TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

[redacted]

No. 201

# WIND DRIVEN-PROPELLERS (or "WINDMILLS").

By Max M. Munk.

From Zeitschrift für Flugtechnik und Motorluftschiffahrt,
August 15, 1920.



April, 1923.

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# WIND DRIVEN PROPELLERS (or "WINDMILLS").*

By Max M. Munk.

Wind-driven propellers are much used as small sources of power, e.g. for radio instruments. In principle, they are nothing but developments of windmills of smaller scale. They differ considerably from ordinary windmills, however, because they are constructed from other view-points. In a windmill, the problem is to obtain the greatest possible horsepower at the lowest cost, it being a matter of indifference as to how much wind is utilized, for the wind costs nothing. In employing windmills on airplanes, however, all unnecessary retarding of the airplanes must be avoided and the efficiency of such windmills can by no means be disregarded. Since, moreover, in the employment of these windmills, the velocity of the wind is nearly constant and is known beforehand, it is worth while to establish the principles involved and to get acquainted with the rules for the design of such windmills.

The transfer of energy from the moving air to the windmill (regarded stationary), is accomplished chiefly by that portion of the air passing through the propeller disk. This circle has the area $S = \pi D^2/4$, in which $D$ is the diameter of the windmill. The air is retarded, not simply while passing through the disk, but also when in front and behind the disk. The original veloc-

* From Zeitschrift für Flugtechnik und Motorluftschiffahrt, August 15, 1920, pp. 220-223.



ity of the air at some distance in front of the windmill (regarded as stationary) may be denoted by $V$, which is equal to the speed of the airplane. While passing through the windmill disk, the velocity of the air may be reduced to the magnitude $V_m$. The velocity is still further reduced, after having passed through the windmill, to a minimum value, denoted by $V_2$. The velocities at particular points may differ somewhat from these average values. The air passing through the windmill per second has a volume $SV_m$ and a mass $SV_m\rho$, in which $\rho$ denotes the density of the air. Therefore less air is passing through the disk than would be the case without the presence of the windmill, since the latter obstructs the air stream to a certain degree. Numerical statements can be made regarding the measure to which this occurs, as soon as the magnitude of the thrust (i.e. the resistance of the windmill) is known or assumed to be known. Between $V$, $V_m$ and $V_2$ there is always, independent of this force, a very simple relation, which also holds true in the theory of the propeller. The air flowing through the windmill transfers, per unit of time, the energy

$$P = S\ V_m\ \rho/2\ [V^2 - V_2^2] \qquad (1)$$

to the windmill, since its own kinetic energy is diminished by this amount. It therefore exerts at the same time a thrust

$$T = S\ V_m\ \rho\ [V - V_2] \qquad (2)$$

on the windmill, since the latter is the diminution of its momentum. This thrust, multiplied by the length of path, will also give the work. We thus obtain the equation

$$S\, V_m \frac{1}{2}\rho\, [V^2 - V_2^{\,2}] = S\, V_m\, \rho\, [V - V_2] \qquad (3)$$

and

$$V_m = \frac{1}{2}\, [V + V_2] \qquad (4)$$

The velocity of flow is therefore always the arithmetic mean between the initial and the final velocity. The latter depends on the magnitude of the thrust. From equation (3) we obtain directly its relation to the flight speed

$$V_2/V = \sqrt{1 - \frac{T}{S\, \frac{1}{2}\, \rho\, V^2}} \qquad (5)$$

This equation furnishes a basis for the choice of proper coefficients, since evidently the relation of the final to the initial velocity, i.e. the degree of braking is decisive for the principal relations regarding the energy transfer. The expression $\frac{1}{2}\rho V^2$ occurring in equation (5) is exactly the dynamic pressure (otherwise designated by $q$) referring to the motion of the airplane. Hence, in equation (5) there occurs the thrust divided by the product of the propeller surface and the dynamic pressure. This ratio is obtained in a manner similar to that of the usual lift and drag coefficients of airfoils. It may be used to indicate the magnitude of the propeller loading. $T/S\,q$ may be designated by $C_T$.

