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SOME EFFECTS OF FUSELAGE FLEXIBILITY ON LONGITUDINAL STABILITY AND CONTROL

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SUMMARY

Some effects of fuselage flexibility on longitudinal stability and control of a large airplane are determined by application of a semirigid analytical technique. The dynamic characteristics are examined by studying the period and damping of the short-period longitudinal modes of oscillation of the fuselage and airplane. The static characteristics are examined by studying the longitudinal stability margins and the elevator-control deflections required for balance in straight or steady maneuvering flight.

The consideration of fuselage flexibility in the airplane studied appears to introduce no serious problems insofar as dynamic longitudinal stability in the subcritical speed range is concerned. It is indicated that, if desired, future designs may incorporate somewhat more flexible fuselages than that studied and still have dynamic longitudinal characteristics approximately equal to those predicted by quasi-static theory and roughly equivalent to those predicted by rigid-body theory.

The changes in static characteristics caused by increased fuselage flexibility appear, in general, as decreased straight-flight and increased maneuvering-flight stability margins and increased elevator-control deflections required for balance. The need for a means of longitudinal balance in steady maneuvering flight other than elevator or horizontal-tail deflection is indicated if flight is desired at low altitude and high speed with airplanes that have fuselage natural frequencies much below those representative of current design practice.

INTRODUCTION

Several modern high-speed airplanes incorporate structural components that are relatively more flexible than those previously used. There has been concern that the increased flexibility might appreciably modify the dynamic stability characteristics as predicted by rigid-airplane theory. (See, for example, ref. 1.) Particular concern was felt for the possibility of interaction between structural and stability vibratory modes because the natural frequencies of the major structural components are approaching the natural frequencies of the short-period stability mode.

The problem of the effect of wing flexibility on dynamic longitudinal stability of large airplanes has been treated analytically in the subcritical speed range by a simplified semirigid method in reference 2. In the present paper, the effects of fuselage flexibility are studied by the same method for the same class of airplanes. The major aspects of this study include

(1) Dynamic stability as described by period and damping of the fuselage and airplane longitudinal modes of oscillation

(2) Static stability as described by longitudinal stability margins

(3) Static longitudinal control as described by elevator-control deflections required for steady straight or maneuvering flight

Implications of the results with regard to the design of future airplanes of the class considered are discussed.

SYMBOLS

C_F	fuselage force coefficient, positive upward
C_L	lift coefficient
C_{L_O}	straight-flight balance lift coefficient
C_m	pitching-moment coefficient, positive nose upward
C_{m_O}	airplane pitching-moment coefficient at zero lift
$\bar{c}$	mean aerodynamic chord, ft
D	differential operator, $d/d\left(t\frac{V}{\bar{c}}\right)$
E	energy, ft-lb
EI	bending stiffness, lb-ft^2

G recurring group of terms, $2\mu\left(\bar{c}\omega_{f_e}\right)^2 \frac{M_3}{M_A}$, which for a given airplane and altitude varies as the square of ω_{f_e}

g acceleration due to gravity, 32.2 ft/sec^2

H nondimensional bending deflection at tail, $h_t/\bar{c}$

h deflection due to bending, positive downward, ft

$h(x)$ shape of fuselage deflection curve which for this paper is assumed to be $(x/x_t)^2$

I_Y moment of inertia about Y-axis, ft-lb-sec^2

K longitudinal stability margin, fraction of $\bar{c}$, positive when stable

k_Y radius of gyration about Y-axis, $\sqrt{\frac{I_Y}{M_A}}$, ft

L lift, positive upward

l_t tail length, ft

M mass, slugs

$M_1 = \int_{x_t}^{x_n} mh(x)dx$, slugs

$M_2 = \int_{x_t}^{x_n} mh(x)x\,dx$, slug-ft

$M_3 = \int_{x_t}^{x_n} m\left[h(x)\right]^2dx$, slugs

M_o free-stream Mach number

m mass per running foot, slugs/ft

Q generalized force or moment

q	generalized coordinate; or pitching velocity, $\frac{d\theta}{dt}$
q_o	free-stream dynamic pressure, lb/sq ft
S_w	wing area, sq ft
t	time, sec
V	true airspeed, fps
X,Z	stability coordinate axes (defined in fig. 1)
X',Z'	earth coordinate axes
x,z	distances along stability coordinate axes, ft
x',z'	distances along earth coordinate axes, ft
$\bar{x}$	nondimensional distance between $0.25\bar{c}$ and center of gravity, $\bar{c}$ lengths, positive when center of gravity is rearward of $0.25\bar{c}$
α	angle of attack, positive nose upward, radians
δ_e	elevator deflection, positive when trailing edge is downward, radians
ϵ	downwash angle, radians
θ	angle of pitch, positive nose upward, radians
μ	airplane relative-density coefficient, $M_A/\rho S_w\bar{c}$
ρ	density, slugs/cu ft
τ	elevator effectiveness factor, $C_{L_{\delta_e}}/C_{L_{\alpha_t}}$
ω_f	fuselage natural frequency, cps
ω_{f_e}	effective fuselage frequency, $2\pi\omega_f\left(1 - \frac{M_2^2}{I_Y M_3} - \frac{M_1^2}{M_3 M_A}\right)^{1/2}$, radians/sec

Subscripts:

A	entire airplane
cg	center of gravity
e	effective
F	flexible
f	fuselage
ff	fuselage forward of center of gravity
fr	fuselage rear of center of gravity
k	kinetic
n	nose of airplane
p	potential
R	rigid
SF	straight flight, flexible fuselage
SR	straight flight, rigid fuselage
TF	turning flight, flexible fuselage
TR	turning flight, rigid fuselage
t	tail of airplane
typ	typical current design practice
w	wing
X	X. stability axis

Dots are used to indicate differentiation with respect to time; for example, $\dot{\alpha} = \frac{d\alpha}{dt}$.

The subscripts α, $D\alpha$, q, H, DH, and δ_e indicate differentiation; for example, with respect to the lift parameter, $C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha}$,
$C_{L_{D\alpha}} = \frac{\partial C_L}{\partial \frac{\dot{\alpha}\bar{c}}{2V}}$, $C_{L_q} = \frac{\partial C_L}{\partial \frac{q\bar{c}}{2V}}$, $C_{L_H} = \frac{\partial C_L}{\partial H}$, $C_{L_{DH}} = \frac{\partial C_L}{\partial \frac{\dot{H}\bar{c}}{2V}}$, and $C_{L_{\delta_e}} = \frac{\partial C_L}{\partial \delta_e}$.

All coefficients, except those designated by suitable subscripts, refer to the entire airplane.

ANALYSIS

Basic Considerations

Some effects of fuselage flexibility on the longitudinal characteristics of a large airplane are examined by means of the Lagrangian formulation of the differential equations of motion. Three degrees of freedom are allowed: namely, angle of pitch θ, vertical displacement z, and fuselage bending h. (See fig. 1.)

Assumptions.- Unless otherwise stated, the following assumptions are made throughout the paper:

(1) The airplane forward velocity is constant.

(2) All drag forces are negligible.

(3) All lift forces on the fuselage are grouped in the wing coefficients.

(4) Wing and tail mass and lift forces are concentrated at their respective quarter-chord fuselage stations.

(5) Wing and tail are considered rigid.

(6) The fuselage is perfectly elastic.

(7) The fuselage deflects parabolically regardless of applied load distribution.

(8) Small classical perturbations are allowed.

Lagrangian formulation.- The Lagrangian equation within the assumption made is

$$\frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}} - \frac{\partial E_k}{\partial q} + \frac{\partial E_p}{\partial q} = Q \tag{1}$$

This equation requires only that the energies (both kinetic and potential) and the generalized forces of the system under study be known.

Energies.- Figure 1 shows that for small perturbation the vertical displacement at any fuselage station is

$$z'_f = z'_{cg} - x\theta + h(x)h_t \tag{2}$$

where

$$x \approx x' \qquad z \approx z' \qquad \cos\theta \approx 1$$

Differentiation with respect to time yields the vertical velocity

$$\dot{z}'_f = \dot{z}'_{cg} - x\dot{\theta} + h(x)\dot{h}_t \tag{3}$$

Thus, the kinetic energy of the entire airplane is

$$E_k = \int_{x_t}^{x_n} \frac{1}{2}m\left(\dot{z}'_f\right)^2 dx \tag{4}$$

and the potential energy is

$$E_p = -\int_{x_t}^{x_n} mgz'_f \, dx + \frac{1}{2}\int_{x_t}^{x_n} EI \left\{\frac{d^2\left[h(x)h_t\right]}{dx^2}\right\}^2 dx \tag{5}$$

where the first integral represents the stored energy due to vertical position and the second integral, which may be calculated according to the method of appendix A, represents the stored energy due to the elastic deformation.

Generalized forces.- The generalized forces are the aerodynamic forces on the wing and tail which, for small perturbations, may be assumed to act vertically. The forces work when they act through a distance. Therefore, for a displacement involving all three degrees of freedom,

$$\Delta \text{Work} = -\left\{\Delta z_{cg} - x_w \Delta\theta + \left[h(x)\right]_w \Delta h_t\right\} L_w - \left(\Delta z_{cg} - x_t \Delta\theta + \Delta h_t\right) L_t \quad (6)$$

Thus, the generalized forces for each degree of freedom are

$$Q_z = \frac{\Delta \text{Work}}{\Delta z} = -L_w - L_t \quad (7)$$

$$Q_\theta = \frac{\Delta \text{Work}}{\Delta\theta} = x_w L_w + x_t L_t \quad (8)$$

$$Q_{h_t} = \frac{\Delta \text{Work}}{\Delta h_t} = -\left[h(x)\right]_w L_w - L_t \quad (9)$$

where the lift on the wing and tail, respectively, are

$$L_w = C_{L_w} q_o S_w = C_{L_{\alpha_w}} \alpha_w q_o S_w \quad (10)$$

and

$$L_t = C_{L_t} q_o S_w = C_{L_{\alpha_t}} \alpha_t q_o S_w \quad (11)$$

Expressions for α_w and α_t, the angles of attack of the wing and tail in terms of the three degrees of freedom allowed, are derived in appendix B.

Equations of Motion

Four sets of differential equations of motion were used in describing the effects of fuselage flexibility on longitudinal stability and control: the semirigid, rigid, quasi-static, and static-balance equations.

