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A METHOD FOR THE DETERMINATION OF THE TIME LAG IN PRESSURE MEASURING SYSTEMS INCORPORATING CAPILLARIES

By Archibald R. Sinclair and A. Warner Robins

Langley Aeronautical Laboratory
Langley Field, Va.

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A METHOD FOR THE DETERMINATION OF THE TIME LAG IN PRESSURE MEASURING SYSTEMS INCORPORATING CAPILLARIES

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SUMMARY

A method is presented for the determination of the time lag in pressure measuring systems incorporating capillaries; this method is a convenient and systematic means of selecting, designing, or redesigning a pressure measuring system to meet the time requirements of a particular installation. Experimental data are shown and a step-by-step sample application is presented.

Calculated and experimental data are in reasonable agreement and show that response time in a pressure measuring system incorporating capillaries is a function of the orifice pressure, the initial pressure differential, and the system volume, is directly proportional to capillary length and to the viscosity of the gas in the capillary, and is inversely proportional to the fourth power of the capillary inside diameter.

INTRODUCTION

The time lag in pressure measuring systems has become more important with the advent of supersonic and transonic wind tunnels in which the pressures are low and the capillaries in the systems are generally small. In such capillary systems, the time required to approach equilibrium and obtain a sufficiently accurate measurement of the orifice pressure at the pressure measuring device may be considerable and increases as the pressures decrease. The importance of reducing the time lag in such installations is associated with the high cost of tunnel operation and with the limitation of running time as in the case of intermittent wind tunnels. A method that would offer a convenient and systematic means of selecting, designing, or redesigning a system to meet the time requirements of a particular installation therefore would be especially useful.

Most of the available literature concerning time lag in pressure measuring systems is developed specifically for application to aircraft instrument systems (refs. 1 to 5). Kendall (ref. 6) has developed a theory for use with a rather specialized type of manometer. The purpose of the present investigation of the problem of time lag is to develop a method of theoretical analysis which could be readily applied to the multitube manometers normally used in wind-tunnel work, to check this method with experimental data, and to investigate when possible the practical limitations of the method. The ranges of variables are chosen to approximate the conditions ordinarily encountered in manometer installations for wind tunnels. The method, though it is developed for liquid manometers, is applicable to various types of measuring devices.

SYMBOLS

t time

p pressure in measuring device at any time t

p_0 initial pressure in measuring device at $t = 0$

p_1 pressure at orifice

V entire air volume of system from orifice to and including measuring device at any time t

V_1 entire air volume of system from orifice to and including measuring device at $p = p_1$ (or $t = \infty$)

V_1' V_1 less volume of capillary, the optimum diameter of which is to be determined

V_m air volume between fluid surface and connection to capillary system in manometer, or volume of pressure capsule at $p = p_1$

V_d volume displaced by motion of manometer fluid or by deflection of capsule walls

d internal diameter of capillary (used with subscripts 1, 2, 3, . . . n to denote various capillaries in series-connected system)

d_x	internal diameter of capillary, the optimum diameter of which is to be determined
l	length of capillary (used with subscripts 1, 2, 3, . . . n to denote various capillaries in series-connected system)
l_x	length of capillary, the optimum diameter of which is to be determined
l_e	length of capillary of diameter d which is equivalent in flow resistance to total resistance of all series-connected capillaries in system
l_e'	length of capillary of diameter d which is equivalent in flow resistance to all series-connected capillaries in system except that capillary, the optimum diameter of which is to be determined
D	internal diameter of manometer tube
T	temperature of air in capillary
w	specific weight of manometer fluid
μ	coefficient of viscosity of air in system
m	mass of air in pressure measuring system
R	universal gas constant

Any consistent set of units may be used.

THEORETICAL ANALYSIS

Analysis of Problem and Development of Method

A simplified diagram of a manometer is shown in figure 1. The total volume of the entire system from the surface of the manometer fluid at equilibrium to the orifice is considered a single volume V_1. The orifice and all connecting tubing are considered as a length of tubing of length l_e and constant diameter d between the orifice and the manometer. The inside diameter of the manometer tube is represented by D. The pressure at any instant in the manometer is p and the pressure at the orifice is p_1. The pressure in the manometer when

$t = 0$ is p_0; then $p_0 - p_1$ is the initial unbalance in the system. Thus the pressure in the manometer starts as p_0 at $t = 0$ and changes progressively with time until it becomes equal to p_1 after an infinitely long time.

In the analysis of the problem, the flow in the capillary is assumed to be laminar. For practical purposes, this assumption appears to be justifiable since the capillaries are smooth-walled and the flow velocities are usually such that the Reynolds numbers are below critical.

In order to analyze the flow in the capillary, the rate of flow in a tube of circular cross section may be related to the pressures at its ends by (see ref. 7, with zero slip assumed)

$$\frac{dm}{dt} = \frac{\pi d^4}{256\mu l_e RT}\left(p^2 - p_1^2\right) \tag{1}$$

Equation (1) is a form of Poiseuille's formula.

If the entire process is assumed to be isothermal, the mass rate of flow of the entire system can also be expressed in terms of the volume in the system, the pressure within this volume, and the rates of change of each as

$$RT\frac{dm}{dt} = -p\frac{dV}{dt} - V\frac{dp}{dt} \tag{2}$$

Since V_1 is the volume of the system at equilibrium $\left(p = p_1\right)$ and V_d is the volume displaced by the fluid as the indicated pressure changes from p_0 to p_1, the changing volume from which the gas must expand may be written as

$$V = V_1 + \frac{p - p_1}{p_0 - p_1}V_d$$

or, since $V_d = \frac{\pi D^2}{4}\frac{p_0 - p_1}{w}$,

$$V = V_1 + \frac{\pi D^2}{4w}\left(p - p_1\right)$$

Then,

$$dV = \frac{\pi D^2}{4w}\,dp$$

Substituting these equations for V and dV into equation (2) results in

$$RT\,\frac{dm}{dt} = -p\,\frac{\pi D^2}{4w}\,\frac{dp}{dt} - V_1\,\frac{dp}{dt} - \frac{\pi D^2}{4w}\left(p - p_1\right)\frac{dp}{dt}$$

Dividing this equation by the constant RT and equating to equation (1) yields

$$-p\,\frac{\pi D^2}{4w}\,\frac{dp}{dt} - V_1\,\frac{dp}{dt} - \frac{\pi D^2}{4w}\left(p - p_1\right)\frac{dp}{dt} = \frac{\pi d^4}{256\mu l_e}\left(p^2 - {p_1}^2\right)$$

and

$$dt = -\frac{256\mu l_e}{\pi d^4}\left[V_1\,\frac{dp}{\left(p + p_1\right)\left(p - p_1\right)} + \frac{\pi D^2}{4w}\,\frac{dp}{p + p_1} + \frac{\pi D^2}{4w}\,\frac{p\,dp}{p^2 - {p_1}^2}\right]$$

Integrating this equation results in

$$t = -\frac{256\mu l_e}{\pi d^4}\left[\frac{V_1}{2p_1}\log_e\frac{p - p_1}{p + p_1} + \frac{\pi D^2}{4w}\log_e\left(p + p_1\right) + \frac{\pi D^2}{8w}\log_e\left(p^2 - {p_1}^2\right)\right] + C_1$$

After the constant C_1 is evaluated at $t = 0$ when $p = p_0$ and terms are collected, t becomes

$$t = \frac{128\mu l_e}{\pi d^4}\left[\frac{V_1}{p_1}\log_e\frac{(p_0 - p_1)(p + p_1)}{(p - p_1)(p_0 + p_1)} + \frac{3\pi D^2}{4w}\log_e\frac{p_0 + p_1}{p + p_1} + \right.$$

$$\left.\frac{\pi D^2}{4w}\log_e\frac{p_0 - p_1}{p - p_1}\right] \qquad (3)$$

Replacing the $\pi D^2/4w$ terms with their equivalent $\frac{V_d}{p_0 - p_1}$ so that the equation may apply to various types of pressure measuring devices yields

$$t = \frac{128\mu l_e}{\pi d^4}\left[\frac{V_1}{p_1}\log_e\frac{(p_0 - p_1)(p + p_1)}{(p - p_1)(p_0 + p_1)} + \frac{3V_d}{p_0 - p_1}\log_e\frac{p_0 + p_1}{p + p_1} + \right.$$

$$\left.\frac{V_d}{p_0 - p_1}\log_e\frac{p_0 - p_1}{p - p_1}\right] \qquad (4)$$

A value of l_e, the equivalent length based on a common diameter d for a set of connected capillaries, may be determined by finding the equivalent lengths of each and adding the resulting lengths to that of the capillary of diameter d. In a system having only two capillaries (of diameters d_1 and d_2 and lengths l_1 and l_2, respectively) in which d_1 is chosen as the basic tube $(d = d_1)$, the equivalent length of the capillary of diameter d_2 and length l_2 is the length of capillary of diameter d to which it is equivalent on point of flow resistance. This length may be found by equating the expressions for t for both capillaries and dropping the bracketed terms. Then, $\frac{l_e}{d^4} = \frac{l_2}{{d_2}^4}$ or, since $d = d_1$, $l_e = l_2 \frac{{d_1}^4}{{d_2}^4}$ where l_e is the equivalent length of capillary 2 in terms of a capillary of diameter d. Then the total equivalent length of a system of connected capillaries when d_1 is chosen arbitrarily as the basic diameter $\left(d = d_1\right)$ is

$$l_e = l_1 + l_2 \frac{d^4}{d_2{}^4} + l_3 \frac{d^4}{d_3{}^4} + \cdot \cdot \cdot + l_n \frac{d^4}{d_n{}^4}$$

Consideration of Optimum Capillary Size

Equation (3) shows that the greatest gain in attempting to reduce the time lag of an existing pressure measuring system can be had by increasing the bore of the smaller capillaries, consistent with space and structural limitations, or by replacing as large a portion of the smaller capillaries as possible with larger tubing.

System volume, which has a direct relation to response time, is influenced by the capillary volumes, and, for the larger capillaries, an optimum exists between large bore with its large volume, and small bore with its greater flow resistance. For a system incorporating only two sizes of capillaries, the problem of mathematically determining this optimum would be relatively simple, but such a system is rarely found in practice. In the case of a wind-tunnel model installation, a system can generally be divided into two groups of capillaries: those in the model and support system, the sizes of which are determined principally by space limitations and which are usually the greatest source of time lag, and those which connect them to the pressure measuring device and on which no space limitations are imposed. By resolving the former group into an equivalent length l_e' of a single capillary of diameter d, the system becomes a two-capillary one which lends itself more readily to calculation.

