NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2964

A REVISED FORMULA FOR THE CALCULATION OF GUST LOADS

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SUMMARY

A revised gust-load formula with a new gust factor is derived to replace the gust-load formula and alleviation factor widely used in gust studies. The revised formula utilizes the same principles and retains the same simple form of the original formula, but provides a more appropriate and acceptable basis for gust-load calculations. The gust factor is calculated on the basis of a one-minus-cosine gust shape and is presented as a function of a mass-ratio parameter in contrast to the ramp gust shape and wing loading, respectively, used for the alleviation factor. The National Advisory Committee for Aeronautics will make use of the revised formula in the evaluation of relevant gust data.

INTRODUCTION

A gust-load formula, embodying a number of simplifying assumptions, has long been used in this country for the calculation of design gust loads on ordinary airplanes by military and civilian regulating agencies (see, for example, ref. 1). This formula was developed and has been utilized by the NACA in the evaluation and interpretation of gust and gust-loads data obtained from measurements of accelerations experienced during routine and some special flights through turbulent air (see, for example, refs. 2 to 4). The formula may be written as

$$\Delta n_{max} = \frac{m\rho_o S V_e U_e}{2W} K$$

where the quantities and customarily used units are as follows:

Δn_{max} airplane maximum nondimensional normal-acceleration increment

m wing lift-curve slope per radian

ρ_o air density at sea level, slugs/cu ft

S wing area, sq ft

V_e equivalent airspeed, ft/sec

U_e "effective" gust velocity, ft/sec

W airplane weight, lb

K dimensionless "alleviation factor"

The formula serves to relate the peak accelerations due to gusts to be expected on a given airplane to the peak accelerations measured on another airplane for flight through the same rough air. The underlying concept is that a measured acceleration increment may be used to derive an "effective" gust velocity which in turn is used to calculate the acceleration on another airplane by reversing the process. The effective gust velocity U_e is not, therefore, a direct physical quantity but is rather a gust-load transfer factor definable in terms of the formula.

The nondimensional parameter K depends on such factors as gust shape and resulting airplane motions. In order to allow for some of these factors and for simplicity in practical application, K has been calculated on the basis that the gust shape is of a ramp type (gust velocity increasing linearly with distance up to a limit of 10 chords) and by taking into account effects of gust penetration and of the resulting vertical motion of the airplane. A small adjustment was then made to the K curve on the basis of model tests and analyses to allow for over-all effects of pitching motion on the normal-acceleration increment. The correction made implied that on all aircraft the acceleration is affected to about the same degree by the pitching motion, this assumption being reasonable only for conventional aircraft having satisfactory flying qualities. On this basis, K is dependent only on a nondimensional mass-ratio parameter which is defined by the mass of the airplane divided by that of a cylinder of air about the wing. For design purposes, however, K was expressed in terms of wing loading and was normalized by dividing by its value for $W/S = 16$ lb/sq ft. This procedure had two effects which now can be considered undesirable. The use of wing loading rather than mass ratio ignored certain effects of altitude and airplane size, and the normalization produced effective gust velocities that are not referred directly to the maximum velocity of the ramp profile but rather to a constant times this value.

Over the years, the alleviation factor K has been modified by the various regulating agencies in their design requirements. As a result, there now exist several different alleviation factors and correspondingly different design gust velocities. This situation has resulted in some confusion when the design gust velocities used by the various agencies are compared with each other or with NACA gust data.

In order to provide for uniformity of gust-load calculations, the interested regulating agencies and the NACA, at a meeting of the ANC-1 Panel on Flight Loading Conditions, agreed to the desirability of adopting a new standard alleviation factor. This new alleviation factor to be referred to as "gust factor" was to be based on the more fundamental parameter, mass ratio, instead of wing loading and also on a new gust profile represented by a one-minus-cosine curve.

The NACA agreed to calculate the new gust factor and to use it in a revised gust-load formula for the reduction of relevant gust data. A point of interest is that the new gust factor as calculated is not normalized to any given value and hence the gust velocity can be conveniently referred directly to the maximum of the gust profile. It is the purpose of this paper to present the revised gust-load formula with the new gust factor.