Semirigid forms.- Performing the operation of equation (1) on equations (2) to (11) and then converting to nondimensional notation (see ref. 2) gives the semirigid forms of the equations of motion for the three degrees of freedom allowed: namely,

vertical force

$$2\mu D(\alpha - \theta) + 2\mu \frac{M_1}{M_A} D^2 H + \alpha C_{L_\alpha} + D\alpha\frac{1}{2}C_{L_{D\alpha}} + D\theta\frac{1}{2}C_{L_q} +$$

$$HC_{L_H} + DH\frac{1}{2}C_{L_{DH}} + \delta_e C_{L_{\delta_e}} - C_{L_O} = 0 \tag{12}$$

pitching moment

$$2\mu \frac{k_Y{}^2}{\bar{c}} D^2\theta - 2\mu \frac{M_2}{M_A\bar{c}} D^2 H - \alpha C_{m_\alpha} - D\alpha\frac{1}{2}C_{m_{D\alpha}} - D\theta\frac{1}{2}C_{m_q} -$$

$$HC_{m_H} - DH\frac{1}{2}C_{m_{DH}} - \delta_e C_{m_{\delta_e}} - C_{m_O} = 0 \tag{13}$$

and fuselage bending force

$$2\mu \frac{M_1}{M_A} D(\alpha - \theta) - 2\mu \frac{M_2}{M_A\bar{c}} D^2\theta + 2\mu \frac{M_3}{M_A} D^2 H + \alpha C_{F_\alpha} + D\alpha\frac{1}{2}C_{F_{D\alpha}} +$$

$$D\theta\frac{1}{2}C_{F_q} + H\left(C_{F_H} + \frac{G}{V^2}\right) + DH\frac{1}{2}C_{F_{DH}} + \delta_e C_{F_{\delta_e}} - C_{L_O} \frac{M_1}{M_A} = 0 \tag{14}$$

The generalized masses (e.g., M_1) in equations (12) to (14) are recurring constants and are derived and defined in appendix C. The equations used for evaluating the aerodynamic coefficients are given in appendix D.

Rigid forms.- A special case of the equations of motion, the familiar rigid-body case, is found when the fuselage bending is zero. Equations (12) to (14) then reduce to

vertical force

$$2\mu D(\alpha - \theta) + \alpha C_{L_\alpha} + D\alpha\frac{1}{2}C_{L_{D\alpha}} + D\theta\frac{1}{2}C_{L_q} + \delta_e C_{L_{\delta_e}} - C_{L_0} = 0 \tag{15}$$

and pitching moment

$$2\mu\left(\frac{k_Y}{\bar{c}}\right)^2 D^2\theta - \alpha C_{m_\alpha} - D\alpha\frac{1}{2}C_{m_{D\alpha}} - D\theta\frac{1}{2}C_{m_q} - \delta_e C_{m_{\delta_e}} - C_{m_0} = 0 \tag{16}$$

Quasi-static solution.- Another solution to the equations of motion of general interest, called the quasi-static solution, is found when the effects of velocity and acceleration in bending on the forces in the system are zero. Equations (12) to (14) then reduce to

vertical force

$$2\mu D(\alpha - \theta) + \alpha C_{L_\alpha} + D\alpha\frac{1}{2}C_{L_{D\alpha}} + D\theta\frac{1}{2}C_{L_q} + HC_{L_H} + \delta_e C_{L_{\delta_e}} - C_{L_0} = 0 \tag{17}$$

pitching moment

$$2\mu\left(\frac{k_Y}{\bar{c}}\right)^2 D^2\theta - \alpha C_{m_\alpha} - D\alpha\frac{1}{2}C_{m_{D\alpha}} - D\theta\frac{1}{2}C_{m_q} - HC_{m_H} - \delta_e C_{m_{\delta_e}} - C_{m_0} = 0 \tag{18}$$

and fuselage bending force

$$2\mu \frac{M_1}{M_A} D(\alpha - \theta) - 2\mu \frac{M_2}{M_A\bar{c}} D^2\theta + \alpha C_{F_\alpha} + D\alpha\frac{1}{2}C_{F_{D\alpha}} + D\theta\frac{1}{2}C_{F_q} +$$

$$H\left(C_{F_H} + \frac{G}{V^2}\right) + \delta_e C_{F_{\delta_e}} - C_{L_0}\frac{M_1}{M_A} = 0 \tag{19}$$

Static-balance form.- Still another form of the semirigid equations of motion, which describes static balance or trim of an airplane with a flexible fuselage in steady maneuvering flight, is obtained when the rate of change of angle of attack and the pitching acceleration as well as the velocity and acceleration in bending are all set equal to zero. Equations (12) to (14) then reduce to

vertical force

$$-2\mu D\theta + \alpha C_{L_\alpha} + D\theta\frac{1}{2}C_{L_q} + HC_{L_H} + \delta_e C_{L_{\delta_e}} - C_{L_0} = 0 \tag{20}$$

pitching moment

$$-\alpha C_{m_\alpha} - D\theta\frac{1}{2}C_{m_q} - HC_{m_H} - \delta_e C_{m_{\delta_e}} - C_{m_0} = 0 = C_m \tag{21}$$

and fuselage bending force

$$-2\mu D\theta \frac{M_1}{M_A} + \alpha C_{F_\alpha} + D\theta\frac{1}{2}C_{F_q} + H\left(C_{F_H} + \frac{G}{V^2}\right) + \delta_e C_{F_{\delta_e}} - C_{L_0}\frac{M_1}{M_A} = 0 \tag{22}$$

Equations (20) to (22) describe the static balance or trim of an airplane with a flexible fuselage in steady straight flight if all the terms involving $D\theta$ are deleted.

Dynamic Longitudinal Stability

An exponential solution to the equations of motion was assumed. If such a solution exists, the determinant of the coefficients must vanish. This determinant when written as an equation yields what is commonly called the characteristic equation of the system. The roots of the operator D contained in the characteristic equation determine the period and the time to damp of the various modes of motion in the system under study if the mode is oscillatory or simply the rate of convergence or divergence if the mode is nonoscillatory.

The foregoing procedure is described in detail in reference 3 and in other literature and was used herein to determine the dynamic longitudinal stability characteristics of the semirigid, rigid, and quasi-static airplane systems considered.

Static Longitudinal Stability and Control

The effects of fuselage flexibility on the static longitudinal stability and control are examined by comparing the classic straight-flight and maneuvering-flight stability margins and elevator angles necessary for balance for rigid airplanes with those for airplanes in which fuselage bending occurs.

Stability margins.- The straight-flight stability margin is defined herein as the longitudinal distance expressed as a fraction of the length of the wing aerodynamic chord between the center of gravity and the point at which the lift associated with incremental lift coefficient acts;[1] the incremental lift coefficient is produced by an infinitesimal change in steady forward flight speed along a straight and level flight path with controls fixed in the position required for balance at the particular initial speed under consideration. When expressed analytically, the straight-flight stability margin is

$$K_S = -\frac{\partial C_m/\partial V}{\partial C_{L_0}/\partial V} \tag{23}$$

The static-balance equations (20) to (22) may be differentiated with respect to velocity to obtain $\partial C_m/\partial V$ and $\partial C_L/\partial V$ where the pitching

[1]It should be noted that this point is not necessarily a fixed point on the airframe in the case of a flexible airplane but, rather, is dependent on center-of-gravity location and stiffness characteristics.

velocity $D\theta$ may be neglected in level unaccelerated flight. Substitution of these values into equation (23) yields the straight-flight stability margin for an airplane with a flexible fuselage as

$$K_{SF} = \frac{C_{m_\alpha} \frac{G}{V^2}}{C_{L_H} C_{F_\alpha} - C_{L_\alpha} C_{F_H} - C_{L_\alpha} \frac{G}{V^2}} + \left(1 + \frac{C_{L_\alpha} \frac{G}{V^2}}{C_{L_H} C_{F_\alpha} - C_{L_\alpha} C_{F_H} - C_{L_\alpha} \frac{G}{V^2}}\right) \frac{C_{m_0}}{C_{L_0}} \tag{24}$$

which, for infinite stiffness $(G = \infty)$, reduces to the well-known rigid-body straight-flight stability margin

$$K_{SR} = - \frac{C_{m_\alpha}}{C_{L_\alpha}} \tag{25}$$

The maneuvering-flight stability margin is herein defined as the longitudinal distance expressed as a fraction of the length of the wing mean aerodynamic chord between the center of gravity and the point at which the lift associated with incremental lift coefficient acts; the incremental lift coefficient is produced by an infinitesimal change in steady angle of attack at constant forward velocity with controls fixed in the position required for either the initial straight or the initial steady curvilinear flight path at the flight speed under consideration. When expressed analytically, the maneuvering-flight stability margin is

$$K_T = - \frac{\partial C_m / \partial \alpha}{\partial C_L / \partial \alpha} \tag{26}$$

The static-balance equations (20) to (22) may be differentiated with respect to angle of attack α to determine the solution of the maneuvering-flight stability margin K_T as defined by equation (26). For an airplane with a flexible fuselage, K_T becomes

$$K_{TF} = \frac{\frac{M_1}{M_A}\left(C_{L_\alpha}C_{m_H} - C_{m_\alpha}C_{L_H}\right) - C_{F_\alpha}C_{m_H} + C_{m_\alpha}C_{F_H} + C_{m_\alpha}\frac{G}{V^2}}{C_{L_H}C_{F_\alpha} - C_{L_\alpha}C_{F_H} - C_{L_\alpha}\frac{G}{V^2}} - \frac{\frac{G}{V^2}\left(\frac{1}{2}C_{L_q}C_{m_\alpha} - \frac{1}{2}C_{m_q}C_{L_\alpha}\right)}{2\mu\left(C_{L_H}C_{F_\alpha} - C_{L_\alpha}C_{F_H} - C_{L_\alpha}\frac{G}{V^2}\right)} \tag{27}$$

which, for infinite stiffness ($G = \infty$), reduces to the well-known rigid-body maneuvering-flight stability margin

$$K_{TR} = K_{SR}\left(1 - \frac{\frac{1}{2}C_{L_q}}{2\mu}\right) - \frac{\frac{1}{2}C_{m_q}}{2\mu} \tag{28}$$

The stability margins presented herein for airplanes with flexible fuselages represent real stability rather than apparent stability which normally is measured directly in flight tests from the variation of elevator angle with speed or acceleration. In this connection it should be noted that, for an airplane with a flexible fuselage, some of the change in elevator angle needed to change speed or acceleration is absorbed by fuselage deflection so that the elevator deflection measured in flight tests cannot, in general, be used as a measure of real stability.

