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EFFECT OF CHANGING THE MEAN CAMBER OF AN AIRFOIL SECTION.

By A. Toussaint.

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EFFECT OF CHANGING THE MEAN CAMBER OF AN AIRFOIL SECTION.*

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Methodical experiments with the series of airfoil sections of the same relative thickness and of variable relative cambers can be utilized for determining the effect of the camber on the aerodynamic properties of airfoil sections.

Technical Bulletin No. 12, of the S. T. Aé. (Service Technique de l'Aéronautique) contains the results obtained at the Aerotechnical Institute with a series of Royer airfoils derived from an initial biconvex section. These airfoils are numbered from 13A to 21A (or SC96 to SC104) and the 9 sections tested are represented in Fig. 1. The mean maximum thickness, of all these sections, is 17.4%. As shown in Fig. 1, all these sections have a common forward portion, the shape of which is practically semi-elliptical. The mean camber of each section was first determined from an accurate drawing; then its corresponding chord was drawn (as defined by the "Normalization Commission") and the following measurements were made:

1. The angle i' between the chord bitangent to the lower camber and the corresponding chord of the mean camber.

2. The ordinate o/c of the mean camber with reference to its corresponding chord.

* From La Technique Aéronautique, 1923, Oct. 15 pp. 780-788, and Nov. 15, pp. 807-818.

Note- The C_z, C_x and associated symbols have been retained.

The results of these measurements are given in the following table:

Airfoil No.	13 A	14 A	15 A	16 A	17 A	18 A	19 A	20 A	21 A
o/c max. %	0	1.4	3.3	4.6	6.4	7.15	8.0	8.85	9.25
i' (degrees)	0	+6.20	+5.10	+4.50	3.60	3.20	2.80	2.40	2.20

Then taking for reference, the angles i formed by the corresponding chord of the mean camber with the relative wind, the unitary curves C_z and C_x were plotted against the angles $i = \alpha_c + i'$ Figs. 2 and 3 give these unitary curves, i being the angle of incidence with reference to the chord bitangent to the lower camber.

<u>1. Effect of o/c on C_z</u>.- The unitary curves in Fig. 3 are regularly staggered with respect to one another. There is no overlapping, except for the curve relative to airfoil 21 A, for which o/c = 9.25%.

Let us first consider the unitary curve relative to the biconvex airfoil No. 13 A. According to the theory of Joukowski, the theoretical lift of an airfoil section is given by the formula

$$C_{z_0} = 2\pi \left[\sqrt{1 + \left(\frac{2o}{c}\right)^2} + \frac{2\delta}{c} \right] \sin \text{arc} \tan \left(\frac{2o}{c} + i_0 \right)$$

in which o/c is the maximum ordinate of the mean camber of the section with respect to its corresponding chord and δ/c is a ratio which depends essentially on the maximum relative thickness (m/c) of the section.

In the Joukowski sections we have

$$\frac{m}{c} = 2.2 \text{ to } 2.6 \frac{\delta}{c}$$

Let us suppose this formula can be applied to the Royer airfoils. For the biconvex airfoil the formula becomes

$$C_{z_0} = 3\pi\left(1 + \frac{2\delta}{c}\right) \sin i.$$

On making

$$\frac{m}{c} = 2.4 \frac{\delta}{c}$$

we have

$$\frac{2\delta}{c} = \frac{0.174}{1.2} = 0.145$$

The value 2.4 was chosen by comparison with Joukowski sections of similar relative thickness to that of 13A. The theoretical lift for airfoil No. 13A (for infinite aspect ratio) is

$$C_{z_0} = 7.2 \sin i_0$$

For a limited aspect ratio A, we must add to the angle i_0 the corresponding value of the induced angle

$$i_i = \frac{C_{z_0}}{\pi} \times \frac{1}{A} \times 57.3.$$

For the Royer airfoil, A = 6 and the induced angle is

$$i_i = C_{z_0} = 3.05$$

We can therefore calculate the unitary curve of the theoretical lifts for the aspect ratio 6. Fig. 2 gives these theoretical lifts. It is evident that the theoretical lifts for the aspect ratio 6 can be represented by

$$C_{z_0}\ th = 5.22 \sin i_0$$

Lastly the experimental lifts for airfoil No. 13A were represented in the same figure. They can also be represented by the expression

$$C_{z_0}\ m = 4.22 \sin i_0$$

In short, it is obvious that the experimental lifts, in the domain of the angles of attack for which the flow takes place without much discontinuity (say up to $i_0 = 14$ to 15°), are equal to $4.22/5.22 = 0.8$ of the theoretical lifts.

The value of this relation has already been demonstrated for other airfoils and it would seem very admissible for the characteristic value for tests in which $Vl = 5\ m^2$ sec.

Let us now consider the other Royer airfoil. The identification of the experimental lifts with the theoretical can be made. The mean camber of the section and the corresponding relative ordinate are obtained, however, from measurements which only take account of the geometrical shape of the sections. From the aerodynamic standpoint, it may happen that the flow around these sections corresponds to a mean camber and relative camber different from those measured. It seems that this is indeed the case, since, if we consider the angles γ (airfoil-section angles) corresponding to zero lift, we find that these angles differ from those corresponding to the geometric relative camber. In other terms, we have

$$\tan\gamma \neq \frac{2o}{c}$$

It is therefore important to find an empirical relation between $\tan\gamma$ and $2\,o/c$ which will take account of the experimental result, i.e. of the actual flow around the airfoil section under consideration. We thus obtain the following table:

Airfoil No.	13A	14A	15A	16A	17A	18A	19A	20A	21A
2 o/c %	0	2.8	6.6	9.2	12.8	14.3	16.0	17.7	18.50
arc tan 2 o/c	0	1.6°	3.80°	5.30°	7.30°	8.15°	9.10°	10.05°	10.50°
γ	0	-1.6°	-3°	-4°	-5.50°	-6.20°	-6.55°	-7.1°	-6.20°
$\tan\gamma$ %	0	2.8	5.24	7.0	9.63	10.85	11.4	12.3	10.85

The variations γ in terms of arc tan $2\,o/c$ are shown in Fig. 2. We may closely approximate these variations by such an expression as

$$\gamma = 0.75 \text{ arc tan } 2\,o/c$$

or,

$$\tan\gamma = 0.75 \times \frac{2o}{c} = 1.50\,o/c$$

In short, the theoretical lift of Royer airfoils for infinite aspect ratio is expressed by the formula

$$C_{z_{th}} = 2\pi\left[\sqrt{1 + (\tan\gamma)^2} + \frac{2\delta}{c}\right]\sin(\gamma + i_0)$$

$$\tan\gamma = 0.75 \times \frac{(2o)}{c}$$

The maximum value of $\tan\gamma$ is about 0.12, or $(\tan\gamma)^2 = 0.0144$. The coefficient $2\pi\sqrt{1 + (\tan\gamma)^2}\ \frac{2\delta}{c}$ differs but little, therefore, from $2\pi\left(1 + \frac{2\delta}{c}\right)$, but the value of δ/c is connected with m/c by a relation to be determined by experience.

For this purpose we have calculated the values of the ratio $C_{ze}: \sin(\gamma + i_0)$ for airfoils No.17A (o/c = 6.4%) and No.20A (o/c = 8.85%). We obtained a mean value of about 4.2. The same coefficient can therefore be adopted as for the biconvex airfoil. It is then readily shown that the experimental lifts $C_{ze} = 4.2 \sin(\gamma + i_0) = 4.2 \sin^{\alpha}$ are still equal to 0.8 of the theoretical lifts for the aspect ratio 6. The effect of o/c on the lift will then be given by the difference

$$C_z - C_{z_0} = 4.20 \left[\sin(\gamma + i_0) - \sin i_0\right]$$

Within the limits of the angles of attack employed in aviation and for maximum cambers of less than 10%, $C_z - C_{z_0}$ is practically equal to

$$C_z - C_{z_0} = 4.20 \times \tan\gamma = 6.3\ o/c$$

Fig. 1 gives the experimental variations of $C_z - C_{z_0}$ in terms of o/c and it is evident that the straight line representing the above relation passes through the mean of the experimental points for all values of o/c below 8.5% and for the α angles between -5° and 12°. It may be noted that the accuracy of $C_z - C_{z_0}$ is much less than that of the total coefficients C_z and C_{z_0}. The accidental deviations from the mean straight are therefore perfectly justified.

This ratio is not accurate above o/c = 8.5%. We have found this anomaly to be due to the particular drawing of the S.T.Aé. airfoils 20A and 21A.

