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MECHANISM OF GENERATION OF PRESSURE WAVES AT FLAME FRONTS

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SUMMARY

Pressure waves are known to be generated at a flame front whenever there is a change in the flame speed or the heating value or density of a combustible mixture. It is shown that if the specific-heats ratio γ of the burned and unburned gases are the same, the pressure waves generated at the flame front are really caused by a change in the rate of heat release at the flame. In the matter of generation of pressure waves, therefore, a flame behaves essentially like a heater. The performance of a flame and a heater is then compared and the conditions under which the two are dynamically equivalent are stated.

INTRODUCTION

When a flame front experiences a change in the flame speed or a change in the heating value or density of the mixture it consumes, pressure waves are generated. For a plane flame front, two families of pressure waves of essentially the same strength are produced, one propagating into the fresh combustible mixture and the other propagating into the burned gas. The strength of the pressure waves generated has been calculated (ref. 1). These pressure waves play an important role in the explanation of many wave phenomena associated with the flame, such as flame oscillation and the development of detonation waves.

In reference 1, the analysis is principally of a mathematical nature. No discussion or demonstration is given to show the mechanism involved in the generation of the pressure waves. In this report, it is shown that the pressure waves generated at the flame front as a result of changes in the flame speed, heating value, density, and so forth can be attributed to a change in the rate of heat release at the flame front. Consequently, these pressure waves are generated as a result of the changes in the rate of expansion of the burned gas behind the flame front.

Pressure waves generated by the expansion of a heated gas have been previously investigated by several authors in connection with different problems. Taylor (ref. 2) in his study of the blast generated by an atomic explosion examined the flow field generated by the instant release of a large amount of heat at a point in space. Lin (ref. 3) solved the same problem for the two-dimensional case and applied the result to the discussion of shock waves generated by thunderbolts and by meteors or missiles moving at hypersonic speeds. Wu (ref. 4) studied the linearized theory of pressure waves generated by heat release, taking into account both the compressibility effects and the heat conductivity. Chu (ref. 5) examined the pressure waves produced by heat addition at a constant rate in tubes and at a parabolic rate in space and applied the results to the shock waves sustained by a uniformly expanding flame front.

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SYMBOLS

A	cross-sectional area of a tube
A_f	flame area
C_p	specific heat at constant pressure
c	velocity of sound
$M_1 = \frac{u_1}{c_1}$	
p	pressure
Q	heat release per unit mass of medium
R	gas constant
S	entropy
S_a	apparent flame speed (relative to given coordinate system)
S_t	flame speed
T	temperature
u	velocity

Δ operator signifying "finite increment of"

δ operator signifying "small change of"

γ ratio of C_p to specific heat at constant volume

ρ density

ω rate of heat release per unit flame (or heater) area

Subscripts 1 and 2 indicate, respectively, conditions of flow ahead of and behind the flame (or the heater).

MECHANISM OF GENERATION OF PRESSURE WAVES

Let us consider a plane flame front propagating into a combustible mixture. A flame front considered as a surface of discontinuity is characterized by its speed of propagation relative to the combustible medium (i.e., the flame speed S_t) and by the amount of heat it releases per unit mass of the fresh mixture it consumes (i.e., the heating value Q). The flame speed and heating value of a given mixture are supposed to be known functions of the thermodynamic state of the mixture.

Quantitative analysis of the pressure waves generated by the flame can be carried out simply as follows (ref. 1). Suppose that the medium ahead of the flame front has a pressure p_1, a temperature T_1, a density ρ_1, a velocity u_1, and a velocity of sound c_1. The corresponding quantities for the medium behind the flame are denoted, respectively, by p_2, T_2, ρ_2, u_2, and c_2. The flame itself is seen to be propagating in the direction of the negative x-axis (fig. 1) with a speed S_a (so that the flame speed $S_t = S_a + u_1$). Suppose that there is a small change in flame speed from S_t to $S_t + \delta S_t$ and/or a small change in the heating value of the mixture from Q to $Q + \delta Q$ and/or a small change in the entropy (or density) of the fresh mixture from S_1 to $S_1 + \delta S_1$, where $\frac{\delta S_t}{S_t}$, $\frac{\delta Q}{Q}$, and $\frac{\delta S_1}{S_1} \ll 1$. At the flame front, in order to maintain the conservation laws, pressure waves must be generated. These pressure waves increase the pressure immediately ahead of and behind the flame front to $p_1 + \delta p_1$ and $p_2 + \delta p_2$, respectively, the velocity immediately ahead of and behind the flame front to $u_1 + \delta u_1$ and $u_2 + \delta u_2$, respectively, and so forth. When $\frac{\delta S_t}{S_t}$, $\frac{\delta Q}{Q}$, and $\frac{\delta S_1}{S_1} \ll 1$, then $\frac{\delta p_1}{p_1}$, $\frac{\delta p_2}{p_2}$, $\frac{\delta u_1}{S_t}$, and $\frac{\delta u_2}{S_t} \ll 1$, so that the δp's and δu's are related by the characteristic relations for plane waves

$$\frac{\delta p_1}{p_1} = -\gamma_1 \frac{\delta u_1}{c_1} \tag{1}$$

$$\frac{\delta p_2}{p_2} = \gamma_2 \frac{\delta u_2}{c_2} \tag{2}$$

In addition, the equations representing the conservation of mass, momentum, and energy at the flame front can be combined into two equations involving δp_1, δp_2, δu_1, and δu_2 which are

