# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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# AERODYNAMIC PROPERTIES OF CRUCIFORM-WING AND BODY COMBINATIONS AT SUBSONIC, TRANSONIC, AND SUPERSONIC SPEEDS

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## SUMMARY

The aerodynamic forces and moments exerted on pitched and yawed wing-body combinations consisting of a slender body of revolution and a cruciform arrangement of thin wings have been investigated by two theoretical methods. One method, an extension of the slender wing-body theory of NACA TN No. 1662, makes possible the determination of simple closed expressions for the load distribution, the forces, and the moments for slender cruciform-wing and body combinations in which the wings may be of dissimilar plan form. The results are applicable at subsonic and transonic speeds, and at supersonic speeds, provided the entire wing-body combination lies near the center of the Mach cone. The second method treats cruciforn-wing and body combinations consisting of a body of revolution and identical wings of arbitrary aspect ratio and plan form. The results of this section are also independent of Mach number.

The main conclusions may be stated as follows: The lift and pitching moment are independent of the angle of yaw, and the side-force and yawing moment are independent of the angle of attack. If the vertical and horizontal wings are identical, the rolling moment is zero for all angles of pitch and yaw.

## INTRODUCTION

A great amount of research is being done on missile configurations utilizing cruciform arrangements of wings and relatively large bodies of revolution. Although the aerodynamic characteristics of the components of such configurations may be well known, the mutual interference resulting from combining the wings, as well as the wings and body, may be so great that it is desirable to study the aerodynamic properties of the complete configurations. Two methods of handling this problem are presented in this report.

The first method is essentially an extension of the theory for slender wing-body combinations of reference 1 to determine the load distribution, forces, and moments exerted on slender cruciform-wing and body combinations inclined simultaneously at small angles in pitch and yaw. Except for specifying that the wings have a pointed leading apex and an unswept trailing edge, the wings are otherwise of arbitrary plan form. Further, the vertical and horizontal wings may be of different size or plan form. The results of this analysis are applicable to combinations having wings of very low aspect ratio at all Mach numbers and to combinations having wings of moderate aspect ratio at Mach numbers near one. Examples are included that illustrate the calculation of the load distribution, the forces, and the moments on several elementary cruciform-wing and body combinations.

The second method, based on symmetry considerations, is capable of treating cruciform-wing and body combinations possessing complete rotational symmetry at any Mach number. For this treatment, the wings may be of completely arbitrary plan form and aspect ratio. The horizontal and vertical wings, however, must be identical and must be mounted at the same longitudinal station of the body. Although no explicit formulas for the aerodynamic loading or for other aerodynamic properties of specific configurations are presented, several general conclusions are found regarding the total forces and moments exerted on any cruciform-wing and body combination possessing rotational symmetry.

## SYMBOLS

$A$ aspect ratio $\left(\frac{\text{span squared}}{\text{area}}\right)$

$B$ cross-section area of body of revolution $(\pi a^2)$

$B_b$ cross-section area of base of body of revolution

$B_m$ mean cross-section area of body of revolution $\left(\frac{\text{volume}}{\text{length}}\right)$

$C_L$ lift coefficient $\left(\frac{L}{qS_H}\right)$

| | |
|---|---|
| $C_{L_\alpha}$ | lift-curve slope $\left(\dfrac{dC_L}{d\alpha}\right)$ |
| $C_l$ | rolling-moment coefficient $\left(\dfrac{L'}{2qS_H s_{max}}\right)$ |
| $C_m$ | pitching-moment coefficient $\left(\dfrac{M}{qS_H c_H}\right)$ |
| $C_n$ | yawing-moment coefficient $\left(\dfrac{N}{qS_V c_V}\right)$ |
| $C_Y$ | side-force coefficient $\left(\dfrac{Y}{qS_V}\right)$ |
| $C_{Y_\beta}$ | rate of change of side force with angle of sideslip $\left(\dfrac{dC_Y}{d\beta}\right)$ |
| $L$ | lift |
| $L'$ | rolling moment |
| $L'_L$ | rolling moment due to lifting forces |
| $L'_Y$ | rolling moment due to side forces |
| $M$ | pitching moment about apex of horizontal wing |
| $M_o$ | free-stream Mach number |
| $N$ | yawing moment about apex of vertical wing |
| $S$ | wing area |
| $V_o$ | free-stream velocity |
| $W$ | complex potential function $(\varphi + i\psi)$ |
| $X$ | complex variable $(y + iz)$ |
| $Y$ | side force |
| $a$ | radius of body |

| | |
|---|---|
| $c$ | maximum wing chord |
| $d$ | semispan of flat plate |
| $i$ | imaginary unit $(\sqrt{-1})$ |
| $l$ | over-all length of wing-body combination |
| $p$ | static pressure |
| $\Delta p_L$ | local pressure difference between lower and upper surfaces $(p_l - p_u)$ |
| $\Delta p_Y$ | local pressure difference between port and starboard surfaces $(p_p - p_s)$ |
| $q$ | free-stream dynamic pressure |
| $r,\theta$ | polar coordinates |
| $s$ | local semispan of horizontal wing |
| $s_{max}$ | maximum semispan of horizontal wing |
| $t$ | local semispan of vertical wing |
| $t_{max}$ | maximum semispan of vertical wing |
| $u$ | perturbation velocity component in the free-stream direction |
| $x,y,z$ | Cartesian coordinates |
| $(x_{c.p.})_L$ | distance from apex of horizontal wing to center of pressure of lifting forces |
| $(x_{c.p.})_Y$ | distance from apex of vertical wing to center of pressure of side forces |
| $\Phi$ | angle of bank |
| $\psi$ | stream function |
| $\alpha$ | angle of attack |
| $\beta$ | angle of sideslip |

$\eta, \zeta$ transformed rectangular coordinates

$\xi$ complex variable $(\eta + i\zeta)$

$\rho$ density of air

$\varphi$ perturbation velocity potential

$\varphi'$ perturbation velocity potential corresponding to unit angles of pitch or yaw

Subscripts

B body

H horizontal wing

V vertical wing

W basic wing without body

a pertains to angle-of-attack case having zero yaw

b pertains to sideslip case having zero angle of attack

$l$ lower side

p port side

s starboard side

u upper side

## ANALYSIS OF SLENDER CRUCIFORM-WING AND BODY COMBINATIONS

### General

The prime problem to be treated in this section is the prediction of the load distribution and aerodynamic properties for slender cruciform-wing and body combinations inclined at small angles of pitch $\alpha$ and yaw $\beta$. (See fig. 1.) The wings must have pointed leading apexes, swept-back leading edges, and unswept trailing edges.

otherwise their plan forms and relative size may be arbitrary. It should be noted that the problem of determining the same characteristics for a slender cruciform-wing and body combination inclined a small angle in pitch $\alpha'$ and banked an angle $\Phi$ is equivalent to the present problem and may be treated by writing $\alpha'\cos\Phi$ and $\alpha'\sin\Phi$ for $\alpha$ and $\beta$ throughout the analysis.

The approach to the problem will be based on the general arguments advanced in references 1, 2, 3, and 4 for slender wings, bodies, and wing-body combinations. The assumptions involved in the study of these configurations permit the Prandtl linearized differential equation for the perturbation velocity potential of a compressible flow in three dimensions

$$(1-M_o^2)\ \varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \tag{1}$$

to be reduced to a particularly simple parabolic differential equation in three dimensions

$$\varphi_{yy} + \varphi_{zz} = 0 \tag{2}$$

which is the two-dimensional Laplace's equation in the transverse yz plane.

