NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4271

MAXIMUM THEORETICAL TANGENTIAL VELOCITY COMPONENT POSSIBLE FROM STRAIGHT-BACK CONVERGING AND CONVERGING-DIVERGING STATORS AT SUPERCRITICAL PRESSURE RATIOS

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SUMMARY

An analytical investigation of the maximum theoretical tangential velocity component possible by expansion about straight-back stator blade trailing edges has been made by the method of characteristics for converging and converging-diverging stators.

The results of the investigation showed that converging stators are limited to exit tangential components of critical velocity ratios less than 1.714, while values up to the theoretical limit of 2.449 corresponding to an infinity Mach number might be realized for converging-diverging stators.

The greatest theoretical gains in tangential velocity due to expansion about stator blade trailing edges occur at small blade exit angles as measured from tangential and relatively low velocities at the exit of the guided-channel portion of the flow passage.

Unless design requirements are rigorous or unless extremely small blade exit angles as measured from the tangential are to be used, an approximate method of assuming uniform flow at the stator exit appears adequate in determining the maximum tangential velocity component possible from stators.

INTRODUCTION

In many present day turbine applications there exists the desirability of minimum turbine weight and low mass flow of working fluid through the turbine. These design requirements result in few turbine stages and high specific work per stage. The specific work output per turbine stage is determined by the change in tangential velocity, or change in "whirl," across the rotors. Because the whirl entering the rotors is established by the stators, it is apparent that the level of work output is greatly influenced by the level of whirl leaving the stators. It therefore appears desirable to establish the maximum limits of exit whirl possible from a row of stator blades. These limits were established in reference 1 for straight-back converging stators as a function of stator blade exit angles from 90° to about 15°, where the blade exit angle was measured from the tangential.

This investigation determines analytically the maximum limits of the tangential component of critical velocity ratio at the exit of both converging and converging-diverging stators. The results may then be used as an aid in preliminary turbine design work to determine the maximum stator exit whirl possible for selected flow parameters. If the design exit whirl selected is less than maximum, the theoretical margin in whirl possible by expansion about the trailing edges may also be determined from this analysis for turbine off-design considerations.

The analysis is concerned only with that portion of the stator passage from the exit of the guided channel to the stator exit. It is in this region that supersonic expansion about the blade trailing edge occurs and results in an increase in exit whirl over that at the exit of the guided channel prior to expansion. The method of characteristics was used to determine the static-pressure variation on the suction surface due to expansion about the blade trailing edge. This was done in a manner similar to that used in reference 1 where converging stators were studied. The major assumptions include: two-dimensional isentropic flow, no boundary-layer growth, uniform flow across the passage at the exit of the guided channel, and straight suction-surface blading from the exit of the guided channel downstream to the blade trailing edge.

The limits of whirl were established as a function of two variables: the critical velocity ratio at the exit of the guided channel and the blade exit angle. The complete supersonic range of critical velocity ratio was covered, and, for each value of critical velocity ratio selected, a range of blade exit angles was selected to include all expansion possibilities from zero to maximum.

SYMBOLS

A area, sq ft

B point at blade trailing edge about which the flow expands

D ratio of suction-surface length downstream of station 2 to the passage width at station 2, also $D = \cot \alpha$

F force, lb

g acceleration due to gravity, 32.17 ft/sec^2

l length

M Mach number

p static pressure, lb/sq ft

V absolute velocity, ft/sec

w weight-flow rate, lb/sec

α stator blade exit angle, measured from tangential direction, deg

γ ratio of specific heats

ρ density, lb/cu ft

μ Mach angle corresponding to critical velocity ratio at station 2, deg

Subscripts:

ap approximate

cr conditions at Mach number of 1.0

s suction surface of blade downstream of station 2

u tangential component

x axial component

1 station at stator throat

2 station at exit of guided channel

3 station at stator exit

Superscript:

' stagnation state

METHOD OF ANALYSIS

The method of analysis in determining the maximum tangential component of critical velocity ratio for both converging and converging-diverging stators was similar to that used in reference 1 for converging stators.

Fundamental Assumptions

Figure 1 shows expansion about the trailing edge of a typical configuration investigated along with the station numbers used. The following assumptions were made for this analysis:

(a) two-dimensional isentropic flow

(b) $\gamma = 1.40$

(c) no boundary-layer growth

(d) uniform flow across the passage at station 2

(e) constant velocity and flow angle from station 2 to Mach line

(f) straight suction surface downstream of station 2

Description of Expansion

Figure 2 shows an example flow passage selected to describe the process of expansion around point B, the blade trailing edge. Between station 2 and the Mach line there is no expansion, and the velocity and flow angle remain constant and equal to their corresponding values at station 2. As indicated, the static pressure on the suction surface also remains constant in this region. Between the Mach line and the exit the flow expands around point B toward the axial. When the expansion waves that fan out from point B reach the suction surface, they are reflected as expansion waves which bend the flow in that area back along the suction surface.

Because the flow bends toward the axial during expansion, it might be inferred that the tangential velocity should decrease rather than increase. However, the increase in flow angle as measured from the tangential is more than offset by the velocity increase so that the tangential component of velocity actually increases. This can also be realized from the fact that as the pressure decreases along the suction surface there must also exist a net increase in tangential momentum of the flow and hence an increase in tangential velocity.

Sketches of typical stators investigated are shown in figure 3 for an arbitrarily selected blade exit angle of about 17°. This figure shows that as the velocity at station 2 prior to expansion is increased, the Mach angle decreases, and, hence, for a given blade angle there is less area for expansion between the Mach line and the exit. All blade configurations used in this analysis are the sharp-cornered minimum-length type as described in reference 2. However, because the analysis is concerned with only that portion of the passage from station 2 to the exit, any other type of stator design for the region upstream of station 2 would produce the same theoretical results. Blade trailing-edge thickness was not considered in the analysis because it has no effect on the tangential component of velocity.

