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TURBULENT BOUNDARY-LAYER TEMPERATURE RECOVERY FACTORS IN TWO-DIMENSIONAL SUPERSONIC FLOW

By Maurice Tucker and Stephen H. Maslen

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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TURBULENT BOUNDARY-LAYER TEMPERATURE RECOVERY FACTORS IN TWO-DIMENSIONAL SUPERSONIC FLOW

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SUMMARY

An analytical method is presented for obtaining the turbulent temperature recovery factors for a thermally insulated surface in supersonic flow. The method is an extension of Squire's analysis for incompressible flow. The boundary-layer velocity profile is represented by the power law and a similarity is postulated for the squared-velocity and static-temperature-difference profiles.

The analysis indicated that the recovery factor decreased with increasing Mach number. For the range of Prandtl numbers considered (0.65 to 0.75), the recovery factors at a stream Mach number of 10 were, on the average, about 5 percent lower than the limiting values at zero Mach number. The Reynolds number effect on recovery factor was of secondary importance. An approximation formula that represents the computations to within 1 percent is included for engineering calculations.

INTRODUCTION

Determination of the temperature attained at the surface of a thermally insulated plate in laminar boundary-layer flow in the absence of radiation (a particular case of the plate-thermometer problem) has been the subject of several analytic investigations. The surface temperature T_{aw} may be presented in terms of the stream static temperature t_1, the temperature recovery factor r, and the local stream Mach number parameter m^2 as

$$T_{aw} = t_1(1 + rm^2) \tag{1}$$

(All symbols used in this report are defined in appendix A.) For very low speed flows, Pohlhausen (reference 1, pp. 627-631) found that r was a function only of the laminar Prandtl number Pr and could be approximated as $r = Pr^{1/2}$. References 2 and 3 indicate that the Pohlhausen approximation is also applicable to supersonic laminar boundary-layer flow.

Existing analyses of the plate-thermometer problem for turbulent boundary-layer flow (see, for example, reference 4) are limited to the case of constant fluid properties. The results of references 5 and 6 indicate that, for the low-speed flows considered, the turbulent recovery factor may be represented by the approximation $r = Pr^{1/3}$. The analysis conducted at the NACA Lewis laboratory and presented herein essentially extends the method of reference 6 to supersonic two-dimensional turbulent boundary-layer flow.

ANALYSIS

Development

The principle of conservation of energy requires the total-energy increment transported through any section normal to a thermally insulated surface along which a two-dimensional boundary-layer flow exists to be constant; that is,

$$\int_0^\infty \rho u c_p(T-T_1)\, dy = \text{constant} \tag{2}$$

where the specific heat c_p is taken as a constant. Inasmuch as no energy transport occurs at the leading edge of the plate, the constant appearing in equation (1) must be identically zero. Squire's basic equation (reference 6) for obtaining the temperature recovery factor is then given in the present notation as

$$\int_0^\infty \frac{\rho u}{\rho_1 u_1} c_p(T-T_1)\, dy = 0 \tag{3}$$

The treatment of reference 6 is confined to the case of low-speed flow by assuming that $\rho = \rho_1$. The method presented herein does not contain this limitation.

For fluids with Prandtl number less than 1, the thermal boundary-layer thickness Δ exceeds the dynamic boundary-layer thickness δ.

For a Prandtl number of 1, in which case $\Delta = \delta$, the following static-temperature profile (from equation (7) of reference 7) is obtained:

$$\frac{t}{t_1} = 1 + m^2\left[1 - \left(\frac{u}{u_1}\right)^2\right] \equiv 1 + m^2\left[1 - g^2(y)\right] \tag{4}$$

which indicates a similarity of the squared-velocity and static-temperature-difference profiles. Squire (reference 6) makes the assumption that such similarity also exists for fluids with Prandtl number differing from 1. The assumption appears plausible when the Prandtl number is relatively constant through the boundary layer and does not differ too greatly from 1.

