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AN ENGINEERING METHOD FOR ESTIMATING NOTCH-SIZE EFFECT IN FATIGUE TESTS ON STEEL

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SUMMARY

Neuber's proposed method of calculating a practical factor of stress concentration for parts containing notches of arbitrary size depends on the knowledge of a "new material constant" which can be established only indirectly. In this paper, the new constant has been evaluated for a large variety of steels from fatigue tests reported in the literature, attention being confined to stresses near the endurance limit; reasonably satisfactory results were obtained with the assumption that the constant depends only on the tensile strength of the steel. Even in cases where the notches were cracks of which only the depth was known, reasonably satisfactory agreement was found between calculated and experimental factors. It is also shown that the material constant can be used in an empirical formula to estimate the size effect on unnotched specimens tested in bending fatigue.

INTRODUCTION

It has long been known that the stress concentration factors developed in fatigue tests increase (for geometrically similar specimens) as the size of the specimen increases (ref. 1, first ed., p. 688). For reasons of economy, standard fatigue tests are run on rather small specimens; the direct application of such data to the design of large parts may lead to rather large unconservative errors to such an extent that many practical engineers decry standard laboratory fatigue tests as being of little value for design.

Size effect is only one of several factors that may result in unconservative strength predictions, but it is a very important one; quantitative rules for estimating it are therefore imperative if the predictions of fatigue strength are to be improved. This paper presents an engineering rule for estimating the effect of size of a notch, or more specifically, a rule for converting the theoretical factor of stress concentration into the actual fatigue factor. The rule utilizes a relation proposed by Neuber in reference 2 which involves the use of a new material constant; the new contribution consists in evaluating a comprehensive array of fatigue tests, collected from the literature, to show that the material constant may be taken as a function of the tensile strength of the material. The evaluation was confined to steel as material and to nominal stresses near the endurance limit. In appendix A, the rule is shown to yield reasonably satisfactory results even in the limiting case where the notch is an artificially produced crack. In appendix B, a simple empirical relation is given for estimating the size effect on unnotched fatigue specimens in bending with the aid of the new material constant.

The material contained in this paper was presented in preliminary form to an aircraft industry group during March 1951. Since that time some of the theoretical factors have been recalculated, and some material has been added.

SYMBOLS

a	distance from axis of symmetry to base of notch
A	new material constant (Neuber constant)
d	minimum diameter
D	maximum diameter
I	moment of inertia
K_D	stress concentration factor for deep notch
K_F	stress concentration factor effective in fatigue
K_N	stress concentration factor corrected for size of notch (Neuber technical factor)
K_S	stress concentration factor for shallow notch
K_T	theoretical stress concentration factor
M	applied moment
N	number of cycles to failure
R	radius of curvature at base of notch

S	stress
S_{AL}	endurance limit for axially loaded specimens
S_{RB}	endurance limit for rotating beams
t	depth of notch
ω	flank angle

DEFINITIONS

The results of fatigue tests on simple specimens are commonly presented by plotting a stress S against the number N of cycles to failure (fig. 1). The stress S is computed by elementary formulas for the smallest cross section of the specimen; for instance, for a notched (grooved) specimen tested in bending, the stress is computed as Ma/I for the cross section containing the bottom of the notch. The symbols used in describing the geometry of a notch are defined in figure 2.

The term "fatigue factor" K_F is used in this paper to denote the stress concentration factor effective under fatigue conditions. The factor is defined for a given value of N (see fig. 1) as the stress carried by the smooth specimen divided by the stress carried by the notched specimen. This definition is general and includes, as a limiting case, the factor obtained in a static test which may be regarded as a fatigue test with $N = \frac{1}{4}$ (for fully reversed stress). In this paper, however, attention is confined to the fatigue factor at the endurance limit, defined herein as the fully reversed stress which leads to fracture in $N = 10^7$ cycles $\left(S_A/S_B\right.$ as indicated in fig. 1$\left.\right)$. This restriction automatically confines attention to peak stresses that are within the engineering elastic range.

The theoretical factor K_T is defined as the factor of stress concentration derived by the conventional theory of elasticity, in which the material is assumed to be elastic, homogeneous, and isotropic. In practice, this factor is often obtained by means of photoelastic tests. The most complete and systematic mathematical theory of stress concentration is given in reference 2.

The term "notch size effect" is used to denote an effect attributable to changes in the absolute size of the notch. Distinct from it is the "material size effect," attributable to the fact that a thin sheet undergoes more forming work in the manufacturing process than a thick slab and that there is a mass effect when a large specimen is undergoing heat treatment, particularly in the quenching operation. The material-size effect can be fairly well eliminated in many investigations of the notch-size effect; for instance, small and large specimens may be made from the same thickness of sheet.

THE NEUBER TECHNICAL FACTOR

The configurations of the notches dealt with by the theory of elasticity (refs. 1 and 2) are such that the bottom of the notch may be considered as a portion of a circle having a radius R. All the formulas for stress concentration contain a term with the square root of the reciprocal of this radius; as R becomes smaller and smaller, this term causes the theoretical factor to increase indefinitely. For a radius which is small but within the range actually used sometimes for test specimens (of the order of 1×10^{-4} inch), the theoretical factor may be of the order of 50, whereas the corresponding experimental fatigue factor may be only one-tenth as large or even less. The use of the theoretical factor for design would therefore be entirely too pessimistic in many cases.

Neuber's book (ref. 2) is devoted largely to a systematic mathematical theory which gives the theoretical factors of stress concentration (denoted in this paper by K_T) for many basic types of notches. Recognizing that the theoretical factors are not acceptable for design, however, Neuber also developed a formula for converting any given theoretical factor K_T into a technical factor (hereinafter termed "Neuber factor" and designated by K_N) intended to be directly applicable in design. This formula is

$$K_N = 1 + \frac{K_T - 1}{1 + \frac{\pi}{\pi - \omega}\sqrt{\frac{A}{R}}} \tag{1}$$

The quantities ω and R are the flank angle and the radius at the bottom of the notch, respectively. The quantity A, which has the dimension of a length, constitutes the key idea in the formula and is called the "Neuber constant" in this paper; it is discussed in the following section.

Inspection of formula (1) shows that the factor K_N lies between two limits as the constant A varies. If A is zero, then $K_N = K_T$; in terms of the widely used concept of notch sensitivity (see ref. 1, second ed., p. 448), the material has 100-percent notch sensitivity. If the constant A becomes very large, $K_N = 1$ regardless of the value of K_T; this value indicates that the material is completely insensitive to notches.

THE NEUBER CONSTANT

In the classical theory of elasticity, the material is considered as a continuum. Pointing to the fact that engineering metals have a granular structure, Neuber stated that this concept must be abandoned when a stress gradient is present. He proposed instead the concept that the material is an aggregate of "building blocks" and postulated that no stress gradient can develop across such a block; the quantity A is the half-length of a block. Neuber stated that the length A should be considered as a new material constant and that it must be determined by experiment.

Neuber's very brief argument may be elaborated somewhat as follows. It is well-known that the different types of grains of which an engineering metal generally consists may have very different properties and that the properties of any one grain may be highly directional. The standard test bars used to determine the properties of the material, however, are sufficiently large to contain an immense number of grains, and the properties measured are the average taken over this large number. Under these conditions, the average is subject to relatively small fluctuations, and the assumption that the material is homogeneous is a useful simplification. But if the test bar is made smaller and smaller until such proportions are reached that the cross section contains only a few grains or finally a single grain, the properties measured will fluctuate more and more between the limits set by the properties of the individual grains. It is evident, then, that the assumption of homogeneity becomes less and less useful. This consideration leads to the interpretation of the building block as the minimum volume of material the behavior of which may be correlated to an acceptable degree of accuracy with the standard engineering properties of the material (or, more precisely phrased, with the properties observed on the standard engineering scale of magnitude).

It is clear that Neuber's building block is not a physical entity directly observable, for instance, by means of a microscope; it is a conceptual quantity that can be determined only by calculation from tests.

Moreover, the preceding interpretation implies a difficulty not evident from Neuber's definition: the Neuber constant A for a given material may have different values, depending on whether the property to be correlated is strain, yield stress, static strength, or fatigue strength.

DETERMINATION OF THE NEUBER CONSTANT FROM FATIGUE TESTS

Within the frame of a broad-scale attack on the problem of putting fatigue design on a more secure basis, an investigation on size effect has been initiated. The Neuber factor appeared to offer promise of being a useful engineering method of estimating this effect; in order that the factor may be used, however, it is necessary that the Neuber constant be known for the materials of interest.

Neuber has determined the constant only for mild steel from two sets of static strain measurements on notched specimens made by another experimenter and arrived at a value of $A \approx 0.02$ inch (ref. 2). Very few measurements of this type have been made since these measurements must be made with extremely small gage lengths; they are thus very difficult to make and are of uncertain accuracy. Furthermore, as pointed out in the preceding section, values of the constant derived from static measurements may not be applicable to fatigue tests. It was decided, therefore, to obtain the constants for various materials from an analysis of published fatigue tests.

The analysis was limited to steel specimens because the number of relevant tests on other materials was inadequate. The fatigue factor was evaluated only for the endurance limit, as stated previously, in order to avoid the complication of corrections for plasticity effects. The data were taken from references 3 to 17. Particular attention was paid to tests in which the size of the specimen was varied systematically, but all individual tests available were also used. (A test means a companion pair of S-N curves, one curve for smooth specimens, one for notched specimens.) No usable tests were discarded for any reason whatever, but many published tests were not usable either because the shape of the notch was not given or because the material was not described adequately.

The analysis included tests on specimens with fillets, semicircular notches, V-notches, and transverse holes; most of the tests were rotating-beam tests, but a fair number of axial-load tests on circular and flat specimens were also available. A large variety of carbon and alloy steels with tensile strengths ranging from 50 to 230 ksi were included.

The direct calculation of the constant A from known values of K_F is very sensitive and, consequently, results in large scatter. A much more practical procedure is to assume trial values of A and to calculate K_N from them.

As a first approximation, the value A = 0.02 inch obtained by Neuber was used, regardless of material. Obviously, a constant value of A can be, at best, only a crude approximation for the entire range of materials. Nevertheless, for most cases, the use of K_N constituted an improvement over the use of K_T as an estimate of the fatigue factor.

A second approximation was obtained by considering the constant A to be a function of the tensile strength of the material. This relation was expected to be reasonable, at least qualitatively, on the basis of the following general observations:

(1) Notch sensitivity increases with increasing tensile strength.

(2) The Neuber building block might be expected to be related to grain size which decreases with increasing tensile strength.

(3) Endurance limits appear to be more closely related to tensile strengths than to other mechanical properties.

The curve obtained by a trial-and-error process is shown in figure 3.

COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL FATIGUE FACTORS

The results obtained by applying formula (1) and the curve of figure 3 to some of the systematic series are shown in figures 4 and 5. These figures show the theoretical factor K_T, the technical factor K_N, and the experimental values K_F. Figure 4 shows the results for four sets of tests on grooved shafts tested as rotating beams. The computed values of K_N are in excellent agreement with the tests. Figure 5 shows the results for three sets of tests on filleted shafts tested as rotating beams. The agreement is very good for two sets; for the third set, the prediction is conservative.

Many of the tests do not constitute systematic series and are therefore not suitable for individual plots. Information on all the tests is presented in tables 1 to 5. The final results for all tests are shown in figures 6 to 9 as plots of the ratio K_N/K_F against the notch radius R. Two vertical lines are drawn at K_N/K_F equal to 0.9 and 1.1, respectively, as an aid in assessing the scatter. The reason for plotting against the notch radius is that small notch radii are often only rather inaccurately established; consequently, K_T is likely to be inaccurate, and increased scatter in the ratio K_N/K_F may be expected for small radii for this reason (and possibly for other reasons).

Because the number of tests is quite large, some groups of points for a given notch radius have been averaged; the circle indicates the average ratio, the number above it the number of points averaged, and the ticks at the ends of the horizontal line indicate the lowest and the highest ratio in the group. In some tests the ultimate strength was not given for the materials used; the data for these tests were analyzed on the basis of estimated strengths, and points obtained in this manner are plotted with tailed symbols.

