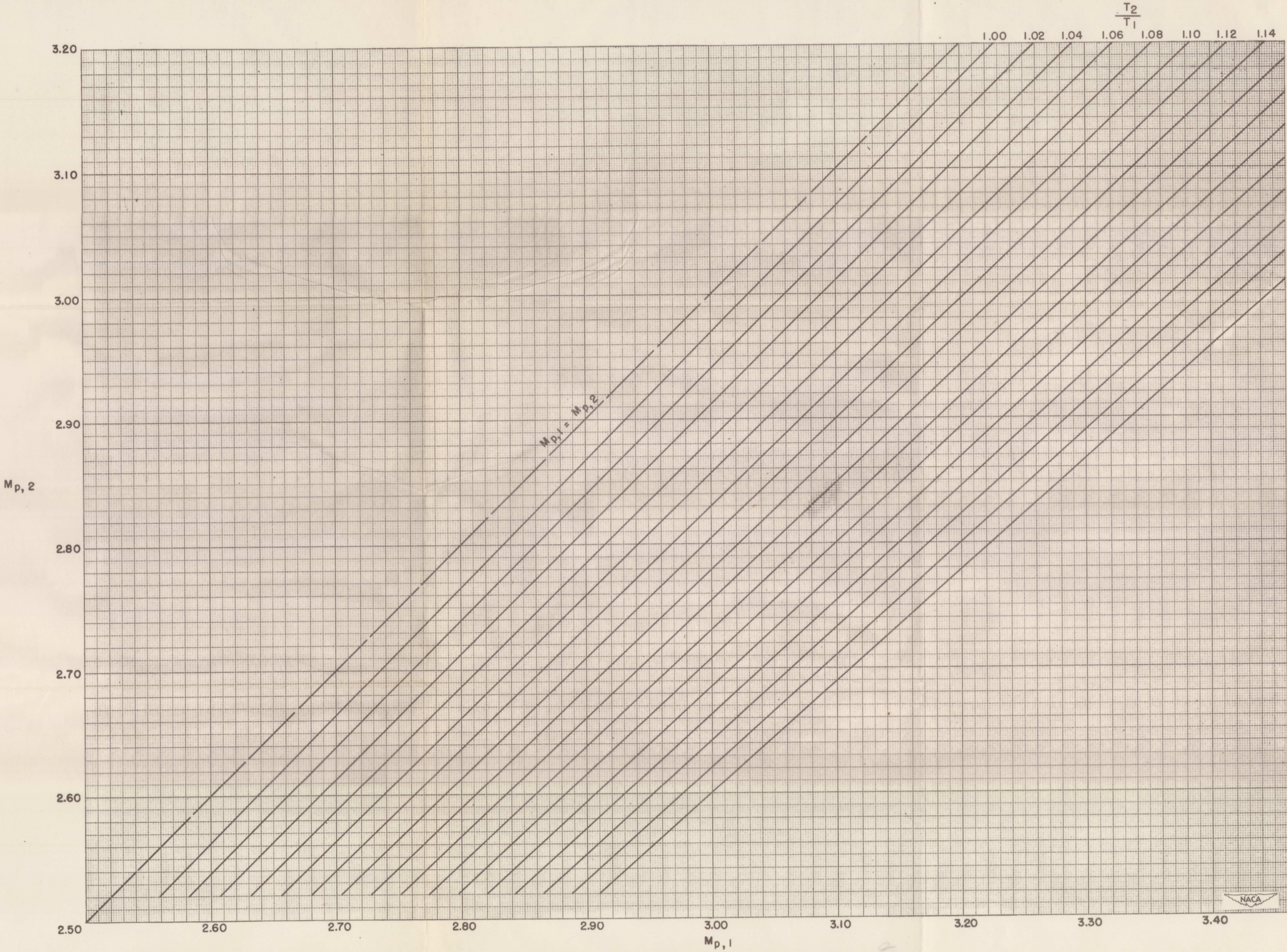

(b) Increment in friction-distance parameter $(F\Delta x/D_e)$, 0.04; range of total-momentum parameter at station 2, 2.52 to 3.01.

Figure 1.- Continued. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition (γ=1.40).

