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ANALYSIS OF PURE-BENDING FLUTTER OF A CANTILEVER

SWEPT WING AND ITS RELATION TO BENDING-TORSION

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By H. J. Cunningham

Langley Aeronautical Laboratory
Langley Field, Va.

NACA

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SUMMARY

The present analysis deals with the possibility that a sweptback wing which is considered to have infinite torsional stiffness can still flutter in bending only. The analysis is implicit in that of NACA Rep. 1014 and employs aerodynamic coefficients of two-dimensional incompressible and compressible flow. The effects of wing mass-density ratio, angle of sweepback, length-semichord ratio, and Mach number for pure-bending flutter are studied. Flutter in bending alone is shown to be the limiting case of bending-torsion flutter as the ratio of bending frequency to torsional frequency approaches zero, provided that the wing is heavy enough compared to its surrounding medium and its sweep parameter is sufficiently large.

Calculations based on two-dimensional compressible-flow coefficients for a normal-component Mach number of 0.7 indicate that compressibility has a much more marked effect on the speed of pure-bending flutter and of closely related coupled flutter of swept wings than has been found in previous work for coupled flutter of unswept wings. In the case of pure-bending flutter, the bending frequency is all-important in setting the flutter speed, and in the case of coupled flutter of a heavy, swept, stubby wing with a low ratio of bending to torsional frequency, the bending frequency again has more influence on flutter speed than the torsional frequency does.

Effects of finite span have not been considered but are expected to be appreciable for low-aspect-ratio wings.

INTRODUCTION

It is possible that a sweptback wing which is considered to have infinite torsional stiffness can still flutter. Flutter of such a wing would occur in bending alone and is a limiting case of the more widely

studied flutter involving both bending and torsion. Flutter in bending alone has been referred to as single-degree bending flutter (and is at times so designated hereinafter), although it should be recognized that the motion may be given in terms of one or several bending degrees of freedom or may be treated by a differential-equation approach which would include all possible bending-mode combinations.

Although no actual airplane wing will ever have infinite torsional stiffness, study of the bending-alone limiting case is of practical interest since an analysis of bending-alone flutter is more quickly and easily made than one for coupled flutter, and it is expected that parameters affecting the bending-alone-flutter speed have a similar qualitative effect on the coupled-flutter speed over significant ranges of those parameters. For example, study of the limiting case of bending-alone flutter can point out ranges of parameters for which the speed of the associated coupled flutter is much more affected by wing bending stiffness than by torsional stiffness.

The theoretical possibility of bending-alone flutter of a swept wing has been noticed by investigators in England and Germany. In this country the possibility was mentioned by Smilg in his study of oscillations of an unswept wing in pitching alone; furthermore, it is implicit in the analysis of reference 1. It is well to point out that pure-pitching flutter of an unswept wing and pure-bending flutter of a swept wing have an important point of similarity; namely, that wing sections in the stream direction pivot about axes near or ahead of the leading edge.

The present paper is intended to give some features of flutter in bending alone and contains results of analyses made to determine some effects of wing mass-density ratio, angle of sweepback, length-semichord ratio, and Mach number. Some relations of pure-bending flutter to the conventional bending-torsion flutter are also given.

The analysis is made on the basis of assumed two-dimensional compressible and incompressible flow over wing sections as in reference 1. Effects of aerodynamic mutual induction due to finite span or spanwise variation of deflection amplitude or both are expected to be appreciable and it must be kept in mind that such corrections have not been included. Moreover, before general conclusions can be made, effects of other degrees of freedom, including body motions, should be investigated.

SYMBOLS

A_{ch}	part of complex coefficient of aerodynamic lifting force on a wing section which is oscillating harmonically in bending in a two-dimensional stream $\left(R_{ch} + iI_{ch}\right)$
A_2, B_2, D_2, E_2	complex coefficients of total lifting force and of total moment in equations of equilibrium, defined following equation (20b) of reference 1
b	semichord of wing measured normal to elastic axis (Y'-axis; see fig. 1)
b_r	semichord of wing at a reference station, feet
$F_h(\eta)$	amplitude function of wing in bending
$F_\theta(\eta)$	amplitude function of wing in torsion
g_h	structural damping coefficient for bending vibration
g_α	structural damping coefficient for torsional vibration
$\underline{h}$	generalized coordinate in bending degree of freedom, feet $\left(h_o e^{i\omega t}, \text{ where } h_o \text{ is a constant}\right)$
I_{ch}	imaginary part of A_{ch}
k_n	reduced frequency, based on velocity component perpendicular to elastic axis $\left(\omega b/v_n\right)$
l'	effective length of wing, measured along elastic axis, feet
m	mass of wing per unit length along Y'-axis, slugs per foot
M	stream Mach number
N_1, N_2	ratios involving $F_h(\eta)$ defined following equation (6)
r_α	nondimensional radius of gyration of wing about elastic axis $\left(\sqrt{I_\alpha/mb^2}, \text{ where } I_\alpha \text{ is mass moment of inertia of wing about its elastic axis per unit length in slug-feet}^2 \text{ per foot}\right)$

$\left(\dfrac{r_\alpha{}^2}{\kappa}\right)_e$ effective or mean value of section mass-moment-of-inertia parameter; see equation (5)

R_{ch} real part of A_{ch}

t time, seconds

v free-stream speed, feet per second

v_n component of free-stream speed normal to elastic axis, feet per second $(v \cos \Lambda)$

x_{ea} distance of elastic axis behind leading edge, taken perpendicular to elastic axis, percent chord

x_{cg} distance of center of gravity of wing section behind leading edge, taken perpendicular to elastic axis, percent chord

$\dot{\gamma}$ sweep parameter $\left(\dfrac{\tan \Lambda}{l'/b_r}\right)$

Δ flutter determinant; see equation (3)

η nondimensional coordinate along elastic axis; 0 at root, 1.0 at wing tip

$\underline{\theta}$ generalized coordinate in torsional degree of freedom, radians $\left(\theta_0 e^{i\omega t},\text{ where }\theta_0\text{ is a constant}\right)$

$1/\kappa$ ratio of mass of wing section to that of an equal-length cylinder of air of diameter equal to wing chord $\left(m/\pi\rho b^2\right)$

$1/\kappa_e$ effective or mean value of section mass ratio obtained by integrating over length; see equation (4)

Λ angle of sweep of elastic axis, positive for sweepback, degrees

ρ density of medium surrounding wing, slugs per cubic foot

ω angular frequency of vibration, radians per second

ω_h angular uncoupled bending frequency, radians per second

ω_α angular uncoupled torsional frequency about elastic axis, radians per second

ANALYTICAL INVESTIGATION

In an analysis of flutter of a cantilever swept wing, the shape of the deflection mode must be taken into account. In the present analysis, as in reference 1, the wing is considered in the following manner: Wing sections are normal to an elastic axis (or locus of flexural centers) which results from considering the wing to have an effective root as in figure 1, and bending is considered to be the deflection of these wing sections so that their displaced positions are parallel to their unstrained positions. .

Also involved in the problem are considerations of unsteady aero-dynamic forces for wings of finite span. Unresolved complications arise for low-length-chord-ratio wings when attempts are made to take into account analytically effects of spanwise mutual induction which originate in spanwise variation of deflection amplitude, in finite span, or in both. Such analytical attempts, even for simple configurations, are exceedingly complex and have been rather inconclusive in their agreement with experimental results. The analysis of the present paper therefore is, for convenience and simplicity, the two-dimensional-flow approach of reference 1 which should be reasonably useful for wings of at least moderate aspect ratio but may require appreciable correction for low aspect ratios.

In addition to calculations for incompressible flow, an attempt has been made to determine effects of compressibility by substituting aerodynamic coefficients for two-dimensional compressible flow (for example, as from references 2 and 3) according to the Mach number of the subsonic flow component normal to the leading edge. That is, for each wing section aerodynamic coefficients for $M = 0$ are replaced by those for the normal-component Mach number $M \cos \Lambda$.