Development of Equations

The maximum exit whirl at station 3 was determined by the equations of momentum conservation as applied to the area bounded by station 2, station 3, and the straight suction surface. The maximum whirl at station 3 for a given velocity at station 2 and a given blade exit angle α will occur when the pressure ratio across the stators is sufficient to cause the Prandtl-Meyer expansion around point B to extend along the suction surface to the blade trailing edge. At this condition the increase in tangential force F_u on the fluid caused by the decrease in static pressure along the suction surface is expressed by the momentum equation

$$F_u = \frac{w}{g}\left(V_{u,3} - V_2 \cos\alpha\right) \tag{1}$$

Applying the continuity relation and reducing to dimensionless terms give

$$\frac{F_u}{A_2 p_2'} = \left[\frac{2\gamma}{\gamma + 1}\left(\frac{\rho V}{\rho' V_{cr}}\right)_2\right]\left[\left(\frac{V_u}{V_{cr}}\right)_3 - \left(\frac{V}{V_{cr}}\right)_2 \cos\alpha\right] \tag{2}$$

The geometric length ratio D is shown in figure 4 and is defined as the ratio of the length of the straight suction surface downstream of station 2 to the passage width at station 2 (also $D = \cot\alpha$). By normalizing with respect to the passage width at station 2, D becomes equal to the suction-surface length. For simplicity, the remainder of the analysis considers the passage width equal to unity and refers to D as the suction-surface length. This suction-surface length D is used in evaluating the increase in tangential force exerted on the fluid by the decrease in static pressure along the suction surface. This increase in force may be evaluated by integrating the pressure variation along the suction surface:

$$F_u = p_2 A_2 \cos\alpha - A_2 \sin\alpha \int_0^D p \, dD \qquad (3)$$

where p is the static pressure anywhere along the suction surface. Again, reducing to dimensionless terms gives

$$\frac{F_u}{A_2 p_2'} = \left(\frac{p}{p'}\right)_2 \cos\alpha - \sin\alpha \int_0^D \frac{p}{p'} \, dD \qquad (4)$$

Substituting equation (4) into equation (2) results in a final expression for the desired exit whirl at station 3:

$$\left(\frac{V_u}{V_{cr}}\right)_3 = \left(\frac{V}{V_{cr}}\right)_2 \cos\alpha + \frac{\left(\frac{p}{p'}\right)_2 \cos\alpha - \sin\alpha \int_0^D \frac{p}{p'} \, dD}{\frac{2\gamma}{\gamma + 1}\left(\frac{\rho V}{\rho' V_{cr}}\right)_2} \qquad (5)$$

Procedure

Inspection of equation (5) indicates that the exit whirl at station 3 is a function only of the blade exit angle α, the critical velocity ratio at the exit of the guided channel $(V/V_{cr})_2$, and the variation in static pressure along the suction surface $\int_0^D \frac{p}{p'} \, dD$. The following procedure was therefore used:

(1) The entire supersonic range of $(V/V_{cr})_2$ was selected in steps of 0.2 from 1.0 to the maximum theoretical value of 2.449 (corresponding to $M = \infty$).

(2) For a given value of $(V/V_{cr})_2$ a range of blade exit angles α was selected from zero to a value equal to the Mach angle μ corresponding to the given value of $(V/V_{cr})_2$. These limits imposed on exit angle are discussed later.

(3) For each value of $(V/V_{cr})_2$ and α selected, the variation in static pressure along the suction surface between the Mach line and the trailing edge was determined by the method of characteristics and integrated to evaluate $\int_0^D \frac{p}{p'} \, dD$. This pressure variation is shown in figure 5.

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(4) Finally, the exit whirl $(V_u/V_{cr})_3$ was calculated by equation (5).

Limits of Blade Exit Angle

For a given velocity at station 2, the amount of expansion possible increases as the blade exit angle decreases. It is reasonable then that the maximum amount of expansion possible occurs at a blade exit angle of 0°. This can be seen from inspection of equation (5). At an exit angle of 0° both terms on the right side of equation (5) are maximum, and the equation reduces to

$$\left(\frac{V_u}{V_{cr}}\right)_3 = \left(\frac{V}{V_{cr}}\right)_2 + \frac{\left(\frac{p}{p'}\right)_2}{\frac{2\gamma}{\gamma + 1}\left(\frac{\rho V}{\rho' V_{cr}}\right)_2} \tag{6}$$

Thus, the lower limit of blade exit angle α for all velocities at station 2 is established as 0°.

The maximum blade angle for a given velocity at station 2 was selected such that the axial Mach number at station 3 was limited to unity. This limit occurs when the blade exit angle is set equal to the Mach angle corresponding to the critical velocity ratio at station 2. At this condition the Mach line spans the exit, the exit axial Mach number is unity, no expansion occurs about the trailing edge, and the exit whirl is the same as that at station 2. In equation form,

$$\left(\frac{V_u}{V_{cr}}\right)_3 = \left(\frac{V_u}{V_{cr}}\right)_2 = \left(\frac{V}{V_{cr}}\right)_2 \cos \mu \tag{7}$$

For blade angles greater than this, the exit axial Mach number is greater than unity. Thus, the upper limit of blade angle for each velocity at station 2 is established as being equal to the Mach angle corresponding to the critical velocity ratio at station 2.

RESULTS OF ANALYSIS

The maximum theoretical exit whirl possible from both converging and converging-diverging straight-back stators due to expansion about stator blade trailing edges is shown in figure 6. The figure presents maximum stator exit whirl $(V_u/V_{cr})_3$ as a function of blade exit angle for selected supersonic values of critical velocity ratio at the exit of the guided channel. A curve was drawn through the points corresponding to the limiting axial Mach number of unity for each selected value of $(V/V_{cr})_2$ and results in an envelope of $M_x = 1.0$.

The maximum expansion for all values of $(V/V_{cr})_2$ occurs at a blade exit angle of 0°. Thus, the area, which is bounded by zero exit angle, the envelope of axial Mach number of unity, and the lowest curve representing a converging stator, includes the maximum whirl for all types of stators within the assumptions of this analysis for all exit blade angle settings and all levels of velocity at station 2.

A theoretical curve of the maximum whirl possible from converging stators is presented in reference 1 for blade exit angles from 90° to about 15° and is verified experimentally with three different blade configurations in the same reference. This theoretical curve is the same as the lowest curve in figure 6 between the same range of blade exit angles. An important feature of this curve is the limitation imposed on converging stators. The maximum tangential component of critical velocity ratio under ideal conditions is 1.714 at an exit blade angle of 0°. For applications requiring higher levels of whirl from the stator blade row, converging-diverging stators would have to be employed, in which case whirl components of critical velocity ratio up to the theoretical limit of 2.449 might be realized.