In the analysis, the turbulent Prandtl number $\left(\frac{\mu+\epsilon}{k+\beta}\right) c_p$ is assumed to be constant along any boundary-layer section and equal in value to the laminar Prandtl number $\mu c_p/k$ at the surface. These assumptions are consistent with the conclusions of references 8 to 10, namely, that the turbulent Prandtl number has a constant value of about 0.7 regardless of the laminar value. Although these assumptions restrict the analysis to a consideration of fluids having a laminar Prandtl number near 0.7, the restriction is not a serious one inasmuch as the Prandtl numbers of most gases are in this range.

The static-temperature-difference profile for $\Delta \neq \delta$ is assumed to be related to the profile for $\Delta = \delta$ by a constant scaling factor $\eta = \Delta/\delta$ and the recovery factor r. For compatability with equations (1) and (4), the static-temperature profile for fluids with Prandtl number different from 1 is then written as

$$\frac{t}{t_1} = 1 + rm^2\left[1-g^2\left(\frac{y}{\eta}\right)\right] = 1 + \frac{ru_1^2}{2c_pt_1}\left[1-g^2\left(\frac{y}{\eta}\right)\right] \tag{5}$$

This temperature relation, postulated by Squire (reference 6), will be used in the analysis. The following expresssion is thereby obtained for the quantity $c_p(T-T_1)$:

$$c_p(T-T_1) = \frac{u_1^2}{2}\left\{r\left[1-g^2\left(\frac{y}{\eta}\right)\right] - \left[1-g^2(y)\right]\right\} \tag{6}$$

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When the perfect gas law and equations (5) and (6) are used, equation (3) takes the form

$$\int_0^{\Delta} \frac{\left\{r\left[1-g^2\left(\frac{y}{\eta}\right)\right] - \left[1-g^2(y)\right]\right\} g(y)\, dy}{1 + rm^2\left[1-g^2\left(\frac{y}{\eta}\right)\right]} = 0 \tag{7}$$

The assumption is now made that the velocity profile can be represented by the power law

$$\frac{u}{u_1} = \left(\frac{y}{\delta}\right)^{1/N} \equiv p \tag{8}$$

The quantity $q \equiv \left(\frac{y}{\eta\delta}\right)^{1/N}$ is defined and

$$g(y) = p \quad \text{for} \quad 0 \leqq y \leqq \delta$$

$$g(y) = 1 \quad \text{for} \quad y \geqq \delta$$

$$g\left(\frac{y}{\eta}\right) = q \quad \text{for} \quad 0 \leqq y \leqq \Delta$$

$$g\left(\frac{y}{\eta}\right) = 1 \quad \text{for} \quad y \geqq \Delta$$

and equation (7) can be written as

$$\eta^{2/N}\left[(r-1) + \left(\eta^{2/N}-r\right)A\right]\int_0^1 \frac{p^N\, dp}{A\eta^{2/N}-p^2} - \frac{\eta}{m^2}\int_{\left(\frac{1}{\eta}\right)^{1/N}}^1 \frac{q^{N-1}\, dq}{A-q^2} - \frac{\left(\eta^{2/N}-r\right)}{N+1} + \frac{\eta r}{N}\left(1-\frac{1}{\eta}\right) = 0 \tag{9}$$

where $A \equiv \frac{1 + rm^2}{rm^2}$.

As is shown in appendix B, the scaling factor η is related to the recovery factor r by

$$r = \eta^2 Pr_t = \eta^2 Pr \tag{10}$$

This relation was also obtained for incompressible flow in reference 6. The effect of a variable scaling factor on the recovery factor was also considered. By use of the conditions of reference 6 ($\rho = \rho_1$ and $\Delta < \delta$), computations were made for the scaling factor S given by

$$S = \eta^{(L-1)/L} \left(\frac{y}{\delta}\right)^{1/L} \qquad \beta\eta < \frac{y}{\delta} < \eta$$

$$S = \beta^{1/L}\eta \qquad 0 < \frac{y}{\delta} < \beta\eta$$

For variable scaling factors, equation (10) takes the form

$$r = S^2_{(y=0)} Pr \tag{10a}$$

When it is assumed that $L = r$ and $\beta = 0.2$, the recovery factor obtained is 98.6 percent of the value obtained for constant scaling factor $S = \eta$. In view of the relative insensitivity of the computed recovery factor to relatively large changes in scaling factor for values of N that are appropriate to turbulent boundary-layer velocity profiles, use of a constant scaling factor was considered adequate.