Elevator-control characteristics in straight and steady maneuvering flight.- The static-balance equations (20) to (22) may be solved for elevator angle necessary for balance in level 1g flight where the pitching velocity $D\theta$ is 0. Thus, for an airplane with a flexible fuselage,

$$\delta_{e_{SF}} = \frac{\left[\frac{M_1}{M_A}\left(C_{L_\alpha}C_{m_H} - C_{m_\alpha}C_{L_H}\right) - C_{F_\alpha}C_{m_H} + C_{m_\alpha}C_{F_H} + C_{m_\alpha}\frac{G}{V^2}\right]C_{L_0}}{C_{L_{\delta_e}}\left(C_{m_\alpha} - C_{L_\alpha}\frac{l_t}{\bar{c}}\right)\frac{G}{V^2}} - \frac{\left(C_{L_H}C_{F_\alpha} - C_{L_\alpha}C_{F_H} - C_{L_\alpha}\frac{G}{V^2}\right)C_{m_0}}{C_{L_{\delta_e}}\left(C_{m_\alpha} - C_{L_\alpha}\frac{l_t}{\bar{c}}\right)\frac{G}{V^2}} \tag{29}$$

which, for infinite stiffness $(G = \infty)$, reduces to

$$\delta_{e_{SR}} = \frac{C_{m_\alpha} C_{L_0} + C_{L_\alpha} C_{m_0}}{C_{L_{\delta_e}}\left(C_{m_\alpha} - C_{L_\alpha} \frac{l_t}{\bar{c}}\right)} \tag{30}$$

The elevator-angle increment necessary to achieve steady flight different from 1g is obtained from the static-balance equations (20) to (22) for $D\theta \neq 0$ with all 1g terms neglected. Thus, for an airplane with a flexible fuselage,

$$\Delta\delta_{e_{TF}} = \frac{\left[\frac{M_1}{M_A}\left(C_{L_\alpha} C_{m_H} - C_{m_\alpha} C_{L_H}\right) - C_{F_\alpha} C_{m_H} + C_{m_\alpha} C_{F_H} + C_{m_\alpha} \frac{G}{V^2}\right] 2\mu D\theta}{C_{L_{\delta_e}}\left(C_{m_\alpha} - C_{L_\alpha} \frac{l_t}{\bar{c}}\right)\frac{G}{V^2}} -$$

$$\frac{D\theta \, \frac{G}{V^2}\left(\frac{1}{2} C_{L_q} C_{m_\alpha} - \frac{1}{2} C_{m_q} C_{L_\alpha}\right)}{C_{L_{\delta_e}}\left(C_{m_\alpha} - C_{L_\alpha} \frac{l_t}{\bar{c}}\right)\frac{G}{V^2}} \tag{31}$$

which, for infinite stiffness $(G = \infty)$, reduces to

$$\Delta\delta_{e_{TR}} = \frac{C_{m_\alpha} 2\mu D\theta - D\theta\left(\frac{1}{2} C_{L_q} C_{m_\alpha} - \frac{1}{2} C_{m_q} C_{L_\alpha}\right)}{C_{L_{\delta_e}}\left(C_{m_\alpha} - C_{L_\alpha} \frac{l_t}{\bar{c}}\right)} \tag{32}$$

It should be noted that any discussion pertinent to the effects of zero-lift pitching moment C_{m_0} on static stability or control is based on the assumption that the wing and tail are rigid.

NUMERICAL CONSTANTS

Several constants were chosen to identify the study with a typical current large airplane:

$$I_{Y_A} \cong 1.12 \times 10^6 \text{ lb-ft-sec}^2 \qquad C_{L_{\alpha_{w+f}}} = 4.927 \text{ per radian}$$

$$C_{L_{\alpha_t}} = 0.802 \text{ per radian (based on wing area)} \qquad \tau = 0.5 \qquad \frac{d\epsilon}{d\alpha} = 0.45$$

$$\omega_f = 2.72 \text{ cps} \qquad \bar{c} = 13 \text{ ft} \qquad S_w = 1{,}428 \text{ sq ft}$$

The dimensions and mass distribution shown in appendix C were selected such that (1) the moment of inertia of the assumed airplane approximated 1.12×10^6 lb-ft-sec^2 over the center-of-gravity range investigated ($0.25\bar{c}$ to $0.544\bar{c}$), and (2) the mass distribution was describable analytically. This second criterion reduced the evaluation of the mass integrals from a tedious graphical method to one in which the values are determined by expressions obtained from simple analytical integrations. (See appendix C.)

RESULTS AND DISCUSSION

The effect of fuselage flexibility on the characteristics of a bomber airplane is discussed with regard to

(1) Dynamic stability as described by period and damping of the fuselage and airplane longitudinal modes of oscillation

(2) Static stability as described by longitudinal stability margins

(3) Static longitudinal control as described by elevator-control deflections needed in steady straight or maneuvering flight

Period and Damping

Aerodynamic coefficients, mass distributions, and a range of center-of-gravity positions were chosen to represent a current high-speed bomber airplane. These values, shown in table I, were substituted into equations (12) to (19) to determine period and damping of the airplane and fuselage modes of oscillation at altitudes of 8,000 and 30,000 feet. (See ref. 3 for the method.) The results are summarized in figure 2 over the entire possible fuselage natural-frequency range at a Mach number of 0.7. Figure 3 summarizes the period and damping of the airplane mode as a function of dynamic pressure for a center-of-gravity location at $0.25\bar{c}$ for four representative fuselage natural frequencies. The frequencies selected were (1) $\omega_f = \infty$ or rigid body, (2) $\omega_{f_{typ}} = 2.72$ cps or typical, current bomber practice, (3) $\omega_f = 1.36$ cps or one-half the frequency of $\omega_{f_{typ}}$, and (4) $\omega_f = 0$ or completely flexible body.

Rigid-body solution.- The rigid-body characteristics as obtained from equations (15) and (16) may be identified in the figures as those at $\omega_f = \infty$. In figure 2, the center-of-gravity location appears to have little effect on the time to damp but causes large changes in the period of oscillation. The increase in period with rearward movement of the center of gravity is caused by the decrease in restoring pitching moments associated with the reduced stability margins. When the center of gravity is at the static neutral point $(0.544\bar{c})$, the motion consists of two aperiodic modes, both of which have positive damping.

Airplane mode of the semirigid body.- The airplane short-period mode of oscillation appears to have reduced damping as the fuselage becomes more flexible. This is probably caused by the reduced tail rotary damping which becomes zero at $\omega_f = 0$. However, the airplane mode is still damped at $\omega_f = 0$ because the vertical motion of the wing and fuselage supplies damping of the plunging motion involved in the short-period oscillation. Because of the assumptions made in the analysis, the wing motion contributes no rotary damping when the center of gravity is at $0.25\bar{c}$ and $\omega_f = 0$ but, as the center of gravity moves rearward, some rotary damping due to the wing is probably introduced and may account for the increased damping shown by figure 2.

The decrease in period of the airplane mode with increasing fuselage flexibility is believed to be due to an apparent forward movement of the effective center of gravity as the fuselage becomes more flexible. When the fuselage becomes completely flexible, the airplane probably tends to behave like two "tailless" airplanes hinged together flying in tandem. Each tailless airplane of the combination would have a relatively large stability margin and much smaller moment of inertia than the original airplane. It might be expected that the airplane mode characteristics would be determined mostly by the largest and heaviest tailless airplane, which would consist of most of the fuselage and the wing of the original conventional airplane.

Fuselage mode of the semirigid body.- Inclusion of a third degree of freedom, that of fuselage flexibility, introduces a new mode of oscillation. This mode represents the motion the flexible fuselage would assume. Although not shown in figure 2, the period of the rigid fuselage $(\omega_f = \infty)$ is zero because the structural restoring forces are infinite. As the fuselage natural frequency ω_f decreases, the structural restoring forces decrease and disappear at $\omega_f = 0$. Existence of a fuselage oscillation at $\omega_f = 0$ (see fig. 2) therefore indicates that a restoring force still exists that may be attributed to the air forces present.

Also, although not shown in figure 2, the time for the fuselage oscillation to damp to 0.1 amplitude for $\omega_f = \infty$ would be infinite because structural damping is not considered, and the effect of air damping would be negligible in comparison with the infinite structural restoring forces. As ω_f decreases, air damping becomes important and causes a gradual increase in damping as shown in figure 2. At $\omega_f = 0$, the damping as well as the period of the fuselage oscillation is determined entirely by the air forces.

Quasi-static solution.- Characteristics based on equations (17) to (19), wherein the effects of velocity and acceleration due to bending on the forces of the system are neglected, are much more readily obtained than the semirigid solutions since the characteristic equation is reduced from the fourth to the second order. Agreement between the two solutions appears to be excellent (see fig. 2) except for exceedingly low fuselage natural frequencies for which the quasi-static method predicts too much damping.

Effect of altitude.- The rigid-body characteristics indicate that both the period and the time to damp to 0.1 amplitude increase with altitude at constant Mach number (fig. 2). This increase is due to the increased ratio between the airplane mass forces and the aerodynamic restoring and damping forces which is brought about by a reduction in dynamic pressure at the higher altitudes.

The period of oscillation is essentially independent of altitude at any given dynamic pressure (fig. 3). However, the time to damp to 0.1 amplitude at a given dynamic pressure increases with altitude because the increase in true speed leads to smaller tail and wing angles of attack because of pitching velocity and, hence, smaller rotary damping moments. The inclusion of flexibility does not seem to alter these basic trends.

Significance of flexibility.- It is apparent from either figure 2 or figure 3 that fuselage flexibilities associated with natural frequencies above the order of 1 cps do not markedly change the period of damping from that expected with a rigid fuselage. Thus, within the framework of the present analysis, designers may possibly incorporate fuselages with appreciably lower natural frequencies than that of the typical fuselage chosen (2.72 cps) without great concern as to the adverse effects on the dynamic longitudinal stability characteristics of the proposed airplane.

However, when the period of the short-period oscillation approaches that of the fuselage mode which occurs in flight as $\omega_f \longrightarrow 0$, there is some evidence of adverse interaction between structural and stability modes. This adverse interaction is indicated by the rapid increase in time to damp to 0.1 amplitude as $\omega_f \longrightarrow 0$ shown by the results obtained with the semirigid method. It should be noted that the quasi-static method, which does not admit the existence of a fuselage oscillation, does not show a corresponding rapid increase in time to damp to 0.1 amplitude as $\omega_f \longrightarrow 0$.

The aerodynamic restoring forces which act on an airplane, as pointed out previously, are indicated to be sufficient to increase the frequency of the fuselage mode in flight so that the latter frequency is always higher than either the fuselage ground structural frequency or the airplane-stability natural frequency. Therefore, the probability of instability associated with proximity of the frequencies of the fuselage and airplane modes is minimized; in the present case, indeed, there was no indication of instability due to interaction between the two modes.

Longitudinal Stability Margins

Straight- and maneuvering-flight stability margins were calculated according to equations (24) and (27) and are presented in figures 4 and 5, respectively. The results are given as the ratio of the stability margin of an airplane flying at a Mach number of 0.7 at 8,000 and 30,000 feet with a flexible fuselage to the stability margin of this airplane operating under identical conditions but with a rigid body.

Straight flight.- The straight-flight stability margin of an airplane that has zero pitching moment at zero lift $\left(C_{m_0} = 0\right)$ is reduced as fuselage flexibility is increased and, in fact, becomes zero for an airplane with an infinitely flexible fuselage. (See fig. 4.) Center-of-gravity location over the range investigated has little if any effect on the stability-margin ratio at a given fuselage frequency.

However, the situation is markedly changed when there is a finite pitching moment about the wing quarter chord at zero lift $\left(C_{m_0} \neq 0\right)$.

