NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4315

AN ESTIMATE OF THE FLUCTUATING SURFACE PRESSURES ENCOUNTERED IN THE REENTRY OF A BALLISTIC MISSILE

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SUMMARY

Calculations are made of the magnitude of the surface sound pressure levels that will occur on blunt-nosed vehicles during reentry into the earth's atmosphere. The results presented cover a wide range of reentrance velocities and reentrance angles into the atmosphere and sizes and weights of the vehicle.

The analysis presented is based on current subsonic results, which can be extrapolated to high supersonic speeds for a blunt-nosed body since the values of local Mach number behind the shock but outside the boundary layer are quite low. The analysis is limited to the particular point on the body where the maximum local sound pressure level is obtained. A constant value of the ratio of the surface root-mean-square pressure to the local dynamic pressure, which is based on current subsonic data, is assumed (4.5×10^{-3}).

The results indicate that surface sound pressure levels of the order of 150 decibels or higher are likely to be encountered. The time of exposure to these levels is found to be of the order of 25 seconds.

INTRODUCTION

The problems associated with the reentry of a ballistic missile are numerous. In some cases even the order of magnitude of the problem is unknown, such as the internal and external noise levels to which the nose cone is subjected. High external fluctuating pressures in the boundary layer may damage the external structure of the vehicle if the times involved are of long enough duration. On the other hand, high external sound pressure levels may result in high internal sound pressure levels (inside the cone) and may interfere with the proper action of internal mechanical or electronic devices.

The analysis presented herein was made to determine the maximum value of sound pressure level that would occur during the reentry of a blunt-nose cone.

Although there are very little data on the fluctuating pressures in a boundary layer even at subsonic speeds (refs. 1 and 2), the necessity of making an order-of-magnitude estimate of the sound pressure levels inside and outside the nose cone is obvious. The results reported herein represent an extrapolation of current knowledge and data in the subsonic speed range to the very high hypersonic speeds encountered by ballistic missiles. 4900

Such an extrapolation would appear to be possible for blunt-nosed vehicles, which are of current interest, since the Mach numbers around the body (behind the shock wave) are only slightly supersonic. Furthermore, the measurement and use of acoustic data on the decibel scale (6 db is a ratio of 2 in rms pressures) to a probable accuracy of no better than ±2 decibels is usually sufficient and, hence, errors involved in data extrapolation may not be any larger than the usual accuracy to which the final result is desired.

SYMBOLS

A area, sq ft

a speed of sound, ft/sec

C_D drag coefficient

e base of Naperian logarithims, 2.718 . . .

g acceleration due to gravitational force, 32.2 ft/sec^2

M Mach number

m mass, slug/cu ft

P total pressure, lb/sq ft

p pressure, lb/sq ft

$\overline{p}$ root-mean-square pressure at surface

q dynamic pressure, $(\gamma/2)M^2p$

R gas constant

Re	Reynolds number
SPL	sound pressure level, db(re 2×10^{-4} dynes/sq cm)
T	temperature, °R
t	time, sec
V	velocity, ft/sec
V_E	velocity at entrance to earth's atmosphere, ft/sec
V_y	vertical component of V, ft/sec
y	altitude, ft
β	constant, 1/22,000, 1/ft
γ	ratio of specific heat for air, 1.4
ϵ	time parameter
θ_E	angle of flight path at reentrance with respect to horizontal, deg
μ	air viscosity, slugs/ft-sec
ρ	air density, slug/cu ft
ρ_0	air density at sea level, 0.0034 slug/cu ft
φ	reentrant parameter, $C_DA/2\beta m \sin\theta_E$

Subscripts:

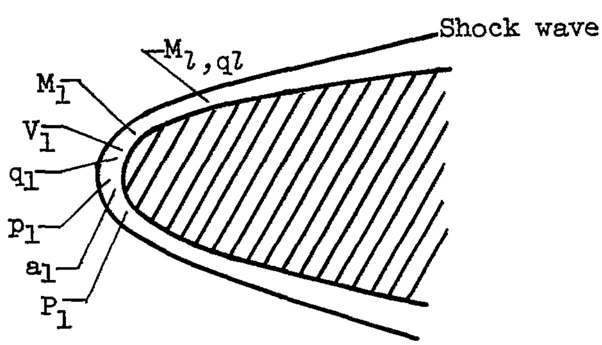
l	local
max	maximum value encountered along trajectory
1	behind shock at stagnation point