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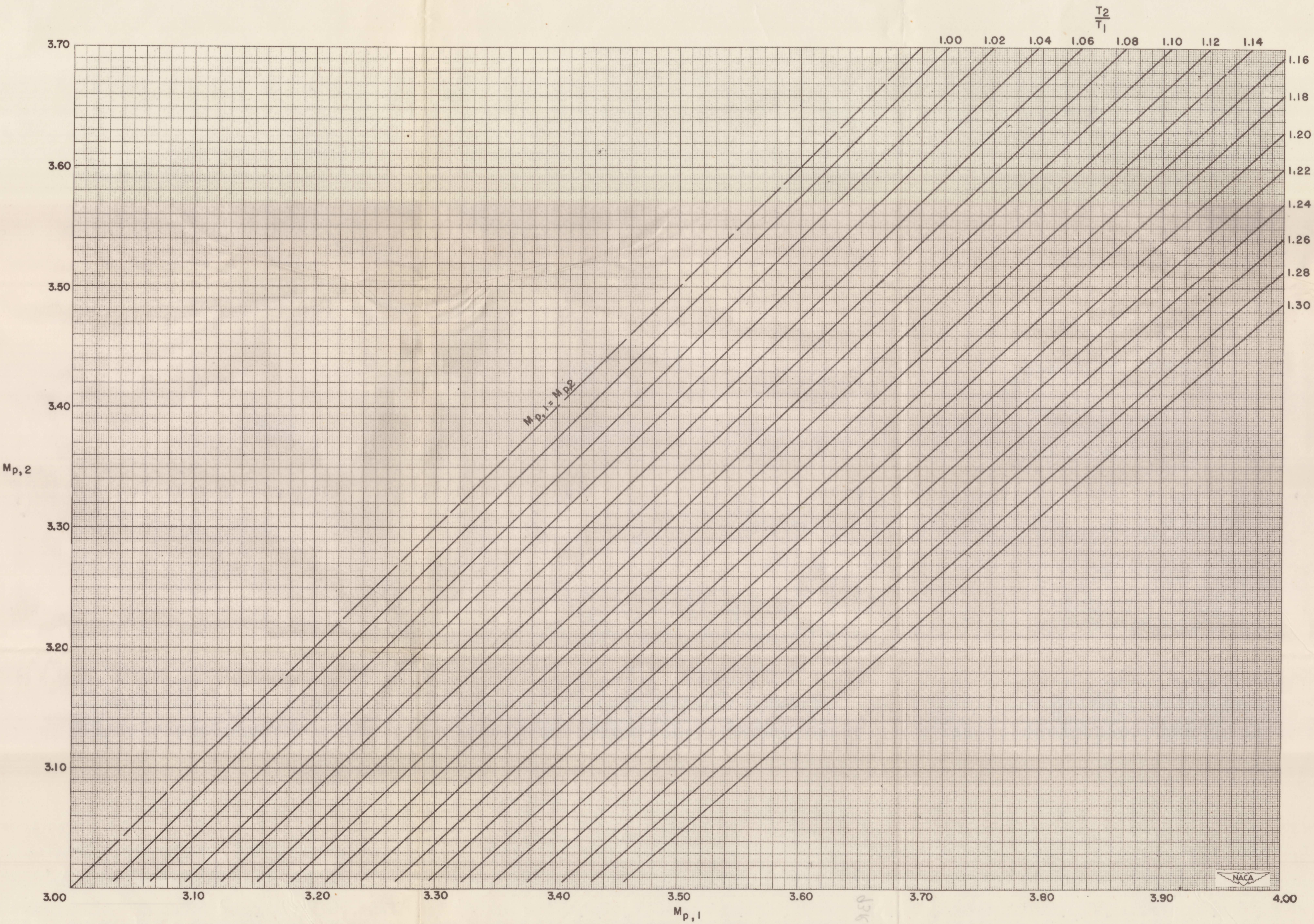

(c) Increment in friction-distance parameter ($F\Delta x/D_e$), 0.04; range of total-momentum parameter at station 2, 3.01 to 3.48.

Figure 1.- Continued. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition ($\gamma = 1.40$).