For a value of C_{m_0} given by

$$C_{m_0} = -\frac{C_{m_\alpha}}{C_{L_\alpha}} C_{L_0}$$

the stability margin for a flexible fuselage remains constant over the entire range of flexibility from an infinitely rigid to an infinitely flexible body. Any value of C_{m_0} above this shows an increased stability margin with increasing flexibility, and, conversely, any value below this shows a decrease in stability margin with increased flexibility.

The center-of-gravity location has an effect on the degrees of stability increase or decrease when $C_{m_0} \neq 0$. When the center of gravity is at the rigid-airplane neutral point, the ratio of stability margins is, of course, infinite (either positively or negatively depending on the sign of C_{m_0}) with respect to the rigid-body margin. At more forward center-of-gravity locations, the ratio of stability margins approaches the values for $C_{m_0} = 0$. Therefore, it is seen that the importance of a given value of C_{m_0} is inversely proportional to the initial rigid-airplane stability margin.

Increasing altitude at constant Mach number reduces the effects of flexibility on the static margins for any airplane configuration. This effect is believed to be solely due to the relative reduction in air forces compared with the structural forces because of the reduced dynamic pressure at any constant Mach number that occurs with increasing altitude.

Maneuvering flight.- As the fuselage becomes flexible $(\omega_f \rightarrow 0)$, the maneuvering-flight stability margin shows a marked increase over the rigid-body values. Although not readily seen from figure 5, the increased margins amount to about 20 percent chord over the center-of-gravity range investigated at $\omega_f = 0$. The increase in maneuvering-flight stability margin ratio with increased altitude probably stems from the reduction in rigid-airplane stability margin which normally occurs with increasing altitude and is associated with reduced damping in pitch. Because the turning-flight stability margins involve no change in speed, the zero-lift pitching-moment coefficient C_{m_0} has no effect on these margins even when flexibility is considered.

Significance of flexibility.- The conclusion often reached is that increasing the flexibility of a fuselage will always result in decreased stability margins because of the tendency of the tail to stream with the relative wind and thereby produce less lift than the same tail on a rigid fuselage. Such an effect is apparently predominant when straight-flight stability margins are considered. However, although such a streaming tendency exists in steady maneuvering flight, the present analysis indicates that the effect of normal acceleration on the distributed fuselage mass for airplanes with conventional tails causes both ends of the fuselage to deflect downward sufficiently such that the tail of the flexible airplane produces more lift than that of the rigid airplane. The resulting increase in tail lift draws the centroid of the total airplane lift rearward which is synonomous with increased stability margin. Thus, although the straight-flight stability margin goes to zero $\left(\text{for } C_{m_0} = 0\right)$ as the fuselage flexibility is increased, the maneuvering-flight stability margin is increased.

For flexible airplanes, the problem of straight-flight stability margin may be minimized through careful design practice. Advantage can be taken of the situation when

$$C_{m_0} = -\frac{C_{m_\alpha}}{C_{L_\alpha}} C_{L_0}$$

If the wing sections are selected such that this equation is obeyed at the design speed and altitude condition, flexibility will have no effect and the airplane will have the same stability it would have if it had a rigid fuselage. As the airplane operates off the design-speed conditions, the effects of fuselage flexibility become important.

Except for large negative values of C_{m_0}, which in most cases can be avoided in airplane design, the effects of the inclusion of fuselage-flexibility effects on current airplane design $\left(\omega_{f_{typ}} = 2.72 \text{ cps}\right)$ with regard to stability margins are either negligible or beneficial in both straight and maneuvering flight. Indeed, from figures 4 and 5, designers may expect to use fuselage natural frequencies appreciably less than is the current practice without expecting very large differences.

Elevator-Control Motion

The elevator angles necessary for balance in either straight or maneuvering flight were calculated according to equations (29) and (31) and are presented in figures 6 and 7, respectively. In the case of straight flight, the elevator angles necessary for balance are given as absolute values. In the case of maneuvering flight, elevator-angle increments necessary for balance at any given normal acceleration different from 1 g are given as the ratio of the elevator-angle increment required for an airplane with a flexible fuselage to the elevator-angle increment required for an airplane with a rigid body. In every case, the airplane is flying at a Mach number of 0.7 and at either 8,000- or 30,000-foot altitudes.

Rigid-body control.- The elevator angles at $\omega_f = \infty$ in figure 6 represent those expected from a rigid airplane of the assumed geometry. The static stick-fixed neutral point occurs when $\delta_e = 0$ for $C_{m_0} = 0$ which is at the center-of-gravity location of $0.544\bar{c}$. In this condition, a positive lift necessary for balance is generated on the entire tail when the rigid airplane is at an angle of attack. As the center of gravity moves forward, less tail lift is required for balance. This is achieved by increasing up-elevator (negative) angle as may be seen in figure 6. When the center of gravity reaches the wing quarter-chord position, the up-elevator angle required is just sufficient to eliminate completely all lift on the horizontal tail for the configuration which has no zero-lift pitching moments or distributed fuselage lift. Of course, positive C_{m_0} causes a down-elevator angle necessary for balance and negative C_{m_0} causes an up-elevator angle necessary for balance with respect to that required when $C_{m_0} = 0$.

Straight-flight control of flexible fuselage.- The elevator angle necessary for balance in straight flight for $C_{m_0} = 0$ becomes increasingly negative (trailing edge up) as fuselage flexibility is increased and, in fact, becomes infinite for an airplane with an infinitely flexible fuselage. (See fig. 6.) However, when zero-lift pitching moments are considered, it can be shown that fuselage flexibility has no effect on the elevator angle necessary for balance when

$$C_{m_0} = \frac{\left[\frac{M_1}{M_A}\left(C_{L_\alpha}C_{m_H} - C_{m_\alpha}C_{L_H}\right) + C_{m_\alpha}C_{F_H} - C_{F_\alpha}C_{m_H}\right]C_{L_0}}{C_{L_H}C_{F_\alpha} - C_{L_\alpha}C_{F_H}}$$

More positive values of C_{m_0} require an increasing positive elevator angle until as $\omega_f \rightarrow 0$ the elevator angle necessary for balance becomes infinite. Conversely, for more negative values of C_{m_0}, the elevator angle necessary for balance becomes infinite in the opposite direction.

Increasing altitude requires increased up-elevator angles necessary for balance. This is probably due primarily to the reduced dynamic pressure which requires that the airplane be flown at higher lift coefficients at high altitudes.

Maneuvering-flight control of flexible fuselage.- The maneuvering-flight elevator-angle-increment ratio necessary for balance for an airplane with a flexible fuselage increases as fuselage flexibility is increased and becomes infinite for an airplane with an infinitely flexible fuselage. (See fig. 7.)

Significance of flexibility.- The deflection curve which a flexible fuselage assumes depends on the distribution of applied aerodynamic loads, the mass distribution, and the stiffness distribution. In normal flight, that is, with the center of gravity at or near $0.25\bar{c}$, the air load required on the horizontal tail for balance in straight flight is zero if $C_{m_O} = 0$. Under these conditions, the fuselage may be thought of as a long flexible beam having distributed mass which is supported by a single concentrated lift force supplied somewhere near the middle of the beam. The result is that both ends of the fuselage will droop, and it is apparent that increasing up elevator will be required to preserve balance as the fuselage increases in flexibility. When the center of gravity is moved rearward from $0.25\bar{c}$, up tail loads are needed for balance. If the up tail load for balance exceeds the weight of the tail assembly (which occurs for the airplane studied when the center of gravity is about $0.32\bar{c}$), the fuselage deflection curve begins to have an inflection point at the rear end which moves forward with further rearward movement of the center of gravity. Therefore, the assumption of a simple deflection curve such as the parabola becomes less satisfactory as the center of gravity approaches the neutral point. However, it is apparent from the foregoing discussion that the assumption of a parabolic deflection curve is at least qualitatively realistic and should allow some idea to be gained in regard to the onset of serious trim problems.

The increases in elevator angle compared with those required for a rigid body $\left(\omega_f = \infty\right)$ are not marked for the design studied $\left(\omega_{f_{typ}} = 2.72 \text{ cps}\right)$ for normal center-of-gravity locations (approximately $0.25\bar{c}$) for the airplane in level flight at both 8,000 and 30,000 feet and also for the airplane at 30,000 feet in maneuvering flight. In fact, from figures 6 and 7 it appears that designers can reduce appreciably current fuselage structural frequencies before any serious elevator-control balance problems will be expected to occur. However, the elevator-control balance problem in low-altitude high-speed flight ($M_O \geqq 0.70$) appears in figure 7 to be becoming serious for current airplanes in maneuvering flight and will probably become intolerable for more flexible designs. If high-speed flight becomes necessary at low altitudes for airplanes having fuselage natural frequencies much below those of present-day practice, some means other than elevator or stabilizer control may become necessary to balance the airplane longitudinally in maneuvering flight.

It is interesting to note from figure 6 or figure 7 that the elevator is, in general, completely incapable of balancing the airplane with a completely flexible fuselage in either straight or turning flight. Thus, any discussions of static stability for fuselage stiffness very close to zero are primarily of academic interest.

CONCLUDING REMARKS

The consideration of fuselage flexibility in a current large high-speed airplane design appears to introduce no serious problems insofar as dynamic longitudinal stability in the subcritical speed range is concerned. It is indicated that, if desired, future designs may incorporate fuselages with natural frequencies appreciably less than that studied and still have dynamic longitudinal characteristics approximately equal to those predicted by quasi-static theory and roughly equivalent to those predicted by rigid-body theory.

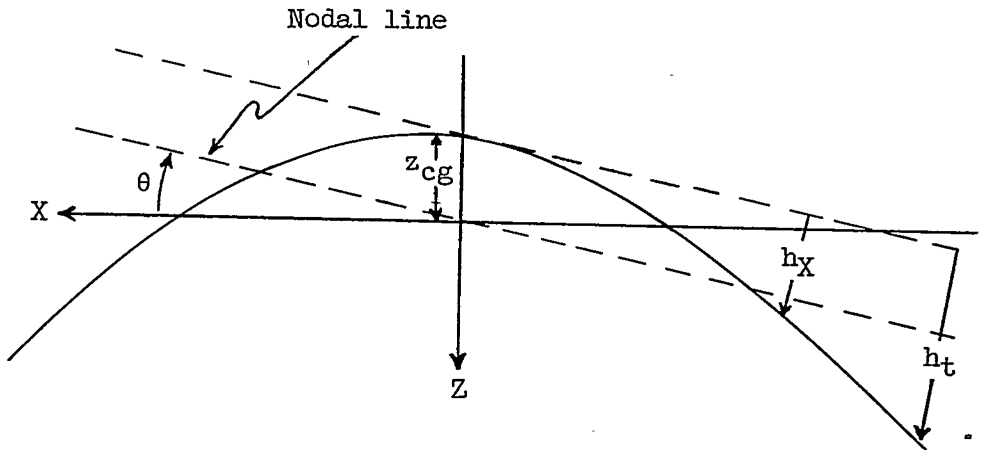
The aerodynamic restoring forces which act on the airplane are sufficient to increase the frequency of the fuselage oscillatory mode in flight so that the latter frequency is always higher than either the fuselage ground structural frequency or the airplane-stability natural frequency. Thus, the occurrence of a resonant condition is avoided.