When the substitutions

$$p = A^{1/2} \left(\frac{r}{Pr}\right)^{1/2N} z \equiv A^{1/2} B^{1/2N} z$$

and

$$q = A^{1/2} z$$

are made and equation (10) is used, equation (9) takes the form

$$\left(AB^{1/N} - \frac{1+m^2}{m^2}\right)\left(A^{1/2}B^{1/2N}\right)\int_0^{A^{-1/2}B^{-1/2N}} \frac{z^N\,dz}{1-z^2} -$$

$$\frac{1}{m^2}\int_{A^{-1/2}B^{-1/2N}}^{A^{-1/2}} \frac{z^{N-1}\,dz}{1-z^2} -$$

$$\frac{A^{(2-N)/2}B^{(2-N)/2N}}{N+1} - \frac{rA^{(2-N)/2}B^{-1/2}}{N(N+1)} + \frac{rA^{(2-N)/2}}{N} \equiv F = 0 \qquad (11)$$

The temperature recovery factor r as a function of Mach number parameter m, velocity-profile parameter N, and laminar Prandtl number Pr is obtained from equation (11). The effect of chordwise pressure gradient on recovery factor enters only to the extent that the velocity profile is dependent upon the gradient.

Methods of Solution

An explicit solution of equation (11) for the recovery factor r is obviously impossible inasmuch as r appears in both the quantities A and B. The following procedure was employed: A method was devised for obtaining a first approximation to the recovery factor. Values obtained by this method were then improved by use of the Newton-Raphson iteration method (reference 11). Details of the procedure follow:

First approximation method. - For Prandtl numbers near 1, it is expedient to write the recovery factor as

$$r = Pr^{\alpha} = \left[1-(1-Pr)\right]^{\alpha} \equiv (1-H)^{\alpha} \qquad (12)$$

and to approximate the quantity $(1-H)^{\alpha}$ as $1-\alpha H$. The integrals appearing in equation (11) may be denoted by $I = I(m,N,H)$. This functional relation may be expanded in a Taylor's series, consistent with the assumption that the Prandtl number is near 1, as $I(m,N,H) = I(m,N,0) + H\left[\frac{\partial I(m,N,H)}{\partial H}\right]_{H=0} + \ldots$. Introducing these substitutions in equation (11) yields

$$\alpha = \frac{1 - (N+1)Q}{1 - N + (N+1)Q\left[\frac{N-1-m^2}{1+m^2}\right]} \tag{13}$$

where

$$Q \equiv \left(\frac{1+m^2}{m^2}\right)^{(N+1)/2} \int_0^{\left(\frac{1+m^2}{m^2}\right)^{-1/2} \equiv L} \frac{z^N\,dz}{1-z^2}$$

For application of equation (12), punch-card equipment was used to evaluate the definite integral $\int_0^L \frac{z^N dz}{1-z^2}$ for integer values of N from 4 through 11.

Iteration method. - With an approximate solution r_a of equation (11) obtained from the method just described, an improved value is given by the Newton-Raphson method as

$$r_b = r_a - \frac{F(r_a)}{\frac{\partial F(r_a)}{\partial R}} \tag{14}$$

The quantity F is defined by equation (11) and the partial derivative is obtained as

$$\frac{\partial F}{\partial r} = \frac{A^{-1/2} B^{1/2N}}{2Nr}\left[A+N(1-A)\right]\left[3AB^{1/N} - \left(\frac{1+m^2}{m^2}\right)\right]\int_0^{A^{-1/2}B^{-1/2N}} \frac{z^N\,dz}{1-z^2} +$$

$$\frac{A^{-N/2}}{2N(N+1)}\left\{2(N+1) + B^{1/2}\left[A+N(1-A)-2\right] - \frac{3}{r}\left[A+N(1-A)\right] B^{(2-N)/2N}\right\} \tag{15}$$

Procedure. - First approximations for r using equations (12) and (13) were obtained for Prandtl numbers of 0.65, 0.70, and 0.75 at local stream Mach numbers of 3, 6, and 10 with values of the profile parameter N of 5, 7, 9, and 11. Two iterations, obtained by use of equation (14), were used for each case to give results correct to four decimal places.