The expectation is that primary effects of wing parameters and compressibility will be predicted and that, as a result, areas where additional efforts should be expended will be delineated.

The succeeding sections are intended to provide insight into the phenomenon of pure-bending flutter by: (1) examining a sequence of results of calculations for bending-torsion flutter, (2) determining the conditions under which bending-torsion flutter contains pure-bending flutter and then showing that pure-bending flutter is, in general, the limit of bending-torsion flutter as the frequency ratio ω_h/ω_α approaches zero under those conditions, (3) investigating effects of some significant parameters on pure-bending flutter, and (4) giving some sample quantitative relations of pure-bending to bending-torsion flutter.

Bending-Torsion Flutter of a Swept Wing

Method of solution.- The flutter of a nonuniform swept wing in two degrees of freedom, one bending and one torsion, is treated as in reference 1. The two equations of equilibrium (from reference 1) are as follows:

$$\left(\underline{h}A_2 + \underline{\theta}B_2\right)\pi\rho b_r{}^3\omega^2 = 0 \qquad (1)$$

$$\left(\underline{h}D_2 + \underline{\theta}E_2\right)\pi\rho b_r{}^4\omega^2 = 0 \qquad (2)$$

The quantities A_2, B_2, D_2, and E_2 (defined subsequent to equation (20b) of reference 1) depend in part on the unknown reduced frequency k_n and on the unknown flutter frequency ω. Real combinations of k_n and ω which cause the determinant of coefficients to vanish

$$\Delta = \begin{vmatrix} A_2 & B_2 \\ D_2 & E_2 \end{vmatrix} = 0 \qquad (3)$$

are characteristic values and define the condition of flutter.

Some typical qualitative results of flutter speed.- For the purpose of gaining some insight into the case of pure-bending flutter, a sequence of curves of bending-torsion-flutter speed obtained by varying certain parameters is of interest. Figure 2 presents such a sequence, in progressive order from figure 2(a) to figure 2(d), of interrelated pairs of curves of flutter-speed coefficients $v/b\omega_\alpha$ and $v/b\omega_h$ as functions of wing frequency ratio ω_h/ω_α $\left(\text{that is, } \dfrac{v}{b\omega_\alpha} = \dfrac{v}{b\omega_h}\dfrac{\omega_h}{\omega_\alpha}\right)$. These curves are intended to show only qualitative results and they cover a fairly wide range of classes of swept cantilever wings.

Two relevant wing parameters that can bring about the progressive changes from figure 2(a) to figure 2(d) are: (1) a sweep parameter γ defined by the sweep angle and length-semichord ratio as

$$\gamma = \frac{\tan \Lambda}{l'/b_r}$$

and (2) a mass parameter $1/\kappa_e$ which is an integrated mean value, based on energy considerations, of the section mass-density ratio and is defined as

$$\frac{1}{\kappa_e} = \frac{\displaystyle\int_0^1 \left(\frac{b}{b_r}\right)^2 \frac{1}{\kappa}\left[F_h(\eta)\right]^2 \, d\eta}{\displaystyle\int_0^1 \left(\frac{b}{b_r}\right)^2 \left[F_h(\eta)\right]^2 \, d\eta} \tag{4}$$

where $F_h(\eta)$ is the amplitude function or shape of the bending mode. Since the mass parameter represents a ratio of wing mass to air mass, it can also give effects of altitude.

The flutter-speed coefficients $v/b\omega_\alpha$ and $v/b\omega_h$ as functions of ω_h/ω_α can change from those of figure 2(a) to those of figure 2(d) provided that: (1) $1/\kappa_e$ is sufficiently large and γ progresses from a small value (fig. 2(a)), through an intermediate value (fig. 2(b)), through a transitional value (fig. 2(c)), to a larger-than-transitional value (fig. 2(d)); or (2) γ is sufficiently large and $1/\kappa_e$ increases from a small value (fig. 2(a)), through an intermediate value (fig. 2(b)), through a transitional value (fig. 2(c)), to a larger-than-transitional value (fig. 2(d)). The values of γ and $1/\kappa_e$ which give curves as in figure 2(c) ($v/b\omega_\alpha$ has infinite slope at the origin) are designated transitional values and have the significance that they are crucial or borderline values separating a region in which a swept cantilever wing considered infinitely stiff in torsion cannot flutter and a region in which such a wing can flutter. In figure 2 only the shapes of the curves, particularly for small values of ω_h/ω_α, are of immediate interest, and the curves have no quantitative significance.

A feature of the curves of $v/b\omega_\alpha$ of figure 2 to be pointed out is that in figure 2(a) torsional frequency ω_α has the predominant positive influence on v for values of ω_h/ω_α less than unity but that bending frequency ω_h has a predominant influence on v for portions of the curves of figures 2(b), 2(c), and 2(d) at small values of ω_h/ω_α. This feature is brought out as follows: Any incremental segment of the curves of $v/b\omega_\alpha$ of figure 2 can be represented approximately by its

tangent given by

$$\frac{v}{b\omega_\alpha} = m_1 + m_2 \frac{\omega_h}{\omega_\alpha}$$

or

$$\frac{v}{b} = m_1\omega_\alpha + m_2\omega_h$$

where m_1 and m_2 are constants. Most of the experience to date with bending-torsion flutter has led generally to curves of $v/b\omega_\alpha$ of the type in figure 2(a), especially with regard to the feature that, for small values of ω_h/ω_α, the slope of the curve is small; that is, m_1 is large compared to m_2. Consequently, torsional frequency ω_α has a much greater influence on flutter speed than ω_h does, hence the usual torsional-stiffness criterion for prevention of bending-torsion flutter.

With a curve of $v/b\omega_\alpha$ as shown in figure 2(b) (as well as in figs. 2(c) and 2(d)) a portion of the curve with large slope at a small value of ω_h/ω_α (m_1 small compared to m_2) delineates a region where bending frequency ω_h has more influence on flutter speed than ω_α does. Thus, flutter speed v can be more influenced by bending frequency ω_h than by torsional frequency ω_α, even for lower-than-transitional values of $1/\kappa_e$ and γ (fig. 2(b)) as well as for larger values (figs. 2(c) and 2(d)).

The succeeding sections verify that the condition for bending-alone flutter is at a limit of the condition for coupled bending-torsion flutter (under certain provisions to be discussed) and effects of some parameters are evaluated.

Bending Flutter as a Limit of Bending-Torsion Flutter

If single-degree bending flutter does exist, the torsional amplitude $\underline{\theta}$ must be zero in equation (1); it then follows that the complex coefficient A_2 must be zero. In order that equation (3) also be satisfied, the coefficient E_2 must be infinite such that $A_2E_2 = B_2D_2$, since B_2 and D_2 are in general finite. Each of the coefficients A_2

and E_2' consist of an inertia and elastic part and an aerodynamic part. The inertia and elastic part of A_2 is

$$\frac{1}{\kappa_e}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2 \left(1 + ig_h\right)\right]$$

and of E_2 is

$$\left(\frac{r_\alpha^2}{\kappa}\right)_e \left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2 \left(1 + ig_\alpha\right)\right]$$

The mass parameter $1/\kappa_e$ is defined by equation (4) and, similarly, the mass-moment-of-inertia parameter $\left(r_\alpha^2/\kappa\right)_e$ is an integrated mean value defined as follows:

$$\left(\frac{r_\alpha^2}{\kappa}\right)_e = \frac{\int_0^1 \left(\frac{b}{b_r}\right)^4 \frac{r_\alpha^2}{\kappa}\left[F_\theta(\eta)\right]^2 \, d\eta}{\int_0^1 \left(\frac{b}{b_r}\right)^4 \left[F_\theta(\eta)\right]^2 \, d\eta}. \tag{5}$$

where r_α^2/κ is a section property.

