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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2553

PITCHING-MOMENT DERIVATIVES C_{m_q} AND $C_{m_{\dot{\alpha}}}$ AT SUPERSONIC SPEEDS FOR A SLENDER-DELTA-WING AND SLENDER-BODY COMBINATION AND APPROXIMATE SOLUTIONS FOR BROAD-DELTA-WING AND SLENDER-BODY COMBINATIONS

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CHARTS AND TABLES FOR USE IN CALCULATIONS OF DOWNWASH OF WINGS OF ARBITRARY PLAN FORM

By Franklin W. Diederich

May 1951

Page 4: In the second line of equation (2), the quantity Δy_v^2 which appears in the numerator of the fraction under the radical in the first term within the brackets should be Δz_v^2.

Page 6: In equation (8), the minus sign inside the parentheses should be a plus sign.

Page 6: In equation (9), the minus sign inside the parentheses should be a plus sign.

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SUMMARY

The pitching-moment derivatives C_{m_q} and $C_{m_{\dot{\alpha}}}$ at supersonic speeds are developed for a slender-delta-wing and slender-body combination having no afterbody. By drawing an analogy between the aerodynamics of the wing-body section of the combination and the aerodynamics of a delta wing alone, the results for the slender-delta-wing and slender-body combination are modified to the extent that approximate solutions for C_{m_q} and $C_{m_{\dot{\alpha}}}$ for broad-delta-wing and slender-body combinations can be obtained.

INTRODUCTION

Various methods, based on linear theory, for obtaining solutions for the flow about wing-body combinations have been developed for the determination of the lift and moment due to angle of attack. References 1 to 7 comprise a fairly comprehensive list of most of the significant of these methods, which include both approximate and exact solutions. All the exact solutions to the linearized differential equation of steady supersonic flow, however, employ iteration processes, infinite series, or both, and their practical application results in approximate solutions although the error is often negligible, depending upon the particular problem, rate of convergence, number of iterations, and so forth. Spreiter (reference 7) has presented solutions in closed form to the two-dimensional Laplace equation of potential flow for the lift and moment of wing-body combinations. These solutions apply to the supersonic range for the limiting case of a slender wing-body configuration.

For the stability derivatives of wing-body combinations, there are a few papers on the damping-in-roll characteristics (see, for example, references 8 and 9) but none for the damping in pitch.

The purpose of the present paper is to extend the method used by Spreiter in reference 7 to the calculation of the pitching-moment derivatives due to constant rate of pitch $\left(C_{m_q}\right)$ and due to constant accelerated motion in the vertical direction $\left(C_{m_{\dot{\alpha}}}\right)$ for a slender-delta-wing and slender-body combination. In addition, an approximate solution to these derivatives is developed for a broad-delta-wing and slender-body combination in supersonic flow by introducing certain modifying factors into the slender-delta-wing and slender-body results.

Certain conditions are placed upon the configuration. The body ahead of the wing is slender, has a circular cross section, and is pointed at the nose, and the slope of the body meridian section is continuous. For the wing-body section, the wing semiapex angle is small; along the wing-body juncture, the body radius is a maximum and is constant; and finally, the configuration has no afterbody (see fig. 1).

SYMBOLS

ϕ,ϕ'	potential functions
ψ,ψ'	stream functions
Z	complex variable $(y + iz)$
R	body radius ($R = R(x)$ on body ahead of wing and $R = a$ along wing-body section)
a	body radius along wing-body section
s	y-coordinate of wing leading edge
w	velocity in positive z-direction
r,θ	polar coordinates
q	constant angular velocity of pitch
$\dot{\alpha}$	constant time rate of change of angle of attack $\left(\frac{1}{V}\frac{dw}{dt}\right)$

p perturbation pressure (difference in pressure between body surface and free stream)

ρ density of fluid

t time

x,y,z Cartesian coordinates

V free-stream velocity

x_o point of rotation measured from nose

$\bar{n}$ inward-drawn unit normal vector

M pitching moment

A area of basic wing (including portion enclosed by body)

C_m pitching-moment coefficient $\left(\frac{M}{\frac{1}{2}\rho V^2 A\bar{c}}\right)$

C_{m_q} nondimensional stability derivative due to constant rate of pitch $\left(\left(\frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}}\right)_{q\rightarrow 0}\right)$

$C_{m_{\dot{\alpha}}}$ nondimensional stability derivative due to constant accelerated motion in vertical direction $\left(\left(\frac{\partial C_m}{\partial \frac{\dot{\alpha}\bar{c}}{2V}}\right)_{\dot{\alpha}\rightarrow 0}\right)$

c root chord of basic wing

$\bar{c}$ mean aerodynamic chord of basic wing $\left(\frac{2}{3}c\right)$

c' root chord of exposed wing

l total length of wing-body configuration

ϵ semiapex angle of basic wing

s_o value of s at $x = x_o$ or at c_o

s_{max} maximum value of s (value of s at $x = l$)

c_o point of rotation measured from apex of basic wing; positive in positive x-direction

e,f,g, h,m interference factors

M Mach number

$\beta = \sqrt{M^2 - 1}$

$K = \tan \epsilon$

Q constant of integration

$k = \dfrac{a}{s_{max}}$

$E'(\beta K)$ complete elliptic integral of second kind

$$\left(\int_0^{\pi/2} \sqrt{1 - \left(1 - \beta^2 K^2\right)\sin^2\theta}\, d\theta\right)$$

$F'(\beta K)$ complete elliptic integral of first kind

$$\left(\int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \left(1 - \beta^2 K^2\right)\sin^2\theta}}\right)$$

$$\lambda_1 = \frac{1 - \beta^2 K^2}{\left(1 - 2\beta^2 K^2\right)E'(\beta K) + \beta^2 K^2 F'(\beta K)}$$

$$\lambda_2 = \frac{1}{E'(\beta K)}$$

$$\lambda_3 = \frac{3 + 2\beta^2}{\beta^2}\lambda_2 - \frac{3\left(1 + \beta^2\right)}{\beta^2}\lambda_1$$

Subscripts:

W	wing
B	body
q	due to q
$\dot{\alpha}$	due to $\dot{\alpha}$

ANALYSIS

The linearized differential equation of steady supersonic flow is

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r}\frac{\partial \phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$$

At present an exact solution to this equation does not exist in closed form for wing-body combinations. However, if the term $\beta^2 \frac{\partial^2 \theta}{\partial x^2}$ becomes very small with respect to the other terms of this equation, it may be neglected. Solutions to the Laplace equation which results from dropping the term $\beta^2 \frac{\partial^2 \phi}{\partial x^2}$ have been found in closed form for the lift and moment due to angle of attack (reference 7). It has been found that the condition necessary for $\beta^2 \frac{\partial^2 \phi}{\partial x^2}$ to be negligible for the angle-of-attack case is that the configuration be slender and that β^2 be not excessive. For a delta-wing and body combination, the term slender implies that $\frac{dR}{dx}$, $\frac{d^2R}{dx^2}$, and K are very small.

In the present paper, which treats the steady-pitching and the time-dependent, constant-acceleration cases of delta-wing and body combinations, a velocity potential satisfying the two-dimensional Laplace equation is used. In the appendix it is shown that the conditions to be satisfied for the Laplace solution to be applicable to the supersonic range are that $\frac{dR}{dx}$, $\frac{d^2R}{dx^2}$, K, q, and $\dot{\alpha}$ be very small.

After a velocity potential which satisfies the Laplace equation is found, the next step in the analysis is the determination of the pressure distributions over the slender-delta-wing and slender-body combination resulting from the two types of motion which give rise to C_{m_q} and $C_{m_{\dot{\alpha}}}$, namely, constant rate of pitch and constant accelerated motion in the positive z-direction, respectively. When the pressure distribution is known, the moment may be calculated about any axis of the configuration, and, from their respective definitions, C_{m_q} and $C_{m_{\dot{\alpha}}}$ may then be determined. The configuration to be considered and the coordinate system employed are shown in figure 1.

Velocity Potential

Spreiter (reference 7) shows that the complex potential for a uniform stream of velocity w at infinity flowing vertically downward over a stationary two-dimensional circular cylinder symmetrically located on a horizontal flat plate is

$$\phi' + i\psi' = iw\left[\left(Z + \frac{R^2}{Z}\right)^2 - \left(s + \frac{R^2}{s}\right)^2\right]^{1/2} \tag{1}$$

where

$Z = y + iz$

R radius of cylinder

s semispan of plate measured from center of cylinder

For a slender configuration describing a slow, steady pitching motion, the cross-flow velocity distribution is, to the first order, proportional to x. Inasmuch as potential flow is assumed, this velocity distribution must be looked upon as being generated by the motion of the configuration in fluid which is at rest, because, if the distribution were due to the motion of the fluid about a stationary body, the flow must be rotational and the assumption of potential flow is then violated.