DISCUSSION OF RESULTS

Inspection of figures 6 to 9 indicates, as expected, that there is more scatter when the notch radius is small. In particular, the group of 72 tests with $R = 0.004$ inch in figure 9 shows a rather wide scatter band. This series includes tests at 20° C and at -78° C, but no systematic difference attributable to the temperature difference could be found.

Figure 6 shows a group, totaling 11 points, at a radius of about 0.01 inch for which the predictions are unconservative. Nine of these points were obtained in one investigation where unusual heat treatments were used to produce widely different grain sizes for essentially the same ultimate tensile strength. The five most conservative predictions are for the specimens with the smallest notch radius shown in the entire figure (0.002 inch); moreover, the tensile strengths of the materials were not given and had to be estimated. The inaccuracy of the conservative prediction may therefore be attributable to inaccuracy of the basic data used.

The theoretical factors for the specimens with transverse holes (fig. 8) were obtained by the laminar-action theory of reference 18, but with the use of the theoretical values of reference 19 as a basis rather than photoelastic values. The laminar-action theory converts the three-dimensional stress problem into a two-dimensional one by means of a simplifying assumption. The result fills a bad gap in the knowledge of stress concentrations, but its accuracy is open to some question in view of the simplifying assumption.

The results shown in figures 6 to 9 may be summarized as follows: The Neuber formula (1), used in conjunction with the curve of figure 3, predicts the fatigue factor with an accuracy of ±10 percent for 69 percent of the tests if specimens with notch radii equal to or less than 0.01 inch are excluded and for 56 percent of the tests if no specimens are excluded. These percentages are increased to 81 and 59 percent, respectively, if the results on transverse holes are disregarded on account of the uncertainty concerning the theoretical factors.

The following facts should be remembered when an evaluation of the results is made:

(1) The S-N curves from which the factor K_F is calculated are often not well established in the region of concern herein (that is, $N = 10^7$).

(2) The S-N curves are subject to statistical fluctuations; consequently, the ratio K_N/K_F is also subject to such fluctuations.

(3) The theoretical factor K_T is known accurately only for a few special cases.

(4) For most cases, only a nominal value of the notch radius was given, without indications of probable accuracy.

(5) Machining stresses may affect the factor K_F. (Stress measurements by means of X-rays suggest that these stresses may be large, particularly on V-grooves. See ref. 20.)

(6) In some cases, the tensile strength of the material was not given and had to be estimated from the type of steel and heat treatment.

In view of all these uncertainties, the degree of correlation achieved may be considered as very satisfactory.

CONCLUSION

An evaluation of the Neuber constant for a large number of fatigue tests on steel specimens for stresses near the endurance limit was made. The large number of tests analyzed is felt to justify the conclusion that the fatigue factor K_F at the endurance limit can be estimated for steels with reasonable accuracy by using Neuber's formula (eq. (1) of this paper) in conjunction with the Neuber constant A (evaluated from fatigue tests and defined by fig. 3 of this paper).

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 29, 1952.

APPENDIX A

STRESS CONCENTRATIONS PRODUCED BY CRACKS

The stress concentrations produced by cracks have been used in a number of attempts to explain various phenomena encountered in the behavior of materials. Although many of these problems are chiefly of theoretical interest, some may be of practical interest. An attempt was therefore made to analyze cracks, considered as limiting cases of notches, by an extension of the method developed in the main body of this paper.

Fatigue tests with artificial cracks as sources of stress concentration have been reported by Peterson (ref. 9) and by Mailänder (ref. 21). Peterson produced the cracks (in 0.44-percent-carbon steel) by turning a narrow vee-groove in a round bar, heating and compressing the bar to close the groove, and finally annealing and machining the specimen. Mailänder used three different methods of producing cracks. In the first, nitrided steels were carefully stretched until the nitrided surface cracked. In the second, specimens containing a groove were subjected to repeated impacts (25 blows) until cracks were visible at the bottom of the groove. The specimens were then turned down practically to the bottom of the groove. In both of these methods, penetrating dyes were applied so that the depth of the cracks could be measured after the test. In the third method, an austenitic stainless steel "was treated for grain disintegration by boiling in a suitable solution for different lengths of time. The grain-boundary cracks generated were apparently so fine that they were not penetrated by the dye; the depth of crack was therefore estimated from the appearance of the fractured surface." Both Peterson and Mailänder calculated the nominal stress on the assumption that the cracked area could transmit compressive stress but not tensile stress. (Stresses computed on the basis of the full section differed by as much as 50 percent.)

Stress-concentration factors for the test specimens were computed in the following manner, with the notation shown in figure 2. For a deep notch, Neuber gives a formula (ref. 2, ch. V, eqs. (72) and (73)) which can be reduced to the simple form

$$K_D = \frac{3}{4}\sqrt{\frac{a}{R}} \tag{A1}$$

when the notch radius R becomes very small; the subscript D denotes deep notch. For a shallow notch, Neuber gives another equation (ref. 2, ch. IV, eq. (131)) which can be reduced to

$$K_S = 2\sqrt{\frac{t}{R}} \tag{A2}$$

when the radius R becomes small; the subscript S denotes shallow notch.

Up to this point, the theory used is classical theory of elasticity for an isotropic homogeneous material. The transition to the actual material is now made by substituting A for R in formulas (A1) and (A2). The final factor of stress concentration is obtained by applying Neuber's interpolation formula (ref. 2, ch. II, eq. (3))

$$K_N = 1 + \frac{\left(K_D - 1\right)\left(K_S - 1\right)}{\sqrt{\left(K_D - 1\right)^2 + \left(K_S - 1\right)^2}} \tag{A3}$$

Detailed data on the tests and on the results of applying formula (A3) are shown in table 6. The agreement between calculated and experimental factors of stress concentration as indicated by the ratio K_N/K_F in table 6 is quite satisfactory in view of the following considerations:

(a) The depth of notch is rather uncertain in some cases.

(b) The method of arriving at the calculated factors involves some debatable steps.

(c) Factors estimated by the classical theory of elasticity are too high by a factor of about 10. (In order to make such an estimate possible, the width of a crack in a Mailänder nitrided-steel specimen was estimated from a photomicrograph.)

The Mailänder tests in which the cracks were produced by stretching nitrided-steel specimens are open to the objection that the stretching may have affected the fatigue strength. Mailänder forestalled this objection by check tests on specimens which had been stretched just short of cracking; no effect on fatigue strength was found in these tests.

APPENDIX B

SIZE EFFECT ON SMOOTH ROTATING BEAMS

Stress gradients exist not only in the vicinity of notches; a beam subjected to bending also exhibits a stress gradient across its depth. This observation suggests that the Neuber constant A might be useful as a correlation parameter in the study of size effects on smooth rotating beams.

Let S_{RB} denote the endurance limit shown by smooth rotating beams of a given material; the endurance limit is the stress in the extreme fiber of the beam computed by the elementary beam formula

$$S_{RB} = \frac{Ma}{I} \tag{B1}$$

The Neuber concept implies that the endurance limit is a function of the stress gradient and thus of the radius of the beam, and it is known from experiments that the endurance limit does appear to vary with the radius. A limiting value of the endurance limit may be expected in the limiting case of zero stress gradient. A rotating beam would require an infinite radius in order to have zero stress gradient, but a zero gradient can easily be realized on a specimen of finite radius by resorting to axial loading. Let the endurance limit under axial loading be denoted by S_{AL}.

The results of fatigue tests on rotating beams of varying sizes are given in references 3, 4, 6, and 22. Preliminary analysis of the results suggested the empirical relation

$$S_{RB} = S_{AL}\left(1 + \sqrt{\frac{A}{a}}\right) \tag{B2}$$

where a is the radius of the specimen and A the Neuber constant from figure 3. For most of the test sets, the value S_{AL} had not been determined experimentally; it was therefore calculated from the experimental values of S_{RB} by using formula (B2) in conjunction with the method of least squares. For the sake of consistency, this procedure was also applied in the final analysis to those test sets in which S_{AL} had been determined experimentally.

The results of the final analysis are shown in figure 10 for the tests of reference 22, and in figure 11 for the tests of references 3, 4, and 6. More detailed information is given in table 7. The agreement is considered reasonably satisfactory.

The relation (B2) is clearly appropriate only for comparing small beams and large beams in which the material has undergone reasonably similar amounts of hot and cold work. Thus, use of the relation is appropriate if the small beams are machined from the same bar stock as the large beams, and if the beams are cut from the bar in such a way that the weakest fibers of all bars are from equivalent locations. One scheme for cutting beams of various sizes from 3-inch-diameter stock used by Moore and Morkovin (ref. 3) is illustrated in figure 12. It is possible, of course, that even large differences in the amount of hot or cold work may be insignificant for some materials.

The size effect predicted by relation (B2) is rather small in the usual range of interest. It is therefore easily conceivable that this effect might be masked in some tests by unrecognized differences in surface conditions which are known to be capable of producing powerful effects.

An equation similar to (B2) should be applicable to the bending of plates or sheet, but no tests were found that could be used to substantiate this belief.

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TABLE I.- ROTATING BEAMS WITH CIRCUMFERENTIAL GROOVES

Type of steel	Maximum diameter, in.	Minimum diameter, in.	Shape of notch	Root radius, in.	K_T	Ultimate tensile strength, ksi	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Reference
SAE 1020	0.145	0.125	Semicircle	0.010	2.00	62.0	21.2	33.0	1.55	0.010	1.50	0.97	3
SAE 1020	.290	.250	Semicircle	.020	2.00	62.0	20.2	32.0	1.59	.010	1.59	1.00	3
SAE 1020	.580	.500	Semicircle	.040	2.00	62.0	17.1	28.0	1.63	.010	1.67	1.02	3
SAE 1020	1.160	1.000	Semicircle	.080	2.00	62.0	17.0	28.0	1.64	.010	1.74	1.06	3
SAE 1020	2.175	1.875	Semicircle	.150	2.00	62.0	17.7	29.0	1.64	.010	1.74	1.06	3
SAE 1035	.145	.125	Semicircle	.010	2.00	87.6	26.0	40.0	1.54	.0054	1.59	1.03	3
SAE 1035	.290	.250	Semicircle	.020	2.00	87.6	24.7	40.0	1.62	.0054	1.66	1.02	3
SAE 1035	.580	.500	Semicircle	.040	2.00	87.6	20.0	34.6	1.73	.0054	1.73	1.00	3
SAE 1035	1.160	1.000	Semicircle	.080	2.00	87.6	19.8	35.0	1.77	.0054	1.79	1.01	3
SAE 1035	2.175	1.875	Semicircle	.150	2.00	87.6	19.8	35.0	1.77	.0054	1.84	1.04	3
SAE 4130	.145	.125	Semicircle	.010	2.00	141.8	45.0	70.0	1.93	.0015	1.72	.89	3
SAE 4130	.290	.250	Semicircle	.020	2.00	141.8	37.0	69.5	1.88	.0015	1.78	.95	3
SAE 4130 Heat treated	.580	.500	Semicircle	.040	2.00	141.8	35.8	65.0	1.82	.0015	1.84	1.01	3
SAE 4130	1.160	1.000	Semicircle	.080	2.00	141.8	35.0	63.7	1.82	.0015	1.88	1.03	3
SAE 4130	2.030	1.750	Semicircle	.140	2.00	141.8	34.9	63.5	1.82	.0015	1.90	1.04	3
SAE 4340	.145	.125	Semicircle	.010	2.00	163.8	51.7	82.5	1.60	.00084	1.78	1.11	4
SAE 4340	.290	.250	Semicircle	.020	2.00	163.8	48.0	81.0	1.69	.00084	1.83	1.08	4
SAE 4340	.580	.500	Semicircle	.040	2.00	163.8	48.0	78.0	1.62	.00084	1.87	1.15	4
SAE 4340	1.160	1.000	Semicircle	.080	2.00	163.8	46.0	74.0	1.61	.00084	1.90	1.18	4
SAE 4340	2.030	1.750	Semicircle	.140	2.00	163.8	42.0	74.0	1.76	.00084	1.93	1.10	4
SAE 1045	.375	.300	60°V	.010	2.90	104.7	26.0	61.5	2.37	.0036	1.99	.84	5
SAE 1045	.375	.300	60°V	.010	2.90	119.9	27.0	67.0	2.48	.0026	2.07	.83	5
SAE 3140	.375	.300	60°V	.010	2.90	108.2	28.0	65.5	2.34	.0034	2.00	.86	5
SAE 3140	.375	.300	60°V	.010	2.90	109.0	22.0	62.5	2.84	.0032	2.07	.73	5
SAE 2340	.375	.300	60°V	.010	2.90	115.7	30.0	71.0	2.37	.0028	2.05	.86	5
SAE 2340	.375	.300	60°V	.010	2.90	122.1	22.0	68.0	3.09	.0024	2.09	.68	5
SAE 2340	.375	.300	60°V	.010	2.90	118.9	21.0	67.0	3.19	.0026	2.06	.65	5
SAE 2340	.375	.300	60°V	.010	2.90	130.1	26.5	75.0	2.83	.0020	2.12	.75	5
SAE 2340	.375	.300	60°V	.010	2.90	134.2	26.5	79.5	3.00	.0018	2.13	.71	5
SAE 1020	.250	.125	Semicircle	.0625	1.26	59.9	23.0	29.0	1.26	.012	1.18	.94	6
SAE 1020	.200	.160	Semicircle	.0200	1.74	59.9	20.5	29.0	1.41	.012	1.42	1.01	6
SAE 1020	.500	.250	U	.0625	1.47	59.9	21.25	29.0	1.36	.012	1.33	.98	6
SAE 1020	.625	.500	Semicircle	.0625	1.74	59.9	19.5	28.0	1.44	.012	1.52	1.05	6
SAE 1020	.540	.500	Semicircle	.0200	2.38	59.9	16.5	28.0	1.70	.012	1.78	1.04	6
SAE 1020	1.040	1.000	Semicircle	.0200	2.60	59.9	15.5	28.0	1.81	.012	1.90	1.05	6
SAE 1020	1.250	1.000	Semicircle	.125	1.74	59.9	18.0	28.0	1.55	.012	1.57	1.01	6
SAE 1020	2.040	2.000	Semicircle	.020	2.80	59.9	15.0	28.0	1.87	.012	2.01	1.07	6
SAE 1020	2.500	2.000	Semicircle	.250	1.74	59.9	17.5	28.0	1.60	.012	1.61	1.01	6
SAE 1035	.290	.250	Semicircle	.020	2.00	77.6	39.0	23.5	1.66	.0069	1.63	.98	6
SAE 2345	.250	.125	Semicircle	.0625	1.26	125.5	56.5	70.25	1.24	.0023	1.22	.98	6
SAE 2345	.200	.160	Semicircle	.020	1.75	125.5	44.5	70.75	1.59	.0023	1.56	.98	6