Whereas figure 6 presents the limits of expansion, figure 7 shows how much expansion could occur between stations 2 and 3. Figure 6 is repeated as the group of solid curves. For each solid curve there is a corresponding dashed curve also originating at the envelope of $M_x = 1.0$. Both of these curves represent a constant indicated velocity at station 2 prior to expansion. The dashed curve represents the whirl velocity at station 2 prior to expansion as the blade angle is varied and is therefore simply a cosine curve. The solid curve represents the maximum exit whirl possible due to expansion as the blade angle is varied. Therefore, for a given $(V/V_{cr})_2$, the maximum increase in whirl possible by expansion for a selected blade angle is simply the vertical difference between the two curves. For example, if $(V/V_{cr})_2$ is selected as being equal to 1.40, the maximum increase in the whirl component of critical velocity ratio that could be expected for a blade exit angle of 20° would be 1.423 minus 1.313, or 0.110. Similarly, for the same velocity at station 2 but a blade exit angle of 10°, the maximum increase possible becomes 0.234, or more than twice as much. This again illustrates the fact that, for a given velocity at station 2, more expansion is possible as the blade exit angle α decreases because there is more area in which to expand between the Mach line and the exit.

For preliminary turbine design work, two uses are suggested for figure 7. First, it establishes the maximum limits of whirl possible for given conditions of $(V/V_{cr})_2$ and α. Second, the figure establishes the theoretical margin in whirl velocity for off-design considerations when the design-point whirl is selected as less than maximum.

The difference between the solid and dashed curves of figure 7 was calculated as percentage increase in whirl and is presented as figure 8. This figure shows the maximum percentage increase in whirl possible by expansion as a function of blade exit angle for each selected critical velocity ratio at station 2. Figure 8 indicates that when the required exit whirl is relatively low a high percentage of the whirl may be realized by expansion about the trailing edge. However, when the required exit whirl is high, only a small percentage of the whirl can be realized by expansion about the trailing edge, while the greatest percentage must come from expansion in the guided passage from the throat to station 2. For example, at a blade exit angle of 15°, the increase in whirl due to expansion about the trailing edge is 45 percent for a converging stator and only 23 percent for a converging-diverging stator with a critical velocity ratio of 1.2 at station 2. The percentage increase drops rapidly as the critical velocity ratio at station 2 is increased further.

The maximum whirl at station 3 as determined by the method of characteristics and used as the basis of this analysis is an average value of whirl resulting from momentum conservation. There is a velocity variation across the exit annulus at station 3, with the velocity greater at the suction-surface trailing edge than at the pressure-surface trailing edge. The maximum exit whirl as determined by the method of characteristics results from momentum conservation and includes this flow variation.

Approximate Method of Determining Maximum Stator Exit Whirl

An approximate method commonly used in determining maximum exit whirl at stator limiting-loading does not consider this flow variation at station 3. This method assumes uniform flow at station 3, the specific weight-flow parameter $\rho V_x/\rho' V_{cr}$ equal at stations 2 and 3, and the flow expanded to an axial Mach number of 1.0 at station 3. If the critical velocity ratio at station 2 and the exit blade angle are known, the maximum exit whirl may be calculated or read directly from flow charts such as that developed in reference 3. However, this method does not satisfy momentum conservation, and considerable error may exist in resulting limits of exit whirl. In view of this, the validity of assuming uniform flow at the stator exit was determined by calculating the limits of whirl by the approximate method and comparing the results with those previously obtained by the method of characteristics. The same range of selected values of $(V/V_{cr})_2$ and α was used as was used by the method characteristics.

Comparison of Approximate Method with Method of Characteristics

The results of the two methods of determining maximum stator exit whirl are compared in figure 9. Figure 6 was again repeated as the set of solid curves of figure 9. For each solid curve representing a selected $(V/V_{cr})_2$ there is a corresponding dashed curve also originating at the envelope of $M_x = 1.0$. The dashed curves are the maximum limits of whirl as determined by the approximate method discussed previously. It can be seen from figure 9 that for very small blade angles a considerable difference exists between the two methods; the approximate method produces higher tangential velocities than could possibly be expected. However, as the blade angle increases, this difference decreases rapidly.

This is shown more clearly in figure 10, which is the result of figure 9 plotted as percentage difference. Figure 10 shows the percentage error in stator exit whirl at limiting-loading introduced by the approximate method of assuming uniform flow at the stator exit. If a practical minimum blade exit angle of 10° is chosen, the maximum error resulting from the approximate method would be 8.5 percent for a converging stator and would decrease as the velocity at station 2 increased. At a blade exit angle of 20° the maximum error drops to only 3.5 percent. It appears then that, unless the design procedure requirements are very rigorous or unless extremely small exit angles are to be employed, the approximate method of determining maximum stator exit whirl by assuming uniform flow at the stator exit is adequate.

SUMMARY OF RESULTS

An analytical investigation of the maximum theoretical whirl possible by expansion about stator blade trailing edges has been made by the method of characteristics for converging and converging-diverging stators. The results may be summarized as follows:

1. The maximum possible tangential component of critical velocity ratio from a row of converging stators is limited to 1.714 at the stator exit. For applications requiring higher stator exit whirl, converging-diverging stators would have to be employed, in which case whirl components of critical velocity ratio up to the theoretical limit of 2.449 might be realized.

2. The greatest theoretical gains in whirl due to expansion about stator blade trailing edges occur at small blade exit angles and relatively low velocities at the exit of the guided channel. At an exit angle of 15°, the percentage increase for a converging stator is twice as great as that of a converging-diverging stator with a critical velocity ratio of 1.2 at the exit of the guided channel.

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3. Unless design requirements are rigorous or unless extremely small blade exit angles are to be used, an approximate method of assuming uniform flow at the stator exit appears adequate in determining maximum stator exit whirl for all types of configurations used in this analysis.

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REFERENCES

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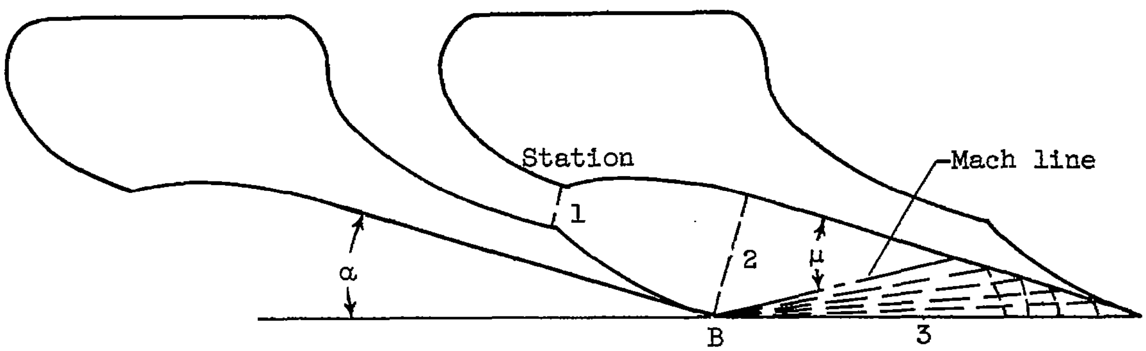

Figure 1. - Typical expansion about stator trailing edge.