The slender body and low-aspect-ratio wing theories (references 2 and 3) neglect the term $(1-M_o^2)\varphi_{xx}$ in comparison with $\varphi_{yy}$ and $\varphi_{zz}$ because $\varphi_{xx}$ is very small for slender wings and bodies. Therefore, the loading and aerodynamic properties of plan forms having very low aspect ratios are independent of Mach number. As pointed out in reference 1 and discussed in greater detail in reference 5, the theory is also valid for swept-back plan forms of moderate aspect ratio at sonic velocity, since the term $(1-M_o^2)\ \varphi_{xx}$ can be neglected because $1-M_o^2$ is zero, provided that $\varphi_{xx}$ does not become very large in comparison with the other velocity gradients $\varphi_{yy}$ and $\varphi_{zz}$. In both the low-aspect-ratio and sonic applications, it should be noted that, with the sole exception of the infinitely long swept wing, it is possible to solve only problems involving the differential pressure arising from an asymmetry of the flow field. Thus, lifting problems may be treated satisfactorily. Thickness problems cannot be handled in general, however, since the theory predicts infinite longitudinal perturbation velocities and hence infinite pressures at all points on the surfaces of wings and bodies other than the infinitely long swept wing, thereby violating the basic assumption of linearized small perturbation theory.

The form of equation (2) permits the analysis to be undertaken as a two-dimensional potential-flow problem at this point; each vertical plane, therefore, may be treated independently of the adjacent planes in the determination of the velocity potential. Thus, the potential is determined for an arbitrary $x = x_o$ plane; then, since the $x_o$ plane may represent any plane normal to the fuselage center line, the potential distribution is known for the entire wing-body combination.

## Velocity Potential

For the problems about to be analyzed, the perturbation velocity field in the $x_o$ plane is similar to the velocity field around an infinitely long cylinder, the cross section of which corresponds to the trace of the wing-body combination in the $x_o$ plane. For a slender cruciform-wing and body combination inclined an angle $\alpha$ in pitch and $\beta$ in yaw, the flow in the $x_o$ plane is as shown in figure 2(a) where the component of the flow velocity at infinity in the z direction is $V_o\alpha$ and that in the y direction is $-V_o\beta$. The flow field of figure 2(a) can obviously be considered to be the sum of the two flow fields shown in figures 2(b) and 2(c), since the vertical wing in figure 2(b) and the horizontal wing in figure 2(c) lie in planes of symmetry and cannot affect the flow shown in their respective figures. Thus the flow fields shown in figures 2(b) and 2(c) are identical to those of figures 2(d) and 2(e), respectively. These component flow fields have been treated in reference 1. Briefly, the derivation of the expression for the velocity potential of the flow field shown in figure 2(d) proceeds as follows:

The transverse flow around a section such as shown in figure 2(b) or 2(d) may be derived from the transverse flow around an infinitely long flat plate (fig. 2(f)), by application of the principles of conformal mapping using the Joukowski transformation. Thus, we consider the mapping of the $\eta\zeta$ plane of figure 2(f) onto the yz plane of figure 2(b) by the relation

$$\xi = X + \frac{a^2}{X} \tag{3}$$

where

$$\xi = \eta + i\zeta$$

and

$$X = y + iz$$

The complex potential function for the flow in the $\eta\zeta$ plane is (see, for instance, reference 6)

$$W_a' = \varphi_a' + i\psi_a' = -iV_o\alpha\sqrt{\xi^2-d^2} \tag{4}$$

where the primed symbols indicate values in the $\eta\zeta$ plane as opposed to the yz plane. If $d = 2a$, the flow around the flat plate expressed by equation (4) transforms by equation (3) into the vertical flow around a circle of radius $a$ having its center at the origin. If $d$ is taken larger than $2a$, the flow transforms into the desired vertical flow around a cylinder consisting of a circular cylinder of radius $a$ with thin flat plates extending outward along the extension of the horizontal diameter to a distance $s$ from the origin. When the $\eta\zeta$ plane is transformed into the yz plane in this manner, the complex potential for the flow in the yz plane is

$$W_a = \varphi_a + i\psi_a = -iV_o\alpha\sqrt{\left(X + \frac{a^2}{X}\right)^2 - d^2} = -iV_o\alpha\sqrt{\left(X + \frac{a^2}{X}\right)^2 - \left(s + \frac{a^2}{s}\right)^2} \tag{5}$$

since the point $d$ in the $\eta\zeta$ plane corresponds to the point $s$ in the yz plane. The velocity potential $\varphi_a$ for the flow in the yz plane may then be determined by squaring equation (5), substituting $X = r(\cos\theta + i\sin\theta)$, and solving. Thus is obtained the expression for the velocity potential of the flow field shown in figure 2(b),

$$\varphi_a = \pm\frac{V_o\alpha}{\sqrt{2}}\sqrt{-\left(1+\frac{a^4}{r^4}\right)r^2\cos 2\theta+s^2\left(1+\frac{a^4}{s^4}\right)+\sqrt{r^4\left(1+\frac{a^8}{r^8}\right)+2a^4\cos 4\theta+s^4\left(1+\frac{a^4}{s^4}\right)^2-2s^2\left(1+\frac{a^4}{s^4}\right)\left(1+\frac{a^4}{r^4}\right)r^2\cos 2\theta}} \tag{6}$$

where the sign is positive in the upper half plane $(0<\theta<\pi)$ and negative in the lower half plane $(\pi<\theta<2\pi)$. The expression for the velocity potential for the flow field shown in figure 2(c) is found in a similar manner to be

$$\varphi_b = \pm\frac{V_o\beta}{\sqrt{2}}\sqrt{\left(1+\frac{a^4}{r^4}\right)r^2\cos 2\theta+t^2\left(1+\frac{a^4}{t^4}\right)+\sqrt{r^4\left(1+\frac{a^8}{r^8}\right)+2a^4\cos 4\theta+t^4\left(1+\frac{a^4}{t^4}\right)^2+2t^2\left(1+\frac{a^4}{t^4}\right)\left(1+\frac{a^4}{r^4}\right)r^2\cos 2\theta}} \tag{7}$$

where the sign is positive in the left half plane $\left(\frac{\pi}{2}<\theta<\frac{3\pi}{2}\right)$ and negative in the right half plane $\left(-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right)$. The perturbation velocity potential for the flow field about a cruciform-wing and body combination inclined in both pitch and yaw (fig. 2(a)) is then given by

$$\varphi = \varphi_a + \varphi_b \tag{8}$$

## Load Distribution

The differential pressure coefficient in linearized potential flow is given by

$$\frac{\Delta p}{q} = \frac{4u}{V_o} \tag{9}$$

where u is the component of the perturbation velocity in the free-stream direction. Since body axes are used in the present treatment rather than wind axes, the component of the perturbation velocity in the free-stream direction will be approximated by

$$u = \frac{\partial\varphi}{\partial x} - \beta\,\frac{\partial\varphi}{\partial y} + \alpha\,\frac{\partial\varphi}{\partial z} \tag{10}$$

The last two terms of equation (10) are often omitted since they each represent the product of a small angle and a small perturbation velocity and are generally much smaller than the first term of the right-hand side, which represents only a perturbation velocity. For the long slender wings, bodies, and wing-body combinations considered here, however, $\frac{\partial\varphi}{\partial x}$ is much smaller than $\frac{\partial\varphi}{\partial y}$ and $\frac{\partial\varphi}{\partial z}$; therefore, all three terms are retained. The pressure coefficient is then given by

$$\frac{\Delta p}{q} = \frac{4}{V_o}\left(\frac{\partial\varphi}{\partial x} - \beta\,\frac{\partial\varphi}{\partial y} + \alpha\,\frac{\partial\varphi}{\partial z}\right) \tag{11}$$