The complex potential of the aforementioned configuration moving upward through still air with the vertical velocity w then is

$$\phi + i\psi = iw\left\{\left[\left(Z + \frac{R^2}{Z}\right)^2 - \left(s + \frac{R^2}{s}\right)^2\right]^{1/2} - Z\right\} \tag{2}$$

Transforming to polar coordinates $(Z = r(\cos\theta + i\sin\theta))$ and solving for the velocity potential gives

$$\phi = -w\left\{\frac{1}{\sqrt{2}}\left[\left(\frac{r^8 + R^8}{r^4} + \frac{s^8 + R^8}{s^4} + 4R^4\cos^2 2\theta - 2\,\frac{r^4 + R^4}{r^2}\,\frac{s^4 + R^4}{s^2}\cos 2\theta\right)^{1/2} - \frac{r^4 + R^4}{r^2}\cos 2\theta + \frac{s^4 + R^4}{s^2}\right]^{1/2} - r\sin\theta\right\} \tag{3}$$

Equation (3) is the general expression for the velocity potential. Whether ϕ pertains to the constant-pitching or the constant-acceleration case depends upon the value of w. For a wing-body configuration pitching about a point x_o from the nose, the vertical velocity w varies along the length of the configuration according to $w = q\,(x - x_o)$. For constant acceleration in the positive z-direction, the velocity varies with time according to $w = \dot{\alpha}Vt$.

Pressure Distribution

The equation for the pressure distribution is

$$p = \rho\left[V\frac{\partial\phi}{\partial x} + \frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2 + \frac{\partial\phi}{\partial t}\right] \tag{4}$$

The term $\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2$ does not contribute to either the lift or moment since on the body it is symmetric and on the wings, although $\frac{\partial\phi}{\partial r}$ is antisymmetric, $\left(\frac{\partial\phi}{\partial r}\right)^2$ is symmetric; therefore, for the configuration considered,

$$p = \rho\left(V\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial t}\right) \tag{5}$$

For the case of pitching with constant angular velocity,

$$p_q = \rho V\left(\frac{\partial\phi_q}{\partial R}\frac{dR}{dx} + \frac{\partial\phi_q}{\partial s}\frac{ds}{dx} + \frac{\partial\phi_q}{\partial w}\frac{dw}{dx}\right) \tag{6}$$

and for constant acceleration, evaluated at time $t = 0$,

$$p_{\dot{\alpha}} = \rho \frac{\partial \phi_{\dot{\alpha}}}{\partial t} \tag{7}$$

In order to determine the loading over the wing-body combination as given by equations (6) and (7), the pressure distributions in two regions must be considered for each expression. They are:

(a) $p_{q,\dot{\alpha}}$ on the body where $r = R$

(b) $p_{q,\dot{\alpha}}$ on the wing where $\theta = 0$ and $a \leqq r \leqq s$

For the pitching wing-body combination with $w = q(x - x_o)$ and the preceding conditions, equation (6) gives for the pressure over the body and wing, respectively,

$$\left(p_q\right)_B = -\rho V q \left[\frac{-Rs \sin\theta + \sqrt{s^4 + R^4 - 2R^2 s^2 \cos 2\theta}}{s} + \frac{2R(x - x_o)\left(R^2 - s^2\cos 2\theta\right)}{s\sqrt{s^4 + R^4 - 2R^2 s^2 \cos 2\theta}} \frac{dR}{dx} + \frac{(x - x_o)\left(s^4 - R^4\right)}{s^2\sqrt{s^4 + R^4 - 2R^2 s^2 \cos 2\theta}} \frac{ds}{dx}\right] \tag{8a}$$

$$\left(p_q\right)_W = -\rho V q \left[\frac{\sqrt{\left(r^2 - s^2\right)\left(R^4 - r^2 s^2\right)}}{rs} + \frac{2R^3(x - x_o)\left(r^2 - s^2\right)}{rs\sqrt{\left(r^2 - s^2\right)\left(R^4 - r^2 s^2\right)}} \frac{dR}{dx} + \frac{r(x - x_o)\left(s^4 - R^4\right)}{s^2\sqrt{\left(r^2 - s^2\right)\left(R^4 - r^2 s^2\right)}} \frac{ds}{dx}\right] \tag{8b}$$

Similarly, for the case of constant acceleration, in which $w = V\dot{\alpha}t$ is used, equation (7) yields

$$\left(p_{\dot{\alpha}}\right)_B = -\rho V \dot{\alpha} \frac{\sqrt{s^4 + R^4 - 2R^2 s^2 \cos 2\theta} - Rs \sin\theta}{s} \tag{9a}$$

$$\left(p_{\dot{\alpha}}\right)_W = -\rho V\dot{\alpha}\,\frac{\sqrt{\left(r^2 - s^2\right)\left(R^4 - r^2s^2\right)}}{rs} \tag{9b}$$

C_{m_q} and $C_{m_{\dot{\alpha}}}$ for Slender-Delta-Wing and Slender-Body Combinations

The moment on the wing-body configuration measured about a point x_o from the nose is

$$M = \int_A (x - x_o)\bar{n} \cdot p\, dA \tag{10}$$

where $\bar{n}$ is an inward-drawn unit vector, normal to the surface, and A represents the surface area of the configuration. Now

$$C_m = \frac{M}{\frac{1}{2}\rho V^2 A\bar{c}}$$

$$C_{m_q} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}} = \frac{\partial}{\partial q}\left(\frac{4M_q}{\rho VA\bar{c}^2}\right)$$

and

$$C_{m_{\dot{\alpha}}} = \frac{\partial C_m}{\partial \frac{\dot{\alpha}\bar{c}}{2V}} = \frac{\partial}{\partial \dot{\alpha}}\left(\frac{4M_{\dot{\alpha}}}{\rho VA\bar{c}^2}\right)$$

Therefore C_{m_q} and $C_{m_{\dot{\alpha}}}$ are respectively,

$$C_{m_q} = \frac{\partial}{\partial q}\left(\frac{4}{\rho VA\bar{c}^2}\left\{\int_0^{l-c'}\int_0^{2\pi} (x - x_o)\left(p_q\right)_B R \sin\theta\, d\theta\, dx + \right.\right.$$

$$4\left[\int_{l-c'}^{l}\int_0^{\pi/2} (x - x_o)\left(p_q\right)_B R \sin\theta\, d\theta\, dx + \right.$$

$$\left.\left.\left.\int_{l-c'}^{l}\int_a^s (x - x_o)\left(p_q\right)_W\, dr\, dx\right]\right\}\right) \tag{11}$$

$$C_{m_{\dot{\alpha}}} = \frac{\partial}{\partial\dot{\alpha}}\left(\frac{4}{\rho V A \bar{c}^2}\left\{\int_0^{l-c'}\int_0^{2\pi} (x - x_o)\left(p_{\dot{\alpha}}\right)_B R \sin\theta \, d\theta \, dx + \right.\right.$$

$$4\left[\int_{l-c'}^{l}\int_0^{\pi/2} (x - x_o)\left(p_{\dot{\alpha}}\right)_B R \sin\theta \, d\theta \, dx + \right.$$

$$\left.\left.\left.\int_{l-c'}^{l}\int_a^{s} (x - x_o)\left(p_{\dot{\alpha}}\right)_W dr \, dx\right]\right\}\right) \tag{12}$$

where the first integral in each expression is the contribution of the body ahead of the wing and the last two in each expression are the contributions of the wing-body section.

The conditions to be imposed in evaluating these integrals are:

(a) On the body ahead of the wing, $s = R$

(b) On the body at the wing-body section, $R = a$ and $\frac{dR}{dx} = 0$

(c) On the wing, $\frac{ds}{dx} = \text{Constant} = \tan\epsilon$

Integration of the terms for the wing-body section of the configuration may be simplified by making the substitutions for x and x_o which are suggested by condition (c). Since $\frac{ds}{dx} = \tan\epsilon$,

$$s = x\tan\epsilon + Q$$

where Q is a constant. Therefore,

$$x = \frac{s - Q}{\tan\epsilon}$$

$$x_o = \frac{s_o - Q}{\tan\epsilon}$$

$$dx = \frac{ds}{\tan\epsilon}$$

and the limits of integration are now from $s = a$ to $s = s_{max}$. From the geometry of the configuration (see fig. 1), $s_{max} = c \tan \epsilon$ and $s_o = c_o \tan \epsilon$, where c_o is the location of the point of rotation measured from the apex of the basic wing, positive in the positive x-direction.