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TABLE I.- ROTATING BEAMS WITH CIRCUMFERENTIAL GROOVES - Concluded

Type of steel	Maximum diameter, in.	Minimum diameter, in.	Shape of notch (a)	Root radius, in.	K_T	Ultimate tensile strength, ksi (a)	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Reference
SAE 2345	0.500	0.250	U	0.0625	1.46	125.5	50.0	66.75	1.34	0.0023	1.39	1.04	6
SAE 2345	.425	.300	Semicircle	.0625	1.54	125.5	48.25	70.0	1.45	.0023	1.45	1.00	6
SAE 2345	.340	.300	Semicircle	.020	2.10	125.5	37.0	70.0	1.89	.0023	1.82	.96	6
SAE 2345	.375	.300	Semicircle	.0375	1.75	125.5	43.5	70.0	1.61	.0023	1.60	.99	6
SAE 2345	.540	.500	Semicircle	.020	2.38	125.5	33.55	66.5	1.99	.0023	2.03	1.02	6
SAE 2345	.625	.500	Semicircle	.0625	1.75	125.5	44.5	66.5	1.59	.0023	1.63	1.03	6
SAE 2345	1.000	.875	Semicircle	.0625	2.06	125.5	35.0	64.0	1.83	.0023	1.89	1.03	6
SAE 2345	1.750	1.500	U	.0625	2.54	125.5	29.5	66.5	2.24	.0023	2.29	1.02	6
SAE 1020	1.875	1.750	Semicircle	.0625	2.42	59.9	15.25	28.0	1.83	.012	1.99	1.09	7
SAE 1020	1.750	1.500	U	.0625	2.54	59.9	14.75	28.0	1.90	.012	2.07	1.09	7
SAE 1020	1.000	.875	Semicircle	.0625	2.07	59.9	17.0	28.0	1.65	.012	1.74	1.05	7
SAE 1020	.875	.625	U	.0625	1.92	59.9	18.0	28.0	1.55	.012	1.64	1.06	7
SAE 1020	.375	.300	Semicircle	.0375	1.76	59.9	19.5	28.0	1.44	.012	1.49	1.03	7
SAE 1020	.340	.300	Semicircle	.020	2.10	59.9	18.0	28.0	1.55	.012	1.62	1.05	7
SAE 1020	.425	.300	Semicircle	.0625	1.51	59.9	20.75	28.0	1.35	.012	1.36	1.01	7
SAE 2345	2.187	1.750	Semicircle	.2187	1.76	125.5	39.0	66.5	1.70	.0023	1.69	.995	7
SAE 2345	1.790	1.750	Semicircle	.020	2.80	125.5	28.5	66.5	2.33	.0023	2.34	1.005	7
SAE 2345	1.500	1.375	Semicircle	.0625	2.30	125.5	31.0	66.5	2.14	.0023	2.09	1.02	7
SAE 2345	1.250	1.000	Semicircle	.125	1.76	125.5	38.5	64.0	1.66	.0023	1.66	1.00	7
SAE 2345	1.040	1.000	Semicircle	.020	2.65	125.5	29.0	64.0	2.21	.0023	2.23	1.01	7
SAE 2345	.875	.625	U	.0625	1.92	125.5	37.0	66.5	1.80	.0023	1.77	.98	7
SAE 4130	.480	.355	45°V	.010	3.20	120	23.5	54.5	2.32	.0027	2.30	.99	8
0.1 % C	.600	.584	(0°)	.002	4.50	(60)	-----	-----	1.07	.0112	2.04	1.91	9
0.1 % C	.300	.294	(0°)	.002	4.24	(60)	-----	-----	1.21	.0112	1.65	1.36	9
0.29 % C	.480	.404	(0°)	.010	3.13	(80)	-----	-----	2.62	.0065	2.18	.83	9
0.29 % C	.480	.328	(0°)	.015	2.73	(80)	-----	-----	2.50	.0065	2.04	.82	9
0.29 % C	.591	.405	(0°)	.063	1.65	(80)	-----	-----	1.53	.0065	1.49	.97	9
0.29 % C	.716	.404	(0°)	.125	1.37	(80)	-----	-----	1.31	.0065	1.30	.99	9
0.29 % C	.966	.404	(0°)	.250	1.20	(80)	-----	-----	1.22	.0065	1.17	.96	9
0.36 % C	.340	.316	(0°)	.006	3.04	(85)	-----	-----	1.75	.0058	2.03	1.16	9
0.42 % C	.600	.584	(0°)	.002	4.50	(90)	-----	-----	1.43	.0052	2.34	1.63	9
0.62 % C	.340	.316	(0°)	.006	3.04	(115)	-----	-----	1.78	.0029	2.21	1.24	9
Cr-Ni Annealed	.340	.316	(0°)	.006	3.04	(65)	-----	-----	1.95	.0097	1.90	.97	9
Cr-Ni Heat treated	.340	.316	(0°)	.006	3.04	(100)	-----	-----	2.20	.0041	2.12	.96	9
Cr-Ni Heat treated	.600	.584	(0°)	.002	4.5	(150)	-----	-----	2.00	.0012	2.97	1.48	9
Cast Steel	.300	.284	(0°)	.002	4.04	(60)	-----	-----	1.31	.0121	1.91	1.46	9

[a]Numbers in parenthesis are estimated values

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TABLE II.- ROTATING BEAMS WITH FILLETS

Type of steel	Maximum diameter, in.	Minimum diameter, in.	Root radius, in.	K_T	Ultimate tensile strength, ksi (a)	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Reference
0.65 % C	0.600	0.400	0.200	1.21	(90)	----	----	1.03	0.0052	1.16	1.13	9
0.65 % C	.600	.400	.080	1.49	(90)	----	----	1.14	.0052	1.32	1.16	9
0.65 % C	.600	.400	.040	1.76	(90)	----	----	1.29	.0052	1.44	1.11	9
Spring	.433	.295	.118	1.25	(225)	----	----	1.13	.0001	1.22	1.08	9
Spring	.433	.295	.065	1.44	(225)	----	----	1.40	.0001	1.44	1.03	9
Spring	.433	.295	.0384	1.65	(225)	----	----	1.57	.0001	1.61	1.03	9
0.20 % C	2.000	1.000	.380	1.29	(65)	----	----	1.11	.0096	1.24	1.12	9
0.45 % C	.820	.410	.1025	1.43	76	27.3	32.5	1.19	.0072	1.28	1.07	10
0.45 % C	.500	.250	.0625	1.43	76	27.5	32.5	1.19	.0072	1.26	1.06	10
0.45 % C	3.20	1.600	.400	1.43	76	26.5	32.5	1.23	.0072	1.34	1.09	10
0.45 % C	.500	.250	.0156	2.04	76	22.5	32.5	1.44	.0072	1.44	1.00	10
0.45 % C	.820	.410	.0256	2.04	76	21.3	32.5	1.53	.0072	1.50	.98	10
0.45 % C	3.200	1.600	.100	2.04	76	17.5	32.5	1.86	.0072	1.68	.90	10
Ni-Mo	.500	.250	.0425	1.6	97	40.0	53.0	1.33	.0044	1.37	1.03	10
Ni-Mo	.820	.410	.0615	1.65	97	37.0	53.0	1.43	.0044	1.42	.99	10
Ni-Mo	3.195	2.130	.3195	1.6	97	35.0	53.0	1.51	.0044	1.49	.98	10
Ni-Cr	.800	.400	.028	1.98	120	35.0	60.0	1.71	.0026	1.61	.94	10
Ni-Cr	2.000	1.000	.080	1.90	120	34.5	60.0	1.74	.0026	1.66	.95	10
0.45 % C	.375	.250	.0156	2.00	76	----	----	1.45	.0072	1.42	.98	10
0.45 % C	.615	.410	.0258	2.00	76	----	----	1.52	.0072	1.49	.98	10
0.45 % C	2.400	1.600	.100	2.00	76	----	----	1.85	.0072	1.65	.89	10
Ni-Mo	.375	.250	.0375	1.60	97	----	----	1.32	.0044	1.36	1.03	10
Ni-Mo	.615	.410	.0615	1.60	97	----	----	1.45	.0044	1.39	.96	10
Ni-Mo	3.195	2.130	.3195	1.60	97	----	----	1.51	.0044	1.49	.98	10
0.45 % C	.375	.250	.0625	1.40	76	----	----	1.17	.0072	1.24	1.06	10
0.45 % C	.615	.410	.1025	1.40	76	----	----	1.18	.0072	1.26	1.07	10
0.45 % C	2.400	1.600	.400	1.40	76	----	----	1.24	.0072	1.32	1.06	10
0.13 % C	.450	.300	.039	1.64	(61)	----	----	1.39	.0109	1.31	.94	10
0.13 % C	.450	.300	.060	1.48	(61)	----	----	1.28	.0109	1.26	.98	10
0.13 % C	.450	.300	.120	1.28	(61)	----	----	1.09	.0109	1.17	1.08	10
0.18 % C	.450	.300	.039	1.64	(65)	----	----	1.36	.0097	1.32	.97	10
0.18 % C	.450	.300	.120	1.28	(65)	----	----	1.05	.0097	1.18	1.12	10
0.28 % C	.450	.300	.039	1.64	(75)	----	----	1.28	.0074	1.34	1.05	10
0.28 % C	.450	.300	.120	1.28	(75)	----	----	1.06	.0074	1.20	1.13	10
0.28 % C	2.27	1.18	.118	1.75	(75)	----	----	1.50	.0074	1.50	1.00	10
0.28 % C	2.27	1.18	.590	1.21	(75)	----	----	1.05	.0074	1.17	1.12	10
0.28 % C	.555	.370	.0777	1.47	(75)	----	----	1.20	.0074	1.29	1.08	10
0.28 % C	.555	.370	.1665	1.23	(75)	----	----	1.05	.0074	1.17	1.12	10
0.30 % C	.555	.370	.185	1.21	(80)	----	----	1.59	.0066	1.15	.72	10
0.30 % C	.555	.370	.0777	1.47	(80)	----	----	1.12	.0066	1.30	1.16	10
0.33 % C 0.65 % Mn	.900	.600	.06	1.75	(85)	----	----	1.52	.0058	1.46	.96	10
0.33 % C 1.97 % Mn	.900	.600	.06	1.75	(100)	----	----	1.45	.0041	1.49	1.03	10
Ni-Cr	.570	.380	.019	2.12	(150)	----	----	2.30	.00123	1.75	.76	10
Ni-Cr	.900	.600	.060	1.75	(120)	----	----	1.67	.0026	1.53	.92	10

[a]Numbers in parentheses are estimated values.