ANALYSIS

The fluctuating surface pressures created by a turbulent boundary layer have been measured in a duct (ref. 1) and on the surface of a wing (ref. 2). In both cases the measurements were made by using microphones embedded in the surface and exposed to the airstream. Both sets of data showed that the root-mean-square pressure $\bar{p}$ at the surface varied linearly with a dynamic pressure q of the flow. The ratio $\bar{p}/q$ was relatively independent of the Reynolds number or of the boundary-layer thickness. The data of reference 1 showed no effect of Mach number, whereas the flight data (ref. 2) indicated a Mach number effect only at low Mach numbers (below 0.55). Since the data of reference 2 were taken on an airfoil, the low Mach number effects could well result from angle-of-attack effects rather than from any real Mach number effect of itself. Unpublished data taken both in flight and in a large acoustic channel agree quite well with the results of reference 1; that is, no effect was noted of Mach number, Reynolds number, or boundary-layer thickness on $\bar{p}/q$. A study of all the available data indicates that $\bar{p}/q$ is approximately 4 to 5×10^{-3} for Mach numbers up to 0.8. For the case of a blunt-nosed reentry vehicle traveling at hypersonic speeds $(M > 5)$ the local values of dynamic pressure may be quite large even though the local Mach numbers (behind the shock) are not high (less than 2.0).

Since the value of $\bar{p}$ depends on local quantities it would be expected that the linear dependence of $\bar{p}$ on q would not be altered greatly by doubling the Mach number, that is, an increase from 0.8 to 1.6.

Consider the case of a blunt-nosed body reentering the earth's atmosphere at extremely high speeds as shown in the following sketch:



At high Mach numbers an extremely strong bow wave precedes the body and the value of M_1 is approximately 0.4 for a range of free-stream Mach numbers from 4 to 20. The flow behind the shock proceeding downstream from the stagnation point speeds up and at some point on the body aft of the stagnation point the maximum value of the local dynamic pressure q_l outside the boundary layer is encountered. This maximum value in uniquely related to a particular value of M_l provided that the total pressure of the flow around the body between the shock and the boundary layer remains constant and equal to the stagnation-point total pressure P_l. Then the maximum value of q_l/P_l is 0.4312 and occurs at $M_l = 1.415$ (e.g., ref. 3). It should be noted that the calculation procedures used herein do not account for any effects associated with extremely high temperatures such as dissociation or variable ratio of specific heats.

To calculate q_l at any point on the trajectory, it is only necessary to know the value of P_l. In order to determine the maximum possible value of P_l for a given set of conditions, the following analysis was undertaken. The usual normal shock relations give

$$\frac{P_l}{p} = \left(\frac{6\ M^2}{5}\right)^{7/2} \left(\frac{7\ M^2 - 1}{6}\right)^{-5/2}$$

The maximum value of P_l occurs at a particular combination of p and M, that is, at some particular point along the trajectory of the vehicle. Setting the derivative $dP_l/dy = 0$ and solving for P_l will yield the maximum value of $(P_l)_{max}$ along the trajectory. The result of such a differentiation and algebraic rearrangement for $M_0 >> 1$ yields

$$\frac{p}{M} \frac{dM}{dp} = -0.5$$

Using the gas law $p = \rho gRT$ and $M = V/a$, then, yields

$$\frac{\rho}{V} \frac{dV}{d\rho} = -0.5$$

Allen (ref. 4) shows that the velocity at any point along the reentrant trajectory can be expressed as

$$V = V_E e^{-\varphi\rho}$$

where

$$\varphi = \frac{C_D A}{2\beta m \sin\ \theta_E}$$

where C_D is the body drag coefficient, A is the body area corresponding to C_D, m is the mass of the body, and θ_E is the reentrant angle of the body as it enters the atmosphere. The constant β appears in the atmospheric relation

$$\rho = \rho_0 e^{-\beta y}$$

The derivation of reference 4 assumes that C_D is essentially constant, that is, the parameter φ does not vary appreciably. Furthermore, the values of θ_E must be large enough so that gravitational effects on the flight path are small.