$$\frac{\delta p_1}{p_1} = \frac{\delta p_2}{p_2} \tag{3}$$

$$\frac{\delta u_2 - \delta u_1}{S_t} + \frac{\gamma_2 - 1}{\gamma_2}\left(\frac{C_{p_2}T_2}{C_{p_1}T_1} - 1\right)\frac{\delta p_1}{p_1} = \frac{\delta S_t}{S_t}\left(\frac{R_2T_2}{R_1T_1} - 1\right) + \frac{\gamma_2 - 1}{\gamma_2}\frac{\delta Q}{R_1T_1} - \frac{\gamma_2 - 1}{\gamma_2}\left(\frac{C_{p_2}T_2}{C_{p_1}T_1} - 1\right)\frac{\delta S_1}{R_1} \tag{4}$$

(cf. eqs. (16a) and (16b) of ref. 1) provided that terms of the order of the square (or higher power) of the Mach number of the flame are neglected and products of $\frac{\delta S_t}{S_t}$, $\frac{\delta Q}{Q}$, $\frac{\delta S_1}{S_1}$, $\frac{\delta p_1}{p_1}$, $\frac{\delta p_2}{p_2}$, $\frac{\delta u_1}{S_t}$, and $\frac{\delta u_2}{S_t}$ are dropped out. In these equations, R_1, R_2, C_{p_1}, C_{p_2}, γ_1, and γ_2 are, respectively, the gas constants, specific heats at constant pressure, and specific-heats ratios of the medium immediately ahead of and behind the flame front. From equations (1) to (4) solutions may be obtained for δp_1, δp_2, δu_1, and δu_2, thus giving the strength and sign of the pressure waves generated at the flame front.

Although quantitative details describing the flow field following changes in the flame speed, heating value, and/or entropy of the combustible mixture have been given and discussed in reference 1, as yet nothing concerning the basic physical mechanism responsible for the generation of pressure waves at the flame front has appeared. A remarkable property of equation (4) clarifies the reason. It is shown subsequently that the effect of changes in the flame speed S_t, the heating value Q, and/or the entropy S_1 can be accounted for by the effect resulting from the change in a single physical variable ω, the rate of heat release per unit area of the flame front, provided that $\gamma_1 = \gamma_2$; for, of the four equations (eqs. (1) to (4)) which determine the four unknowns δp_1, δp_2, δu_1, and δu_2, the changes δS_t, δQ, and δS_1 occur only in the last equation (eq. (4)) and yet these changes can be combined into the change of the single variable ω.

The rate of heat release per unit flame area, by definition, is

$$\omega = S_t \rho_1 Q \tag{5}$$

so that there will, in general, be a change in ω whenever there are changes in S_t, ρ_1, and/or Q. Assuming that these changes are small, then

$$\frac{\delta\omega}{\omega} = \frac{\delta S_t}{S_t} + \frac{\delta\rho_1}{\rho_1} + \frac{\delta Q}{Q} \tag{6}$$

Now, $\delta\rho_1/\rho_1$ is related to the change in entropy by the equation of state

$$\frac{\delta\rho_1}{\rho_1} = -\frac{\delta S_1}{C_{p_1}} + \frac{1}{\gamma_1}\frac{\delta p_1}{p_1} \tag{7}$$

and Q is related to the change of temperature across the flame by

$$Q = C_{p_2}T_2 - C_{p_1}T_1 \tag{8}$$

(provided that terms of order of the square of Mach number of the flame are neglected). Substituting equations (7) and (8) into equation (6) and multiplying the result by $\left(\frac{C_{p_2}T_2}{C_{p_1}T_1} - 1\right)$ give

$$\left(\frac{C_{p_2}T_2}{C_{p_1}T_1} - 1\right)\frac{\delta S_t}{S_t} + \frac{\delta Q}{C_{p_1}T_1} - \left(\frac{C_{p_2}T_2}{C_{p_1}T_1} - 1\right)\frac{\delta S_1}{C_{p_1}} = \left(\frac{C_{p_2}T_2}{C_{p_1}T_1} - 1\right)\left(\frac{\delta\omega}{\omega} - \frac{1}{\gamma_1}\frac{\delta p_1}{p_1}\right) \tag{9}$$

Comparing equation (9) with the right-hand side of equation (4), it is at once evident that if $\gamma_1 = \gamma_2$, equation (4) can be simplified to

$$\frac{\delta u_2 - \delta u_1}{S_t} = \left(\frac{R_2T_2}{R_1T_1} - 1\right)\left(\frac{\delta\omega}{\omega} - \frac{\delta p_1}{p_1}\right) \tag{10}$$

Thus, δp_1, δp_2, δu_1, and δu_2 can be calculated from the four equations (1), (2), (3), and (10) instead of equations (1) to (4); furthermore, it is proved that the pressure waves generated by changes in flame speed, heating value, and density of the combustible mixture are really caused by a change in the rate of heat release at the flame front provided that $\gamma_1 = \gamma_2$. The physical mechanism involved in the generation of pressure waves at the flame front is now clear. A change in the flame speed, heating value, and density of the fresh combustible mixture generally leads to a change in the rate of heat release at the flame front. When the rate of heat release by the flame is changed, the rate of expansion of the volume of the burned gas is altered. This can best be seen by following a volume (or a lump) of combustible mixture through the flame (see fig. 2). As far as the fluid particles outside the volume of gas are concerned, the fictitious boundaries of the volume (shown by dashed lines in fig. 2) are not different from a solid wall expanding at the same speed as the boundaries. Consequently, when there is a change in the rate of expansion of the burned gas, pressure waves are produced.