Since the equivalent length l_e, based on a capillary of diameter d, may be expressed as

$$l_e = l_e' + l_x \frac{d^4}{d_x{}^4}$$

(where the subscript x refers to the capillary, the optimum diameter of which is to be determined) and if the volume V_1' is known, the equation for settling time becomes

$$t = \frac{128\mu}{\pi} \frac{d_x^{4} l_e' + d^4 l_x}{d_x^{4} d^4} \left[\left(V_1' + \frac{\pi}{4} d_x^{2} l_x\right) \frac{1}{p_1} \log_e \frac{p_0 - p_1}{p - p_1} \frac{p + p_1}{p_0 + p_1} + \right.$$

$$\left. \frac{3V_d}{p_0 - p_1} \log_e \frac{p_0 + p_1}{p + p_1} + \frac{V_d}{p_0 - p_1} \log_e \frac{p_0 - p_1}{p - p_1}\right]$$

By letting

$$A = \frac{128\mu}{\pi}$$

$$B = \frac{1}{p_1} \log_e \frac{p_0 - p_1}{p - p_1} \frac{p + p_1}{p_0 + p_1}$$

and

$$C = \frac{3V_d}{p_0 - p_1} \log_e \frac{p_0 + p_1}{p + p_1} + \frac{V_d}{p_0 - p_1} \log_e \frac{p_0 - p_1}{p - p_1}$$

the equation for t may be written as

$$\frac{t}{A} = \frac{d_x^{4} l_e' + d^4 l_x}{d_x^{4} d^4} \left[\left(V_1' + \frac{\pi}{4} d_x^{2} l_x\right) B + C\right]$$

Differentiating this equation with respect to d_x, setting the result equal to zero, and simplifying yields

$$\frac{\pi}{4} l_e' d_x^{6} - \frac{\pi}{4} l_x d^4 d_x^{2} - 2Ed^4 = 0 \tag{5}$$

where

$$E = V_1' + \frac{C}{B}$$

Equation (5) is an expression for the diameter d_x required for a minimum value for time t. Substituting y for d_x^2 into equation (5) and simplifying yields

$$y^3 = \frac{l_x d^4}{l_e'} y + \frac{8Ed^4}{\pi l_e'}$$

and, if the two imaginary roots are neglected,

$$y = \sqrt[3]{\frac{4Ed^4}{\pi l_e'} + \sqrt{\left(\frac{4Ed^4}{\pi l_e'}\right)^2 - \left(\frac{l_x d^4}{3l_e'}\right)^3}} + \sqrt[3]{\frac{4Ed^4}{\pi l_e'} - \sqrt{\left(\frac{4Ed^4}{\pi l_e'}\right)^2 - \left(\frac{l_x d^4}{3l_e'}\right)^3}}$$

or

$$d_x = \sqrt{\sqrt[3]{\frac{4Ed^4}{\pi l_e'} + \sqrt{\left(\frac{4Ed^4}{\pi l_e'}\right)^2 - \left(\frac{l_x d^4}{3l_e'}\right)^3}} + \sqrt[3]{\frac{4Ed^4}{\pi l_e'} - \sqrt{\left(\frac{4Ed^4}{\pi l_e'}\right)^2 - \left(\frac{l_x d^4}{3l_e'}\right)^3}}} \qquad (6)$$

This equation is somewhat unwieldy, however, and may be replaced for all practical purposes by a simpler expression which arises from neglecting the two displacement terms of the original equation for t, which leaves

$$t \approx \frac{128\mu}{\pi} \frac{d_x^4 l_e' + d^4 l_x}{d_x^4 d^4} \left[\left(V_1' + \frac{\pi}{4} d_x^2 l_x\right) \frac{1}{p_1} \log_e \frac{p_0 - p_1}{p - p_1} \frac{p + p_1}{p_0 + p_1}\right]$$

The simpler expression for d_x which then results is

$$d_x \approx \sqrt{\sqrt[3]{\frac{4V_1'd^4}{\pi l_e'} + \sqrt{\left(\frac{4V_1'd^4}{\pi l_e'}\right)^2 - \left(\frac{l_x d^4}{3l_e'}\right)^3}} + \sqrt[3]{\frac{4V_1'd^4}{\pi l_e'} - \sqrt{\left(\frac{4V_1'd^4}{\pi l_e'}\right)^2 - \left(\frac{l_x d^4}{3l_e'}\right)^3}}} \qquad (7)$$

For the range of variables considered in this paper, the term $\left(\frac{l_x d^4}{3l_e'}\right)^3$ is negligible compared to $\left(\frac{4V_1' d^4}{\pi l_e'}\right)^2$, and equation (7) can then be further reduced to

$$d_x \approx \sqrt[6]{\frac{8V_1' d^4}{\pi l_e'}} \tag{8}$$

which shows that, for practical purposes, the optimum diameter of the capillary is independent of its length.

Table I presents a comparison of the optimum-tube internal diameters computed by equations (6), (7), and (8) throughout the practical ranges of volume, length, and pressure and indicates that equation (8) is sufficiently accurate for conventional manometer systems, particularly in the critical case of very low pressures.

Application of the preceding method to an example is shown in the appendix.

EXPERIMENTAL INVESTIGATION

Apparatus and Procedure

A schematic drawing of the main part of the test apparatus is shown in figure 2. Related equipment not shown includes a sensitive barometer, a thermometer, and a manually operated device for recording the settling time. The capillaries used ranged in inside diameter from 0.0205 to 0.039 inch and were of stainless steel and Monel metal.

The orifice pressures p_1, ranging from 1/50 to 2 atmospheres, were maintained in the air tank (see fig. 2), which was of sufficient volume to take up the air introduced at point C with no discernable change in pressure. Tube 2 was the reference tube which determined the setting of the scale. Tube 1, in which the initial pressure p_0 was set, simulated a typical manometer tube. Tubes 1 and 2 were of 0.160-inch internal diameter.

For each test the procedure was essentially the same. After the orifice pressure p_1 (indicated by the mercury U-tube) was set in the tank, air was introduced at point C (see fig. 2) to establish the initial unbalance in pressure $p_0 - p_1$ across the test capillary so that the manometer fluid moved down past point B. When the air at point C was valved off, the fluid moved upward past point B as a result of flow through the capillary, and the return toward equilibrium at point A was then timed at selected intervals on the scale.

The tests consisted of two main groups: one for a volume between the orifice and manometer fluid (within C, D, and A in fig. 2) of 0.00382 cubic foot and the other for a volume of 0.00247 cubic foot. This change was accomplished by using a different size of air chamber for each group. These groups were subdivided into tests of individual capillaries and various combinations of capillaries which, in turn, were divided into runs at the various orifice static pressures p_1. The initial unbalance between points A and B was 15 inches for all the tests. Since Alkazene 42 (x-dibromoethylbenzene) of specific gravity 1.749 at 72° F was used, the pressure unbalance was 136.4 pounds per square foot.

Accuracy

The accuracy of the data is mainly dependent on the accuracy of the measurement of the internal diameter d of the capillaries, since the variable d enters the equation for response time in the fourth power. These measurements were made near the ends of the capillary with plug gages; some error besides that of reading would therefore result if the bores were not uniform throughout the lengths of the capillaries. The over-all errors introduced by inaccuracies in any of the remaining variables would, by comparison, be negligible. Since no practical means were available for reducing the possible error in the measurement of capillary bore, a considerable amount of data was obtained so that an average might be established. The good agreement throughout the tests between theory and experiment indicates that, for the purposes of this paper, no significant errors occurred.

RESULTS AND DISCUSSION

Figures 3(a) and 3(b), which show response time t as a function of pressure unbalance $p - p_1$, are representative of the plots of all the single-capillary tests and show the generally good agreement between calculated and experimental results. Figures 4 to 6 are a breakdown and comparison of the data to show time t plotted against each of the principal variables d, p_1, and l for both values of V_1. Each plot shows similarly good correlation between theory and experiment, and no significant trend is revealed which might indicate the approach of a limiting condition within the practical range of the theory.

A comparison between theoretical and experimental results for two connected capillaries is shown in figure 7. In this case, various lengths ranging from 0.002 inch to 6 feet of very small capillary (0.0205-inch internal diameter) were added to a 6-foot length of larger capillary (0.039-inch internal diameter) and the resulting configurations were tested at various orifice pressures. The difference between the experimental and calculated data appears to be due to an orifice or tube end effect which decreases as the length or thickness of the orifice approaches zero. A related effect is shown in figure 8, in which the calculated values of response time for a plain capillary are compared with experimentally obtained values for a capillary of the same dimension but to which a very thin orifice has been attached. A thin orifice is shown to have only a slight effect on the time lag until its diameter becomes less than about 0.25 of that of the capillary to which it is fixed. An orifice of any appreciable thickness (or length) might best be treated as an additional length of capillary, although no account in theory is made of total-pressure losses resulting from orifice and capillary junctions.

Several straight capillaries were tested and then retested with a number of small-radius turns and kinks. The resulting differences in response time were almost negligible although considerable flattening of the capillaries was noted at each bend. The effect of a bend appears to be similar to that of an orifice.

Reduction of the response time by lowering the initial pressure unbalance does not in many cases appear to be practical. For example, reducing the initial unbalance to 10 percent of its original value will only reduce the response time by a factor of 1/2 to 1/3, as may be seen in figure 3.

In the present tests, neglecting the time lag due to the viscous effect of the liquid in the manometer appears justifiable since theory and experiment agree for response times as low as 14 seconds. The geometry of the liquid system in the manometer used approximated that of a conventional system so that its resistance to flow should also be comparable. Since a relatively light manometer medium was used, that is, for a given pressure differential a greater volume of flow must occur than for a more dense fluid, it follows that the occurrence of a noticeable mechanical lag in a conventional system is unlikely, unless unusually viscous liquids are used or very short response times are required.

Since, in the equations, the entire air volume of the system between the orifice and the surface of the manometer fluid is assumed to be lumped together (as shown in fig. 1), some doubt may exist about whether the equations hold for a system which is greatly dissimilar in volume distribution. It appears that the equations, based on the assumed lumped volume, will yield pessimistic answers when applied to an actual system since, theoretically, the gases expanded from or into the system will have to pass through all the capillaries. The error resulting from the assumption of lumped volume depends then on the flow resistance of that part of the system containing most of the system volume and on how great a part this volume is of the whole, if, in the case of interconnected capillaries of different sizes, the smallest capillaries are assumed to be located at the orifice end of the system. For example, for a manometer system in which there are 1, 25, and 190 feet of capillaries of 0.020-, 0.032-, and 0.090-inch inside diameters, respectively, and in which approximately 97 percent of the entire system volume is within the largest capillary, the difference in response time is approximately 8.7 percent if, first, the entire volume is assumed to be lumped together at the measuring device and, second, the largest capillary itself is assumed to be a chamber (with no flow resistance) containing the entire system volume. Conditions in the actual manometer system fall somewhere between these two assumed conditions, and the error due to the assumption of a lumped volume must therefore be less than 8.7 percent. In most manometer systems, it should be considerably less.