Limiting solutions. - The limiting solutions of equation (11) are of interest. For the limiting case of infinite Mach number, the recovery factor r_∞ is obtained as

$$r_\infty = \frac{B^{1/2N}\left(B^{1/N}-1\right)\displaystyle\int_0^{B^{-1/2N}} \frac{z^N\,dz}{1-z^2} - \frac{B^{(2-N)/2N}}{N+1}}{\dfrac{B^{-\frac{1}{2}}}{N(N+1)} - \dfrac{1}{N}} \tag{16}$$

Computations indicate that r_∞ is practically independent of the profile parameter N and can be represented within 1 percent by the relation

$$r_\infty = 0.670\ Pr + 0.322 \tag{17}$$

for Prandtl numbers in the range from 0.65 to 1.

For the limiting case of zero Mach number, expansion of the integrands in equation (11) permits writing the equation as

$$\frac{-r_0}{N(N+1)} + \frac{2Pr^{-1/2} r_0^{3/2}}{N(N+2)} + \frac{Pr^{1/N} r_0^{(N-1)/N}}{(N+2)(N+3)} - \frac{2}{(N+1)(N+3)} = 0$$

Approximating $Pr^G = (1-H)^G$ as $1-GH + \frac{G(G-1)H^2}{2}$ and solving for r_0 by the Newton-Raphson method gives the result

$$r_0 = Pr^{\frac{N+1}{3N+1}} \tag{18}$$

The expression for r_0 is identical with Squire's result (reference 6) for incompressible flow of a fluid with Prandtl number greater than 1. The limiting values of recovery factor obtained by the first approximation method are $r_0 = Pr^{\frac{N+1}{3N+1}}$ and $r_\infty = Pr$. The approximation method thus overestimates the effect of Mach number on recovery factor.

RESULTS AND DISCUSSION

The computed recovery factors are given to three decimal places in table I. It may be noted here that the first approximations obtained by use of equations (12) and (13) were found to be within 1.4 percent of the values tabulated. The maximum error occurred at a local stream Mach number of 10 for a Prandtl number of 0.65 and a profile parameter N of 5. The following expression

$$r = Pr^{\frac{N+1+0.528 M_1^2}{3N+1+M_1^2}} \tag{19}$$

represents the tabulations to within 1 percent and may prove of convenience in engineering calculations. With the recovery factor known, the stagnation temperature profile for constant scaling factor may be obtained from equation (6) as

$$\left.\begin{aligned} \frac{T}{T_1} &= 1 + \frac{m^2}{1+m^2}\left\{r\left[1-\left(\frac{Pr}{r}\right)^{1/N}\left(\frac{y}{\delta}\right)^{2/N}\right]-\left[1-\left(\frac{y}{\delta}\right)^{2/N}\right]\right\}; \quad 0 < y \leqq \delta \\ \frac{T}{T_1} &= 1 + \frac{m^2}{1+m^2}\left\{r\left[1-\left(\frac{Pr}{r}\right)^{1/N}\left(\frac{y}{\delta}\right)^{2/N}\right]\right\}, \quad \delta \leqq y \leqq \Delta \end{aligned}\right\} \tag{20}$$

A typical profile is shown in figure 1. The discontinuities in slope at $y/\delta = 1$ and $y/\delta = \eta$ and the incorrect value of the slope at the wall result from use of a power-law profile throughout the boundary layer.

Effect of Mach Number

Table I indicates that the turbulent temperature recovery factor decreases with an increase in local stream Mach number. For the range of Prandtl numbers and velocity profile parameters considered, the decrease is about 5 percent for a change in Mach number from 0 to 10. A typical variation of turbulent recovery factor with Mach number is shown in figure 2. The corresponding laminar recovery factor obtained by the method of reference 2 is also shown. The invariance of laminar recovery factor with change in Mach number apparently results from use of the assumption $\mu \sim t$. The results of reference 12 indicate that use of the assumption $\mu \sim t^{0.8}$ leads to a decrease of laminar recovery factor with increase in stream Mach number. For a Prandtl number of 0.70, the recovery factor at a nominal Mach number of 5.4 is 0.815 as compared with the value 0.837 for incompressible flow.