The fact that the nonhomogeneous terms on the right-hand side of equation (4) can be nicely combined into a single physical variable ω, the rate of heat release per unit area of the flame (for the case $\gamma_1 = \gamma_2$), suggests that the whole analysis might have become simpler if ω had been introduced at an earlier stage. This may be demonstrated and a better insight into the role played by ω may be acquired if equation (10) is derived directly from the first principles. In so doing, the accuracy of the conclusions already reached and the factors neglected may be seen more clearly.

Using the notations already introduced (fig. 1), consideration of the conservation of mass, momentum, and energy at the flame front leads to the three equations

$$\rho_2\left(u_2 + S_a\right) = \rho_1\left(u_1 + S_a\right) \tag{11}$$

$$p_2 + \rho_2\left(u_2 + S_a\right)^2 = p_1 + \rho_1\left(u_1 + S_a\right)^2 \tag{12}$$

$$\omega = \rho_2\left(u_2 + S_a\right)\left(C_{p_2}T_2 + \frac{1}{2}u_2^2\right) - \rho_1\left(u_1 + S_a\right)\left(C_{p_1}T_1 + \frac{1}{2}u_1^2\right) \tag{13}$$

If the flow conditions immediately ahead of the flame front (indicated by the subscript 1), as well as ω, R_2, and C_{p_2}, are assumed to be given, the three equations ((11) to (13)), together with the gas law and the definition of the flame speed S_t (also assumed given) which are

$$p_2 = \rho_2 R_2 T_2 \tag{14}$$

$$S_t = S_a + u_1 \tag{15}$$

can be used to solve for the five unknowns p_2, ρ_2, T_2, u_2, and S_a. Now, under normal conditions the speed of propagation of a deflagration wave is much smaller than the local speed of sound, the ratio being of the order of 10^{-2} to 10^{-3}. If a coordinate system is chosen so that u_1 is also small compared with the local sound speed, then, by solving equations (11) to (15), it can be easily shown that both u_2 and S_a are small compared with the local sound speed. Hence, equations (12) and (13) can be simplified to

$$p_2 = p_1 \tag{16}$$

$$\omega = \rho_2\left(u_2 + S_a\right)C_{p_2}T_2 - \rho_1\left(u_1 + S_a\right)C_{p_1}T_1 \tag{17}$$

Using the gas law, equation (17) can be rewritten as

$$\omega = \frac{\gamma_2}{\gamma_2 - 1}p_2\left(u_2 + S_a\right) - \frac{\gamma_1}{\gamma_1 - 1}p_1\left(u_1 + S_a\right) \tag{18}$$

It is observed that equations (18) and (16) do not contain the variables ρ and T but only contain p and u.

Now, suppose that there is a sudden change in the rate of heat release from ω to $\omega + \Delta\omega$, where $\Delta\omega$ need not be small compared with ω. There will be corresponding changes in p_1, p_2, u_1, u_2, and S_a to $p_1 + \Delta p_1$, $p_2 + \Delta p_2$, $u_1 + \Delta u_1$, $u_2 + \Delta u_2$, and $S_a + \Delta S_a$, where the changes again need not be small. The conservation laws must hold at all instants; therefore,

$$p_2 + \Delta p_2 = p_1 + \Delta p_1$$

$$\omega + \Delta\omega = \frac{\gamma_2}{\gamma_2 - 1}\left(p_2 + \Delta p_2\right)\left(u_2 + S_a + \Delta u_2 + \Delta S_a\right) - \frac{\gamma_1}{\gamma_1 - 1}\left(p_1 + \Delta p_1\right)\left(u_1 + S_a + \Delta u_1 + \Delta S_a\right)$$

which may be simplified to

$$\left.\begin{aligned}
&\Delta p_2 = \Delta p_1 \\
&\Delta\omega = \frac{\gamma_2}{\gamma_2 - 1}\Delta p_1\left(u_2 - u_1\right) + \frac{\gamma_2}{\gamma_2 - 1}\left(p_1 + \Delta p_1\right)\left(\Delta u_2 - \Delta u_1\right) + \\
&\qquad \left(\frac{\gamma_2}{\gamma_2 - 1} - \frac{\gamma_1}{\gamma_1 - 1}\right)\left[\Delta p_1\left(u_1 + S_a\right) + \left(p_1 + \Delta p_1\right)\left(\Delta u_1 + \Delta S_a\right)\right]
\end{aligned}\right\} \tag{19}$$

if use is made of equations (16) and (17). Equation (15) must also be satisfied at all instants; therefore,

$$\Delta S_t = \Delta S_a + \Delta u_1 \tag{20}$$

Substituting this equation and equation (15) into equations (19) gives

$$\left.\begin{aligned}
&\Delta p_2 = \Delta p_1 \\
&\Delta\omega = \frac{\gamma_2}{\gamma_2 - 1}\Delta p_1\left(u_2 - u_1\right) + \frac{\gamma_2}{\gamma_2 - 1}\left(p_1 + \Delta p_1\right)\left(\Delta u_2 - \Delta u_1\right) + \\
&\qquad \left(\frac{\gamma_2}{\gamma_2 - 1} - \frac{\gamma_1}{\gamma_1 - 1}\right)\left[S_t\,\Delta p_1 + p_1\,\Delta S_t + \Delta p_1\,\Delta S_t\right]
\end{aligned}\right\} \tag{21}$$

