TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 223

ON THE STABILITY OF OSCILLATIONS OF AN AIRPLANE WING.

By A. C. Von Baumhauer and C. Koning.

Report of the Ryks-Studiedienst Voor de Luchtvaart, Amsterdam.

(Paper read at International Air Congress, London, June, 1923.)

August, 1923.

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Summary.

During a flight with a Van Berkel W.B. seaplane it was observed that the wing could perform violent oscillations. A theoretical and experimental investigation led to the conclusion, that in some cases an unstable oscillation of the wing-aileron system under the influence of the elastic and aerodynamic forces is possible without further external causes. A special case is described in details in the report and a way, in which unstability of motion may be prevented, is given.

(A) General Considerations.

During a flight with a Van Berkel W.B. seaplane** the wings were seen oscillating violently. As was reported by the pilot, he suddenly felt strong blows in his aileron control; the oscillation died out at a change of motion of the airplane, which was not clearly described. As the oscillations were considered dangerous, an investigation of this question was undertaken by the R.S.L.

The oscillations of a wing during flight are governed by the elasticity of the wing-structure and the aerodynamic forces. If the amplitude is small, the oscillations will not interfere with the motion of the airplane as a whole, nor do they cause dangerous

* Report of the Ryks-Studiedienst Voor de Luchtvaart, Amsterdam. Paper read at International Air Congress, London, June, 1923.
** The W.B. seaplane is a semicantilever monoplane. A description of it is given in Flight, 1921, p.260, and Aeronautics, 1921, p.339.

stresses in the wing structure. If, on the other hand, the amplitude is increasing, the oscillations may become dangerous by influencing the stability of the airplane or causing some structural failure. Oscillations of this kind may be caused by:

1. Resonance with some periodical disturbance, such as an insufficiently balanced engine or periodical gusts;

2. Unstable oscillations of the wing.

It is intended to consider only the latter. In the case mentioned above, an external cause could not be traced, nor was there any abnormal vibration of the engine, so that apparently the oscillations of the wing itself were unstable.

From a dynamical point of view the oscillations of a wing may be divided into two groups:

1. The aileron has no motion relative to the wing;

2. Some motion of the aileron relative to the wing is possible and the motions of wing and aileron interfere.

The motion of the wing may be considered as pure bending, pure torsional, or a combination of both. In practice only the latter will be the case, but it has some advantages to consider both other cases, as they are abstractions of wings, in which the bending or torsional motion supersedes and the treatment of those cases is more convenient.

A simplification of the mathematical investigation of these problems is obtained by considering small oscillations only. In this case the equations of motion are linear differential equations, the stability can be determined by application of the theory of Routh.*

* Routh, Stability of motion.

It is intended to give here only the investigation in a special case, which is as yet available for publication. The treatment of the more general case is in course of development and will be published later.

(B) Investigation in a Special Case.

1. Introduction.

The investigation of wing-oscillations was initiated by the observation of the swinging of the wing of a Van Berkel W.B. seaplane. To collect experimental data a model of this wing was tested in the wind channel, and it was observed that this model could perform an unstable oscillation. A further theoretical and experimental investigation was carried out and led to the conclusion, that in this case the oscillation could be made stable by a displacement of the center of gravity of the aileron.

2. Mathematical Investigation.

(a) Assumptions.

To simplify the mathematical treatment the following assumptions have been made, which are supposed to hold good in the case, which was observed in the wind channel:-

The wing is rigid, but has an elastic fixing to the channel wall, so that the only possible motion is turning about the fixed axis AA parallel to the wind (ss Fig.1);

The aileron can freely move about its hinge BB;

The dimensions and mass of the aileron are small compared with those of the wing, so that the force on the hinge may be neglected against the other forces on the wing.

By these assumptions the problem is reduced to a motion with two degrees of freedom. The coordinates, which determine the motion, are:

α = the angle through which the wing turns round the axis AA.

β = the angle through which the aileron turns round the axis BB.

The coordinates are measured from the position of equilibrium, the signs are indicated in Fig. 1

(b) Notations.

The following notations are used:-

M_1 = mass of the wing.

M_2 = mass of the aileron.

r_1 = radius of inertia of the wing about the axis AA.

r_2 = radius of inertia of the aileron about the axis BB.

a = distance of the center of gravity of the aileron behind the axis BB.

b = distance of the center of gravity of the aileron from the axis AA.

V = velocity of the wind.

c_1, c_2, c_3, c_4 and c_5 = constants, the meaning of which is mentioned in point c. The differential coefficients with regard to time are indicated by dots.

(c) Equations of motion.

Restricting the investigation to small oscillations, the relations between the moments and the coordinates may be assumed as follows:-

Elastic moment on the wing about AA = $-c_1 a$

Aerodynamic moment on the wing about AA =

$$-(c_2 \dot{a}/V + c_3\beta)\, V^2$$

Aerodynamic moment on the aileron about BB =

$$-(c_4\,\beta + c_5\dot{\beta}/V^2$$

Only the parts of the moments that govern the motion are reckoned with, as the other parts which are constant are in equilibrium. The meaning of the coefficients c_2, etc., is the following: c_2V denotes the change of rolling moment with the angular velocity of roll; c_3V^2 the change of rolling moment, c_4V^2 that of hinge moment with aileron angle, whereas c_5 is the damping factor of the aileron.

Now the equations of motion are:-

$$M_1 r_1^2\,\ddot{a} + c_2 V\ddot{a} + c_1 a \qquad\qquad + c_3 V^2 \beta = 0 \qquad (1)$$

$$M_2 ab\ddot{a} \qquad\qquad + M_2 r_2^2\ddot{\beta} + c_5 V\dot{\beta} + c_4 V^2\beta = 0 \qquad (2)$$

which may be written in the form:-

$$A_1\ddot{a} + A_2\dot{a} + A_3 a \qquad\qquad + B_3\,\beta = 0 \qquad (1^1)$$

$$D_1\ddot{a} \qquad\qquad + E_1\ddot{\beta} + E_2\dot{\beta} + E_3\beta = 0 \qquad (2^1)$$

(d) Solution of the equations of motion.

