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BEST RECTANGULAR AND I-SHAPED CROSS-SECTIONS

FOR AIRPLANE WING SPARS.

By Dr. R. Sonntag.

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FOR AIRPLANE WING SPARS.*

By Dr. R. Sonntag.

In No. 6 of this magazine* (March 31, 1922, p. 77), Eric
Thomas discusses the influence of the weight of the wings and
especially of the wing-spars on the descending speed of soaring
airplanes or gliders. This weight G_h usually increases with
the width b, but is also largely dependent on the kind of spar.

$$G_h = \int_0^{b/2} \gamma \cdot F \cdot dx,$$

in which F denotes the area of cross-section of the spar and
γ the specific gravity of the material employed. F must have
the smallest possible value.

In determining the cross-section, F is found after the de-
termination of the resistance moment W required to withstand
the bending moment M. With a permissible bending stress σ,
$W = \frac{M}{\sigma}$ in which

$$M = \frac{1}{2} \cdot \frac{G_r}{b} \cdot \left(\frac{b}{2} - x \right)^2$$

when G_r denotes the weight of the fuselage, including pilot and
accessories.

W can be determined in such manner as to give F a minimum
value. I discussed this problem in an article "Wirtschaftliche
Querschnittformen für I-Träger" (Best cross-section shapes for I
girders) appearing on pp. 206-207 of the 1922 volume of the

"Zentrallblatt der Bauverwaltung," after I had already broached
the subject in my habilitation lecture on I-iron.*

For rolled-iron girders, the expression $n_W = W/F$ is termed
the coefficient of strength. This serves as a criterion for the
determination of its efficacy. $n = cm^3/cm^2 = cm$ is a quantity of
the first order and occasionally shows with how great an F in
cm^2 the desired W in cm^3 was obtained. That shape is the best,
for which the required W necessitates the smallest possible F.

For rectangular and I-shaped cross-sections (Figs. 1 and 2)

$$F = BH - bh$$

$$W = \frac{BH^3 - bh^3}{6H}$$

$$n_W = \frac{1}{6H} \cdot \frac{BH^3 - bh^3}{BH - bh} \ .$$

n_W must be a maximum. The problem may also be solved ana-
lytically by trial.

After a rough determination of the cross-section, the thick-
ness of the web $d = B - b$ is made as small as possible, since
every increase in d means a decrease in n_W. In this connec-
tion, however, limitations are established by the properties of
the material employed, and indeed first by the fact that the per-
missible shearing stress of the web

$$\tau = \frac{Q \cdot S}{J \cdot d}$$

* "I-Eisen unter besonderer Berücksichtigung der breitflanschigen
und der parallelflanschigen I-Eisen," No. 225, of the "Forschung-
sarbeiten auf dem Gebiete des Ingenieurwesens," published by the
"Verein deutscher Ingenieure," Berlin, 1920.

must not be exceeded and second by the manner of making the spars
Buckling of the web hardly comes into the question. In the above
equation, Q denotes the transverse force, S the static moment and J the inertia moment of the cross-section. Aside from the maximum shearing stress in the zero line, under certain conditions, the resulting stress* on the flanges of the web comes into question. In the process of manufacture, care must be taken that, in the junction of web and flange, when the spar consists of more than one piece, the combining substance (glue) is strong enough to withstand the shearing force and that, when the spar consists of only a single piece, the working stresses are not too great. On the other hand, the thickness d of the web may be computed so small as to be impracticable.