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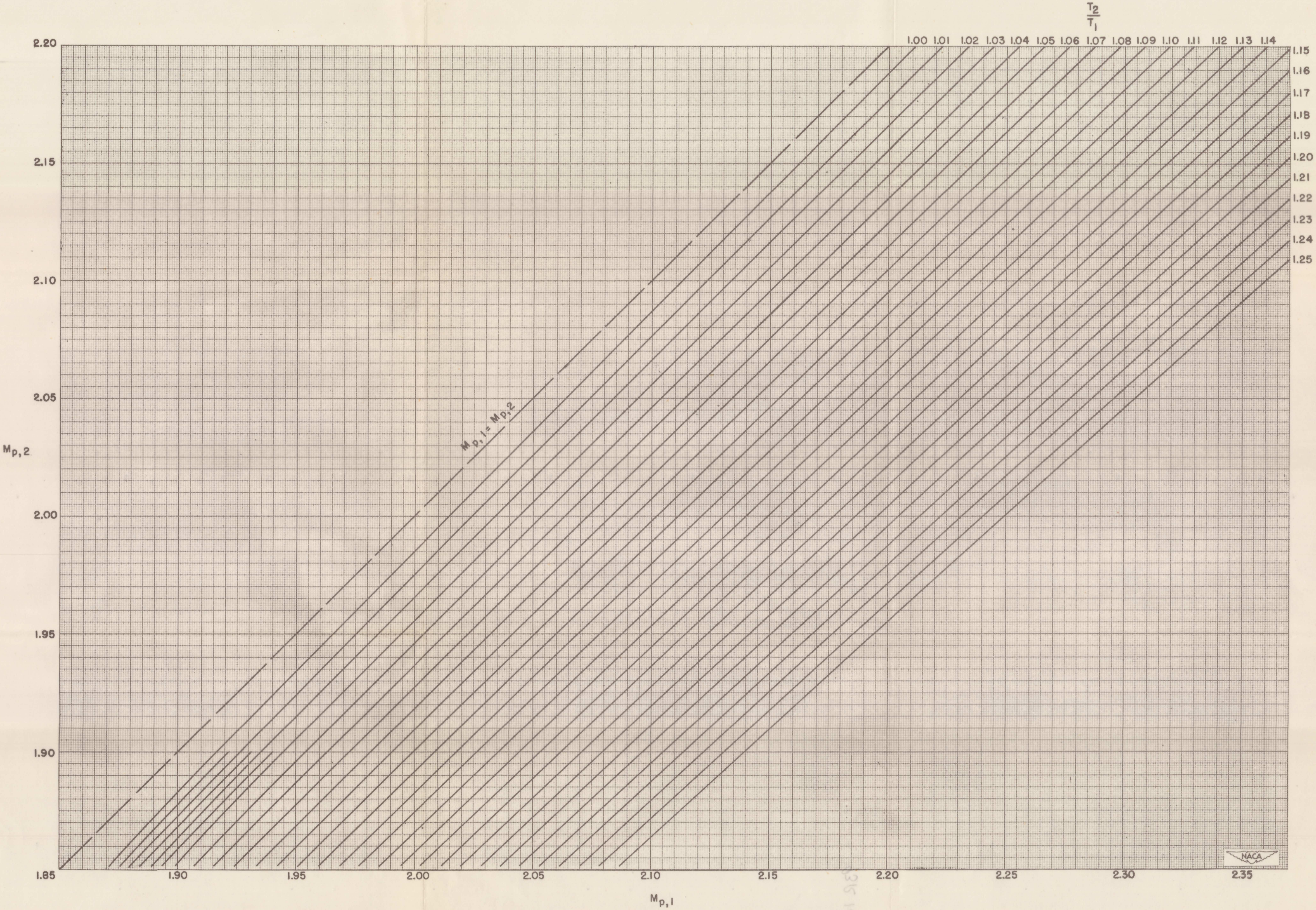

(d) Increment in friction-distance parameter ($F\Delta x/D_e$), 0.01; range of total-momentum parameter at station 2, 1.852 to 2.109.

Figure 1.- Continued. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition ($\gamma = 1.40$).