When these relations are used, the maximum value of P_1 occurs at $\rho\varphi = 0.5$. Hence, for $M >> 1$,

$$(P_1)_{max} = \frac{0.1695}{\varphi} V_E^2$$

and $(q_l)_{max}$ on the body corresponding to $(P_1)_{max}$ is given by

$$(q_l)_{max} = \frac{0.073\ V_E^2}{\varphi}$$

The maximum q_1 immediately behind the shock at the stagnation point is also obtained at the condition $(P_1)_{max}$ and is given by

$$(q_1)_{max} = 0.01533\ \frac{V_E^2}{\varphi}$$

The values of $(q_l)_{max}$ and $(q_1)_{max}$ are unique functions of the altitude y for specified values of φ (see appendix A). The altitude y corresponding to $(q_l)_{max}$ or $(q_1)_{max}$ is plotted as a function of φ in figure 1. It is evident that the peak dynamic pressures occur between 40,000 and 95,000 feet for a range of φ values from 1000 to 10,000. A plot of $(q_l)_{max}$ as a function of the reentrant velocity for various values of φ is given in figure 2.

The fact that the local dynamic pressures are high during reentry (500 lb/ft or greater) does not necessarily infer that a turbulent boundary layer exists on the vehicle. One important consideration that determines the existence of a turbulent boundary layer is the Reynolds number associated with the flow. The relation for Reynolds number per foot (based on conditions immediately behind the shock at the nose $Re_{1/ft} = \rho_1 V_1/\mu_1$) is shown in appendix B to be

$$Re_{1/ft} = \frac{219 \times 10^9}{\varphi\sqrt{V_E}} \quad \text{at } (q_l)_{max}$$

Furthermore, it can be readily shown (appendix B) that the number per foot Re/ft based on free-stream conditions at condit. corresponding to $(q_l)_{max}$ is given by

$$Re/ft = \frac{V_E}{\varphi} \times 10^6$$

Figure 3 shows a plot of $Re_{1/ft}$ as a function of φ for reentrant velocities from 10,000 to 25,000 feet per second. From the figure, it is evident that over the whole range of φ and V_E the value of $Re_{1/ft}$ varies by only a little more than a decade. Furthermore, even for a moderate size missile, values of Reynolds number well over one million will be achieved, which indicates a high probability of the existence of a turbulent boundary layer.

As pointed out previously, the value of $(q_l)_{max}$ occurs at a local Mach number of about 1.4, and since $\overline{p}/q$ is essentially constant over a wide range of subsonic Mach numbers, it would be expected that there would be no large changes in $\overline{p}/q$ at low supersonic speeds. There is, of course, the possibility that local shock-wave - boundary-layer interactions could greatly alter local values of $\overline{p}/q$. Such an effect is quite unlikely in the nose-cone region under discussion because the pressure gradients around the nose are favorable.

It would appear, therefore, that assuming a constant value for p/q equal to 4.5×10^{-3} would be justified for most preliminary calculations. Even with a change in $\overline{p}/q$ of 2, the sound pressure levels would vary by only 6 decibels. The sound pressures associated with $(q_l)_{max}$ can be calculated using this assumption, and a graph of the maximum sound pressure levels SPL_{max} as a function of φ is given in figure 4 for various values of initial reentrant velocity. Current values of φ and V_E would give values of SPL_{max} of approximately 150 to 160 decibels for a long-range ballistic missile. It should be remembered that the values of sound pressure level will vary over the entire surface and that SPL_{max} corresponds only to the maximum value that will occur only at one location aft of the stagnation point. However, since the magnitude of the sound pressures is quite large compared with the usual values it would appear that consideration should be given during the designing of long-range ballistics missiles to minimize the effects of these sound pressures properly.

One factor that would tend to alleviate both the external and internal noise problems is the extremely short duration of exposure to the earth's atmosphere. The total time elapsed from the entrance of the nose cone into the earth's atmosphere until its impact on the surface would probably not exceed 120 seconds. Only during a portion of this time would the local dynamic pressures be high enough to result in high noise levels. Calculations of the variation of the maximum value of local dynamic pressure with altitude for a range of φ values are shown in figure 5. In each case the q_l value increases sharply below an altitude of 140,000 feet, reaches a peak value $(q_l)_{max}$ at some altitude between 100,000 and 50,000 feet (dependent on φ), and then decreases sharply as the altitude decreases. For values of φ between 10,000 and 3000, the high values of q_l exist between 140,000 and 50,000 feet. For lower φ values, the high values of q_l are encountered down to 30,000 feet. A method for calculating the time during which high values of q_l exist is given in appendix C.