Verification of the above Ratio.- It is extremely interesting to verify the above ratio for another series of airfoils of variable camber with a different relative thickness. The tests recently made in the Eiffel laboratory, with a series of airfoils proposed by Mr. Lachassagne, have provided us the data for this verification. This series comprises five airfoil sections, the characteristics o/c and i' of which are given in the following table.

Airfoil No.	E 429	E 430	E 431	E 432	E 438
o/c max. %	2.67	4.73	5.9	7.67	9.0
i'	0	$+1.45^\circ$	1.33°	1.20°	1.00°

Fig. 4 gives the unitary curves C_z in terms of the angle of attack $i = i' + \alpha_c$ of the different airfoil sections, α_c being the angle of attack with respect to the chord bitangent to the lower camber. It is seen that these unitary curves are regularly staggered and that the ratio between the zero angle of lift (or airfoil-section angle) and the angle corresponding to the maximum camber can be determined as before. Thus we find

$$\gamma = 0.72 \text{ arc tan } 2\, o/c$$

or

$$\tan\gamma = 0.72 \times 2\, o/c = 1.44\, o/c$$

We can then calculate the coefficient of the formula for the theoretical lifts. The maximum thickness of the Lachassagne airfoil sections being about 9.6%, we obtain (as for the Royer foils)

$$\frac{2\delta}{c} = \frac{0.096}{1.2} = 0.08$$

and consequently

$$2\pi\left(1 + \frac{2\delta}{c}\right) = 6.77$$

For theoretical lifts relative to the aspect ratio 6, the same reduction can be admitted as before.

$$6.77 \times \frac{5.22}{7.2} = 4.9$$

Lastly, we find that the experimental lifts are still equal to 79 or 80% of the theoretical lifts and we have

$$C_{ze} = 3.93 \sin (\gamma + i)$$

The differences in lift for two different cambers can therefore be expressed by some such formula as

$$C_z (o_1/c_1) - C_z (o_2/c_2) = 3.92 (\gamma_1 - \gamma_2)$$
$$= 5.65 (o_1/c_1 - o_2/c_2)$$

Experience gives

$$5.85 \times \Delta o/c$$

We find, moreover, that the symmetrical biconvex wing, which might be considered as the basis of the Lachassagne series of airfoils, has an experimental lift $C_{z0} = 3.92 \sin i_0$ in the domain of the angles of attack of a good airfoil section.

Another confirmation can be found in the results obtained with a series of Dewoitine airfoils tested at Saint Cyr. These are the S.T.Ae. airfoils 25A to 28A, whose characteristics are given in the following table and represented in Fig. 5.

Airfoil No.	25 A	26 A	27 A	28 A
o/c max. %	3.0	5.0	7.33	11.33
i'	5.0°	4.2°	3.05°	2.65°

In Fig. 6 the unitary curves C_z are plotted against the angle of attack $i = i' + \alpha_c$. It is evident that

$$\gamma = 0.78 \text{ arc tan } \frac{2o}{c}$$

or,

$$\tan \gamma = 0.78 \times 2 \text{ o/c} = 1.56 \text{ o/c}$$

The relative maximum thickness of these airfoils is practically equal to that of the Royer airfoils. The theoretical and experimental lifts must therefore be expressed by the same formulas and we have

$$C_{z_2} - C_{z_1} = 4.18 \ (\gamma_1 - \gamma_2)$$
$$= 6.55 \ (o_1/c_1 - o_2/c_2)$$

Fig. 6 gives the experimental differences which verify the foregoing ratio with a very close approximation. It is also manifest that the imaginary biconvex airfoil corresponding to these airfoils would have experimental lifts $C_{z_0} = 4.18 \sin i_0$, the same as the Royer airfoil No. 13A.

Lastly, let us consider airfoils 429 and431 of the Göttingen Laboratory. These airfoils have practically the same thickness m/c = 11.6% and the biconvex airfoil No. 429 may be considered as the basis of airfoil No. 431, for which we have

$$o/c = 7.0\% \text{ and } i' = 1.70^\circ$$

The experimental data contained in Prandtl's book gives, for airfoil 429, $C_{z_0} = 2.8$ when $i = 0$. This is doubtless due to a slight lack of symmetry in the model. We have corrected the experimental values by reducing the C_z by 2.8 and subtracting the corresponding induced drag from the measured C_x. We will first consider the C_z. The coefficient of the theoretical lifts for the infinite aspect ratio is then

$$2\pi\left(1 + \frac{0.116}{1.2}\right) = 6.88$$

If we add the induced angle corresponding to the aspect ratio 5, this amounts to multiplying the coefficient by 0.703

$$6.88 \times 0.703 = 4.84$$

The experimental lifts of airfoil No. 429 can be represented by the equation

$$C_{z_0} = 4.1 \sin i_0$$

from which we conclude that the ratio of the experimental to the theoretical lifts, for the aspect ratio 5, would be $4.1/4.84 = 0.845$. This value is very admissible, being given the product Vl relative to these tests ($Vl = 6\ m^2$ sec.). Under these conditions, the lifts of airfoil 431 are given by

$$C_z = 4.1 \sin (\gamma + i)$$

we find that

$$\gamma = 9.5^\circ - 1.7 = 7.8^\circ$$

and since

$$\text{arc tan} \frac{2o}{c} = 8.6^\circ$$

we have

$$\tan\gamma = 0.905 \times \frac{2o}{c} = 1.81\ o/c.$$

Hence,

$$\Delta C_Z = 7.4\ o/c.$$

This equation is confirmed by experiment for all angles of attack between -6° and $+13^\circ$.

Conclusions.- The difference of the lifts $(C_{Z_1} - C_{Z_2})$, for all angles of attack at which the flow takes place without much discontinuity, for two wings of the same series, with maximum cambers o_1/c_1 and o_2/c_2, is proportional to the difference $(o_1/c_1 - o_2/c_2)$.

The proportionality coefficient P is a function of the maximum relative thickness m/c and is expressed by the formula

$$P_{(a)} = 5 \times \frac{C_a}{C_\infty}\left(1 + \frac{m}{1.2\ c}\right) \times \frac{\gamma}{o/c}$$

The numerical factor 5 obtained from the product of 2π times the calculated ratio of the theoretical and experimental lifts, or 0.8 for recent wind tunnel experiments (Vl = 3 to 6 m^2 sec) C_a/C_∞ is the calculated ratio of the theoretical lifts for the finite aspect ratio a and for the infinite aspect ratio. For a = 6 we found $C_a/C_\infty = 0.725$, so that

$$P_{(6)} = 3.63\left(1 + \frac{m}{1.2\ c}\right)\left(\frac{\gamma}{o/c}\right)$$

Lastly, the ratio $\frac{\gamma}{o/c}$ is the quotient obtained by dividing the zero angle of lift (expressed in radians) by the maximum relative camber (o/c).

$$C_{Z_1} - C_{Z_2} = 6.28 \times 0.8 \times \frac{C_a}{C_\infty}\left(1 + \frac{m}{1.2\ c}\right) \times \frac{\gamma}{o/c}\,[o_1/c_1 - o_2/c_2]$$

or,

$$\frac{\Delta C_z}{\Delta o/c} = 5 \times \frac{C_a}{C_\infty}\left(\frac{\gamma}{o/c}\right)\left(1 + \frac{m}{1.2\ c}\right)$$

The ratio $\frac{\gamma}{o/c}$ is constant for any given m/c. It seems to increase slightly with m/c.

$$\left(\frac{\gamma}{o/c} = 1.44 \text{ for } \frac{m}{c} = 9.6\% \text{ and } \left(\frac{\gamma}{o/c}\right) = 1.50 \text{ for } \frac{m}{c} = 17.4\%\right)$$

We may conclude that $\frac{\Delta C_z}{\Delta (o/c)}$ increases when m/c increases. The relative increase in thickness is therefore favorable to increase in lift.

<u>Effect of o/c on C_x</u>.- Fig. 3 shows that the unitary curves C_x are regularly staggered with reference to one another, except for the o/c of 8.85 and 9.25%. For the latter there is an overlapping, the same as for the C_z. We can express $C_x - C_{x_0}$, for any given angle, in terms of o/c. Thus we find that, for positive lifts

$$C_x - C_{x_0} = \frac{1}{100}\,(3.7 + 0.086\ C_{z_0})\ (o/c)^2$$

but this relation does not prove strictly true in the neighborhood of C_x minimum. We have tried to discover a more accurate formula by proceeding as follows:

a) <u>Study of unitary drag for the biconvex airfoil</u>.- The unitary drag C_{x_0} of the biconvex airfoil may be considered as constituting the sum of several components, to wit:

1. The minimum drag at the zero angle of lift, $C_{x_{p0}}$. This drag corresponds practically to the friction of the air on the airfoil section for the case of flow without lift.