TABLE II.- ROTATING BEAMS WITH FILLETS - Concluded

Type of steel	Maximum diameter, in.	Minimum diameter, in.	Root radius, in.	K_T	Ultimate tensile strength, ksi (a)	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Reference
Ni-Cr	0.450	0.300	0.0186	2.05	(120)	----	--	2.16	0.0026	1.60	0.74	10
Ni-Cr	.450	.300	.039	1.64	(120)	----	--	1.61	.0026	1.42	.88	10
Ni-Cr	.450	.300	.081	1.38	(120)	----	--	1.21	.0026	1.28	1.06	10
Ni-Cr	1.500	1.000	.080	1.89	120	----	--	1.75	.0026	1.69	.97	10
Ni-Cr	.615	.410	.0308	1.91	120	----	--	1.71	.0026	1.63	.95	10
Cr-Ni-W	1.770	1.180	.118	1.75	(150)	----	--	2.10	.00123	1.62	.77	10
Cr-Ni-W	1.770	1.180	.236	1.48	(150)	----	--	1.58	.00123	1.42	.90	10
Cr-Ni-W	1.770	1.180	.590	1.21	(150)	----	--	1.18	.00123	1.19	1.01	10
Cr-Ni-W	.450	.300	.051	1.56	(150)	----	--	1.40	.00123	1.45	1.04	10
Cr-Ni-W	.450	.300	.144	1.22	(150)	----	--	1.07	.00123	1.19	1.11	10
Cr-Co	.450	.300	.039	1.64	(75)	----	--	1.47	.0074	1.34	.91	10
0.33 % C 0.65 % Mn	.900	.600	.060	1.75	(100)	----	--	1.88	.0041	1.44	.76	10
0.33 % C 1.97 % Mn	.900	.600	.060	1.75	(120)	----	--	1.78	.0026	1.53	.86	10
0.45 % C 1.74 % Si	.450	.300	.039	1.64	(130)	----	--	1.55	.00202	1.44	.93	10
0.45 % C 1.74 % Si	.450	.300	.120	1.28	(130)	----	--	1.11	.00202	1.22	1.10	10
0.46 % C	1.500	1.000	.180	1.54	(95)	----	--	1.48	.0046	1.41	.95	10
0.46 % C	1.500	1.000	.180	1.54	(95)	----	--	1.41	.0046	1.41	1.00	10
0.46 % C	1.500	1.000	.180	1.54	(95)	----	--	1.30	.0046	1.41	1.08	10
0.60 % C	.900	.600	.060	1.75	(118)	----	--	1.58	.0027	1.53	.97	10
0.33 % C	1.250	.600	.0498	2.10	(85)	----	--	1.47	.0058	1.62	1.10	9
0.33 % C	1.250	.600	.0498	2.10	(95)	----	--	1.92	.0046	1.68	.88	9
0.46 % C	2.000	1.000	.188	1.58	(100)	----	--	1.35	.0041	1.30	.96	9
0.53 % C Heat treated	2.000	1.000	.188	1.58	(110)	----	--	1.42	.0033	1.46	1.03	9
0.53 % C	2.500	2.000	.312	1.32	(110)	----	--	1.37	.0033	1.27	.93	9
0.60 % C	1.250	.600	.0498	2.10	(118)	----	--	1.62	.0027	1.75	1.08	9
0.37 % C 0.60 % Cr 1.30 % Ni	1.250	.600	.0498	2.10	(140)	----	--	1.67	.0016	1.81	1.08	9
C	.748	.374	.020	2.10	76.5	----	--	1.73	.0071	1.50	.87	11
Cr-Ni	.748	.374	.020	2.10	152.5	----	--	2.12	.00116	1.74	.82	11
Cr-Ni-W	.748	.374	.020	2.10	230	----	--	2.30	.00008	1.98	.87	11
SAE 1050	6.562	5.250	.281	2.10	102	18.5	30	1.62	.0038	1.89	1.17	12
SAE 1050	6.562	5.250	.843	1.50	102	25.5	30	1.18	.0038	1.46	1.24	12

[a]Numbers in parenthesis are estimated values.

TABLE III.- ROTATING BEAMS WITH TRANSVERSE ROUND HOLES

Type of steel	Diameter, D, in.	$\frac{2R}{D}$	Radius, R, in.	K_T	Ultimate tensile strength, ksi (a)	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Reference
Armco Iron	0.300	0.183	0.0275	2.04	(45)	----	----	1.63	0.0236	1.54	0.95	9
0.49 % C	.300	.183	.0275	2.04	(80)	----	----	1.51	.0066	1.70	1.13	9
0.52 % C	.300	.183	.0275	2.04	(85)	----	----	1.58	.0058	1.72	1.08	9
Cyclops	.300	.183	.0275	2.04	(100)	----	----	1.25	.0041	1.75	1.40	9
0.45 % C	.100	.250	.0125	2.00	76	29.8	33.0	1.11	.0072	1.57	1.41	13
0.45 % C	.500	.250	.0625	2.00	76	24.0	33.0	1.37	.0072	1.75	1.27	13
0.45 % C	.500	.063	.0156	2.38	76	24.8	33.0	1.33	.0072	1.82	1.37	13
0.45 % C	1.000	.250	.125	2.00	76	23.5	33.0	1.40	.0072	1.81	1.29	13
0.45 % C	1.000	.063	.0312	2.38	76	21.2	33.0	1.56	.0072	1.94	1.24	13
0.45 % C	3.000	.250	.375	2.00	76	21.2	33.0	1.56	.0072	1.88	1.20	13
0.45 % C	3.000	.063	.0938	2.38	76	17.5	33.0	1.88	.0072	2.08	1.11	13
0.57 % C Heat treated	.273	.144	.0187	2.11	(90)	34.5	48.0	1.39	.0052	1.73	1.25	13
0.57 % C Heat treated	2.130	.15	.160	2.10	(90)	22.0	49.0	2.22	.0052	1.93	.87	13
Ni-Mo	.178	.09	.0080	2.27	97	44.0	53.0	1.20	.0044	1.73	1.44	10
Ni-Mo	.178	.25	.0223	2.00	97	37.0	53.0	1.43	.0044	1.69	1.18	10
Ni-Mo	.100	.25	.0125	2.00	97	36.5	53.0	1.45	.0044	1.63	1.12	10
Ni-Mo	.500	.25	.0625	2.00	97	30.5	53.0	1.74	.0044	1.79	1.03	10
Ni-Mo	.500	.09	.0225	2.27	97	27.0	53.0	1.96	.0044	1.88	.96	10
Ni-Mo	2.000	.25	.2500	2.00	97	26.2	53.0	2.02	.0044	1.76	.87	10
Ni-Mo	2.000	.09	.09	2.27	97	24.7	53.0	2.15	.0044	2.04	.95	10
0.42 % C	.500	.050	.0125	2.46	70	----	----	1.42	.0085	1.80	1.27	9
0.42 % C	.500	.080	.02	2.31	70	----	----	1.55	.0085	1.80	1.16	9
0.42 % C	.500	.140	.035	2.12	70	----	----	1.62	.0085	1.75	1.08	9
0.42 % C	.500	.250	.0625	2.00	70	----	----	1.70	.0085	1.73	1.02	9
0.45 % C	2.000	.0050	.0050	2.88	76	----	----	1.22	.0072	1.86	1.52	9
0.45 % C	2.000	.060	.060	2.40	76	----	----	1.70	.0072	2.04	1.20	9
0.45 % C	2.000	.250	.25	2.00	76	----	----	1.45	.0072	1.86	1.28	9

[a]Numbers in parenthesis are estimated values.

TABLE IV.- AXIAL-LOAD FATIGUE TESTS OF CYLINDRICAL SPECIMENS CONTAINING CIRCUMFERENTIAL GROOVES

[All specimens have maximum diameter of 0.315 in.; minimum diameter of 0.276 in.; 60° V groove; root radius of 0.004 in.; K_T = 4.4.]

Type of steel	Designation of steel (a)	Ultimate tensile strength, ksi	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Temperature, °C	Reference
Cr-Mo	1452	87.1	19.9	41.2	2.07	0.0056	2.23	1.03	20	14 and 15
Cr-Mo	1452	105.0	22.7	56.8	2.50	.0036	2.45	.98	-78	14 and 15
Cr-Mo	1452	114.0	22.7	56.8	2.50	.0029	2.49	1.01	20	14 and 15
Cr-Mo	1452	130.2	28.4	61.8	2.18	.0021	2.68	1.23	-78	14 and 15
Cr-Mo	1452	135.7	25.6	69.6	2.72	.0018	2.70	.99	20	14 and 15
Cr-Mo	1452	154.0	28.4	76.7	2.70	.0011	2.94	1.09	-78	14 and 15
Cr-Ni-Mo-V	1460	128.0	25.6	62.5	2.44	.0021	2.63	1.08	20	14 and 15
Cr-Ni-Mo-V	1460	142.8	28.4	73.8	2.60	.0015	2.82	1.08	-78	14 and 15
Cr-Ni-Mo-V	1460	171.0	31.2	82.4	2.64	.00065	3.12	1.18	20	14 and 15
Cr-Ni-Mo-V	1460	187.0	34.1	88.0	2.69	.00041	3.34	1.24	-78	14 and 15
Cr-Ni-Mo-V	1460	174.0	28.4	80.9	2.85	.00063	3.04	1.07	20	14 and 15
Cr-Ni-Mo-V	1460	189.5	28.4	86.6	3.05	.00036	3.38	1.11	-78	14 and 15
Cr-Ni-Mo-V	1460	197.0	25.6	85.2	3.33	.00031	3.40	1.02	20	14 and 15
Cr-Ni-Mo-V	1460	209.0	25.6	89.5	3.50	.00022	3.54	1.01	-78	14 and 15
Cr-V	1604	92.0	17.0	44.7	2.63	.0049	2.28	.87	20	14 and 15
Cr-V	1604	106.8	19.9	52.5	2.64	.0035	2.46	.93	-78	14 and 15
Cr-V	1604	108.6	19.9	54.0	2.72	.0034	2.43	.89	20	14 and 15
Cr-V	1604	124.8	19.9	58.9	2.96	.0023	2.64	.89	-78	14 and 15
Cr-V	1604	139.5	19.9	68.2	3.43	.0016	2.75	.80	20	14 and 15
Cr-V	1604	151.3	19.9	78.1	3.93	.0012	2.92	.74	-78	14 and 15
Cr-V	1610	130.8	15.6	56.8	3.63	.002	2.65	.73	20	14 and 15
Cr-V	1610	151.0	15.6	61.1	3.91	.0012	2.90	.74	-78	14 and 15
Cr-V	1610	131.2	17.0	68.2	4.00	.0019	2.67	.67	20	14 and 15
Cr-V	1610	149.8	19.9	72.4	3.64	.0013	2.89	.79	-78	14 and 15
Cr-V	1610	196.1	22.7	79.5	3.50	.00031	3.40	.97	20	14 and 15
Cr-V	1610	215.0	22.7	80.9	3.56	.00015	3.66	1.03	-78	14 and 15
Cr-V	1620	135.5	25.6	67.5	2.64	.0018	2.70	1.02	20	14 and 15
Cr-V	1620	151.0	31.2	75.3	2.41	.0012	2.90	1.20	-78	14 and 15
Cr-V	1620	172.2	28.4	83.8	2.95	.00067	3.10	1.05	20	14 and 15
Cr-V	1620	188.0	31.2	88.0	2.82	.00041	3.34	1.18	-78	14 and 15
Cr-V	1620	174.2	28.4	83.8	2.95	.00063	3.14	1.07	20	14 and 15
Cr-V	1620	189.0	31.2	88.0	2.82	.00040	3.34	1.18	-78	14 and 15
Cr-V	1620	192.5	19.9	85.2	4.28	.00036	3.35	.78	20	14 and 15
Cr-V	1620	209.5	19.9	89.5	4.50	.00020	3.58	.80	-78	14 and 15
StC 45.61	1130	112.6	18.5	49.7	2.69	.0030	2.48	.92	20	14 and 15
StC 45.61	1130	127.5	18.5	62.5	3.38	.0021	2.66	.79	-78	14 and 15
StC 45.61	1130	114.8	15.6	54.0	3.45	.0029	2.49	.72	20	14 and 15
StC 45.61	1130	135.0	15.6	62.5	4.00	.0018	2.74	.68	-78	14 and 15
StC 60.61	1150	136.0	19.9	56.8	2.86	.0018	2.70	.94	20	14 and 15
StC 60.61	1150	151.8	22.7	71.0	3.12	.0012	2.91	.93	-78	14 and 15
NC 80	1207	156.0	22.7	49.7	2.19	.0012	2.91	1.33	20	14 and 15
NC 80	1207	171.0	25.6	60.4	2.36	.00069	3.14	1.33	-78	14 and 15
NC 100	1208	212.0	24.2	85.9	3.56	.00017	3.60	1.01	20	14 and 15

[a]German aircraft construction material designations.