It is seen that, in addition to the variables Δp_1, Δp_2, Δu_1, and Δu_2, there are two types of nonhomogeneous terms in equations (21), namely, $\Delta\omega$ and ΔS_t, unless $\gamma_1 = \gamma_2$, in which case only the term $\Delta\omega$ survives. If, now, only infinitesimal changes are considered, equations (21) become

$$\delta p_2 = \delta p_1$$

$$\delta\omega = \frac{\gamma_2}{\gamma_2 - 1}\delta p_1\left(u_2 - u_1\right) + \frac{\gamma_2}{\gamma_2 - 1}p_1\left(\delta u_2 - \delta u_1\right) + \left(\frac{\gamma_2}{\gamma_2 - 1} - \frac{\gamma_1}{\gamma_1 - 1}\right)\left(S_t\ \delta p_1 + p_1\ \delta S_t\right)$$

which, for the case $\gamma_1 = \gamma_2$, can easily be reduced to equation (10). $\Bigg($In this connection, one need only observe that $u_2 - u_1 = S_t\left(\dfrac{u_2 + S_a}{u_1 + S_a} - 1\right) = S_t\left(\dfrac{\rho_1}{\rho_2} - 1\right) = S_t\left(\dfrac{R_2T_2}{R_1T_1} - 1\right)$ and $\omega = S_t\rho_1 Q = S_t p_1\left(\dfrac{C_{p_2}T_2}{C_{p_1}T_1} - 1\right)\dfrac{\gamma_1}{\gamma_1 - 1}.\Bigg)$ For the case $\gamma_1 \neq \gamma_2$, it is clear that the pressure waves generated at the flame front cannot all be attributed to changes in the rate of heat release at the flame front.

If $\gamma_1 = \gamma_2$, equations (21) become

$$\left.\begin{aligned} \Delta p_2 &= \Delta p_1 \\ \Delta\omega &= \frac{\gamma}{\gamma - 1}\Delta p_1\left(u_2 - u_1\right) + \frac{\gamma}{\gamma - 1}\left(p_1 + \Delta p_1\right)\left(\Delta u_2 - \Delta u_1\right) \end{aligned}\right\} \qquad (22)$$

where the subscripts on γ have been deleted. In deriving these equations it is assumed only that the Mach number of the flame is so small that the square and higher powers of Mach number can be neglected and it is not assumed that the increments Δp_1, Δp_2, Δu_1, and Δu_2 are small. These finite increments are produced by shock waves propagating into the burned and unburned mixture. Now, across a shock wave, the Δp's and Δu's are related by

$$\frac{\Delta u_1}{c_1} = -\frac{\frac{1}{\gamma}\frac{\Delta p_1}{p_1}}{\sqrt{\frac{\gamma + 1}{2\gamma}\frac{\Delta p_1}{p_1} + 1}} \tag{23}$$

$$\frac{\Delta u_2}{c_2} = \frac{\frac{1}{\gamma}\frac{\Delta p_2}{p_2}}{\sqrt{\frac{\gamma + 1}{2\gamma}\frac{\Delta p_2}{p_2} + 1}} \tag{24}$$

instead of equations (1) and (2). Equations (22) to (24) are then used to calculate Δp_1, Δp_2, Δu_1, and Δu_2 in terms of $\Delta\omega$. Thus,

$$\Delta\omega = \frac{\gamma}{\gamma - 1}\Delta p_1\left(u_2 - u_1\right) + \frac{\gamma}{\gamma - 1}\frac{p_1\left(1 + \frac{\Delta p_1}{p_1}\right)}{\sqrt{\frac{\gamma + 1}{2\gamma}\frac{\Delta p_1}{p_1} + 1}}\frac{c_2 + c_1}{\gamma}\frac{\Delta p_1}{p_1} \tag{25}$$

from which one can calculate Δp_1 in terms of $\Delta\omega$. The first term on the right side of the equation is actually small compared with the second term since it is of the order of the Mach number of the flame. A set of curves of $\frac{\Delta p_1}{p_1}$ versus $\frac{\Delta\omega}{p_1 c_1}$ for $\frac{c_2}{c_1} = 1.5$, 2.0, 2.5, and 3.0 and $\gamma = 1.4$ (fig. 3). It is seen from equation (25) that the strength of the shock wave (measured in terms of the pressure ratio across the shock wave) generated at a flame front as a result of a large change in the rate of heat release at the flame front is proportional to the two-thirds power of the nondimensional heat-release parameter $\frac{\Delta\omega}{1/2\left(c_1 + c_2\right)p_1}$.

In fact,

$$\frac{\Delta p_1}{p_1} = \left[\frac{\gamma - 1}{2}\sqrt{\frac{\gamma + 1}{2\gamma}}\frac{\Delta\omega}{1/2\left(c_1 + c_2\right)p_1}\right]^{2/3}$$

This conclusion is analogous to that presented in reference 5 (see eq. (40) of ref. 5).