Performing the operations indicated in equations (11) and (12) and substituting limits, $s_{max} = c \tan \epsilon$, and $s_o = c_o \tan \epsilon$ results in

$$C_{m_q} = -\frac{4\pi}{A\bar{c}^2}\int_0^{l-c'}(x - x_o)R^2 dx - \frac{8\pi}{A\bar{c}^2}\int_0^{l-c'}(x - x_o)^2 R\frac{dR}{dx}dx -$$

$$6\pi \tan \epsilon\left(\frac{9}{8}e - \frac{c_o}{\bar{c}}f\right) + 4\pi \tan \epsilon \frac{c_o}{\bar{c}}\left(f - \frac{c_o}{\bar{c}}g\right) \tag{13}$$

$$C_{m_{\dot{\alpha}}} = -\frac{4\pi}{A\bar{c}^2}\int_0^{l-c'}(x - x_o)R^2 dx - 2\pi \tan \epsilon\left(\frac{9}{8}h - \frac{c_o}{\bar{c}}m\right) \tag{14}$$

where

$$e = 1 - \frac{2}{3}k^2 - \left(\frac{1}{3} + \frac{4}{3}\log_e \frac{1}{k}\right)k^4$$

$$f = 1 - \frac{3}{5}k^2 - \frac{11}{5}k^3 + \frac{9}{5}k^4$$

$$g = 1 - 2k^2 + k^4$$

$$h = 1 - 4k^2 + \left(3 + 4\log_e \frac{1}{k}\right)k^4$$

$$m = 1 - 6k^2 + 8k^3 - 3k^4$$

The variation of these interference factors with k is shown in figure 2.

Equations (13) and (14) are the expressions for C_{m_q} and $C_{m_{\dot{\alpha}}}$ for a slender-delta-wing and slender-body combination corresponding to the conditions stipulated. When these terms are added to obtain the damping-in-pitch parameter $C_{m_q} + C_{m_{\dot{\alpha}}}$, integration by parts allows the resulting expression to be written as

$$C_{m_q} + C_{m_{\dot{\alpha}}} = -\frac{4\pi a^2}{A\bar{c}^2}\left[(l - c') - x_o\right]^2 - 6\pi \tan \epsilon\left(\frac{9}{8} e - \frac{c_o}{\bar{c}} f\right) +$$

$$4\pi \tan \epsilon \frac{c_o}{\bar{c}}\left(f - \frac{c_o}{\bar{c}} g\right) - 2\pi \tan \epsilon\left(\frac{9}{8} h - \frac{c_o}{\bar{c}} m\right) \tag{15}$$

Again, from the geometry of the configuration, when $k \neq 1$,

$$(l - c') - x_o = -\left(c_o - \frac{a}{\tan \epsilon}\right)$$

This relation allows equation (15) to be written as

$$C_{m_q} + C_{m_{\dot{\alpha}}} = -4\pi k^2 \tan \epsilon\left[\frac{9}{4} k^2 - 3k \frac{c_o}{\bar{c}} + \left(\frac{c_o}{\bar{c}}\right)^2\right] - 6\pi \tan \epsilon\left(\frac{9}{8} e - \frac{c_o}{\bar{c}} f\right) +$$

$$4\pi \tan \epsilon \frac{c_o}{\bar{c}}\left(f - \frac{c_o}{\bar{c}} g\right) - 2\pi \tan \epsilon\left(\frac{9}{8} h - \frac{c_o}{\bar{c}} m\right) \tag{16}$$

When $k = 1$, the wing span goes to zero, and for a slender body of revolution

$$C_{m_q} = -\frac{4\pi}{A\bar{c}^2}\int_0^l (x - x_o)R^2 dx - \frac{8\pi}{A\bar{c}^2}\int_0^l (x - x_o)^2 R \frac{dR}{dx} dx \tag{17}$$

$$C_{m_{\dot{\alpha}}} = -\frac{4\pi}{A\bar{c}^2}\int_0^l (x - x_o)R^2 dx \tag{18}$$

and

$$C_{m_q} + C_{m_{\dot{\alpha}}} = -\frac{4\pi a^2}{A\bar{c}^2}(l - x_o)^2 \tag{19}$$

where A and $\bar{c}$ represent some characteristic area and length, respectively, of the body. Equation (19) agrees with Miles' result (reference 10) if $A = \pi a^2$ and $\bar{c} = l$.

When $R = a = 0$, the body radius goes to zero, and for a slender delta wing

$$C_{m_q} = -6\pi \tan \epsilon \left(\frac{9}{8} - \frac{c_o}{\bar{c}}\right) + 4\pi \tan \epsilon \frac{c_o}{\bar{c}}\left(1 - \frac{c_o}{\bar{c}}\right) \tag{20}$$

$$C_{m_{\dot{\alpha}}} = -2\pi \tan \epsilon \left(\frac{9}{8} - \frac{c_o}{\bar{c}}\right) \tag{21}$$

which are the expressions for C_{m_q} and $C_{m_{\dot{\alpha}}}$ for the slender delta wing found by Ribner (reference 11).

From these equations for C_{m_q} and $C_{m_{\dot{\alpha}}}$ the terms for the wing-body section of a slender-delta-wing and slender-body combination are seen to be in the same form as C_{m_q} and $C_{m_{\dot{\alpha}}}$ for the basic wing alone. Each term of the equations for the basic wing alone is modified by a factor which is a function of the ratio of the body diameter to the maximum wing span. This modification is due to the interference effects which result from placing a slender body on a slender delta wing.

C_{m_q} and $C_{m_{\dot{\alpha}}}$ for Broad-Delta-Wing and Slender-Body Combinations

From practical considerations, solutions for C_{m_q} and $C_{m_{\dot{\alpha}}}$ for broad-delta-wing and slender-body combinations in supersonic flow are desired. A method of obtaining an approximate solution to this problem from the preceding development is suggested by the similarities between the expressions for the slender delta wing alone and for the slender-delta-wing and slender-body section of the configuration. An intuitive approach would be to assume that a delta-wing and slender-body section, in going from a slender-delta-wing and slender-body section to a broad-delta-wing and slender-body section, follows the same laws that a delta wing alone follows in making the same transition (see the next section for a discussion of the validity of this assumption).

Investigations by Brown and Adams (reference 12) and by Ribner and Malvestuto (reference 13) made after the publication of Ribner's paper on the stability derivatives of slender delta wings (reference 11) show that the stability derivatives of broad delta wings in compressible supersonic flow such that $\beta \tan \epsilon < 1$ are the same as the results for the slender delta wing multiplied by certain elliptic integrals which are functions of the wing semiapex angle and the Mach number of the flow. Applying these laws to the wing-body section gives

$$C_{m_q} = -\frac{4\pi}{A\bar{c}^2}\int_0^{l-c'}(x - x_o)R^2 dx - \frac{8\pi}{A\bar{c}^2}\int_0^{l-c'}(x - x_o)^2 R\frac{dR}{dx}dx - \lambda_1 6\pi \tan\epsilon\left(\frac{9}{8}e - \frac{c_o}{\bar{c}}f\right) + \lambda_2 4\pi \tan\epsilon\,\frac{c_o}{\bar{c}}\left(f - \frac{c_o}{\bar{c}}g\right) \tag{22}$$

$$C_{m_{\dot{\alpha}}} = -\frac{4\pi}{A\bar{c}^2}\int_0^{l-c'}(x - x_o)R^2 dx + \lambda_3 2\pi \tan\epsilon\left(\frac{9}{8}h - \frac{c_o}{\bar{c}}m\right) \tag{23}$$

where λ_1, λ_2, and λ_3 are the appropriate elliptic integrals (see fig. 3). The damping-in-pitch parameter is

$$C_{m_q} + C_{m_{\dot{\alpha}}} = -4\pi k^2 \tan\epsilon\left[\frac{9}{4}k^2 - 3k\frac{c_o}{\bar{c}} + \left(\frac{c_o}{\bar{c}}\right)^2\right] - \lambda_1 6\pi \tan\epsilon\left(\frac{9}{8}e - \frac{c_o}{\bar{c}}f\right) + \lambda_2 4\pi \tan\epsilon\,\frac{c_o}{\bar{c}}\left(f - \frac{c_o}{\bar{c}}g\right) + \lambda_3 2\pi \tan\epsilon\left(\frac{9}{8}h - \frac{c_o}{\bar{c}}m\right) \tag{24}$$

In order to determine approximate expressions for C_{m_q} and $C_{m_{\dot{\alpha}}}$ for the configuration when the wing leading edges are supersonic ($\beta \tan \epsilon > 1$), the analogy drawn previously between the laws followed by a broadening delta wing alone and a broadening-delta-wing and slender-body section is continued into the region where $\beta \tan \epsilon > 1$.