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TABLE IV.- AXIAL-LOAD FATIGUE TESTS OF CYLINDRICAL SPECIMENS CONTAINING CIRCUMFERENTIAL GROOVES - Concluded

[All specimens have maximum diameter of 0.315 in.; minimum diameter of 0.276 in.; 60°V groove; root radius of 0.004 in.; $K_T = 4.4$.]

Type of steel	Designation of steel (a)	Ultimate tensile strength, ksi	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Temperature, $^{\circ}C$	Reference
EC 100	1208	226.0	24.2	93.0	3.85	0.00010	3.78	0.98	-78	14 and 15
VC 135	1253	140.0	28.4	68.2	2.40	.0016	2.75	1.15	20	14 and 15
VC 135	1253	156.0	28.4	75.3	2.65	.0011	2.96	1.12	-78	14 and 15
VC 135	1253	136.0	21.3	62.5	2.93	.0018	2.70	.92	20	14 and 15
VC 135	1253	153.0	21.3	68.2	3.20	.0012	2.93	.92	-78	14 and 15
Mn	1265	105.3	17.0	48.3	2.83	.0036	2.40	.85	20	14 and 15
Mn	1265	119.7	17.0	58.2	3.42	.0026	2.54	.74	-78	14 and 15
Mn	1267	127.5	19.9	51.1	2.57	.0021	2.62	1.02	20	14 and 15
Mn	1267	144.0	19.9	59.6	3.00	.0014	2.83	.94	-78	14 and 15
Mn	1267	138.2	18.4	62.5	3.38	.0016	2.73	.81	20	14 and 15
Mn	1267	156.8	18.4	66.7	3.61	.0010	2.97	.82	-78	14 and 15
Mn-V	1310	146.0	28.4	64.6	2.28	.0014	2.81	1.23	20	14 and 15
Mn-V	1310	162.2	28.4	72.4	2.55	.0009	3.03	1.19	-78	14 and 15
Cr-Mo Casting	1811	131.0	19.9	42.6	2.14	.0020	2.66	1.24	20	14 and 15
Cr-Mo Casting	1811	145.5	19.9	48.3	2.43	.0014	2.85	1.17	-78	14 and 15
Mn	----	79.2	14.0	38.0	2.71	.0067	2.16	.80	20	15
Mn	----	91.4	16.0	43.0	2.69	.0050	2.27	.84	-78	15
Mn	----	113.8	18.0	48.0	2.67	.0030	2.48	.93	20	15
Mn	----	120.4	18.0	51.0	2.83	.0025	2.55	.90	-78	15
Mn	----	89.7	14.0	41.0	2.93	.0052	2.26	.77	20	15
Mn	----	98.2	14.0	48.0	3.43	.0042	2.34	.68	-78	15
Mn	----	112.8	15.0	54.0	3.60	.0030	2.47	.69	20	15
Mn	----	127.2	15.0	56.0	3.74	.0022	2.62	.70	-78	15
Cr-Mo Casting	----	116.0	15.0	29.0	1.93	.0028	2.51	1.30	20	15
Cr-Mo Casting	----	129.5	15.0	32.5	2.17	.0020	2.64	1.22	-78	15
Mn-V Casting	----	92.8	13.0	24.0	1.85	.0048	2.28	1.24	20	15
Mn-V Casting	----	103.7	16.0	29.0	1.81	.0037	2.39	1.32	-78	15
Cr-Mn Casting	----	125.6	17.0	30.0	1.76	.0023	2.60	1.48	20	15
Cr-Mn Casting	----	134.2	17.0	30.0	1.76	.0018	2.69	1.53	-78	15

[a]German aircraft construction material designations.

TABLE V.- AXIAL-LOAD FATIGUE TESTS OF SHEET SPECIMENS CONTAINING

HOLES, SYMMETRICAL NOTCHES AND FILLETS

Type of steel	Gross width, in.	Net width, in.	Shape of notch	Root radius, in.	K_T	Ultimate tensile strength, ksi	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Reference
SAE 4130 Normalized	4.500	1.500	Hole	1.500	2.0	117	25	47	1.88	0.0027	1.96	1.04	16
SAE 4130 Normalized	2.250	1.500	U	.3175	2.0	117	27	47	1.74	.0027	1.92	1.10	16
SAE 4130 Normalized	2.250	1.500	Fillet	.1736	2.0	117	27	47	1.74	.0027	1.80	1.03	16
SAE 4130 Normalized	2.250	1.500	U	.057	4.0	117	14	47	3.36	.0027	3.46	1.03	16
SAE 4130 Normalized	2.250	1.500	Fillet	.0195	4.0	117	17	47	2.76	.0027	2.72	.99	16
SAE 4130 Normalized	2.250	1.500	U	.0315	5.0	117	11	47	4.27	.0027	4.10	.96	17

TABLE VI.- ROTATING BEAMS CONTAINING ARTIFICIAL CRACKS

Type of steel	Maximum diameter, in.	Minimum diameter, in.	Ultimate tensile strength, ksi (a)	Endurance limit (notched), ksi	Endurance limit (unnotched), ksi	K_F	A from fig. 3, in.	K_N	K_N/K_F	Reference
0.44 % C	3.20	2.36	(76)	----	-----	4.00	0.0072	5.11	1.28	9
0.44 % C	0.80	.594	(76)	----	-----	2.83	.0072	2.74	.97	9
0.44 % C	0.80	.320	(76)	----	-----	6.75	.0072	2.10	.31	9
Nitrided	.294	.240	128	27.7	82.3	3.00	.00212	4.74	1.58	21
Nitrided	.294	.232	101	17.0	68.1	4.00	.00396	4.10	1.02	21
Nitrided	.294	.248	149	28.4	105.0	3.70	.0013	5.84	1.58	21
Nickel	.394	.378	118	12.1	73.8	6.10	.0027	3.20	.52	21
Stainless	.236	.220	88	33.2	45.5	1.94	.00531	2.24	1.15	21
Stainless	.236	.189	79.5	34.4	45.5	1.88	.00655	2.48	1.32	21

[a]Numbers in parenthesis are estimated values.

TABLE VII.- UNNOTCHED ROTATING BEAMS

Type of steel	Diameter, in.	Endurance limit, ksi	Ultimate strength, ksi (a)	A from fig. 3, in.	K_N or $1 + \sqrt{\frac{A}{a}}$	S_{AL}, ksi	S_{RB}, ksi	Reference
VCN 35	0.268	85.2	(100)	0.0041	1.18		58.5	22
VCN 35	.646	79.5	(100)	.0041	1.11	49.82	55.4	22
VCN 35	1.076	73.9	(100)	.0041	1.09		54.2	22
1 % C Normalized	.268	46.9	(80)	.0066	1.22		33.0	22
1 % C Normalized	.646	44.0	(80)	.0066	1.14	27.0	30.9	22
1 % C Normalized	1.076	42.6	(80)	.0066	1.11		30.0	22
1 % C Annealed	.268	44.0	(60)	.0112	1.29		31.5	22
1 % C Annealed	.646	41.2	(60)	.0112	1.19	24.4	28.9	22
1 % C Annealed	1.076	39.8	(60)	.0112	1.14		27.9	22
0.10 % C	.268	39.8	(50)	.013	1.31		27.5	22
0.10 % C	.646	36.9	(50)	.013	1.20	21.5	25.8	22
0.10 % C	1.076	35.5	(50)	.013	1.16		24.8	22
0.04 % C	.0358	43.3	(50)	.013	1.85		33.4	22
0.04 % C	.0716	38.4	(50)	.013	1.60		28.9	22
0.04 % C	.1432	39.0	(50)	.013	1.43	18.0	25.7	22
0.04 % C	.2864	39.0	(50)	.013	1.30		23.5	22
0.04 % C	Axial Load	26.3	(50)	.013	1.00		18.0	22
0.41 % C	.0358	30.5	(60)	.0112	1.79		23.3	22
0.41 % C	.0716	27.0	(60)	.0112	1.56		20.3	22
0.41 % C	.1432	27.0	(60)	.0112	1.39	13.0	18.1	22
0.41 % C	.2864	27.0	(60)	.0112	1.28		16.7	22
0.41 % C	Axial Load	19.9	(60)	.0112	1.00		13.0	22
X4130	.125	75	142	.00152	1.16		71.4	3
X4130	.250	69	142	.00152	1.12		68.1	3
X4130	.500	65	142	.00152	1.08	61.8	66.6	3
X4130	1.000	63	142	.00152	1.06		65.3	3
X4130	2.000	63	142	.00152	1.04		64.1	3
SAE 1035 (as rolled)	.125	39	87.6	.0054	1.29		40.6	3
SAE 1035 (as rolled)	.250	39	87.6	.0054	1.21		37.9	3
SAE 1035 (as rolled)	.500	35	87.6	.0054	1.15	31.4	36.0	3
SAE 1035 (as rolled)	1.000	35	87.6	.0054	1.10		34.6	3
SAE 1035 (as rolled)	2.000	35	87.6	.0054	1.07		33.7	3
SAE 1020 (as rolled)	.125	34	62	.0104	1.41		34.8	3
SAE 1020 (as rolled)	.250	32	62	.0104	1.29		31.8	3
SAE 1020 (as rolled)	.500	28	62	.0104	1.20	24.7	29.8	3
SAE 1020 (as rolled)	1.000	28	62	.0104	1.14		28.2	3
SAE 1020 (as rolled)	2.000	30	62	.0104	1.10		27.2	3

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[a]Numbers in parenthesis are estimated values.