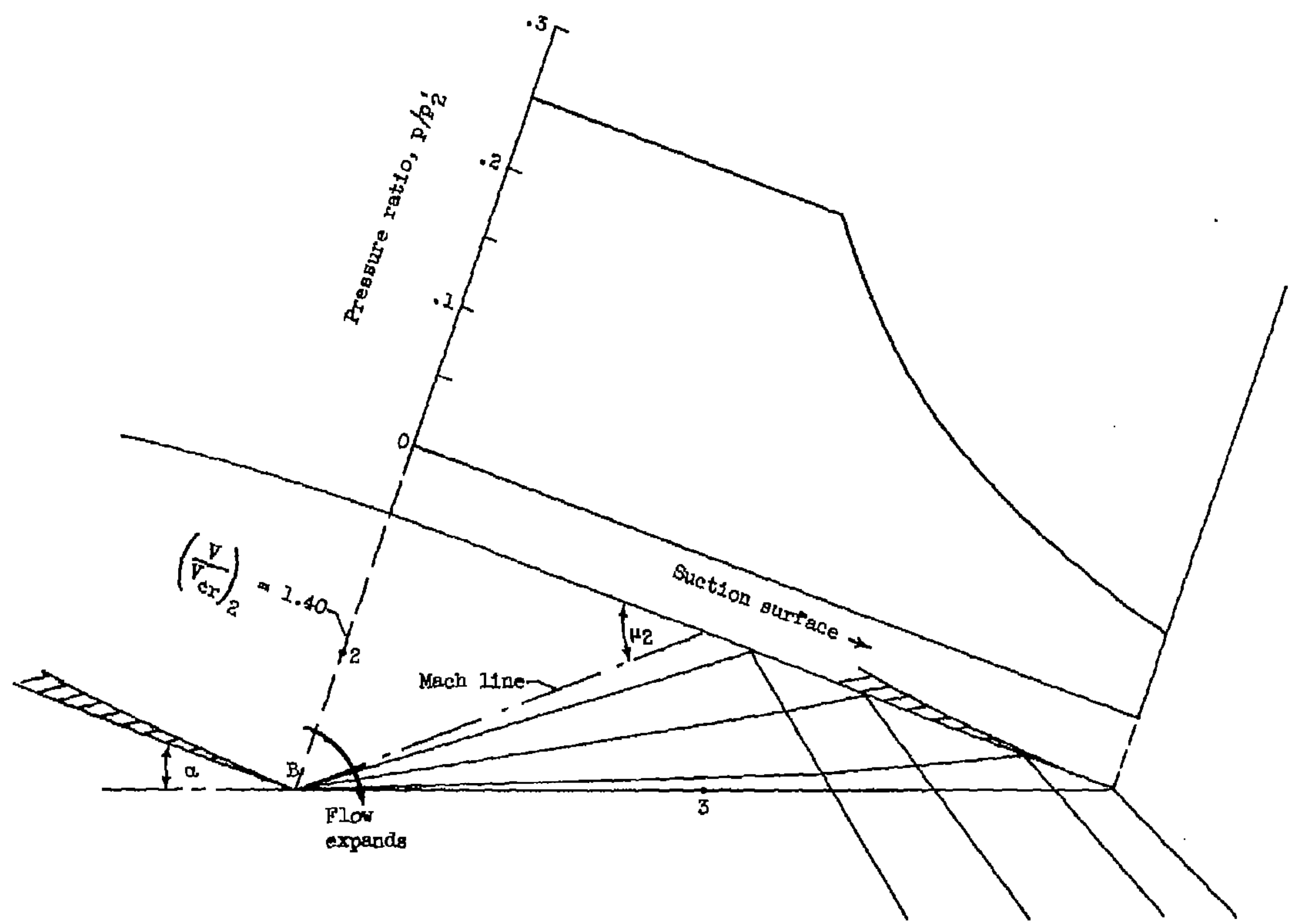

Figure 2. - Characteristic diagram and static-pressure distribution on suction surface for maximum expansion. Stator blade exit angle, 20°; critical velocity ratio, 1.40.