As the differential equations are linear, the solution is given by:-*

$$\alpha = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 c^{\lambda_3 t} + C_4 e^{\lambda_4 t} \qquad (3)$$

$$\beta = C_5 e^{\lambda_1 t} + C_6 e^{\lambda_2 t} + C_7 e^{\lambda_3 t} + C_8 e^{\lambda_4 t} \qquad (4)$$

in which λ_1, etc., are the roots of the equation:-

* Routh, Stability in motion, p.3.

$$\begin{vmatrix} A_1 \lambda^2 + A_2 \lambda + A_3 & B_3 \\ D_1 & E_1 \lambda^2 + E_2 \lambda + E_3 \end{vmatrix} = 0 \qquad (5)$$

The equation (5) may also be written in the form:-

$$F \lambda^1 + G \lambda^3 + H \lambda^2 + J \lambda + K = 0 \qquad (5')$$

in which:-

$$F = A_1 E_1$$

$$G = A_1 E_2 + A_2 E_1$$

$$H = A_1 E_3 + A_2 E_2 + A_3 E_1 - B_3 D_1$$

$$J = A_2 E_3 + A_3 E_2$$

$$K = A_3 E_3$$

The coefficients C_1, etc. may be determined from the initial conditions and from the relations between α and β given by the equations (1) and (2).

(e) Criterion of stability.

According to the theory of Routh the motion described by the equations (1) and (2) will be stable, if the real parts of the roots λ of equation (5) are all negative. This will be the case, if the coefficients F, G, H, J and K and the discriminant $GHJ - FJ^2 - KG^2$ are all positive. One of these being negative, the motion will be unstable. Of the coefficients, F is always positive, whereas in normal cases this will also be the case with the other coefficients, if only the angle of incidence is below the critical and there is no overbalancing of the aileron.

Substituting the values of F, etc., given above, the discriminant becomes:-

$$GHJ - FJ^2 - KG^2$$

$$= (A_2E_2 - B_3D_1)(A_1E_2 + A_2E_1)(A_2E_3 + A_3E_2) + A_2E_2(A_1E_3 - A_3E_1)^2 \qquad (6)$$

This will certainly be positive, if the coefficients A_1, etc., and $(A_2E_2 - B_3D_1)$ are all positive. The first will be so in normal cases. Substituting the original values of the coefficients, we get from the second:

$$A_2E_2 - B_3D_1 = c_2c_5V^2 - M_2abc_3V^2 > 0$$

$$M_2ab < c_2c_6/c_3 \qquad (7)$$

The criterion of stability given in equation (7) is sufficient but not necessary. If in some case it is not satisfied, the original form, given in (6), must be used to decide whether the motion is stable. Moreover, the equation (7) indicates a direction in which stability can be obtained as decreasing M_2a, the static moment of the mass of the aileron about the hinge tends to make the motion stable. In a special case $(a = 0)$ the stability of the motion may directly be seen from the equation (5), as in this case it becomes:

$$(A_1\lambda^2 + A_2\lambda + A_3)\ (E_1\lambda^2 + E_2\lambda + E_3) = 0$$

Now the motion of the system will be stable, if only both the isolated oscillations of the wing and the aileron are stable.

Other ways given by the equation (7) to obtain stability are increase of the ratio c_2/c_3 or the damping factor c_5. Freed from numerical coefficients, the ratio c_2/c_3 may be written in the form:

$$\delta L/\delta p : \delta L/\delta \beta^{*}$$

and from a point of view of controllability it may be undesirable to increase this ratio. As to the damping factor c_g, there is practically nothing known about it, so that it cannot be decided, whether increase of its value is possible and how it may be obtained.

3. Description of the Model.

The model used was copied from the wing of the Van Berkel W.B. seaplane on 1/10 scale. The wing section differs only slightly from the Göttingen 285 section.** A drawing of model and wing section is given in Figs. 2 and 3. The model was made of wood and had a wooden aileron, which could move freely about its hinge. Afterwards an aileron, made of aluminium plate, was also used.

The aileron had a detachable balance-flap of the Avro type. This balance-flap was made of brass, the weight of it could be changed by bending strips of copper plate around it.

The different forms of aileron and balance-flap are indicated by letters, the signification of which is given in Table 1.

The inner side of the model was fixed to a rectangular iron plate of 1.00 × 1.37 mm, and a thickness of 4 mm (as indicated in Fig. 5). The longer sides of this plate were fixed to the channel wall parallel to the wind in such a way that some bending was possible. The angle of incidence of the wing could be changed.

4. Experimental Methods.

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* English standard notations (see Advisory Committee for Aeronautics T.R., 1912-13, p.142).
** Technische Berichte der Flugzeugmeisterei der Fliegertruppen. Vol. II, Table 160.

The experiments were carried out in the R.S.L. wind channel.*

The visual observation of the oscillations was made possible by the use of a stroboscope. The time of oscillation could be determined by regulating the motor of the stroboscope until the model, as observed through the disc, was motionless and reading a tachometer coupled to the motor. By slowing down the motor a little, the model could be seen oscillating very slowly.

To record the observations a photographic method was used, a scheme of which is given in Fig. 4. A small mirror A was fixed in the surface of the aileron. This mirror reflected the light of the arc-lamp B on a white screen C. The lens D concentrated the beam of light in such a way that an image of the crater was formed on the screen C. The movement of this lighting point was photographed by the camera E. As the arc-lamp was supplied with alternating current with 50 periods per second, the light was extinguished every 1/100 second, and the curves also showed these interruptions, giving a time-measure. To record the motion of the system, when oscillating freely, an ordinary camera was used and some kind of Lissajous' figure was obtained. The same method was also used to obtain the damping factor of the aileron. In this case the wing tip was fixed, so that only the aileron could oscillate. The lighting point described a straight line on the screen, but as a photographic apparatus with continuously moving film was used, an oscillation curve was obtained.

* A description of this wind channel, which has a diameter of 1.60 meters (5.25 feet) has been published in Verslagen en Verhandelingen van den Ryks-Studiedienst voor de Luchtvaart, Vol. I, p.11.