TABLE VII.- UNNOTCHED ROTATING BEAMS - Concluded

Type of steel	Diameter, in.	Endurance limit, ksi	Ultimate strength, ksi	A from fig. 3, in.	K_N or $1 + \sqrt{\frac{A}{a}}$	S_{AL}, ksi	S_{RB}, ksi	Reference
SAE 1020 (annealed)	0.160	29	60	0.0110	1.37		31.6	6
SAE 1020 (annealed)	.250	29	60	.0110	1.30		29.8	6
SAE 1020 (annealed)	.500	28	60	.0110	1.21	23.0	27.8	6
SAE 1020 (annealed)	1.000	28	60	.0110	1.15		26.4	6
SAE 1020 (annealed)	1.875	28	60	.0110	1.11		25.6	6
SAE 1035 (annealed)	.125	35	77.6	.0069	1.33		35.8	6
SAE 1035 (annealed)	.250	34	77.6	.0069	1.24	26.9	33.2	6
SAE 1035 (annealed)	.500	31.5	77.6	.0069	1.17		31.4	6
SAE 2345	.125	70.25	125.5	.00226	1.19		72.2	6
SAE 2345	.160	70.75	125.5	.00226	1.17		70.8	6
SAE 2345	.250	66.75	125.5	.00226	1.13		68.9	6
SAE 2345	.300	71.0	125.5	.00226	1.12	60.7	68.2	6
SAE 2345	.500	66.5	125.5	.00226	1.10		66.5	6
SAE 2345	.875	64.0	125.5	.00226	1.07		65.1	6
SAE 2345	1.500	66.5	125.5	.00226	1.05		64.0	6
SAE 4340	.125	82.5	163.5	.0009	1.12		81.8	4
SAE 4340	.250	81	163.5	.0009	1.08		79.2	4
SAE 4340	.500	78	163.5	.0009	1.06	73.0	77.5	4
SAE 4340	1.000	74	163.5	.0009	1.04		76.2	4
SAE 4340	1.750	74	163.5	.0009	1.03		75.5	4

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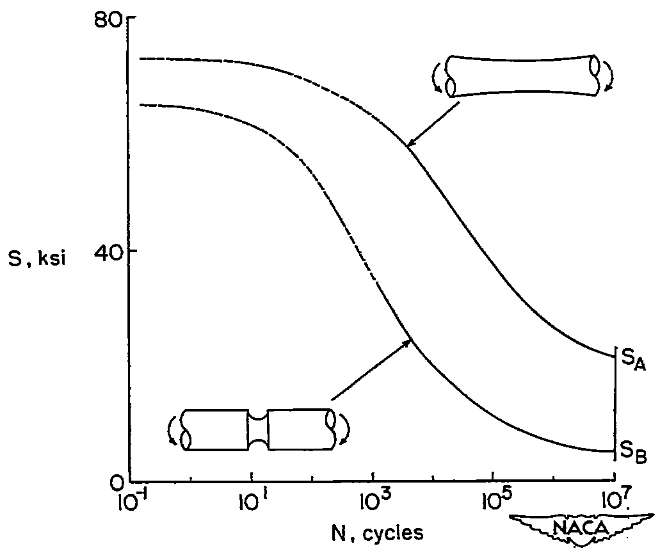


Figure 1.- Typical S-N curves for notched and unnotched specimens.

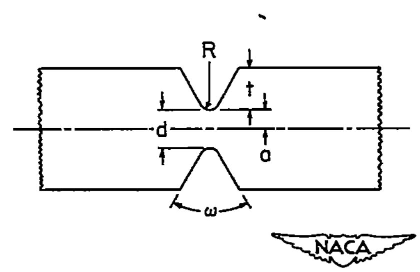


Figure 2.- Symbols defining the geometry of a notched specimen.

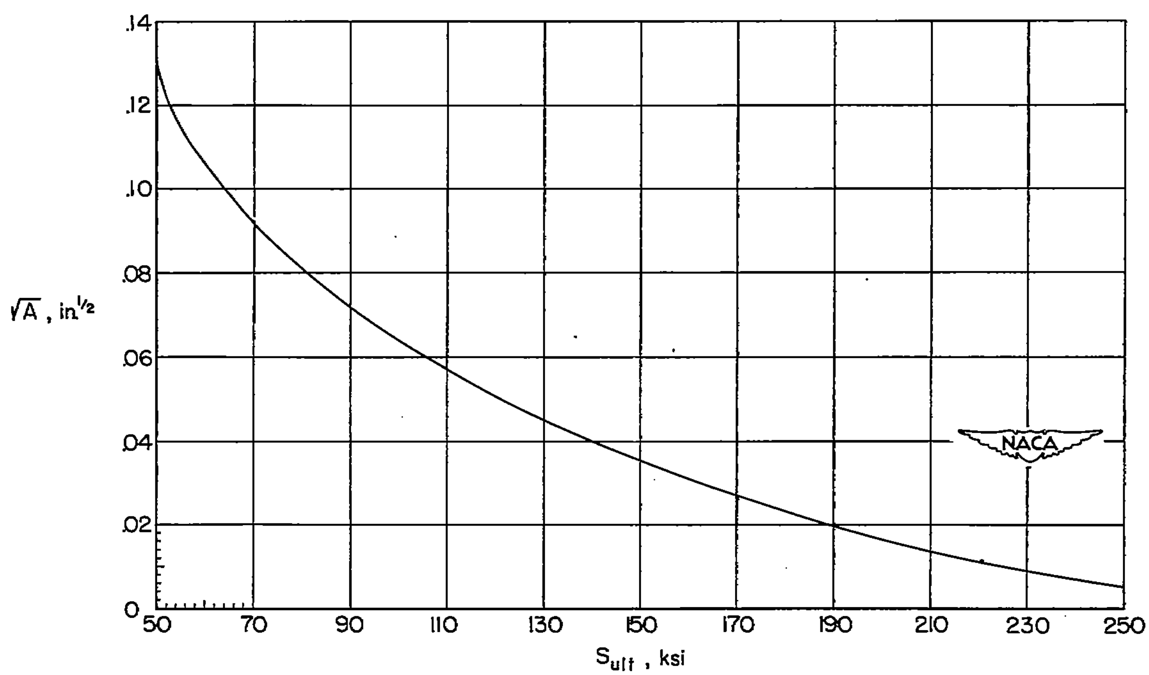


Figure 3.- Proposed relationship between the Neuber constant A and the ultimate strengths of steels.

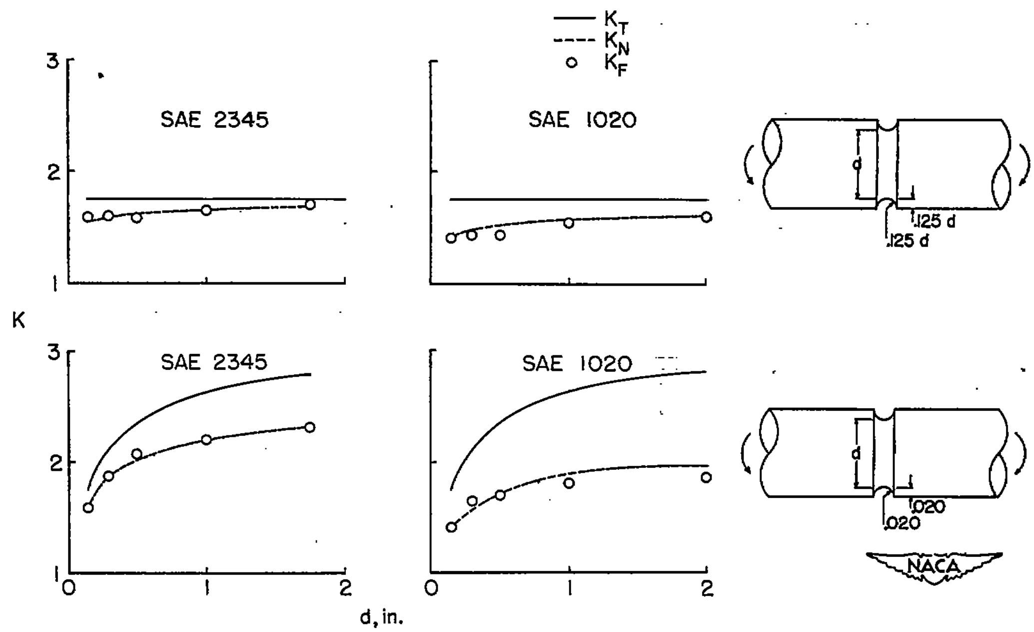


Figure 4.- Typical comparison between theoretical, Neuber, and fatigue factors for rotating beams containing circumferential grooves. (Data taken from ref. 7.)

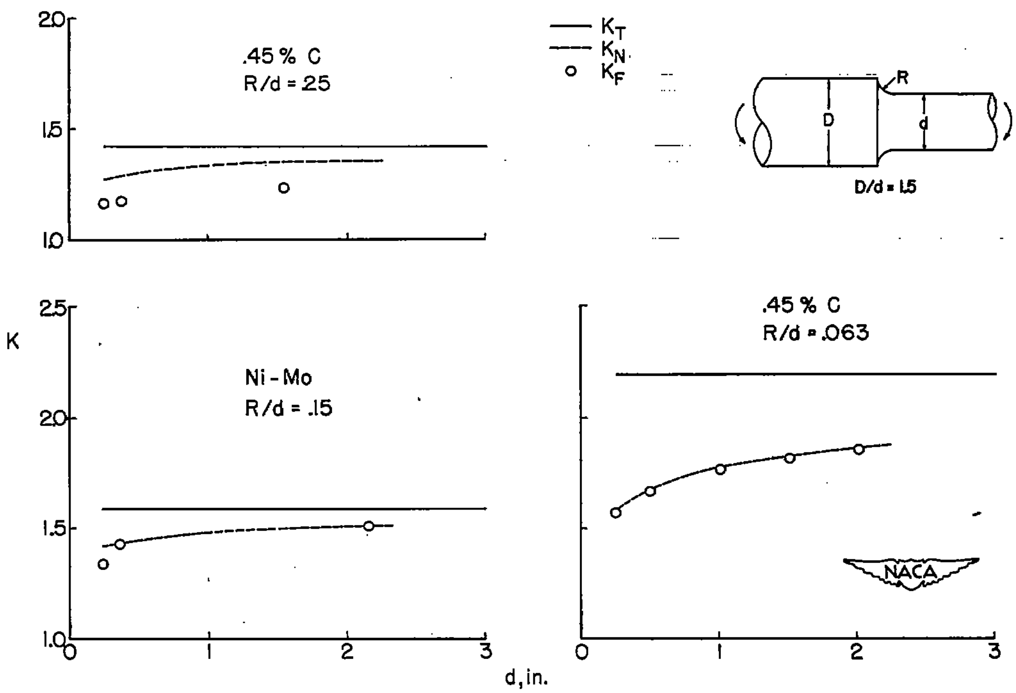


Figure 5.- Typical comparison between theoretical, Neuber, and fatigue factors for rotating beams containing fillets. (Data taken from ref. 10.)

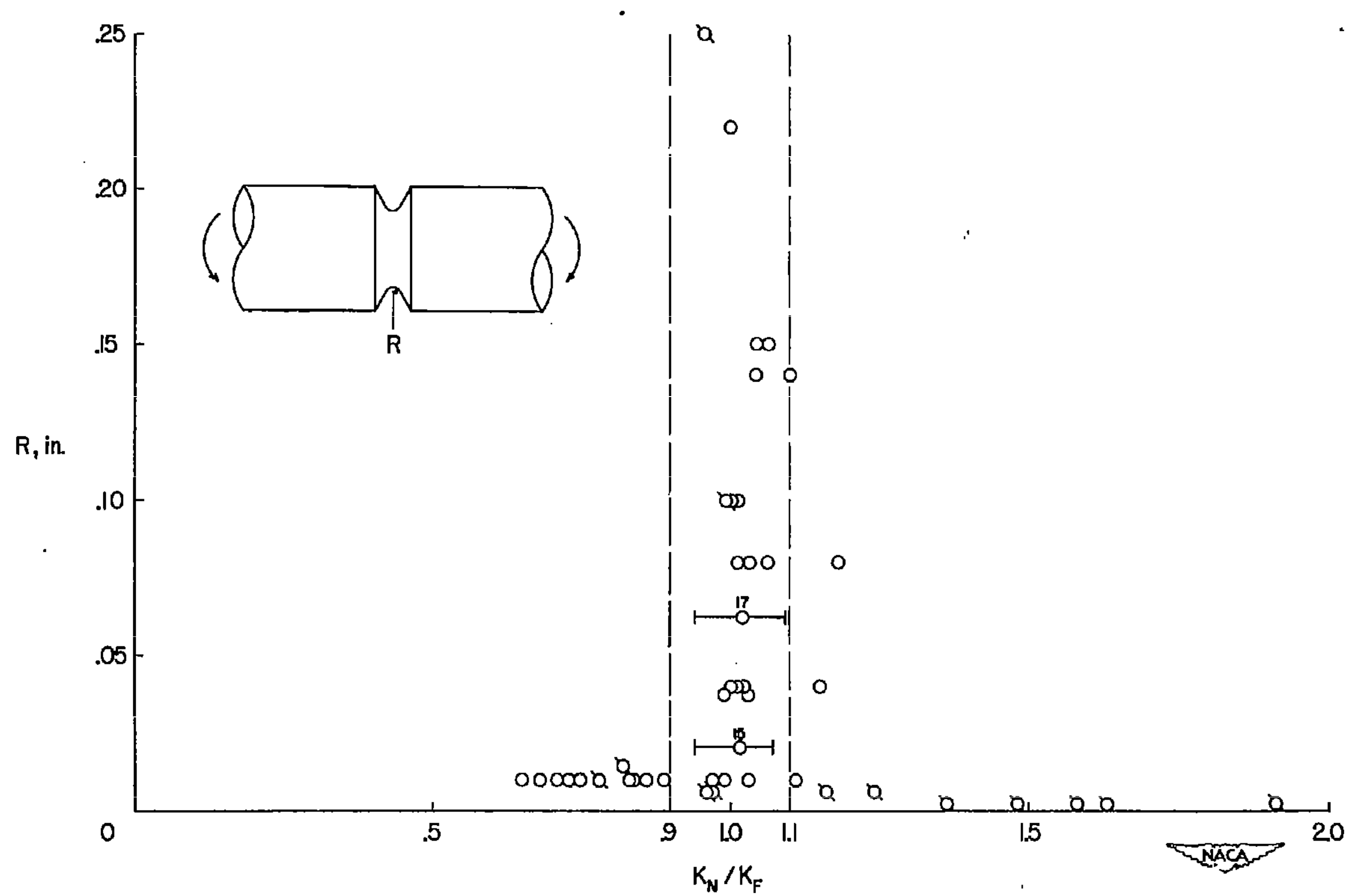


Figure 6.- Comparison between Neuber and fatigue factors for rotating beams containing circumferential grooves. (Points with tails indicate computation of K_N was made on basis of estimated value of ultimate strength. Numbers above symbols indicate the number of points averaged and ticks indicate extreme values in the group.)