Through application of equations (8) and (11) expressions for the lifting differential pressure (lower minus upper) on the horizontal wing and body are found to be, respectively,

$$\left(\frac{\Delta p_L}{q}\right)_H = 4\alpha\left\{\frac{\left[\frac{ds}{dx}\left(1-\frac{a^4}{s^4}\right) + 2\,\frac{a}{s}\,\frac{da}{dx}\left(\frac{a^2}{s^2}-\frac{a^2}{y^2}\right)\right] + \left[\beta\,\frac{y}{s}\left(1-\frac{a^4}{y^4}\right)\right]}{\sqrt{\left(1+\frac{a^4}{s^4}\right) - \frac{y^2}{s^2}\left(1+\frac{a^4}{y^4}\right)}}\right\} \tag{12}$$

$$\left(\frac{\Delta p_L}{q}\right)_B = 4\alpha\left\{\frac{\left[\frac{ds}{dx}\left(1-\frac{a^4}{s^4}\right) + 2\frac{a}{s}\,\frac{da}{dx}\left(\frac{a^2}{s^2}+1-\frac{2y^2}{a^2}\right)\right] + \left[4\beta\frac{y}{s}\left(1-\frac{y^2}{a^2}\right)\right]}{\sqrt{\left(1+\frac{a^2}{s^2}\right)^2 - 4\,\frac{y^2}{s^2}}}\right\} \pm \frac{16\,\alpha\beta\left(\frac{y^2}{at}\right)\sqrt{1-\frac{y^2}{a^2}}}{\sqrt{\left(1-\frac{a^2}{t^2}\right)^2 + 4\,\frac{y^2}{t^2}}} \tag{13}$$

where in equation (13), the plus sign is taken for the starboard side of the body and the minus sign for the port side. Similarly the yawing differential pressure (port minus starboard) on the vertical wing and body are given, respectively, by

$$\left(\frac{\Delta p_Y}{q}\right)_V = 4\beta\left\{\frac{-\left[\frac{dt}{dx}\left(1-\frac{a^4}{t^4}\right)+2\,\frac{a}{t}\,\frac{da}{dx}\left(\frac{a^2}{t^2}-\frac{a^2}{z^2}\right)\right]+\left[\alpha\,\frac{z}{t}\left(1-\frac{a^4}{z^4}\right)\right]}{\sqrt{\left(1+\frac{a^4}{t^4}\right)-\frac{z^2}{t^2}\left(1+\frac{a^4}{z^4}\right)}}\right\} \tag{14}$$

$$\left(\frac{\Delta p_Y}{q}\right)_B = 4\beta\left\{\frac{-\left[\frac{dt}{dx}\left(1-\frac{a^4}{t^4}\right)+2\,\frac{a}{t}\,\frac{da}{dx}\left(\frac{a^2}{t^2}+1-2\,\frac{z^2}{a^2}\right)\right]+\left[4\alpha\,\frac{z}{t}\left(1-\frac{z^2}{a^2}\right)\right]}{\sqrt{\left(1+\frac{a^2}{t^2}\right)^2-4\,\frac{z^2}{t^2}}}\right\} \pm \frac{16\,\alpha\beta\left(\frac{z^2}{as}\right)\sqrt{1-\frac{z^2}{a^2}}}{\sqrt{\left(1-\frac{a^2}{s^2}\right)^2+4\,\frac{z^2}{s^2}}} \tag{15}$$

where, in equation (15), the plus sign is taken for the upper side of the body and the minus sign for the lower side. It should be noted that the expressions for the pressures on the body are not valid for stations behind the trailing edge of the more forward wing since the influence of the downwash field behind the wings has not been considered in the analysis.

## Total Forces and Moments

The total forces and moments exerted on a complete cruciform-wing and body combination may be determined by integrating the loading over the entire surface area. It is convenient to carry out the integration by first evaluating the forces and moments on one transverse strip and then integrating these elemental quantities over the length of the wing-body combination. The lift and side force on a transverse strip of width $dx$ are given, respectively, by

$$dL = q\ dx \int_{-s}^{+s} \left(\frac{\Delta p_L}{q}\right) dy \tag{16}$$

$$dY = q\ dx \int_{-t}^{+t} \left(\frac{\Delta p_Y}{q}\right) dz \tag{17}$$

The rolling moment on this elemental strip is given by

$$dL' = q\ dx \left[ -\int_{-s}^{-a} y \left(\frac{\Delta p_L}{q}\right)_H dy - \int_{a}^{s} y \left(\frac{\Delta p_L}{q}\right)_H dy + \int_{-t}^{-a} z \left(\frac{\Delta p_Y}{q}\right)_V dz + \int_{a}^{t} z \left(\frac{\Delta p_Y}{q}\right)_V dz \right] \tag{18}$$

where the integration is carried only over the surface of the wing since pressures on the body cannot produce a rolling moment. When the indicated operations are performed, the following expressions for the elemental lift, side force, and rolling moment are obtained:

$$\frac{d}{dx}\left(\frac{L}{q}\right) = 4\pi\alpha s \left[\frac{ds}{dx}\left(1 - \frac{a^4}{s^4}\right) + \frac{da}{dx}\left(2\frac{a^3}{s^3}\right)\right] + 2\alpha s \frac{da}{dx}\left[2\left(1 - \frac{a^2}{s^2}\right) - \frac{s}{a}\left(1 + \frac{a^2}{s^2}\right)^2 \sin^{-1}\frac{2\frac{a}{s}}{1 + \frac{a^2}{s^2}}\right] \tag{19}$$

$$\frac{d}{dx}\left(\frac{Y}{q}\right) = -4\pi\beta t \left[\frac{dt}{dx}\left(1 - \frac{a^4}{t^4}\right) + \frac{da}{dx}\left(2\frac{a^3}{t^3}\right)\right] - 2\beta t \frac{da}{dx}\left[2\left(1 - \frac{a^2}{t^2}\right) - \frac{t}{a}\left(1 + \frac{a^2}{t^2}\right)^2 \sin^{-1}\frac{2\frac{a}{t}}{1 + \frac{a^2}{t^2}}\right] \tag{20}$$

$$\begin{aligned} \frac{d}{dx}\left(\frac{L'}{q}\right) = {} & 2\alpha\beta t^2\left[2\frac{a}{t}\left(1 - \frac{a^2}{t^2}\right) + \pi\left(1 + \frac{a^4}{t^4}\right) - \left(1 + \frac{a^2}{t^2}\right)^2 \cos^{-1}\frac{t^2 - a^2}{t^2 + a^2}\right] - \\ & 2\alpha\beta s^2\left[2\frac{a}{s}\left(1 - \frac{a^2}{s^2}\right) + \pi\left(1 + \frac{a^4}{s^4}\right) - \left(1 + \frac{a^2}{s^2}\right)^2 \cos^{-1}\frac{s^2 - a^2}{s^2 + a^2}\right] \end{aligned} \tag{21}$$

The forces, moments, and center of pressure for the complete wing-body combination may now be determined by integration of the forces on all the elemental strips. Expressed in nondimensional form, these characteristics are given by

$$C_L = \frac{1}{S_H}\int \frac{d}{dx}\left(\frac{L}{q}\right)dx \tag{22}$$

$$C_m = \frac{-1}{S_H c_H}\int \frac{d}{dx}\left(\frac{L}{q}\right)x\ dx \tag{23}$$

$$\left(\frac{x_{c.p.}}{c_H}\right)_L = -\frac{C_m}{C_L} \tag{24}$$

$$C_Y = \frac{1}{S_V}\int \frac{d}{dx}\left(\frac{Y}{q}\right)dx \tag{25}$$

$$C_n = \frac{-1}{S_V c_V}\int \frac{d}{dx}\left(\frac{Y}{q}\right)x\ dx \tag{26}$$

$$\left(\frac{x_{c.p.}}{c_V}\right)_Y = -\frac{C_n}{C_Y} \tag{27}$$

$$C_l = \frac{1}{2s_{max}S_H}\int \frac{d}{dx}\left(\frac{L'}{q}\right)dx \tag{28}$$

where the integration interval extends from the most forward to the most rearward part of the wing-body combination. To achieve a unity of expression for side force and lift and for yawing moment and pitching moment, the side-force coefficients have been based upon the area of the vertical wing rather than the more conventional horizontal-wing area, and the yawing-moment coefficients have been based upon the area and root chord of the vertical wing rather than the area and span of the horizontal wing.

The expressions for the forces and moments on the elemental strips (equations (19), (20), and (21)) indicate four important general characteristics of slender cruciform-wing and body combinations. First of all, there is a complete correspondence of the expressions for the lift and side force as would be expected from the geometry of the configuration. Second, the lift is independent of the angle of yaw and the side force is independent of the angle of attack. Third, the expressions for the lift and pitching moment for slender cruciform-wing and body combinations are identical to those given in reference 1 for slender plane-wing and body combinations. Last, if the vertical and horizontal wings are identical, the rolling moment is zero.