Figure 9 is a comparison of the calculated and experimental response times for the manometer system mentioned in the previous paragraph. The discrepancies are attributed mainly to the flexibility of the 190-foot length of plastic capillary which tends to increase greatly the displacement volume of the system. The effect of distributed volume appears in this case to be secondary.

In the practical application of the equations, in many cases the exact geometry of the system and the initial and final pressures imposed may be difficult to ascertain so that the resulting solutions for response time may be only of the correct order. In many cases, the correct order may be all that is required. Some of the probable causes for discrepancies in applying the equations are:

(1) Inaccuracies in measurement of capillary diameter

(2) Unknown system volume

(3) Unknown capillary length, particularly for smallest capillary

(4) Elasticity of flexible tubing

(5) Porosity of capillaries, particularly for flexible tubing

(6) Effect of distributed volume

(7) Effect of orifices and capillary junctions

(8) Kinks or bends which reduce considerably the cross-sectional area of the capillary

CONCLUSIONS

A method has been presented for the determination of the time lag in pressure measuring systems incorporating capillaries, and an experimental investigation has been made to apply this method. The results show that:

1. The calculated and experimental data are in reasonable agreement.

2. Response time in a pressure measuring system incorporating capillaries is a function of the orifice pressure, the initial pressure differential, and the system volume, is directly proportional to capillary length and to the viscosity of the gas in the capillary, and is inversely proportional to the fourth power of the capillary inside diameter. The response time increases as the orifice pressure decreases.

3. In a system of capillaries connecting a model orifice with the pressure measuring device, those capillaries located within the model and support system should be carefully selected to insure an adequate diameter.

4. Where the maximum diameters of part of a set of connected capillaries in a pressure measuring system are determined by space limitations or other considerations, one optimum inside diameter exists for the other capillaries in the system.

5. An orifice in a capillary system has little effect on response time when its diameter is greater than about 0.25 of the internal diameter of the capillary to which it is attached.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX

EXAMPLE OF USE OF THE METHOD

Basic configuration.- The purpose of the following calculations is to show the way a conventional manometer system having poor response might be altered in order to shorten response time to an acceptable amount. The equations are applied to an actual case, the pertinent physical characteristics of which are:

Model tubing,

$d_1 = 0.0205 \text{ in.} = 0.001708 \text{ ft}$ $\qquad l_1 = 3.5 \text{ ft}$

model-support tubing,

$d_2 = 0.0310 \text{ in.} = 0.002583 \text{ ft}$ $\qquad l_2 = 10.5 \text{ ft}$

support-to-wall tubing,

$d_3 = 0.0450 \text{ in.} = 0.003750 \text{ ft}$ $\qquad l_3 = 4.0 \text{ ft}$

wall-to-manometer tubing,

$d_4 = 0.2040 \text{ in.} = 0.017000 \text{ ft}$ $\qquad l_4 = 21.0 \text{ ft}$

and

$$V_m = 0.0016 \text{ cu ft}$$

The assumptions are made that the over-all capillary-system length cannot be changed and that the number of model orifices and associated capillary systems cannot be reduced. For purposes of comparison, the same values of viscosity, displacement volume, and pressure are imposed on each configuration; they are

$$p_0 = 178.82 \text{ lb/sq ft abs}$$

$$p_1 = 42.32 \text{ lb/sq ft abs} = \frac{1}{50} \text{ atm}$$

$$p = 43.32 \text{ lb/sq ft abs}$$

$$V_d = 0.0001745 \text{ cu ft}$$

$$\mu = 3.8 \times 10^{-7} \text{ slug/ft-sec}$$

Low pressures were selected since they are more critical.

All the capillaries can be reduced to an equivalent length of a capillary of diameter d_2 $\left(\text{or } d = d_2\right)$ as follows:

$$l_e = l_2 + l_1 \frac{d_2^4}{d_1^4} + l_3 \frac{d_2^4}{d_3^4} + l_4 \frac{d_2^4}{d_4^4}$$

$$l_e = 29.72 \text{ ft}$$

The volume of the system is

$$V_1 = V_m + \frac{\pi}{4} d_1^2 l_1 + \frac{\pi}{4} d_2^2 l_2 + \frac{\pi}{4} d_3^2 l_3 + \frac{\pi}{4} d_4^2 l_4$$

$$V_1 = 0.006473 \text{ cu ft}$$

The time required to settle to within 1 lb/sq ft abs from equilibrium in the basic configuration found from equation (4) is

$$t = \frac{128\mu l_e}{\pi d_2^4}\left(\frac{V_1}{p_1} \log_e \frac{p_0 - p_1}{p - p_1} \frac{p + p_1}{p_0 + p_1} + \frac{3V_d}{p_0 - p_1} \log_e \frac{p_0 + p_1}{p + p_1} + \right.$$

$$\left.\frac{V_d}{p_0 - p_1} \log_e \frac{p_0 - p_1}{p - p_1}\right)$$

$$t = 6,470 \text{ sec} \tag{A1}$$

<u>First alteration</u>.- If the wall-to-manometer tubing is not of the proper diameter, a reduction in the settling time may be effected with a minimum of effort by determining and applying the optimum size.

The orifice-to-wall capillaries (1, 2, and 3) can be reduced to an equivalent length of a capillary of diameter d_2 $\left(d = d_2\right)$ as follows:

$$l_e' = l_2 + l_1 \frac{d_2^4}{d_1^4} + l_3 \frac{d_2^4}{d_3^4}$$

$$l_e' = 29.71 \text{ ft}$$

The volume is

$$V_1' = V_m + \frac{\pi}{4} d_1^2 l_1 + \frac{\pi}{4} d_2^2 l_2 + \frac{\pi}{4} d_3^2 l_3$$

$$V_1' = 0.001706 \text{ cu ft}$$

The optimum size of capillary 4 is given by

$$d_4 = d_x$$

$$d_4 = \sqrt[6]{\frac{8V_1'd_2^4}{\pi l_e'}}$$

$$d_4 = 0.004137 \text{ ft}$$

Then for the entire system

$$l_e = l_e' + l_4 \frac{d_2^4}{d_4^4}$$

$$l_e = 32.90 \text{ ft}$$

and

$$V_1 = V_1' + \frac{\pi}{4} d_4^2 l_4$$

$$V_1 = 0.001988 \text{ cu ft}$$

The response time for the revised system then is

$$t = \frac{128\mu l_e}{\pi d_2^4}\left(\frac{V_1}{p_1} \log_e \frac{p_0 - p_1}{p - p_1} \frac{p + p_1}{p_0 + p_1} + \frac{3V_d}{p_0 - p_1} \log_e \frac{p_0 + p_1}{p + p_1} + \right.$$

$$\left. \frac{V_d}{p_0 - p_1} \log_e \frac{p_0 - p_1}{p - p_1}\right)$$

$$t = 2,351 \text{ sec} \tag{A2}$$

Second alteration.- This value of t, however, may still be unsatisfactory with regard to time lag so that revision of the entire capillary system may be necessary. The alteration would involve replacement when possible of the small capillaries by those of greater internal diameter. It is assumed that the original system could be modified so the following characteristics apply:

Model tubing,

$d_1 = 0.0205$ in. $= 0.001708$ ft $\qquad l_1 = 0.0833$ ft

$d_2 = 0.0390$ in. $= 0.003250$ ft $\qquad l_2 = 3.5$ ft

model-support tubing,

$d_3 = 0.0450$ in. $= 0.003750$ ft $\qquad l_3 = 10.5$ ft

support-to-wall tubing,

$d_4 = 0.0550$ in. $= 0.004583$ ft $\qquad l_4 = 4.0$ ft

wall-to-manometer tubing,

$d_5 = d_x$ $\qquad l_5 = 21.0$ ft

The length of capillary of diameter d_2 to which capillaries 1, 2, 3, and 4 are equivalent is

$$l_e' = l_2 + l_1 \frac{d_2^4}{d_1^4} + l_3 \frac{d_2^4}{d_3^4} + l_4 \frac{d_2^4}{d_4^4}$$

$$l_e' = 11.53 \text{ ft}$$

and the volume is

$$V_1' = V_m + \frac{\pi}{4} d_1^2 l_1 + \frac{\pi}{4} d_2^2 l_2 + \frac{\pi}{4} d_3^2 l_3 + \frac{\pi}{4} d_4^2 l_4$$

$$V_1' = 0.001811 \text{ cu ft}$$

The optimum internal diameter for capillary 5 is found to be

$$d_5 = d_x$$

$$d_5 = \sqrt[6]{\frac{8V_1'd_2^4}{\pi l_e'}}$$

$$d_5 = 0.005956 \text{ ft}$$

Then for the entire system the equivalent length is

$$l_e = l_e' + l_5 \frac{d_2^4}{d_5^4}$$

$$l_e = 13.39 \text{ ft}$$

and the volume is

$$V_1 = V_1' + \frac{\pi}{4} d_5^2 l_5$$

$$V_1 = 0.002396 \text{ cu ft}$$

Applying equation (4) yields the settling time for the second revision

$$t = 435.9 \text{ sec} \tag{A3}$$

In a continuous-operation wind tunnel, a response time of 400 to 500 seconds may be acceptable. Such a time lag would be excessive in an intermittent wind tunnel, however, so that, if such a capillary arrangement were applied, further alterations would be necessary.

If capillaries 1, 2, 3, and 4 (of second alteration) are assumed to be of maximum diameter for the space allowed, only two feasible alternatives by which the time lag may be reduced without increasing the error remain. One means would be discarding a number of the orifices and associated capillary systems in order to allow sufficient space for larger diameter capillaries for the remaining systems; the other would require the use of a measuring device of smaller volume located so that a shorter capillary length could be used.