SYMBOLS

$C_{L_g}(s)$ transient lift response to penetration of sharp-edge gust

$C_{L_\alpha}(s)$ transient lift response to unit-jump change in angle of attack

c mean geometric wing chord, $\frac{\text{Wing area}}{\text{Wing span}}$, ft

g acceleration due to gravity, ft/sec^2

H gust-gradient distance (horizontal distance from zero to maximum gust velocity), chords

K alleviation factor, defined in reference 2

K_g gust factor (revised alleviation factor)

m slope of lift curve per radian

M airplane mass, slugs

Δn nondimensional vertical- or normal-acceleration increment, $\frac{d^2z}{dt^2}\Big/g$

Δn_s reference nondimensional vertical- or normal-acceleration increment, $\frac{m\rho SVU}{2W}$

S	wing area, sq ft
s	distance of penetration into a gust, chords
s_1	dummy variable in superposition integral, chords
t	time, sec
t_1	dummy variable in superposition integral, sec
U	gust velocity (maximum value), fps
U_{de}	"derived" gust velocity, fps
U_e	effective gust velocity defined in reference 2, fps
u	gust velocity at any penetration distance, fps
V	airspeed, fps
V_e	equivalent airspeed, $V\sigma^{1/2}$, fps (see ref. 5)
W	airplane weight, lb
z	airplane vertical displacement (positive upward), ft
μ_g	airplane mass ratio (sometimes referred to as "mass parameter" in the past), $\frac{2W}{m\rho cgS}$
ρ	air density, slugs/cu ft
ρ_o	air density at sea level, slugs/cu ft
σ	air-density ratio, ρ/ρ_o

Subscript:

max	maximum value

REVISED GUST-LOAD FORMULA

The revised gust-load formula to be derived herein, like the original formula, was obtained from solutions of an equation of airplane vertical motion in an isolated gust. The use of the formula to transfer accelerations from one airplane to another for continuous rough air implies, therefore, the assumption that the relative loads for single isolated gusts are a measure of the relative loads in a sequence of gusts. In regard to this assumption, it is recognized that some of the more recent methods for analysis of airplane loads in continuous rough air with proper allowance for various degrees of freedom of airplane motion may in due course be adopted; however, for the present, it remains desirable to retain the simplicity of the original method. As in the case of the original formula, the present method will not be suitable for all airplane configurations. Unusual airplanes will require special analysis. After the presentation of the revised gust-load formula, a brief comparison of features of the original and revised formulas is given.

Basic Assumptions and Equation of Motion

The equation of motion is based on the following assumptions commonly used in gust-load problems:

(1) The airplane is a rigid body.

(2) The airplane forward speed is constant.

(3) The airplane is in steady level flight prior to entry into the gust.

(4) The airplane can rise but cannot pitch.

(5) The lift increments of the fuselage and horizontal tail are negligible in comparison with the wing lift increment.

(6) The gust velocity is uniform across the wing span and is parallel to the vertical axis of the airplane at any instant.

Disregarding the forces associated with steady level flight, a summation of vertical or normal forces on the airplane in a gust yields the following equation of motion:

$$M \frac{d^2z}{dt^2} + \frac{\rho}{2} V^2 Sm \int_0^t \frac{1}{2\pi} C_{L_\alpha}(t - t_1) \frac{d^2z}{dt_1^2} \frac{1}{V} dt_1 =$$

$$\frac{\rho}{2} V^2 Sm \frac{U}{V} \int_0^t \frac{1}{2\pi} C_{L_g}(t - t_1) \frac{d\left[\frac{u(t_1)}{U}\right]}{dt_1} dt_1 + \frac{\rho}{2} V^2 Sm \frac{U}{V} \frac{u(0)}{U} \frac{1}{2\pi} C_{L_g}(t) \qquad (1)$$

In equation (1), the first term on the left-hand side is the inertia reaction and the second term is the damping force due to airplane vertical velocity. On the right-hand side, both terms are forces due to the gust; the first term is the force due to a gust having zero velocity at the beginning of penetration by the airplane and the second term is the force due to a gust having an initial velocity other than zero at the beginning of penetration.