The changes in static characteristics caused by increased fuselage flexibility, in general, appear as decreased straight-flight and increased maneuvering-flight stability margins and increased elevator-control deflections required for balance. The need for a means of longitudinal balance in steady maneuvering flight other than elevator or horizontal-tail deflection is indicated if flight is desired at low altitude and high speed with airplanes that have fuselage natural frequencies much below those representative of current design practice.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 23, 1955.

APPENDIX A

CALCULATION OF POTENTIAL STRAIN ENERGY



The potential strain energy is derived in terms of the maximum kinetic energy of a harmonically vibrating free-free beam (the fuselage). Mathematically, this equality can be expressed as maximum potential strain energy equal to maximum kinetic energy or

$$\frac{1}{2}\int_{x_t}^{x_n} EI\left\{\frac{d^2\left[h(x)h_t\right]}{dx^2}\right\}^2 dx = \frac{\omega_f^2}{2}\cdot\int_{x_t}^{x_n} mz_f^2 dx \tag{A1}$$

The problem then is to express z_f in known terms.

The vertical displacement and acceleration of any fuselage station may be expressed as

$$z_f = z_{cg} - x\theta + h(x)h_t \tag{A2}$$

and

$$\ddot{z}_f = \ddot{z}_{cg} - x\ddot{\theta} + h(x)\ddot{h}_t \tag{A3}$$

It is a necessary condition that the external vertical force and pitching moments of a free-free beam be zero at all times. Hence,

$$\int_{x_t}^{x_n} m\ddot{z}_f \, dx \equiv 0 = \ddot{z}_{cg}M_A + \ddot{h}_t M_1 \tag{A4}$$

and

$$\int_{x_t}^{x_n} mx\ddot{z}_f \, dx \equiv 0 = -\ddot{\theta}I_Y + \ddot{h}_t M_2 \tag{A5}$$

Solving equations (A4) and (A5) for $\ddot{z}_{cg}$ and $\ddot{\theta}$, respectively, in terms of $\ddot{h}_t$ yields

$$\left.\begin{aligned} \ddot{z}_{cg} &= -\frac{M_1}{M_A}\ddot{h}_t \\ \ddot{\theta} &= \frac{M_2}{I_Y}\ddot{h}_t \end{aligned}\right\} \tag{A6}$$

or, combining equations (A3) and (A6) gives

$$\ddot{z}_f = \ddot{h}_t\left[-\frac{M_1}{M_A} - \frac{M_2}{I_Y}x + h(x)\right] \tag{A7}$$

Under the assumption of sinusoidal motion it can be shown easily that

$$\ddot{z}_f = -\omega_f^2 z_f \tag{A8}$$

and

$$\ddot{h}_t = -\omega_f^2 h_t \tag{A9}$$

so that equation (A7) becomes

$$z_f = h_t\left[-\frac{M_1}{M_A} - \frac{M_2}{I_Y}x + h(x)\right] \tag{A10}$$

Equation (A10) then is an expression for z_f in known terms. Substitution of equation (A10) into equation (A1) and simplifying yields the potential strain energy expressed as

$$\frac{1}{2}\int_{x_t}^{x_n} EI\left\{\frac{d^2\left[h(x)h_t\right]}{dx^2}\right\}^2 dx = \frac{\omega_f^2}{2} M_3 h_t^2\left(1 - \frac{M_1^2}{M_A M_3} - \frac{M_2^2}{I_Y M_3}\right) \tag{A11}$$

APPENDIX B

DERIVATIONS OF TAIL AND WING ANGLES OF ATTACK

For a rigid airplane, the wing angle of attack is

$$\alpha_{w_R} = \alpha - \frac{x_w\dot{\theta}}{V} \tag{B1}$$

and the tail angle of attack is

$$\alpha_{t_R} = \alpha - \frac{d\epsilon}{d\alpha}\left(\alpha + \dot{\alpha}\,\frac{x_t}{V}\right) - \frac{x_t}{V}\,\dot{\theta} \tag{B2}$$

For the flexible airplane, α_w and α_t are modified because of fuselage bending displacement and velocity; consequently,

$$\alpha_{w_F} = \alpha_{w_R} + \left\{\frac{d\left[h(x)h_t\right]}{dx}\right\}_{x=x_w} + \left[\frac{h(x)\dot{h}_t}{V}\right]_{x=x_w} \tag{B3}$$

and

$$\alpha_{t_F} = \alpha_{t_R} + \left\{\frac{d\left[h(x)h_t\right]}{dx}\right\}_{x=x_t} + \left[\frac{h(x)\dot{h}_t}{V}\right]_{x=x_t} \tag{B4}$$

For the assumed parabolic fuselage deflection under load,

$$h(x) = \left(\frac{x}{x_t}\right)^2 \tag{B5}$$

so that

$$\frac{d\left[h(x)\right]}{dx} = \frac{2x}{{x_t}^2} \tag{B6}$$

Substitution of equations (B1) into (B3) and (B2) into (B4) and nondimensionalizing gives, respectively,

$$\alpha_w = \alpha - \frac{x_w}{\bar{c}} D\theta - \frac{2x_w}{x_t}\frac{\bar{c}}{x_t} H + \left(\frac{x_w}{x_t}\right)^2 DH \tag{B7}$$

and

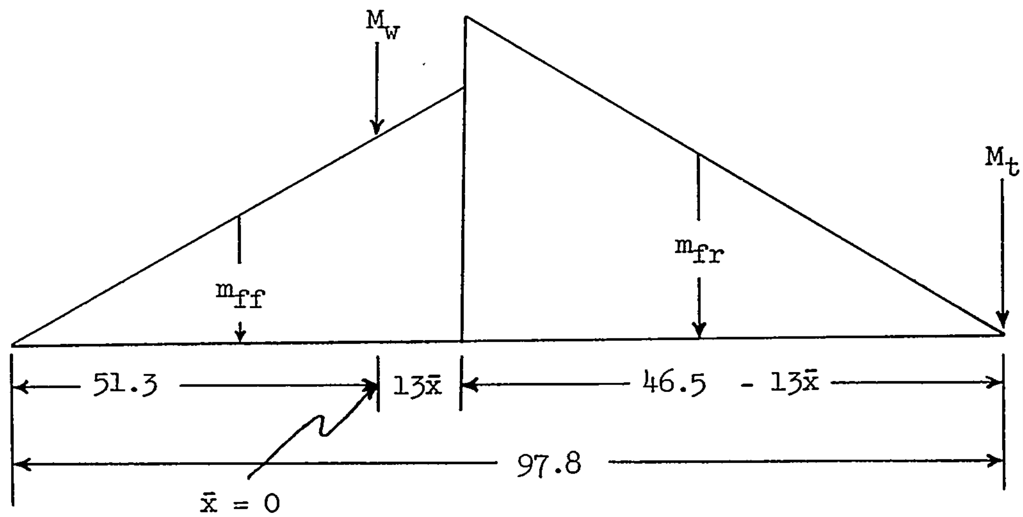
$$\alpha_t = \alpha - \frac{d\epsilon}{d\alpha}\left(\alpha + D\alpha \frac{x_t}{\bar{c}}\right) - \frac{x_t}{\bar{c}} D\theta - 2H \frac{\bar{c}}{x_t} + DH \tag{B8}$$

Equations (B7) and (B8) are used herein to describe the angles of attack of the wing and tail for the several degrees of freedom allowed.

APPENDIX C

DERIVATION OF GENERALIZED MASSES

The assumed loading for the airplane under consideration may be pictured as



where all distances shown are in feet and

$$\bar{c} = 13 \text{ ft}$$

$$M_A = 3{,}882 \text{ slugs } (125{,}000 \text{ lb})$$

$$M_w = 1{,}273 \text{ slugs } (41{,}000 \text{ lb})$$

$$M_f + M_t = M_A - M_w = 2{,}609 \text{ slugs}$$

$$M_t = 621 \text{ slugs}$$

$$M_f = M_{ff} + M_{fr} = 1{,}988 \text{ slugs}$$

and where, for the assumed fuselage geometry and triangular load distribution in slugs,

$$M_{ff} = 1{,}299 - 870\bar{x}$$

The mass per running foot is

$$m_{ff} = 2M_{ff} \frac{x_n - x}{x_n^2} \qquad (\text{for} \quad x = x_n \quad \text{to} \quad x = 0) \tag{C1}$$

and

$$m_{fr} = 2M_{fr} \frac{-x_t + x}{x_t^2} \qquad (\text{for} \quad x = 0 \quad \text{to} \quad x = x_t) \tag{C2}$$

for

$$x_n = 51.3 + 13\bar{x} \tag{C3}$$

and

$$x_t = -46.5 + 13\bar{x} \tag{C4}$$

Proceeding to the mass integrals gives

$$M_1 = \int_{x_t}^{x_n} mh(x)dx \tag{C5}$$

Combining equations (C1) to (C5) yields

$$M_1 = \int_0^{x_n} 2M_{ff} \frac{x_n + x}{x_n^2}\left(\frac{x}{x_t}\right)^2 dx + M_w\left(\frac{x}{x_t}\right)^2_{x=x_w} +$$

$$\int_{x_t}^{0} 2M_{fr} \frac{-x_t + x}{x_t^2}\left(\frac{x}{x_t}\right)^2 dx + M_t\left(\frac{x}{x_t}\right)^2_{x=x_t} \tag{C6}$$

When the integrations have been performed,

$$M_1 = \frac{M_{ff}}{6}\left(\frac{x_n}{x_t}\right)^2 + M_w\left(\frac{x_w}{x_t}\right)^2 + \frac{M_{fr}}{6} + M_t \tag{C7}$$

Similarly,

$$M_2 = \frac{M_{ff}}{10}\left(\frac{x_n}{x_t}\right)^2 x_n + M_w\left(\frac{x_w}{x_t}\right)^2 x_n + \frac{M_{fr}}{10} x_t + M_t x_t \tag{C8}$$

and

$$M_3 = \frac{M_{ff}}{15}\left(\frac{x_n}{x_t}\right)^4 + M_w\left(\frac{x_w}{x_t}\right)^4 + \frac{M_{fr}}{15} + M_t \tag{C9}$$

Equations (C7) to (C9) are used to evaluate the mass integrals over the center-of-gravity range of the investigation.