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TABLE I

VALUES OF OPTIMUM-TUBE INTERNAL DIAMETERS d_x AS COMPUTED

BY EQUATIONS (6), (7), AND (8) FOR EIGHT PRESSURE MEASURING SYSTEMS

Configuration	V_1', cu ft	V_d, cu ft	l_e', ft	l_x, ft	d, ft	p_0, lb/sq ft abs	p_1, lb/sq ft abs	p, lb/sq ft abs	d_x, ft		
									Equation (6)	Equation (7)	Equation (8)
1	0.0025	0.0001745	15	15	0.002	178.82	42.32	43.32	0.004381	0.004352	0.004352
2	.0025	.0001745	15	15	.002	157.70	21.20	22.20	.004371	.004352	.004352
3	.0010	.0001745	15	15	.002	178.82	42.32	43.32	.003792	.003735	.003735
4	.0050	.0001745	15	15	.002	178.82	42.32	43.32	.004900	.004885	.004885
5	.0025	.0001745	5	30	.002	178.82	42.32	43.32	.005263	.005227	.005227
6	.0025	.0001745	30	5	.002	178.82	42.32	43.32	.003904	.003877	.003877
7	.0025	.0001745	15	15	.002	2116	1981	1982	.004904	.004352	.004352
8	.0025	.0001745	15	15	.002	4232	4097	4098	.005266	.004352	.004352

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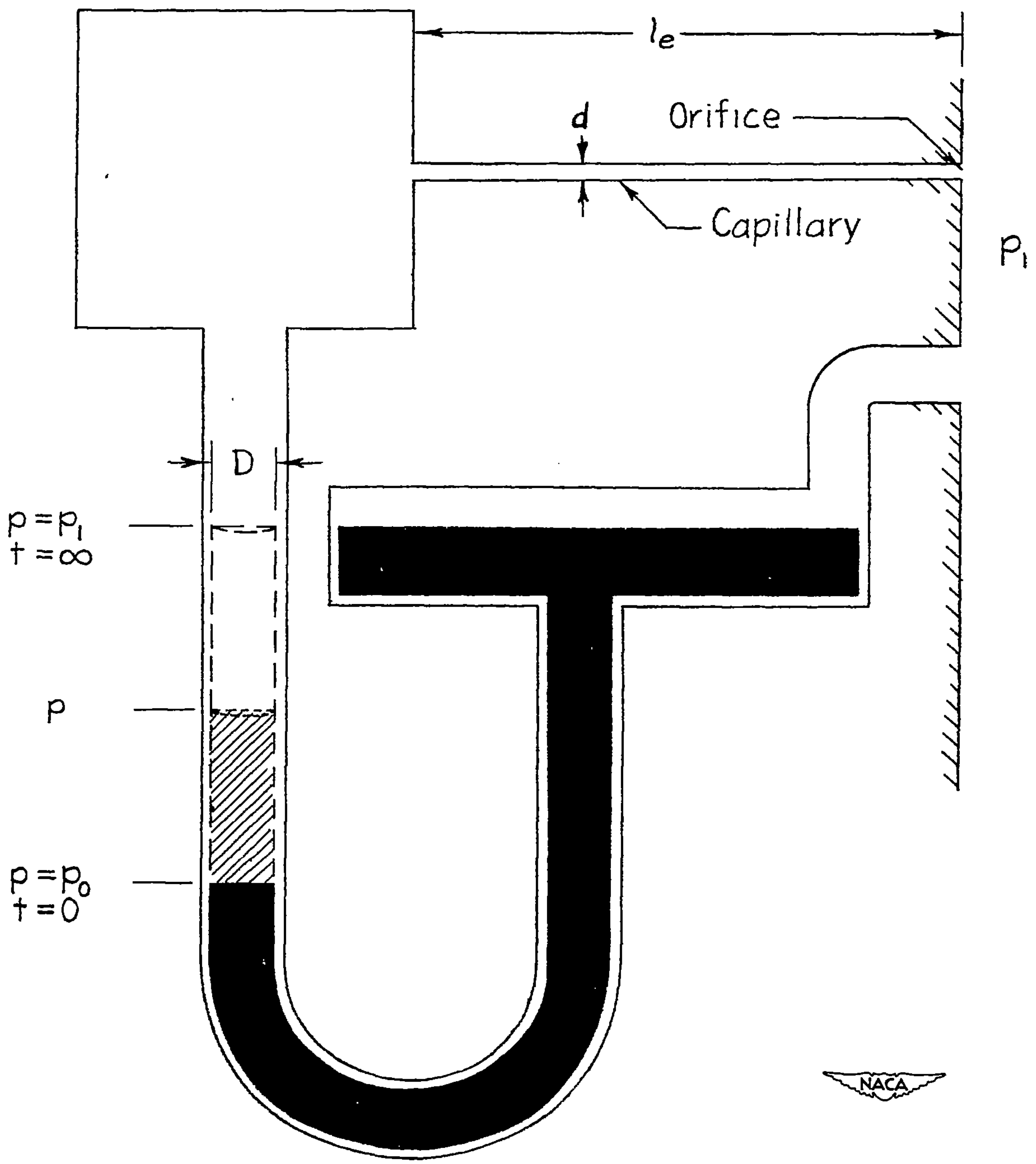


Figure 1.- Simplified sketch of a manometer.