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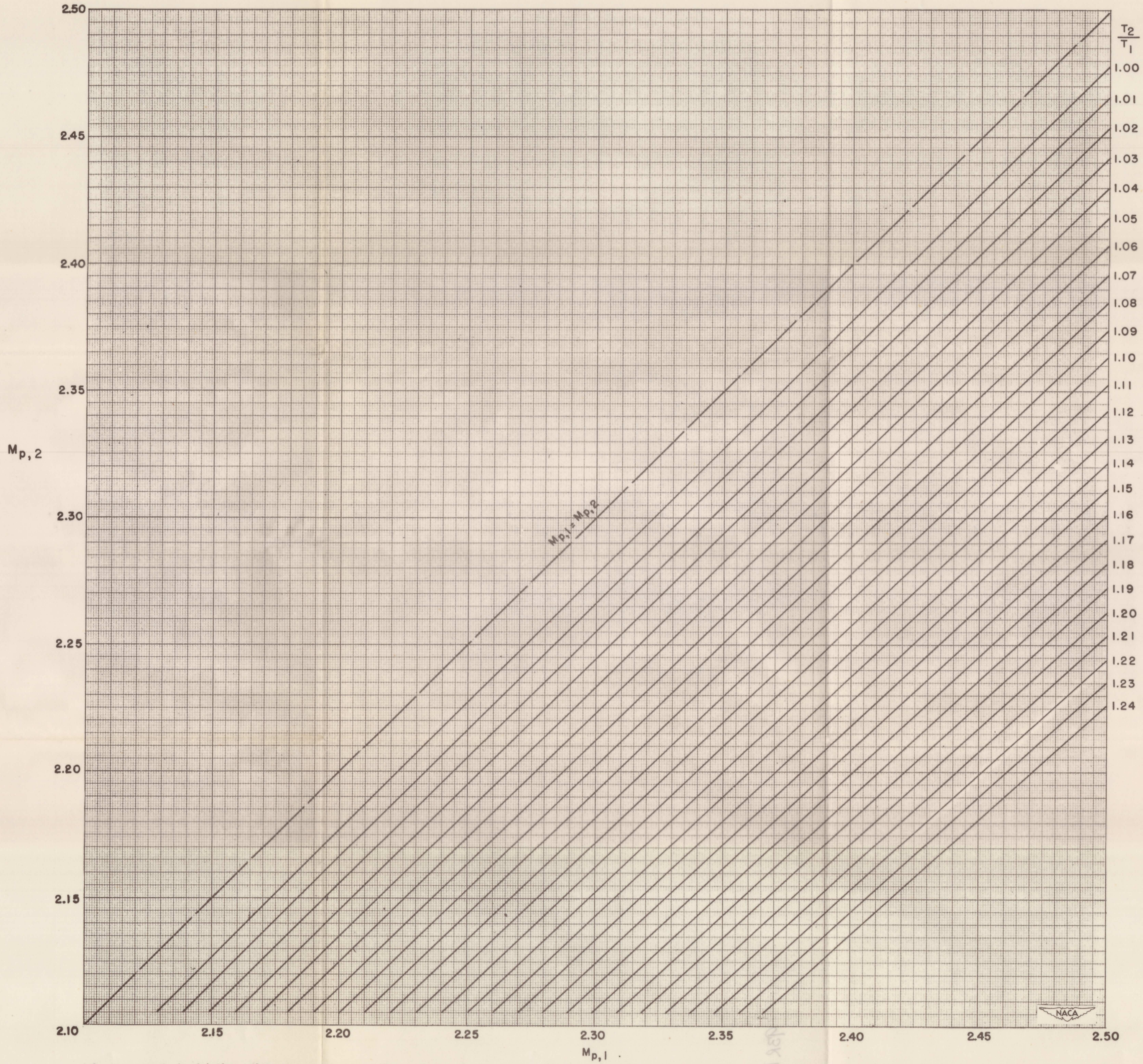

(e) Increment in friction-distance parameter ($F\Delta x/D_e$), 0.01; range of total-momentum parameter at station 2, 2.105 to 2.226.

Figure 1.- Concluded. Charts describing compressible-flow process in a constant-area duct with simultaneous friction and arbitrary heat addition ($\gamma = 1.40$).