The coefficient E_2 can be infinite only if ω_α/ω is infinite for a wing of finite $\left(r_\alpha^2/\kappa\right)_e$. The quantity ω_α/ω can be infinite if: (a) ω_α is infinite and ω is finite or (b) ω_α is finite and ω is zero. The coefficient A_2 can be zero only if ω_h/ω is not infinite for a wing of finite $1/\kappa_e$. Thus, under condition (a) ω_h must be finite since ω is finite, and under condition (b) ω_h must be zero since ω is zero. Examination of possible values of ω_h and ω_α reveals that under both conditions (a) and (b) the frequency ratio ω_h/ω_α is zero. The conclusion is thereby reached that the condition for coupled flutter (equation (3)) can contain the condition $A_2 = 0$ for pure-bending flutter only if ω_h/ω_α is zero. This is a necessary but not a sufficient condition and the further requirement exists that with $A_2 = 0$ the solution for ω_h/ω be real and positive.

It was found from a study of the aerodynamic part of A_2 that ω_h/ω is positive and real if $1/\kappa_e$ and the sweep parameter γ are

larger than certain interdependent minimum values, which turn out to be the transitional values discussed in the previous section. These transitional values give curves of flutter-speed coefficient as in figure 2(c), while larger-than-transitional values give curves as in figure 2(d).

Another facet of pure-bending flutter to be brought out is that the frequency ratio ω_h/ω_α can be zero in two ways, as might be deduced from the two conditions (a) and (b) just given; namely, for condition (a) ω_h is finite and ω_α infinite, and for condition (b) ω_h is zero and ω_α finite and greater than zero. The second possibility would seem more easily approached than the first in an experimental program on pure-bending flutter through the use of a bending hinge at the wing root restrained by a vanishingly weak spring.

Application to a Uniform Wing

<u>Determination of transitional values</u>.- For the purpose of simplicity of example, a uniform cantilever sweptback wing, the root of which is considered to behave structurally as shown in figure 1, is treated. The coefficient A_2 is set equal to zero and real solutions of v and ω represent flutter in bending alone. The result of setting A_2 of reference 1 equal to zero and multiplying by a convenient constant is

$$\left[1 - \left(\frac{\omega_h}{\omega}\right)^2 (1 + ig_h)\right]\frac{1}{\kappa} - A_{ch} + N_1 \gamma \left(\frac{i}{k_n}\right)(-1 + A_{ch}) - N_2 \gamma^2 \frac{1}{k_n^2} = 0 \qquad (6)$$

where

$$N_1 = \frac{\displaystyle\int_0^1 F_h(\eta) \frac{dF_h}{d\eta} \, d\eta}{\displaystyle\int_0^1 \left[F_h(\eta)\right]^2 d\eta}$$

and

$$N_2 = \frac{\displaystyle\int_0^1 F_h(\eta) \frac{d^2F_h}{d\eta^2} \, d\eta}{\displaystyle\int_0^1 \left[F_h(\eta)\right]^2 d\eta}$$

If the bending-mode shape $F_h(\eta)$ is chosen as that of an ideal uniform cantilever beam in first bending, N_1 is 2.0000 and N_2 is 0.85837.

Since equation (6) is complex, it can be divided into two equations as follows:

Equation of imaginaries (damping-force coefficients)

$$\frac{1}{\kappa}\left(\frac{\omega_h}{\omega}\right)^2 g_h + I_{ch} - N_1\gamma\left(-1 + R_{ch}\right)\frac{1}{k_n} = 0 \qquad (7)$$

Equation of reals (inphase-force coefficients)

$$\frac{1}{\kappa}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right] - R_{ch} - N_1\gamma\frac{1}{k_n}I_{ch} - N_2\gamma^2\frac{1}{k_n^2} = 0 \qquad (8)$$

Consider first the condition of zero structural damping $\left(g_h = 0\right)$. For any chosen value of k_n and the associated values of R_{ch} and I_{ch}, solutions can be made for γ and $\frac{1}{\kappa}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]$. Figure 3 gives

$\frac{1}{\kappa}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]$ for $g_h = 0$ (plotted on a log scale) as a function of γ for incompressible flow and for the compressible flow $M\cos\Lambda = 0.7$. Values of $1/k_n$ are indicated on each curve. All values of k_n which result in positive values of γ (indicating sweepback) also result in positive values for $\frac{1}{\kappa}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]$. This circumstance, together with the physical conditions that $1/\kappa_e$ and ω_h/ω are positive and real, requires ω_h/ω to fall in the range $0 \leqq \omega_h/\omega < 1.0$. Then, for any value of ω_h/ω between 0 and 1.0, $1/\kappa$ must always be greater than the value of $\frac{1}{\kappa}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]$. It follows therefore that, for a given value of γ, the ordinate $\frac{1}{\kappa}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]$ of figure 3 may be considered the smallest value of $1/\kappa$ which can be had and still allow single-degree bending flutter of a uniform wing to occur; the limiting value of $1/\kappa$ is actually associated with the condition $\frac{\omega_h}{\omega} = 0$. Of course the abscissa in figure 3 defines the lowest value of γ just as the ordinate defines

the lowest value of $1/\kappa$ and these are the transitional values which lead to curves of flutter-speed coefficient $v/b\omega_\alpha$ as in figure 2(c), discussed in a previous section. Thus, even though ω_h/ω_α is zero, the condition must be satisfied that $1/\kappa$ and γ for a uniform wing fall on or above the curves of figure 3 to enable flutter in bending alone. Figure 3 shows that the particular compressible flow $M \cos \Lambda = 0.7$ is predicted to have a very strong relieving effect on the requirements on $1/\kappa$ and γ for single-degree bending flutter. Both parameters can have much lower values than in incompressible flow.

For the condition of structural damping not zero, numerical solution is somewhat more involved. The transitional values of $1/\kappa$ and γ are unaffected by the value of g_h, however, since for these transitional values ω_h/ω is zero and thus the first term of equation (7) is zero.

Effect of some parameters on bending-alone flutter.- The results of figure 3 can be translated into various forms; for example, instead of γ for an abscissa, a specific value of l'/b can be chosen, and the angle of sweep Λ can be utilized as the abscissa. Such a transformation results in the curves of figure 4 for $\dfrac{l'}{b} = 6$. The ordinate of figure 4 is identical to that of figure 3 and the curves show again that the particular compressible flow $M \cos \Lambda = 0.7$ theoretically greatly decreases the required minimum (transitional) values of $1/\kappa$ and γ from those of incompressible flow.

Figure 5 gives results of flutter-speed coefficient $v/b\omega_h$ as a function of angle of sweepback Λ for three values of structural damping coefficient g_h and for the specific parameters $\dfrac{l'}{b} = 6$ and $\dfrac{1}{\kappa} = 1000$. The theoretical flutter-speed coefficient is greatly reduced in the compressible flow $M \cos \Lambda = 0.7$ from that of incompressible flow $M = 0$. For example, at $60°$ sweep and g_h of 0, the value of $v/b\omega_h$ is predicted to be 15.4 for the compressible flow and 51 for incompressible flow. This fact means that a value of $b\omega_h$ for the compressible flow is required to be 331 percent of that for incompressible flow in order that the two absolute flutter speeds be equal.

The curves of figure 5 also predict that for the particular parameters involved, small values of structural damping coefficient have a large effect on the flutter-speed coefficient. For instance, in incompressible flow for a sweepback angle of $60°$, an increase of g_h from 0 to 0.03 increases $v/b\omega_h$ by 85 percent. It was found that for lower values of $1/\kappa$, g_h does not have such a marked effect. The values of

sweep angle to which the two sets of curves of figure 5 are asymptotic are the values of Λ of figure 4 for an ordinate of 1000.