The other experiments were made to obtain some of the coefficients, which are present in the equations of motion. The relation between the elastic moment and the angle α (coefficient c_1) was obtained by hanging a weight at the wing tip and measuring the deflection. The coefficient c_3 (relation between the aerodynamic moment on the wing and the angle β of the aileron) was obtained by measuring the deflection of the wing tip for different angles and a known velocity of the wind. The coefficient c_4 (relation between hinge-moment and aileron-angle) was calculated from the measurement of hinge-moment, the method of which is indicated schematically in Fig. 5.

5. Experimental Results.

The experimental investigation showed that there was a marked difference between the stability of small oscillations and that of larger ones. In the case of unstability of small oscillations a small unobservable disturbance of the wind caused an oscillation with increasing amplitude. At last some kind of stable motion was reached as the amplitude of the aileron was restricted by the tapered form of the hinge-slot or (and) by the balance-flap touching the surface of the wing. In this state of motion the energy supplied by the wind and which (with freely moving aileron) should increase the amplitude, was now dissipated by the knocking of the walls of the aileron-slot against each other, of the balance-flap on the wing surface and perhaps also by elasticity losses in the fixing plate. The knocking of the aileron was so violent that the noise was heard outside of the wind channel and that in some cases the hinge gave away and the aileron was carried along with the wind.*

In some cases, however, there was stability against small oscillations, but by some larger disturbances an oscillation could be initiated, which was unstable and the amplitude of which increased till a steady state of motion, as mentioned above, was reached. In most cases the initial disturbance had to be rather violent, and was obtained by putting a cylindrical rod on the leading part of the upper surface of the wing parallel to the nose and keeping it there until an oscillation with large amplitude was attained. In the following the latter case will be mentioned as unstability of large oscillations.

In Table II the results obtained in the stability observations are collected. The wind velocity was 27 m/sec (88.6 ft/sec). Stability is indicated by the sign +. As to the influence of the velocity it was observed that for the model a the oscillations began at a velocity of 22 m/sec (72.2 ft/sec) for an angle of incidence of 0° and at 21 m/sec (68.9 ft/sec) for $+15^{\circ}$.

The stroboscopic observation of the unstable oscillation and the following steady motion showed that part of the time that the wing was going down the aileron was up and the reverse, so that in this way the moving system could gather energy from the wind. The results of the measurement of the oscillation time are given in Table III.

Two photographs of the oscillations, obtained in the way described in point 4, were taken. One shows the beginning of the

* Some analogy may be noted between this state of motion and that of vibrating R.A.F. wires, as described by Harris in R.M.759.

motion in an unstable case (model a). The movement of the aileron is shown as large in comparison with that of the wing, the amplitude of both increasing constantly. The other shows the motion in a heavily damped case (model d) after an initial disturbance. The amplitude of the wing and aileron are of the same order and the motion dies out rapidly.

The other results, which were used to obtain the coefficients for the theoretical calculations, are given in Table IV.

Of these the coefficients c_1, c_3, c_4, c_5, were determined experimentally as described in point 4, the masses were obtained by weighing. The coefficient c_2 was calculated from the results published by the Göttingen laboratory,* whereas the other values are obtained from the dimensions of the model. In this table all values are given in technical units of the metric system.

6. Results of Theoretical Calculation.

From the formulas obtained in point 2 and the values of the coefficients given in Table IV, some particulars of the oscillations were calculated. The results of the reduced criterium of stability (7) for an angle of incidence 0° are given in Table V. The cases in which the motion is certainly stable are indicated by +, the others by ? as the criterium gives no decision with regards to instability. For an angle of incidence of $+15^\circ$ the criterium is of no use, the coefficient c_2 being equal to zero. The criterium of Routh gives the results tabulated in Table VI for a velocity of 27 m/sec, and in Table VII, for a model a and different velocities.

* Prandtl, Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, p.48, 1921.

In Table VI two cases are indicated as the value of the discriminant was within the experimental error, so that no decision could be made whether the sign was positive or negative.

The oscillation time calculated for the models a and b are given in Table III as number of oscillations per minute. As a matter of course this is the oscillation time of the unstable (or less stable) part of the motion.

In Figs. 6 and 7 the first part of the motion is given for the models a and d. The angle of incidence is 0°, the wind velocity 27 m/sec. The initial conditions in both cases are:

$$\alpha = \beta = \beta = 0; \quad \alpha = 1 \text{ rad/sec.}$$

It should be noted that in order to obtain comparable figures in both cases the scale of β is 1/10 of that of α. From the figures the beginning of the motion may clearly be seen; as the wing is instantly decelerated by the elastic and aerodynamic moments, there is still no force on the aileron (except that at the hinge), so that it is moving further by its inertia, until it is moved back by the aerodynamic moment. As the static moment of the mass about the hinge is much greater for the model a, the relative angular acceleration and the amplitude of the first oscillation are greater too. The further oscillations of the model a have an increasing amplitude, whereas those of the model d are damped. In both cases the wing is moving down with the aileron up and the reverse, but as with the model d the angle β is small, it has but a small influence on the aerodynamic moment on the wing and the motion dies out under the influence of the positive damping forces. On the other hand, the angle β has large values for the model a, so that its influence surpasses that of the damping forces and an unstable motion is produced.

As to the ratio β/α this is very great for the model a and at the beginning of the motion about one for the model d.

7. Comparison of the Experimental and Theoretical Results.

For an angle of incidence of 0° there is a complete agreement between the stability observations and the results of the criterium of Routh (see Tables II and VI). This is not the case with the angle of incidence of 15°; while there is agreement for the models a and e, there is none for the models b and f. In the case of b it should be noted that the theoretical investigation covers small oscillations only, so that the observed instability of the large oscillations is beyond its reach. Moreover, there are the two models c and d for which the theoretical investigation is undecisive for the reasons mentioned in point 6. In all these cases the model is more stable than the theoretical calculation should indicate. An explanation may be the following.

The dissipation of energy by the wing is largely governed by the value of the coefficient A_2 see equation (1). Now at 15° the wing is at its critical angle, so that c_2 and also A_2 are zero.