2. The increase of this frictional drag $\Delta C_{x_{p0}}$, when the distribution of the dynamic pressures and velocities is changed along the airfoil section as a result of the flow with lift.

3. The induced drag C_{x_i} due to the lift.

4. The drag C_{x_d} due to discontinuity of the fluid filaments along the contour, which produce "Karman" vortices with the appearance of a rapidly increasing supplementary drag. Hence we have

$$C_{x0} = C_{x_{p0}} + \Delta C_{x_{p0}} + \frac{C_z^2}{\pi a} + C_{x_d}$$

We shall try to find the value of each of these drags.

1. Minimum drag of wing section.- According to the experiments, this drag is $C_{x_{p0}} = 0.8$. It may generally be calculated with sufficient approximation, when the coefficient of friction of the air on the exterior surface of the wing section is known. (It is known that this coefficient depends also on the characteristic product Vl.) For a closer approximation, it would be well to know also the distribution of the pressures or velocities along the contour, as given farther along for the evaluation of $\Delta C_{x_{p0}}$.

2. Increase of drag due to friction.- When the angle of attack is increased, the distribution of the pressures or velocities is modified. The upper camber of the airfoil is the seat of dynamic negative pressures and the corresponding velocities are greater than the average velocity of the air stream. The reverse is true of the lower camber. On an element dS of the wing section, the elementary drag dF, due to friction, is given by the formula

$$dF = C_o \, dS \frac{a V^2}{2 g} \cos^{\alpha} ,$$

dS $\cos\alpha$ being the projection of the element of the airfoil section on the direction of the air stream. Let us consider the diagram of the dynamic pressures for a certain lift. The abscissa is equal to $\Sigma \, dS \cos\alpha$. Consequently, the air F, of the surface EDABCKE (Fig. 8), corresponding to the negative pressures of the upper camber, will have the value

$$F_1 = \Sigma_m \, dS \frac{a V^2}{2 g} \cos^{\alpha}.$$

In like manner, the air F_2, corresponding to the pressures of the lower camber, will be

$$F_2 = \Sigma_i \, dS \frac{a V^2}{2 g} \cos^{\alpha}$$

The formula

$$\lambda = \frac{F_1 + F_2}{2 \, AB \times AD} = \frac{\Sigma_i + \Sigma_m}{2 V_O^2 \; \Sigma \, dS \cos\alpha}$$

gives the increase in drag due to the friction resulting from the distribution of the velocities along the contour. This formula varies therefore with the lift, i.e., with the angle of attack.

The frictional drag is then

$$F = \lambda \; C_O \frac{a V_O^2}{2 g} \Sigma \, dS \cos\alpha$$

or, in unitary drag,

$$C_{x_{pO}} + \Delta C_{x_{pO}} = \lambda C_O \times \frac{\Sigma \, dS \cos\alpha}{S}$$

$$= \lambda C_{x_{pO}}$$

For a theoretical airfoil section, calculated and investigated experimentally in a horizontal parallel air stream, Betz obtained the values of λ corresponding to different lifts C_z. We have remarked that the values found by Betz can be exactly represented by the simple expression

$$\lambda = 1 + 0.25\ C_z$$

This expression evidently depends on the shape of the airfoil section. For the present investigation, however, we are assuming that it applies to all the airfoil sections. Thus we obtain

$$\Delta\ C_{x_{p0}} = 0.25\ C_{z_0} \times C_{x_{p0}}$$

3. Induced drag.- This is readily calculated, by known formulas, in terms of C_z and of the real or reduced aspect ratio $(k^2\ L^2/S)$ of the given surface.

4. Drag due to discontinuities C_{x_d}.- In our present ignorance of the circumstances producing discontinuity and the correlative vortices, the value of C_{x_d} can only be determined experimentally by the difference between the total drag and the partial calculable drags. The following table gives the calculation data for the biconvex section No. 13A.

i =	0°	2.5°	5.0°	7.5°	10.0°	12.5°	15°
C_z =	0	0.185	0.365	0.55	0.73	0.90	1.05
$(C_{x_{p0}} + \Delta C_{x_{p0}})$ =	0.8	0.837	0.873	0.91	0.945	0.98	1.015
C_{x_i} =	0	0.183	0.705	1.61	2.825	4.30	5.85
$C_{x_{p0}} + \Delta C_{x_{p0}} + C_{x_i}$ =	0.8	1.020	1.578	2.52	3.77	5.28	5.85
$C_{x_{exp}}$ =	0.8	1.05	1.75	2.90	4.32	6.28	9.30
C_{x_d}	0	0.03	0.172	0.38	0.55	1.00	2.44

These values of C_{x_d} for the biconvex airfoil appear to increase in $C_z{}^2$, at least for the angles corresponding to lifts below $C_z = 0.9$, above which they increase still more rapidly, especially after exceeding the angle of maximum lift.

<u>b) Study of unitary drag for airfoil of any ordinate</u>.- We can apply to any airfoil the same reasoning as for a biconvex airfoil and write

$$C_{x_o} = C_{x_{po}} + \Delta C_{x_{po}} + C_{x_i} + C_{x_d}$$

For example, let us consider airfoil No. 17A, for which $o/c = 6.4\%$. We find experimentally that $C_{x_{6.4}} = 1.1$, but we must take the minimum theoretical C_x, or 1.03.* We can then make out the following table.

i =	-5°	-2.5°	0°	2.5°	5°
C_z =	0.035	0.23	0.415	0.60	0.78
$(C_{x_p} + \Delta C_{x_p})$ =	1.01	1.06	1.10	1.155	1.20
C_{x_i} =	0	0.28	0.91	1.90	3.25
$C_{x_p} + \Delta C_{x_p} + C_{x_i}$ =	1.01	1.34	2.01	3.05	4.45
$C_{x\,exp}$ =	1.15	1.30	2.00	3.06	4.45
C_{x_d} =	+0.14	-0.04	-0.01	+0.01	0.00

* We must take $C_{x_{6.4}}$ without initial discontinuities. According to the variation of C_{x_m} with o/c, this $C_{x_{po}}$ is given by the formula

$$C_{x_{po}} = 0.8 + 0.037\ o/c$$

or for

$$o/c = 6.4 \qquad C_{x_{po}} = 0.8 + 0.23 = 1.03$$

Table (Cont.)

i	= 7.5°	10°	12.5°	15°	17.5°
C_z	= 0.955	1.11	1.24	1.35	1.405
$(C_{x_p} + \Delta C_{x_p})$	= 1.24	1.28	1.31	1.340	1.49
C_{x_i}	= 4.85	6.55	8.17	9.70	10.05
$C_{x_p} + \Delta C_{x_p} + C_{x_i}$	= 6.09	7.83	9.48	11.04	11.54
$C_{x_{exp}}$	= 6.2	8.30	10.75	13.80	17.55
C_{x_d}	= +0.11	-0.47	1.27	2.76	6.00

It is seen that the discontinuities are generally zero, down to lifts less than $C_z = 100$. Thus airfoils possessing a certain camber appear to conform better to the lifting flow than the bi-convex airfoil. The values of C_{x_d} again increase very rapidly, when we approach the angle of maximum lift. Let us again take airfoil 19A for which o/c = 8%. We obtain $C_{x_p} = 1.8$, but must take 1.1. We have the following table.

i	= -5°	0°	5°	10°	15°	19°
C_z	= 5.25	0.52	0.905	1.26	1.465	1.465
$C_{x_p} + \Delta C_{x_p}$	= 1.13	1.24	1.35	1.45	1.50	1.50
C_{x_i}	= 0.08	1.44	4.35	8.43	11.40	11.40
$C_{x_p} + \Delta C_{x_p} + C_{x_i}$	= 1.21	2.68	5.70	9.98	12.90	12.90
$C_{x_{exp}}$	= 1.85	3.10	6.20	10.70	15.50	23.60
C_{x_d}	0.64	0.42	0.50	0.82	2.60	10.70

The positive values found for C_{x_d} show that the initial discontinuities continue at all angles of attack. Lastly, let us take airfoil No.15, for which o/c = 3.3%. It has been found experimentally that $C_{x_{p3.3}}$ = 0.92. The following table is compiled like the preceding one.

i	= -2.5°	0°	2.5°	5°	7.5°
C_z	= 0.035	0.32	0.405	0.585	0.77
$(C_{x_p} + \Delta C_{x_p})$	= 0.93	0.97	0.995	1.05	1.096
C_{x_i}	= 0.006	0.26	0.87	1.82	3.15
$C_{x_p} + \Delta C_{x_p} + C_{x_i}$	= 0.936	1.23	1.865	2.87	4.245
C_{x_e}	= 0.92	1.2	1.86	2.95	4.35
C_{x_d}	= -0.016	+0.03	+0.005	+0.08	0.105

i	= 10°	12.5°	15°	17.5°
C_z	= 0.935	1.09	1.225	1.26
$(C_{x_p} + \Delta C_{x_p})$	= 1.133	1.17	1.20	1.21
C_{x_i}	= 4.65	6.30	8.00	8.45
$C_{x_p} + \Delta C_{x_p} + C_{x_i}$	= 5.583	7.47	9.20	9.66
C_{x_e}	= 6.10	8.30	11.01	14.20
C_{x_d}	= 0.52	0.83	1.85	4.54

Here the discontinuities remain zero or negligible for lifts below C_z = 90. The minimum airfoil section drag corresponds therefore to the flow without initial discontinuities. This study leads to the following conclusions.