For the cases of current interest, the critical values of q_l are encountered between 140,000 and 50,000 feet. The time for the missile to traverse this distance should give a reasonable estimate of the duration of exposure. Figure 6 shows a time parameter ϵ for the missile to fall from 140,000 to 50,000 feet as a function of parameter φ. The time parameter ϵ is equal to $\beta t V_E \sin \theta_E$, where t is the actual time (sec) for the missile to traverse the given distance. For missiles of current interest the time is of the order of 25 seconds. For future missiles the times will probably approach approximately one-half this value.

CONCLUDING REMARKS

The results presented herein give an indication of the magnitude of the surface sound pressure levels that may be encountered during reentry. These levels are high (of the order of 150 db) but the time of exposure to the earth's atmosphere is short, that is, approximately 25 seconds. Furthermore, the Reynolds numbers are high enough to yield turbulent boundary layers. Current efforts to reduce heat transfer by maintaining large lengths of laminar boundary layers would greatly reduce any problems that might be created by excessively high sound pressure levels. On the other hand, the use of ablation-type nose cones with their inevitably higher surface roughness should increase the order of magnitude of the sound pressures.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, May 8, 1958

APPENDIX A

ALTITUDE AT WHICH MAXIMUM LOCAL DYNAMIC PRESSURE IS ENCOUNTERED

The condition that yields $(q_l)_{max}$ or $(q_1)_{max}$ is $\rho\varphi = 0.5$ as derived in the ANALYSIS section.

Hence,

$$\rho_0 e^{-\beta y} \varphi = 0.5$$

where

$$\beta = 1/22{,}000; \ \rho_0 = 0.0034$$

Then

$$e^{\beta y} = 2\rho_0\varphi$$

$$\ln e^{\beta y} = \ln 2\rho_0\varphi$$

$$\beta y = \ln 2\rho_0\varphi$$

$$y = \frac{1}{\beta} \ln 2\rho_0\varphi$$

$$= 22{,}000 \ln(0.0068\varphi)$$

APPENDIX B

REYNOLDS NUMBER PER FOOT BASED ON CONDITIONS IMMEDIATELY BEHIND SHOCK AT NOSE ($\rho_1 V_1/\mu_1$) CORRESPONDING TO MAXIMUM LOCAL DYNAMIC PRESSURE

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$$Re_{1/ft} = \frac{\rho_1 V_1}{\mu_1} = \frac{\rho V}{\mu_1} \quad \text{since} \quad \rho_1 V_1 = \rho V$$

$$\mu_1 = 3.44\times10^{-9}\ T_1^{0.75}$$

$$\frac{T_1}{T} \cong \frac{7}{36} M^2 \quad \text{for} \quad M > 5$$

$$\mu_1 \cong 3.44\times10^{-9}\left[\frac{7}{36}\left(\frac{V}{a}\right)^2 T\right]^{0.75}$$

Since the atmospheric relation $\rho = \rho_0 e^{-\beta y}$ is an isothermal atmosphere where T = constant = 392° R,

$$\mu_1 \cong 3.44\times10^{-9}\left[\left(\frac{7}{36}\right)\left(\frac{1}{971}\right)^2 392\right]^{0.75} V^{1.5}$$

$$\mu_1 \cong 2.93\times10^{-12}\ V^{1.5}$$

Since $\rho\varphi = 0.5$ for $(q_l)_{max}$, then $V = V_E e^{-0.5} = 0.606\ V_E$

$$Re_{1/ft} = \frac{\rho V}{2.93\times10^{-12}\ V^{1.5}} = \frac{3.415\times10^{11}\ \rho}{\sqrt{V}} = \frac{3.415\times10^{11}\left(\frac{0.5}{\varphi}\right)}{\sqrt{0.606\ V_E}} = \frac{219\times10^{9}}{\varphi\sqrt{V_E}}$$

Furthermore, since $V = 0.606\ V_E$ at conditions for $(q_l)_{max}$ and $a = 971$, then $M = V_E/1600$.