The efficiency $\eta$ of the device, aside from friction, depends exclusively on the propeller loading $C_T$. It is less than 1, because the force of the aircraft is delivered with a velocity $V$, while the windmill is in an air stream with the smaller velocity $V_m$. The efficiency $V_m/V$ or, transformed,



$$\eta = \frac{1}{2}\, [1 + \sqrt{1 - C_T}] \qquad (6)$$

A more detailed consideration of equation (2) for the thrust, and of equation (6) for the efficiency leads to an understanding of the fundamental principles. The air passing through the windmill cannot be retarded indefinitely, a retardation to $V_2 = 0$ being the utmost conceivable. In this limiting case $V_m = V/2$, $C_T = 1$, and the thrust attains its maximum value. The resistance or drag is then about as great as that of a flat circular disk of the diameter of the windmill. The efficiency drops to 1/2 and the output of the propeller is further decreased. This output attains its maximum value at a retardation 1/3 of the initial velocity, i.e. $V_2 = V/3$. Its value is then

$$P_{max} = 16/27\; q\, V\, S \qquad (7)$$

The load coefficient is then $C_T = 8/9$ and the efficiency $\eta = 2/3$. With a very small load the efficiency is almost 1. The actual losses from air resistance and non-uniform flow through the windmill are not considered here. The former losses prevent the efficiency from ever attaining the calculated value (equation (6)). The other values also differ correspondingly from the computed values. The difference, however, amounts to only a small percentage for the conditions here involved. In a windmill, the conditions are especially favorable in this respect. On the one hand, its load coefficient $C_T$ is usually not very large, being smaller than in regular propellers. Consequently, the loss from non-uniform distribution of the load is not noticeable and the number of blades makes no particular difference. On the other hand, such a windmill is a device in which the flow along the surface takes place in a region of diminishing pressure. According to Prandtl's marginal layer theory, there is then but a very small loss through the resistance of the air and the conditions come closer to the ideal ones.

We will now consider the windmill as a small airplane propeller revolving in an air stream of velocity $V_m$ (not $V$). The velocity of the air stream varies with the distance from the axis. Only approximate data can be determined concerning the distribution of the thrust along the diameter of the propeller. Hence we can only calculate with mean values. In the discussion of the experimental result we will take the velocity of the outermost point as the basis of our calculation, instead of the unknown mean velocity. We shall then have calculated with a too great velocity and accordingly obtained too small a coefficient. This does not interfere with the application of the result to other windmills, but rather facilitates it. If, however, we wish to apply the results obtained with airfoils to windmills, we must remember to at least double the computed lift coefficient of the windmill, in order to obtain the actual average.

Now, as to the actual tests, the experiments were made in the Zeppelin Airship Company's wind tunnel at Friedrichshafen on a six-bladed propeller of 50 cm (19.68 in.) diameter, and 681 mm (36.8 in) pitch. The air stream had a circular cross-section of about one meter (3.28 ft) diameter. The windmill was mounted, during the experiment, directly on the shaft of a dynamo which had an outside



diameter of 150 mm (5.9 in.). The trailing end was well finished and tapered for the smooth flow of the air. The magnetic field was separately excited at constant voltage and the required power for each electrical condition measured immediately before the experiment. The revolution speed was measured on a synchronously revolving disk outside the air stream. The disk had an opening, so that it was possible to observe and regulate the agreement in revolution speed of both devices. The customary instruments were used for measuring the strength of current, the voltage and the air resistance of the whole device, which was supported by wires.

The resistance of the suspension wires was separately determined after the experiment and was taken into account. In like manner, the resistance of the dynamo without windmill is deducted in the accompanying table, although the right to do so could be contested. This resistance is not very large and is given separately in the table. The cross-section of the windmill blades was a modern airplane wing-section almost flat on the bottom, uniformly cambered on top, well pointed on the leading edge.

In the table of experimental values, $T$ denotes the drag after deducting the drag of the dynamo, the latter being measured without the windmill. $P$ gives the power in watts imparted by the windmill to the dynamo. By the power coefficient $k_n$ is understood the ratio of this power to the power $P_0 = S\,q\,V$, in which $S$ denotes the area of the windmill disk, $q$ the dynamic pressure and $V$ the velocity of the air stream. Hence $k_r = P/P_0$. The efficiency $\eta$ is the ratio of the power $P$ to the power $TV$ taken from the wind.