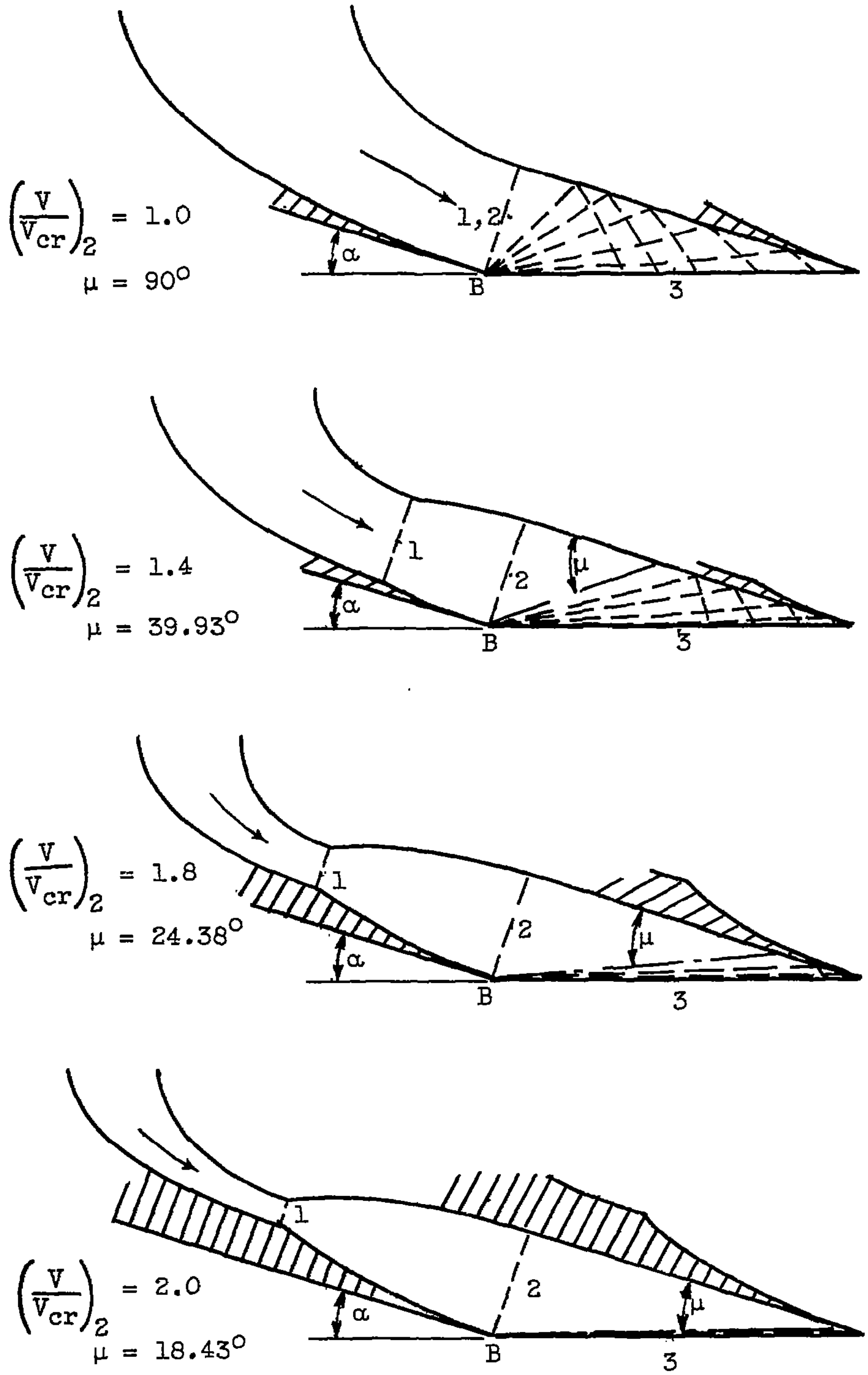

Figure 3. - Sketches of typical stators investigated. Stator blade exit angle, approximately 17°.

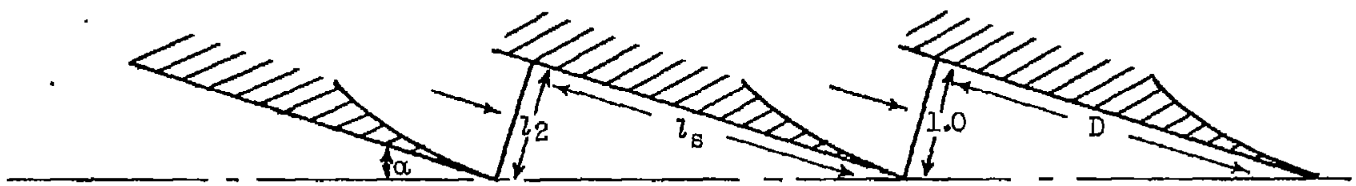

Figure 4. - Geometric length ratio, $D = l_s/l_2 = \cot \alpha$.

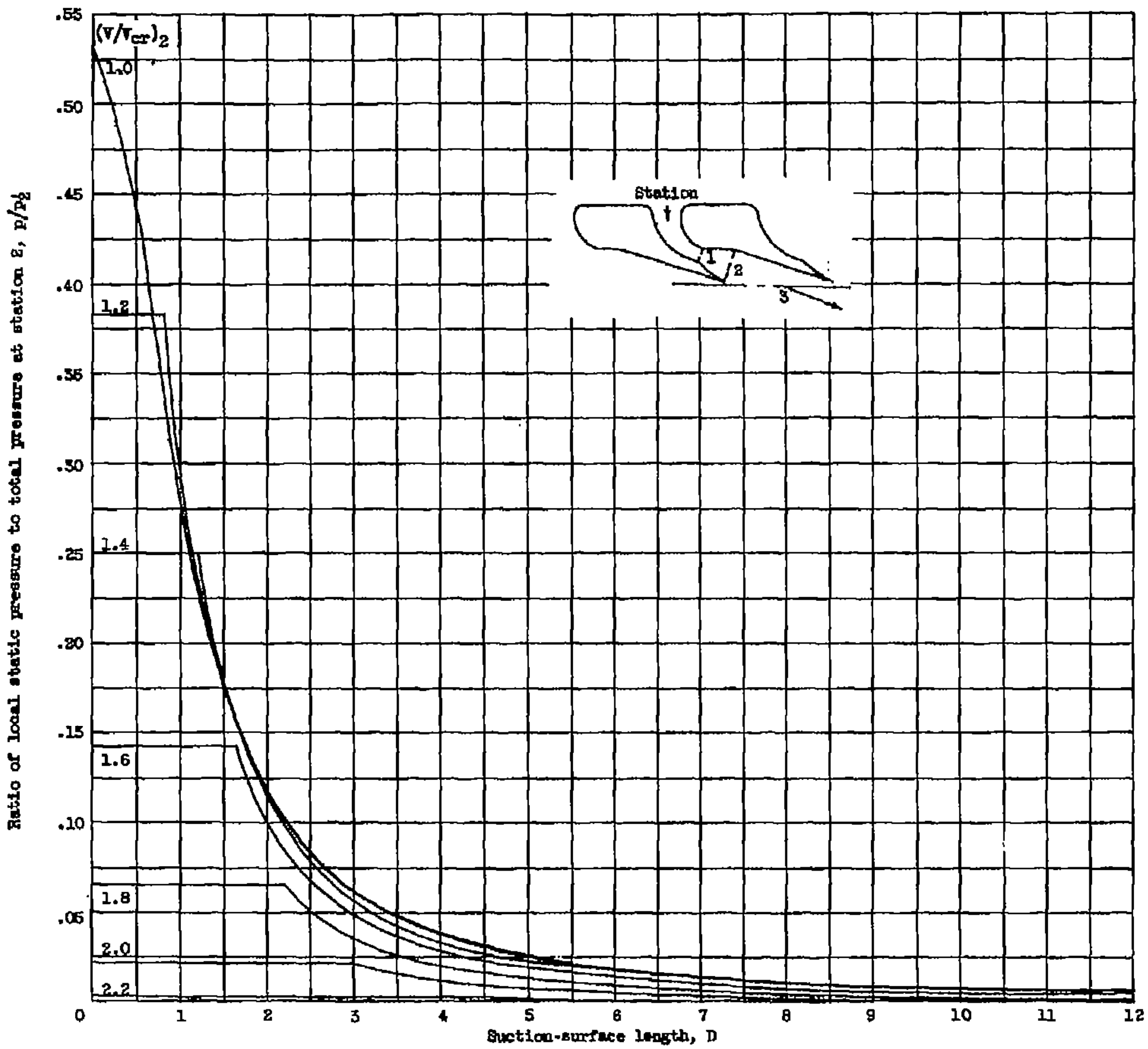

Figure 5. - Variation of static pressure along suction surface downstream of station 2 for various critical velocity ratios at station 2.