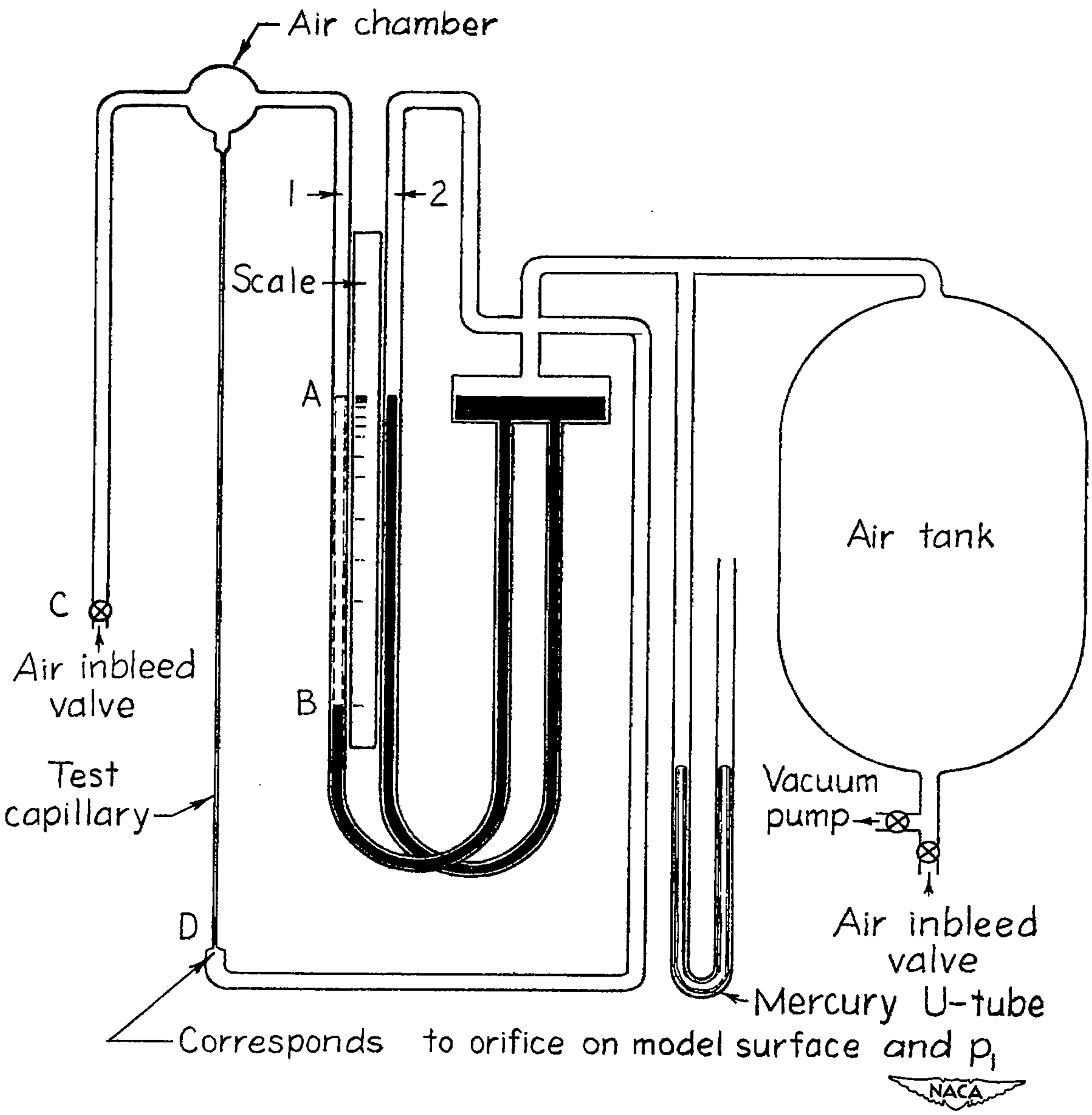
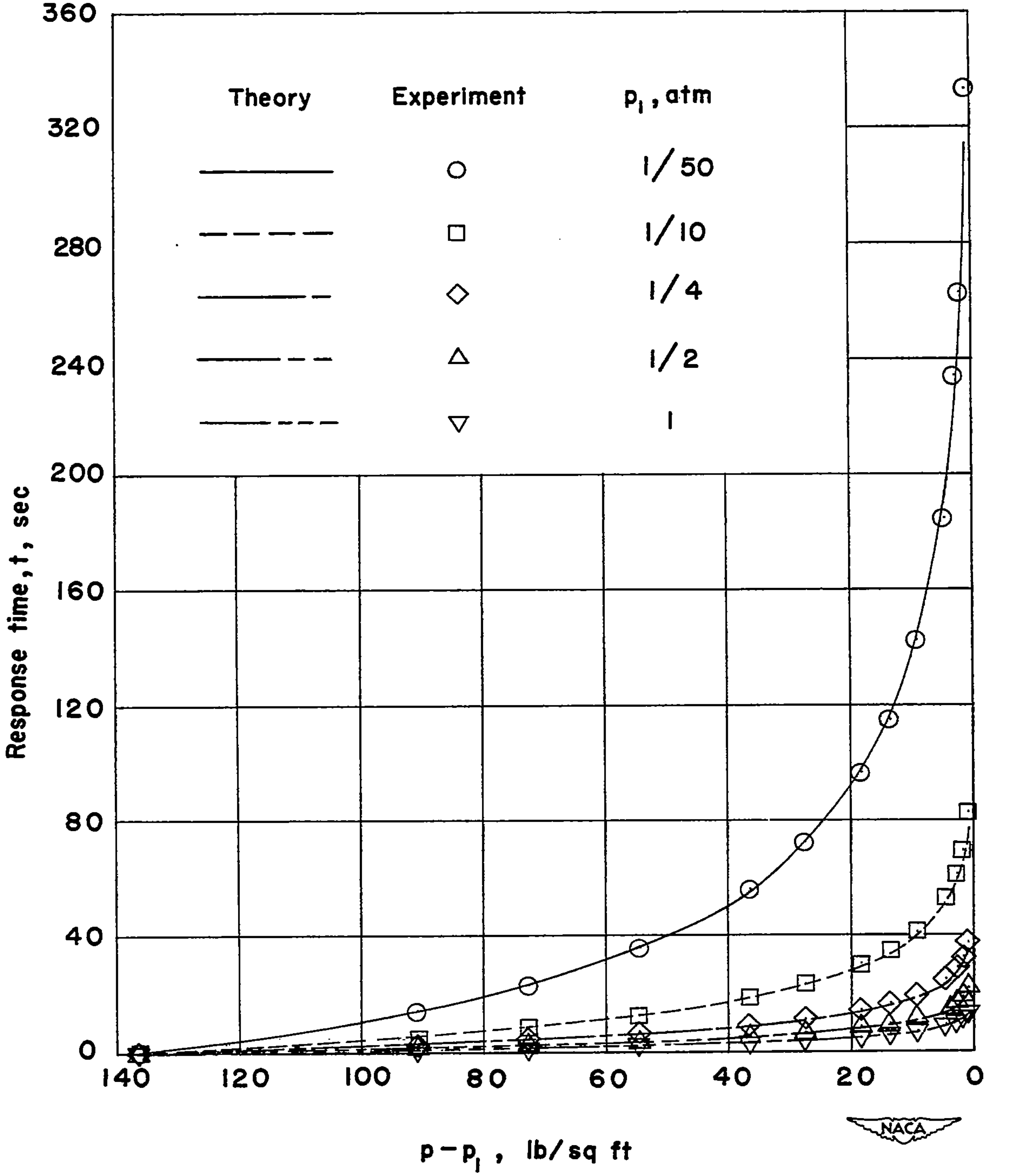
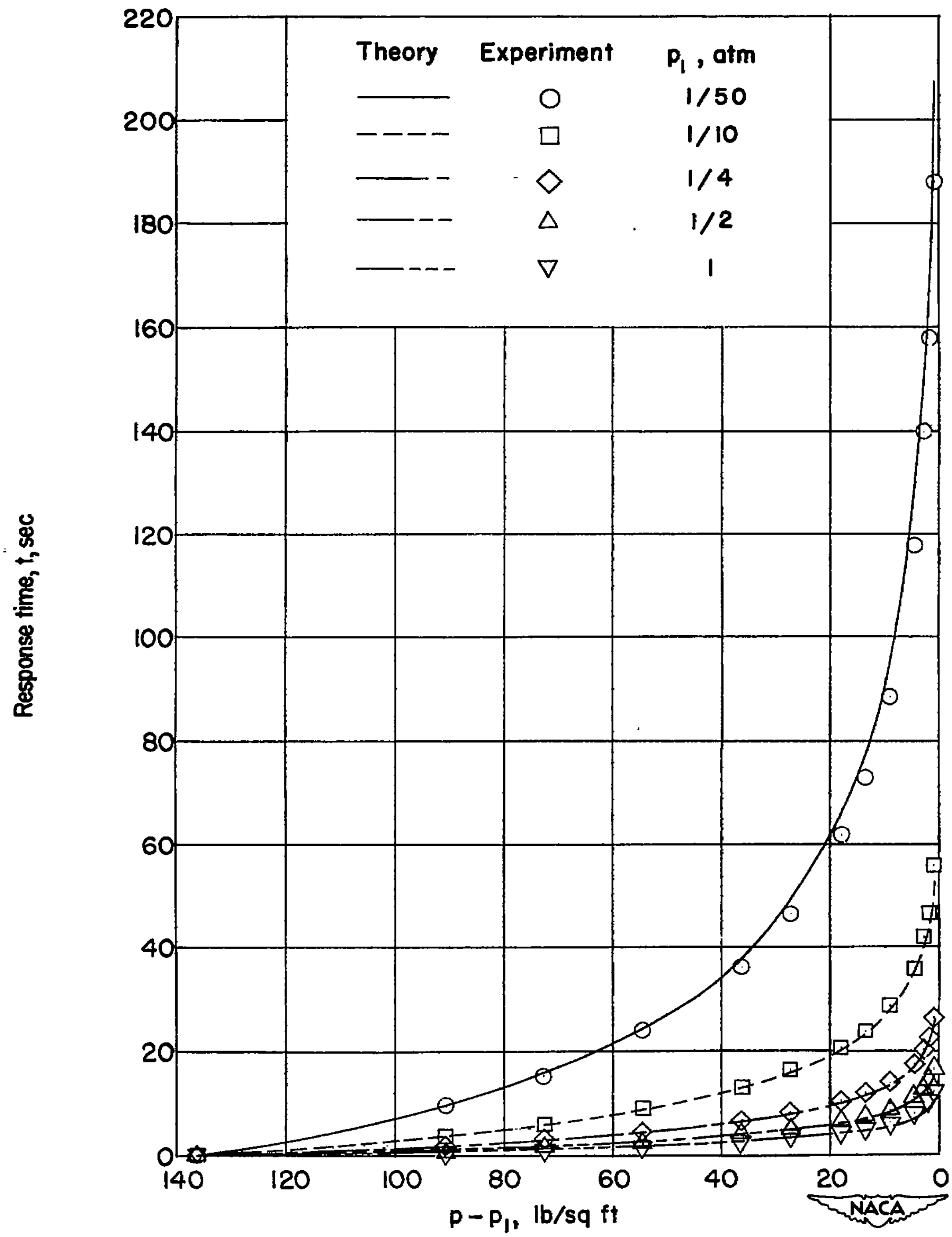


Figure 2.- Diagram of test apparatus.



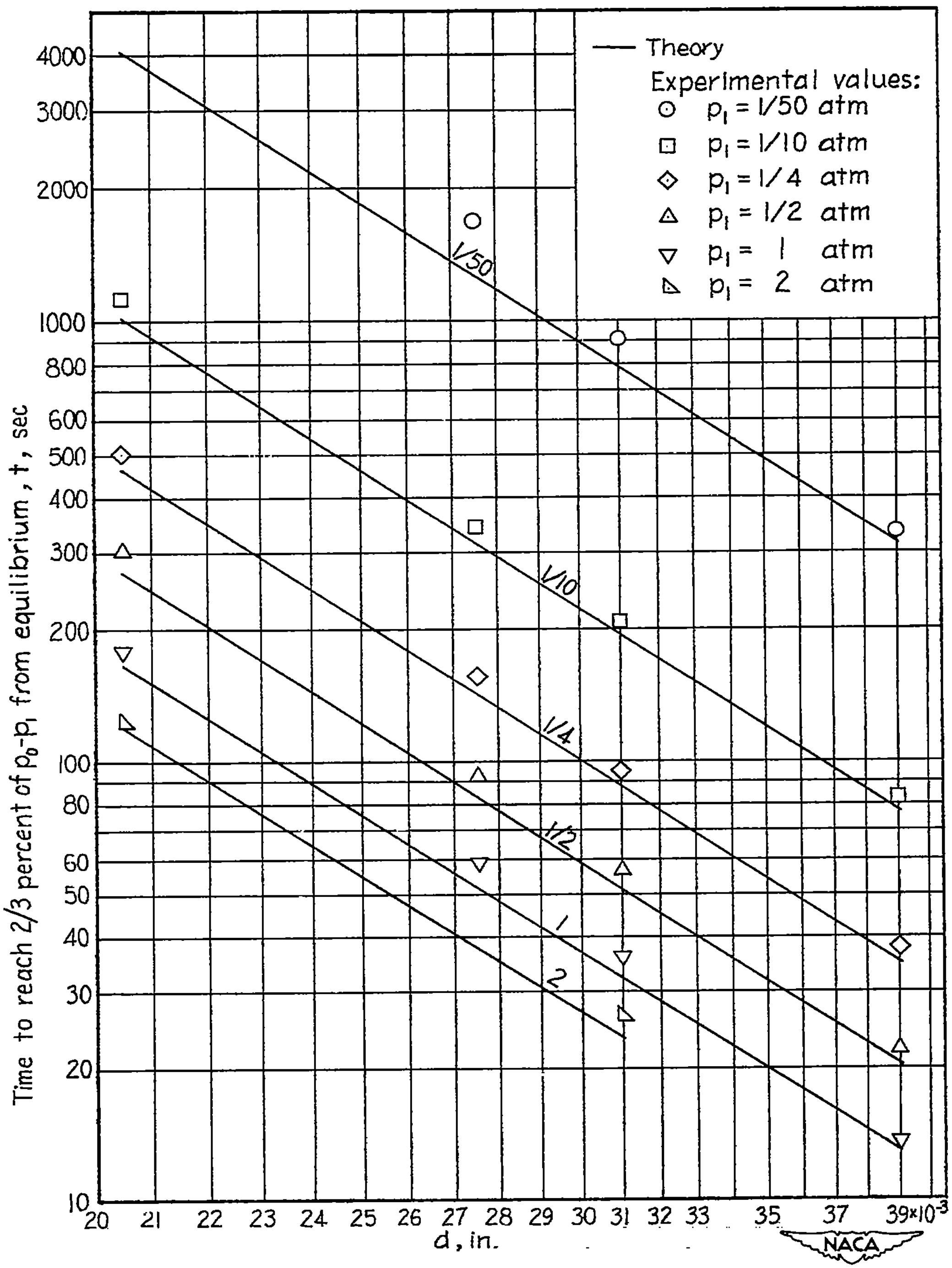
(a) $V_1 = 0.00382$ cubic foot.

Figure 3.- Variation of response time with pressure unbalance. $l = 6$ feet; $d = 0.039$ inch.