1. The airfoil section drag increases in proportion to the lift

2. The minimum airfoil section drag may be taken as initial reference, so long as the section is not too curving and does not produce too large initial discontinuities. It is, moreover, easy to take account of the limit of o/c, which causes an increase in this initial drag due to the airfoil section. In fact, from the experimental curves C_x we obtain the following values:

Airfoil	No. 13 A	14 A	15 A	16 A	17 A	18 A	19 A	20 A	21 A
o/c %	0	1.4	3.3	4.6	6.4	7.15	8.0	8.85	9.25
C_x min.	0.8	1.89	0.92	0.95	1.1	1.45	1.8	2.1	2.2
$i_{C_{xm}}$	0	-1°	-2.4	-3.4	-4.4	-5.2	-5.8	-5	-3.25

The graphic presentation of these data (Fig. 3), shows that:

a) For values of o/c not exceeding 6%, we have

$$C_{x_{po}} = C_{x_{p0}} + 0.037 \left(\frac{o}{c}\right) \quad (o/c) \text{ in } \%$$

or, $C_{x_{po}} = 0.8 + 0.37$ o/c for Royer airfoils.

b) For values of o/c not exceeding 6%, the minimum airfoil section drag given by the above formula must be increased about (o/c - 6.2). Thus we obtain

$$C_{x_{po}} = C_{x_{p0}} + 0.407 \frac{o}{c} - 2.3$$

or

$$C_{x_{po}} = 0.407 \frac{o}{c} - 1.5 \quad \text{for Royer airfoils.}$$

The increase to be applied to $C_{x_{po}}$ corresponds to the initial discontinuities, which are doubtless produced under the intrados of the cambered airfoil sections at angles near zero lift. It is moreover, observed that, for negative coefficients of lift, the same airfoil sections quickly produce large drags, owing to the rapid increase of the initial discontinuities.

c) The angle for which C_x is minimum is practically equal to 85% of the angle of zero lift.

$$\tan i_{C_{xm}} = 1.28\ o/c = 0.85 \tan\gamma$$

For a practical calculation, we might make $i_{C_{xmin}}$ and γ correspond, since the induced drag corresponding to C_z for $i_{C_{xmin}}$ is entirely negligible.

3. The biconvex airfoil presents, in general, moderate but not negligible discontinuities even at angles of lift below 0.9. The cambered airfoil derived from this biconvex airfoil does not present such discontinuities. In order to calculate the effect of the relative o/c on the unitary drags, the experimental drags of the biconvex airfoil should, therefore, be diminished by the amount of the added drag due to the discontinuities. We found that, for airfoil No. 13A, this value of C_{x_d} is practically equal to $1.2\ C_z^2$.

Effect of o/c on C_x (difference between the polars).- The relation found

$$C_{z_1} - C_{z_2} = P\ (\varphi_1 - \varphi_2),$$

enables the plotting of the unitary curves in terms of i, when we know C_{z_2}, φ_2 and φ_1, as likewise the ratio γ_1/φ_1 which is constant for a given series of airfoils. For a given value $C_{z_1} = C_z$, the induced drags are the same and the difference between the polars is

$$C_{x_1} - C_{x_2} = C_{x_{p1}} - C_{x_{p2}} + \Delta C_{x_{p1}} - \Delta C_{x_{p2}} +$$

$$(C_{x_{d1}} - C_{x_{d2}}) = (1 + 0.25\ C_z)\ (C_{x_{p1}} - C_{x_{p2}}) + (C_{x_{d1}} - C_{x_{d2}}).$$

We have seen that, when o/c does not exceed 6%,

$$C_{x_{p1}} - C_{x_{p2}} = 0.037 \left(\frac{o_1}{c_1} - \frac{o_2}{c_2}\right)$$

We then have

$$C_{x_1} - C_{x_2} = 0.037 \left(1 + \frac{C_z}{4}\right) (\varphi_1 - \varphi_2) + C_{x_{d1}} - C_{x_{d2}}$$

the difference due to the airfoil section being very small.

So long as $C_{x_{d1}} - C_{x_{d2}} = 0$, the polars will be regularly staggered toward the right, in proportion as o/c increases, the stagger tending to increase slightly with C_z, for

$$C_z = 0 \quad C_{x_8} - C_{x_0} = 0.037 \times 8 = 0.296 = 3\text{ mm}$$

$C_{x_{d1}}$, $C_{x_{d2}}$ and $C_{x_{d1}} - C_{x_{d2}}$ generally differ, however, from zero. $C_{x_{d1}} - C_{x_{d2}}$ may even become negative, when airfoil 2 attains the higher lifts for which the more arched airfoil 1 would be better adapted. These discontinuities cause the majority of the staggers between the polars.

On several of the experimental polars, there is observed a point of increase. This increase occurs when C_z = about 1.14. At this point

$$(C_{x_{d1}} - C_{x_{d2}}) = 0.0475 \; (\varphi_1 - \varphi_2).$$

For airfoils whose o/c is less than 6.2%,

$$C_{x_{p1}} - C_{x_{p2}} = 0.407 \left(\frac{o_1}{c_1} - \frac{o_2}{c_2}\right) = 0.407 \; (\varphi_1 - \varphi_2)$$

$$C_{x_1} - C_{x_2} = 0.407 \; (\varphi_1 - \varphi_2) + \frac{C_z}{4} \times 0.037 \; (\varphi_1 - \varphi_2) + C_{x_{d1}} - C_{x_{d2}}$$

since

$$\Delta C_{x_{p1}} = 0.250\ C_{z_1}\ (C_{x_{p0}} + 0.037\ \varphi_1)$$

$$\Delta C_{x_{p2}} = 0.25\ C_{z_2}\ (C_{x_{p0}} + 0.037\ \varphi_2)$$

$$= C_z \times 0.25 \times 0.037\ (\varphi_1 - \varphi_2)$$

whence

$$C_{x_1} - C_{x_2} = (0.407 + 0.009\ C_z)\ (\varphi_1 - \varphi_2) + (C_{x_{d1}} - C_{x_{d2}})$$

Study of Discontinuities.

a) Discontinuities in the vicinity of zero lift.- We have seen that $C_{x_{min}}$ varies directly as o/c up to o/c = 6.2%, but beyond this value the $C_{x_{min}}$ given by 0.037 o/c must be increased by 0.37 (o/c - 6.2). This increase is due to discontinuities in the vicinity of the angle of zero lift. These discontinuities should rapidly diminish, when C_z increases slightly and the airfoil section is better adapted to the lifting flow, so that the value of the initial discontinuities takes the form of 0.37 (o/c - 6.2) - N C_z.

In the case of the biconvex airfoil, we have seen that the discontinuities at sustaining speeds are at first of the form 1.25 $C_{z_0}^2$ to C_z = 0.9, when they begin to increase much more rapidly. For other airfoils, the calculations already indicated render it possible to plot the curve of the C_{x_d} against C_z or C_x. We thus find that the discontinuities are zero or negligible up to values of C_z as much larger as o/c is larger. The result of this study may be represented by a simple relation such as $C_{z_{LD}}$ = 0.145 to 0.15 o/c.

On the unitary curves $C_z = o(i)$ we then observe that the angles of attack corresponding to the values of $C_{z_{LD}}$ thus determined are exactly equal to arc tan 2 o/c (Fig. 2). Moreover, we have

$$C_{z_{LD}} = 0.147\ o/c = 0.042 \times 0.75\ \ 2\ o/c + 0.042\ i$$

whence

$$i = \frac{0.147 - 0.063}{0.042}\frac{o}{c} = \frac{0.084}{0.042}\frac{o}{c}$$

whence

$$i = \frac{2o}{c}$$

This simple relation expresses an important property of cambered airfoil sections from the viewpoint of the origin of the discontinuities.