Figure 6 presents flutter-speed coefficients as a function of length-semichord ratio for $g_h = 0$, $\frac{1}{\kappa} = 1000$, and sweep angle of 60°. The curves of figure 5 for $g_h = 0$ and those of figure 6 can be considered part of a three-dimensional plot with mutually perpendicular coordinates of sweep angle, length-semichord ratio, and flutter-speed coefficient where the curves of the two figures have a juncture at $\Lambda = 60^\circ$ and $\frac{l'}{b} = 6$. As might be expected after noting figures 3, 4, and 5, the compressible flow $M \cos \Lambda = 0.7$ is seen in figure 6 to have a strong detrimental effect compared to incompressible flow.

The surprising feature of figure 6 is that as l'/b decreases, the flutter-speed coefficient is predicted to decrease. (If b is held constant and l' reduced, flutter speed v itself will increase, however, since ω_h is approximately inversely proportional to l'^2.) This result is in contrast to that for coupled flutter of unswept wings where a decrease in aspect ratio has led generally to an increase in flutter-speed coefficient. The effect given in figure 6 is based on two-dimensional flow and the actual behavior of flutter-speed coefficient for heavy swept wings as l'/b decreases may be a compromise between the predicted variation shown in the figure and the increase found for unswept wings in three-dimensional flow.

Quantitative relation of pure-bending to bending-torsion flutter.- Quantitative relations of flutter speed, frequency, and other characteristics can be obtained through solution of the flutter determinant (equation (3)) as the bending-torsion frequency ratio is considered to increase from its zero limit (ω_α decreases from an infinite value for finite values of ω_h). The flutter departs from its pure-bending limit and becomes of a coupled type. For the present examples only two modes, first bending and first torsion, are included.

The point in figure 5 for incompressible flow, $g_h = 0$, and $\Lambda = 60^\circ$ has been chosen and ω_h/ω_α varied from 0 to about 1.1. The results of flutter-speed coefficients $v/b\omega_h$ and $v/b\omega_\alpha$ as well as frequency ratio ω/ω_h as functions of ω_h/ω_α are shown in figure 7. Qualitative similarity of the curves of $v/b\omega_h$ and $v/b\omega_\alpha$ to those of figure 2(d) can be noticed. It is observed that a decrease of ω_α from an infinite value to a value five times ω_h $\left(\frac{\omega_h}{\omega_\alpha} = 0.2\right)$ effects a decrease in $v/b\omega_h$

of only about 22 percent. Thus, in the range $0 \leq \dfrac{\omega_h}{\omega_\alpha} \leq 0.2$ torsional

rigidity, represented by ω_α, theoretically has a relatively small effect on the flutter speed for the wing parameters of figure 7. The ratio ω/ω_h of flutter frequency to bending frequency is seen to

decrease from about 1.3 at $\dfrac{\omega_h}{\omega_\alpha} \doteq 0$ to about unity for $\dfrac{\omega_h}{\omega_\alpha} = 1.0$.

Another example of the effect of finite ω_α is shown in figure 8 which applies to a different set of wing parameters both in incompressible flow and in the compressible flow $M \cos \Lambda = 0.7$. The significantly differing wing parameter is the mass ratio $1/\kappa$, which has a value of 112. Such a wing is lighter or is at a lower altitude than is the wing of figures 5 and 6, which has $\dfrac{1}{\kappa} = 1000$. The sweep param-

eter for the case in figure 8 is 0.2. It is apparent from the curve for the compressible flow in the region of ω_h/ω_α between 0 and about 0.03 that a decrease of ω_α from infinity has little effect on the predicted flutter-speed coefficient, but a further decrease of ω_α has a somewhat greater effect. Solution for incompressible flow does not predict a bending-alone flutter for ω_h/ω_α of zero since $1/\kappa$ of 112 falls below

the minimum required value for γ of 0.2 ($\Lambda = 45^\circ$, $\dfrac{l'}{b} = 5$) shown in

figure 3, and both $v/b\omega_h$ and ω/ω_h approach infinite values as ω_h/ω_α approaches zero. The results of figure 8 predict that the compressible flow dealt with has a marked detrimental effect on flutter-speed coefficient for small values of ω_h/ω_α. For a value of ω_h/ω_α of 0.2, for example, $v/b\omega_h$ in the compressible flow $M \cos \Lambda = 0.7$ is only 53 percent of that for incompressible flow. It follows that, in order for borderline flutter to occur at the same absolute speeds in incompressible and in the compressible flow, the wing stiffness in terms of $b\omega_h$ must be 188 percent as great in the compressible flow as in incompressible flow.

For this swept-wing example as well as in the other figures, the compressibility (Mach number) effect was found to be greater than has been found for unswept wings in previous work. It is reemphasized that the present results are based on two-dimensional flow and that finite span is expected to have an appreciable effect.

DISCUSSION

The over-all findings of the present analysis seem to indicate that calculations of single-degree bending flutter, based on coefficients for two-dimensional incompressible flow, are of interest at least as a theoretical limiting condition. Such calculations predict that a wing, in order to flutter in single-degree bending, must possess a combination of very large mass-density ratio $1/\kappa$ and large sweep parameter as well as satisfy the condition of zero frequency ratio ω_h/ω_α. This specified combination of parameters conceivably could be approached by a highly swept, stubby wing, which is very stiff in torsion and which carries a heavy fuel tank on its tip. As a result bending frequency ω_h could be more influential on the flutter speed than torsional frequency ω_α.

A simplified attempt to approximate some effect of compressibility for a particular compressible flow, $M \cos \Lambda = 0.7$, resulted in more serious predictions. It is predicted that compressibility has a strikingly unfavorable effect both on single-degree flutter (frequency ratio ω_h/ω_α of zero) and on related coupled flutter (frequency ratio not zero) of a heavy sweptback wing with low length-semichord ratio.

The use of aerodynamic coefficients for two-dimensional compressible flow may result in overestimation of effects of compressibility on the flutter of sweptback wings. Compressibility may, however, have a greater effect on a low-length-semichord-ratio swept wing than on a low-aspect-ratio unswept wing. It remains for experimental investigation to give effects of three-dimensional compressible flow.

CONCLUSIONS

The possibility of flutter in bending alone is examined. The analysis is essentially that given in NACA Rep. 1014 and employs aerodynamic coefficients of two-dimensional incompressible and compressible flow. As a result of the analysis and application to uniform wings, the following conclusions are made:

1. A cantilever sweptback wing will theoretically flutter in bending alone in potential flow even though it is infinitely stiff in torsion if it is heavy enough relative to the surrounding medium and has a sufficiently large sweep parameter.

2. Flutter in bending alone is a limiting case of the better-known coupled flutter as the bending-torsion frequency ratio approaches zero under the two conditions stated in the first conclusion.

3. Computations based on two-dimensional compressible-flow coefficients for the normal-component Mach number of 0.7 indicate that compressibility has a marked detrimental effect on bending-alone flutter of a heavy sweptback wing of low length-semichord ratio, first in that the conditions on mass ratio and sweep parameter are greatly relaxed, and second, that a wing is required to have considerably higher bending frequency to prevent flutter in the compressible flow than in incompressible flow.

4. Even though a heavy wing of high sweep parameter does not satisfy the condition of zero bending-torsion frequency ratio, if that ratio is low, coupled-flutter speed may depend more on wing bending than on torsional frequency. Furthermore, the effect of compressibility on the coupled flutter of such a sweptback wing may be considerably greater than has been found for low-aspect-ratio unswept wings.