Effect of Reynolds number

It is well known (reference 1, p. 340) that use of the power law (equation (8)) to represent the turbulent boundary-layer velocity profile requires that the parameter N increase with Reynolds number. Equation (4b) of reference 7 presents the following approximate guide for such variation:

$$N = 2.6\, R_{s,w}^{\ 1/14} \tag{21}$$

The analysis herein, as do low-speed flow analyses, thus requires the recovery factor to increase with increasing Reynolds number for a given Mach number. This increase is greater at high Mach numbers. In general, however, the effect of Reynolds number upon recovery factor is, according to the analysis, a secondary one. From the relative insensitivity of the results to changes in profile parameter N, the effect of moderate chordwise pressure gradients upon the recovery factor should also prove of secondary importance.

Comparison With Experiment

Experimental turbulent recovery factors obtained at a Mach number of 2.4 are presented in reference 13 for steady flow over a flat plate with natural transition in a tunnel having test-section dimensions of $5\frac{1}{2}$ by $5\frac{1}{2}$ inches. As the Reynolds number based on distance from the plate leading edge and on free-stream kinematic viscosity was increased from 2×10^6 to 6.7×10^6, the turbulent recovery factors decreased from 0.897 to 0.884. The trend with Reynolds number is thus in the opposite direction to that predicted by the various analyses.

In reference 13, the Prandtl number at the plate surface was taken as 0.72 and the profile parameter N as 5. Using equation (19) as an interpolation formula and substituting these values of Prandtl number and profile parameter yields, at a Mach number of 2.4, a recovery factor of 0.872, which is from 1.4 to 2.8 percent lower than the extremes of the experimental values. The profile-parameter value $N = 5$ chosen in reference 13 appears somewhat low for the range of Reynolds numbers used. A value of $N = 7$ obtained from equation (21) by averaging with respect to plate-surface and stream Reynolds number yields a recovery factor of 0.877 for $Pr \equiv 0.72$ and $M = 2.4$. This value of recovery factor is from 0.8 to 2.2 percent lower than the extreme values measured. The theoretical value of laminar recovery factor given in reference 2 for $Pr = 0.72$ is 0.845 as compared with the experimental value of 0.881 given in reference 13. More extensive tests, particularly at very high Mach numbers, are required for a decisive check of the assumptions involved in the analysis.

SUMMARY OF RESULTS

The present work essentially extends the low-speed analysis of Squire to the case of two-dimensional supersonic flow along a thermally insulated surface. The analysis makes use of the following simplified model of turbulent boundary-layer flow:

The turbulent Prandtl number was assumed to be constant along any boundary-layer section and equal in value to the laminar Prandtl number at the surface. The boundary-layer velocity profile was approximated by a power law. The presence of the laminar sublayer was not considered except for evaluation of the surface Prandtl number and the similarity of the squared-velocity and the static-temperature-difference profiles postulated by Squire was assumed.

Under these conditions, the temperature recovery factor in turbulent flow was shown to decrease with increasing Mach number. For the range of Prandtl numbers considered (0.65 to 0.75), the recovery factor at a Mach number of 10 was about 5 percent lower than the limiting value at a Mach number of 0. As in low-speed analyses, the recovery factor was shown to increase with an increase in Reynolds number. The Reynolds number effect upon recovery factor was, in general, of secondary importance. The approximation for recovery factor $r = Pr^{\frac{N+1+0.528 M_1^2}{3N+1+M_1^2}}$ (where Pr is the laminar Prandtl number, N is the profile parameter, and M_1 is the local stream Mach number) was found to represent the computed results to within 1 percent.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, October 27, 1950

APPENDIX A

Symbols

The following symbols are used in this report:

$A \equiv$	$\dfrac{1+rm^2}{rm^2}$
$B \equiv$	$\dfrac{r}{Pr}$
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
F	defined by equation (11)
G	generalized exponent
g(y)	velocity-ratio function, u/u_1
$H \equiv$	1 - Pr
k	thermal conductivity for laminar flow
L	limit of integration
M	Mach number
m^2	Mach number parameter, $\frac{\gamma-1}{2} M_1{}^2$
N	velocity-profile parameter, $u/u_1 - \left(\frac{y}{\delta}\right)^{1/N}$
Pr	Prandtl number for laminar flow, $\frac{\mu c_p}{k}$
Pr_t	Prandtl number for turbulent flow, $\frac{\mu+\epsilon}{k+\beta}c_p$
$p \equiv$	$\left(\frac{y}{\delta}\right)^{1/N}$
$q \equiv$	$\left(\frac{y}{\eta\delta}\right)^{1/N}$