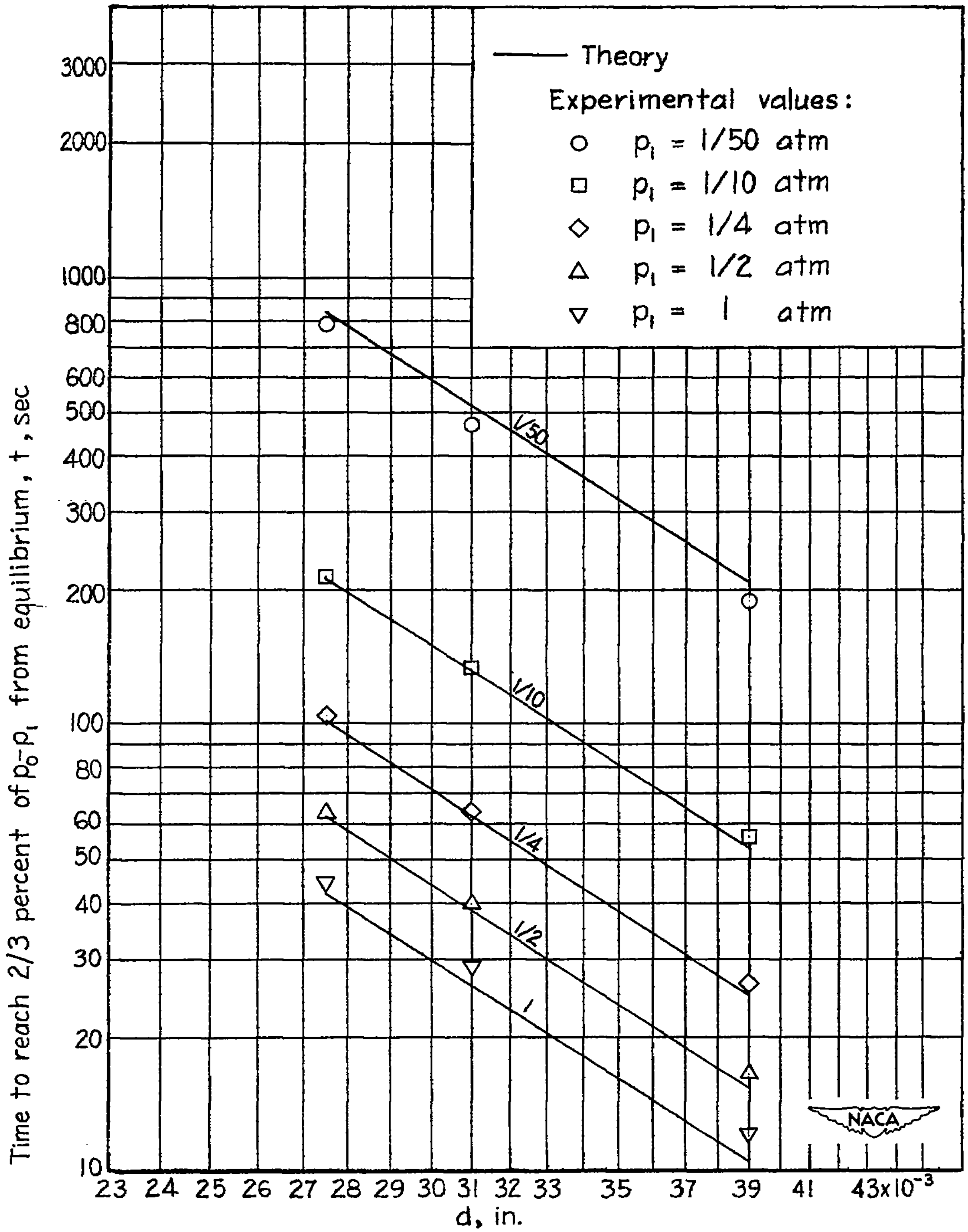
(b) V_1 = 0.00247 cubic foot.

Figure 3.- Concluded.



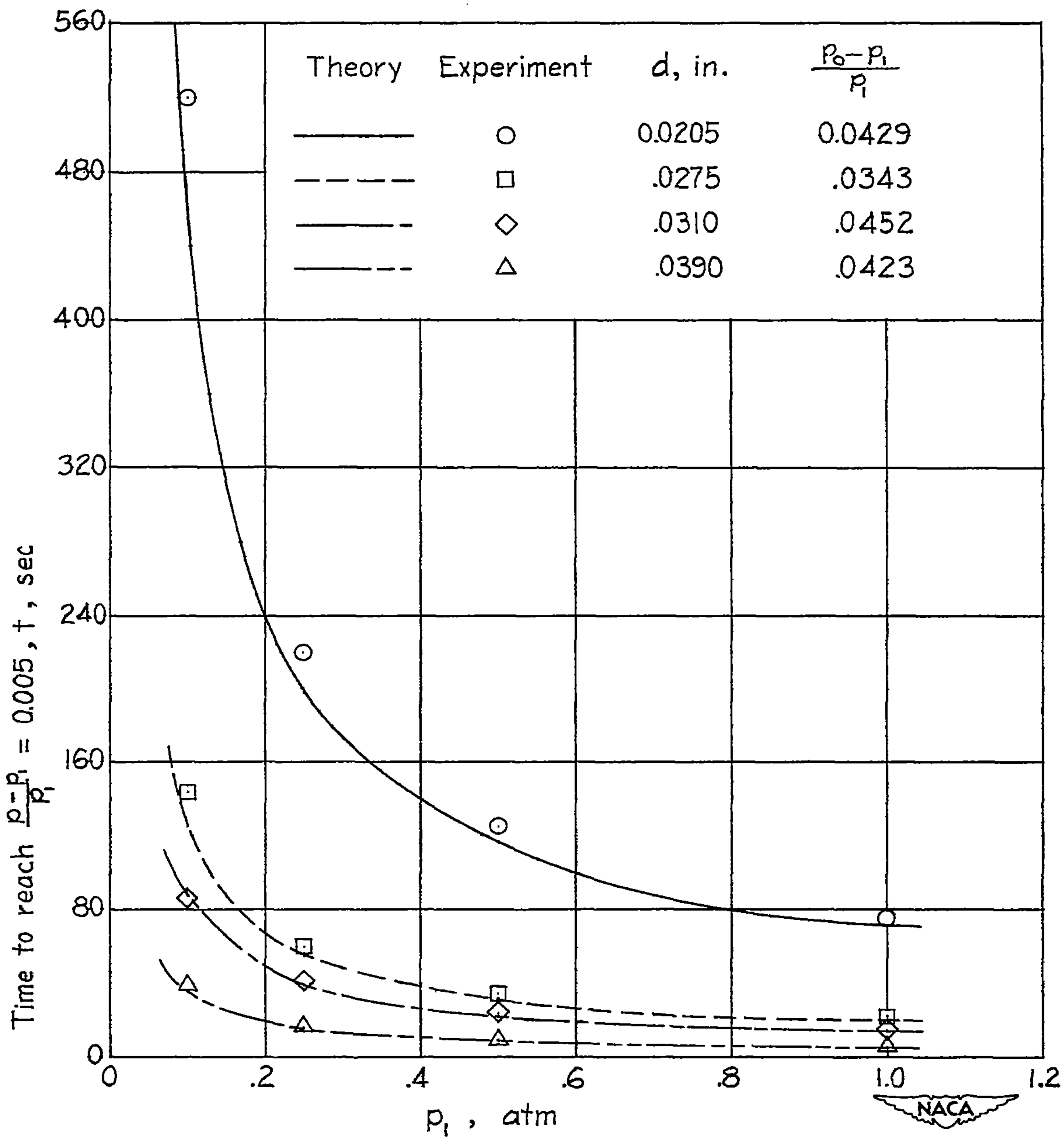
(a) V_1 = 0.00382 cubic foot.

Figure 4.- Variation of response time with capillary diameter d. l = 6 feet; t = 0 when $p_0 - p_1$ = 136.4 pounds per square foot.



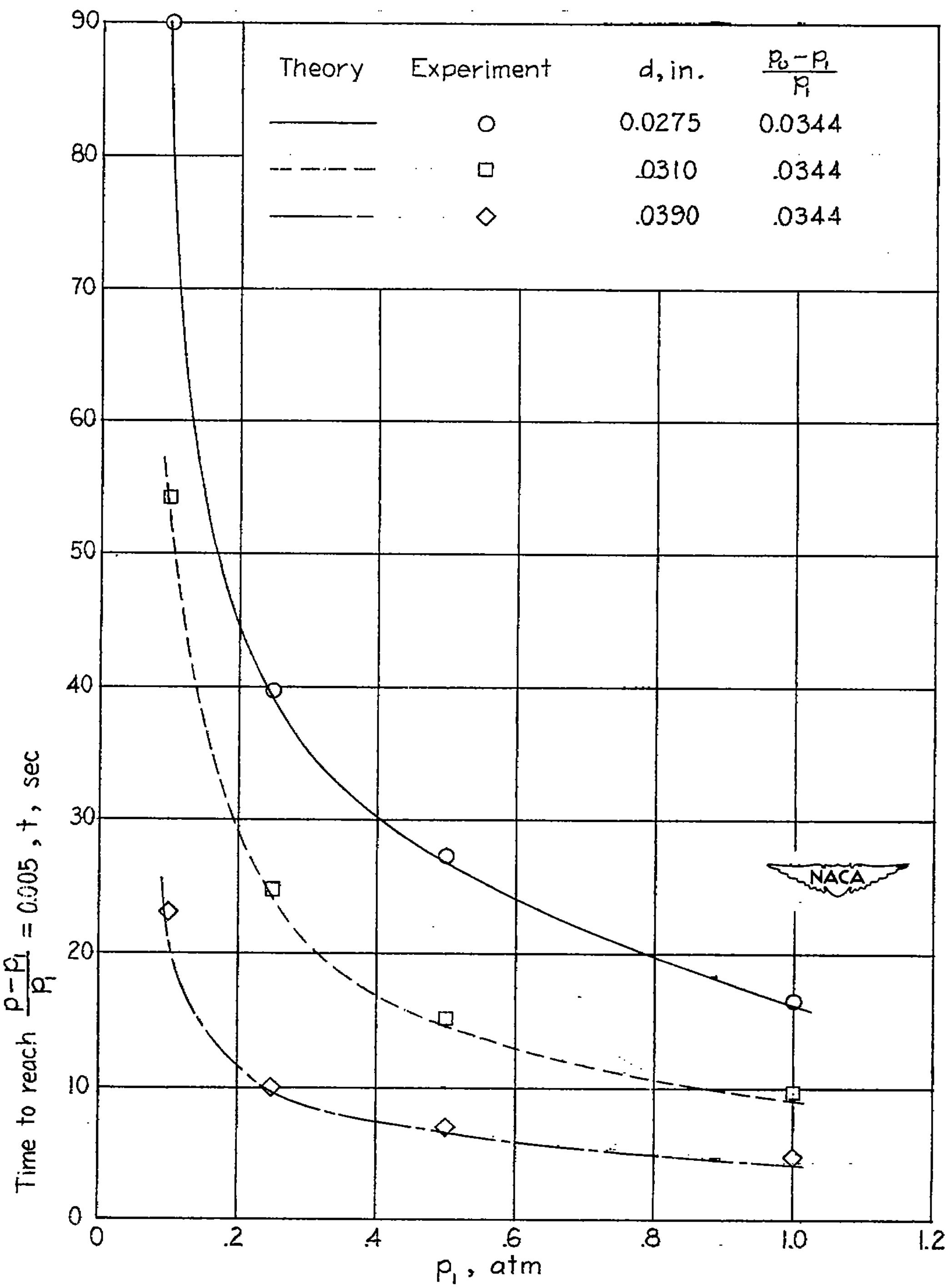
(b) V_1 = 0.00247 cubic foot.