The ratio of the wind velocity to the peripheral velocity of the windmill is denoted by $\lambda$ as usual.

The diagram shows (plotted against the load coefficient $C_T$) the efficiency $\eta$ and the power coefficient $k_n$, both as found by measurement and also as presented for the ideal conditions discussed at the beginning of this article. Lastly, the curve $\lambda$ is drawn and shows the variation of the revolution speed, since $\lambda$ is inversely proportional to the revolution speed, at constant flight speed. It appears that, within a certain region (or range) the curves of the observed values closely approach the corresponding curves of the calculated values. This is the region in which the angle of attack of the propeller blade is best adapted. With further increase of load, the power does not increase and the efficiency falls rapidly. In the most favorable region, the latter is only about 5% smaller than the computed value. The diminution due to the resistance of the dynamo is not taken into account.

In the region of high efficiency, the peripheral speed was 78.5 m/sec (257.55 ft/sec) which corresponds to a dynamic pressure of 372 kg/m² (76.19 lb/sq.ft). The dynamic pressure of the resultant speed is increased by the amount of the dynamic pressure due to the flight speed and equals 450.4 kg/m² (92.25 lb/sq.ft). The thrust was 4.33 kg (9.55 lb/sq.ft) and the total area of all the blades was 0.066 m² (0.71 sq.ft). From this we obtain a lift coefficient of $C_L = 0.14$. The actual lift coefficient is probably twice as large. The angle of attack is almost exactly 0°. The motion of the air, relative to the blade, is parallel to the chord



of the section, which is not inconsistent with the value of the lift coefficient.

For designing such windmill blades, in accordance with the theory presented, as confirmed by the result of the experiment, we proceed as follows. Under the assumption of 80% efficiency, we first estimate the thrust and choose the diameter so that the coefficient of load $C_T$ is 0.3 to 0.5. In the latter case, we already have a further diminution of the efficiency by a small percentage. We may also start with the power and make $k_n = 0.25$. The total area of all the wings is then to be so determined that, with reference to the dynamic pressure at the blade tips, the lift coefficient $C_L$ becomes 0.15 (up to 0.2). The pitch is so to be determined that, at the velocity $V_m = \frac{1}{2} v \left(1 + \sqrt{1 - C_T}\right)$, there is no slip, but this velocity is equal to $p\,N/60$, in which $p$ denotes the pitch. The number of blades is determined by structural considerations.

<u>Example</u>.- It may be desired to design a windmill which, at 1000 meters (3280.83 ft) altitude with $V = 36$ m/sec (118.11 ft/sec), shall have a revolution speed of $N = 3500$ R.P.M. at and a performance of $P = 3.5$ HP.

1. The air density at the altitude $A$ is

$$\rho = \frac{1}{8}\, 0.9^{A/1000} = \frac{1}{8}\, 0.9^{1} = \frac{1}{8.9}$$

2. The dynamic pressure is

$$q = V^2\rho/2 = \frac{36^2}{2\ 8.9} = 73\ \text{kg/m}^2.$$

3. With the first assumed efficiency of $\eta = 0.8$

$$T = \frac{75\ P}{V \times \eta} = \frac{75 \times 5.5}{36 \times 0.8} = 14.3\ \text{kg.}$$

4. $C_T$ is given the value 0.4. Then

$$S = \frac{T}{q \times C_T} \quad \frac{14.3}{73 \times 0.4} = 0.49\ \text{qm} = D^2\ \frac{\pi}{4};\ D = 0.791\ \text{m.}$$

D is chosen at 0.8 m (2.63 ft).

5. The peripheral speed is

$$U = D \times \pi \times \frac{N}{60} = 0.8 \times \pi \times \frac{3500}{60} = 146.5\ \text{m/s}$$

6. The corresponding dynamic pressure is

$$q' = U^2 \times \rho/2 = \frac{146.5^2}{2 \times 8.9} = 1220\ \text{kg/m}^2,$$

The total dynamic pressure $q'' = q + q' = 1220 + 73 = 1293\ \text{kg/m}^2$.