It might be noted at this time that the value of zero for the rolling moment in the case of identical wings results from a complete balancing of the rolling moment exerted on the horizontal wing by an equal but opposite rolling moment on the vertical wing rather than by having zero rolling moment on each wing. Since such a complete balancing may be easily disturbed by factors neglected in the analysis (for instance, higher-order terms neglected in the analysis or separation along the wing-body junction), particularly at large angles of inclination, the pitch and yaw range over which this conclusion is expected to apply may be more limited than that of the conclusions regarding lift and side force.

Several applications of the foregoing theory are presented in detail in the appendix.

## ANALYSIS OF CRUCIFORM-WING AND BODY COMBINATIONS HAVING IDENTICAL WINGS

An analysis of some of the aerodynamic properties of cruciform-wing and body combinations having identical wings may be undertaken on the basis of symmetry considerations. For this treatment the wings may be of any plan form or aspect ratio, provided the vertical and horizontal wings are identical and are mounted at the same longitudinal position of the body. The concepts of linearized theory are used in this treatment; therefore, the usual restrictions that the body is slender and that the angles of pitch and yaw are small must be observed. The conclusions are applicable at all speeds, since the Mach number does not enter the problem directly. Consider the cruciform-wing and body combinations as being inclined small angles $\alpha$ in pitch and $\beta$ in yaw from the free-stream direction, the free-stream velocity being $V_0$. Since superposition is a valid principle in linearized theory, the perturbation velocity potential $\varphi$ may be considered to be the sum of the two components

$$\varphi = \alpha\varphi'_a + \beta\varphi'_b \tag{29}$$

where $\varphi'_a$ and $\varphi'_b$ are the perturbation velocity potentials of the flow about a cruciform-wing and body combination inclined unit angles of pitch and yaw, respectively.

Consider now the differential pressure between corresponding points on the upper and lower surfaces of the body and the horizontal wing

$$\frac{\Delta p_L}{q} = \frac{2}{V_o}\left[\left\{\alpha\left[\left(\frac{\partial\varphi'_a}{\partial x}\right)_u - \left(\frac{\partial\varphi'_a}{\partial x}\right)_l\right] + \beta\left[\left(\frac{\partial\varphi'_b}{\partial x}\right)_u - \left(\frac{\partial\varphi'_b}{\partial x}\right)_l\right]\right\} + \right.$$
$$\beta\left\{\alpha\left[\left(\frac{\partial\varphi'_a}{\partial y}\right)_l - \left(\frac{\partial\varphi'_a}{\partial y}\right)_u\right] + \beta\left[\left(\frac{\partial\varphi'_b}{\partial y}\right)_l - \left(\frac{\partial\varphi'_b}{\partial y}\right)_u\right]\right\} +$$
$$\left.\alpha\left\{\alpha\left[\left(\frac{\partial\varphi'_a}{\partial z}\right)_u - \left(\frac{\partial\varphi'_a}{\partial z}\right)_l\right] + \beta\left[\left(\frac{\partial\varphi'_b}{\partial z}\right)_u - \left(\frac{\partial\varphi'_b}{\partial z}\right)_l\right]\right\}\right] \qquad (30)$$

Ordinarily, only the potential gradients, or perturbation velocities, in the x direction would be included in equation (30). For long slender objects, however, the perturbation velocities in the x direction are so much smaller than those in the y and z directions that the products of small angles and perturbation velocities in the y and z directions must be retained as well as the perturbation velocities in the x direction. Since the inclusion of these terms does not introduce any particular restrictions into the problem, they will be retained throughout the present discussion even though in many instances, such as with high-aspect-ratio unswept wings, it is unnecessary to do so. In general, none of the individual terms in equation (30) is zero and the pressure at every point depends upon both $\varphi'_a$ and $\varphi'_b$, or, what is equivalent, upon both the angle of attack and the angle of yaw. However, several of the terms are equal and will cancel. Thus, remembering that equation (30) represents the lifting differential pressure, it is apparent from symmetry considerations that

$$\left(\frac{\partial\varphi'_b}{\partial x}\right)_u - \left(\frac{\partial\varphi'_b}{\partial x}\right)_l = 0 \qquad \left(\frac{\partial\varphi'_b}{\partial y}\right)_l - \left(\frac{\partial\varphi'_b}{\partial y}\right)_u = 0 \qquad \left(\frac{\partial\varphi'_a}{\partial z}\right)_u - \left(\frac{\partial\varphi'_a}{\partial z}\right)_l = 0$$

everywhere and that

$$\left(\frac{\partial\varphi'_b}{\partial z}\right)_u = \left(\frac{\partial\varphi'_b}{\partial z}\right)_l = 0$$

on the horizontal wing. Therefore, equation (30) simplifies to

$$\frac{\Delta p_L}{q} = \frac{2}{V_o}\left\{\alpha \underbrace{\left[\left(\frac{\partial\varphi'_a}{\partial x}\right)_u - \left(\frac{\partial\varphi'_a}{\partial x}\right)_l\right]}_{1} + \alpha\beta \underbrace{\left[\left(\frac{\partial\varphi'_a}{\partial y}\right)_l - \left(\frac{\partial\varphi'_a}{\partial y}\right)_u\right]}_{2} + \alpha\beta \underbrace{\left[\left(\frac{\partial\varphi'_b}{\partial z}\right)_u - \left(\frac{\partial\varphi'_b}{\partial z}\right)_l\right]}_{3}\right\} \tag{31}$$

where the third bracketed term differs from zero only on the body. The second and third bracketed terms are odd in $y$; hence, they cannot contribute to the lift. The lift is therefore evaluated by integrating the first bracketed term over the entire projected area of the wing-body combination.

$$L = \rho V_o \alpha \iint_{B,H} \left[\left(\frac{\partial\varphi'_a}{\partial x}\right)_u - \left(\frac{\partial\varphi'_a}{\partial x}\right)_l\right] dx\, dy \tag{32}$$

The pitching moment and longitudinal center-of-pressure position are given similarly by

$$M = -\rho V_o \alpha \iint_{B,H} \left[ \left( \frac{\partial \varphi'_a}{\partial x} \right)_u - \left( \frac{\partial \varphi'_a}{\partial x} \right)_l \right] x \, dx \, dy \tag{33}$$

$$\left( \frac{x_{c.p.}}{c_H} \right)_L = \frac{-M}{Lc_H} \tag{34}$$

An expression for the rolling moment due to lifting differential pressures may be obtained in a similar manner. The first and third bracketed terms in equation (31) need not be considered; the first because it is even in $y$, the third because it is different from zero only on the surface of the body. Hence, the rolling moment due to the lifting differential pressure is given by

$$L'_L = -\rho V_o \alpha \beta \iint_H \left[ \left( \frac{\partial \varphi'_a}{\partial y} \right)_l - \left( \frac{\partial \varphi'_a}{\partial y} \right)_u \right] y \, dx \, dy \tag{35}$$

where the integration is carried over only the area of the horizontal wing.

In a similar manner, expressions for the side force, yawing moment, and rolling moment may be developed from the differential pressure between corresponding points on the port and starboard sides of the wing-body combination, thus

$$Y = \rho V_o \beta \iint_{B,V} \left[ \left( \frac{\partial \varphi'_b}{\partial x} \right)_p - \left( \frac{\partial \varphi'_b}{\partial x} \right)_s \right] dy \, dz \tag{36}$$

$$N = -\rho V_o \beta \iint_{B,V} \left[ \left( \frac{\partial \varphi'_b}{\partial x} \right)_p - \left( \frac{\partial \varphi'_b}{\partial x} \right)_s \right] x \, dy \, dz \tag{37}$$

$$\left( \frac{x_{c.p.}}{c_V} \right)_Y = \frac{-N}{Yc_V} \tag{38}$$

$$L'_Y = \rho V_o \alpha \beta \iint_V \left[ \left( \frac{\partial \varphi'_b}{\partial z} \right)_s - \left( \frac{\partial \varphi'_b}{\partial z} \right)_p \right] z \, dy \, dz \tag{39}$$

where the integration is carried over the projected area of the wing-body combination in equations (36) and (37) and only over the area of the vertical wing in equation (39).