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$R_{s,w}$ Reynolds number based on distance from leading edge of plate and stream or wall value of kinematic viscosity

r temperature recovery factor, $\frac{T_{aw}-t_l}{T_l-t_l}$

S scaling factor

T stagnation temperature

t static temperature

u,v velocity components parallel and normal to surface, respectively

x,y cartesian coordinates parallel and normal to surface, respectively

z variable of integration

α recovery-factor exponent, $R = Pr^{\alpha}$

β eddy conductivity

γ ratio of specific heats, c_p/c_v

Δ thermal boundary-layer thickness

δ dynamic boundary-layer thickness

ϵ eddy viscosity

η scaling factor, Δ/δ

μ laminar coefficient of viscosity

ρ density

Subscripts:

l local stream

aw at surface of adiabatic or thermally insulated plate

0 at limiting case $M_1 = 0$

∞ at limiting case $M_1 = \infty$

APPENDIX B

Relation Between Scaling Factor η and Recovery Factor r

The following equation for the conservation of energy is assumed to hold for steady two-dimensional compressible turbulent boundary-layer flow:

$$\rho u c_p \frac{\partial t}{\partial x} + \rho v c_p \frac{\partial t}{\partial y} = u \frac{\partial p}{\partial x} + (\mu+\epsilon)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left[(k+\beta)\frac{\partial t}{\partial y}\right] \tag{B1}$$

Adding to this equation the corresponding equation of motion multiplied through by the velocity u yields, after some manipulation,

$$\rho u\left[c_p \frac{\partial t}{\partial x} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)\right] + \rho v\left[c_p \frac{\partial t}{\partial y} + \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]$$

$$= \frac{\partial}{\partial y}\left\{(\mu+\epsilon)\left[c_p \frac{\partial t}{\partial y} + \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]\right\} + \left(\frac{1}{Pr_t} - 1\right)\frac{\partial}{\partial y}\left[(\mu+\epsilon)c_p \frac{\partial t}{\partial y}\right] \tag{B2}$$

In deriving equation (B2), the assumption has been made that the turbulent Prandtl number $Pr_t \equiv \frac{(\mu+\epsilon)}{(k+\beta)} c_p$ is constant. At the plate surface where $u = v = 0$, equation (B2) can be written as

$$\left\{\frac{\partial}{\partial y}\left[(\mu+\epsilon)\frac{\partial}{\partial y}(c_pT - c_pT_1)\right]\right\}_{y=0} = (1-Pr_t)_{y=0}\left\{\frac{\partial}{\partial y}\left[(\mu+\epsilon)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]\right\}_{y=0}$$

or, noting that $\frac{\partial}{\partial y}\left[c_pT - c_pT_1\right]_{y=0} = 0$, then

$$(\mu+\epsilon)\left[\frac{\partial^2}{\partial y^2}(c_pT - c_pT_1)\right]_{y=0} = (1-Pr_t)_{y=0}\ (\mu+\epsilon)\left[\left(\frac{\partial u}{\partial y}\right)^2\right]_{y=0} \tag{B3}$$

The following relation may be obtained from equation (6):

$$\left[\frac{\partial^2}{\partial y^2}\left(c_pT-c_pT_1\right)\right]_{y=0} = \left(1-\frac{r}{\eta^2}\right)\left[\left(\frac{\partial u}{\partial y}\right)^2\right]_{y=0} \tag{B4}$$

The desired relation between η and r is obtained from equations (B3) and (B4) as

$$r = \eta^2 \, Pr_t = \eta^2 \, Pr \tag{10}$$

in view of the assumption regarding the equality of the turbulent and laminar Prandtl numbers at the surface $(y=0)$.