As a delta wing alone continues to broaden to the extent that $\beta \tan \epsilon > 1$, the equations for C_{m_q} (see reference 12) and $C_{m_{\dot{\alpha}}}$ are

$$C_{m_q} = -\frac{8}{\beta}\left(\frac{9}{8} - \frac{c_o}{\bar{c}}\right) + \frac{8}{\beta}\frac{c_o}{\bar{c}}\left(1 - \frac{c_o}{\bar{c}}\right) \qquad (25)$$

$$C_{m_{\dot{\alpha}}} = \frac{4}{\beta^3}\left(\frac{9}{8} - \frac{c_o}{\bar{c}}\right) \qquad (26)$$

($C_{m_{\dot{\alpha}}}$ was obtained by use of equation (15) in reference 13 and agrees with Miles' result (reference 14).) Therefore the derivatives for a broad-delta-wing and slender-body combination in supersonic flow, such that $\beta \tan \epsilon > 1$, may be approximated by

$$C_{m_q} = -\frac{4\pi}{A\bar{c}^2}\int_0^{l-c'} (x - x_o)R^2 dx - \frac{8\pi}{A\bar{c}^2}\int_0^{l-c'} (x - x_o)^2 R \frac{dR}{dx} dx -$$

$$\frac{8}{\beta}\left(\frac{9}{8} e - \frac{c_o}{\bar{c}} f\right) + \frac{8}{\beta}\frac{c_o}{\bar{c}}\left(f - \frac{c_o}{\bar{c}} g\right) \qquad (27)$$

and

$$C_{m_{\dot{\alpha}}} = -\frac{4\pi}{A\bar{c}^2}\int_0^{l-c'} (x - x_o)R^2 dx + \frac{4}{\beta^3}\left(\frac{9}{8} h - \frac{c_o}{\bar{c}} m\right) \qquad (28)$$

provided the body ahead of the wing-body section remains slender with respect to the Mach cone emanating from its nose.

Because of the nature of the factor λ_3 and the values of C_{m_q} and $C_{m_{\dot{\alpha}}}$ for $\beta \tan \epsilon > 1$, a general curve, such as $C_{m_q} + C_{m_{\dot{\alpha}}}$ plotted against $\beta \tan \epsilon$, cannot be drawn. Certain basic delta wings have therefore been chosen and curves of $C_{m_q} + C_{m_{\dot{\alpha}}}$ plotted against M have been drawn for different values of k. These curves are presented in figure 4.

DISCUSSION AND CONCLUDING REMARKS

By an extension of the method used by Spreiter in reference 7 the pitching-moment derivatives C_{m_q} and $C_{m_{\dot{\alpha}}}$ for supersonic speeds have been developed for a slender-delta-wing and slender-body combination having no afterbody. By drawing an analogy between the aerodynamics of the wing-body section of the configuration and the aerodynamics of a delta wing alone, the results for the slender-delta-wing and slender-body combination were modified to the extent that approximate solutions for C_{m_q} and $C_{m_{\dot{\alpha}}}$ for broad-delta-wing and slender-body combinations were also obtained.

In order to check the validity of the reasoning used in arriving at the assumption by which the approximate solutions were obtained, the same reasoning was applied to Spreiter's results for the lift-curve slope C_{L_α} of a wing-body combination for which an exact solution to the linearized supersonic-flow equation also exists (reference 6).

In reference 6, Browne, Friedman, and Hodes have presented an exact solution to the linearized equation of steady supersonic flow for a delta-wing and slender-conical-body combination for which the apexes are coincident. Spreiter (reference 7) has presented a solution to the two-dimensional Laplace equation for the same configuration. In order to obtain some indication as to the reliability of the assumption made, the same reasoning was applied to Spreiter's results for C_{L_α} of the delta-wing and conical-body configuration as was applied to the C_{m_q} and $C_{m_{\dot{\alpha}}}$ results of this paper, and the modification of Spreiter's results were then compared with the results of reference 6. The results of this comparison are shown in figure 5 wherein βC_{L_α} is plotted against $\beta \tan \epsilon$ for different values of k. For $k = 0.70$ the curve from reference 6 is incorrect for high values of $\beta \tan \epsilon$ because an insufficient number of terms of the series results were taken.

From the results of this comparison it appears that values of C_{m_q} and $C_{m_{\dot{\alpha}}}$ for broad-delta-wing and slender-body combinations will give fairly good approximations up to at least $k = 0.50$.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., August 21, 1951

APPENDIX

CONDITIONS FOR LAPLACE SOLUTION TO APPLY TO SUPERSONIC RANGE

In the limit, as $\beta^2 \frac{\partial^2 \phi}{\partial x^2}$, $\frac{2M^2}{V} \frac{\partial^2 \phi}{\partial x \, \partial t}$, and $\frac{M^2}{V^2} \frac{\partial^2 \phi}{\partial t^2}$ approach zero, a solution to the two-dimensional Laplace equation

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2}$$

is a solution to the linearized equation of supersonic flow

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{1}{r^2} \frac{\partial^2 \phi}{\partial r^2} + \frac{2M^2}{V} \frac{\partial^2 \phi}{\partial x \, \partial t} + \frac{M^2}{V^2} \frac{\partial^2 \phi}{\partial t^2}$$

Therefore these two equations are compatible if the above limiting conditions are satisfied.

In equation (3) a solution to the two-dimensional Laplace equation is given as

$$\phi = \phi(w,R,s,r,\theta) \tag{29}$$

where $R = R(x)$ and $s = s(x) = Kx + Q$. If, for the present, the assumption is made that $w = w(x,t)$, from equation (29)

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial w^2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial^2 \phi}{\partial R^2}\left(\frac{\partial R}{\partial x}\right)^2 + \frac{\partial^2 \phi}{\partial s^2}\left(\frac{\partial s}{\partial x}\right)^2 + 2\left(\frac{\partial^2 \phi}{\partial w \, \partial R} \frac{\partial w}{\partial x} \frac{dR}{dx} + \frac{\partial^2 \phi}{\partial w \, \partial s} \frac{\partial w}{\partial x} \frac{ds}{dx} + \frac{\partial^2 \phi}{\partial R \, \partial s} \frac{dR}{dx} \frac{ds}{dx}\right) + \frac{\partial \phi}{\partial w} \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi}{\partial R} \frac{d^2 R}{dx^2} + \frac{\partial \phi}{\partial s} \frac{d^2 s}{dx^2} \tag{30}$$

$$\frac{\partial^2 \phi}{\partial x \, \partial t} = \frac{\partial^2 \phi}{\partial w^2} \frac{\partial w}{\partial x} \frac{\partial w}{\partial t} + \frac{\partial^2 \phi}{\partial w \, \partial R} \frac{\partial w}{\partial t} \frac{dR}{dx} + \frac{\partial^2 \phi}{\partial w \, \partial s} \frac{\partial w}{\partial t} \frac{ds}{dx} + \frac{\partial \phi}{\partial w} \frac{\partial^2 w}{\partial x \, \partial t} \tag{31}$$

and

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial w^2}\left(\frac{\partial w}{\partial t}\right)^2 + \frac{\partial \phi}{\partial w}\frac{\partial^2 w}{\partial t^2} \tag{32}$$