APPENDIX D

EQUATIONS FOR EVALUATING THE AERODYNAMIC PARAMETERS

The aerodynamic parameters in the general case may be expressed as

$$f(n)C_{L_{f(n)}} = \alpha_{w_{f(n)}}C_{L_{\alpha_w}} + \alpha_{t_{f(n)}}C_{L_{\alpha_t}}$$

$$f(n)C_{m_{f(n)}} = \alpha_{w_{f(n)}}C_{L_{\alpha_w}}\frac{x_w}{\bar{c}} + \alpha_{t_{f(n)}}C_{L_{\alpha_t}}\frac{x_t}{\bar{c}}$$

$$f(n)C_{F_{f(n)}} = \alpha_{w_{f(n)}}C_{L_{\alpha_w}}\left(\frac{x_w}{x_t}\right)^2 + \alpha_{t_{f(n)}}C_{L_{\alpha_t}}$$

For example, the airplane lift-curve slope C_{L_α}, the damping in pitch $\frac{1}{2}C_{m_q}$, and the bending-force coefficient due to velocity in bending $\frac{1}{2}C_{F_{DH}}$ are expressed, respectively, as

$$\alpha C_{L_\alpha} = \alpha C_{L_{\alpha_w}} + \alpha\left(1 - \frac{d\epsilon}{d\alpha}\right)C_{L_{\alpha_t}}$$

$$D\theta\frac{1}{2}C_{m_q} = -D\theta\left(\frac{x_w}{\bar{c}}\right)^2 C_{L_{\alpha_w}} - D\theta\left(\frac{x_t}{\bar{c}}\right)^2 C_{L_{\alpha_t}}$$

$$DH\frac{1}{2}C_{F_{DH}} = DH\left(\frac{x_w}{x_t}\right)^4 C_{L_{\alpha_w}} + DHC_{L_{\alpha_t}}$$

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TABLE I.- CONSTANTS USED IN NUMERICAL SOLUTIONS

[Two values of μ were used: 223.9 and 470.7, corresponding to altitudes of 8,000 and 30,000 feet, respectively]

Parameter	Values of parameter for center-of-gravity locations of -			
	$0.25\bar{c}$	$0.35\bar{c}$	$0.45\bar{c}$	$0.544\bar{c}$
M_1/M_A	0.1375	0.1441	0.1510	0.1580
$(k_Y/\bar{c})^2$	1.762	1.736	1.710	1.685
$M_2/M_A\bar{c}$	-.01117	-.00377	.00376	.01148
M_3/M_A	.07056	.07718	.08433	.09180
C_{L_α}	5.368	5.368	5.368	5.368
$\frac{1}{2}C_{L_{D\alpha}}$	1.292	1.256	1.220	1.186
$\frac{1}{2}C_{L_q}$	2.871	2.298	1.725	1.185
C_{L_H}	.4482	.3797	.3021	.2197
$\frac{1}{2}C_{L_{DH}}$	.8021	.8062	.8193	.8416
$C_{L_{\delta_e}}$	.4010	.4010	.4010	.4010
C_{m_α}	-1.579	-1.042	-.5060	0
$\frac{1}{2}C_{m_{D\alpha}}$	-4.624	-4.370	-4.123	-3.896
$\frac{1}{2}C_{m_q}$	-10.280	-9.758	-9.356	-9.082
C_{m_H}	-1.604	-1.612	-1.639	-1.683
$\frac{1}{2}C_{m_{DH}}$	-3.135	-3.055	-2.972	-2.888
$C_{m_{\delta_e}}$	-1.865	-1.813	-1.762	-1.712
C_{F_α}	.4412	.4453	.4584	.4807
$\frac{1}{2}C_{F_{D\alpha}}$	1.292	1.256	1.220	1.186
$\frac{1}{2}C_{F_q}$	2.871	2.790	2.707	2.623
C_{F_H}	.4482	.4610	.4741	.4862
$\frac{1}{2}C_{F_{DH}}$	.8021	.8062	.8194	.8419
$C_{F_{\delta_e}}$	.4010	.4010	.4010	.4010

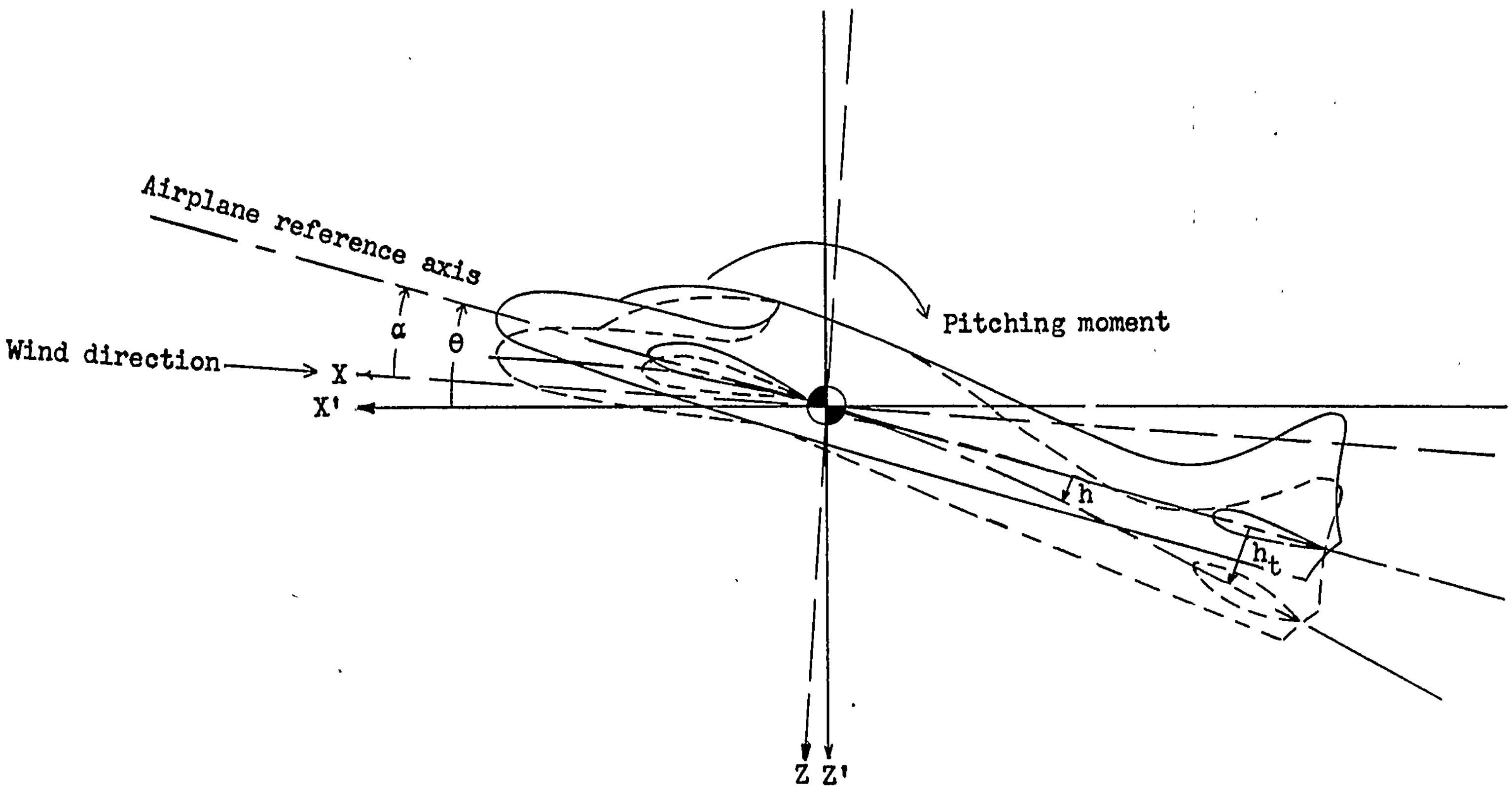
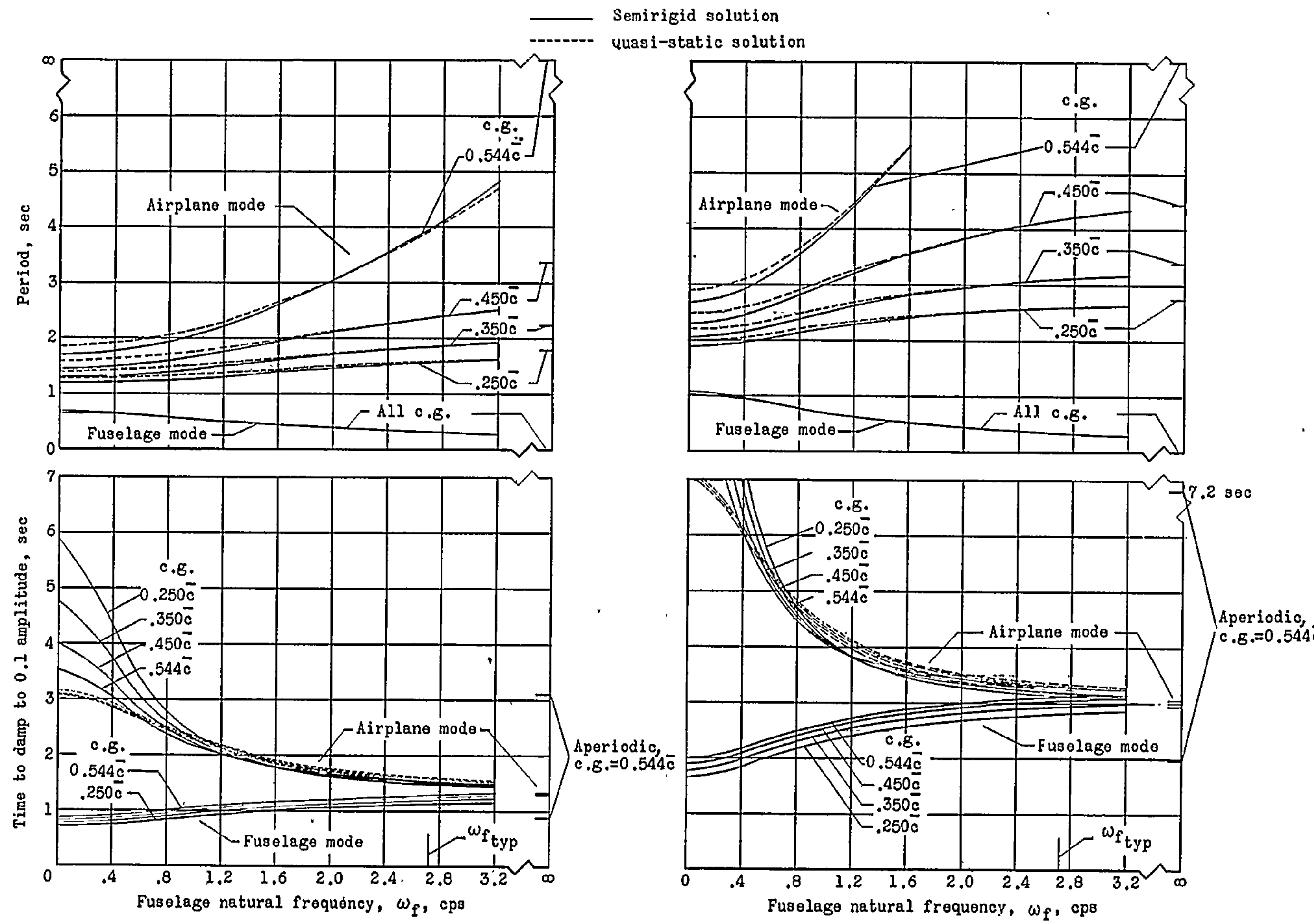