The latter will also be the case when the wing is oscillating with no wind and fixed aileron. If in this case the equation (1) was applied, an oscillation without any damping would result, and yet in reality the motion dies out fairly rapidly. So there seems to exist still another cause of dissipation of energy, which may consist in elasticity losses of the fixing-plate and friction in its fixings. This dissipating agent will also exist with the model moving with wind, though it is neglected in the equations of motion. It will always tend to stabilize the motion. It may occur that at an angle of incidence of 0° the neglection of this factor is right, the energy losses caused by it being small as compared with others. At an angle of incidence of 15° the latter are largely reduced, the coefficient A_2 being zero, so that here it may have some measurable influence. As to the influence of the velocity the agreement is fairly good, the theoretical investigation giving a change from stability to instability between 15 and 20 m/sec (49.2 to 65.6 ft/sec), whereas in the experiments the unstable oscillations begin at about 21 or 22 m/sec (68.9 to 72.2 ft/sec).

The calculation of the number of oscillations for the model a gives a result which agrees fairly good with those determined experimentally (see Table III). For the model b there is a large divergence between the two values. It should be noticed, however, that in this case the theoretical value is that for small oscillations, whereas the experiment gives the value for the large unstable oscillations. In the latter case the application of the equations of motion may be inadequate, as both the hinge moment and the damping moment need not be linear functions of β and $\dot{\beta}$. For instance, there may be burbling at the balance-flap.

Moreover, the frictional losses caused by the balance-flap when knocking on the wing surface and gliding over it by the elastic deformation may have some influence.

The oscillation curves, given in Figs. 6 and 7, give a qualitatively good agreement with the stroboscopic observations and the photographs. The relative position of the aileron to the wing is the same. For the model a the values of α are small as compared with those of β, and both are increasing. For the model d the values of a and b are of the same order and the motion dies out rapidly.

As a whole, it may be said that the theory gives a good agreement with the experiments, so far as small angles of incidences and small oscillations are concerned. For large angles the agreement is less good, but it may be supposed that it is caused by neglecting the energy-dissipation by the fixing plate and the character of the motion is the same.

8. Practical Application.

There are some divergences between the case, the investigation of which has been described in the present report, and that of the airplane, as in the latter it is possible that torsional oscillations with the axis parallel to the longerons of the wing may occur. Though the report does not deal with this case, it may be noticed that there is an analogy with respect to the influence of the location of the center of gravity of the ailerons with regard to the hinge. For the normal type of aileron (at the trailing edge of the wing) no unstable oscillations seem to be possible if the center of gravity is sufficiently in front of the aileron. As it was probable that in the case of the Van Berkel seaplane stability could be obtained with the center of gravity at the hinge, it

The oscillation curves, given in Figs. 6 and 7, give a sufficiently good agreement with the stroboscopic observations and the photographs. The relative position of the aileron to the wing is the same. For the model A the values of α are small as compared with those of β, and both are increasing. For the model C the values of α and β are of the same order and the motion dies rapidly.

As a whole, it may be said that the theory gives a good agreement with the experiments, so far as small angles of incidence and small oscillations are concerned. For large angles the agreement is less good, but it may be supposed that it is caused by neglecting the energy-dissipation by the fixing plate and the character of the motion is the same.

<u>Practical application</u>

There are some differences between the case, the investigation of which has been described in the present report, and that of the airplane, as in the latter it is possible that torsional oscillations with the axis parallel to the longerons of the wing may occur. Though the report does not deal with this case, it may be mentioned that there is an analogy with respect to the influence of the location of the center of gravity of the ailerons with regard to the hinges. For the normal type of aileron (at the trailing edge of the wing) no unstable oscillations seem to be possible if the center of gravity is sufficiently in front of the aileron. It is probable that in the case of the torsional oscillations stability could be obtained with the center of gravity of the wing, if was advised to alter the weight of the balance-flap in such a way that the static moment of the weight of the aileron about the hinge shall be zero. The experiments with the modified airplane have not yet been carried out.

Table I.

Different Forms of the Model.

No.	Aileron	Balance Flap
a	wood	none
b	"	normal weight
c	"	with increased witght 1
d	"	" " " 2
e	aluminium	none
f	"	normal weight

Table II.

Stability Observations Wind Velocity 27 m/sec (88.58 ft/sec)

	Angle of incidence 0°.		Angle of incidence 15°	
No.	Small osc.	Large osc.	Small osc.	Large osc.
a	--	--	--	--
b	+	+	+	--
c	+	+	+	+
d	+	+	+	+
e	--	--	--	--
f	+	+	+	+

+ = stable; - = unstable.

Table III.

Number of Oscillations per Minute.

	Angle of incidence 0°		Angle of incidence 15°	
No.	Observed	Calculated	Observed	Calculated
a	610	560	620	592
b	--	--	550	780

Table V.

Reduced Criterium of Stability.

Angle of incidence 0°.

No.	a	b	c	d	e	f
Stability ...	?	?	+	+	?	?

+ = stable; ? = undecided.

Table VI.

Stability Criterium of Routh. Wind Velocity 27 m/sec (88.58 ft/sec)

Angle of incidence.	No. of the model.					
	a	b	c	d	e	f
0°	--	+	+	+	--	+
$+15^{\circ}$	--	--	?	?	--	--

+ = stable; - = unstable.

Table VII.

Stability Criterium of Routh. Model No. a.

Angle of incidence	Wind Velocity.				
	82	65.6	49.2	32.8	16.4 ft/sec.
	25	20	15	10	5 m/sec
0° ...	--	--	+	+	+
$+15^{\circ}$ ...	--	--	+	+	+

Figures.

Table IV.

Numerical Values of Coefficients (kg/m/sec Units).

