NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2546

VISCOSITIES OF AIR AND NITROGEN AT LOW PRESSURES

By Herrick L. Johnston, Robert W. Mattox, and Robert W. Powers

The Ohio State University

Washington
November 1951

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2546

VISCOSITIES OF AIR AND NITROGEN AT LOW PRESSURES

By Herrick L. Johnston, Robert W. Mattox, and Robert W. Powers

SUMMARY

Information is presented of the results of an investigation to measure the true viscosities of nitrogen and air at 306° and 273° K at pressures from 500 to 0.0005 millimeter of mercury and at 194° and 79° K at pressures down to 0.3 millimeter of mercury.

In the region in which the thickness of the gas layer is many times the mean free path of the gas (the region of slip), the "slip coefficient" - defined as the proportionality constant existing between the slip velocity and the velocity gradient - was found to show a linear dependence on the inverse pressure down to about 0.02 millimeter of mercury.

In the region in which the pressures are sufficiently low to permit the neglect of collisions between molecules, theory has indicated that the viscous drag should be directly proportional to the pressure and inversely proportional to the square root of the temperature. Results of the present paper indicate that this relationship was reached at pressures a little below 0.02 millimeter of mercury.

INTRODUCTION

Ordinary gas kinetic theory leads to the conclusion that the viscosity of a gas at low and moderate pressures is independent of its pressure. This conclusion has been confirmed experimentally for pressures between about 0.1 and a little more than 1 atmosphere. At higher pressures, there is an increase in viscosity, that in the case of carbon dioxide (CO_2), for example, amounts to about 8 percent at 24 atmospheres and to about 200 percent at 100 atmospheres. At the latter pressure, the viscosity of CO_2 is almost linear with its density. (See reference 1, p. 150.)

At pressures below about 0.1 atmosphere, the measured viscosity decreases with decreasing pressure, gradually approaching a linear relationship. Down to pressures of about 0.001 atmosphere, this decrease is usually attributed to "slip" of gas layers over adjacent surfaces.

It is interpreted as being due to a sharp decrease in the velocity gradient, normal to the surface, of gas flow parallel to the surface. (See reference 1, p. 292.) There is an apparent reduction of viscous drag on the surface due to slip. This effect appears as an apparent reduction in the viscosity of the gas, since viscous drag always occurs on some surface in viscosimetry. It is probable, however, that the gas maintains its ordinary viscosity at some distance from the surface, in the sense that there is no reduction in the shearing force between laminar layers of gas that are moving parallel to each other with different velocities.

The measured values of the viscosity in the intermediate pressure region decrease with pressure and depend upon the dimensions of the apparatus, particularly upon the distance between the disks or cylinders in an oscillating-disk or cylinder viscosimeter. Such measurements, however, can be used to obtain a coefficient of slip ζ which can then be used to compute the viscous drag of an apparatus of different dimensions.

The conception of slip is only applicable in the interpretation of viscous drag at low pressures when the thickness of the gas layer is many times the mean free path of the gas. When the pressure becomes so low that collisions in the gas phase are infrequent compared with collisions with the surface, neither the ordinary concept of a shearing force between laminar gas layers nor the concept of surface slip is applicable. However a viscous drag still exists and can be explained in terms of an exchange of momentum between the gas molecules and the moving surface. Values computed from viscosimeter data in the region of very low pressures, usually below 10^{-5} atmosphere, are termed "free-molecule viscosities." They are directly proportional to the pressure and are independent of the dimensions of the apparatus, provided there is a high proportion of surface collisions.

Measurements that established the foregoing concepts of viscous drag at low pressures were made by Kundt and Warburg (reference 2), by Timiriazeff (reference 3), by Hogg (reference 4), by Stacy (reference 5), by Van Dyke (reference 6), and by Blankenstein (reference 7). However, the total literature on low-pressure viscosities is relatively scanty. An investigation was therefore initiated at this Laboratory to determine the viscosities of air and of nitrogen at low pressures and low temperatures. Apparent viscosities have been measured at 306° K and at 273° K over the pressure range 0.0005 to 500 millimeters of mercury (Hg) and at 194° K and at 79° K down to pressures of 0.3 millimeter of Hg. The latter measurements could not be extended to lower pressures because of erratic drifts of the "zero point" of the suspension in the oscillating-disk viscosimeter.

This work was done by The Ohio State University under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

APPARATUS AND PROCEDURE

The gases used in this investigation were carefully purified. The air was freed of carbon dioxide and of moisture by passing it through successive towers of concentrated potassium hydroxide (KOH), concentrated sulfuric acid $\left(H_2SO_4\right)$, and stick potassium hydroxide (KOH), and thence through a dry-ice trap and over a drier of phosphorous pentoxide $\left(P_2O_5\right)$. Hy-Purity nitrogen was obtained from the Linde Air Products Co., who represented it to be better than 99.99 percent pure, and was passed through a dry-ice trap and over P_2O_5. Measurements were made in an oscillating-disk viscosimeter described in an earlier paper (reference 8) from this Laboratory. It has a moving disk of burnished silver, 1 millimeter thick and 3 centimeters in diameter, that executes rotational oscillation between two fixed silver disks. There is a clearance of 1 millimeter between the movable disk and each of the fixed disks.

The constantan suspension wire used in the original apparatus (reference 8) was replaced by a 0.001-inch manganin wire, in order to reduce the energy dissipated in torsion of the wire. The method used to anneal constantan suspensions, heating electrically in vacuo to a dull red, was unsuccessful with manganin and resulted in a brittle manganin wire. The manganin wire was, therefore, annealed by heating at 150° C in a vacuum oven for 48 hours.

The pressure inside the apparatus was measured by two methods. A mercury manometer was used for pressures between 10 and 500 millimeters of Hg, and a McLeod gage for pressures below 10 millimeters of Hg.

Experiments were conducted at 306° K by immersing the bulb of the apparatus in a manually controlled constant-temperature bath of water, at 273.2° K in a mixture of ice and water, at 195° K in a dry-ice - acetone bath, and at 79° K in liquid nitrogen.

CALIBRATION

Maxwell (reference 9) derived the following fundamental equation for the oscillating-disk viscosity apparatus:

$$\eta = (\lambda - x)/Ct \qquad (1)$$

where η is the viscosity, λ the logarithmic decrement, x the wire constant, t the period of one oscillation in seconds, and C the apparatus constant.