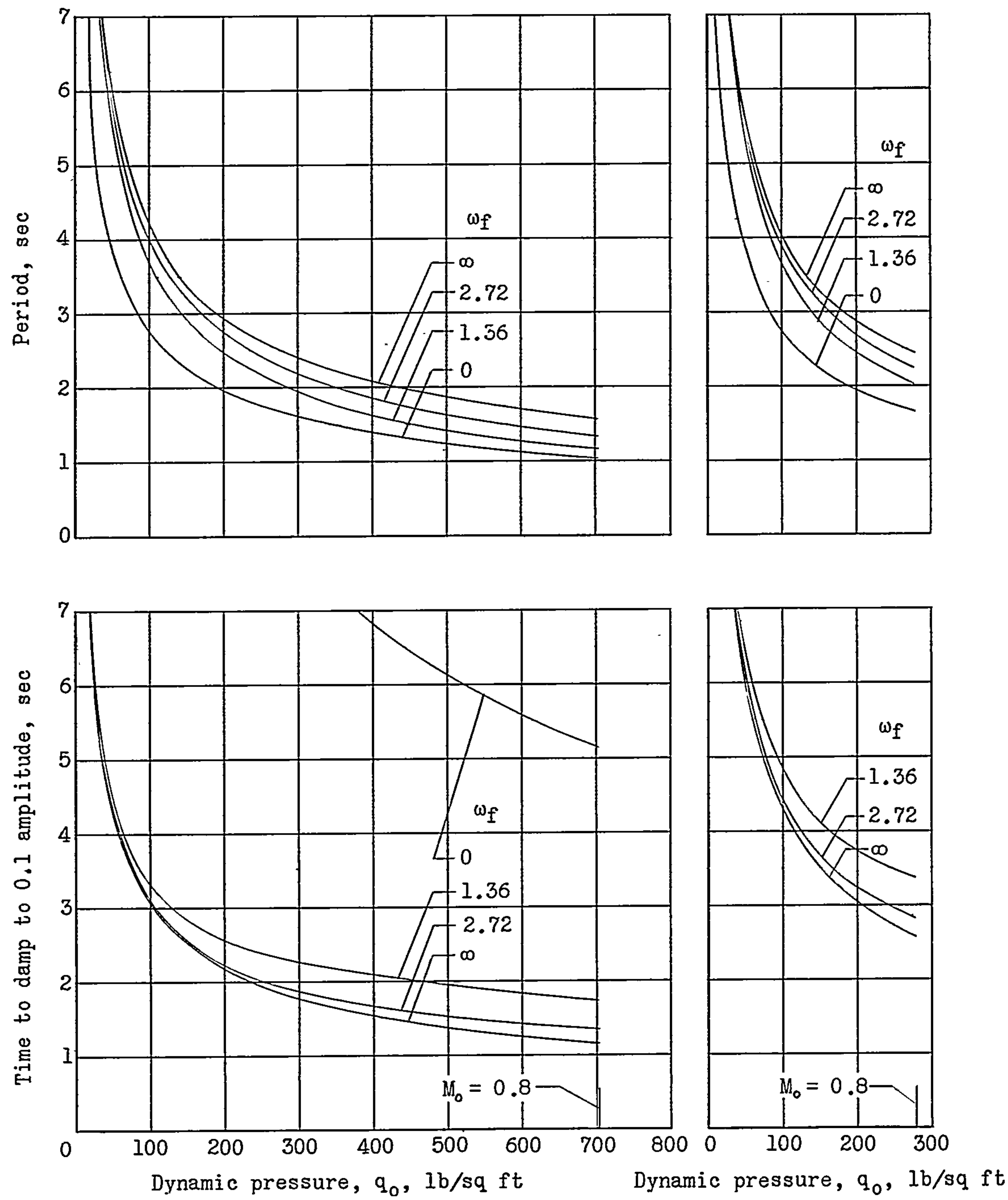
Figure 1.- Stability- and earth-axes systems employed in analysis showing positive directions of forces, moments, and displacements.



(a) Altitude, 8,000 feet.

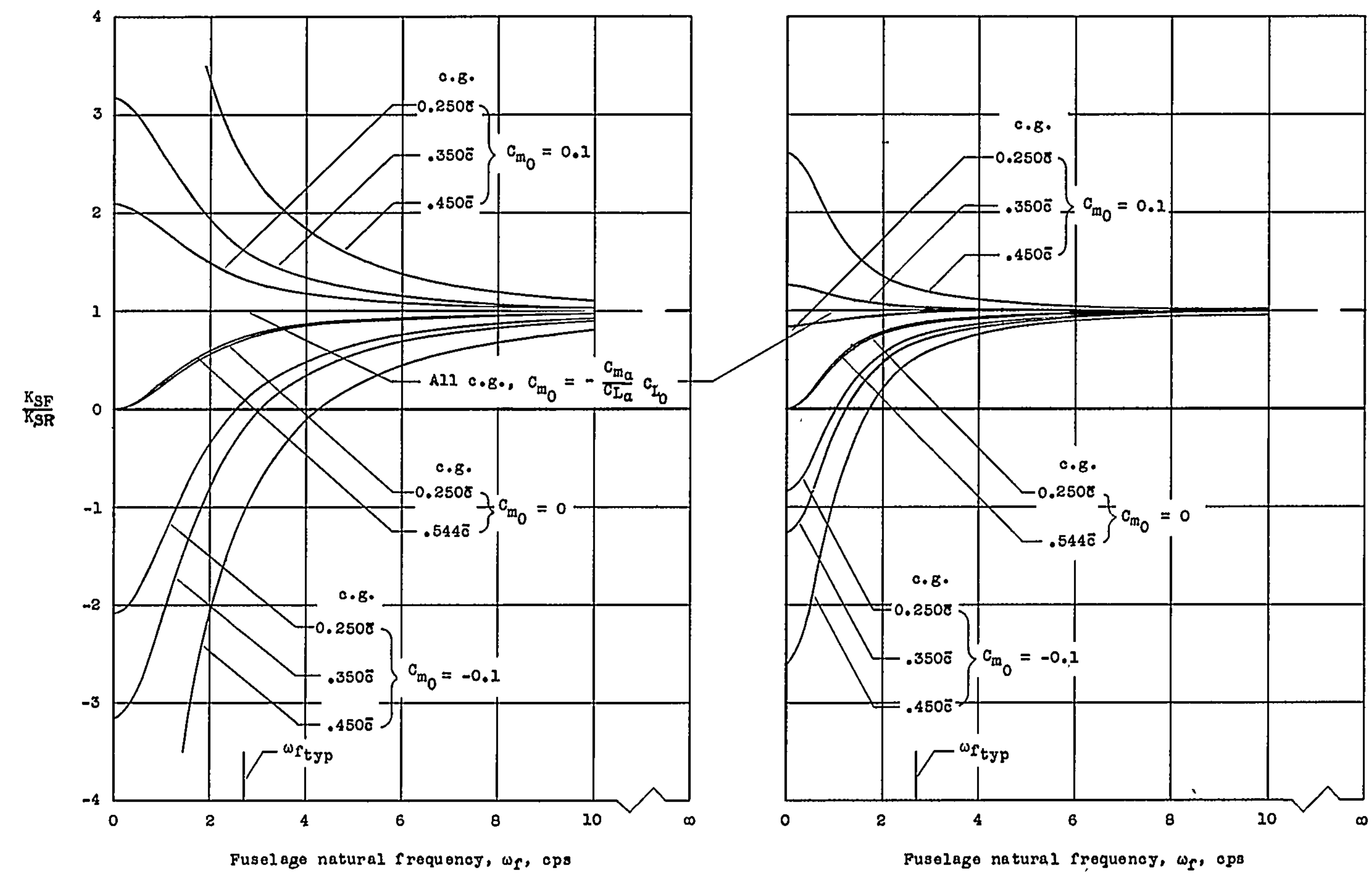
(b) Altitude, 30,000 feet.

Figure 2.- Semirigid and quasi-static solutions for period and damping of short-period longitudinal mode and fuselage mode of a large airplane as a function of fuselage natural frequency for various airplane center-of-gravity locations. Mach number, 0.7; rigid-body values at $\omega_f = \infty$.



(a) Altitude, 8,000 feet. (b) Altitude, 30,000 feet.

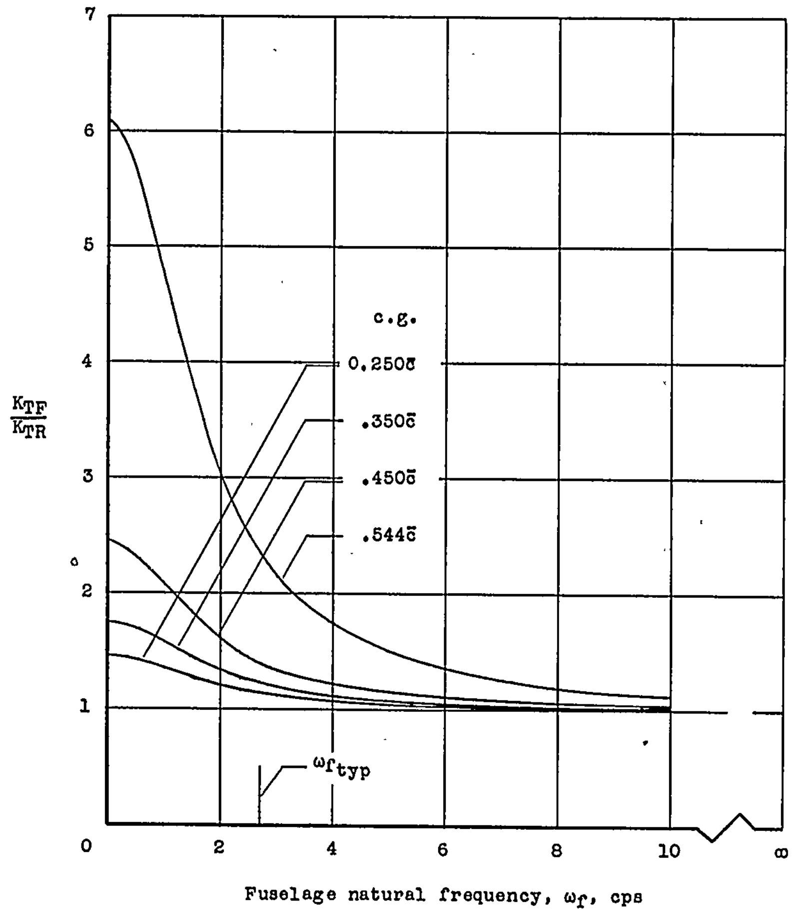
Figure 3.- Semirigid solution for period and damping of short-period longitudinal mode of a large airplane as a function of dynamic pressure for various fuselage natural frequencies. Airplane center of gravity, $0.25\bar{c}$.