The total rolling moment is

$$L' = L'_Y + L'_L = \rho V_0 \alpha\beta \left\{ \iint_V \left[ \left( \frac{\partial \varphi'_b}{\partial z} \right)_s - \left( \frac{\partial \varphi'_b}{\partial z} \right)_p \right] z \, dy \, dz - \iint_H \left[ \left( \frac{\partial \varphi'_a}{\partial y} \right)_l - \left( \frac{\partial \varphi'_a}{\partial y} \right)_u \right] y \, dx \, dy \right\} = 0 \tag{40}$$

since it can be seen that the two integrals have identical values because the flows they represent are identical save for orientation in the coordinate system.

From examination of equations (32) through (40), it may be seen that the aerodynamic properties of cruciform-wing and body combinations having identical vertical and horizontal wings of arbitrary plan form and aspect ratio may be summarized in the following statements. The lift and pitching moment are independent of the angle of yaw and the side force and yawing moment are independent of the angle of attack. Further, the rolling moment is zero for all combinations of angles of pitch and yaw. For the corresponding problem relating to a cruciform-wing and body combination inclined in pitch and bank, the conclusions may be restated as follows: The lift and longitudinal center-of-pressure position are independent of the angle of bank and the rolling moment is zero for all angles of bank.

## CONCLUSIONS

The forces and moments exerted on pitched and yawed wing-body combinations consisting of a slender body of revolution and a cruciform arrangement of thin wings have been investigated by two theoretical methods.

One method made possible the determination of simple closed expressions for the load distribution, the forces, and the moments for slender cruciform-wing and body combinations of which the wings may be of dissimilar plan form. These results indicate four general characteristics of slender cruciform-wing and body combinations. First, there is a complete correspondence of the expressions for the lift and side force. Second, the lift is independent of the angle of yaw and the side force is independent of the angle of attack. Third, the expressions for the lift and pitching moment for slender cruciform-wing and body combinations are identical to those given in NACA TN No. 1662 for slender plane-wing and body combinations. Last, if the vertical and horizontal wings are identical, the rolling moment is zero.

The second method treats cruciform-wing and body combinations consisting of a body of revolution and identical wings of arbitrary aspect ratio and plan form. Although explicit formulas for the aerodynamic properties are not given in terms of the physical dimensions of the wings and body, the following general relationships are found for all Mach numbers for such cruciform-wing and body combinations inclined simultaneously in pitch and yaw. The lift and pitching moment are independent of the angle of yaw, and the side force is independent of the angle of attack. The rolling moment is zero for all angles of pitch and yaw. For the corresponding problem relating to cruciform-wing and body combinations inclined in pitch and bank these conclusions may be restated in the following manner. The lift and pitching moment are independent of the angle of bank and the rolling moment is zero for all angles of bank.

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## APPENDIX

## EXAMPLES OF SLENDER CRUCIFORM-WING AND BODY COMBINATIONS

For any given cruciform-wing and body combination complying with the general requirements of the slender-body theory presented in the first part of this note, the load distribution may be determined directly by substituting the proper values for the body radius and wing semispan and their rate of change with $x$ into equations (12), (13), (14), and (15). By using these expressions for the load distribution, closed expressions for the forces and moments for several elementary configurations may be readily found by simple integration of the integrals indicated by equations (22) through (28). The discussion will be brief since the present results are similar to those given in reference 1.

## Pointed Low-Aspect-Ratio Wings, No Body

The first and simplest example of a slender cruciform configuration to be considered consists of a set of pointed low-aspect-ratio wings, which may have different plan forms and aspect ratios, and no body. The aerodynamic properties of such a configuration may be determined by letting

$$\frac{a}{s} = \frac{a}{t} = 0 \qquad \frac{da}{dx} = 0$$

By substitution of these values into equations (12) and (14), it follows that the load distributions for the horizontal and vertical wings are given by

$$\left(\frac{\Delta p_L}{q}\right)_H = \frac{4\alpha\left(\frac{ds}{dx} + \beta\frac{y}{s}\right)}{\sqrt{1 - \frac{y^2}{s^2}}} \qquad \left(\frac{\Delta p_Y}{q}\right)_V = \frac{4\beta\left(-\frac{dt}{dx} + \alpha\frac{z}{t}\right)}{\sqrt{1 - \frac{z^2}{t^2}}} \tag{A1}$$

These expressions are similar to that given by Ribner (reference 4) for the loading on a single low-aspect-ratio triangular wing inclined in pitch and yaw. The symmetric first terms contribute to lift and side force; the antisymmetric second terms contribute to rolling moments. To illustrate this point further, figure 3 has been prepared showing the load distribution on a cruciform arrangement of triangular wings. The loading on the vertical wing is shown by the two top sketches while that on the horizontal wing is shown by the lower sketches. The sketches on the left represent the contribution of the symmetric first terms of equation (A1); those on the right the contribution of the antisymmetric second terms. In accordance with the stated assumptions, these expressions are invalid when either the angle of pitch or yaw becomes so large that the leading edge passes beyond the stream direction and becomes, effectively, a trailing edge. Mathematically expressed, the expressions are valid when $|\beta| \leq ds/dx$ and $|\alpha| \leq dt/dx$. If it is desired to investigate wings inclined at larger angles, consideration must be given to the influence of the trailing vortices lying outboard of one of the sides of the wing. Such a problem may be treated by an extension of the methods employed in the treatment of a swept-back wing in reference 5. The total load on an elemental strip is found from equations (19) and (20) to be

$$\frac{d}{dx}\left(\frac{L}{q}\right)q = 4\pi\alpha qs\,\frac{ds}{dx} \qquad \frac{d}{dx}\left(\frac{Y}{q}\right)q = -4\pi\beta qt\,\frac{dt}{dx} \tag{A2}$$

The rolling moments exerted on the horizontal and vertical wings are given, respectively, by the corresponding terms of equation (21)

$$\frac{d}{dx}\left(\frac{L'}{q}\right)_H q = -2\pi\alpha\beta qs^2 \qquad \frac{d}{dx}\left(\frac{L'}{q}\right)_V q = 2\pi\alpha\beta qt^2 \tag{A3}$$

The lift and side-force coefficients for the cruciform wing are found by integration of the forces on the elemental strips between the leading apex and the trailing edge as indicated by substituting equation (A2) into equations (22) and (25)

$$C_{L_W} = \frac{1}{S_H}\int_0^{c_H} 4\pi\alpha s\,\frac{ds}{dx}\,dx = \frac{\pi}{2}A_H\alpha \qquad C_{Y_W} = -\frac{\pi}{2}A_V\beta \tag{A4}$$

where $A_H$ and $A_V$ are the aspect ratios of the horizontal and vertical wings, respectively.

Similarly, the pitching-, yawing-, and rolling-moment coefficients are found by substituting equation (A2) into equations (23), (26), and (28), respectively, and integrating:

$$\begin{aligned} C_{m_W} &= -\frac{1}{S_H c_H}\int_0^{c_H} 4\pi\alpha s\,\frac{ds}{dx}\,x\,dx = -\frac{\pi}{2}\alpha\left[\frac{4s^2_{max}}{S_H} - \frac{4(s^2)_m}{S_H}\right] \\ &= -\frac{\pi}{2}A_H\alpha\left[1 - \frac{(s^2)_m}{s^2_{max}}\right] \qquad C_{n_W} = \frac{\pi}{2}A_V\beta\left[1 - \frac{(t^2)_m}{t^2_{max}}\right] \end{aligned} \tag{A5}$$

$$C_{l_W} = \frac{1}{2s_{max}S_H}\left[\int_0^{c_V} 2\pi\alpha\beta t^2 dx - \int_0^{c_H} 2\pi\alpha\beta s^2 dx\right]$$

$$= \frac{\pi\alpha\beta}{S_H s_{max}}\left[c_V(t^2)_m - c_H(s^2)_m\right] \tag{A6}$$

where

$$(s^2)_m = \frac{1}{c_H}\int_0^{c_H} s^2 dx \qquad (t^2)_m = \frac{1}{c_V}\int_0^{c_V} t^2 dx$$

Attention is called to the fact that the pitching- and yawing-moment coefficients represent moments about the leading apexes and are nondimensionalized through use of the area and root chord of the horizontal and vertical wings, respectively.