5. Effects of finite span have not been considered but are expected to be appreciable for low-aspect-ratio wings.

Langley Aeronautical Laboratory
    National Advisory Committee for Aeronautics
        Langley Field, Va., June 19, 1951

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Actual root

Section A-A

A

Effective root

Λ

Elastic axis

B

Midchord line for
sections normal
to the elastic axis

A

Y'

$\underline{h}F_h(\eta)$

$\underline{\theta}\,F_\theta\,(\eta)$
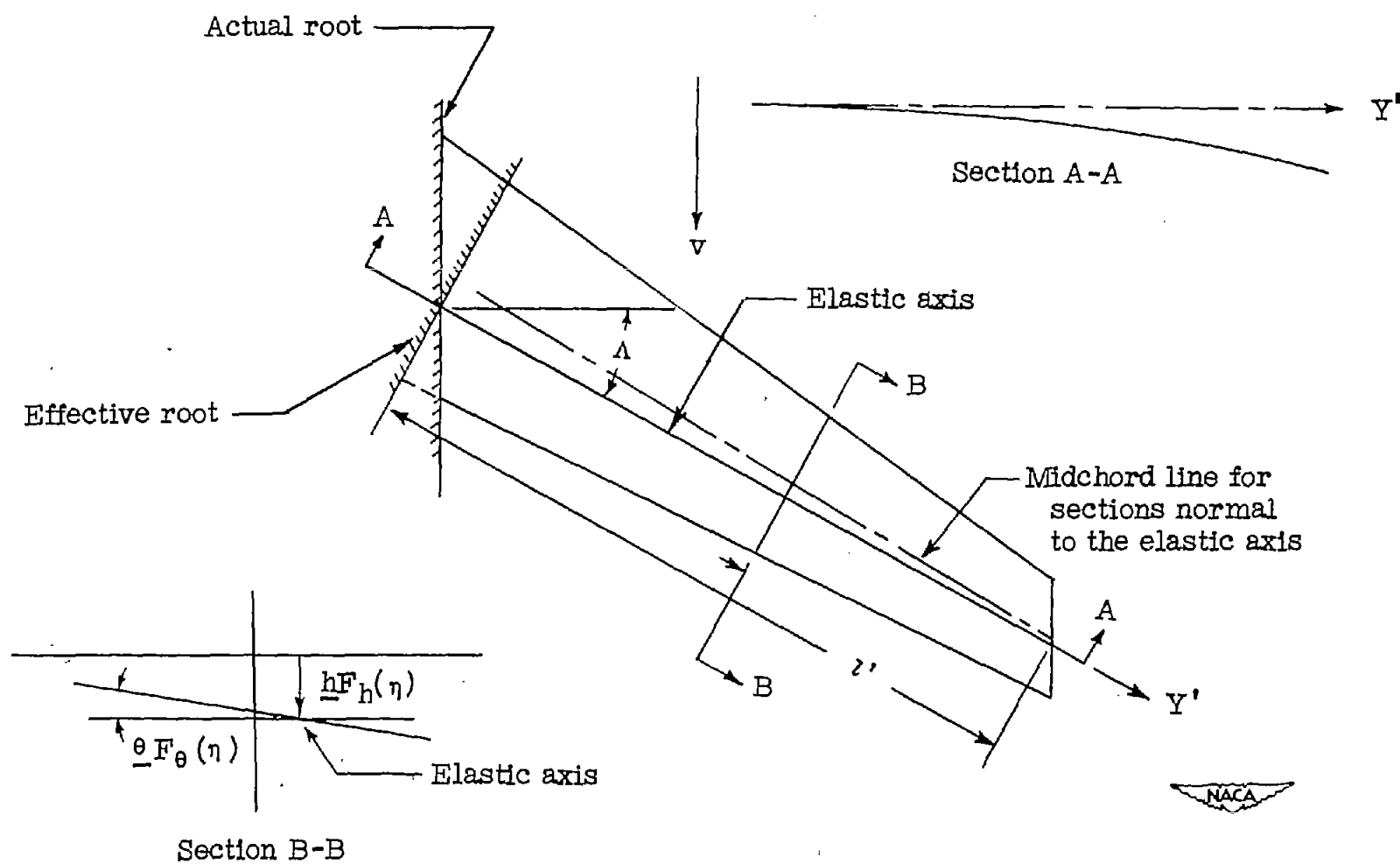
Elastic axis

B

ℓ'

Y'

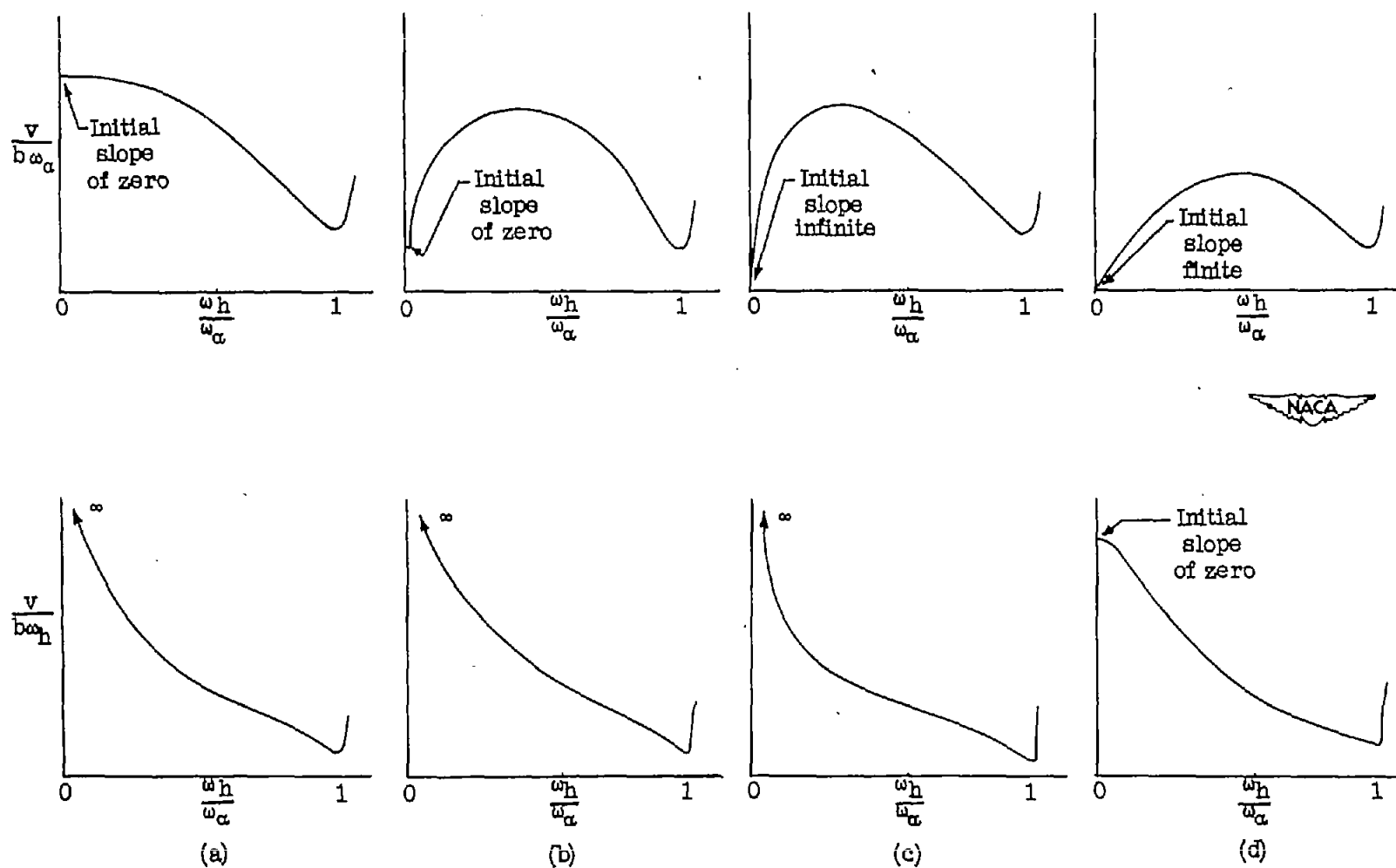Section B-B

Figure 1.- Swept wing of the present analysis.