The logarithmic decrement can be represented as

$$\lambda = \left[\log L_n - \log L_{(n+i)}\right] / i \qquad (2)$$

where L_n is the full amplitude of the nth oscillation and n takes on the values 1, 2, . . . , i. The present custom has been to take series of 10 oscillations ($i = 5$) for gas runs. The five values for λ thus obtained ordinarily agreed to within 0.1 percent.

As in the earlier work (reference 8), the apparatus constant was determined by measurements of logarithmic decrements in air at temperatures close to 296.1° K. The viscosity of air at 296.1° K was assumed to be 1833.0×10^{-7} poise. Wire constants were determined from logarithmic decrements in vacuo. Values of the wire constants for the three different manganin suspensions used in the course of this investigation and the corresponding apparatus constants are given in table I.

EXPERIMENTAL RESULTS

The data obtained for nitrogen are given in table II and figure 1, and those for air in table III and in figure 2. The viscosities shown in the tables were computed by equation (1), using the values of the apparatus constant listed in table I. Corrections were applied for the changes in dimensions due to thermal contraction of the metal disks (reference 8) and, at low pressures and at temperatures below 273° K, for slight superheating of the gas between the disks due to heat leak down the suspension system. The latter correction amounted to about 0.2 percent for air at 193° K, but increased to about 1 percent at 80° K.

Data obtained earlier at this Laboratory by Johnston and McCloskey (reference 8) were interpolated to the temperatures employed in this investigation. When the present results for pressures above 200 millimeters of Hg are compared with those of reference 8, which correspond to atmospheric pressure, a mean deviation of -0.17 percent exists for nitrogen and of 0.00 percent for air.

DISCUSSION AND INTERPRETATION OF RESULTS

Region of Slip

Viscosity can be defined as the coefficient of shearing force between laminar layers of fluid that move parallel to each other at different velocities. For layers in the YZ-plane that move in the Y-direction, this quantity can be represented by the relationship

$$P_y = \eta \left(dV_y / dX \right) \tag{3}$$

where V_y is the mass velocity of the fluid and varies from layer to layer, dV_y/dX is the velocity gradient normal to the direction of flow, and η is the coefficient of the Y component of the shearing force P.

In ordinary viscous flow parallel to a solid surface, there is a constant velocity gradient dV_y/dX normal to the surface. This gradient extends from the thin fluid layer adjacent to the surface, whose velocity relative to the surface is zero, to the point of the fluid where the velocity is at a maximum. This gradient produces a viscous drag on the surface.

With slip flow, the velocity gradient is uniform in the bulk of fluid but increases rapidly near the surface. The result is as though the velocity itself decreased uniformly to a finite value u at the surface and then fell abruptly to zero. The effect would correspond to slip of the bulk fluid over the surface at a velocity u. In terms of ordinary frictional forces, a proportionality should exist between the slip velocity u and the velocity gradient dV_y/dX. This relation defines a proportionality constant ζ, termed the "slip coefficient," as

$$u = \zeta \left(dV_y / dX \right) \tag{4}$$

The constant ζ depends upon the nature of the surface, particularly upon its degree of roughness. Probably it is also dependent upon the characteristics of the fluid. Equation (4) shows that ζ also corresponds to the distance by which the coordinate X must be extended beyond the surface to reduce V_y to zero, if the gradient remained constant at its slip value. Maxwell (reference 9) showed that the slip coefficient almost equals the mean free path, for slip over smooth surfaces.

It is possible to evaluate the slip coefficient from measurements with an oscillating-disk viscosimeter if the apparent viscosity, calculated from data taken in the slip region of pressure, is compared with the true viscosity obtained at normal pressures. This evaluation can be made by combining equation (1) with the following equation for the apparatus constant, by Keesom and Van Itterbeek (reference 10):

$$C = \pi r^4 \left(d_u + d_l\right) \Big/ \left(4 I d_u\, d_l\right) \tag{5}$$

where r is the radius of the rotating disk, I the moment of inertia of the suspension system, and d_l and d_u the spacings that separate the rotating disk from the upper and lower fixed disks, respectively. This equation reduces to

$$C = \pi r^4 / 2dI \tag{6}$$

for an apparatus like the present one, in which the rotating disk is centered between the two fixed disks so that $d_u = d_l = d$.

The apparatus constant in equation (1) is not a true constant when slip occurs. Using equation (1) with an apparatus constant determined in the viscous region, therefore, will not give the true viscosity but, instead, will give an apparent viscosity that decreases with pressures. However, the true gas viscosity, which is associated with the uniform velocity gradient away from the wall, can be calculated from equation (1) if the apparatus constant is corrected for the reduction in viscous drag due to slip at the surfaces of the disks. This correction is made by adding 2ζ to the plate separation d of equation (6). If η_p represents the apparent viscosity obtained from the uncorrected form of equation (1) and if η is the true viscosity of the gas obtained by substituting the corrected apparatus constant for slip C_s into equation (1), the following relationship is obtained:

$$\eta/\eta_p = C/C_s = (d + 2\zeta)/d$$

which can be rearranged as

$$\zeta = (d/2)\left[\left(\eta/\eta_p\right) - 1\right] \tag{7}$$

Equation (7) has been used to calculate the slip coefficients of air and of nitrogen from the present data for pressures between 10 and 0.01 millimeter of Hg. The results are summarized in tables IV and V and are plotted in figures 3 and 4, which show a linear dependence on the inverse pressure. Constants in the equation

$$\zeta = K(1/P) \qquad (8)$$

are listed in table VI for both air and nitrogen. Figure 5 is a plot for nitrogen at 306° and at 273° K extended to 0.01-millimeter pressure. As the graph indicates, the linear dependence on $1/P$ continues down to about 0.02 millimeter of Hg where noticeable departure from linearity begins.

It was thought, from the relationships shown in table VI, that the slip coefficients might be proportional to the specific volumes or reciprocals of density at all temperatures.

Figures 6 and 7 are plots of slip coefficient against mean free molecular path for both nitrogen and air. The results with nitrogen appear to bear out the reciprocal density for the higher-temperature isotherms, but the 78° isotherm falls below the others. The 80° isotherm for air also falls below the straight line drawn for the 306° and 275° isotherms, and the lowest pressure point at 194° also falls low.

Region of Free-Molecule Viscosity

Theory (see reference 1, p. 300) indicates that, at pressures sufficiently low to permit neglect of collisions between molecules, the viscous drag exerted by a gas enclosed between two parallel plates moving with a relative velocity U is $-ZU$. The value Z is the free-molecule viscosity and is represented by

$$Z = P/(2\pi RT)^{1/2} \qquad (9)$$

where R and T represent the molecular gas constant and the absolute temperature, respectively. This expression indicates that the drag should be directly proportional to P and inversely proportional to the square root of T. Figure 8 indicates that this relationship was reached at pressures a little below 0.02 millimeter of Hg for both air and nitrogen.