By using the relationships $\frac{d^2z}{dt^2} = \Delta n\, g$ and $t = \frac{sc}{V}$, equation (1) can be written in nondimensional form as

$$\frac{\Delta n(s)}{\Delta n_s} + \frac{1}{\mu_g} \int_0^s \frac{1}{2\pi} C_{L_\alpha}(s - s_1) \frac{\Delta n(s_1)}{\Delta n_s} ds_1 =$$

$$\int_0^s \frac{1}{2\pi} C_{L_g}(s - s_1) \frac{d\left[\frac{u(s_1)}{U}\right]}{ds_1} ds_1 + \frac{u(0)}{U} \frac{1}{2\pi} C_{L_g}(s) \qquad (2)$$

where

$$\Delta n_s = \frac{m\rho SVU}{2W} \qquad (3)$$

$$\mu_g = \frac{2W}{m\rho cgS} \qquad (4)$$

and the functions C_{L_g} and C_{L_α} are the transient lift responses of a wing to a penetration of a sharp-edge gust and to a unit-jump change in angle of attack, respectively. In equation (2), Δn is the vertical-acceleration increment that results from the gust and Δn_s is a convenient reference acceleration increment which may be interpreted as the acceleration increment that would result solely from a lift force equal to the steady-state lift associated with the maximum velocity of the gust. The second term is associated with the damping due to the airplane vertical velocity and the remaining terms are associated directly with the gust. It can be remarked that the mass ratio μ_g is a basic parameter in equation (2).

Solution of the Equation of Motion

Equation (2) was solved for histories of the acceleration ratio $\Delta n(s)/\Delta n_s$ on the basis of the following transient lift functions and gust shape.

The transient lift functions used were

$$\frac{1}{2\pi} C_{L_\alpha}(s) = 1.000 - 0.165e^{-0.090s} - 0.335e^{-0.600s} \tag{5}$$

$$\frac{1}{2\pi} C_{L_g}(s) = 1.000 - 0.236e^{-0.116s} - 0.513e^{-0.728s} - 0.171e^{-4.84s} \tag{6}$$

These are the transient lift functions for infinite aspect ratio given in reference 6, normalized to asymptotic values of unity. These expressions, rather than finite-aspect-ratio functions (such as those given in ref. 6), were used for simplicity in order to provide solutions of the equation of motion independent of aspect ratio except of course as aspect ratio affects the slope of the lift curve. Thus in effect only the shapes of the infinite-aspect-ratio functions are used, the appropriate finite-aspect-ratio lift-curve slope being used in evaluating the mass ratio μ_g. The results obtained through the use of equations (5) and (6), however, are probably less than 5 percent different from the results that would be obtained through the use of the finite-aspect-ratio functions, as indicated by some limited information in reference 7. This reference also indicates that the differences might be slightly larger when the transient lift functions for a Mach number of 0.7 are used.

The gust shape used was that designated by the ANC-1 Panel, that is,

$$\left.\begin{aligned} \frac{u(s)}{U} &= \frac{1}{2}\left(1 - \cos\frac{\pi s}{H}\right) = \sin^2\frac{\pi s}{2H} && (0 < s < 2H) \\ \frac{u(s)}{U} &= 0 && (0 > s > 2H) \end{aligned}\right\} \qquad (7)$$

where H was designated equal to 12.5 chords. (Inasmuch as the initial portion of the revised gust profile is relatively ineffective, the gradient distance of 12.5 chords corresponds roughly to the 10-chord gradient distance for the original ramp profile.)

With these lift functions and gust shape, equation (2) is noted to depend on only one parameter, the mass ratio μ_g. Solutions of the equation were obtained for a range of μ_g, by the numerical recurrence method presented in reference 8 for the case of a rigid airplane. Although solutions of the equation also can be obtained in closed form when equations (5) and (6) are used, the numerical method was chosen because it is much more rapid, is easy to apply, and gives good accuracy (error in $\Delta n/\Delta n_s$ less than ± 0.005). Sample histories of the calculated acceleration ratio for three different values of μ_g are presented in figure 1.