Figure 4.- Concluded.



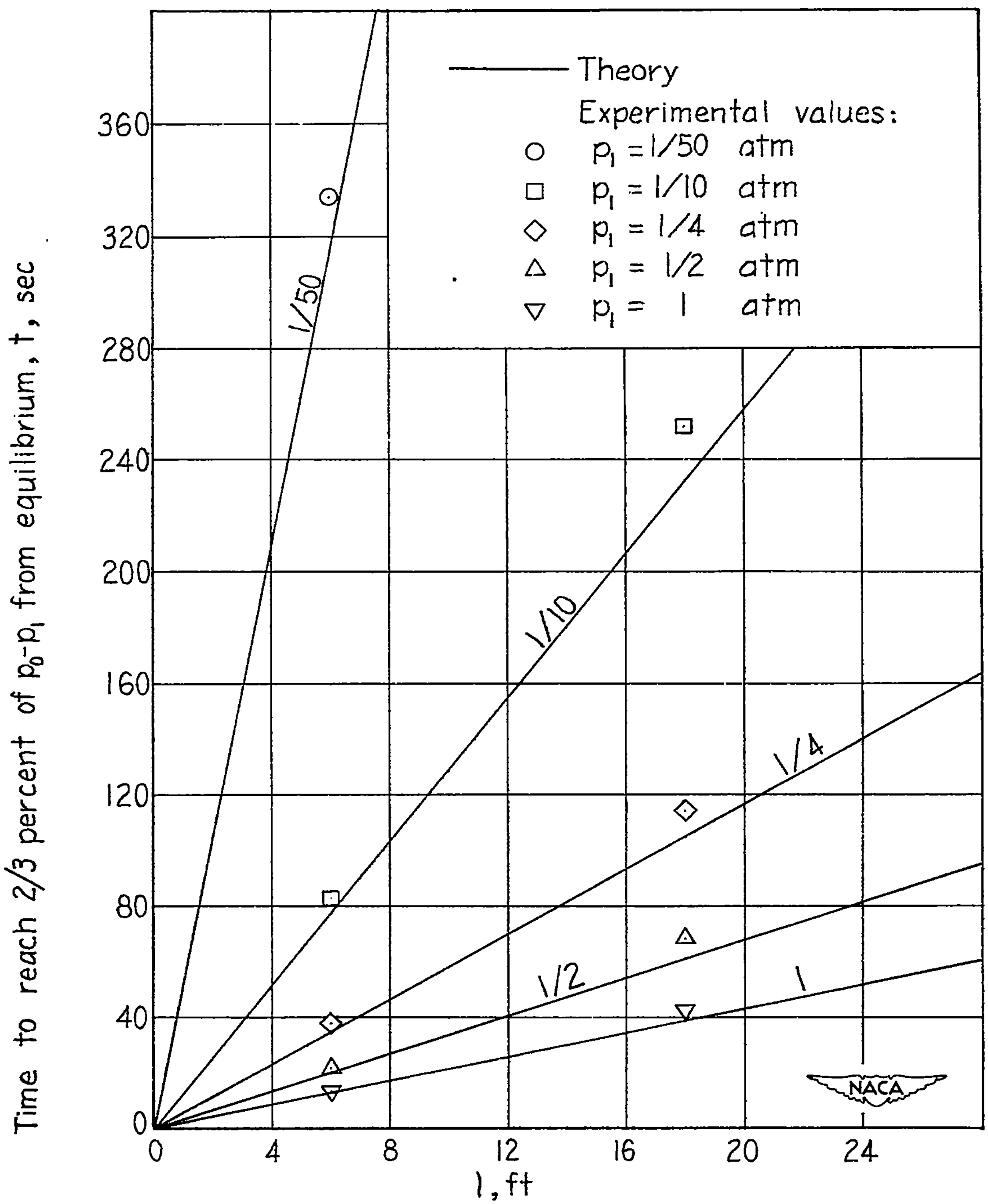
(a) V_1 = 0.00382 cubic foot.

Figure 5.- Variation of response time with orifice pressure. l = 6 feet.



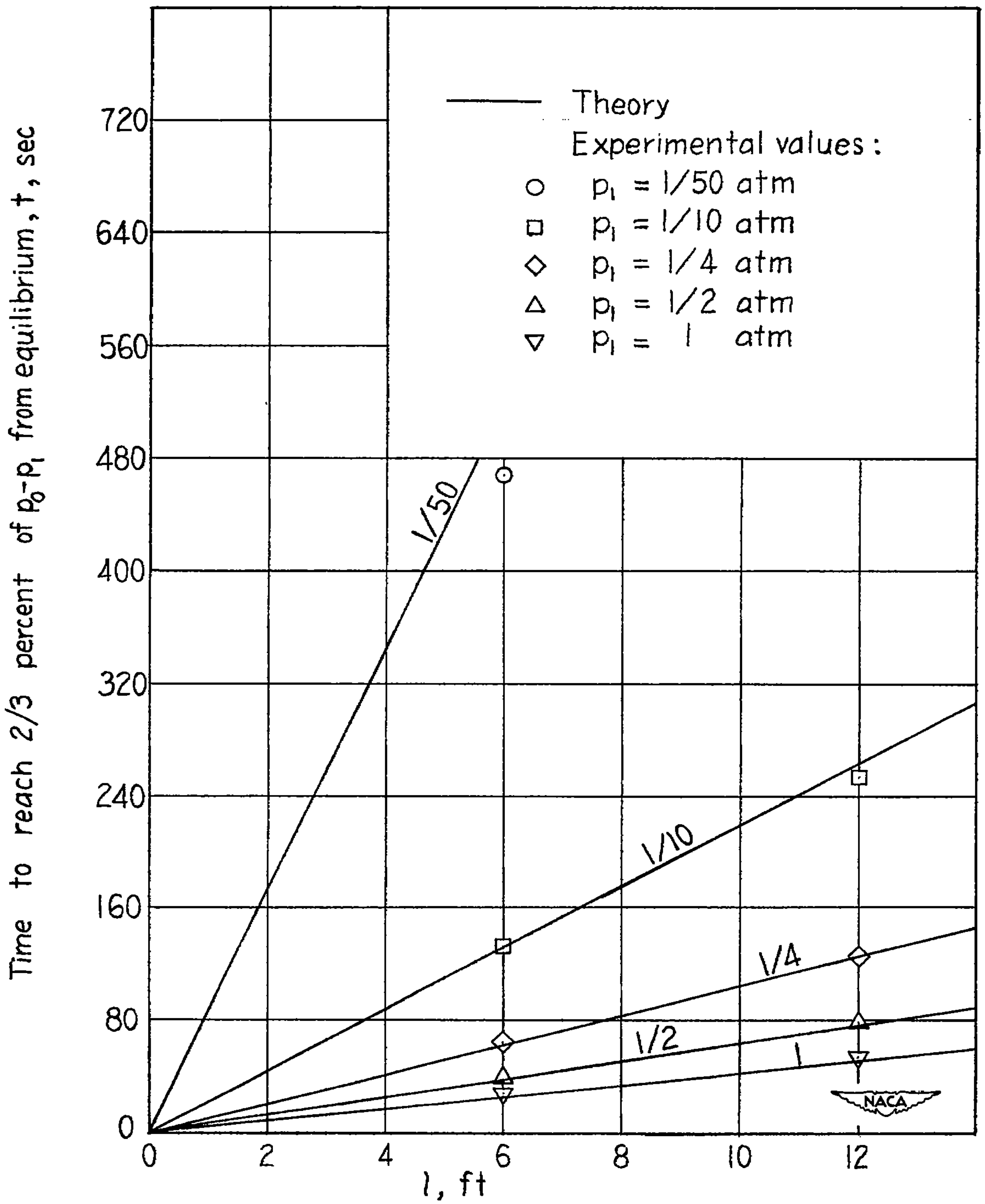
(b) V_1 = 0.00247 cubic foot.

Figure 5.- Concluded.



(a) V_1 = 0.00382 cubic foot.

Figure 6.- Variation of response time with capillary length. d = 0.039 inch; $p_0 - p_1$ = 136.4 pounds per square foot.



(b) V_1 = 0.00247 cubic foot.

Figure 6.- Concluded.