Together with the generation of shock waves at the flame front, a contact surface is produced at the instant when the rate of heat release changes abruptly. The temperature jump across the contact surface can be readily determined from the continuity equation after Δu_2 is known.

For small changes in ω the following equation then applies:

$$\frac{\delta p_1}{p_1} = \frac{\gamma - 1}{c_1 + c_2}\frac{\Delta\omega}{p_1} \tag{26}$$

Using the technique of Friedrichs (ref. 6) and equations (19), one can also calculate the behavior of the shock wave generated by an accelerating flame, provided that the scattering of the pressure waves by the continuous variation in the entropy of the medium behind the flame front can be ignored.

If one wishes to examine the individual effect of changes in flame speed, heating value, and density of the fresh gas, it is only necessary to calculate $\Delta\omega$ in terms of ΔS_t, ΔQ, and $\Delta\rho_1$. Making use of equation (5) and assuming that $\frac{\delta p_1}{p_1}$, $\frac{\delta\rho_1}{\rho_1}$, $\frac{\delta Q}{Q}$, and $\frac{\delta S_t}{c_1}$ $\left(\text{but not } \frac{\delta S_t}{S_t}\right) \ll 1$, equations (19) become

$$\frac{\delta p_2}{p_2} = \frac{\delta p_1}{p_1}$$

$$\frac{\delta u_2 - \delta u_1}{S_t} = \frac{\delta S_t}{S_t}\left(\frac{R_2 T_2}{R_1 T_1} - 1\right) + \frac{R_2}{R_1}\left(1 + \frac{\delta S_t}{S_t}\right)\frac{\delta Q}{C_{p_1} T_1} -$$

$$\frac{R_2}{R_1}\left(1 + \frac{\delta S_t}{S_t}\right)\left(\frac{C_{p_2} T_2}{C_{p_1} T_1} - 1\right)\frac{\delta S_1}{C_{p_2}} - \frac{\gamma_2 - 1}{\gamma_2}\left(1 + \frac{\delta S_t}{S_t}\right)\left(\frac{C_{p_2} T_2}{C_{p_1} T_1} - 1\right)\frac{\delta p_1}{p_1}$$

which should replace equations (14a) and (14b) of reference 1 where a misprint in the derivation of the equations has resulted in a few more terms than are given here. (Fortunately, results of ref. 1 are not affected by the errors indicated here.)

FLAME AND HEATER

Since the pressure waves generated at a flame front can all be attributed to a change in the rate of heat release at the flame front (assuming $\gamma_1 = \gamma_2$), it is natural to ask under what conditions a flame can be considered a heater.

To determine these conditions consider a plane heating element releasing heat at the same rate ω as a given flame (i.e., that given by eq. (5)) and moving with a given speed S_a in the direction of the negative x-axis. If the pressure, temperature, density, and velocity of the flow immediately ahead of and behind the heating element are denoted by p_1, p_2, T_1, T_2, ρ_1, ρ_2, u_1, and u_2, respectively, and if it is assumed that the heating element has the property of changing the gas constant and specific heat at constant pressure of the gas it heated from R_1 and C_{p_1} to R_2 and C_{p_2}, respectively, then the equations of continuity, momentum, and energy which apply at the heater are exactly the same as those which apply at the flame front, that is, equations (11) to (13). The chief difference between a flame and a heater lies in the fact that, for a flame, S_a (see eqs. (11) to (13)) is not known and is related to the flame speed S_t by equation (15), whereas, for a heater, S_a is assumed to be given. Of course, if the heater should move in such a manner that equation (15) is always satisfied (even if u_1 is not constant with time), then there is absolutely no difference between a flame and a heater. However, the performance of such a heater is in no way easier to visualize than the performance of the flame itself when the flow field is not uniform. If, on the other hand, the heater should move in some other way, the performance of the heater and the flame will no longer be the same. The problem is then to determine the extent to which this difference between the flame and heater is important.

This problem can be approached by comparing the "dynamic behavior"[1] of a given flame with that of a heater releasing heat at a rate equal to the rate of heat release of the flame. The flow fields ahead of and behind the heater are therefore identical with those of the flame. Now, suppose that there is a change in the rate of heat release at the flame; the motion of the flame will change in such a manner that equation (15) remains valid. On the other hand, however, for the same change in the rate of heat release at the heater, the motion of the heater may assume any speed one desires. Since all the equations representing the conservation laws applied at the heater and the flame are identical in form, equations (16) to (19) apply equally to the flame as to the heater. When the rate of heat release at the heater is suddenly changed from ω to $\omega + \delta\omega$, pressure waves must be

[1]By the dynamic behavior of a flame (or a heater) is meant the pressure and velocity (but not temperature and density) fields produced by the flame (or heater) as a result of a change in the rate of heat release at the flame (or heater).