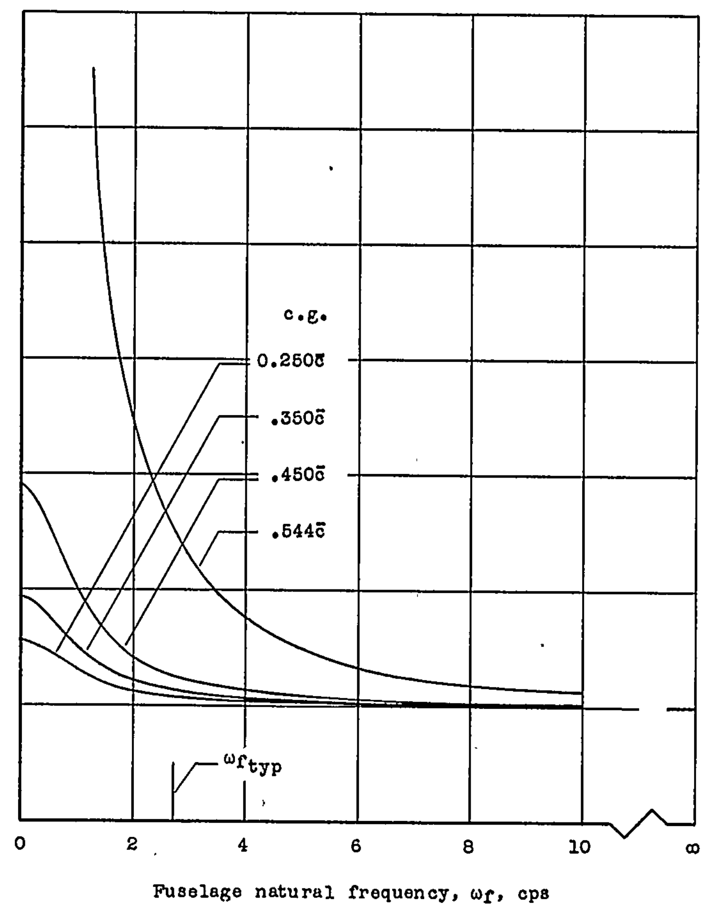
(a) Altitude, 8,000 feet.

(b) Altitude, 30,000 feet.

Figure 4.- Variation of static-stability-margin ratio with fuselage natural frequency for a large airplane in straight flight at several airplane center-of-gravity locations. Mach number, 0.7.

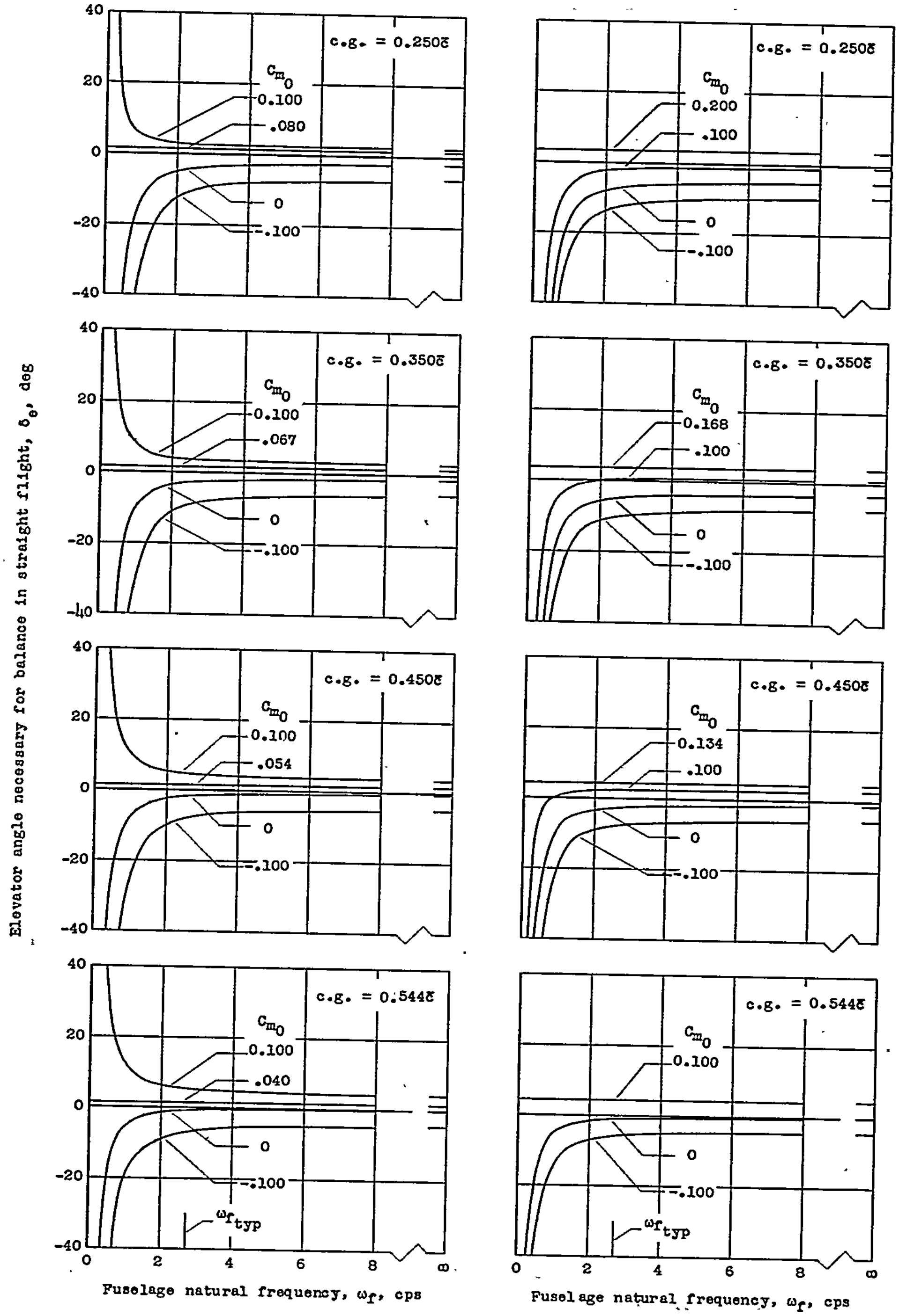


(a) Altitude, 8,000 feet.



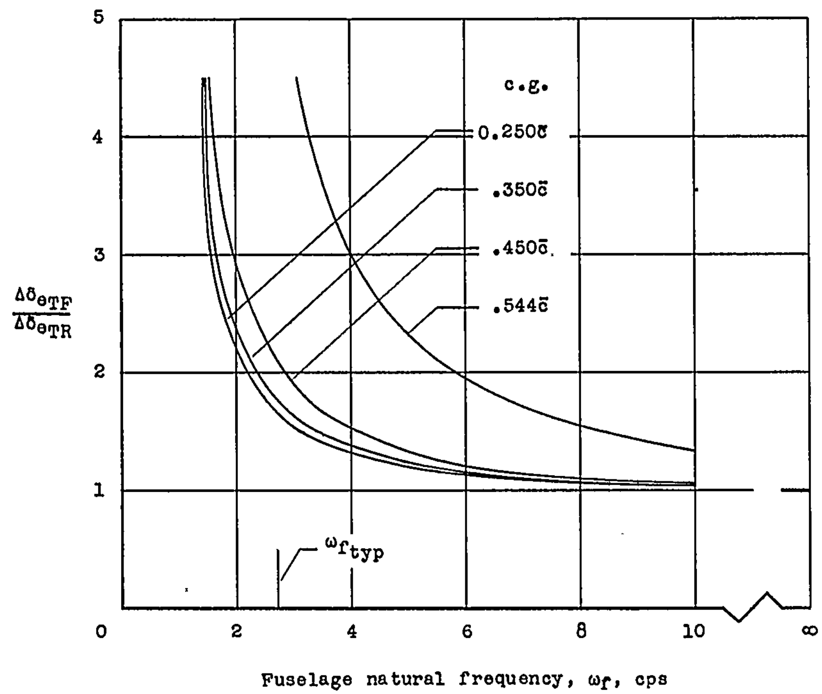
(b) Altitude, 30,000 feet.

Figure 5.- Variation of maneuvering-flight stability-margin ratio with fuselage natural frequency for a large airplane at several airplane center-of-gravity locations. Mach number, 0.7.

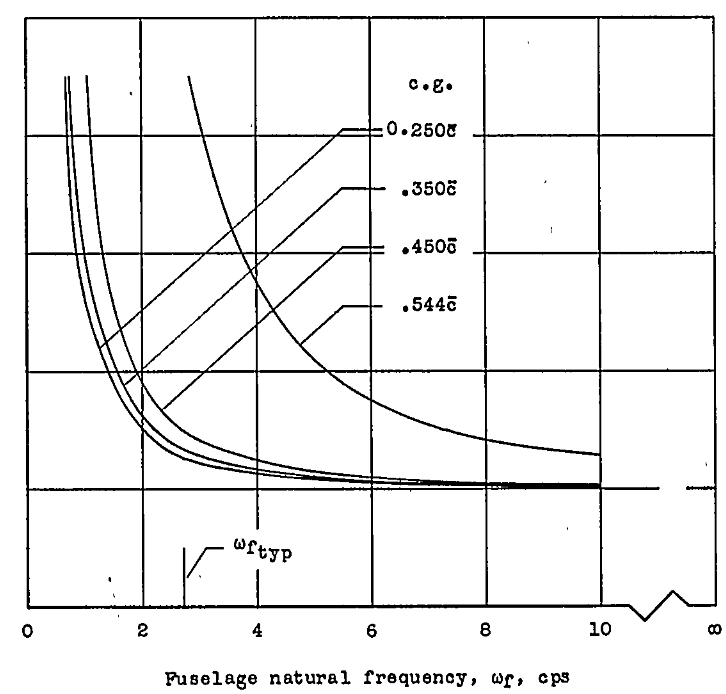


(a) Altitude, 8,000 feet. (b) Altitude, 30,000 feet.

Figure 6.- Variation of elevator angle necessary for balance in straight flight with fuselage natural frequency for a large airplane at several airplane center-of-gravity locations. Mach number, 0.7; rigid-body values at $\omega_f = \infty$.



(a) Altitude, 8,000 feet.



(b) Altitude, 30,000 feet.

Figure 7.- Variation of elevator-angle-increment ratio necessary to maintain steady maneuvering flight with fuselage natural frequency for a large airplane at several airplane center-of-gravity locations. Mach number, 0.7.

NACA - Langley Field, Va.