Therefore, the free-stream Reynolds number per foot Re/ft is given by

$$Re/ft = \frac{\rho V}{\mu} = \frac{\frac{0.5}{\varphi}\ 0.606\ V_E}{3.03\times10^{-7}} = \frac{V_E}{\varphi}\times10^{6}$$

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CU-2 back

APPENDIX C

CALCULATION OF TIME DURING WHICH HIGH VALUES OF LOCAL DYNAMIC PRESSURE ARE ENCOUNTERED

In reference 4 it is shown that the vertical component of velocity V_y of a ballistic vehicle is given by

$$V_y = V \sin \theta_E = \frac{dy}{dt}$$

where $V = V_E e^{-\varphi\rho}$ on all points of the trajectory. Hence, the time for a vehicle to cover the distance between two values of y is given by

$$t = \frac{1}{V_E \sin \theta_E} \int_{y_1}^{y_2} \frac{dy}{e^{-\varphi\rho}}$$

where

$$\rho = \rho_0 e^{-\beta y}$$

$$d\rho = -\beta\rho_0 e^{-\beta y}\, dy$$

$$dy = -\frac{d\rho}{\beta\rho}$$

$$t = \frac{-1}{\beta V_E \sin \theta_E} \int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho e^{-\varphi\rho}}$$

$$t = -\frac{1}{\beta V_E \sin \theta_E} \left[\log \rho + \rho + \frac{(\varphi\rho)^2}{4} + \frac{(\varphi\rho)^3}{18} + \ldots\right]_{\rho_1}^{\rho_2}$$

Since ρ_1 and ρ_2 are unique functions of y, it is possible to calculate t for any chosen values of y_1 and y_2 for particular values of φ, V_E, and $\sin \theta_E$. If the time is expressed in terms of the dimensionless time parameter $\epsilon = \beta t V_E \sin \theta_E$ then ϵ is a function of φ only and can be shown by a single curve as indicated in figure 6.

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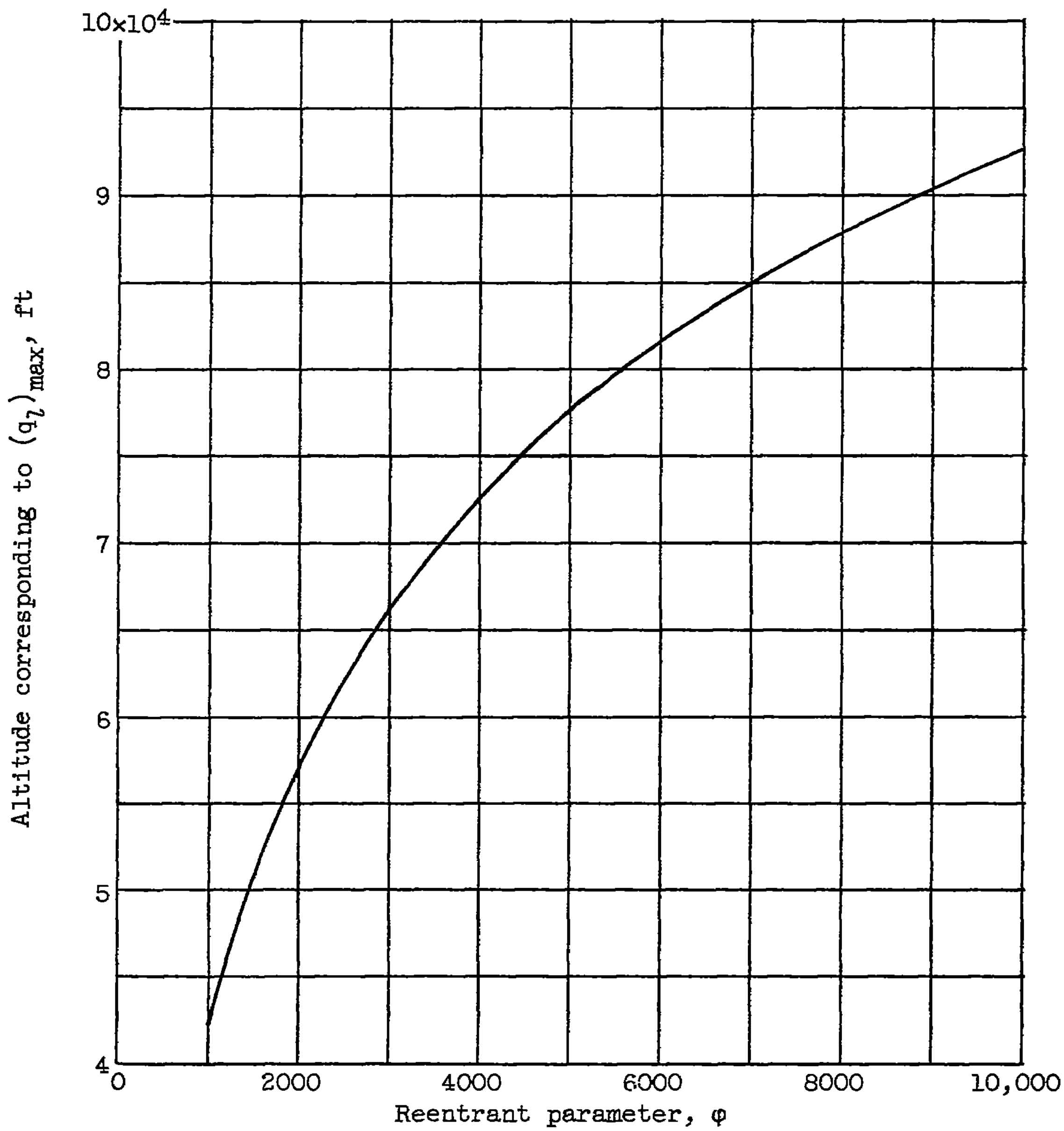