The Ohio State University
Columbus, Ohio, November 20, 1950

REFERENCES

1. Kennard, Earle H.: Kinetic Theory of Gases. First ed., McGraw-Hill Book Co., Inc., 1938.

2. Kundt, A., and Warburg, E.: Ueber Reibung und Waermeleitung verduennter Gase. Ann. Phys. und Chemie, Folge 2, Bd. 155, 1875, pp. 337-365.

3. Timiriazeff, A.: Viscosity of Very Dilute Gases, and the Relation between Gliding and the Temperature Rise at the Boundary between Metal and Gas. Ann. Phys., ser. 4, vol. 40, 1913, pp. 971-990.

4. Hogg, J. L.: Friction and Force Due to Transpiration as Dependent on Pressure in Gases. Proc. Am. Acad. Arts and Sci., vol. 42, 1906, pp. 115-146.

5. Stacy, Leland Johnson: A Determination by the Constant Deflection Method of the Value of the Coefficient of Slip for Rough and for Smooth Surfaces in Air. Phys. Rev., vol. 21, no. 3, March 1923, pp. 239-249.

6. Van Dyke, Karl S.: The Coefficients of Viscosity and of Slip of Air and of Carbon Dioxide by the Rotating Cylinder Method. Phys. Rev., vol. 21, no. 3, March 1923, pp. 250-265.

7. Blankenstein, E.: Coefficients of Slip and Momentum Transfer in Hydrogen, Helium, Air and Oxygen. Phys. Rev., vol. 22, 1923, pp. 582-589.

8. Johnston, Herrick L., and McCloskey, Kenneth E.: Viscosities of Several Common Gases between 90° K and Room Temperature. Jour. Phys. Chem., vol. 44, no. 9, Dec. 1940, pp. 1038-1058.

9. Maxwell, J. C.: V. Illustrations on the Dynamical Theory of Gases - Part I. On the Motions and Collisions of Perfectly Elastic Spheres. Phil. Mag., vol. 19, 1860, pp. 19-32; also, Collected Works. Vol. 2. Cambridge Univ. Press (London), 1890.

10. Keesom, W. H., and Van Itterbeek, A.: Measurement of the Viscosity of Oxygen Gas at Liquid-Oxygen Temperatures. Physica, vol. 2, 1935, pp. 97-103.

TABLE I

APPARATUS AND WIRE CONSTANTS FOR MANGANIN SUSPENSIONS

Wire number	Wire constant	Apparatus constant
1	0.00036	5.354
2	.00031	5.345
3	.00021	5.361

TABLE II

VISCOSITY OF NITROGEN AT VARIOUS TEMPERATURES

Pressure (mm Hg)	η (poises)	Pressure (mm Hg)	η (poises)
306.05° K		273.25° K	
503.0	1809.2×10^{-7}	550.0	1661.7×10^{-7}
310.0	1809.2	403.0	1661.0
100.0	1807.1	98.0	1659.4
68.0	1804.8	70.0	1658.3
9.62	1788.2	6.66	1637.2
5.07	1771.5	3.22	1610.2
2.38	1735.5	2.34	1595.4
1.15	1651.3	1.12	1515.6
.574	1525.7	.485	1398.7
.243	1308.4	.220	1249.8
.100	922.2	.097	937.2
.056	614.2	.034	445.4
.027	359.4	.022	325.7
.012	199.3	.010	185.6
.0062	103.3	.0030	53.4
.0010	23.4	.00041	9.5
.00017	6.75	.000026	3.1
193.90° K		78.60° K	
417.5	1258.7	295.0	549.5
102.0	1257.6	104.0	549.5
72.5	1256.3	70.0	550.2
9.04	1250.3	8.56	548.1
4.32	1243.3	4.76	547.2
1.98	1227.0	2.60	544.2
.816	1177.8	1.17	541.2
.353	1079.0	.440	540.1
		.220	524.4
		.132	516.2

TABLE III

VISCOSITIES OF AIR AT VARIOUS TEMPERATURES

Pressure (mm Hg)	η (poises)	Pressure (mm Hg)	η (poises)
306.05° K		275.00° K	
696.0	1880.9×10^{-7}	195.0	1735.4×10^{-7}
402.0	1879.4	98.0	1735.2
100.0	1877.4	50.5	1724.3
70.0	1876.0	26.5	1724.9
9.70	1857.6	8.38	1712.8
5.38	1838.3	4.46	1698.0
1.76	1751.1	.880	1533.0
.440	1459.3	.507	1351.8
.220	1107.9	.232	1046.9
.088	802.9	.088	770.0
.035	458.4	.044	537.0
.020	300.0	.022	338.4
.010	161.9	.0093	167.7
.0042	66.9	.0042	73.7
.00054	5.7	.00033	10.0
193.90° K		80.30° K	
495.5	1309.2	314.0	575.7
105.0	1304.1	205.0	579.2
70.0	1303.8	49.0	581.6
7.72	1290.6	25.0	581.7
3.57	1279.4	9.35	582.2
1.76	1263.4	4.68	579.9
.970	1239.2	2.18	579.8
.308	1150.2	1.00	571.4
		.397	547.0
		.243	542.5

TABLE IV

SLIP COEFFICIENTS OF NITROGEN AT VARIOUS TEMPERATURES

Pressure (mm Hg)	ζ (mm)	Pressure (mm Hg)	ζ (mm)
306.05° K		273.25° K	
9.62	0.0059	6.66	0.0075
5.07	.0106	3.22	.0160
2.38	.0212	2.34	.0208
1.15	.0478	1.12	.0482
.574	.0929	.485	.0940
.243	.191	.220	.165
.100	.481	.097	.386
.056	.973	.034	1.36
.027	2.02	.022	2.05
.012	4.04	.010	3.98
.0062	8.26		
193.90° K		78.60° K	
9.04	0.0034	1.17	0.0077
4.32	.0062	.440	.0087
1.98	.0129	.220	.0239
.816	.0343	.132	.0322
.353	.0832		