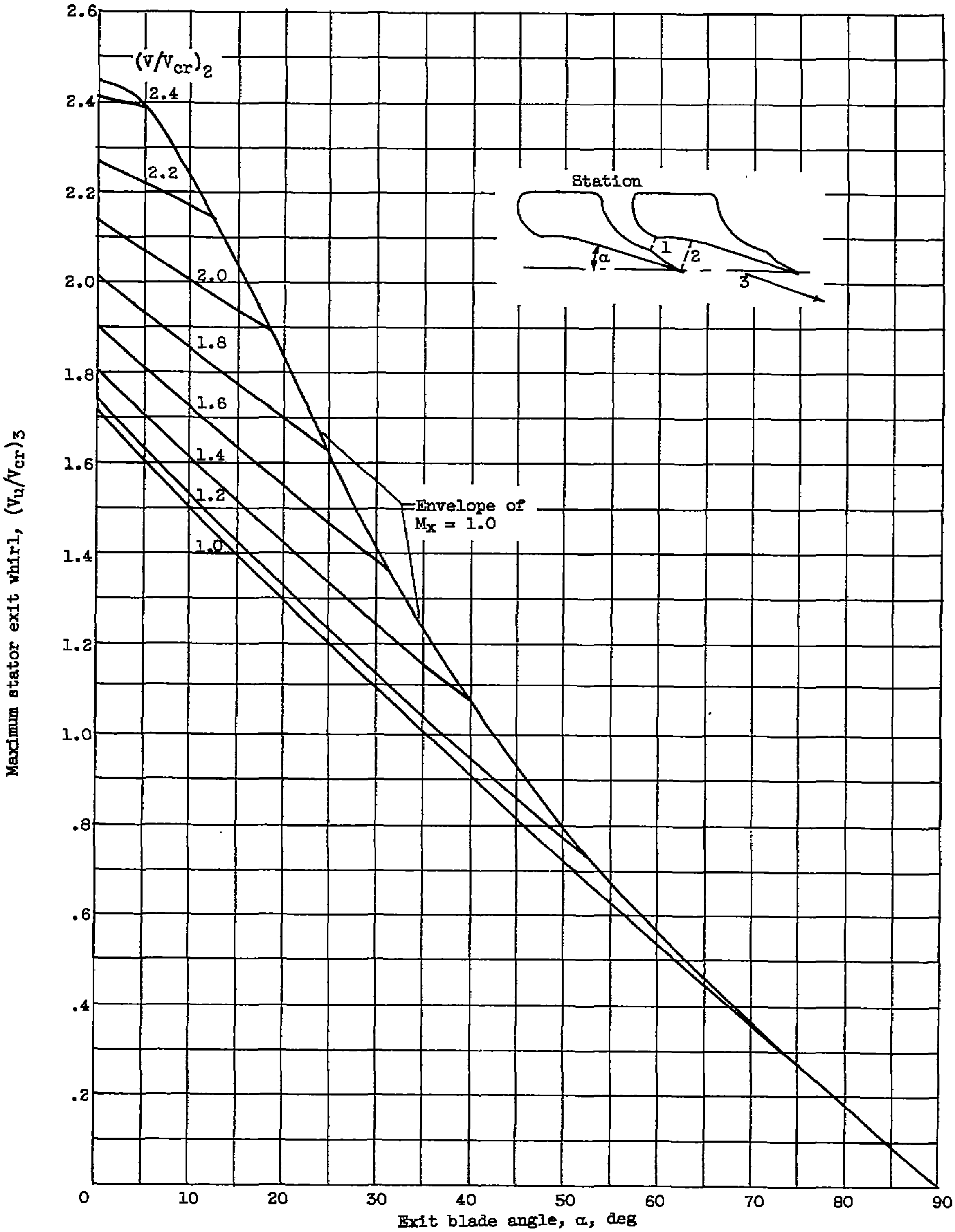

Figure 6. - Maximum tangential velocity possible by expansion about stator blade trailing edges.