	Angle of incidence 0°.					
No.	a	b	c	d	e	f
M_1	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}
M_2	1.45×10^{-2}	2.07×10^{-2}	2.54×10^{-2}	2.69×10^{-2}	1.01×10^{-2}	1.63×10^{-2}
a	23.3×10^{-3}	8.2×10^{-3}	1.7×10^{-3}	0.1×10^{-3}	30.0×10^{-3}	8.3×10^{-3}
b	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}
r_1	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}
r_2	3.12×10^{-2}	3.56×10^{-2}	3.66×10^{-2}	3.70×10^{-2}	4.06×10^{-2}	4.15×10^{-2}
c_1	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}
c_2	1.68×10^{-2}	1.68×10^{-2}	1.68×10^{-2}	1.68×10^{-2}	1.68×10^{-2}	1.68×10^{-2}
c_3	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}
c_4	4.61×10^{-5}	2.65×10^{-5}	2.65×10^{-5}	2.65×10^{-5}	4.61×10^{-5}	2.65×10^{-5}
c_5	1.40×10^{-5}	1.50×10^{-5}	1.50×10^{-5}	1.50×10^{-5}	1.40×10^{-5}	1.50×10^{-5}

Table IV (Cont.)

Numerical Values of Coefficients (kg/m/sec Units).

No.	Angle of incidence +15°.					
	a	b	c	d	e	f
M_1	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}	5.29×10^{-1}
M_2	1.45×10^{-2}	2.07×10^{-2}	2.54×10^{-2}	2.69×10^{-2}	1.01×10^{-2}	1.63×10^{-2}
a	23.3×10^{-3}	8.2×10^{-3}	1.7×10^{-3}	0.1×10^{-3}	30.0×10^{-3}	8.3×10^{-3}
b	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}	6.89×10^{-1}
r_1	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}	5.12×10^{-1}
r_2	3.12×10^{-2}	3.56×10^{-2}	3.66×10^{-2}	3.70×10^{-2}	4.06×10^{-2}	4.15×10^{-2}
c_1	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}	5.37×10^{-2}
c_2	0	0	0	0	0	0
c_3	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}	7.12×10^{-3}
c_4	7.95×10^{-5}	4.14×10^{-5}	4.14×10^{-5}	4.14×10^{-5}	7.95×10^{-5}	4.14×10^{-5}
c_5	1.29×10^{-5}	1.05×10^{-5}	1.05×10^{-5}	1.05×10^{-5}	1.29×10^{-5}	1.05×10^{-5}

Fig.1.

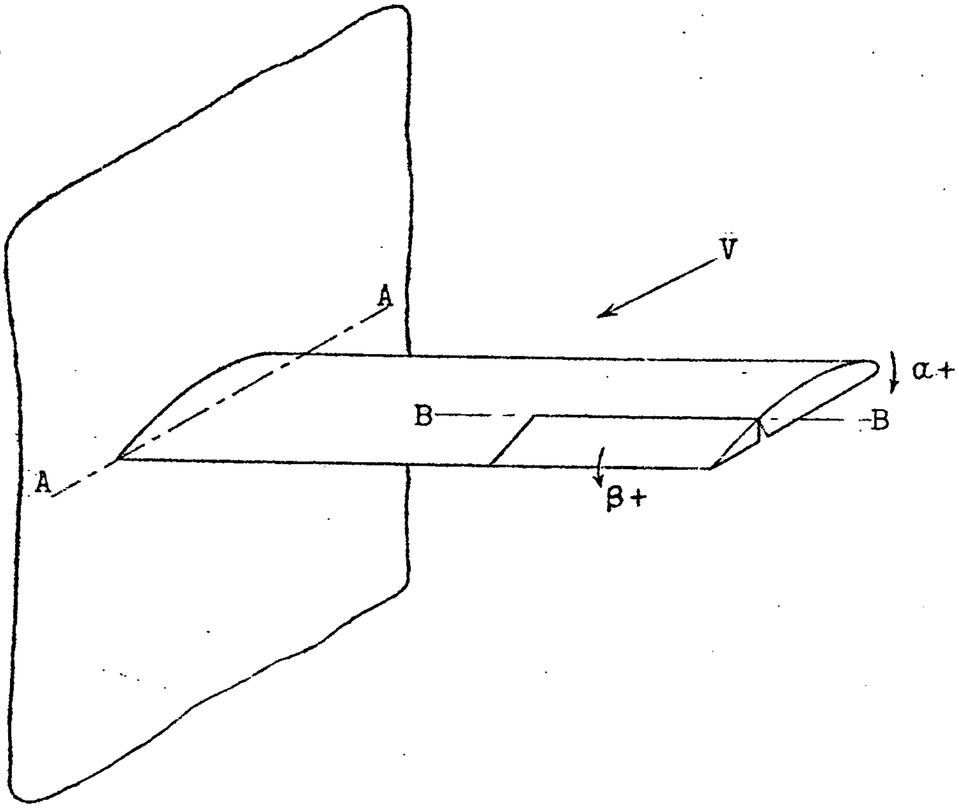
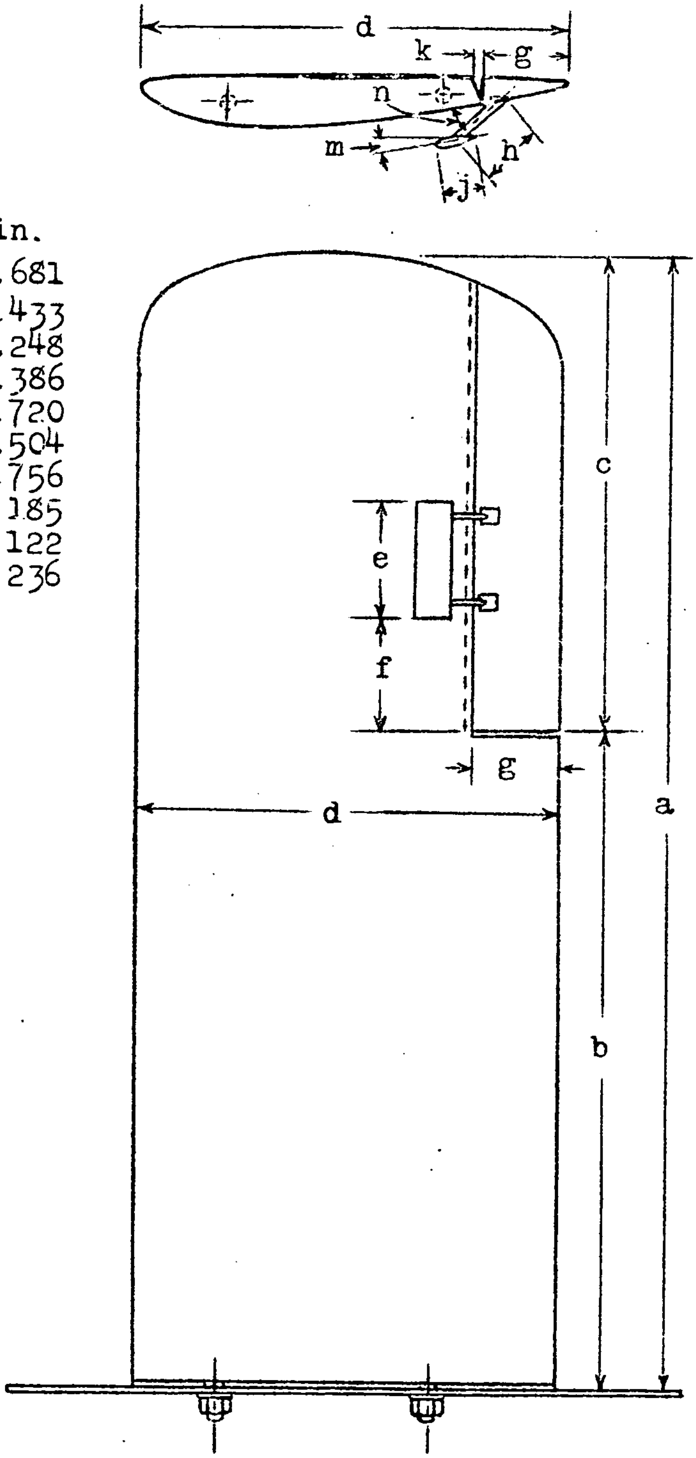