For constant rate of pitch, $v = w(x) = q(x - x_o)$, and from equations (30) to (32)

$$\left.\begin{aligned} \beta^2 \frac{\partial^2 \phi}{\partial x^2} &= \beta^2\left[\frac{\partial^2 \phi}{\partial w^2} q^2 + \frac{\partial^2 \phi}{\partial R^2}\left(\frac{dR}{dx}\right)^2 + \frac{\partial^2 \phi}{\partial s^2} K^2 + 2\left(\frac{\partial^2 \phi}{\partial w\, \partial R} q \frac{dR}{dx} + \right.\right. \\ &\qquad \left.\left. \frac{\partial^2 \phi}{\partial w\, \partial s} qK + \frac{\partial^2 \phi}{\partial R\, \partial s} K \frac{dR}{dx}\right) + \frac{\partial \phi}{\partial R}\frac{d^2 R}{dx^2}\right] \\ \frac{\partial^2 \phi}{\partial x\, \partial t} &= \frac{\partial^2 \phi}{\partial t^2} = 0 \end{aligned}\right\} \tag{33}$$

For constant accelerated motion in the vertical direction, $w = w(t) = \dot{\alpha}Vt$, and from equations (30) to (32)

$$\left.\begin{aligned} \beta^2 \frac{\partial^2 \phi}{\partial x^2} &= \beta^2\left[\frac{\partial^2 \phi}{\partial R^2}\left(\frac{dR}{dx}\right)^2 + \frac{\partial^2 \phi}{\partial s^2} K^2 + 2\frac{\partial^2 \phi}{\partial R\, \partial s} K \frac{dR}{dx} + \frac{\partial \phi}{\partial R}\frac{d^2 R}{dx^2}\right] \\ 2\frac{M^2}{V}\frac{\partial^2 \phi}{\partial x\, \partial t} &= 2M^2\left(\frac{\partial^2 \phi}{\partial w\, \partial R}\dot{\alpha}\frac{dR}{dx} + \frac{\partial^2 \phi}{\partial w\, \partial s}\dot{\alpha}K\right) \\ \frac{M^2}{V^2}\frac{\partial^2 \phi}{\partial t^2} &= M^2 \frac{\partial^2 \phi}{\partial w^2}\dot{\alpha}^2 \end{aligned}\right\} \tag{34}$$

An examination of equations (33) and (34) shows that, in order for the Laplace solution to be a solution to the linearized equation of supersonic flow, $\frac{dR}{dx}$, $\frac{d^2R}{dx^2}$, K, q, and $\dot{\alpha}$ must approach zero.

Within the framework of the small-disturbance theory, however, such stringent conditions as these are not necessary for the Laplace solution to apply to the supersonic range. Rather it is required that $\frac{dR}{dx}$, $\frac{d^2R}{dx^2}$,

K, q, and $\dot{\alpha}$ be of such an order of magnitude that $\beta^2 \frac{\partial^2 \phi}{\partial x^2}$, $2 \frac{M^2}{V} \frac{\partial^2 \phi}{\partial x \, \partial t}$, and $\frac{M^2}{V^2} \frac{\partial^2 \phi}{\partial t^2}$ be negligibly small compared with the remaining terms of the linearized equation of supersonic flow.

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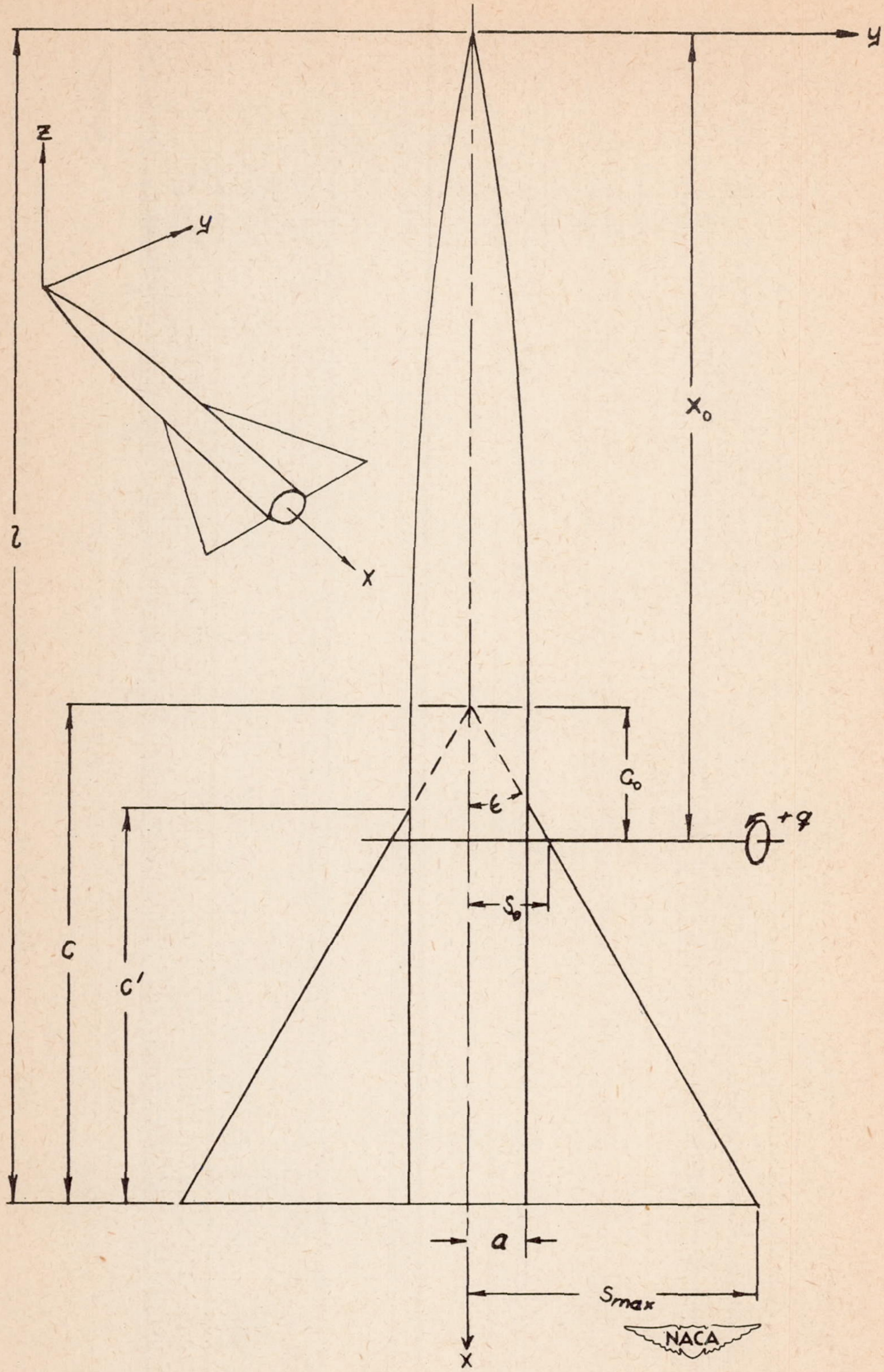


Figure 1.- Geometry of configuration and coordinate system used.