Verification of the above on Dewoitine airfoils.- For these airfoils, we do not have the curves of the initial biconvex airfoil, but we can take airfoil No. 13A, which has the same thickness. Under these conditions, it is seen that the minimum C_x of all these airfoils is greater than given by the formula

$$C_{x_{min}} = 0.8 + 0.037\ o/c$$

This shows that there are always initial discontinuities which must be taken into account. We have, therefore, taken the values of $C_{x_{min}}$ given by the formula and have found that for

Airfoil No. 26A, $C_{x_{min}} = 0.8 + 0.037 \times 5 = 0.95$

" No. 27A " $= 0.8 + 0.037 \times 7.3 = 1.07$

" No. 28A " $= 0.8 + 0.037 \times 11.3 = 1.22$.

We then calculated the resultant of the drags and plotted the polars of the discontinuities (Fig. 6). The discontinuities for airfoil 28A do not become zero, which proves that, by reason of the camber of the airfoil section, the initial discontinuities are maintained until the appearance of the posterior discontinuities. The origin of the latter appears to be characterized for the lifts $C_{z_d} = 1.4$ o/c corresponding to the angles

$$i_{LD} = 0.9 \times 2 \text{ o/c}.$$

Lachassagne airfoils.- We assumed that the minimum C_x of airfoil E429 could be taken as the origin. Since this airfoil has a camber of 2.66% and $C_{x_p} = 1.14$ for $C_z = 0.174$, we have, for the initial biconvex airfoil,

$$C_{x_{p0}} = \frac{(1.14 - C_{x_i}) - (0.037 \times 2.67)}{1 + 0.25 \times 0.17} = 0.81$$

whence we deduced, for

Airfoil No. E 429, $C_{x_p} = 0.81 + 0.1 = 0.91$

" No. E 431, $C_{x_p} = 0.8 + 0.217 = 1.017$

" No. E 432, $C_{x_p} = 0.8 + 0.28 = 1.08$

" No. E 438, $C_{x_p} = 0.8 + 0.33 = 1.13$.

As above, we plotted the discontinuities (Fig. 4) according to the resultant of the drags, these polars being characterized by

$$C_{z_D} = 0.155 \text{ o/c}$$

with

$$i_{LD} = 2.4 \text{ o/c}$$

In short, the polars of the posterior discontinuities are characterized in general by a limiting angle i_{LD} which varies between 0.9×2 o/c and 1.2×2 o/c.

These polars have such a trend that they cannot be calculated. The present paper is simply for the purpose of establishing, by finding the resultant of the drags, the angle and lift at which these posterior discontinuities appear.

Effect of o/c on the Maximum C_z.

If we plot the maximum values of C_z against o/c, we obtain, for the Royer, Dewoitine, and Lachassagne airfoils, a group of three parallel straight lines, the Royer line lying between the other two (Fig. 7), and having for its equation

$$C_{z_m} = 105 + 5.6\ (o/c) \qquad o/c \text{ in } \%.$$

The coefficient 5.6 would therefore be practically the same for all three series of airfoils and the constant at the origin ($C_{z_{m0}}$) would be 111 for the Dewoitine airfoils and 90 for the Lachassagne.

Mr. Margoulis indicated on p.258 of No. 29 (June, 1923) of L'Aeronautique, a straight line which cuts the R, D and L group. Its equation would be

$$C_{z_m} = 80 + 8.6\ (o/c)$$

After what we have already seen, the coefficient 8.6 would seem to be considerably too large.

Effect of the Camber on the Moments.

It is known that, according to the theory of supporting airfoils, the angular coefficient of the straight lines representing C_m in terms of C_z is equal to 0.25.

If we examine these straight lines for the different Royer airfoils, we find they are actually parallel, with this same angular coefficient (0.5 in the case of the S.T.Ae. graphics), and differ only in the abscissa at their origin, i.e. in the value of C_{m0} of the moment at zero angle of lift. On representing the values of C_{m0} in terms of o/c, we obtain $C_{m0} = 1.88$ o/c for the R, L and D airfoils.

It follows that the straight lines of C_m (within the limits where the initial and posterior discontinuities are small) can be plotted in advance from any airfoil by means of the formulas $C_{m_0} = 100\ c_{m_0}$ and

$$C_{m_0} = 0.25\ C_z + 1.88\ o/c \qquad (o/c \text{ in } \%)$$

Mr. Margoulis gave, in the above-mentioned paper, the variations of C_{m_0} in terms of the angle of the bisector of the elements of egress with the theoretical chord. He thus found $C_{m_0} = 1.2\alpha$.

This value, found as the mean for the many airfoils tested at Göttingen, is in accord with the value found by us, since, according to the figures indicated by Mr. Margoulis, we have $o/c = 0.64\,\alpha$, whence $1.88\ o/c = 1.2\,\alpha$. The coefficient 1.88 must, moreover, be a function of the thickness of the airfoil, as indicated by Mr. Margoulis in his paper.

Lastly, the C_{m_0}, thus calculated from experimental results, appear to be in the same relation with the theoretical C_{m_0} as the experimental lifts:

$$\frac{C_{m_{0\,exp.}}}{C_{m_{0\,the.}}} = \frac{C_{z_{exp.}}}{C_{z_{the.}}} = 0.80 \text{ to } 0.85.$$

If we imagine an airfoil automatically deformable (by construction or some suitable mechanism), so that we may pass regularly from the airfoil section with a small o/c to the section with a large o/c, when C_z must be increased, the straight line of the C_m for this automatic airfoil will be more inclined to the axis of the C_m. It will be such that we may have neutral or stable equilibrium, at will, according to the change made.

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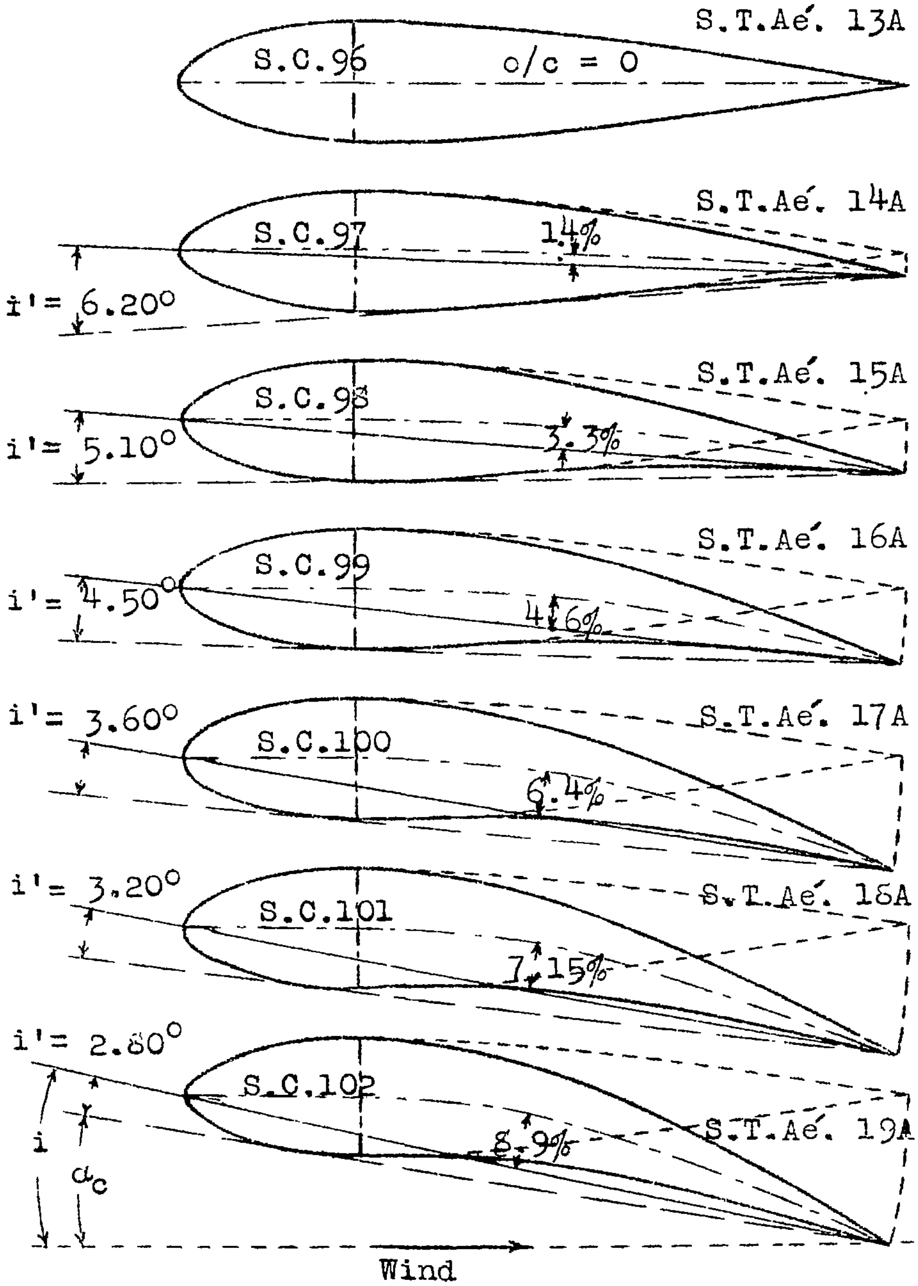
for Aeronautics.