Figure 1. - Altitude at which maximum local dynamic pressure is encountered as a function of reentrant parameter.

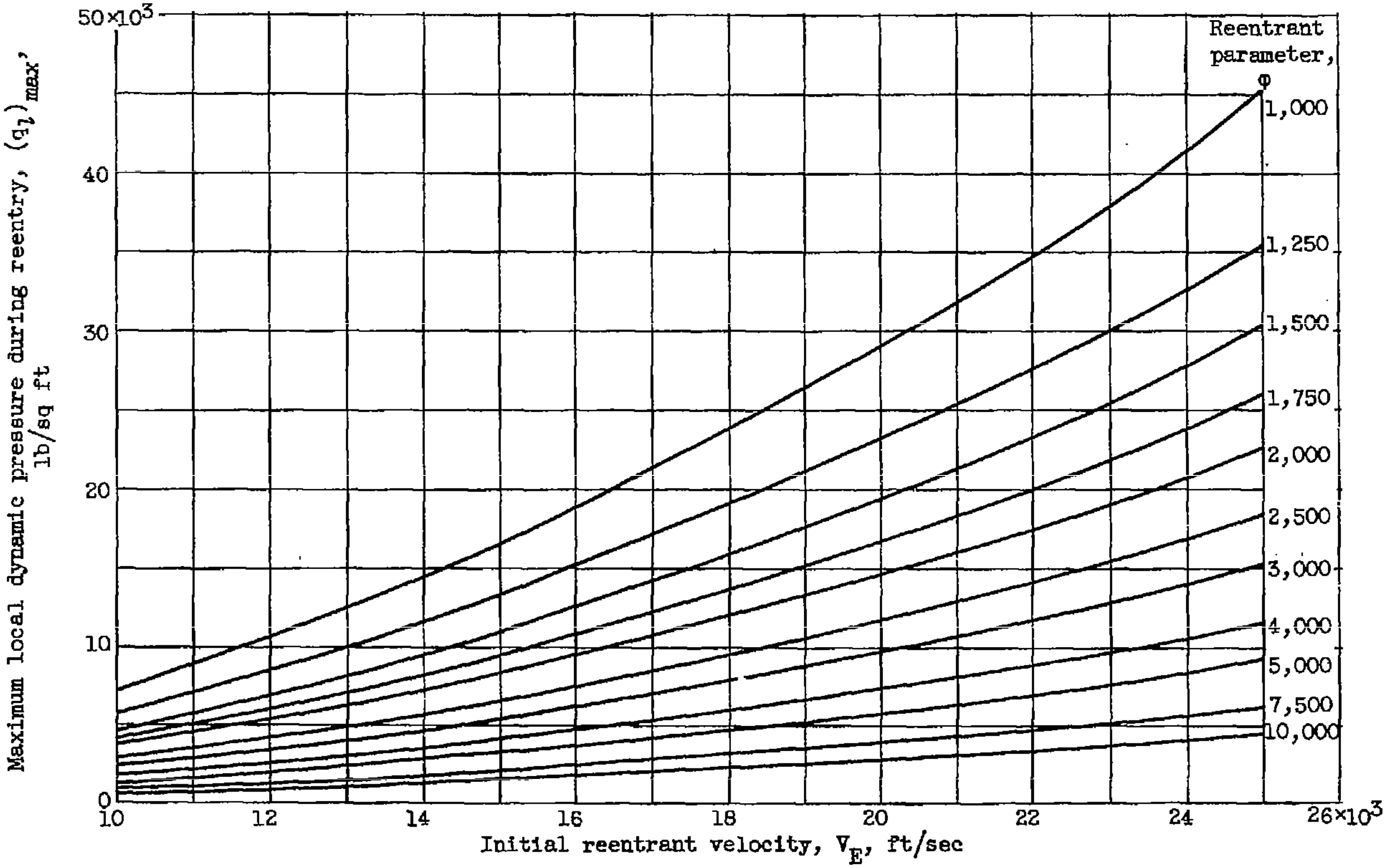

Figure 2. - Maximum local dynamic pressure around body during