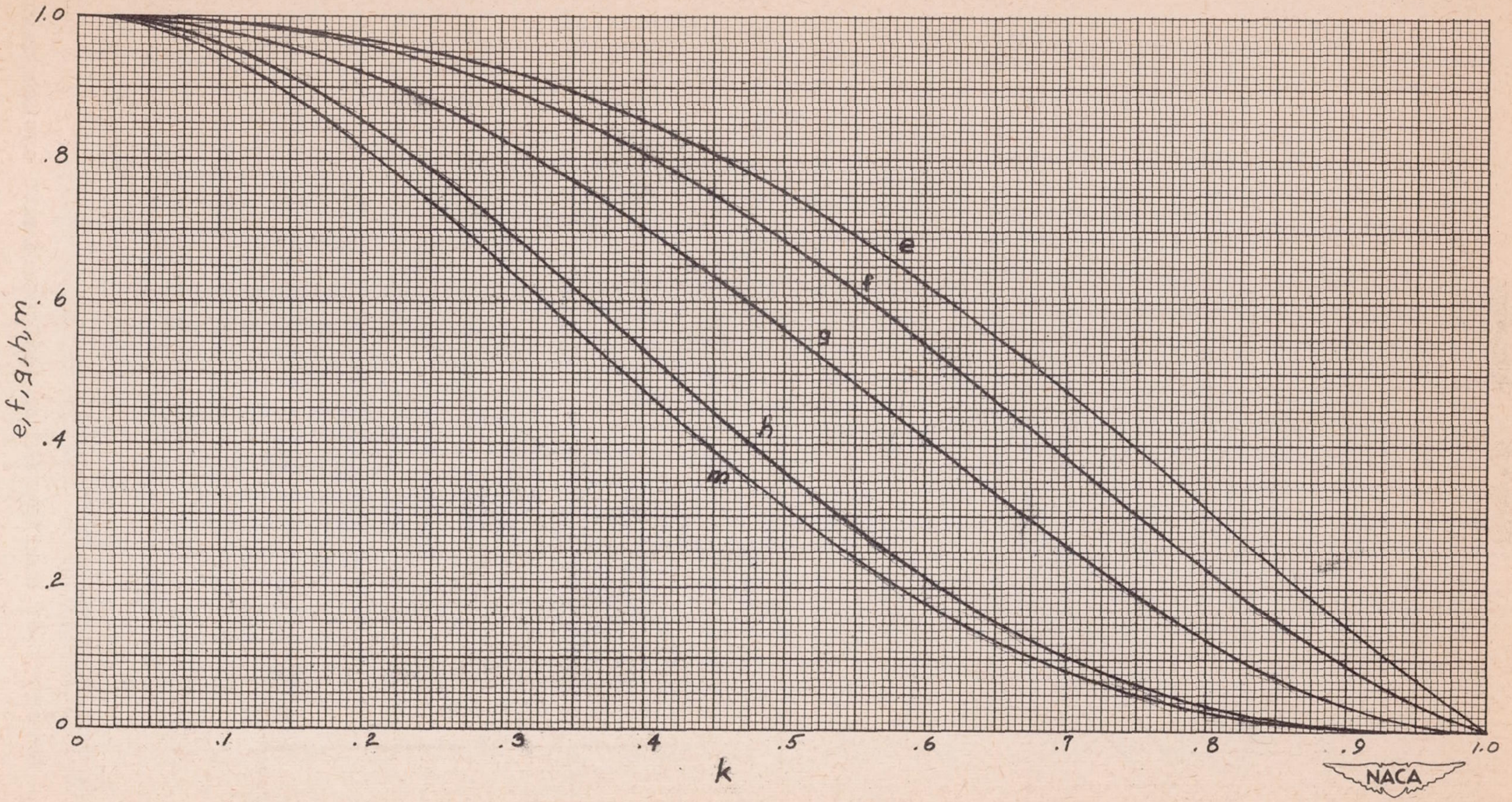


Figure 2.- Variation of interference factors with k.

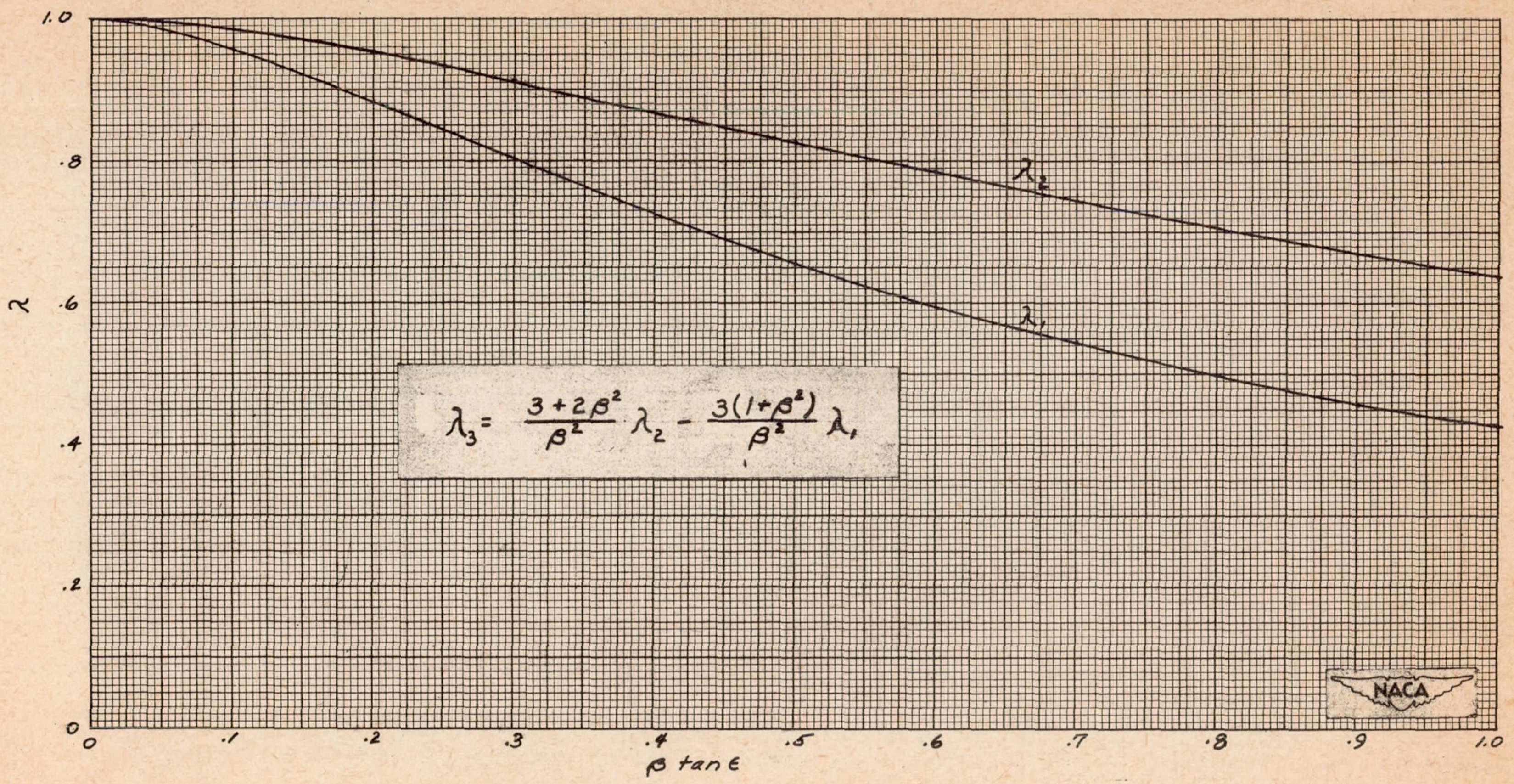
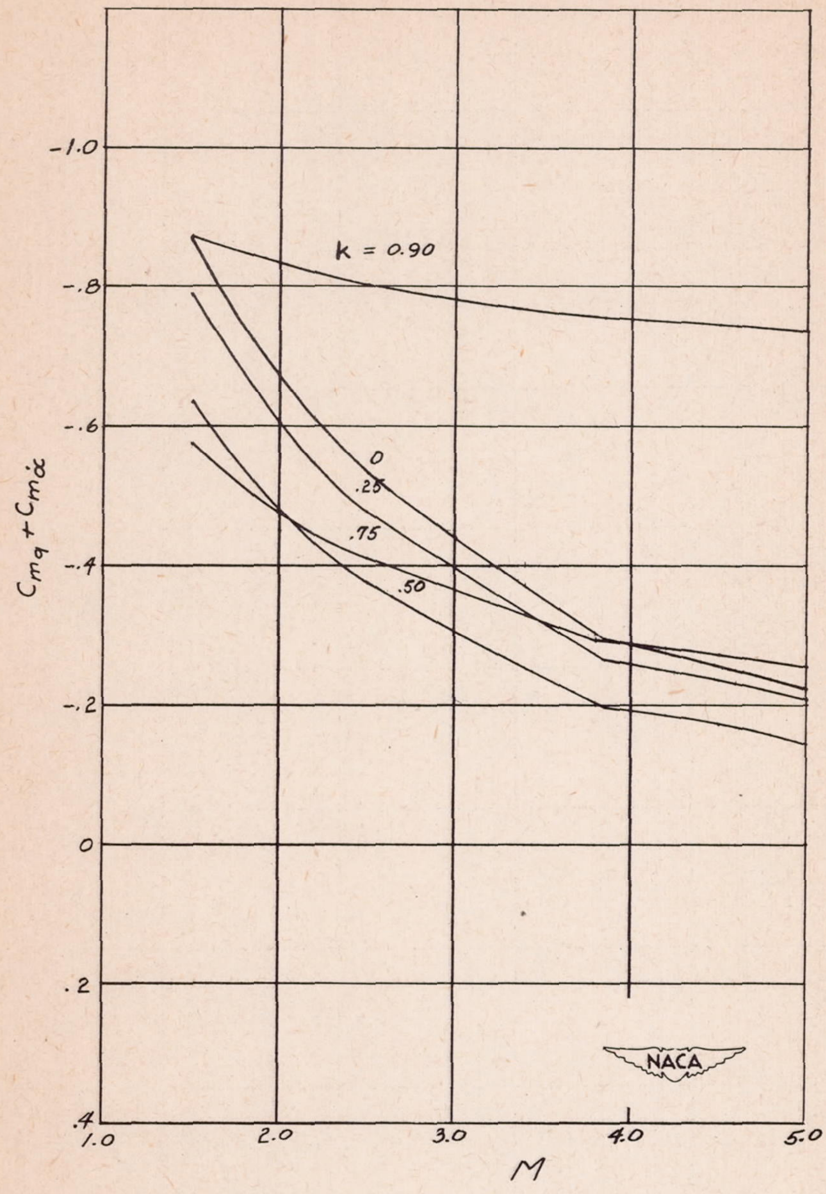
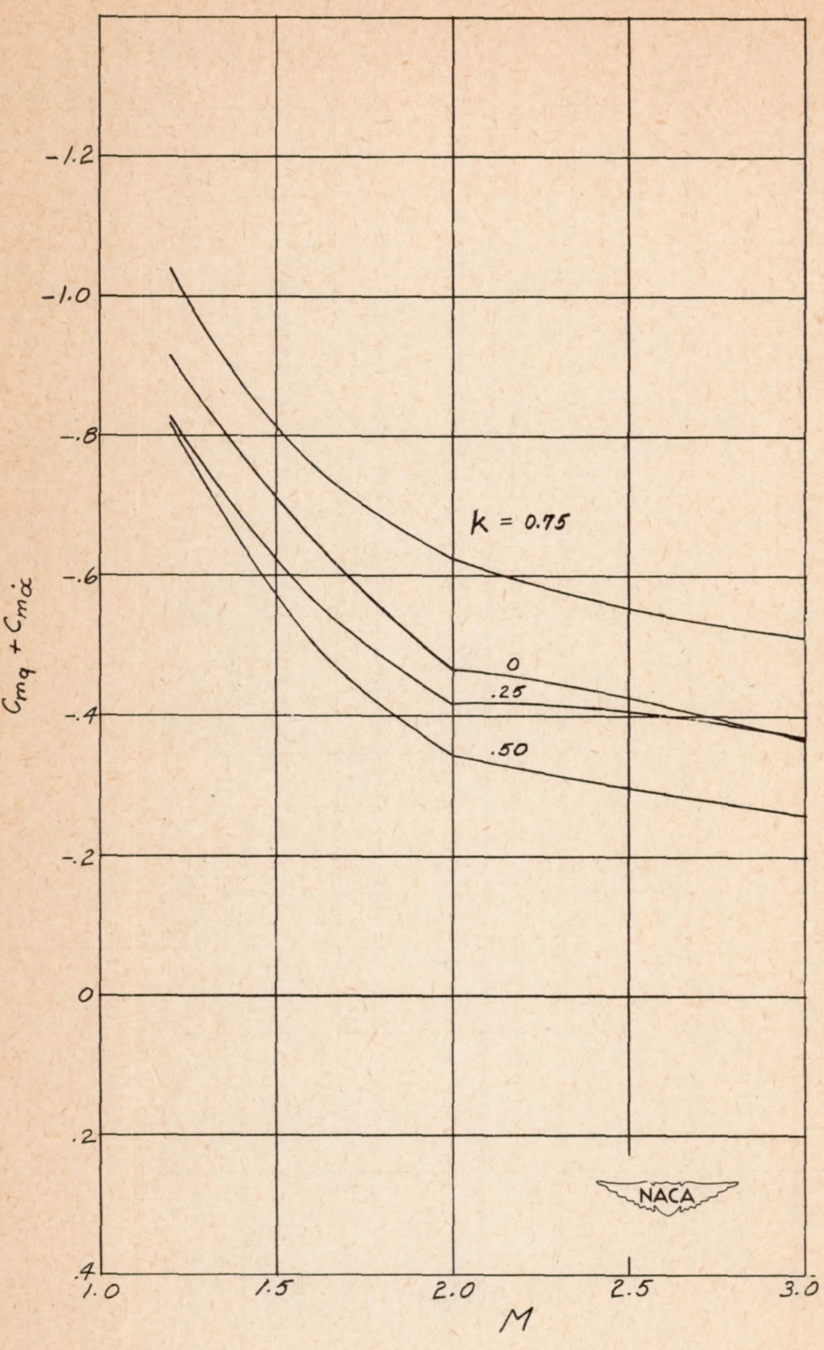