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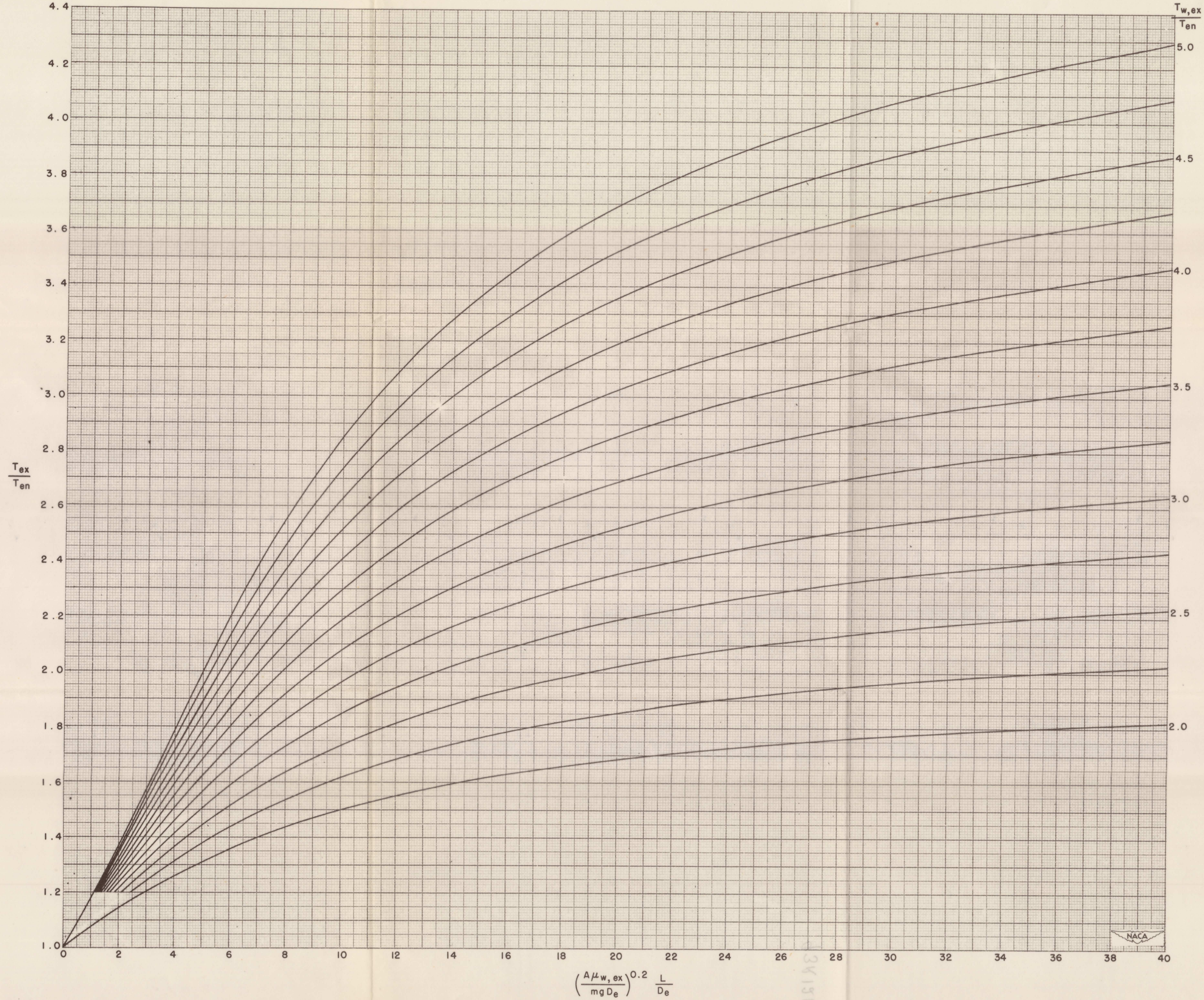

Figure 6.- Variation of ratio of exit fluid temperature to entrance fluid temperature with over-all passage-distance parameter for various ratios of maximum (exit, in this case) wall temperature to entrance fluid temperature. Turbulent flow of air for constant rate of heat-input conditions.