Fig.1.

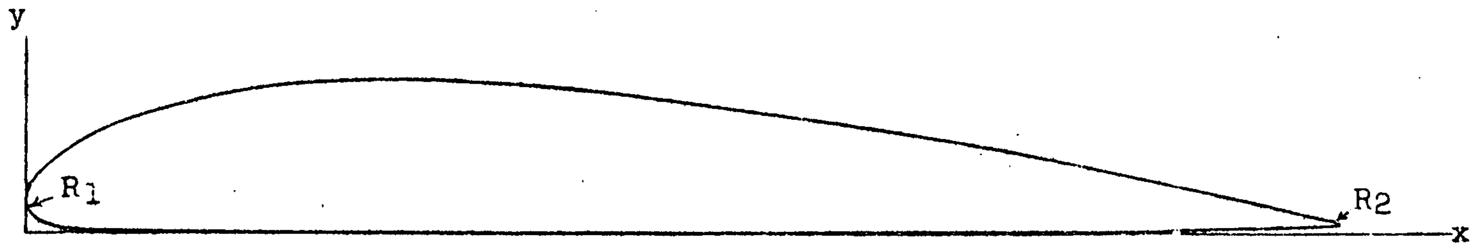
Fig. 2.



	mm		in.
a =	906.3	=	35.681
b =	519.0	=	20.433
c =	387.3	=	15.248
d =	340.0	=	13.386
e =	94.5	=	3.720
f =	89.0	=	3.504
g =	70.0	=	2.756
h =	55.5	=	2.185
j =	28.5	=	1.122
k =	6.0	=	0.236
m	8.5°		
n	26.0°		

Fig. 2.

Fig. 3.



x	y_1	y_2	x	y_1	y_2
0.00	2.42	2.42	50.00	0.00	10.11
1.25	0.60	4.70	60.00	0.00	8.75
2.50	0.40	5.77	70.00	0.00	7.13
5.00	0.29	7.47	80.00	0.20	5.28
7.50	0.20	8.60	90.00	0.32	3.09
10.00	0.11	9.36	95.00	0.45	2.00
15.00	0.00	10.60	100.00	0.70	0.70
20.00	0.00	11.32	R1	2.00	----
30.00	0.00	11.71	R2	0.20	----
40.00	0.00	11.18	------	----	----

Column x gives stations.

Column y_1 gives ordinates for lower surface.

Column y_2 gives ordinates for upper surface.

Fig. 3.

Fig.4.