Revised Gust Factor and Gust-Load Formula

Since the maximum value of $\Delta n/\Delta n_s$ with respect to gust penetration distance (see fig. 1) defines the maximum acceleration experienced by the airplane, it is of primary concern in design. This maximum value is herein designated as the "gust factor" and is labeled K_g, that is,

$$\left(\frac{\Delta n}{\Delta n_s}\right)_{max} = K_g \qquad (8)$$

The variation of this "gust factor" with mass ratio is shown in figure 2. No closed-form analytical expression for the curve of K_g can be written, since it was obtained by a numerical procedure. A convenient expression which closely approximates the curve was found, however, namely

$$K_g = \frac{0.88\mu_g}{5.3 + \mu_g} \tag{9}$$

This simple expression gives K_g with an error less than ±0.01.

The revised gust-load formula follows directly from equation (8), that is,

$$\begin{aligned}\Delta n_{max} &= \Delta n_s K_g \\ &= \frac{m\rho SVU}{2W} K_g \end{aligned} \tag{10}$$

In terms of equivalent speeds this equation becomes

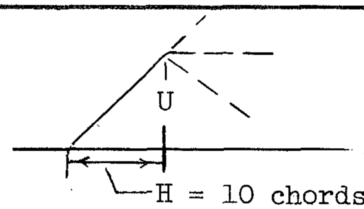
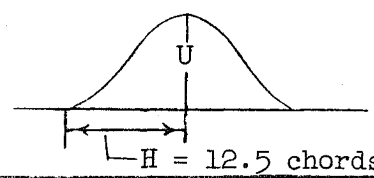
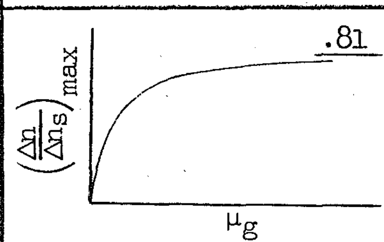
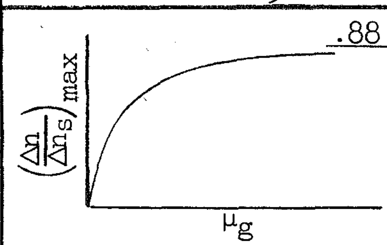
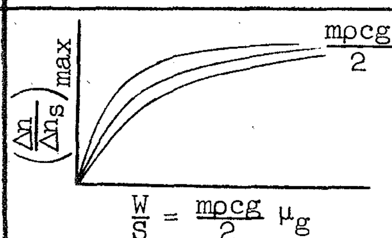
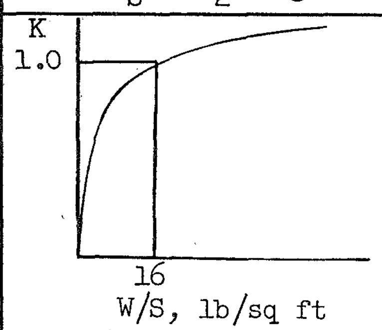
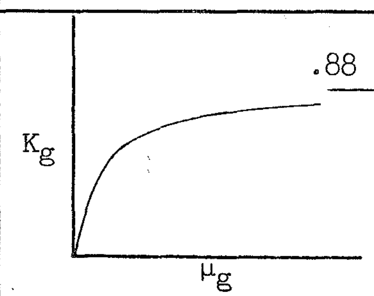
$$\Delta n_{max} = \frac{m\rho_o SV_e U_{de}}{2W} K_g \tag{11}$$

where the subscript e is used to denote that both the airspeed and gust velocity are equivalent speeds. The subscript d has been added also to the gust velocity to denote that, when the formula is used to evaluate gust velocities from measured accelerations, the gust velocities obtained, like U_e in the original formula, are "derived" not measured values. For application in design, however, U_{de} may of course be a stipulated value.

The revised gust-load formula, equation (11), may be noted to be of the same form as the original formula, the gust factor K_g being in effect a revision of the alleviation factor K. A further comparison of the original and new formulas is given in the subsequent section.

COMPARISON OF ORIGINAL AND REVISED GUST-LOAD FORMULAS

The salient features of the revised gust-load formula as compared with those of the original formula are illustrated in the following table: (The description of the original curve to follow is schematic in nature and is not intended to be sufficiently detailed to permit reproduction of the curve.)