reentrance as a function of reentrant velocity.

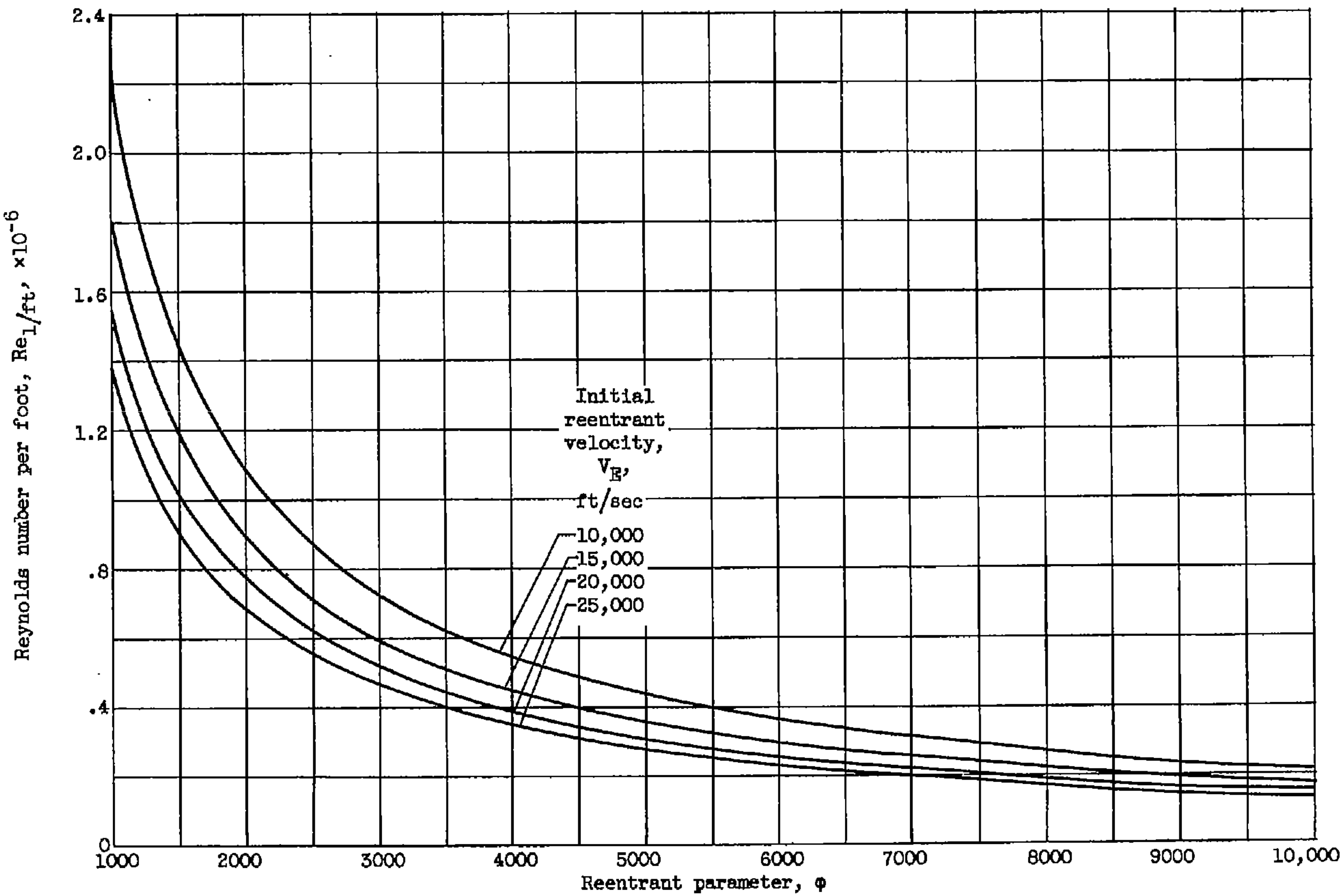

Figure 3. - Reynolds number per foot corresponding to maximum local dynamic pressure as a function of reentrant parameter.