Figure 2.- A sequence of typical qualitative results of associated flutter-speed coefficients $v/b\omega_h$ and $v/b\omega_\alpha$ as functions of wing frequency ratio ω_h/ω_α.
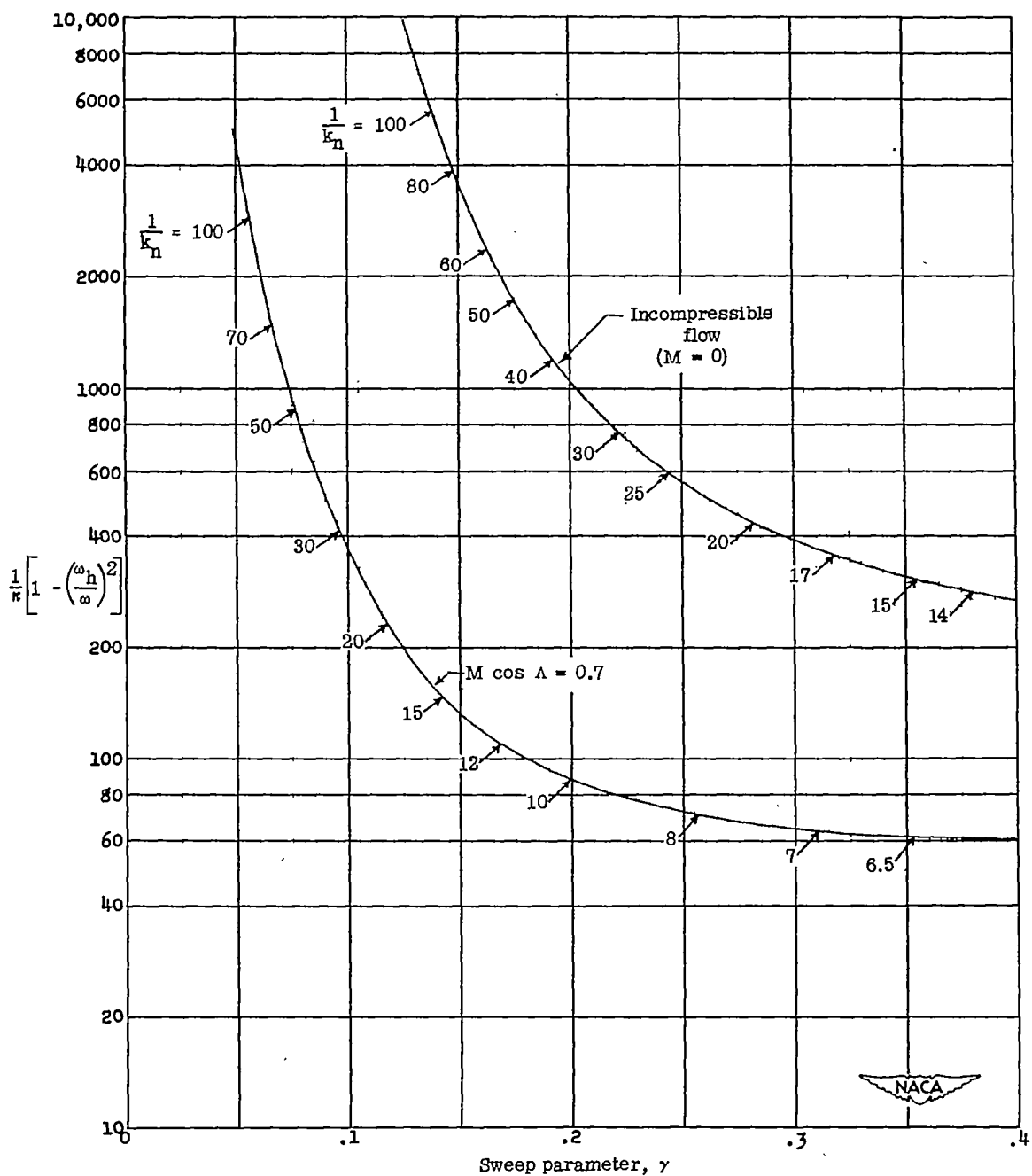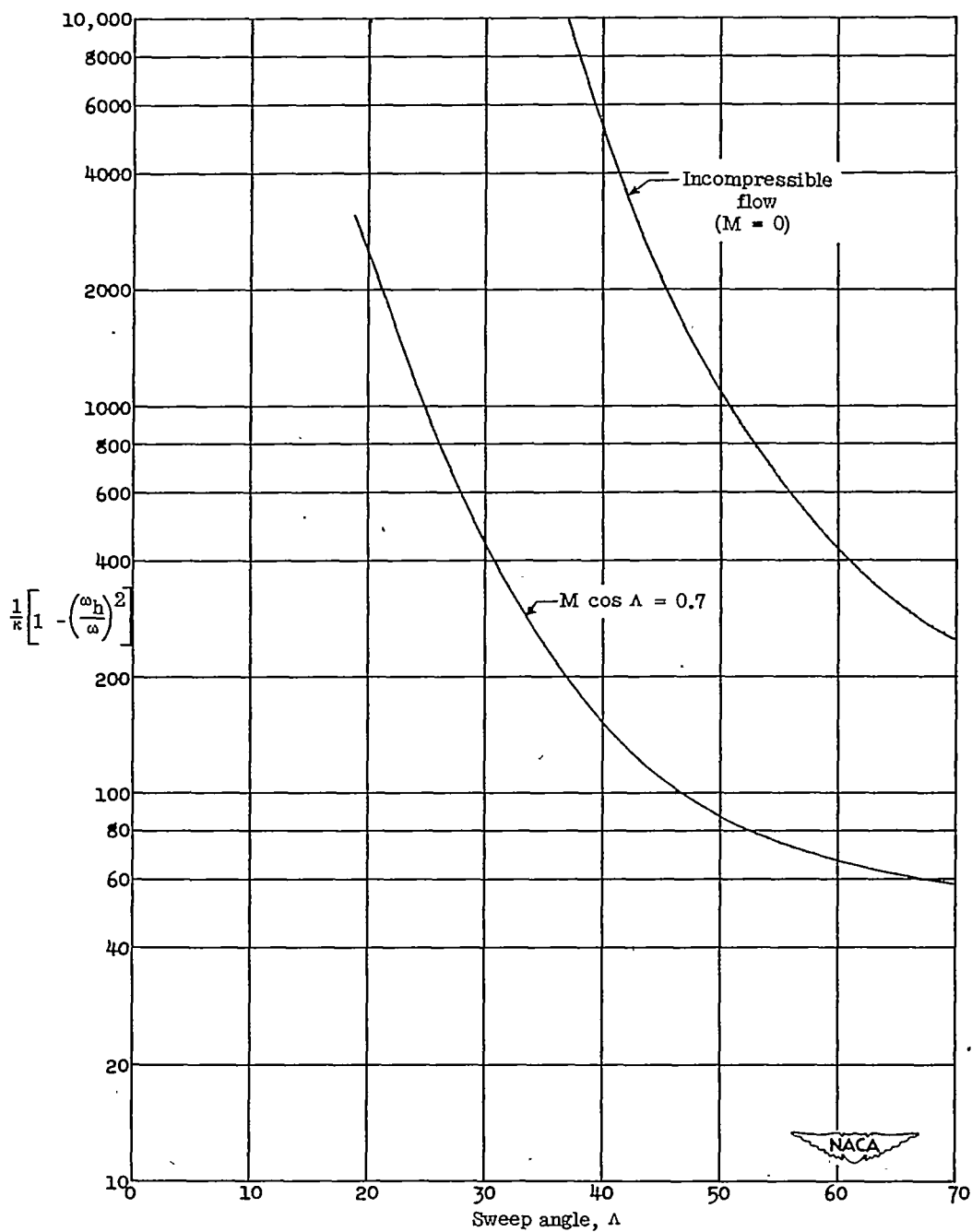
Figure 3.- Curves which give $\frac{1}{\kappa}\left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]$ for $g_h = 0$ as well as the lower limit of mass-density ratio $1/\kappa$ as functions of the sweep parameter γ for single-degree bending flutter of uniform cantilever swept wings in incompressible flow and in the compressible flow $M \cos \Lambda = 0.7$.