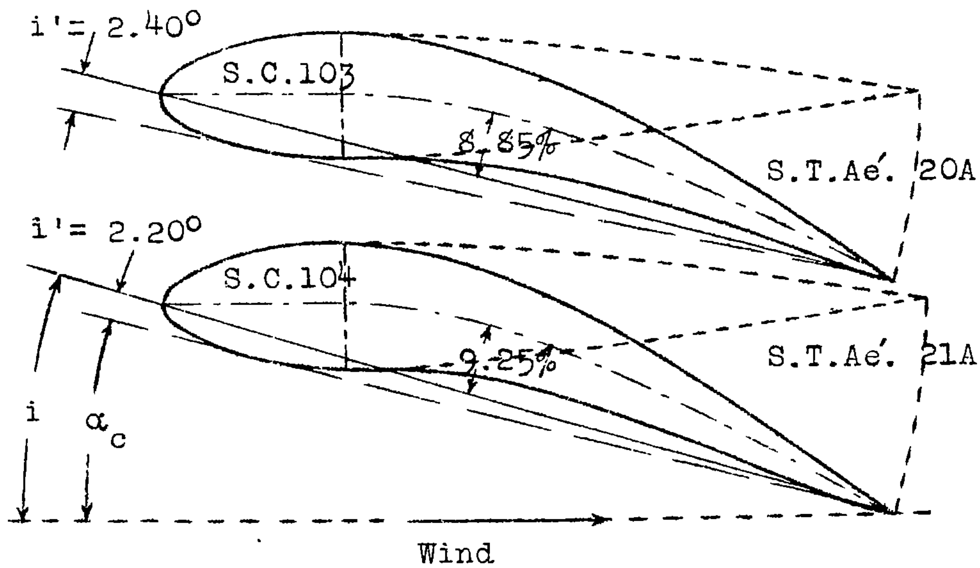
Fig.1.

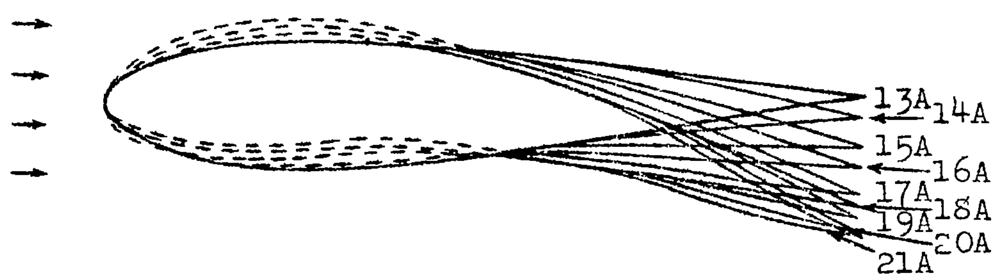


Royer series of airfoil sections derived from a symmetrical biconvex section.

Fig.1. Continued on next page.

Continuation of Fig.1.





Superposition of the different sections, at the respective angles, showing the deviations.

Royer series of airfoil sections derived from a symmetrical biconvex section.

Continuation of Fig.1.

Fig.2

A = Theoretical C_z, (Aspect ratio=∞)
B = Theoretical C_z, (Aspect ratio=6)
C = C_x Experimental, (a=6)
D = id = arc tan, 2o/c. C_{zd}=0.147 o/c.
E = Line of the angles and lifts of discontinuities.

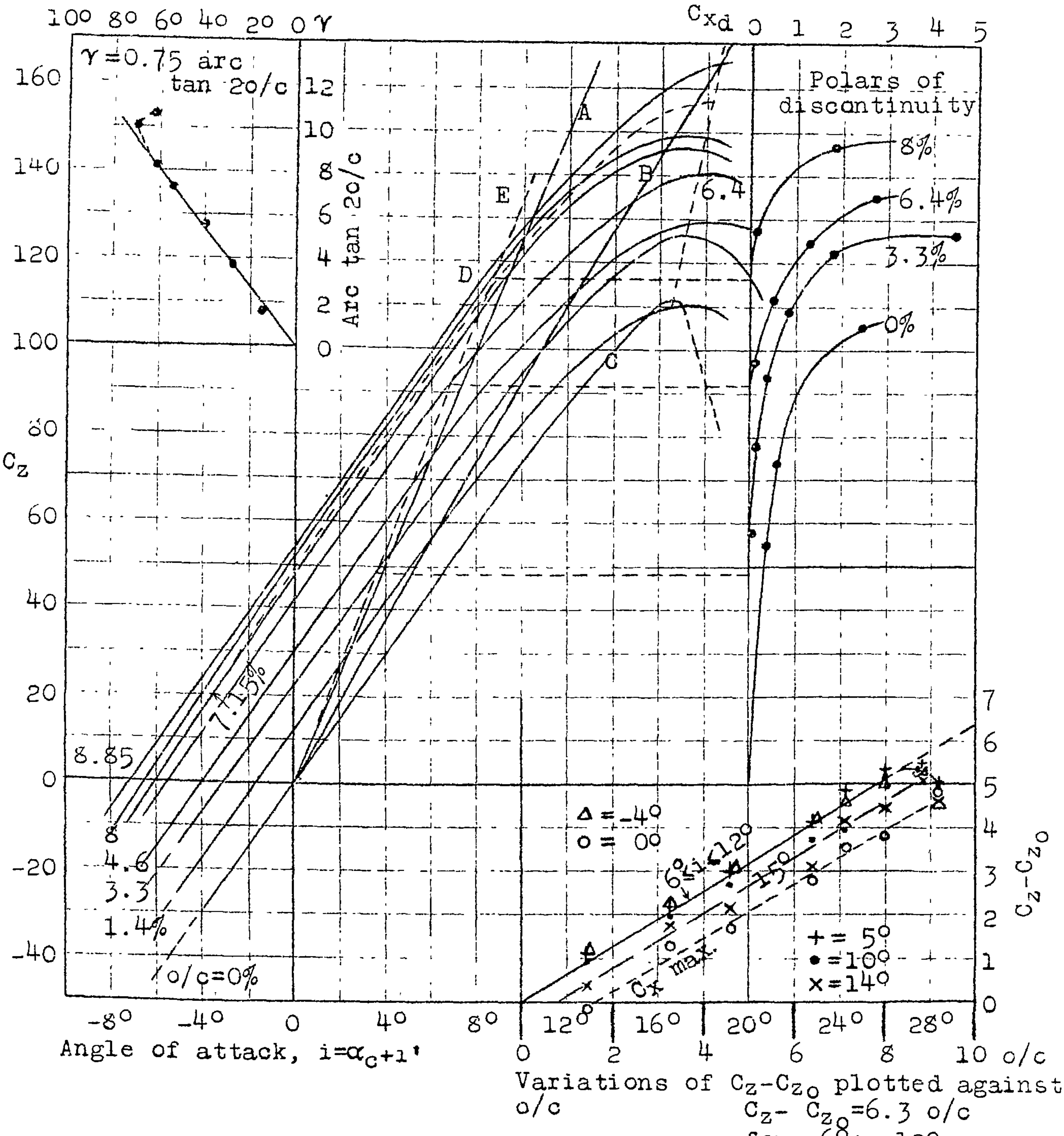


Fig. 2 Effect of maximum camber o/c on C_z. Results for Royer airfoils (S.T.Ae. 13A-21A)