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TABLE I - VARIATION OF TURBULENT TEMPERATURE RECOVERY FACTOR WITH PRANDTL NUMBER Pr, VELOCITY-PROFILE PARAMETER N, AND STREAM MACH NUMBER M_1

Prandtl number Pr	Velocity-profile parameter, N	Mach number, M_1			
		0	3	6	10
0.65	5	0.851	0.834	0.813	0.796
	7	.855	.840	.821	.803
	9	.857	.844	.826	.808
	11	.859	.847	.830	.813
0.70	5	0.875	0.860	0.842	0.826
	7	.878	.865	.848	.833
	9	.880	.869	.853	.838
	11	.882	.872	.856	.841
0.75	5	0.898	0.885	0.870	0.856
	7	.901	.890	.875	.862
	9	.902	.893	.879	.866
	11	.904	.895	.882	.869

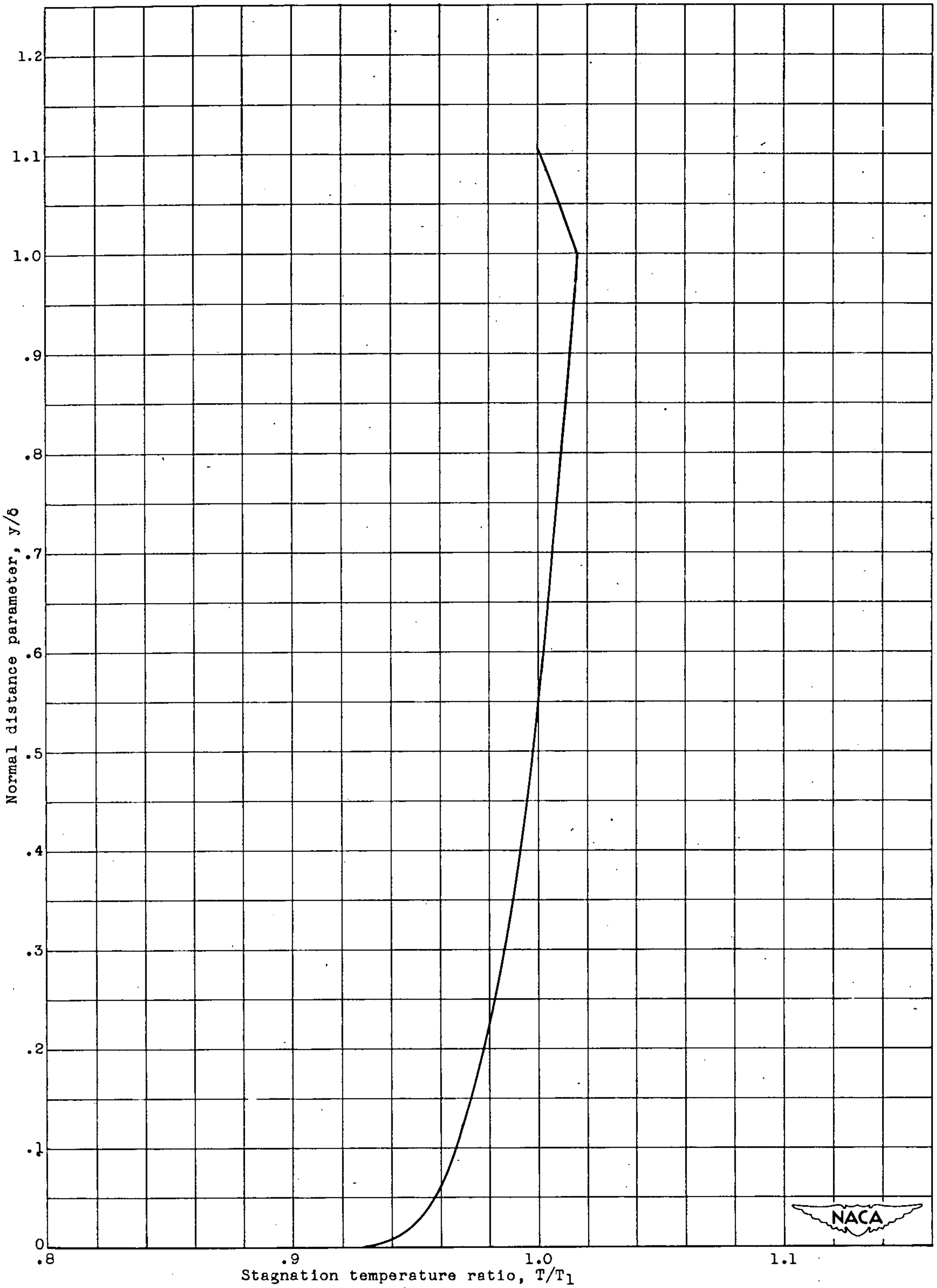

Figure 1. - Stagnation temperature profile for turbulent flow at a stream Mach number of 3 for Prandtl number of 0.70 and boundary-layer velocity profile parameter N = 7.