Figure 4.- Curves which give $\dfrac{1}{\kappa}\left[1 - \left(\dfrac{\omega_h}{\omega}\right)^2\right]$ for $g_h = 0$ as well as the

lower limit of mass-density ratio $1/\kappa$ as functions of sweep angle Λ for single-degree bending flutter of uniform cantilever swept wings of length-semichord ratio 6 in incompressible flow and in the compressible flow $M \cos \Lambda = 0.7$.
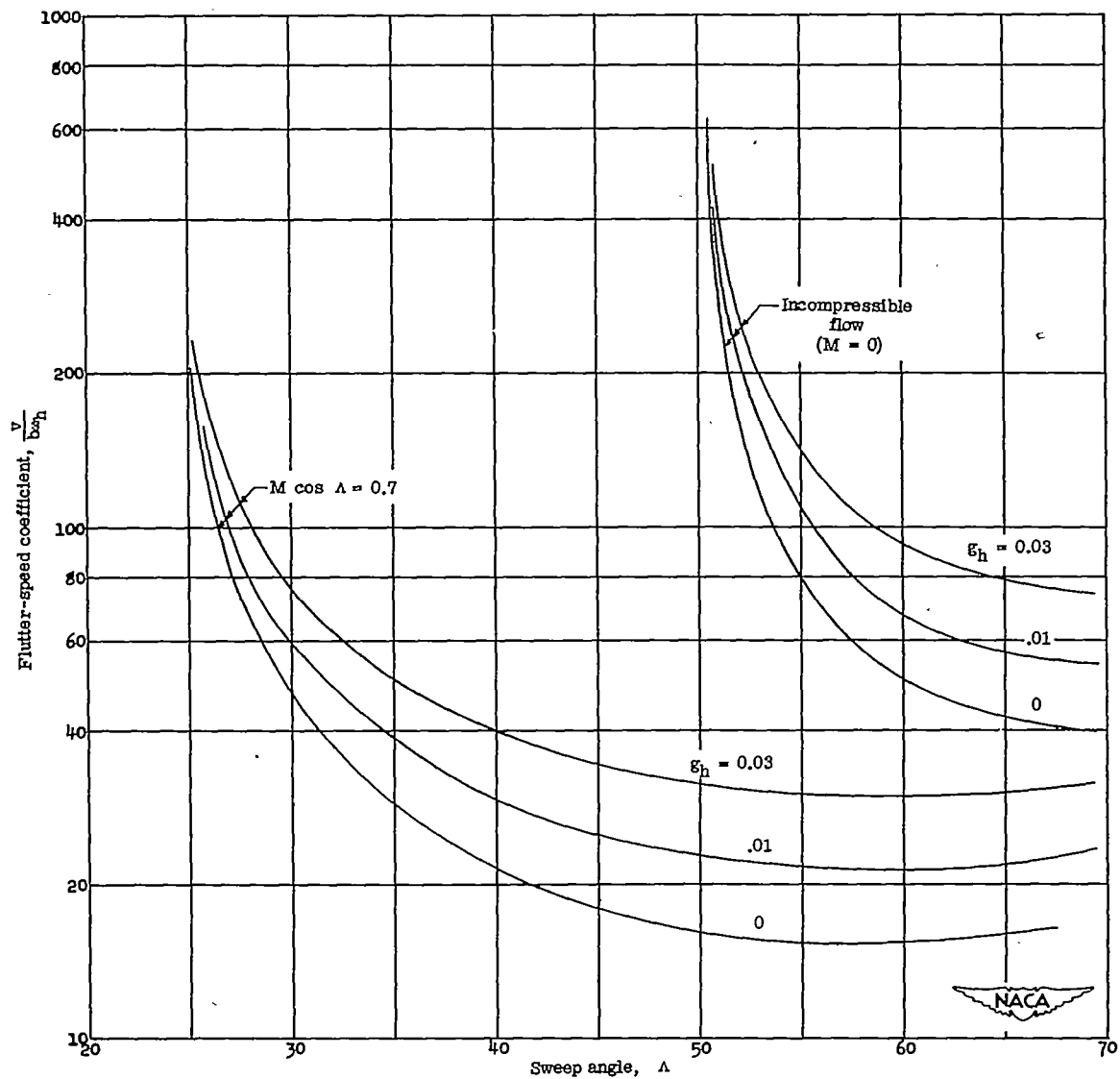
Figure 5.- Flutter-speed coefficient $v/b\omega_h$ as a function of sweep angle Λ for single-degree bending flutter of a uniform cantilever wing of $\frac{1}{\kappa} = 1000$ and $\frac{l'}{b} = 6$ for three values of structural damping coefficient in incompressible flow and in the compressible flow $M \cos \Lambda = 0.7$.
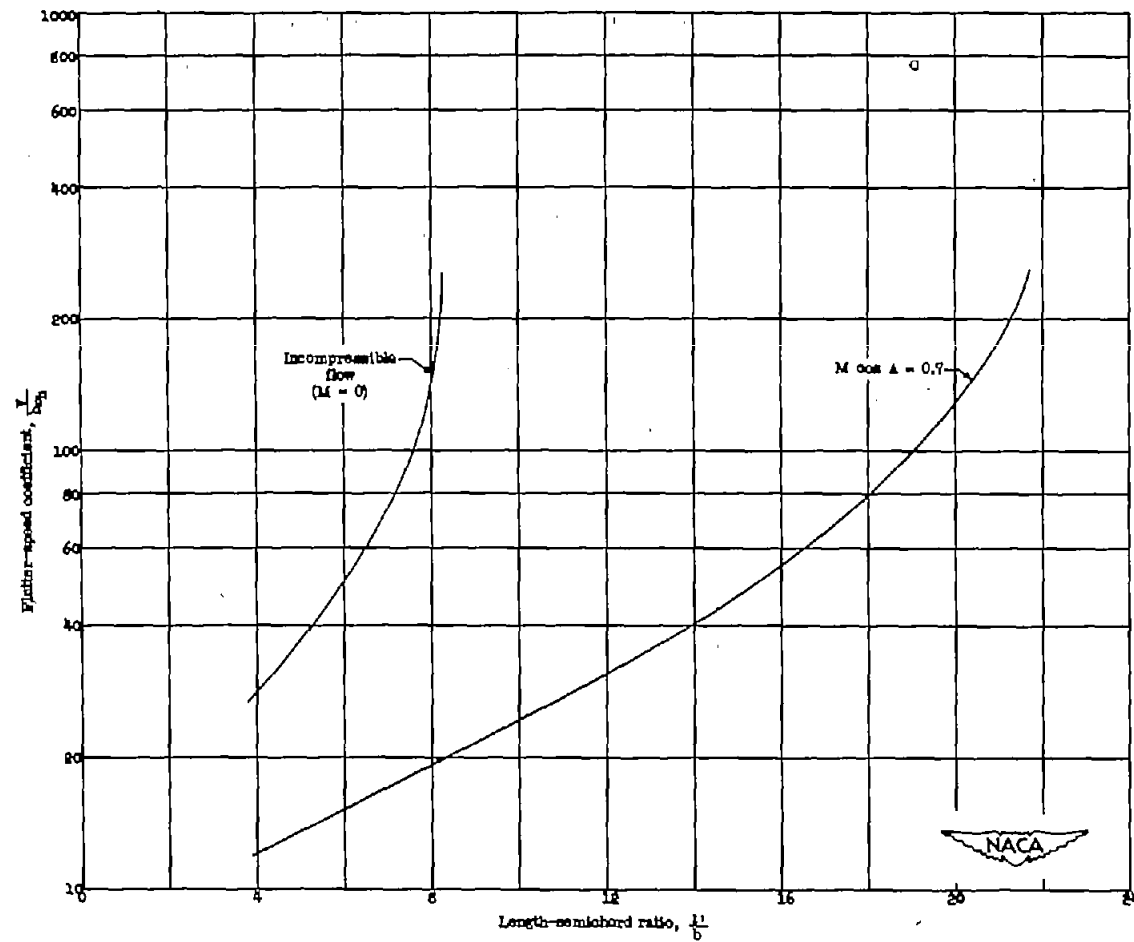
Figure 6.- Flutter-speed coefficient $v/b\omega_h$ as a function of length-semichord ratio l'/b for single-degree bending flutter of a uniform cantilever wing of $\frac{1}{\kappa} = 1000$, $\Lambda = 60^\circ$, and $g_h = 0$ in incompressible flow and in the compressible flow $M \cos \Lambda = 0.7$.