generated at the heater in such a way that equations (19) are satisfied. In these equations, S_a and ΔS_a are known; their values depend on the type of motion assumed for the heater. In the case of the flame, however, pressure waves must be generated at the flame in such a way that equations (21) are satisfied. The strength of the pressure waves generated by the heater and the flame are therefore, in general, different. However, it is observed that if $\gamma_1 = \gamma_2$, equations (19) become identical with equations (21). As a result, exactly the same conditions are imposed on the pressure waves generated at the heater as at a flame front. Furthermore, these conditions are independent of the motion of the heater after the change in rate of heat release has taken place, since the term ΔS_a drops out altogether from the equations. Consequently, it can be concluded that the dynamic behavior of a flame is identical with that of a heating element releasing heat at the same rate as the flame at all instants if (1) the flame propagates with a speed small in comparison with the local sound speed, (2) there is a current of flow through the heater so that the relative velocity between the flow and the heater is just equal to the flame speed, and (3) $\gamma_1 = \gamma_2$.[2]

From this analysis one cannot help but feel intuitively that, as far as the generation of pressure waves is concerned, a flame behaves essentially like a heater. If such intuition is correct, one may even attempt to analyze one-dimensionally certain problems which do not appear to be one-dimensional at first sight. Thus, consider the propagation of a flame in a tube. It is found that, in general, the flame front assumes a curved shape. In fact, this must be so because of the quenching and cooling effects near the wall. After the mixture is ignited, the flame will at first propagate down the tube with a uniform velocity. It then starts to oscillate with increasing amplitude. As the flame oscillates its shape also changes, becoming alternatively fuller and flatter (see ref. 7). Such a problem can be treated with a one-dimensional model if the flame is considered a heater and if the tube is long in comparison with its diameter. Thus, if S_t, A_f, ρ_1, and Q are, respectively, the flame speed, flame area, density, and heating value of the combustible mixture, then the rate of heat release by the flame is $A_f S_t \rho_1 Q$. If the flame is replaced by a plane heater and if A is the cross-sectional area of the tube, then the average rate of heat release per unit area of the plane

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[2]In a prepublication review of reference 5 for the NACA, Dr. Harold Mirels of the Lewis Laboratory, NACA, independently arrived at almost the same conclusion. He showed by rather intuitive reasoning that a flame and a heater are equivalent under conditions (1) and (3) given here. However, in actuality, condition (2) cannot be omitted in establishing the dynamic equivalence of the two since the coefficient $(u_2 - u_1)$ in equations (19) and (21) must have the same value.

heater will be

$$\omega = \frac{A_f}{A} S_t \rho_1 Q \tag{29}$$

If the rate of heat release of the heater can be adjusted so that equation (29) is satisfied at all instants, the pressure and velocity fields generated by the flame may perhaps be reproduced. Furthermore, it also becomes apparent from equation (29) how the observed oscillation must have been induced. A dynamic system will only start to oscillate with increasing amplitude either when it is subjected to repeated external excitations applied at some characteristic frequencies or when the system itself contains an energy source which interacts with the oscillation in such a manner as to reinforce the vibration. The first phenomenon is usually referred to as resonance; the second, as instability. Clearly, the example cited is a case of instability and the energy source is the flame itself. Anyone who has observed a flame carefully is usually struck by the remarkable sensitivity of the flame configuration to flow disturbances. Now, if there are some disturbances in the flow, they will easily cause a change in flame configuration and, in general, an accompanying change in the flame area A_f. When A_f changes, the rate of heat release by the flame (or the equivalent heater, cf. eq. (29)) is also changed. This produces pressure waves which under proper circumstances will reinforce the initial disturbance. In this manner the flame will oscillate with increasing amplitude as it propagates down the tube. It is believed that the same mechanism is responsible for the self-excitation of vibration of practically all systems containing a flame front (excluding diffusive flames).

With the one-dimensional model just mentioned, the pressure waves generated at the flame front as a result of a change in the flame area can easily be calculated. Thus, it is seen from equation (29) that

$$\frac{\Delta\omega}{\omega} = \frac{\Delta A_f}{A_f} \tag{30}$$

Substitution of equation (30) into equations (25) and (26) gives the strength of the pressure waves generated.

An earlier attempt at studying the effects of changes in the flame area was made by Blackshear (ref. 8). In the notation used herein, the boundary conditions at the flame (given in ref. 8) are

$$\delta p_2 - \delta p_1 = O\left(M_1^2\right) \tag{31a}$$

$$\delta u_2 - \delta u_1 = u_1\left(\frac{\rho_1}{\rho_2} - 1\right)\frac{\delta A_f}{A_f} \tag{31b}$$

where $M_1 = \frac{u_1}{c_1}$. The first equation is equivalent to equation (3). If equation (30) is substituted into equation (10), the second boundary condition at the flame is obtained as

$$\delta u_2 - \delta u_1 = u_1\left(\frac{\rho_1}{\rho_2} - 1\right)\left(\frac{\delta A_f}{A_f} - \frac{\delta p_1}{p_1}\right) \tag{31c}$$

which contains one more term than equation (31b). However, this additional term is of the order of M_1 (see eq. (1)) and, hence, is unimportant if $M_1 \ll 1$.

It should be added here that boundary conditions (31a) and (31c) are not sufficient to solve such a problem as the instability induced by the flame. The answer is still needed to the important question of how the change of the flame area δA_f should vary with the fluctuations in the flow. The answer, in general, will depend upon the system considered and must be analyzed for each individual case. It is this dependence which introduces a time lag in the response of a flame front to flow disturbances in many systems where the main feedback mechanism responsible for the self-excited oscillations is known to be due to the fluctuation in flame area with flow disturbances.