TABLE V

SLIP COEFFICIENTS OF AIR AT VARIOUS TEMPERATURES

Pressure (mm Hg)	ζ (mm)	Pressure (mm Hg)	ζ (mm)
306.05° K		275.00° K	
9.70	0.0063	8.38	0.0066
5.30	.0116	4.46	.0110
1.76	.0371	.880	.066
.440	.144	.507	.142
.220	.348	.232	.329
.088	.671	.088	.627
.035	1.55	.044	1.12
.020	2.63	.022	2.06
.010	5.31	.0093	4.67
193.90° K		80.30° K	
7.72	0.0072	1.00	0.0090
3.57	.0116	.397	.0317
1.76	.0181	.243	.0361
.970	.0282		
.308	.0691		

TABLE VI

VALUES OF CONSTANT k IN EQUATION (8)

Temperature (°K)	Constant k for -	
	Nitrogen	Air
306.05	4.81×10^2	5.90×10^2
275.00		5.50
273.25	3.96	
193.90	2.92	2.37
80.30		1.08
78.60	.53	

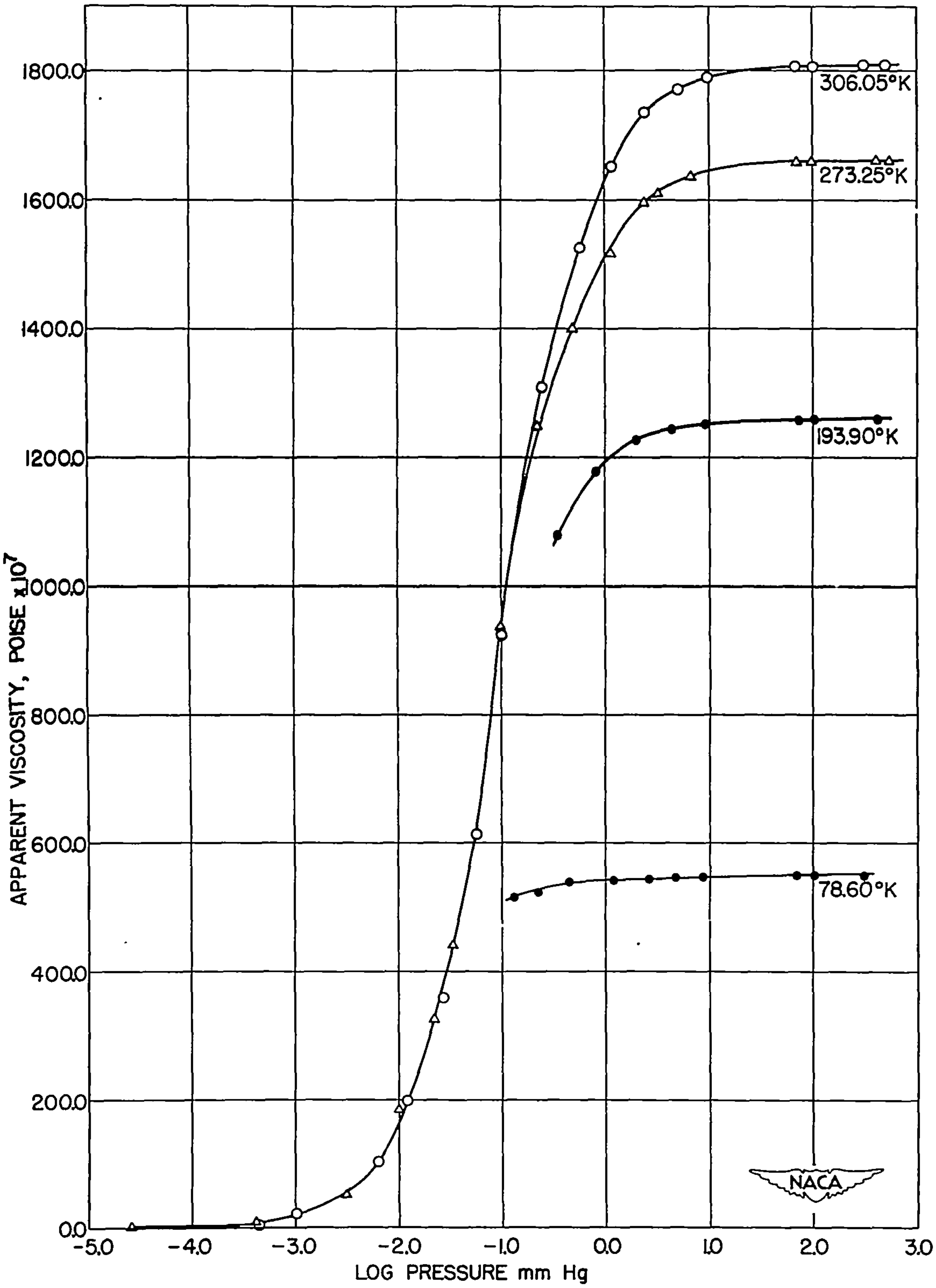

Figure 1.- Viscosity of nitrogen against logarithm of pressure at fixed temperatures.