Fig. 3

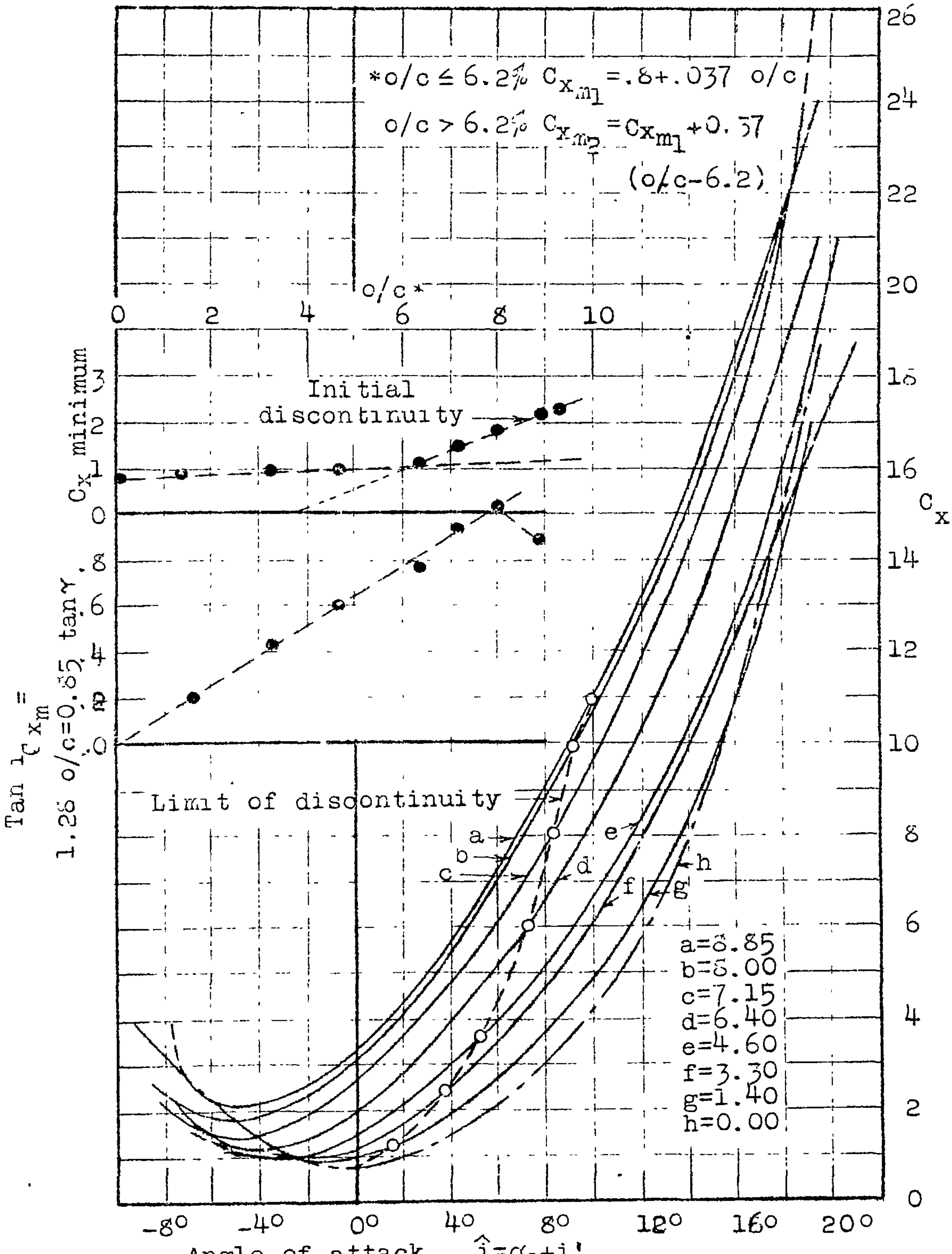


Fig. 3 Effect of maximum camber o/c on the C_x. Results for Royer airfoils (S.T.Aé. 13A - 21A)

Fig.4.

A = Limit of discontinuity.
B = $C_{z_{LD}}$ = 0.155 o/c. Tan, i_{LD} = 1.2×2o/c
C = Theoretical biconvex airfoil.

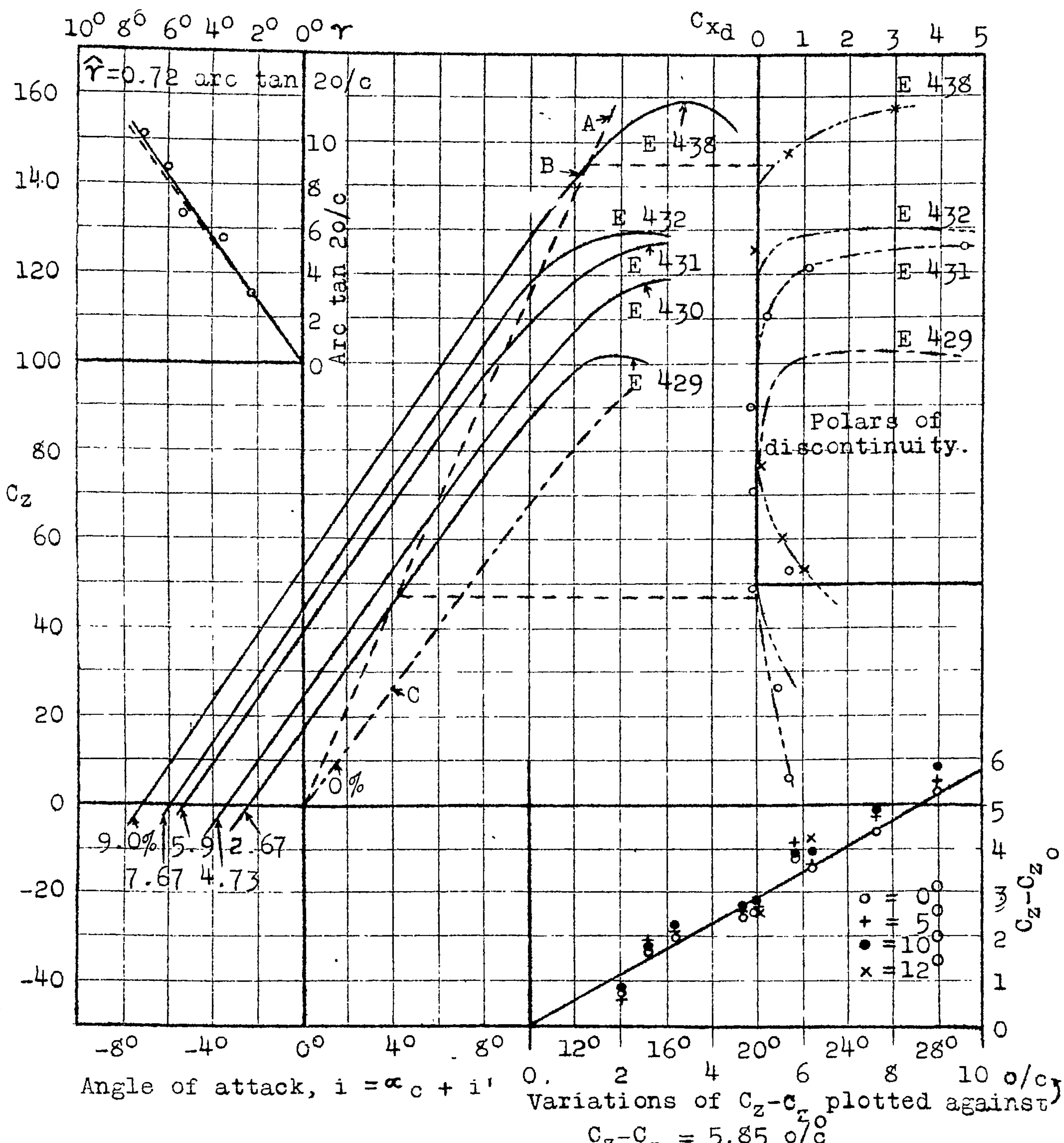


Fig.4. Effect of maximum camber o/c on C_z. Relative results for Lachassagne airfoils (E-429 to 432 and E-438).

Fig.5.