Item	Original	Revised
(a) Gust-load formula	$\Delta n_{max} = \frac{m\rho_o S V_e U_e}{2W} K$	$\Delta n_{max} = \frac{m\rho_o S V_e U_{de}}{2W} K_g$
(b) Gust shape		
(c) Maximum acceleration ratio in gust		
(d) Maximum acceleration ratio plotted against wing loading (corrected for pitch effects)		— — — — —
(e) Alleviation and gust factors		
(f) Gust velocity	$U_e \propto U\sigma^{1/2}$	$U_{de} = U\sigma^{1/2}$

As item (a) shows, the forms of the original and the revised formulas are the same. The respective gust shapes are shown as item (b). The original gust shape was of a ramp type but was effectively undefined beyond a gradient distance of 10 chords as a consequence of an approximation made in solving the equation of motion. This approximation made use of the value of the acceleration increment at a penetration distance of 10 chords as the maximum acceleration increment if the actual maximum did not occur within this distance. The revised gust profile, in comparison, is symmetrical in shape, finite in length, and has a gust gradient distance of 12.5 chords.

For item (c), curves of maximum acceleration ratio $\left(\frac{\Delta n}{\Delta n_s}\right)_{max}$ (ratio of the maximum acceleration increment in the gust to the reference acceleration increment Δn_s) associated with the respective gust shapes are given as a function of mass ratio. As previously mentioned, the original alleviation-factor curve described in reference 2 was not used in terms of mass ratio but rather in terms of the convenient design parameter, wing loading. This use implies a separation of the left-hand curve of item (c) into a family of curves involving the parameter $m\rho cg/2$ as indicated for item (d). The alleviation factor was obtained from this family as a single curve which was not, however, a particular curve of the family but was obtained from the entire family on the basis of various engineering considerations. These considerations included

(1) An assumed variation of wing chord with wing loading

(2) An allowance, based on experiment and analysis, for the effects of pitching motion, consisting of a constant percentage correction to the maximum acceleration ratio (that is, multiplication of $(\Delta n/\Delta n_s)_{max}$ in item (d) by a constant factor)

(3) Normalization of the curve to unity at $W/S = 16$ lb/sq ft

The alleviation-factor curve thus obtained is shown as item (e).

Although the use of the single alleviation-factor curve K based on wing loading does not fully account for variations in the parameter $m\rho cg/2$, at the time of derivation it was considered representative of airplane design and operating practice. At the present time, however, the variations of $m\rho cg/2$ have increased to the point where a single curve based on wing loading cannot be considered representative. In the light of modern airplane practice, it is now desirable to revert to a single curve for the gust factor which is based on the less restrictive and more fundamental parameter, mass ratio. The gust-factor curve K_g is shown on the right-hand side of item (e); it is the same as that in item (c).

The original formula, as indicated in reference 2, has been subject to scrutiny in the form of continuing experiments in regard to usefulness for conventional airplanes and in regard to the effects of various other factors not explicitly taken into account in its derivation. This background of experience can be carried over in the use of the revised formula as well. In the same vein, the allowance for effects of pitching motion made in the derivation of the alleviation factor but not explicitly taken into account in the derivation of the gust factor nevertheless can be included in the use of the gust factor. The pitch correction was not directly applied to the gust factor because it would cancel out of calculations relating the acceleration of one airplane to that of another airplane.

As mentioned earlier, in the use of the formulas to evaluate measured accelerations, the derived gust velocity U_{de} and the effective gust velocity U_e are both derived rather than measured quantities. They differ, however, as indicated by item (f), in that U_{de} corresponds to the maximum equivalent velocity of the gust shape whereas U_e corresponds to only a fraction of the maximum equivalent velocity of the original gust shape. This fraction stems from the value used to normalize the alleviation-factor curve at $W/S = 16$ lb/sq ft. There is no single constant proportional relationship between U_{de} and U_e for all airplanes because of their respective mass-ratio and wing-loading bases. However, it might be noted that the ratio of U_{de} to U_e is of the order of 2:1, being, for example, 55:30 for $W/S = 16$ lb/sq ft and $\mu_g = 8$.