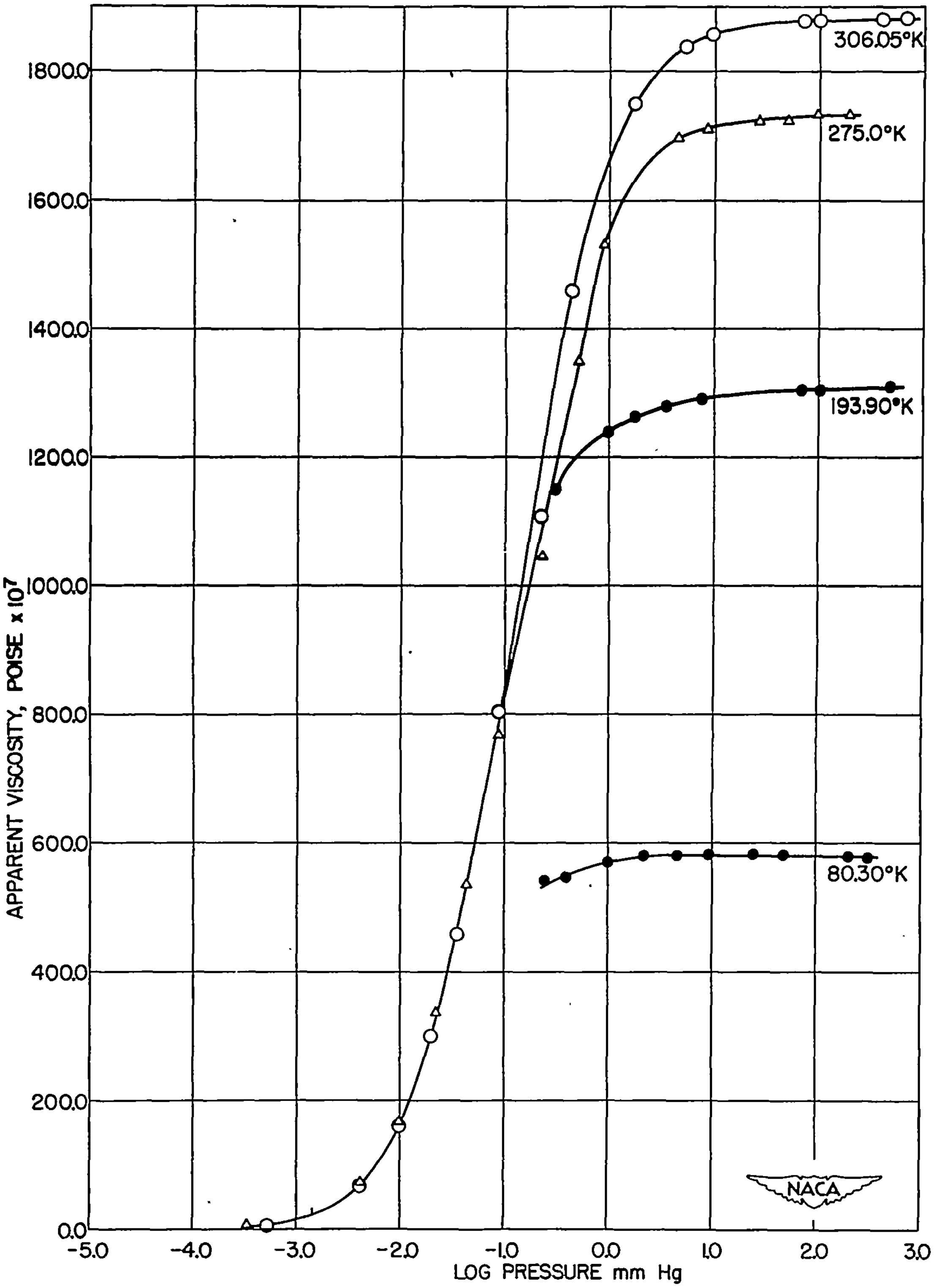

Figure 2.- Viscosity of air against logarithm of pressure at fixed temperatures.

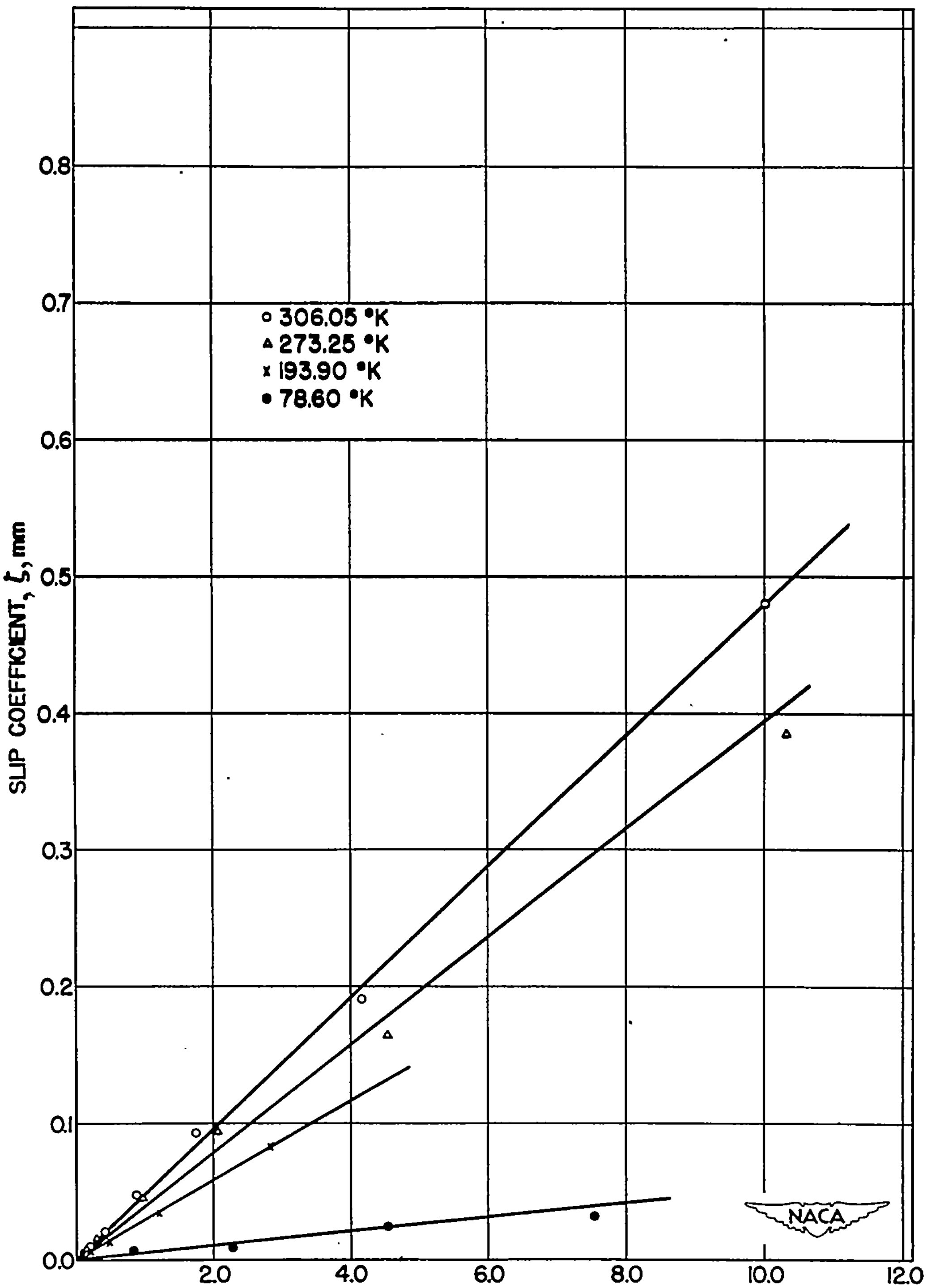

Figure 3.- Slip coefficient of nitrogen to 0.1 millimeter of mercury.