Figure 3.- λ_3 and variation of λ_1 and λ_2 with $\beta \tan \epsilon$.



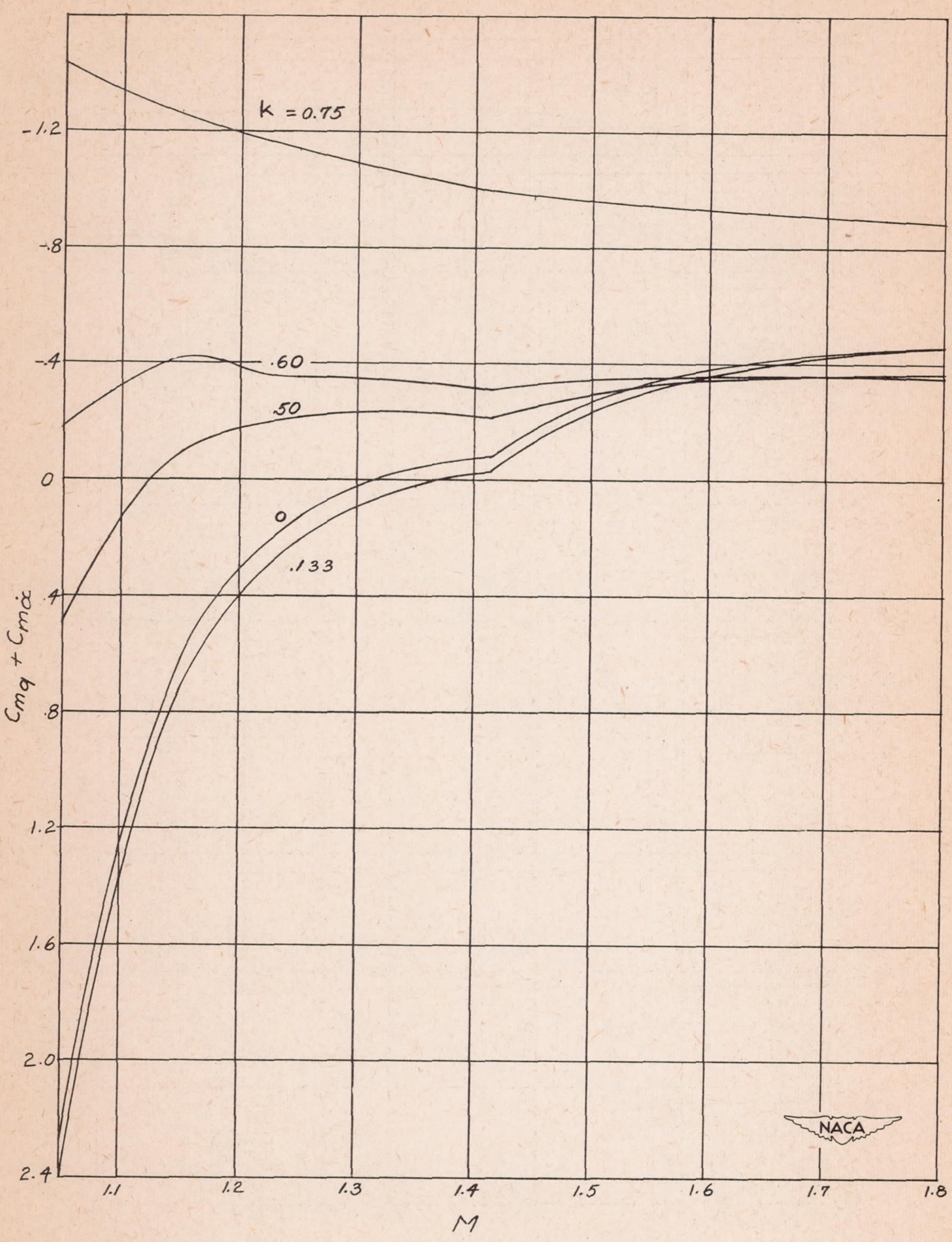
(a) $\epsilon = 15^{\circ}$.