RECAPITULATION AND DISCUSSION

The present analysis provides strong indications that the principal physical cause of the production of pressure waves at a flame front is the change in the rate of heat released by the flame. This change in the rate of heat release could be induced by a change in the flame area (accompanying any change in flame configuration) and/or a change in the flame speed and/or a change in the density of the unburned medium.

It is proved that the pressure waves generated at plane flame fronts of zero thickness are all due to the change in the rate of heat release at the flame front when the specific-heat ratios γ of the burned and unburned gases are the same. The exact conditions of the equivalence of a plane flame front and a heater have been stated previously. The assumption of a flame front of zero thickness necessarily implies an instantaneous response of the flame chemistry to pressure disturbances or other changes. (See footnote, p. 604, ref. 1.) The assumptions of a plane flame front and one-dimensional disturbances eliminate the time lag in the response of the flame to changes in flow conditions.

In practice, the flame front is never exactly plane; neither are the flow disturbances one dimensional. Owing to such lack of one dimensionality the total rate of heat release by a flame does not vary with changes in flow conditions instantaneously. This, or whatever other causes introduce a time lag in the response of the flame to flow disturbances, has an important bearing in the study of the self-excited oscillations in a system containing a heat source. However, if the mechanism of production of pressure waves suggested by this analysis is true, the flame will still be similar to a heater in the production of pressure waves if the rate of heat release by the heater lags behind the flow disturbances in exactly the same manner as that of the flame.

The Johns Hopkins University,
Baltimore, Md., October 1, 1954.

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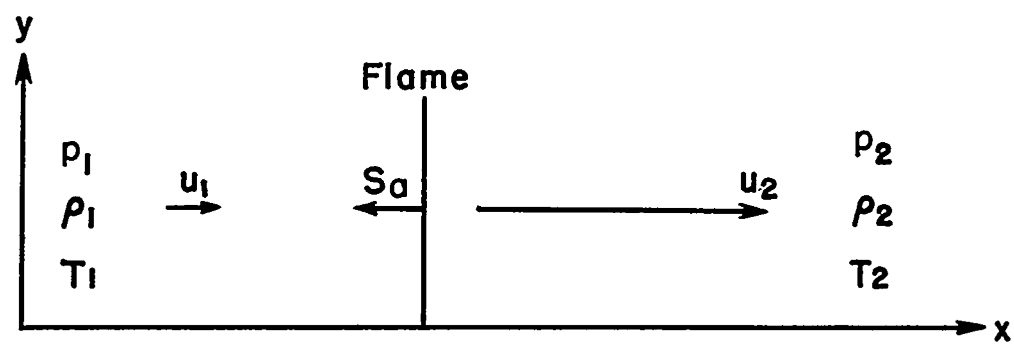

Figure 1.- Flame speed $S_t = S_a + u_1$.