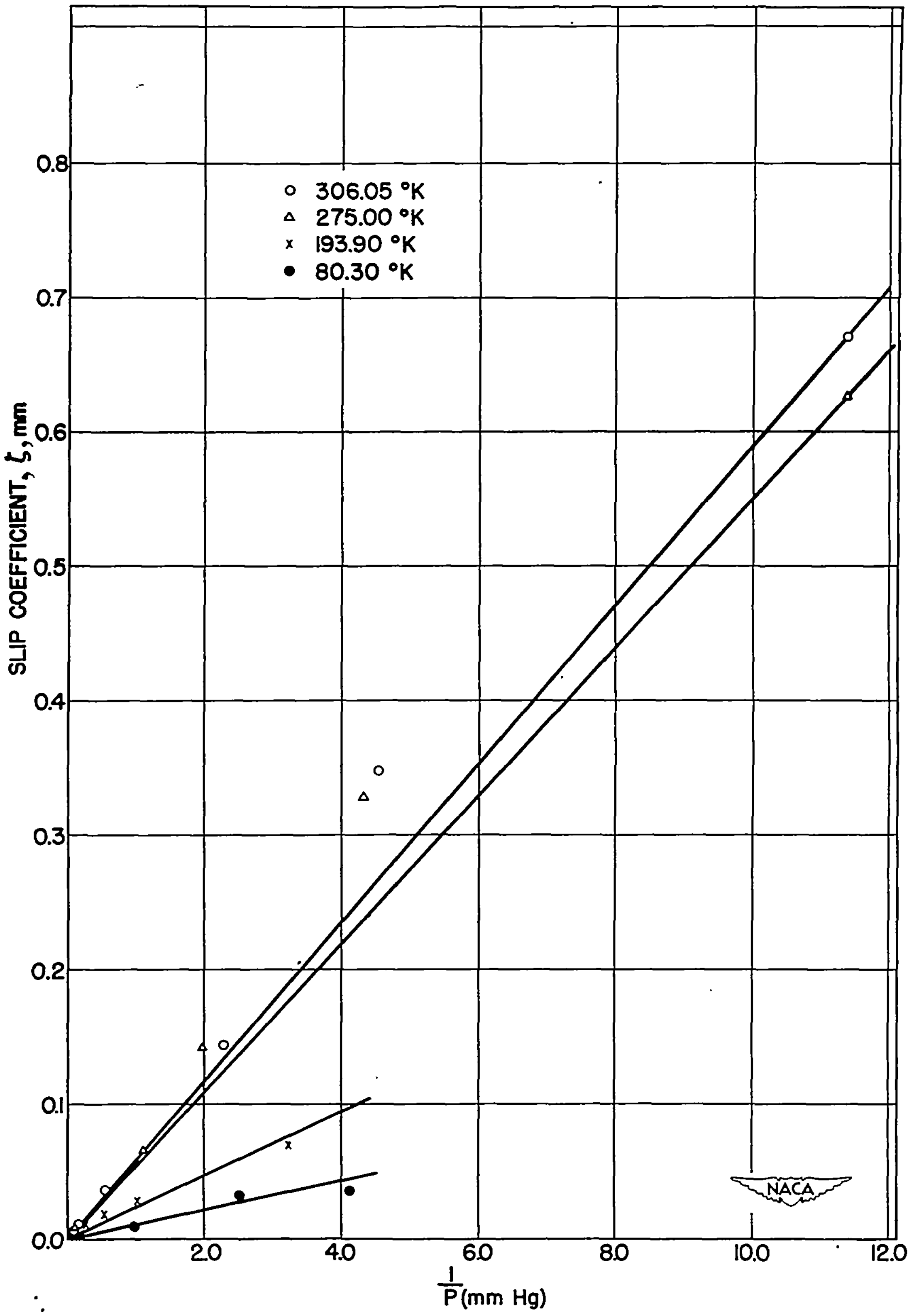

Figure 4.- Slip coefficient of air to 0.1 millimeter of mercury.