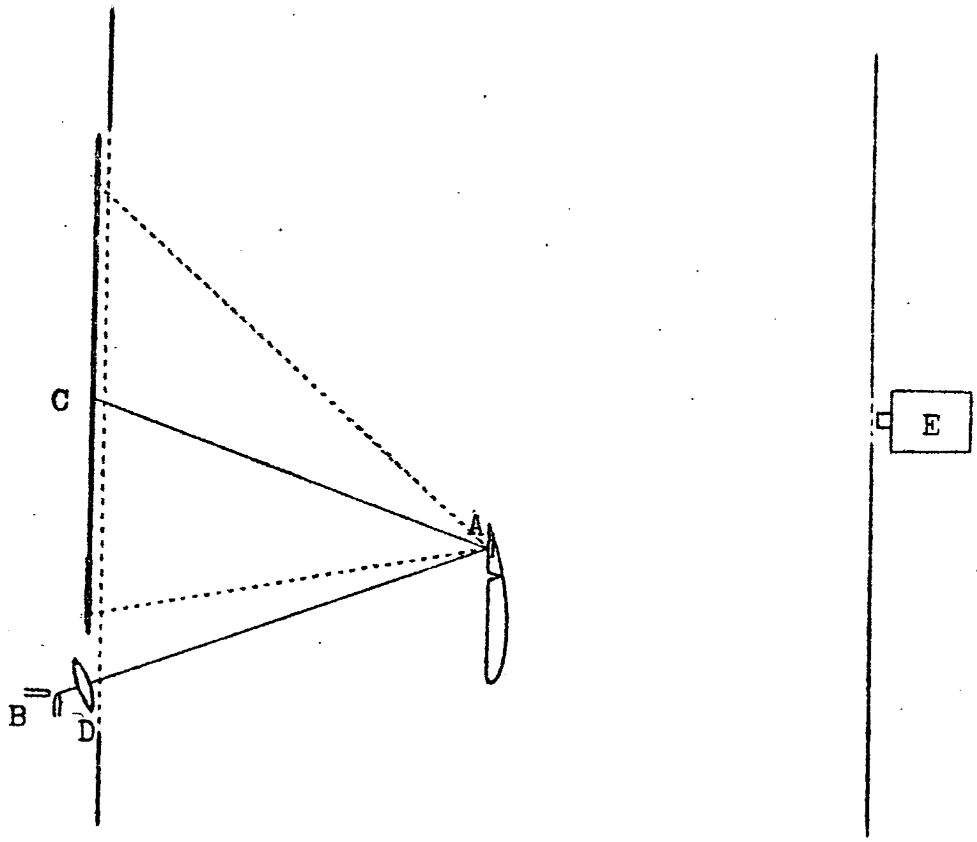
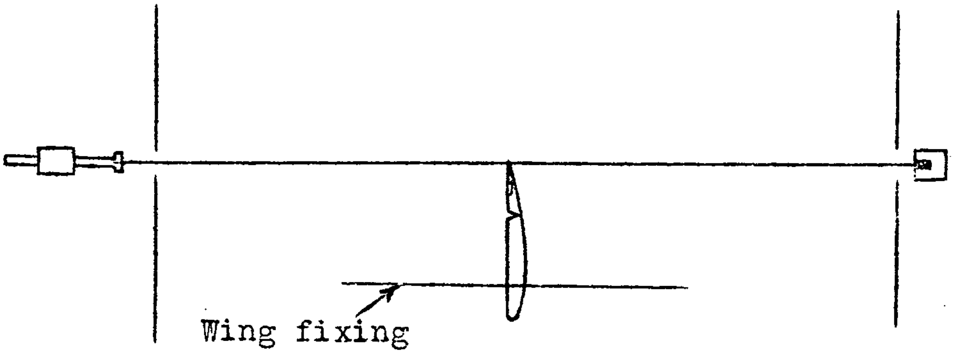


Fig.4.

Fig.5.



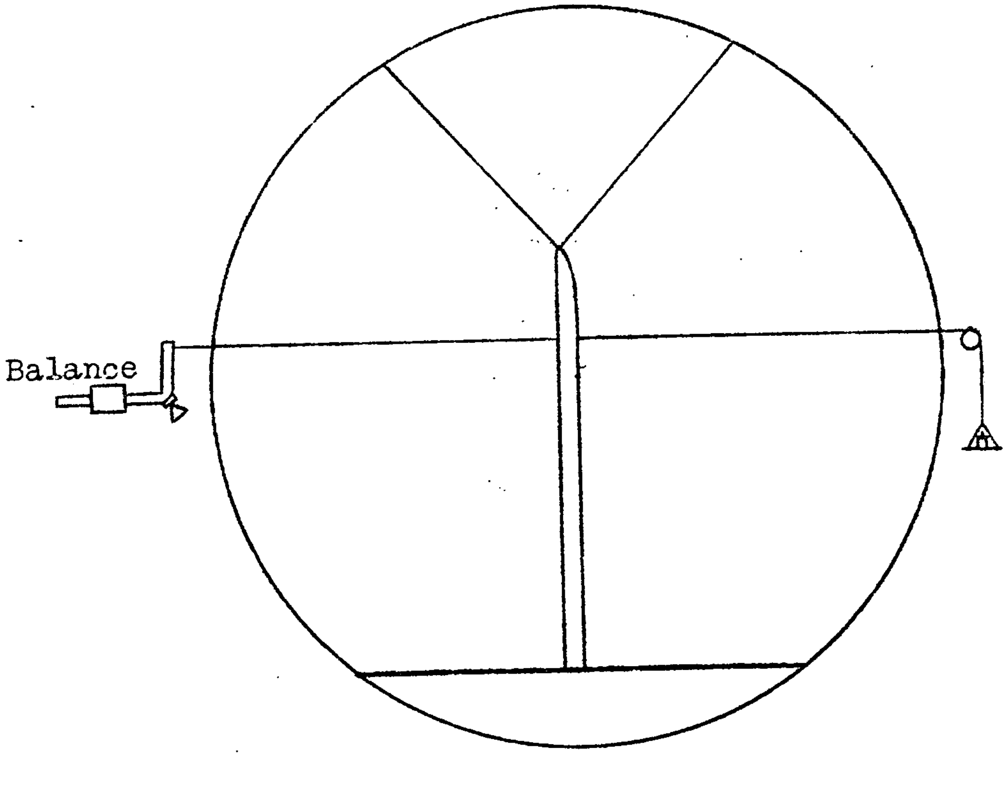
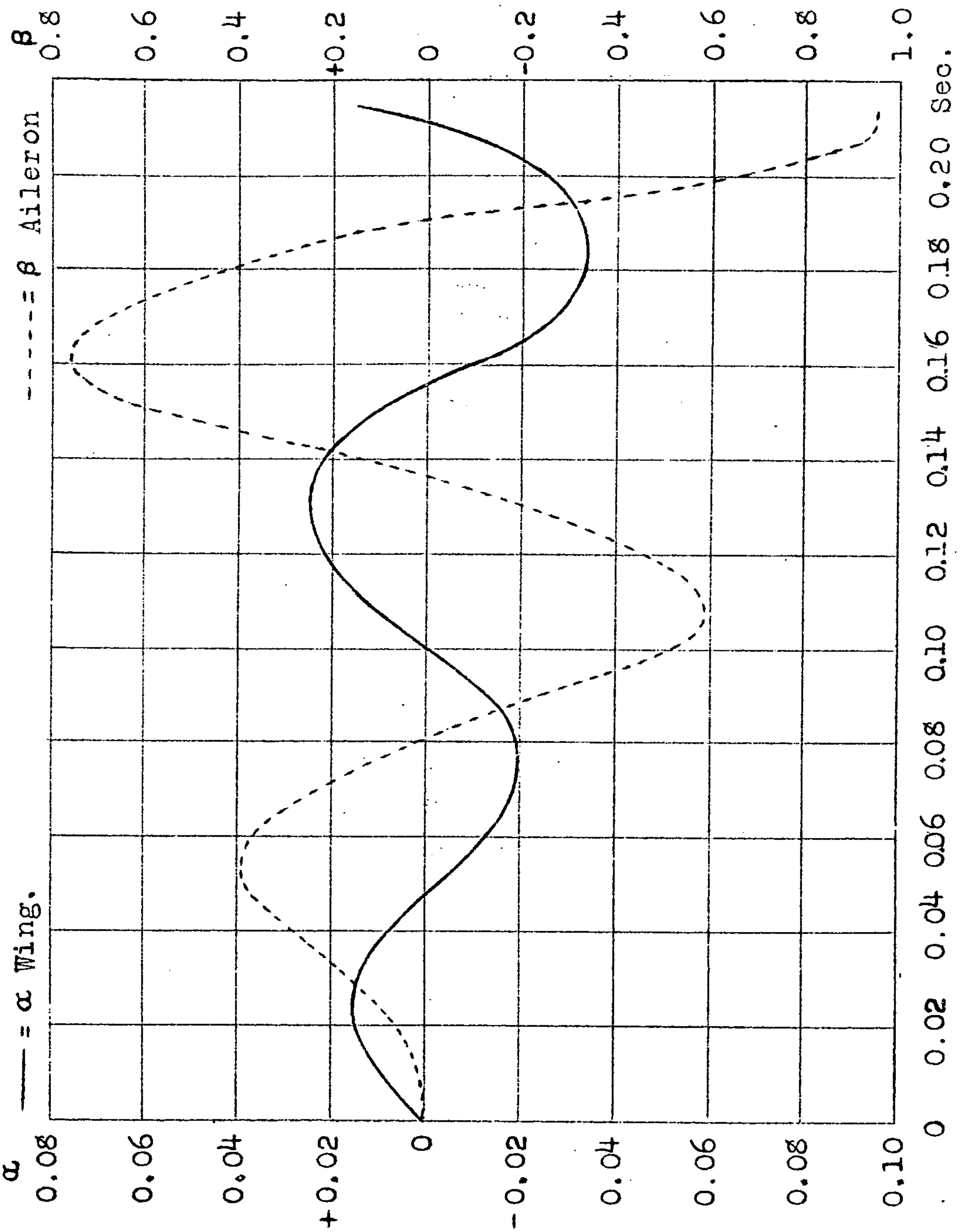