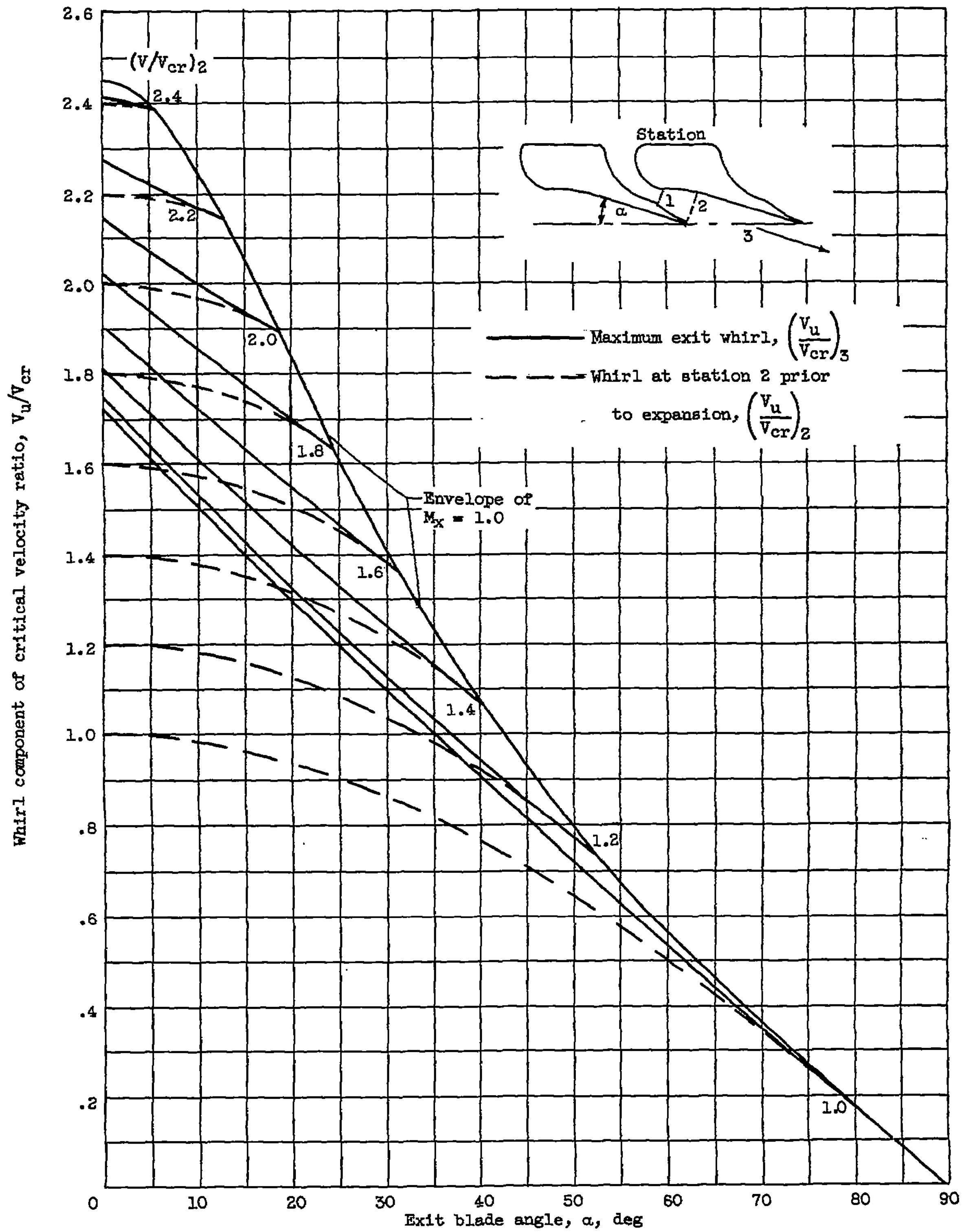

Figure 7. - Whirl characteristics before and after expansion.

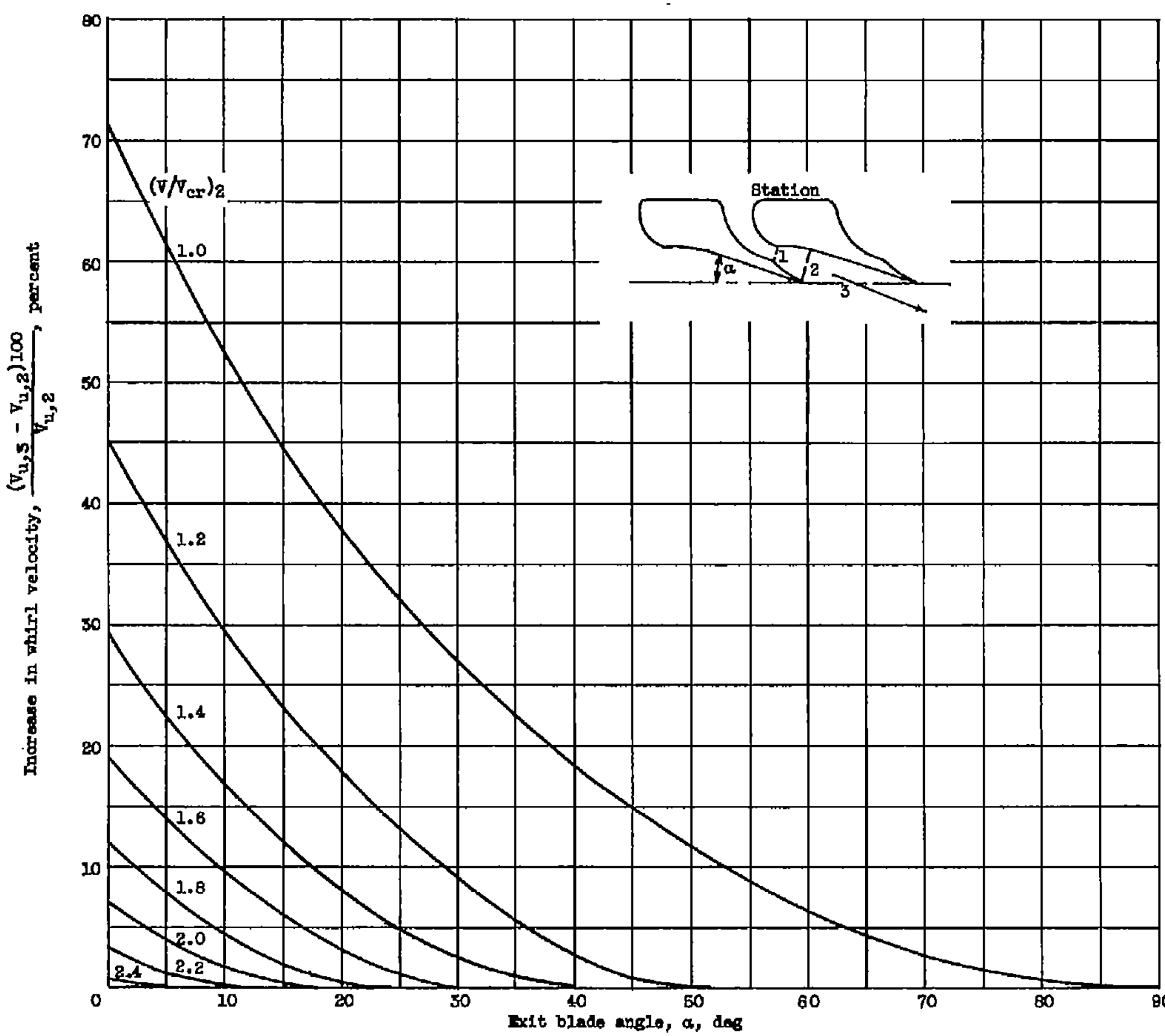

Figure 8. - Increase in tangential velocity possible by expansion about stator blade trailing edges.