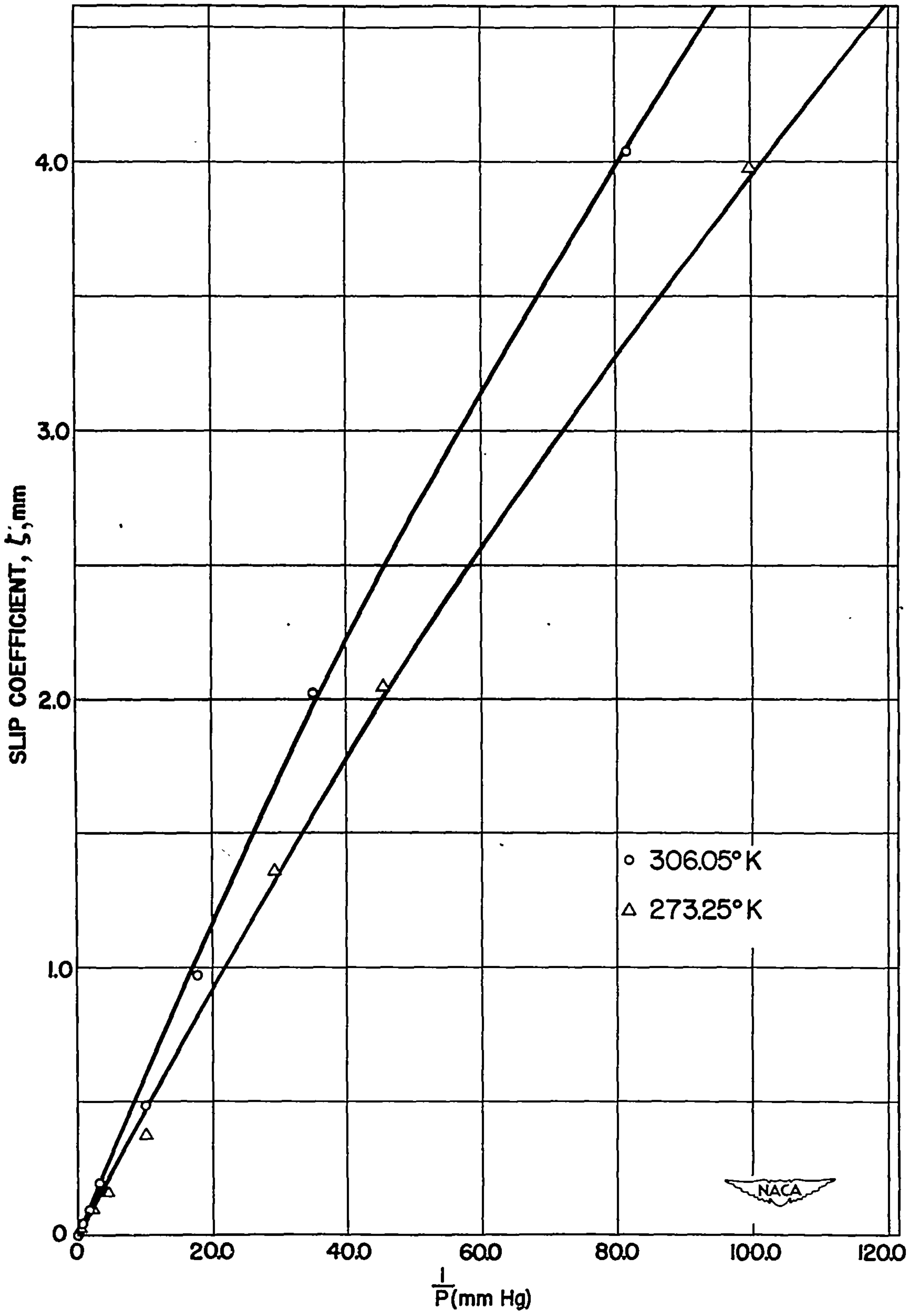

Figure 5.- Slip coefficient of nitrogen to 0.01 millimeter of mercury.

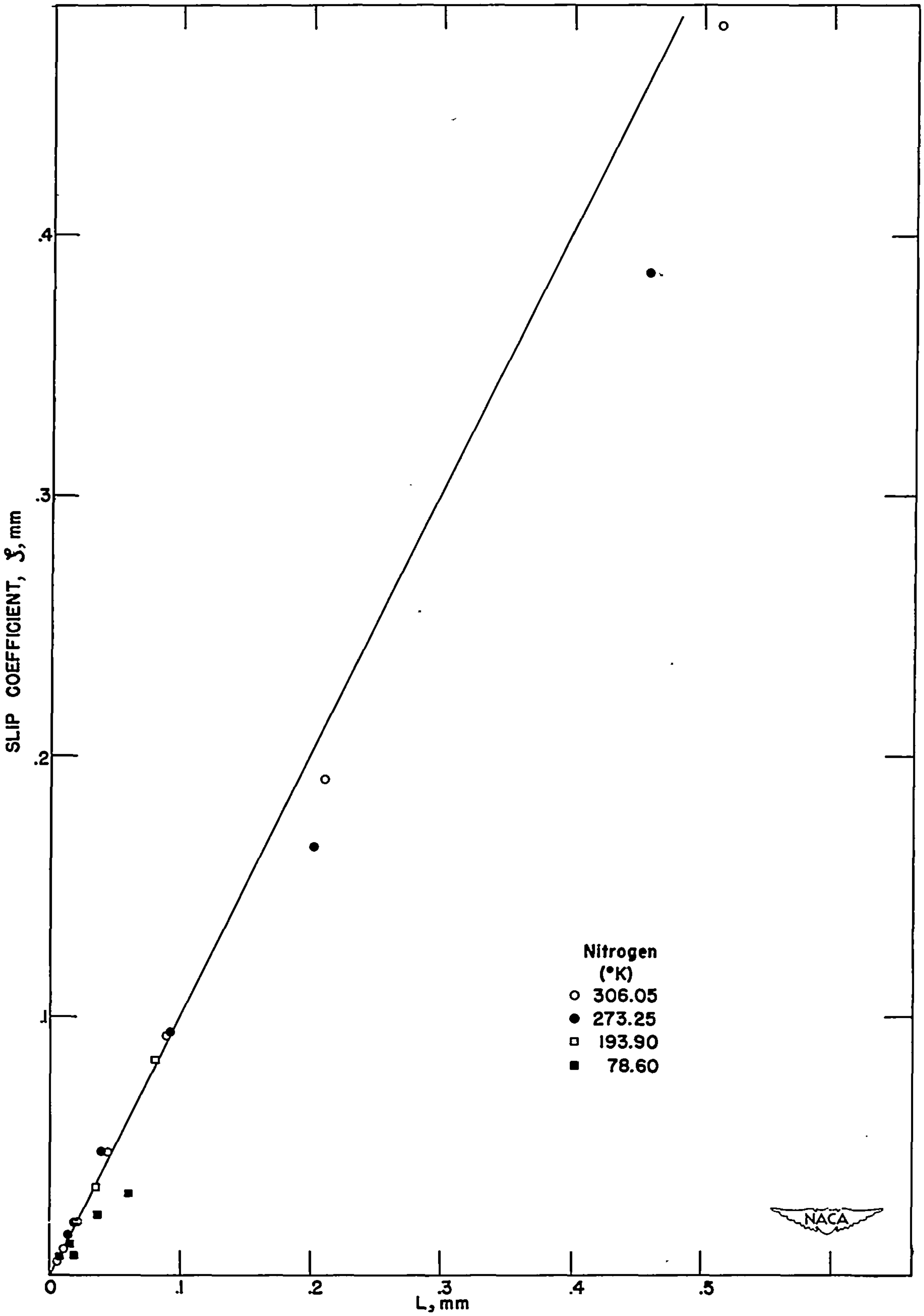

Figure 6.- Slip coefficient of nitrogen against mean free path.