7. $C_L$ is made 0.15. Then

$$f = \frac{T}{C_L \times q'} = \frac{14.3}{0.15 \times 1293} = 0.0738\ \text{m}^2.$$

8. With $i = 4$ blades and a blade length $L = 0.3$ (about 100 mm (3.94 in.) being lost on account of the hub), the blade width is

$$b = \frac{f}{i \times L} = \frac{0.0738}{4 \times 0.3} = 0.0615\ \text{m.},$$

maximum width being 65 mm (2.56 in.).

9. $V_w = \frac{1}{2} V [1 + \sqrt{1 - C_T}] = \frac{1}{2} \times 36 [1 + \sqrt{1 - 0.4}] = 31.9\ \text{m/s.}$

10. The pitch is

$$P = \frac{V_w}{N/60} = \frac{31.9 \times 60}{3500} = 0.546\ \text{m.}$$



Table.

| Dynamic pressure | | Air stream | | Drag | | Parasite drag of dynamo | |
|---|---|---|---|---|---|---|---|
| q kg/m² | q lbs/ft² | V m/s | V ft/sec | T kg | T lbs | $T_1$ kg | $T_1$ lbs |
| 84.0 | 17.20 | 37.4 | 122.70 | 2.02 | 4.45 | 0.37 | 0.82 |
| 80.0 | 16.39 | 36.4 | 119.42 | 3.18 | 7.01 | 0.35 | 0.77 |
| 78.5 | 16.08 | 36.1 | 118.44 | 4.33 | 9.55 | 0.35 | 0.77 |
| 77.0 | 15.77 | 35.8 | 117.45 | 4.80 | 10.58 | 0.34 | 0.75 |
| 74.5 | 15.26 | 35.2 | 115.49 | 5.26 | 11.60 | 0.33 | 0.73 |
| 72.0 | 14.75 | 34.6 | 113.52 | 6.03 | 13.29 | 0.32 | 0.71 |
| 71.0 | 14.54 | 34.4 | 112.86 | 6.13 | 13.51 | 0.32 | 0.71 |
| 70.5 | 14.34 | 34.2 | 112.20 | 6.33 | 13.96 | 0.32 | 0.71 |
| 64.0 | 13.11 | 32.6 | 106.96 | 1.82 | 4.01 | 0.30 | 0.66 |
| 62.5 | 12.80 | 32.2 | 105.64 | 2.85 | 6.28 | 0.29 | 0.64 |
| 60.0 | 12.29 | 31.6 | 103.67 | 3.55 | 7.83 | 0.28 | 0.62 |
| 57.5 | 11.78 | 30.9 | 101.38 | 4.54 | 10.00 | 0.27 | 0.60 |
| 55.2 | 11.31 | 30.3 | 99.41 | 4.95 | 10.91 | 0.26 | 0.57 |

Table (Contd.)

| Power in watts<br>P | Rev. per minute<br>N | Thrust coefficient<br>$C_T = \frac{T}{S\,q}$ | Efficiency<br>$\eta = \frac{F}{P} V_0$ | Coef. of advance<br>$\lambda = \frac{V}{D} \pi N$ | Coef. of power<br>$k_{P} = \frac{P}{q} \ldots$ |
|---|---|---|---|---|---|
| 210 | 3600 | 0.132 | 0.287 | 0.397 | 0.035 |
| 990 | 3280 | 0.309 | 0.874 | 0.423 | 0.143 |
| 1305 | 3000 | 0.282 | 0.850 | 0.460 | 0.240 |
| 1430 | 2820 | 0.318 | 0.850 | 0.483 | 0.271 |
| 1410 | 2420 | 0.360 | 0.779 | 0.506 | 0.280 |
| 1300 | 1980 | 0.427 | 0.635 | 0.670 | 0.272 |
| 1180 | 1800 | 0.439 | 0.570 | 0.730 | 0.251 |
| 1100 | 1800 | 0.459 | 0.518 | 0.727 | 0.238 |
| 190 | 3200 | 0.145 | 0.328 | 0.389 | 0.047 |
| 745 | 2800 | 0.341 | 0.829 | 0.439 | 0.193 |
| 940 | 2350 | 0.302 | 0.855 | 0.514 | 0.258 |
| 935 | 1980 | 0.403 | 0.680 | 0.597 | 0.274 |
| 790 | 1470 | 0.458 | 0.537 | 0.786 | 0.246 |

Translated by the National Advisory Committee for Aeronautics.