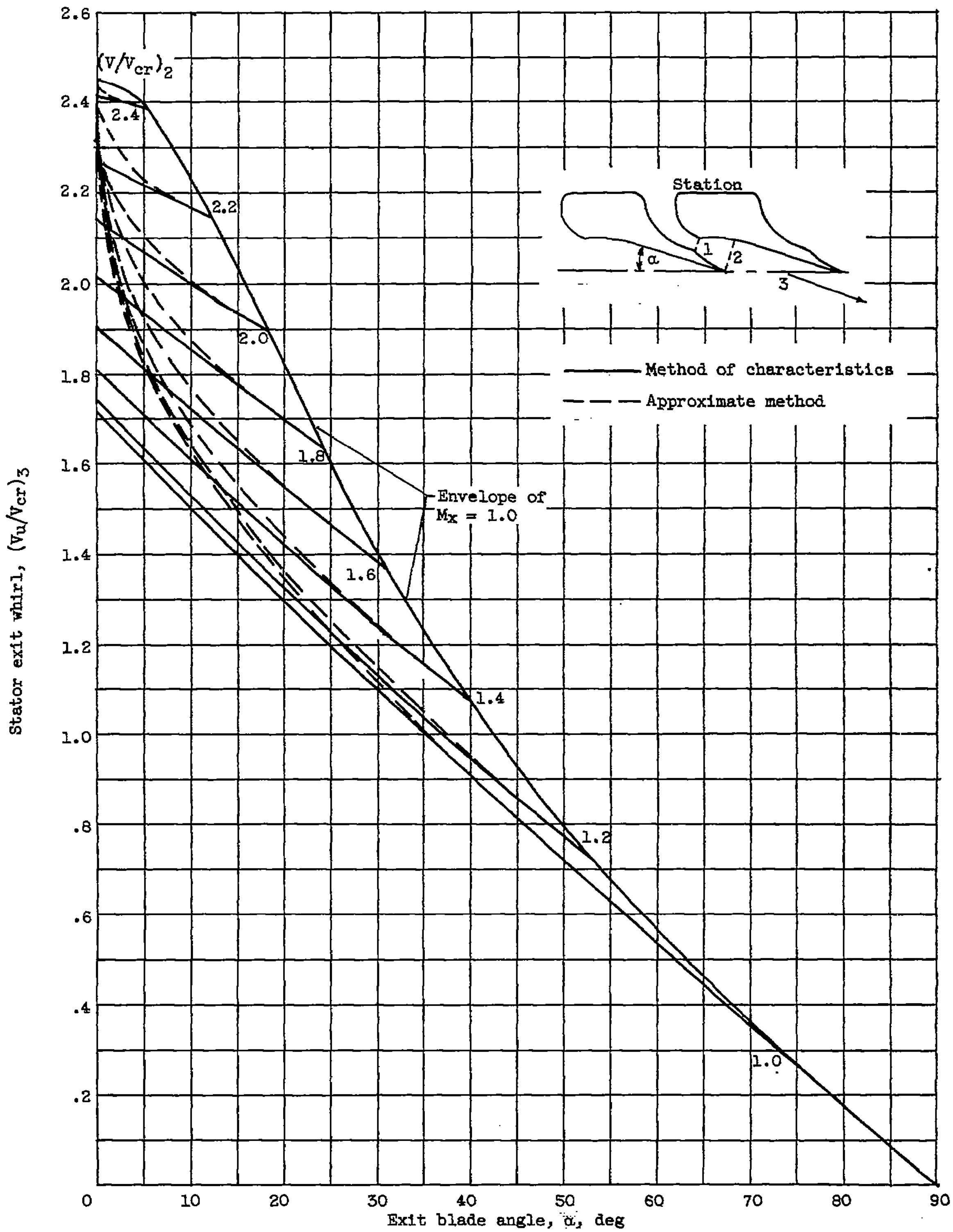

Figure 9. - Deviation in exit whirl between approximate method and method of characteristics in calculating maximum stator exit whirl.

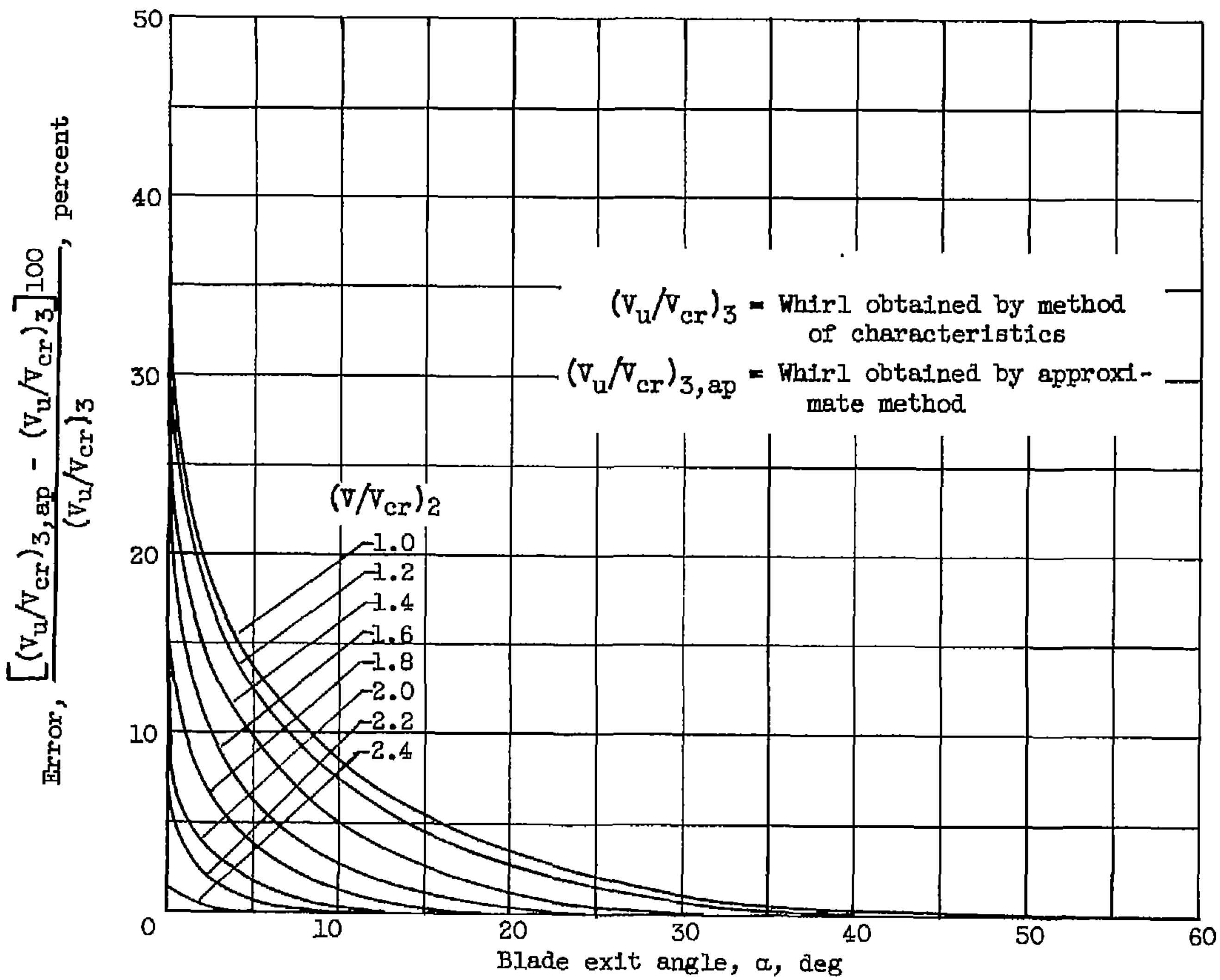

Figure 10. - Error introduced in maximum theoretical stator exit whirl as determined by assuming uniform flow at stator exit.