After the preliminary determination of the web thickness d, the best flange thickness t may be determined by trial or analytically as follows: n_W must be the maximum of the value sought. If $x = h = H - 2t$ is introduced as an unknown quantity we have

$$n_W = \frac{1}{6H} \cdot \frac{BH^3 - b\,x^3}{BH - b\,x} \,.$$

The deduction for $x = 0$ gives for the determination of x an equation of the third degree

$$x^3 - \frac{3}{2} \cdot \frac{BH}{b} \cdot x^2 + \frac{1}{2} \cdot \frac{BH^3}{b} = 0,$$

* See R. Sonntag, "Biegung, Schub und Scherung in Stäben von Zusammengesetzten und mehrteiligen Querschnittsformen mit gleichen und wechselnden Trägheitsmomenten auf Grund der Zerlegung in ihre Einzelteile," Berlin, 1909, and in the "Forschungsarbeiten" of the "Verein deutscher Ingenieure," No. 225, Berlin, 1920.

the solution of which was explained in my article on I-iron.
Lastly

$$t_{n \text{ max}} = \frac{H - x}{2}.$$

If the value thus obtained differs appreciably from the provisional cross-section, it is advisable to verify d or b and redetermine $t_{n \text{ max}}$.

With the adoption of the height H of the cross-section, there comes in question, from the static point of view, the permissible bending or flexibility of the spar and, from the aerodynamic point of view, the desired wing shape. In case d does not have to be made considerably thicker, on account of the material used, than is required by the computation, the efficacy of a rectangular or I-shape increases with H. For the same value of H, it increases with B, but greater values of W can be obtained with like values of F, greater values of H and smaller values of B. The interdependence of H, B, d and t have also been quite thoroughly examined in my article on I-iron already mentioned. Furthermore, rectangular and I-shapes with parallel faces and sharp corners show a greater efficacy than the same with rounded corners and inclined flange surfaces of like cross-sectional area (F). For spars, it will probably be the usual case that B, d and t remain constant while H varie In making the very most of the material, however, variations of B and t may also come into the question.

The examination of three cross-sections will usually suffice

for determining the cross-section of a spar. The dimensions of the other cross-sections may be determined graphically by inter-polation. In carrying out the integration of G_h, the H, B, d and t lines can be so selected as to give favorable F lines. Simplifying assumptions will also be possible, when it is not desired to make the measurements with a planimeter.

If it is desired to integrate in a purely analytical manner, F must be introduced as a variable. If W/n_W is substituted for F and if

$$W = \frac{M}{\sigma} = \frac{1}{2\sigma} \cdot \frac{G_r}{b} \cdot \left(\frac{b}{2} - x \right)^2,$$

then

$$G_h = \gamma \cdot \frac{1}{2\sigma} \cdot \frac{G_r}{b} \cdot \int_o^{b/2} \frac{1}{n_W} \left(\frac{b}{2} - x \right)^2 \cdot dx.$$

This method does not seem practicable, on account of the difficulty of grasping the changeableness of n_W.

Fundamentally, nothing stands in the way of investigating even tube-shaped or other cross-section or girder shapes, with the help of the greatest efficacy $n_{W\,max}$.

This method is not confined to the determination of spar cross-sections, but can also be utilized for finding the cross-section of any girder subject to bending stresses.

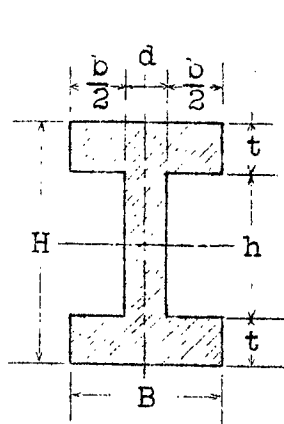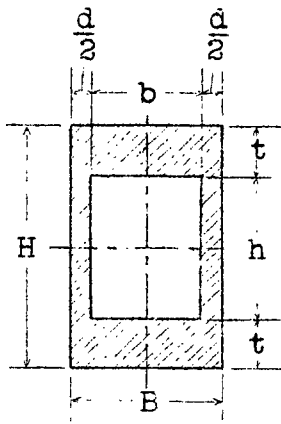Translated by National Advisory Committee for Aeronautics.

Fig. 1   Fig. 2

Figs. 1-2.