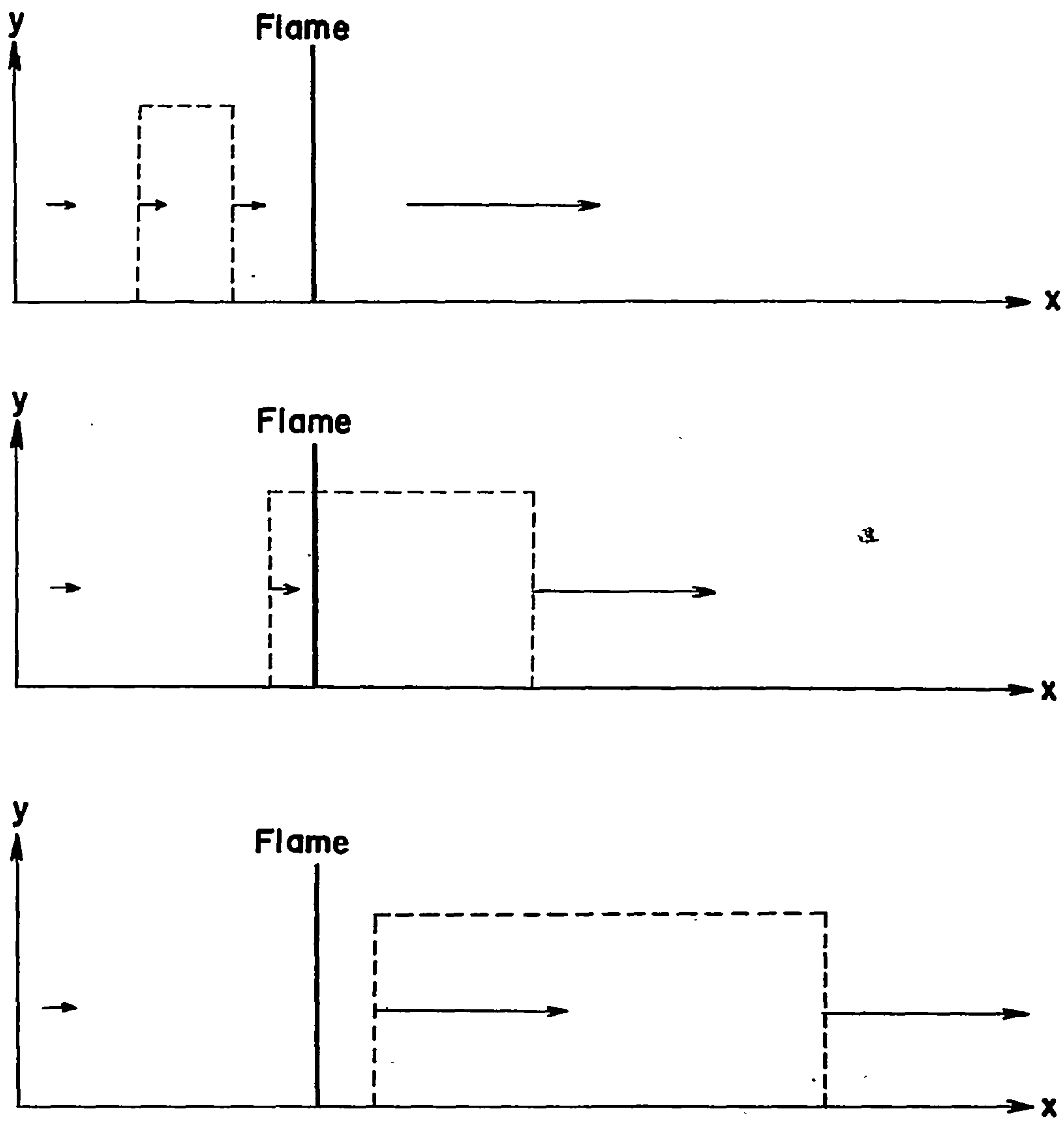

Figure 2.- Expansion of volume of gaseous mixture undergoing combustion.

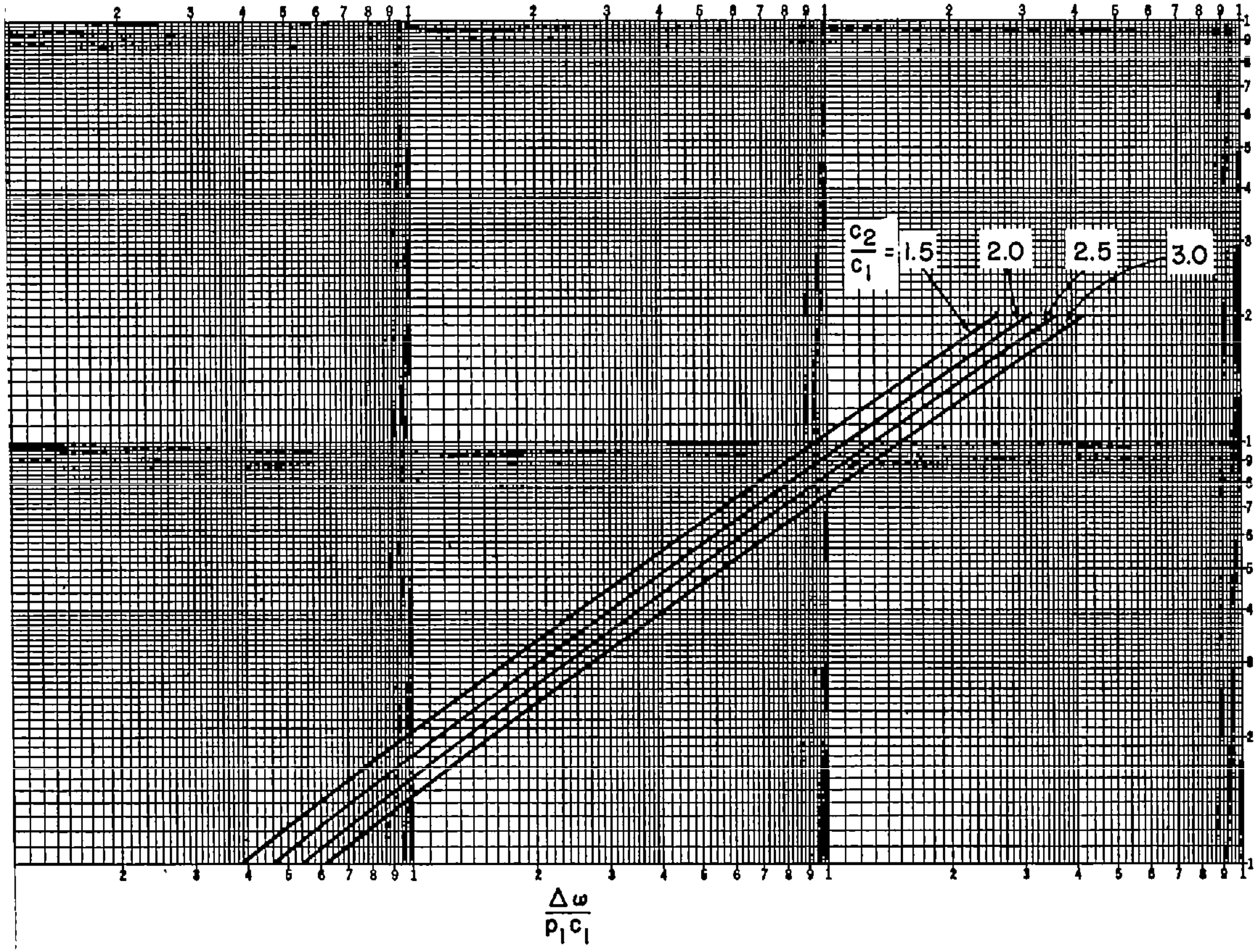

Figure 3.- Shock strength versus change in rate of heat release at flame front.