CONCLUDING REMARKS

A revised gust-load formula with a new alleviation factor termed "gust factor" has been derived herein to replace the gust-load formula widely used for design and gust studies. The revised formula, which is similar in form to the original formula, will be used by the NACA in the evaluation of relevant gust data. A brief comparison of the features of the two formulas has also been presented.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., April 20, 1953.

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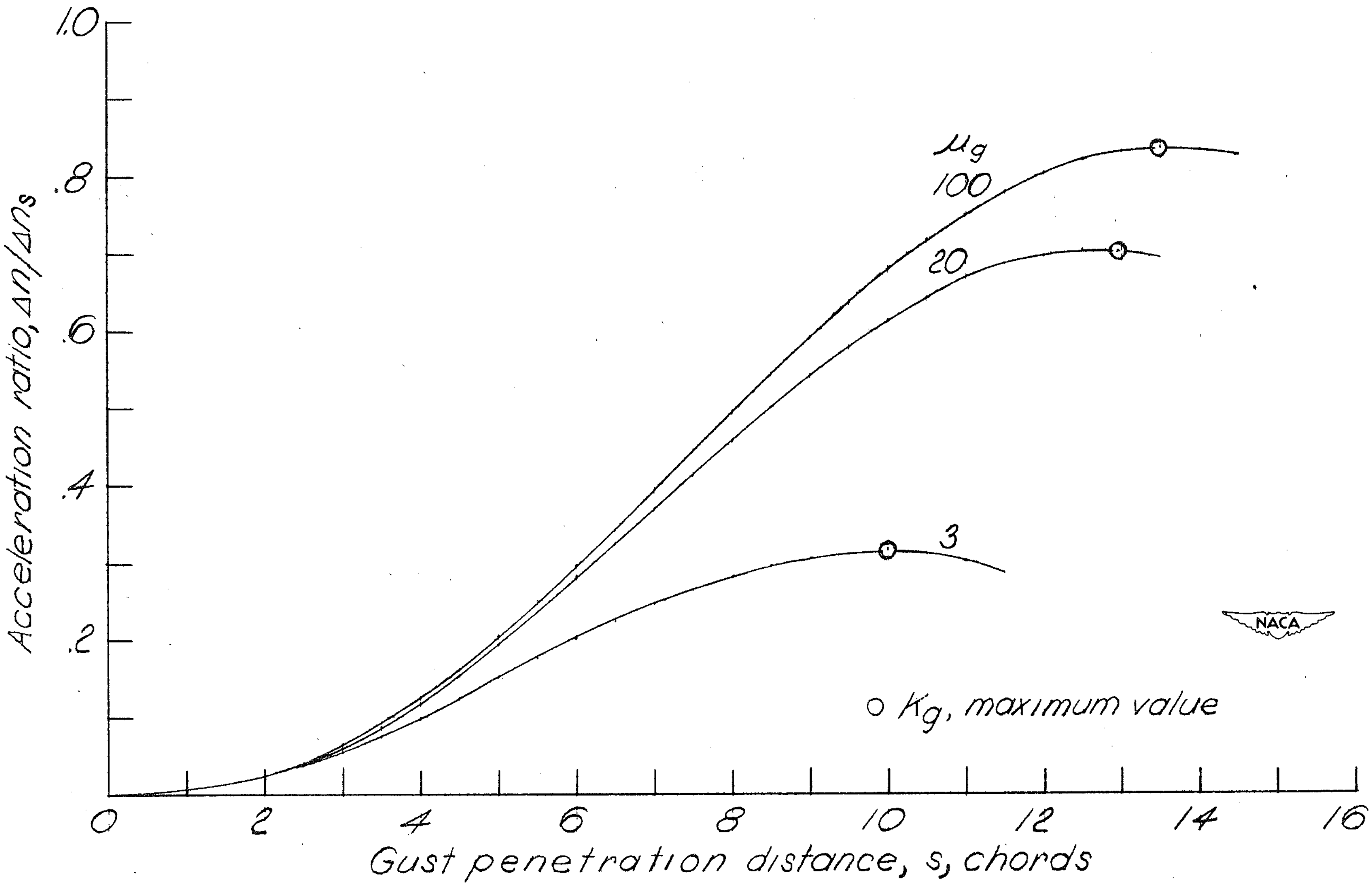

Figure 1.- Representative histories of acceleration ratio.