For a more specific example, consider the wings to be of triangular plan form moving point foremost. Then, since $(s^2)_m = \frac{1}{3} s^2{}_{max}$ and $(t^2)_m = \frac{1}{3} t^2{}_{max}$, the moment coefficients given by equations (A5) and (A6) are

$$C_m = -\frac{\pi}{3} A_H \alpha \qquad C_n = \frac{\pi}{3} A_V \beta \tag{A7}$$

$$C_l = \frac{\pi\alpha\beta}{3}\left(\frac{S_V t_{max}}{S_H s_{max}} - 1\right) \tag{A8}$$

Pointed Slender Body of Revolution

Expressions for the aerodynamic characteristics of a pointed slender body of revolution may be derived from the previously derived equations by letting

$$\frac{a}{s} = 1 \qquad \frac{da}{dx} = \frac{ds}{dx} = \frac{dt}{dx}$$

Since these results have been presented many times (see, for instance, references 1, 2, 7, and 8) and are well known, only the final equations will be listed

$$\left(\frac{\Delta p_L}{q}\right)_B = 8\alpha \frac{da}{dx}\sqrt{1-\frac{y^2}{a^2}} \qquad \left(\frac{\Delta p_Y}{q}\right)_B = -8\beta \frac{da}{dx}\sqrt{1-\frac{y^2}{a^2}} \tag{A9}$$

$$\frac{d}{dx}\left(\frac{L}{q}\right) q = 4\pi\alpha q a \frac{da}{dx} = 2\alpha q \frac{dB}{dx} \qquad \frac{d}{dx}\left(\frac{Y}{q}\right) q = -2\beta q \frac{dB}{dx} \tag{A10}$$

$$C_L = \frac{1}{B_b}\int_0^l 2\alpha\left(\frac{dB}{dx}\right) dx = 2\alpha \qquad C_Y = -2\beta \tag{A11}$$

$$\frac{x_{c.p.}}{l} = \frac{1}{l}\,\frac{\int_0^l (dB/dx)x\,dx}{\int_0^l (dB/dx)\,dx} = 1-\frac{B_m}{B_b} \tag{A12}$$

where $B$ is the local cross-section area, $B_b$ is the area of the base, and $B_m$ is the mean cross-section area (i.e., the volume of the body divided by the length). In the coefficients, the reference area is taken as the area of the base cross section.

For a more specific example, consider a cone moving point foremost. The relationship between the base area and the mean cross-section area is given by

$$B_m = \frac{1}{3} B_b$$

The center-of-pressure position is thus seen to be at the two-thirds point as would be anticipated by the conical nature of the load distribution for this case

$$\frac{x_{c.p.}}{l} = \frac{2}{3}$$

## Triangular Wings With Conical Body

The first example of a wing-body combination to be considered is a conical arrangement (coincident vertices) of a conical body and triangular vertical and horizontal wings of, in general, different aspect ratio. The geometry of such a configuration requires that

$$\frac{a}{s} = \frac{da/dx}{ds/dx} = k_H \qquad \frac{a}{t} = \frac{da/dx}{dt/dx} = k_V$$

where $da/dx$, $ds/dx$, and $dt/dx$ are constants. If these values are substituted in equations (12), (13), (14), and (15), the load distribution along any elemental strip of the wings and body is given by

$$\left(\frac{\Delta p_L}{q}\right)_H = 4\alpha\left[\frac{\frac{ds}{dx}\left(1+k_H{}^4 - \frac{2k_H{}^4 s^2}{y^2}\right) + \beta\frac{y}{s}\left(1-\frac{k_H{}^4 s^4}{y^4}\right)}{\sqrt{(1+k_H{}^4) - \frac{y^2}{s^2}\left(1+\frac{k_H{}^4 s^4}{y^4}\right)}}\right]$$

$$\left(\frac{\Delta p_L}{q}\right)_B = 4\alpha\frac{ds}{dx}\sqrt{(1+k_H{}^2)^2 - 4k_H{}^2\left(\frac{y^2}{a^2}\right)} + \tag{A13}$$

$$16\alpha\beta\left[\frac{k_H\left(\frac{y}{a}\right)\left(1-\frac{y^2}{a^2}\right)}{\sqrt{(1+k_H{}^2)^2 - 4k_H{}^2\left(\frac{y^2}{a^2}\right)}} \pm \frac{k_V\left(\frac{y^2}{a^2}\right)\sqrt{1-\frac{y^2}{a^2}}}{\sqrt{(1-k_V{}^2)^2 + 4k_V{}^2\left(\frac{y^2}{a^2}\right)}}\right]$$

$$\left(\frac{\Delta p_Y}{q}\right)_V = 4\beta\left[\frac{-\frac{dt}{dx}\left(1+k_V{}^4 - \frac{2k_V{}^4 t^2}{z^2}\right) + \alpha\frac{z}{t}\left(1-\frac{k_V{}^4 t^4}{z^4}\right)}{\sqrt{(1+k_V{}^4) - \frac{z^2}{t^2}\left(1+\frac{k_V{}^4 t^4}{z^4}\right)}}\right]$$

$$\left(\frac{\Delta p_Y}{q}\right)_B = -4\beta\frac{dt}{dx}\sqrt{(1+k_V{}^2)^2 - 4k_V{}^2\left(\frac{z^2}{a^2}\right)} +$$

$$16\alpha\beta\left[\frac{k_V\left(\frac{z}{a}\right)\left(1-\frac{z^2}{a^2}\right)}{\sqrt{(1+k_V{}^2)^2 - 4k_V{}^2\left(\frac{z^2}{a^2}\right)}} \pm \frac{k_H\left(\frac{z^2}{a^2}\right)\sqrt{1-\frac{z^2}{a^2}}}{\sqrt{(1-k_H{}^2)^2 + 4k_H{}^2\left(\frac{z^2}{a^2}\right)}}\right]$$

where the plus and minus signs are taken as in equations (13 and (15).

The integrated lift, side force, and rolling moment on an elemental strip are

$$\frac{d}{dx}\left(\frac{L}{q}\right)q = 4\pi\alpha qs \frac{ds}{dx}\ \sigma_H \qquad \frac{d}{dx}\left(\frac{Y}{q}\right)q = -\ 4\pi\beta qt \frac{dt}{dx}\ \sigma_V \tag{A14}$$

$$\frac{d}{dx}\left(\frac{L'}{q}\right)q = 2\alpha\beta q(t^2\tau_V - s^2\tau_H) \tag{A15}$$

where $\sigma$ and $\tau$ are constants given by

$$\sigma = 1+k^4+\frac{1}{2\pi}\left[2k(1-k^2) - (1+k^2)^2 \sin^{-1}\frac{2k}{1+k^2}\right]$$

$$\tau = 2k(1-k^2) + \pi(1+k^4) - (1+k^2)^2 \cos^{-1}\frac{1-k^2}{1+k^2}$$

The subscripts H and V on $\sigma$ and $\tau$ refer to the use of $k_H$ or $k_V$ in the above expressions. The lift, side-force, and rolling-moment coefficients for the entire conical cruciform-wing and body combination are then

$$C_L = \frac{\pi}{2} A_H\alpha\sigma_H \qquad C_Y = -\frac{\pi}{2} A_V\beta\sigma_V \tag{A16}$$

$$C_l = \frac{\alpha\beta}{3}\left(\tau_V \frac{t^2_{max}}{s^2_{max}} - \tau_H\right) \tag{A17}$$

Due to the radial nature of the lines of constant pressure, the center-of-pressure position is independent of the body radius - wing semispan ratios and lies at the two-thirds chord point. The pitching and yawing moments are then given by

$$C_m = -\frac{\pi}{3} A_H\alpha\sigma_H \qquad C_n = \frac{\pi}{3} A_V\beta\sigma_V \tag{A18}$$

The lift, side force, pitching-moment, and yawing-moment results are plotted in figures 4 and 5. Figure 6 presents rolling-moment results for selected ratios of vertical wing span to horizontal wing span. To facilitate the computation of further results, the values of $\tau$ for use in equation (A17) are plotted as a function of $k$ in figure 7.