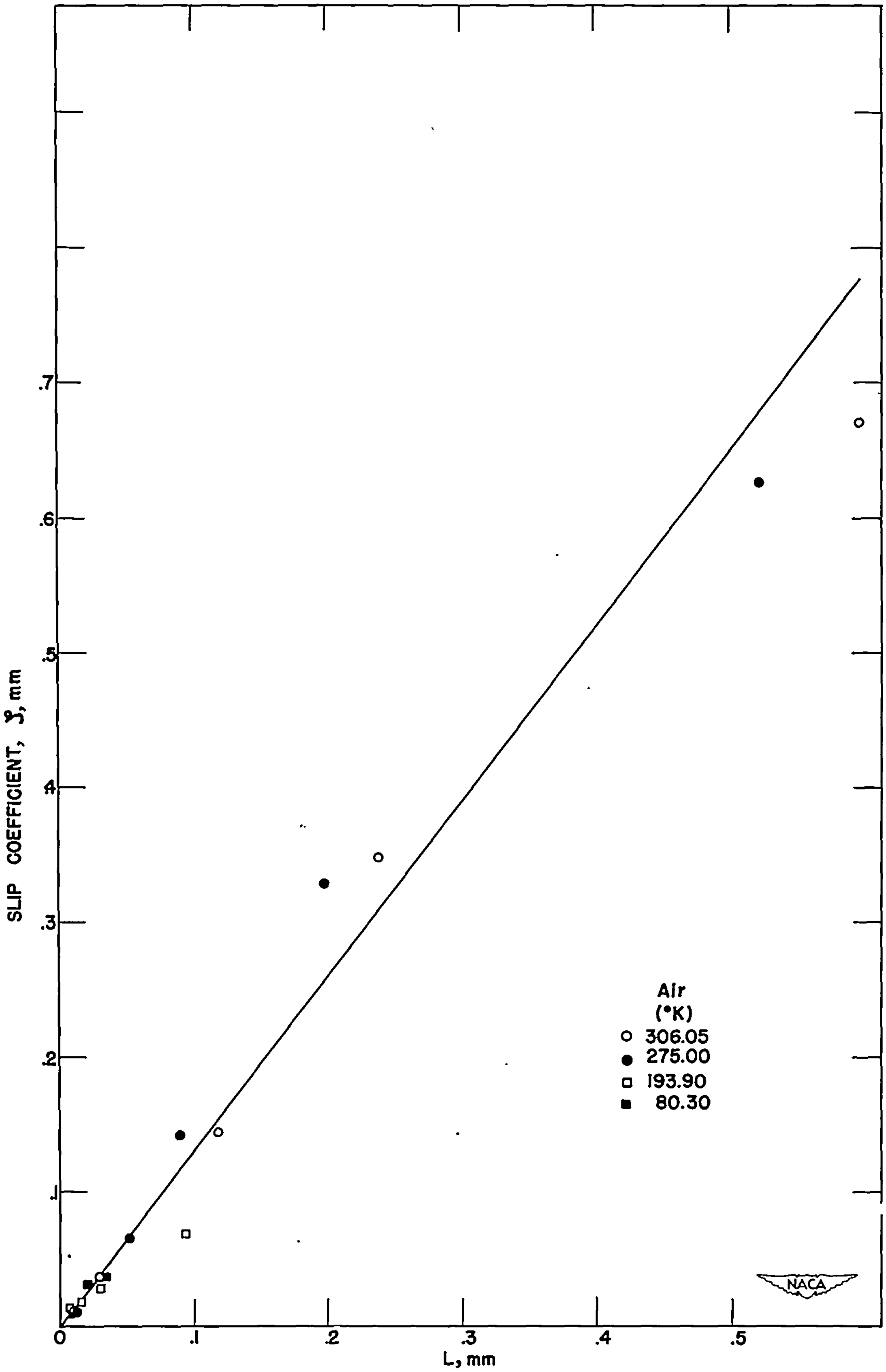

Figure 7.- Slip coefficients of air against mean free path.

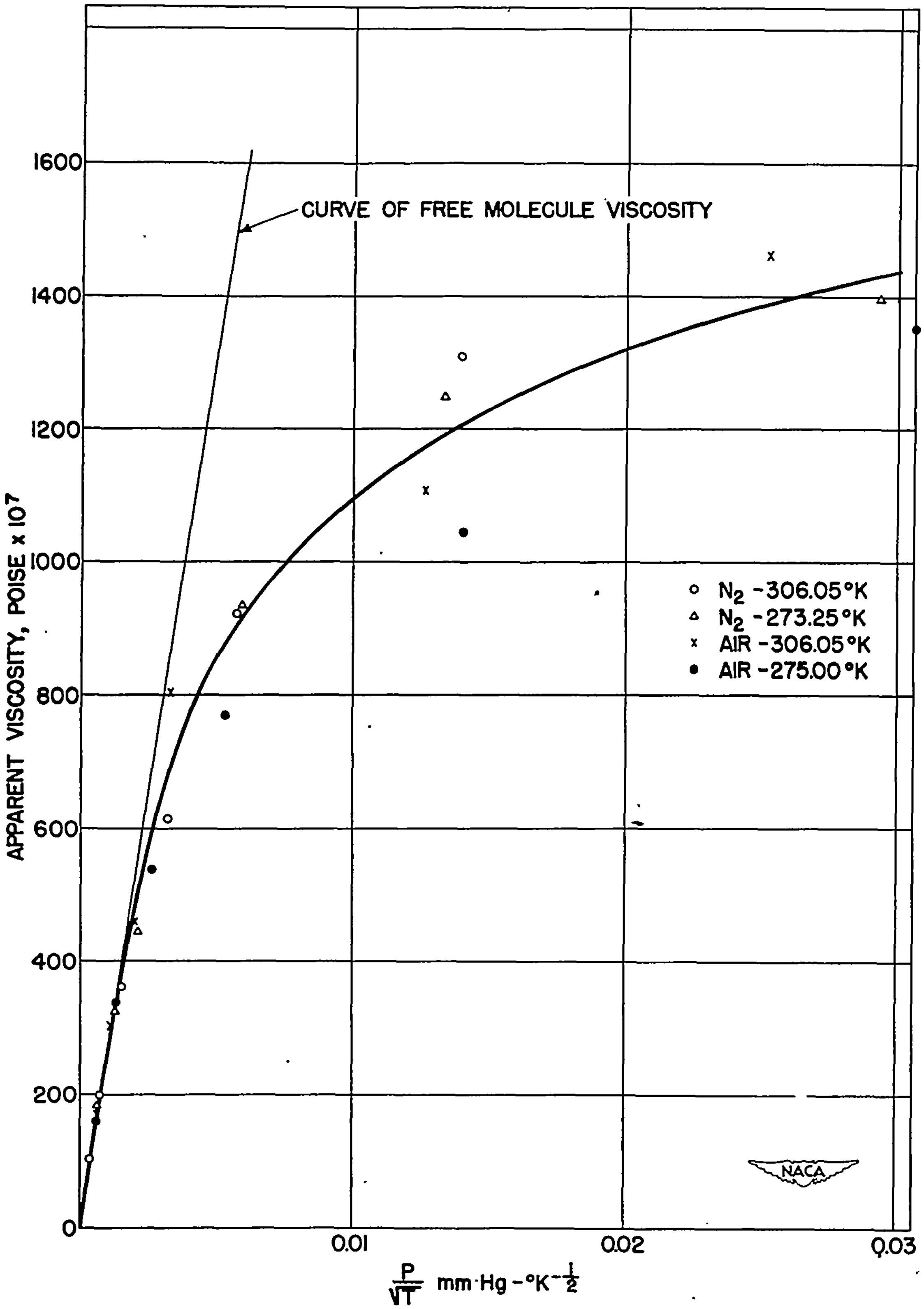

Figure 8.- Free-molecule viscosities of air and of nitrogen.

NACA-Langley - 9-13-54 - 100