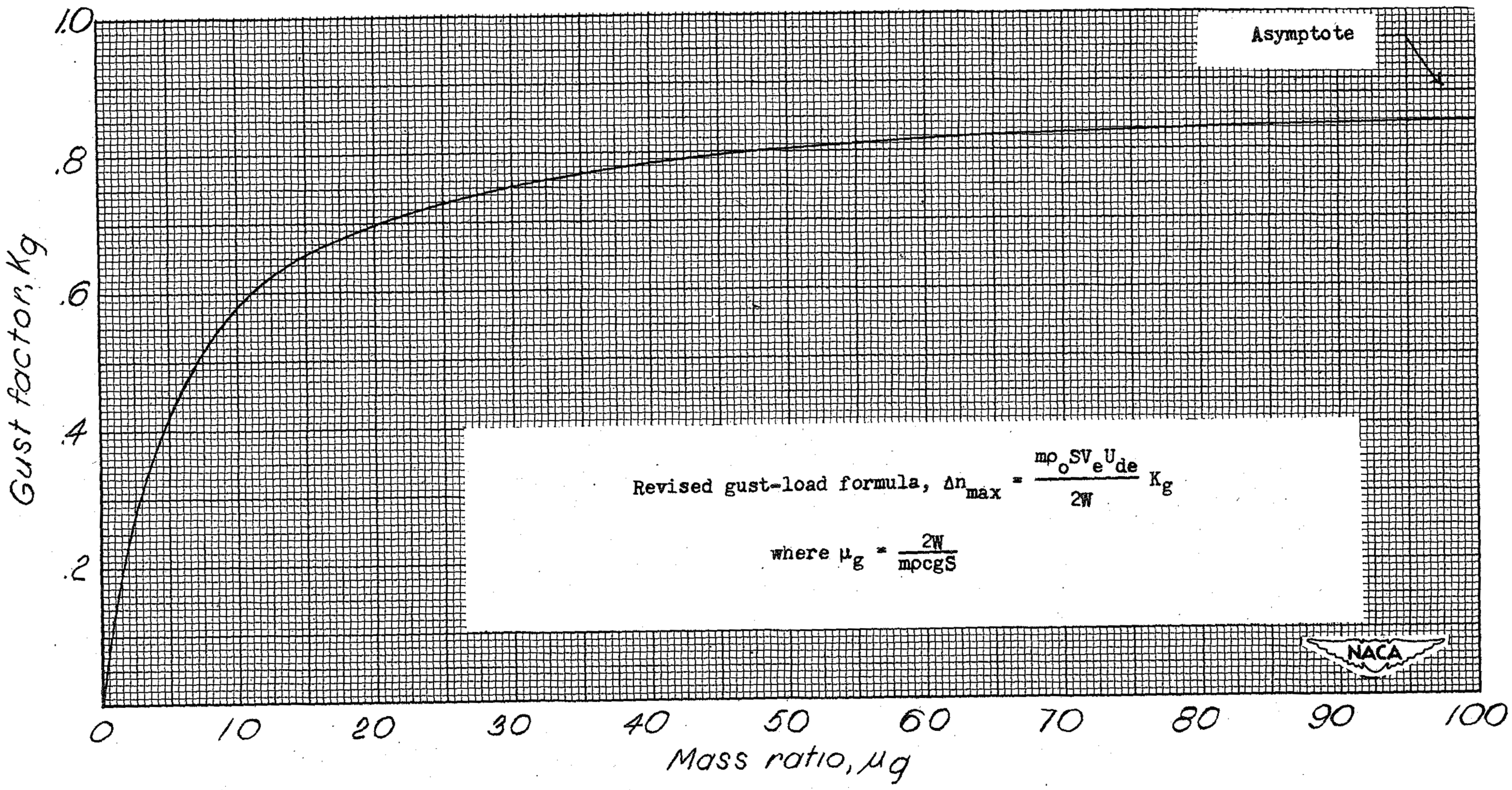

Figure 2.- Gust factor K_g as a function of mass ratio for standard gust shape $\frac{u}{U} = \frac{1}{2}\left(1 - \cos \frac{\pi s}{12.5}\right)$.

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