## Triangular Wings on a Semi-infinite Cylindrical Body

The next example to be considered is that of a triangular cruciform wing mounted on a semi-infinite cylindrical body. The essential relationships associated with this configuration are that $da/dx$ equals zero and that $ds/dx$ and $dt/dx$ are constants. Therefore it is clear from equation (A9) that no forces are exerted on the body ahead of the leading edge of the root chord. Behind this point, pressures are exerted on the wings and body in accordance with the following relations:

$$\left.\begin{aligned}
\left(\frac{\Delta p_L}{q}\right)_H &= 4\alpha\left[\frac{\frac{ds}{dx}\left(1-\frac{a^4}{s^4}\right)+\beta\frac{y}{s}\left(1-\frac{a^4}{y^4}\right)}{\sqrt{\left(1+\frac{a^4}{s^4}\right)-\frac{y^2}{s^2}\left(1+\frac{a^4}{y^4}\right)}}\right]\\
\left(\frac{\Delta p_L}{q}\right)_B &= 4\alpha\left[\frac{\frac{ds}{dx}\left(1-\frac{a^4}{s^4}\right)+4\beta\frac{y}{s}\left(1-\frac{y^2}{a^2}\right)}{\sqrt{\left(1+\frac{a^2}{s^2}\right)^2-4\frac{y^2}{s^2}}}\right]\pm\frac{16\alpha\beta\frac{y^2}{at}\sqrt{1-\frac{y^2}{a^2}}}{\sqrt{\left(1-\frac{a^2}{t^2}\right)^2+4\frac{y^2}{t^2}}}\\
\left(\frac{\Delta p_Y}{q}\right)_V &= 4\beta\left[\frac{-\frac{dt}{dx}\left(1-\frac{a^4}{t^4}\right)+\alpha\frac{z}{t}\left(1-\frac{a^4}{z^4}\right)}{\sqrt{\left(1+\frac{a^4}{t^4}\right)-\frac{z^2}{t^2}\left(1+\frac{a^4}{z^4}\right)}}\right]\\
\left(\frac{\Delta p_Y}{q}\right)_B &= 4\beta\left[\frac{-\frac{dt}{dx}\left(1-\frac{a^4}{t^4}\right)+4\alpha\frac{z}{t}\left(1-\frac{z^2}{a^2}\right)}{\sqrt{\left(1+\frac{a^2}{t^2}\right)^2-4\frac{z^2}{t^2}}}\right]\pm\frac{16\alpha\beta\frac{z^2}{as}\sqrt{1-\frac{z^2}{a^2}}}{\sqrt{\left(1-\frac{a^2}{s^2}\right)^2+4\frac{z^2}{s^2}}}
\end{aligned}\right\}\quad\text{(A19)}$$

where the plus and minus signs are taken as in equations (13) and (15).

The integrated forces and moments on an elemental strip are given by

$$\frac{d}{dx}\left(\frac{L}{q}\right) q = 4\pi\alpha q s\left(1-\frac{a^4}{s^4}\right)\frac{ds}{dx} \qquad \frac{d}{dx}\left(\frac{Y}{q}\right) q = -4\pi\beta q t\left(1-\frac{a^4}{t^4}\right)\frac{dt}{dx} \quad \text{(A20)}$$

$$\frac{d}{dx}\left(\frac{L'}{q}\right) q = 2\alpha\beta q t^2\left[2\frac{a}{t}\left(1-\frac{a^2}{t^2}\right)+\pi\left(1+\frac{a^4}{t^4}\right)-\left(1+\frac{a^2}{t^2}\right)^2\cos^{-1}\frac{t^2-a^2}{t^2+a^2}\right]-$$

$$2\alpha\beta q s^2\left[2\frac{a}{s}\left(1-\frac{a^2}{s^2}\right)+\pi\left(1+\frac{a^4}{s^4}\right)-\left(1+\frac{a^2}{s^2}\right)^2\cos^{-1}\frac{s^2-a^2}{s^2+a^2}\right]$$

(A21)

By integration along the length of the body, force and moment coefficients for the complete wing-body combination, based on the area of the basic triangular wing without fuselage, are found to be

$$C_L = \frac{\pi}{2}A_H\alpha\left(1-\frac{a^2}{s^2_{max}}\right)^2 \qquad C_Y = -\frac{\pi}{2}A_V\beta\left(1-\frac{a^2}{t^2_{max}}\right)^2 \quad \text{(A22)}$$

$$C_m = -\frac{\pi}{3}A_H\alpha\left(1-4\frac{a^3}{s^3_{max}}+3\frac{a^4}{s^4_{max}}\right) \qquad C_n = \frac{\pi}{3}A_V\beta\left(1-4\frac{a^3}{t^3_{max}}+3\frac{a^4}{t^4_{max}}\right) \quad \text{(A23)}$$

$$C_l = \frac{\alpha\beta}{3}\left(\frac{S_V t_{max}}{S_H s_{max}}\nu_V - \nu_H\right) \quad \text{(A24)}$$

where

$$\nu = 2k(1-k^2)+\pi(1+4k^3-3k^4)-(1+6k^2-k^4)\cos^{-1}\frac{1-k^2}{1+k^2}+\frac{7}{3}k^3\log\frac{2k^2}{1+k^2}$$

where, for determining $\nu_V$, $k$ is taken as $a/t_{max}$ and, for $\nu_H$, $k$ is taken as $a/s_{max}$.

It may be seen from the equations and from figures 4 and 5 that the addition of a semi-infinite cylindrical body to a cruciform arrangement of triangular wings decreases the magnitudes of the forces and moments just as in the preceding example with the conical body. In the present example, however, the forces and moments continue to decrease as the body radius increases with respect to the wing semispan until finally, when they are equal (corresponding to a body without wings), the forces and moments are all zero. It is further seen that the rolling moment vanishes when the two wings become identical, as has been shown in general in the text. The rolling-moment results cannot be plotted in general, however, as was done in the case of the conical wing-body combination since there are now three significant parameters instead of two. To facilitate calculation of these results, therefore, the variation of $\nu$ with $k$ has been plotted in figure 7.

## Triangular Cruciform Wing on a Pointed Body

The theoretical characteristics of a triangular cruciform wing mounted on a pointed body of revolution, closed in an arbitrary manner at the nose but cylindrical along the wing root, may be determined by combining the results of two previous examples. The portion of the wing-body combination ahead of the leading edge of the wing root may be considered to be equivalent to the arbitrary body of revolution treated in the second example. The portion of the wing-body combination aft of the leading edge of the wing root is equivalent to a cruciform arrangement of triangular wings mounted on a semi-infinite cylinder discussed in the preceding example. The load distribution and integrated load on an elemental spanwise strip are then the same as those given in the corresponding example.

The lift and side-force coefficients are found by adding the forces on the component parts of the wing-body combination and dividing by the dynamic pressure and the characteristic area, again taken to be the area of the basic triangular wing without fuselage. The lift and side-force coefficients are then

$$C_L = \frac{\pi}{2} A_H \alpha \left( 1 - \frac{a^2}{s^2_{max}} + \frac{a^4}{s^4_{max}} \right) \qquad C_Y = -\frac{\pi}{2} A_V \beta \left( 1 - \frac{a^2}{t^2_{max}} + \frac{a^4}{t^4_{max}} \right) \tag{A25}$$

These relationships are shown graphically in figure 4.

The pitching- and yawing-moment coefficients for this wing-body combination may be found in a manner similar to that used in finding the lift and side-force coefficients, taking care to transfer the moments of both components to the same axis.

$$C_m = -\frac{\pi}{3}A_H\alpha\left[1-\frac{5}{2}\frac{a^3}{s^3_{max}}+3\frac{a^4}{s^4_{max}}-\frac{3}{2}\frac{B_m}{\pi s^2_{max}}\left(\frac{l}{c}-1+\frac{a}{s_{max}}\right)\right]$$

$$C_n = \frac{\pi}{3}A_V\beta\left[1-\frac{5}{2}\frac{a^3}{t^3_{max}}+3\frac{a^4}{t^4_{max}}-\frac{3}{2}\frac{B_m}{\pi t^2_{max}}\left(\frac{l}{c}-1+\frac{a}{t_{max}}\right)\right] \tag{A26}$$

The rolling-moment coefficient is given, of course, by equation (A24).