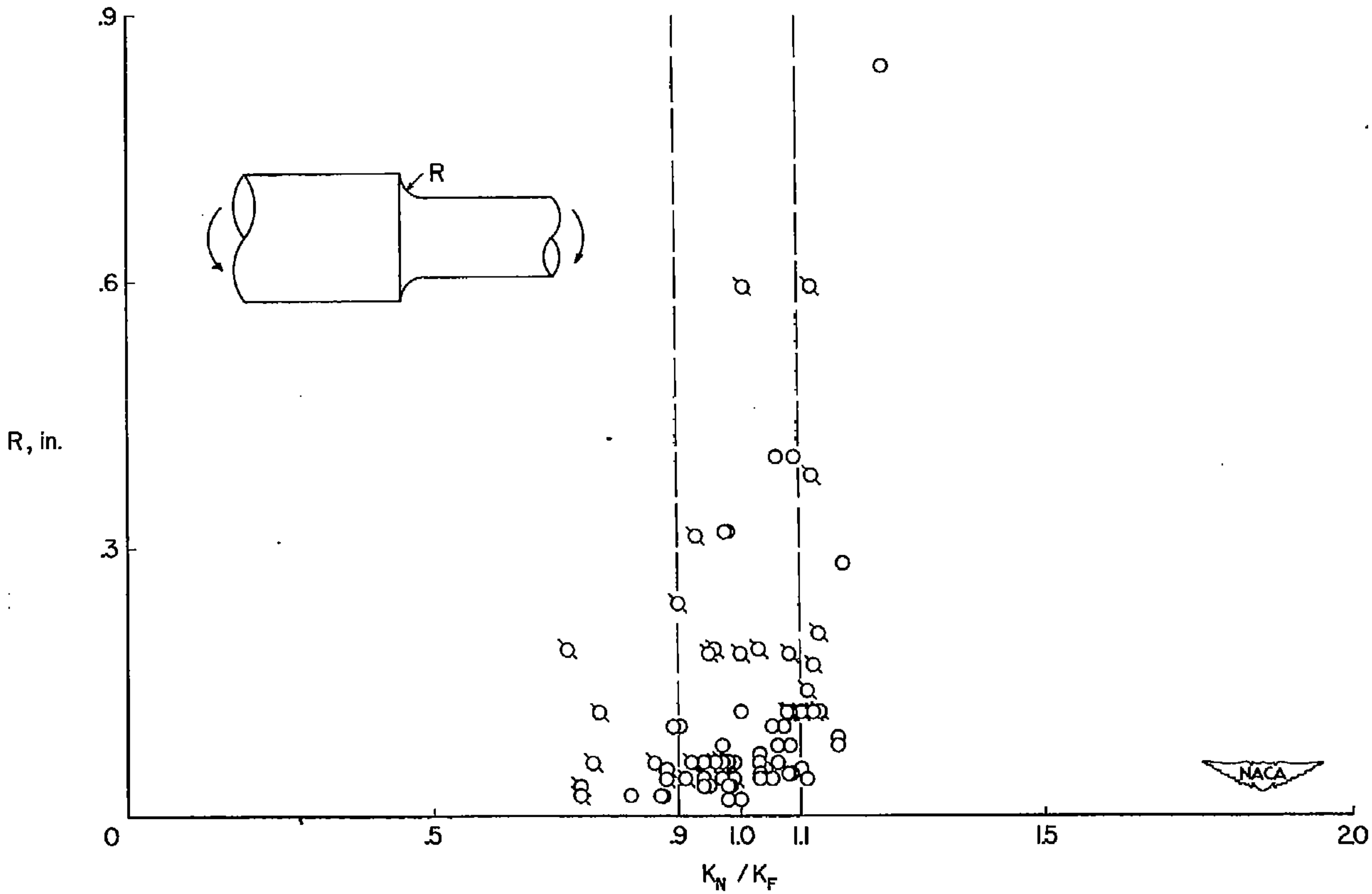


Figure 7.- Comparison between Neuber and fatigue factors for rotating beams containing fillets. (Points with tails indicate computation of K_N was made on basis of estimated value of ultimate strength.)

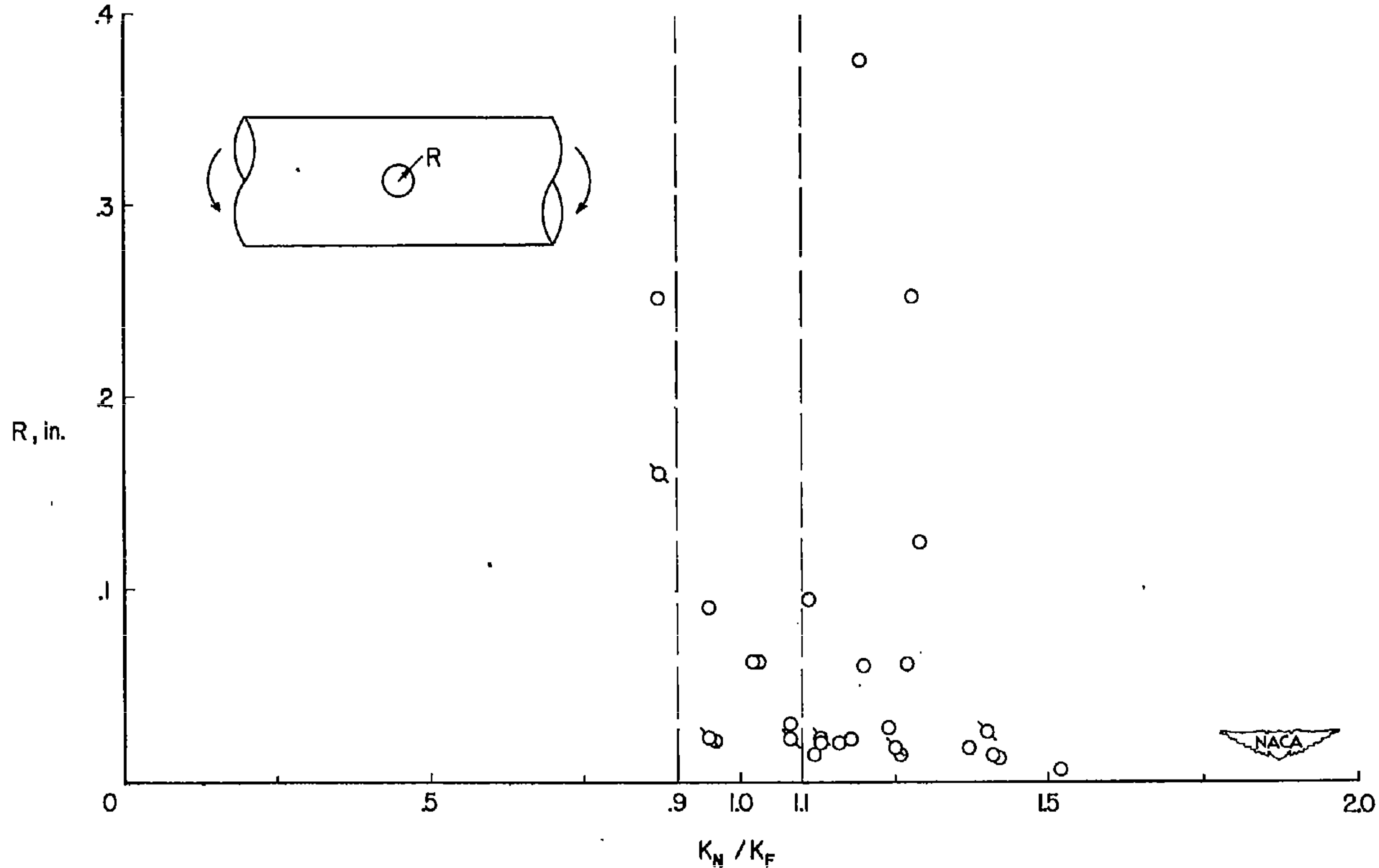


Figure 8.- Comparison between Neuber and fatigue factors for rotating beams containing transverse holes. (Points with tails indicate computation of K_N was made on basis of estimated value of ultimate strength.)

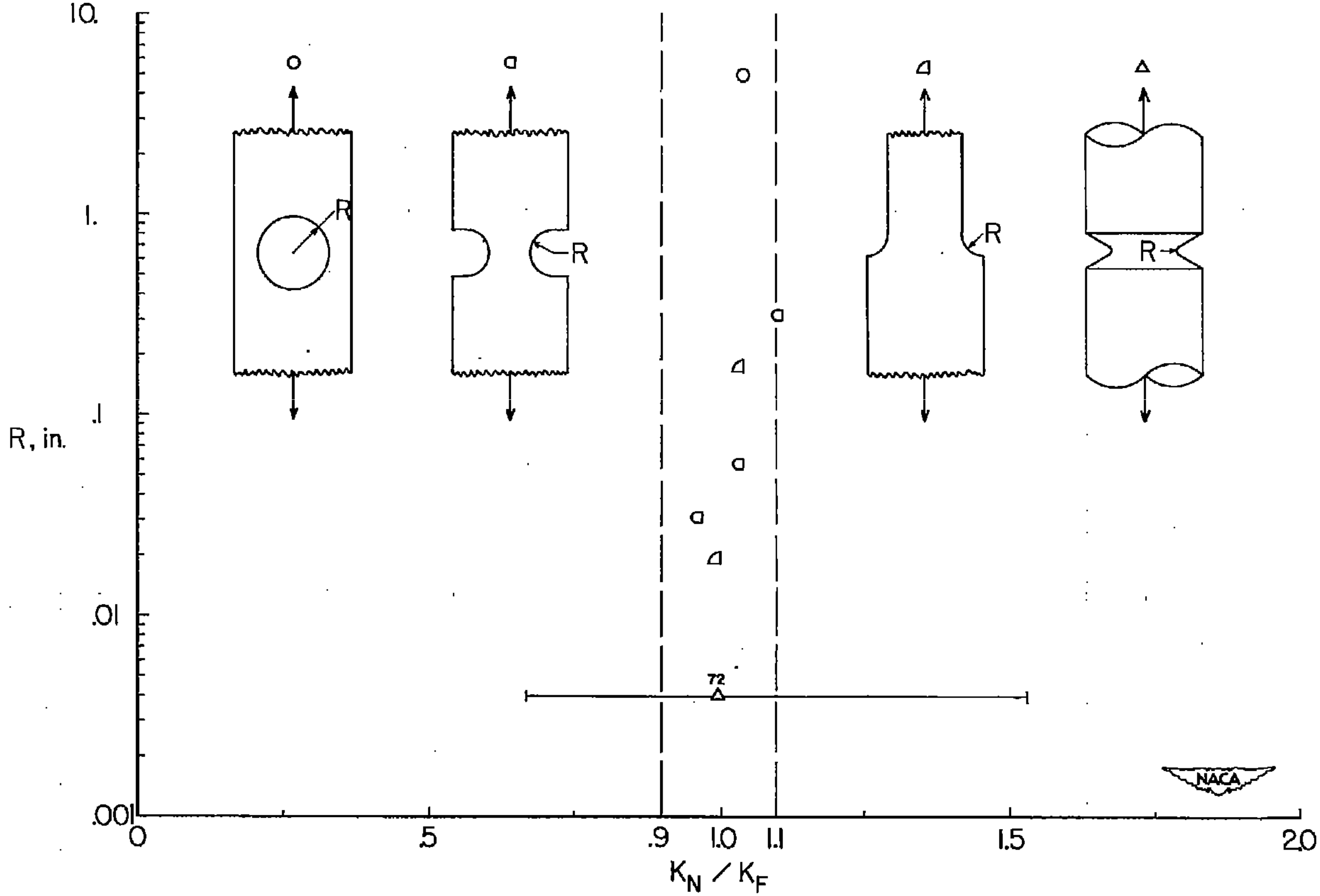


Figure 9.- Comparison between Neuber and fatigue factors for axial load tests. (Numbers above symbols indicate the number of points averaged and ticks indicate extreme values in the group.)