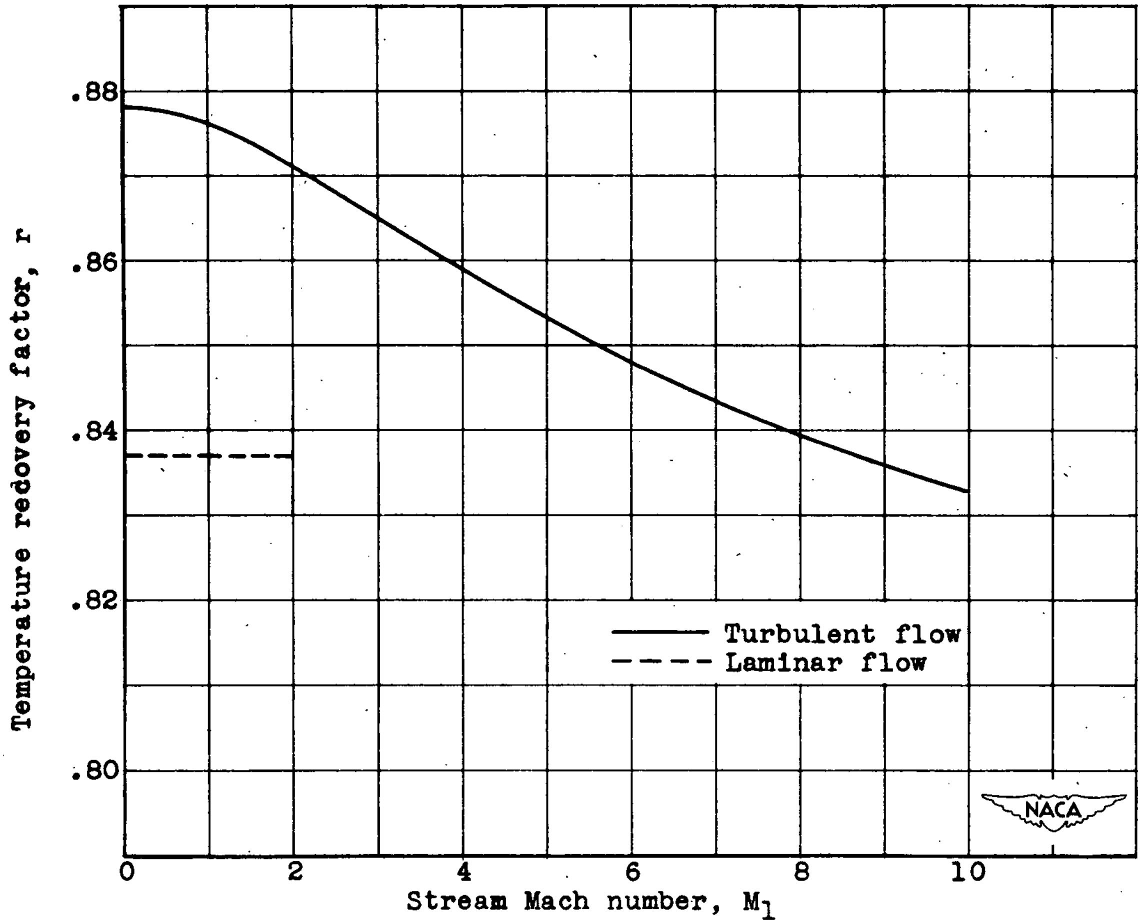

Figure 2. - Variation of temperature recovery factor with stream Mach number in two-dimensional flow of fluid with Prandtl number of 0.7. For turbulent flow, boundary-layer profile parameter N = 7.