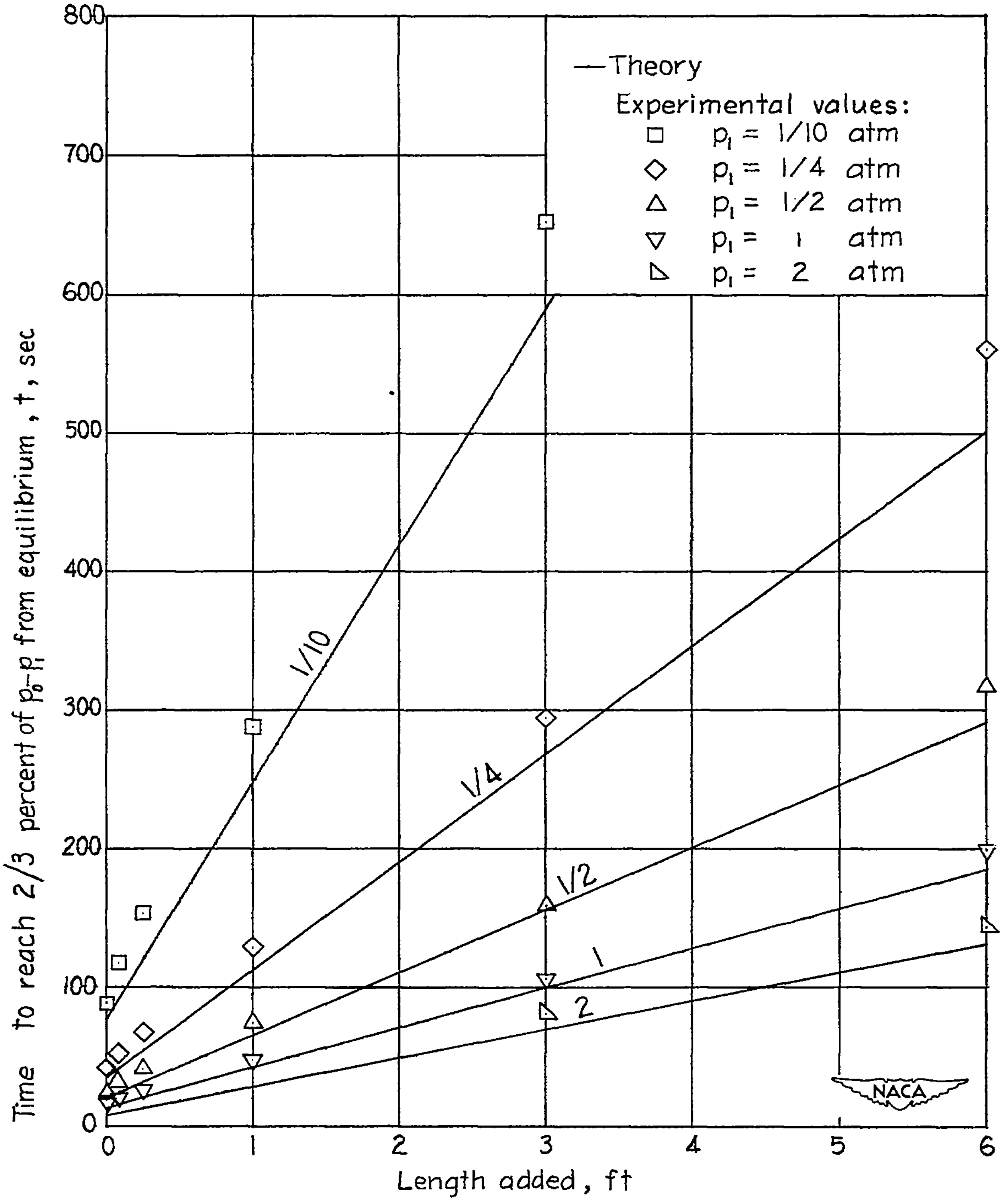


Figure 7.- Variation of response time with length of 0.0205-inch capillary added to 6 feet of 0.039-inch capillary. V_1 = 0.00382 cubic foot; $p_0 - p_1$ = 136.4 pounds per square foot.

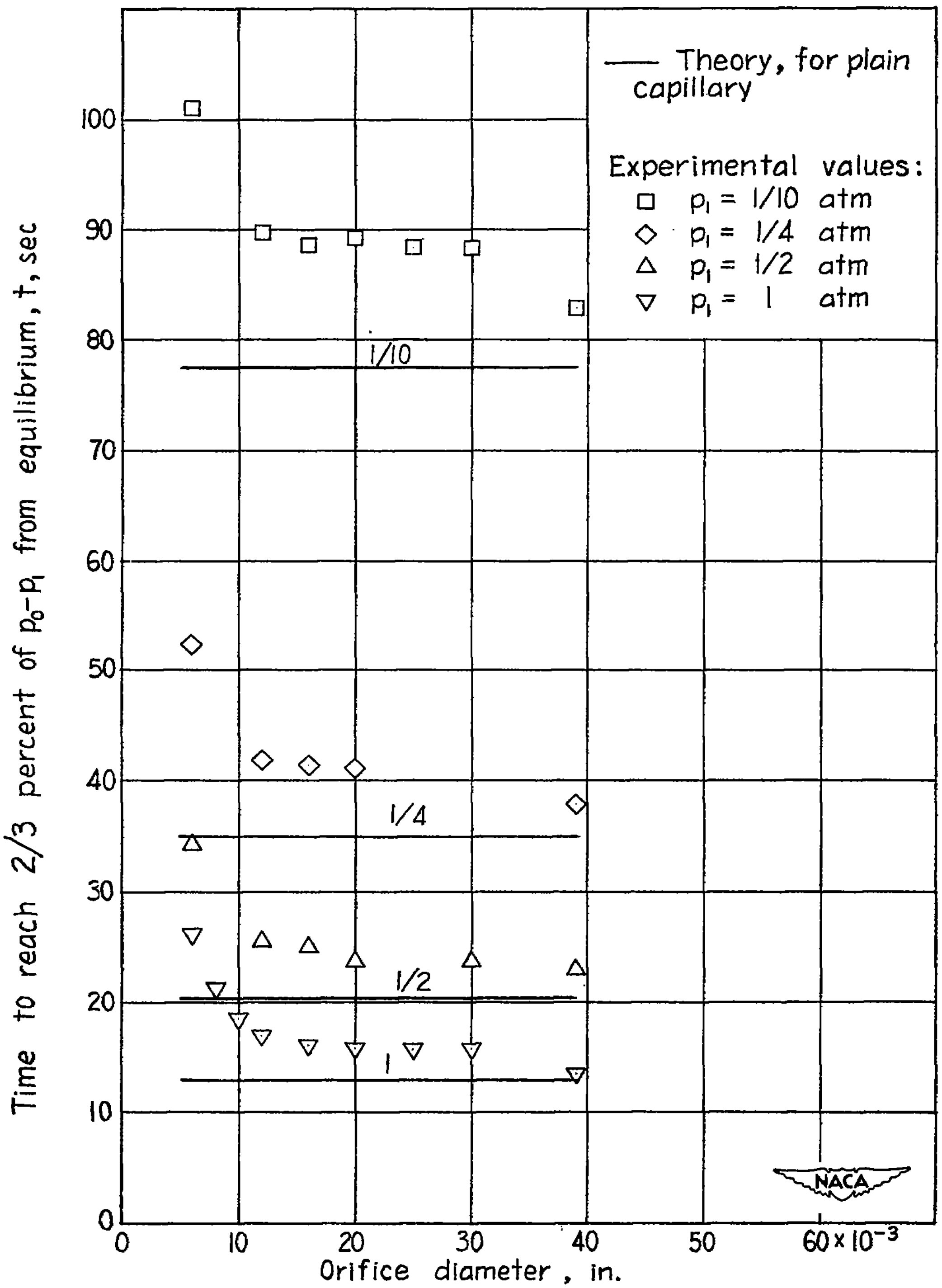


Figure 8.- Variation of response time with orifice diameter for 0.002-inch-thin orifice on 6 feet of 0.039-inch capillary. V_1 = 0.00382 cubic foot; $p_0 - p_1$ = 136.4 pounds per square foot.

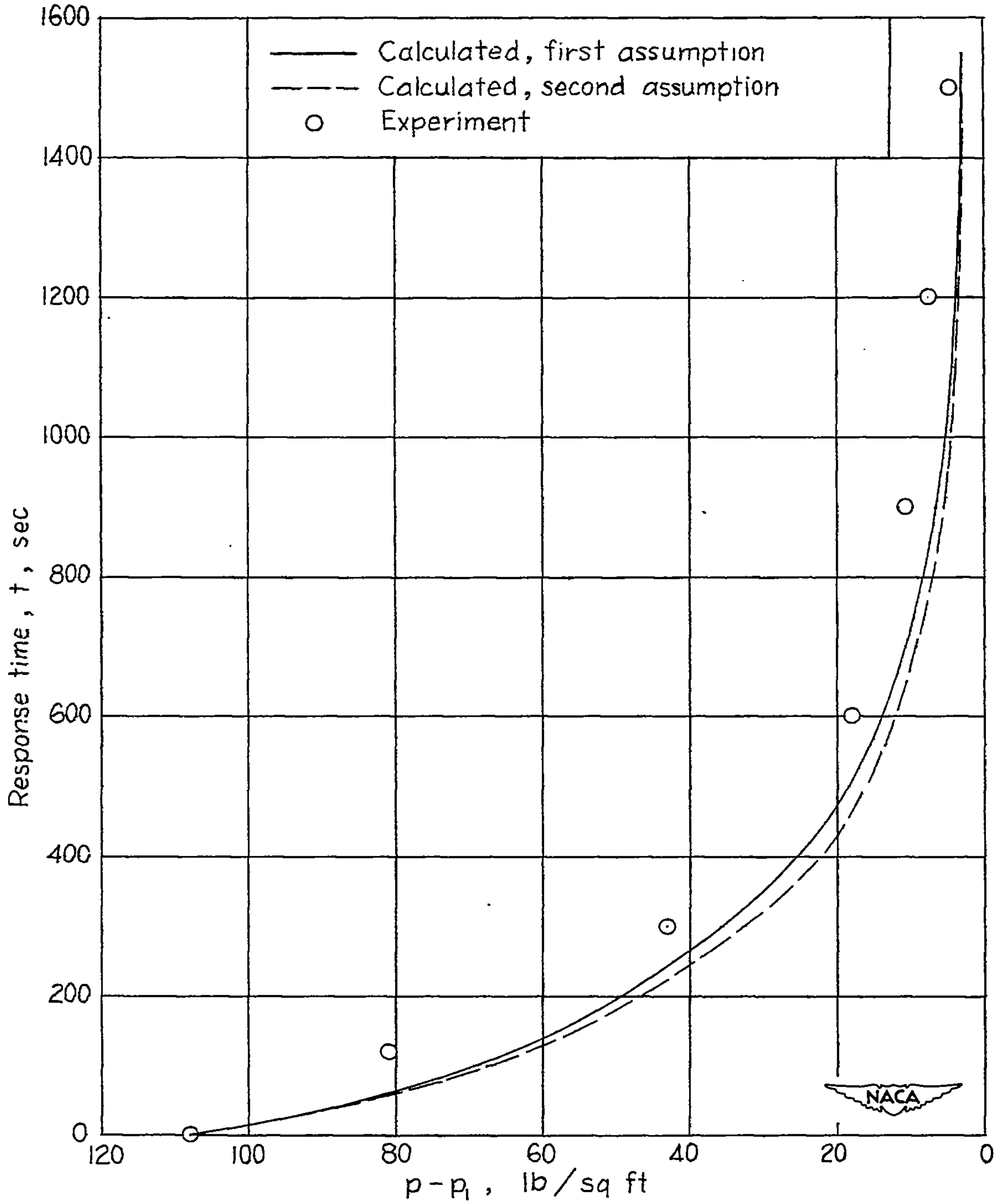


Figure 9.- Variation of response time with pressure unbalance for a manometer system incorporating 1.0, 25.0, and 190.0 feet of capillaries of 0.020-, 0.032-, and 0.090-inch internal diameters, respectively.