Fig.5.

Fig.6.



—— = α Wing.
- - - - = β Aileron
α
0.08
0.06
0.04
+0.02
0
-0.02
0.04
0.06
0.08
0.10
β
0.8
0.6
0.4
+0.2
0
-0.2
0.4
0.6
0.8
1.0
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20 Sec.

Fig.6.

Fig.7.

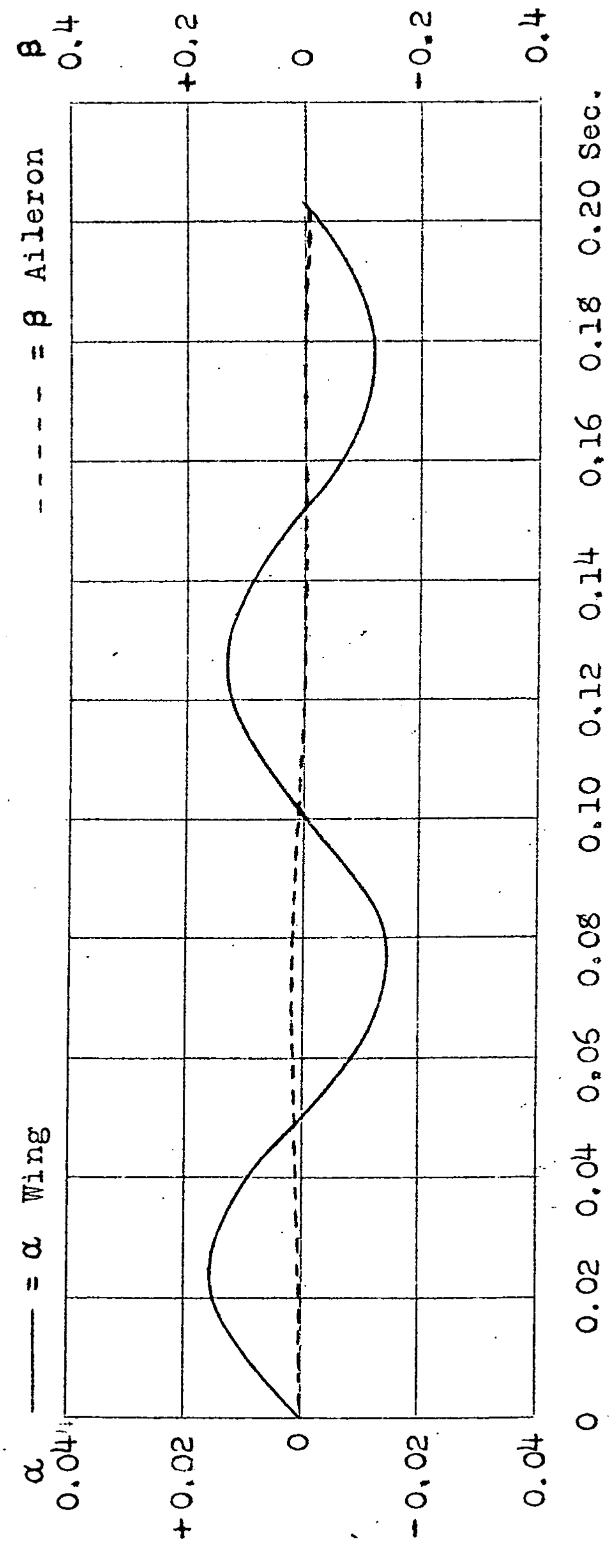

——— = α Wing
- - - - - = β Aileron
α
0.04
+0.02
0
-0.02
0.04
β
0.4
+0.2
0
-0.2
0.4
0
0.02
0.04
0.05
0.08
0.10
0.12
0.14
0.16
0.18
0.20 Sec.

Fig.7.