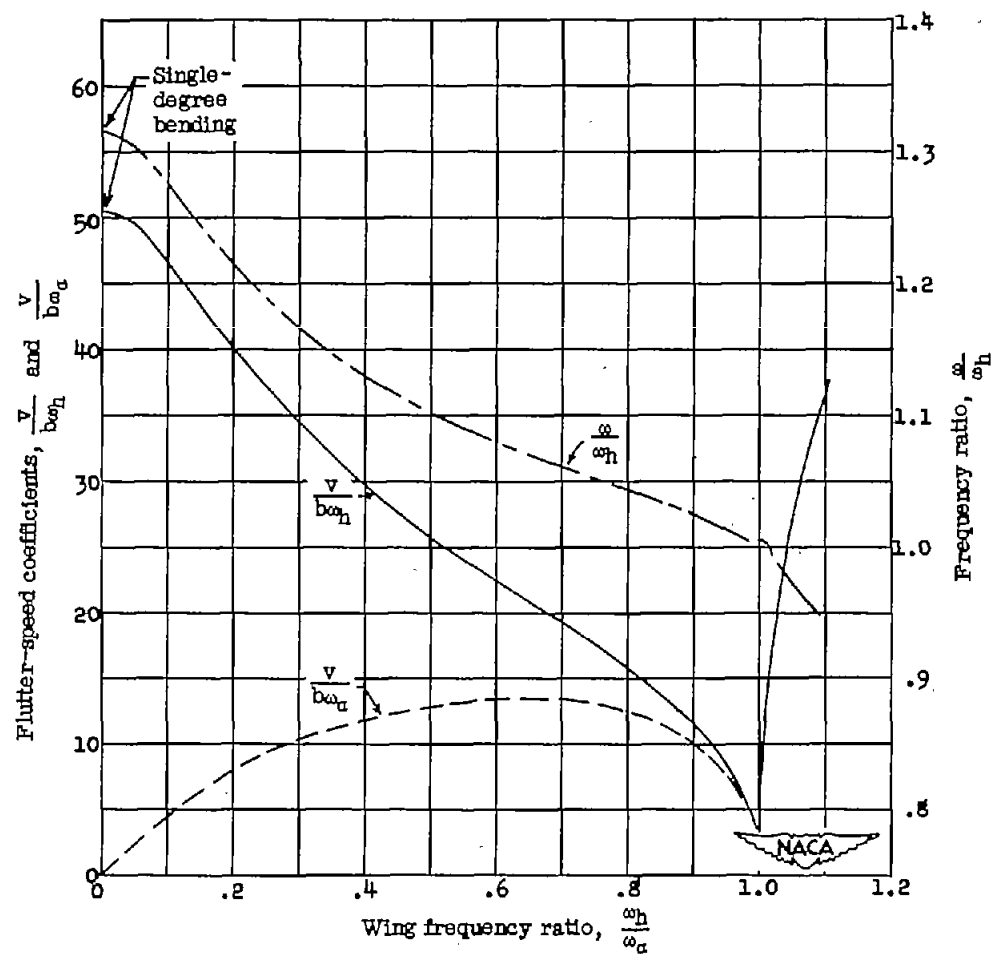
Figure 7.- Flutter-speed coefficients $v/b\omega_h$ and $v/b\omega_\alpha$ and frequency ratio ω/ω_h as functions of wing frequency ratio ω_h/ω_α for a uniform cantilever wing having the following parameters: $\Lambda = 60^\circ$; $\frac{l'}{b} = 6$; $\frac{1}{\kappa} = 1000$; $x_{ea} = 40$; $x_{cg} = 40$; $r_\alpha^2 = 0.25$; and $g_h = g_\alpha = 0$.
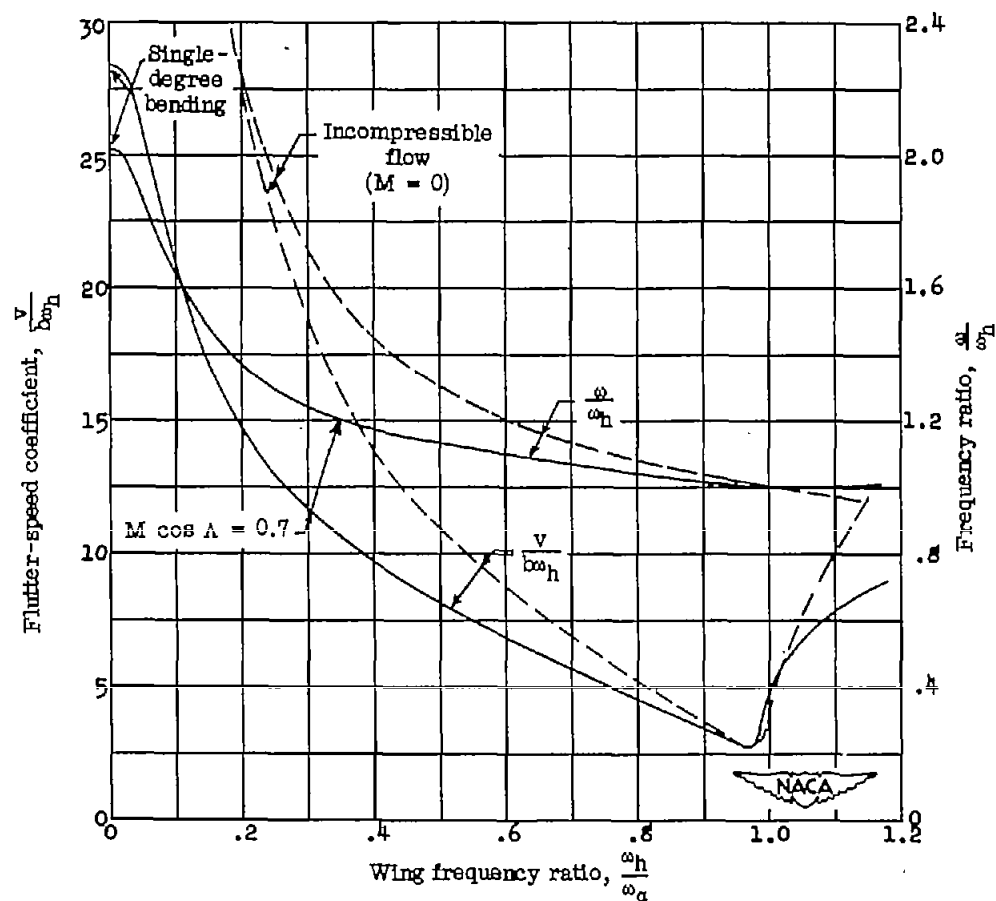
Figure 8.- Flutter-speed coefficient $v/b\omega_h$ and frequency ratio ω/ω_h as functions of wing frequency ratio ω_h/ω_α in incompressible flow and in the compressible flow $M \cos \Lambda = 0.7$ for a uniform cantilever wing having the following parameters: $\Lambda = 45°$; $\dfrac{l'}{b} = 5$; $\dfrac{1}{\kappa} = 112$; $x_{ea} = 40$; $x_{cg} = 40$; $r_\alpha^2 = 0.25$; and $g_h = g_\alpha = 0$.