Figure 4.- Variation of $C_{m_q} + C_{m_{\dot{\alpha}}}$ with Mach number for various values of k for $\frac{c_o}{\bar{c}} = 0.85$.



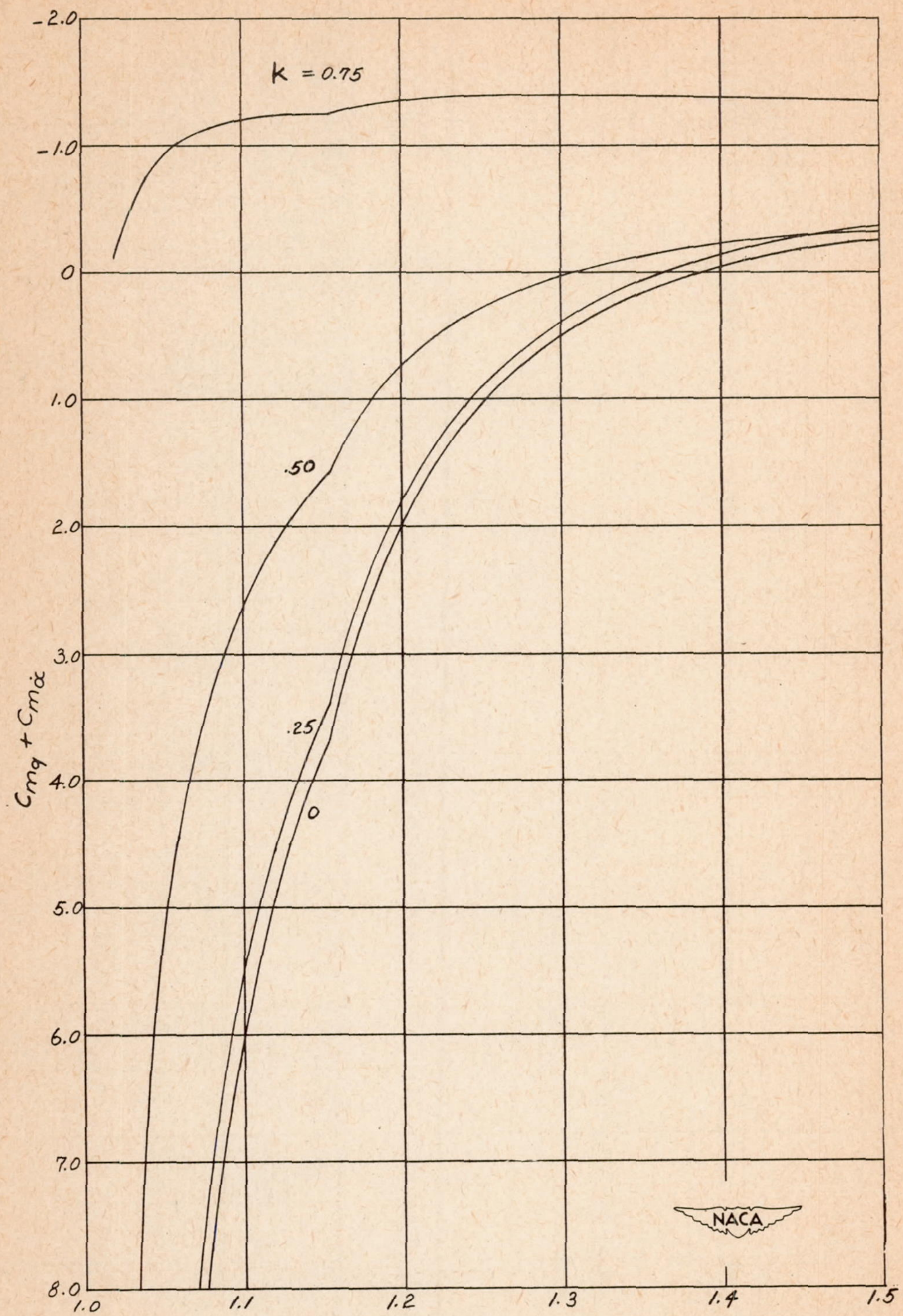
(b) $\epsilon = 30^{\circ}$.

Figure 4.- Continued.



(c) $\epsilon = 45^\circ$.

Figure 4.- Continued.



(d) $\epsilon = 60^{\circ}$.

Figure 4.- Concluded.

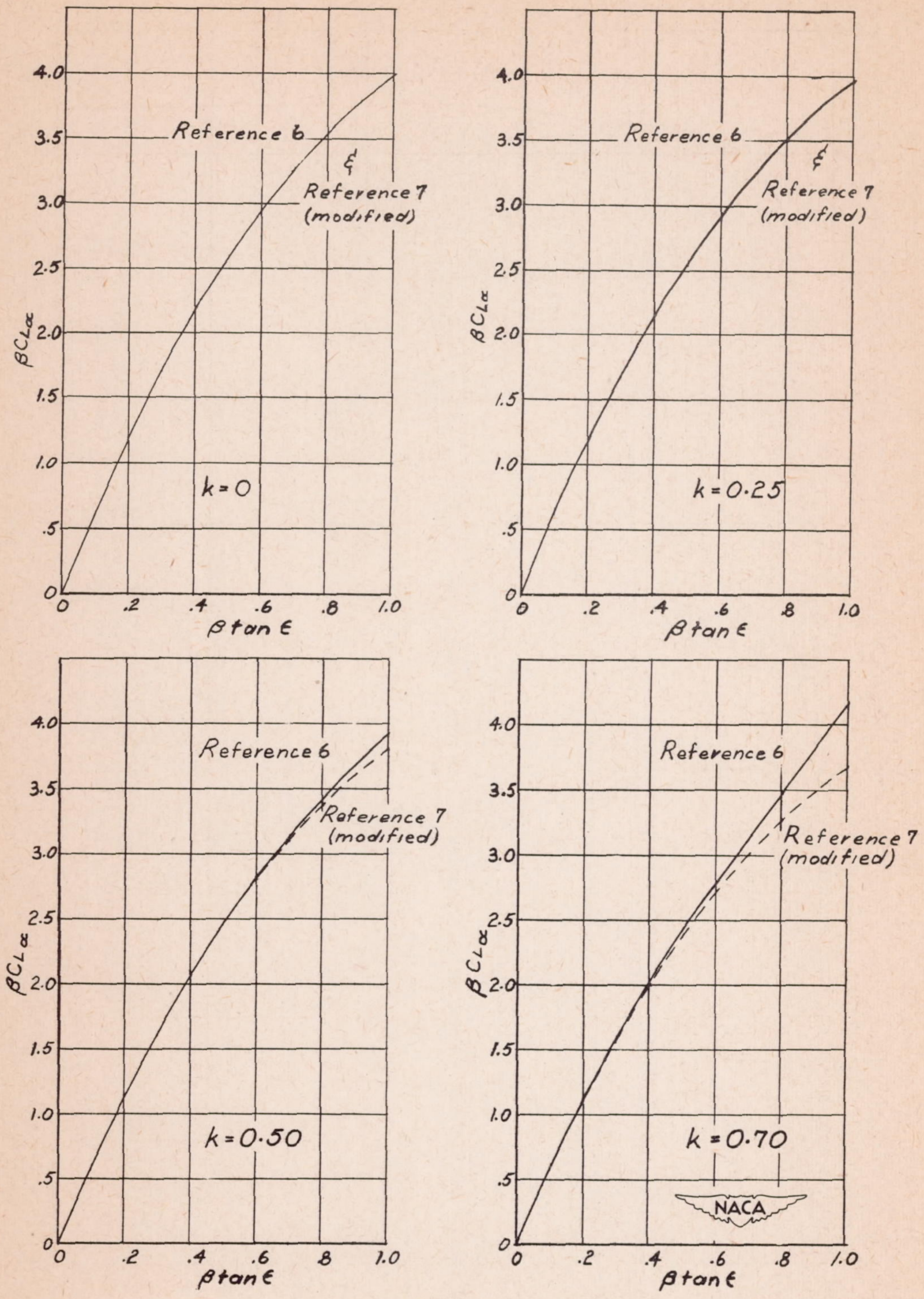


Figure 5.- βC_{L_α} plotted against $\beta \tan \epsilon$ for various values of k for delta-wing and conical-body combination. Comparison of exact solution of reference 6 and modified solution of reference 7.

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