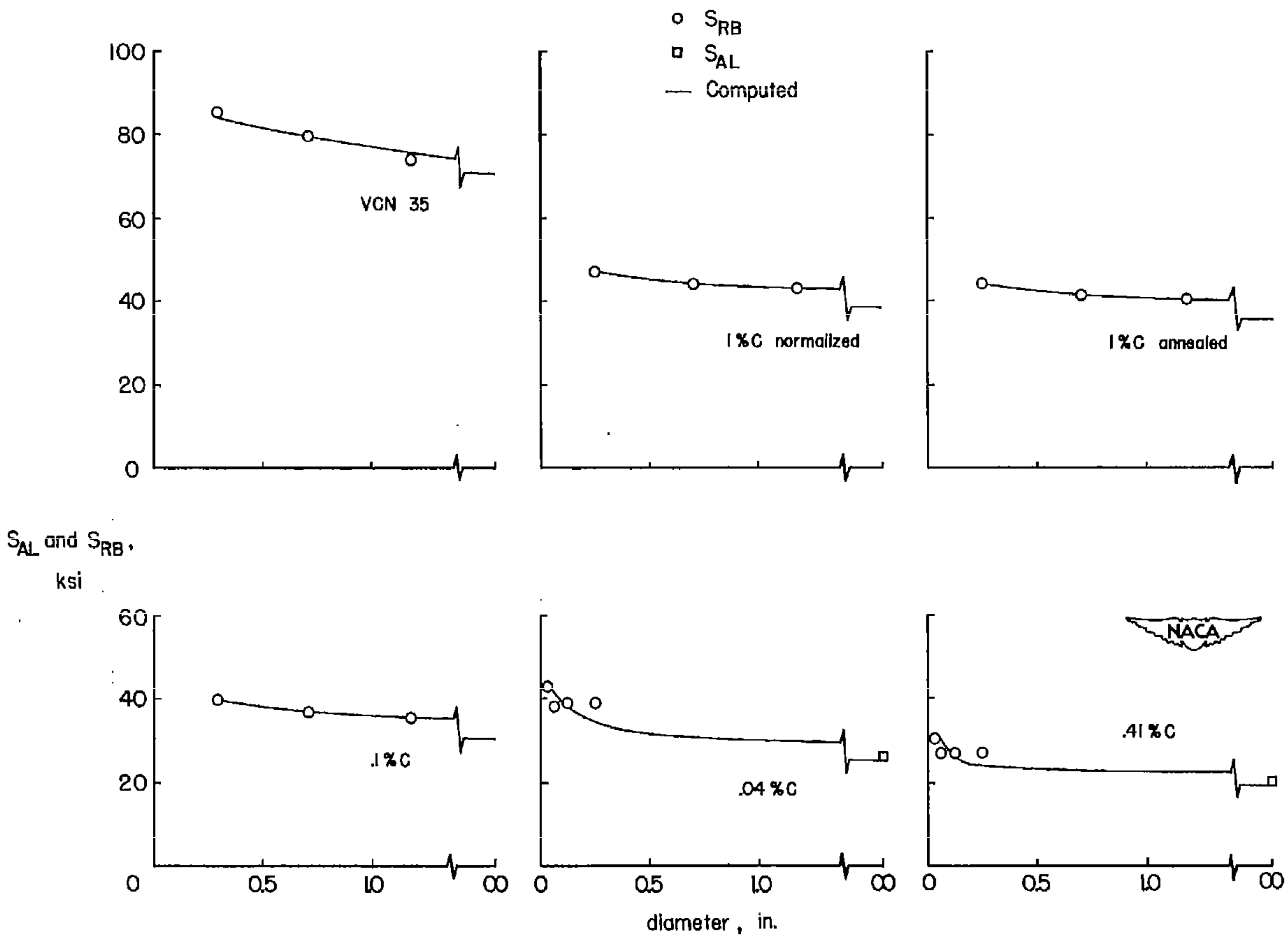


Figure 10.- Comparison of endurance limits for unnotched rotating beams and predictions by formula (B2). (Data from ref. 22.)

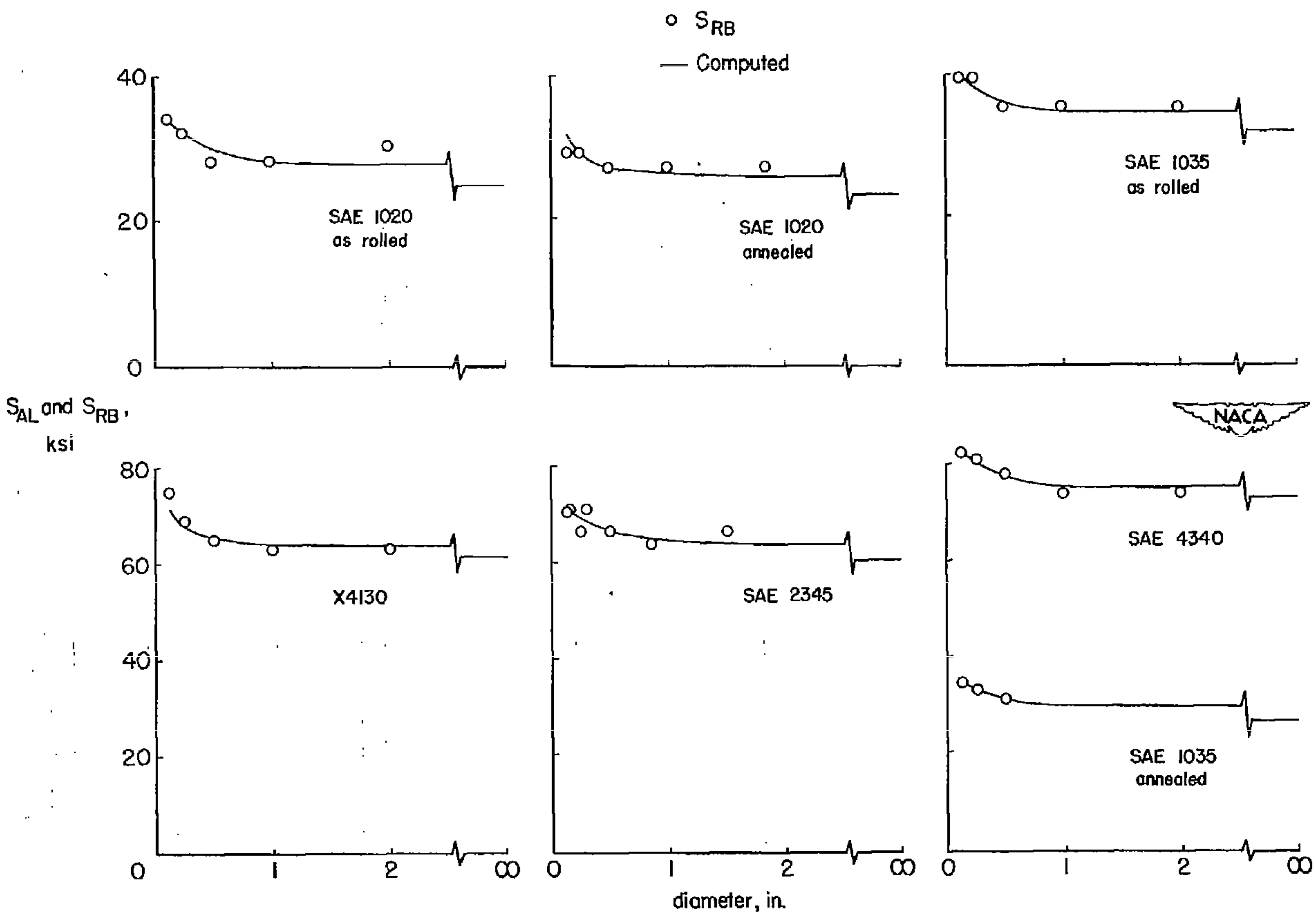
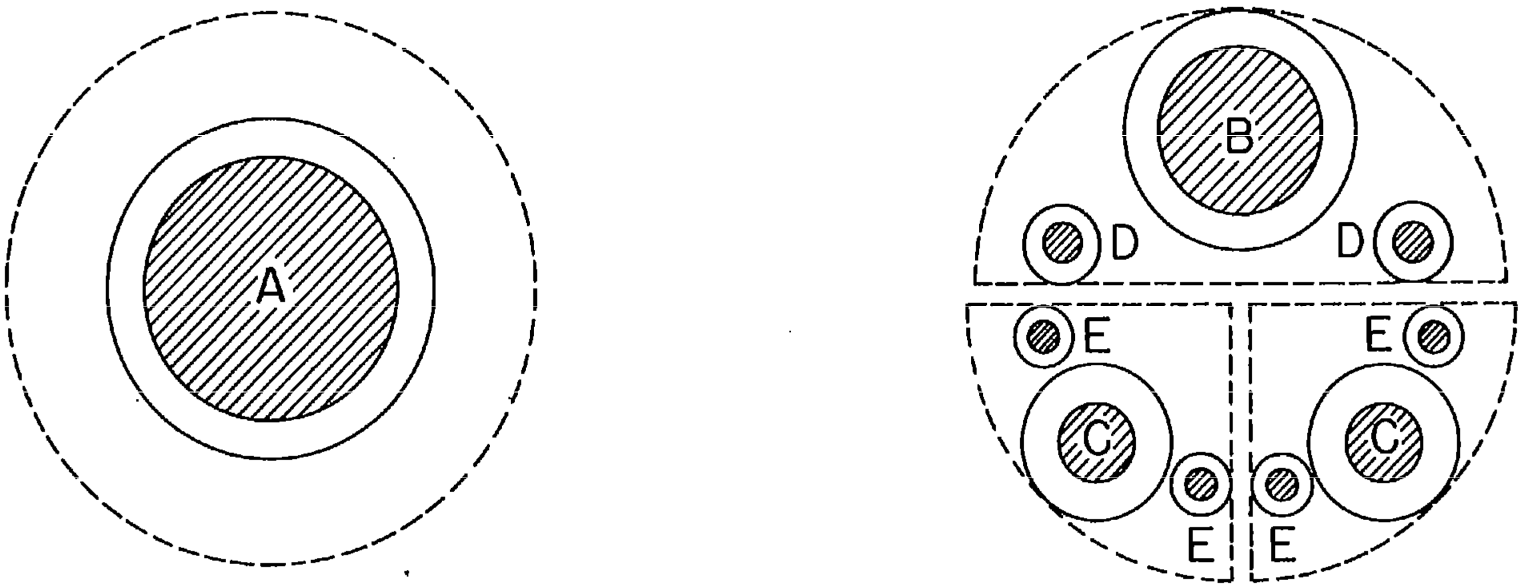


Figure 11.- Comparison of endurance limits for unnotched rotating beams and predictions by formula (B2). (Data from refs. 3, 4, and 6.)



A 1.5 , 1.75 , 1.875 - in. specimens
B 1.0 - in. specimens
C .5 - in. specimens
D .25 - in. specimens
E .125 - in. specimens

Figure 12.- Scheme used by Moore and Morkovin (ref. 3) for cutting specimens from 3-inch-diameter rolled bar stock.

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