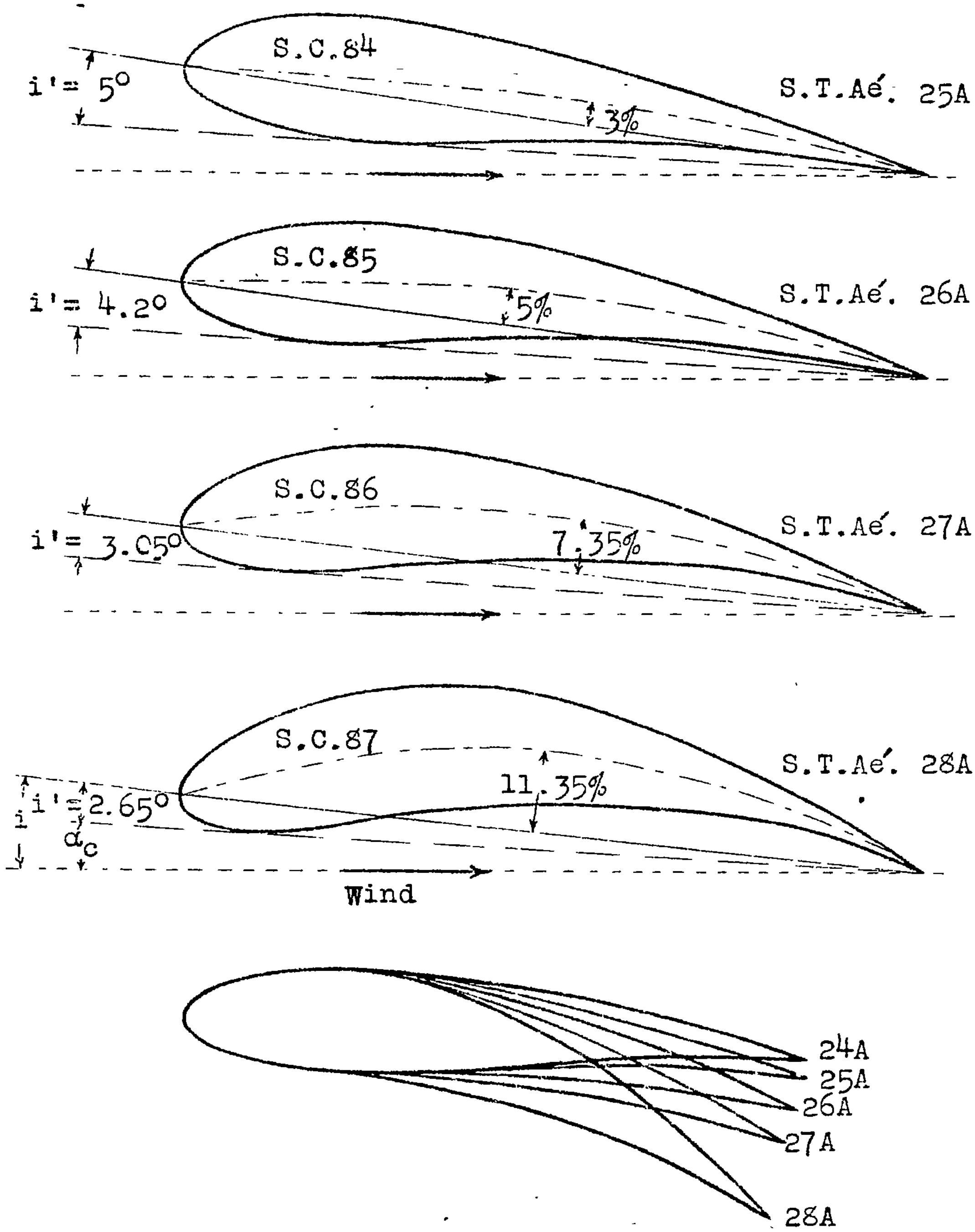


Fig.5. Dewoitine series of airfoils S.T.Aé. 25A to 28A.

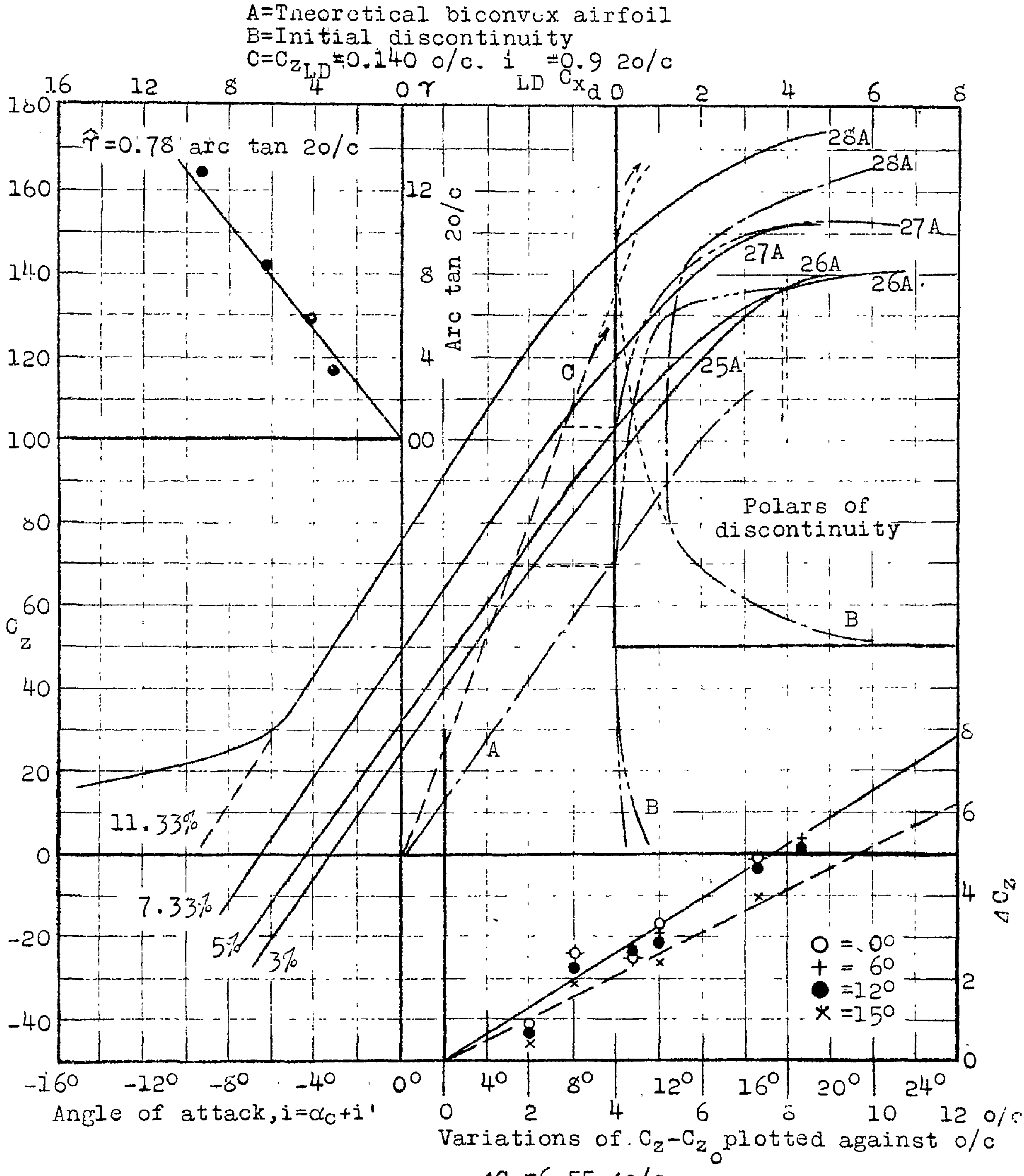


Fig. 6 Effect of maximum camber o/c on C_z. Results for Dewoitine airfoils (S.T.Aé.25A to 28A)

Figs. 7 & 8

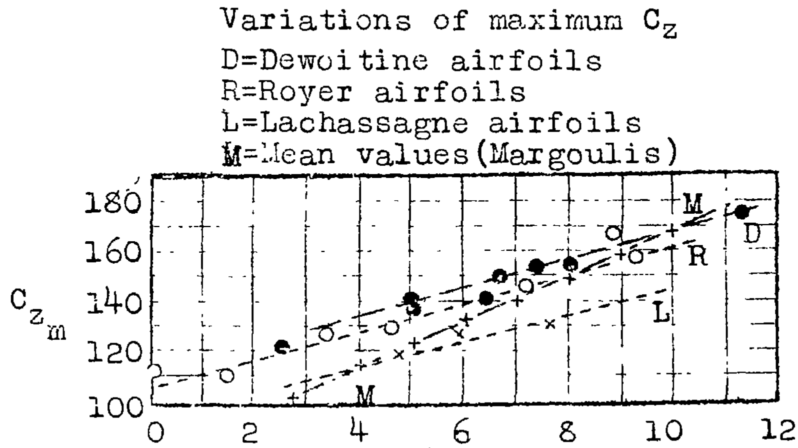


Fig. 7

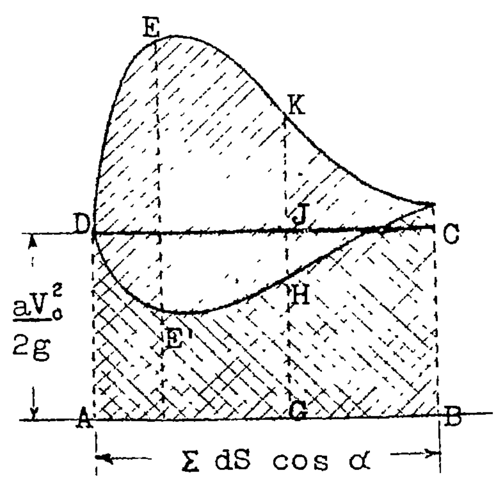


Fig. 8