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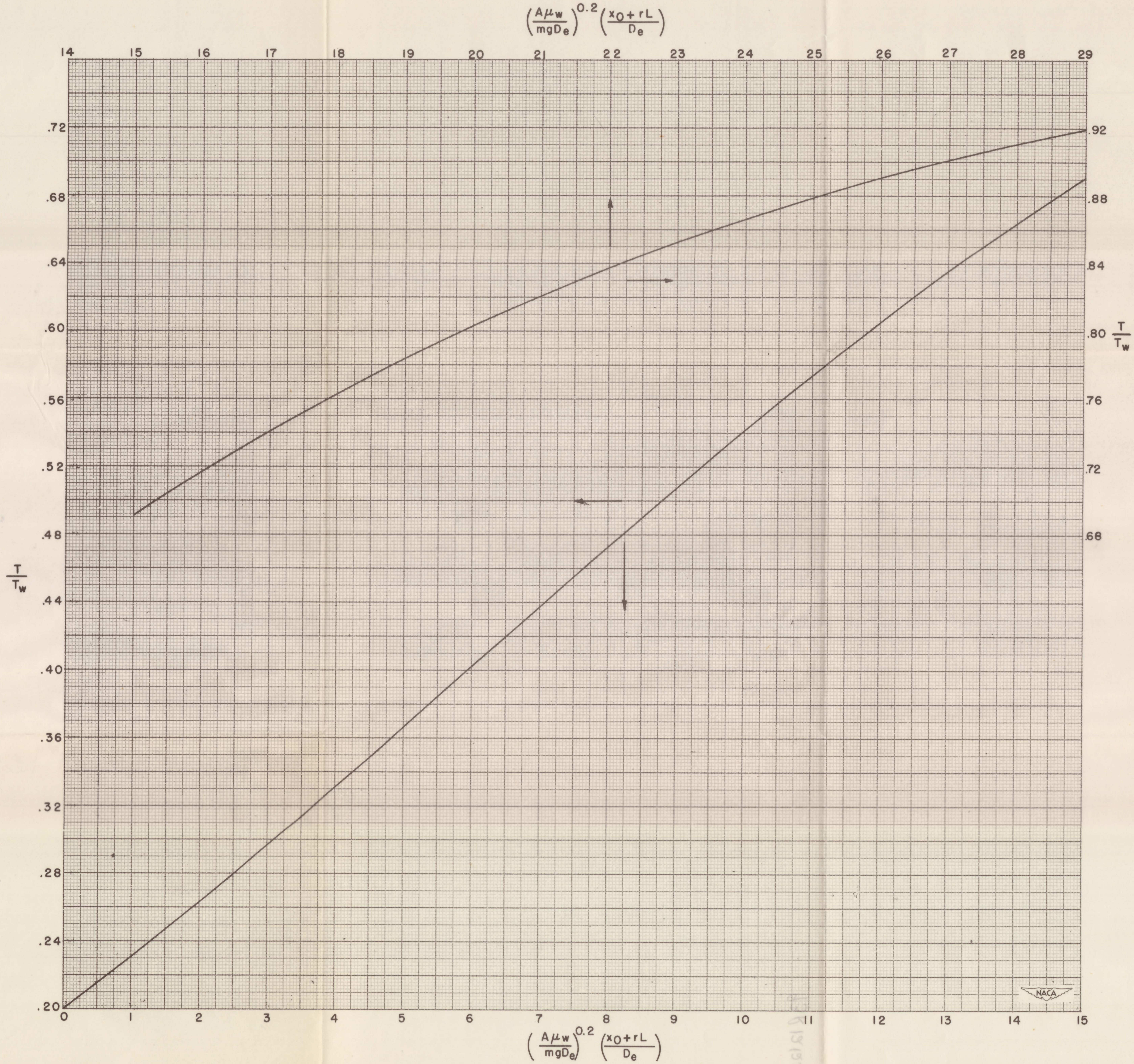

Figure 7. - Variation of ratio of fluid to wall temperature with passage-distance parameter for turbulent flow of air and constant passage-wall temperature conditions.

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