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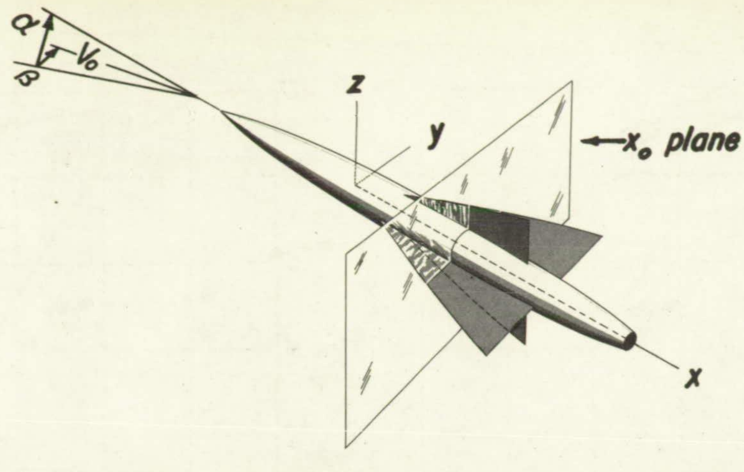


*Figure 1.-View of cruciform-wing and body combination showing coordinate axes.*

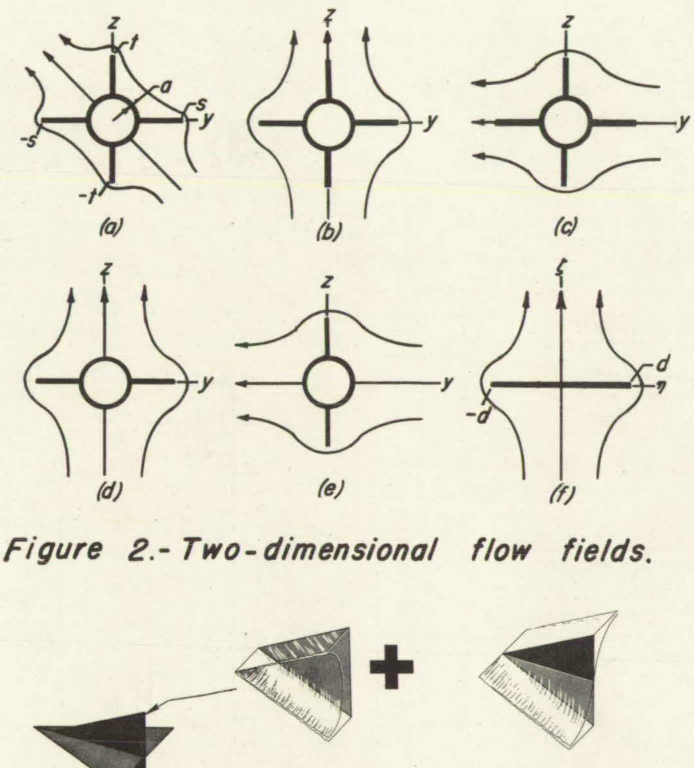


*Figure 2.- Two-dimensional flow fields.*

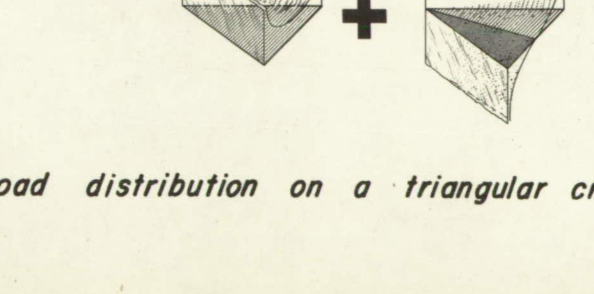

*Figure 3.- Load distribution on a triangular cruciform wing.*

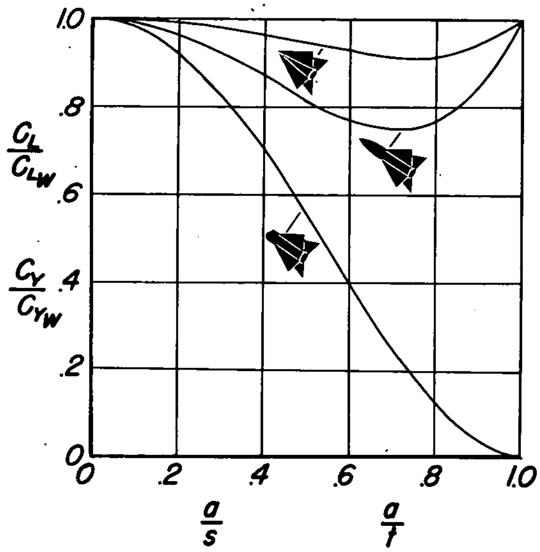


*Figure 4.-Lift and side-force ratios for three cruciform-wing and body configurations.*

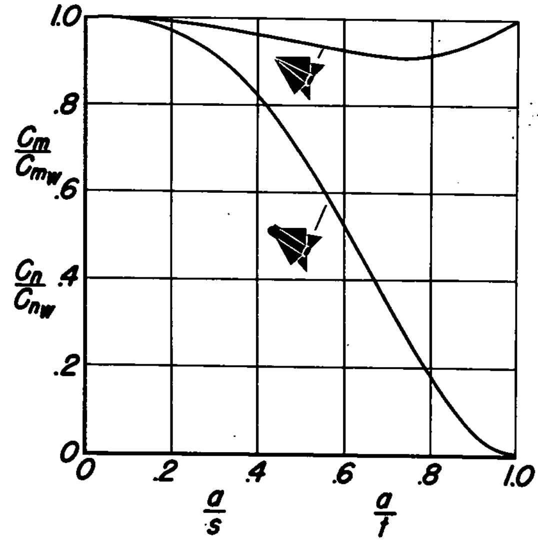


*Figure 5.-Pitching-moment and yawing-moment ratios for two cruciform-wing and body configurations.*

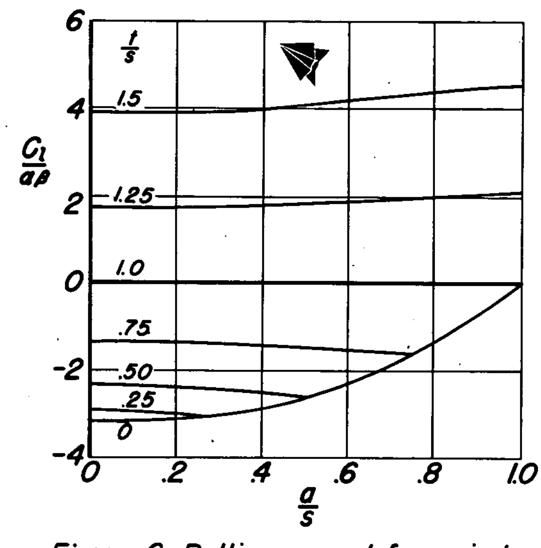


*Figure 6.-Rolling moment for conical cruciform-wing and body configurations.*

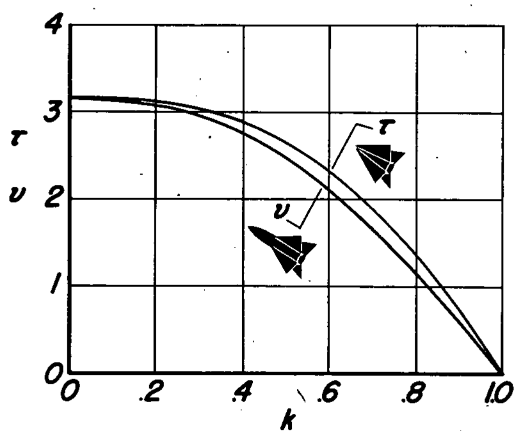


*Figure 7.-Variation of $\tau$ and $\nu$ with k.*