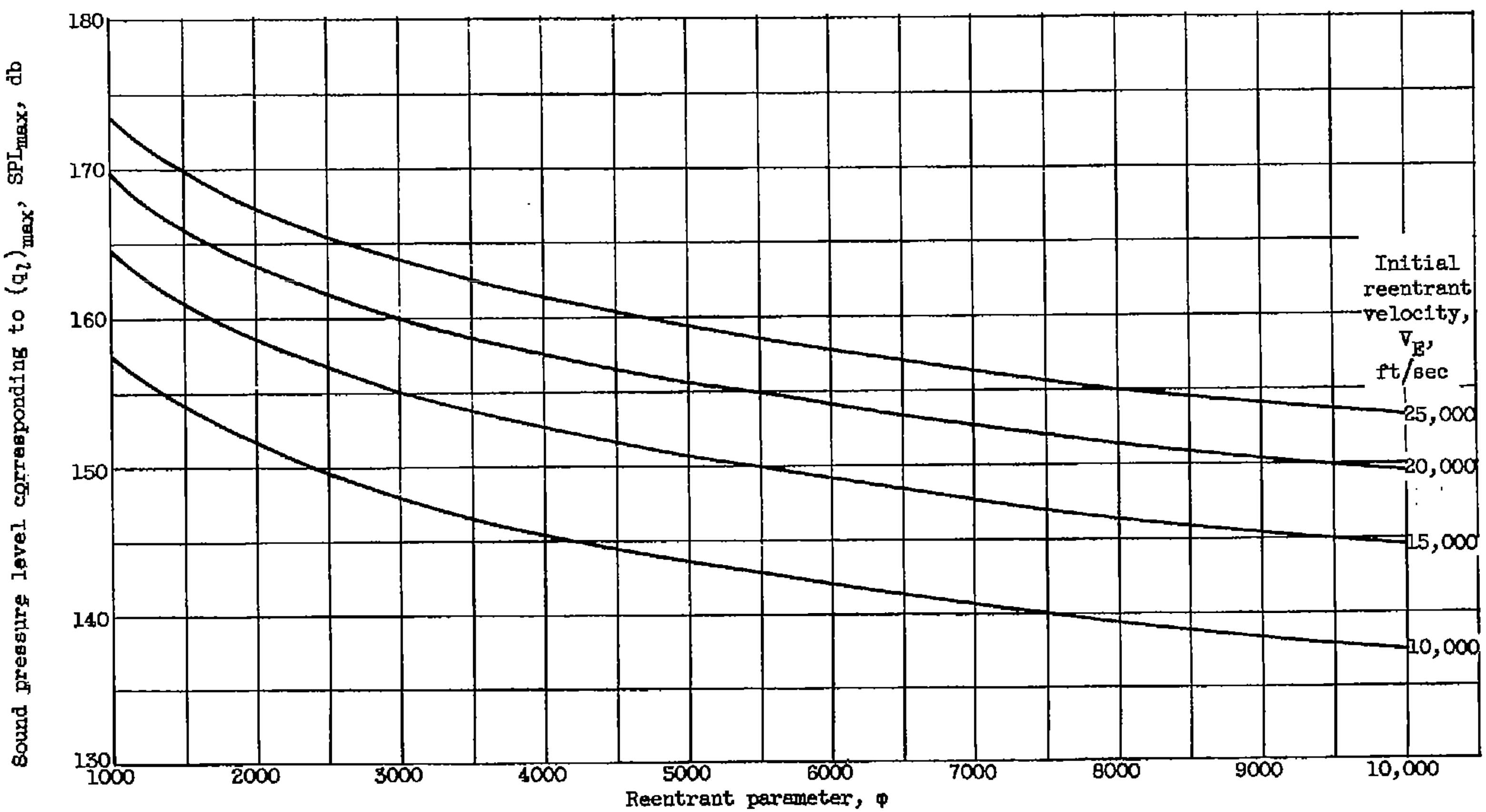

Figure 4. - Sound pressure levels corresponding to maximum local dynamic pressure for various entrance conditions.

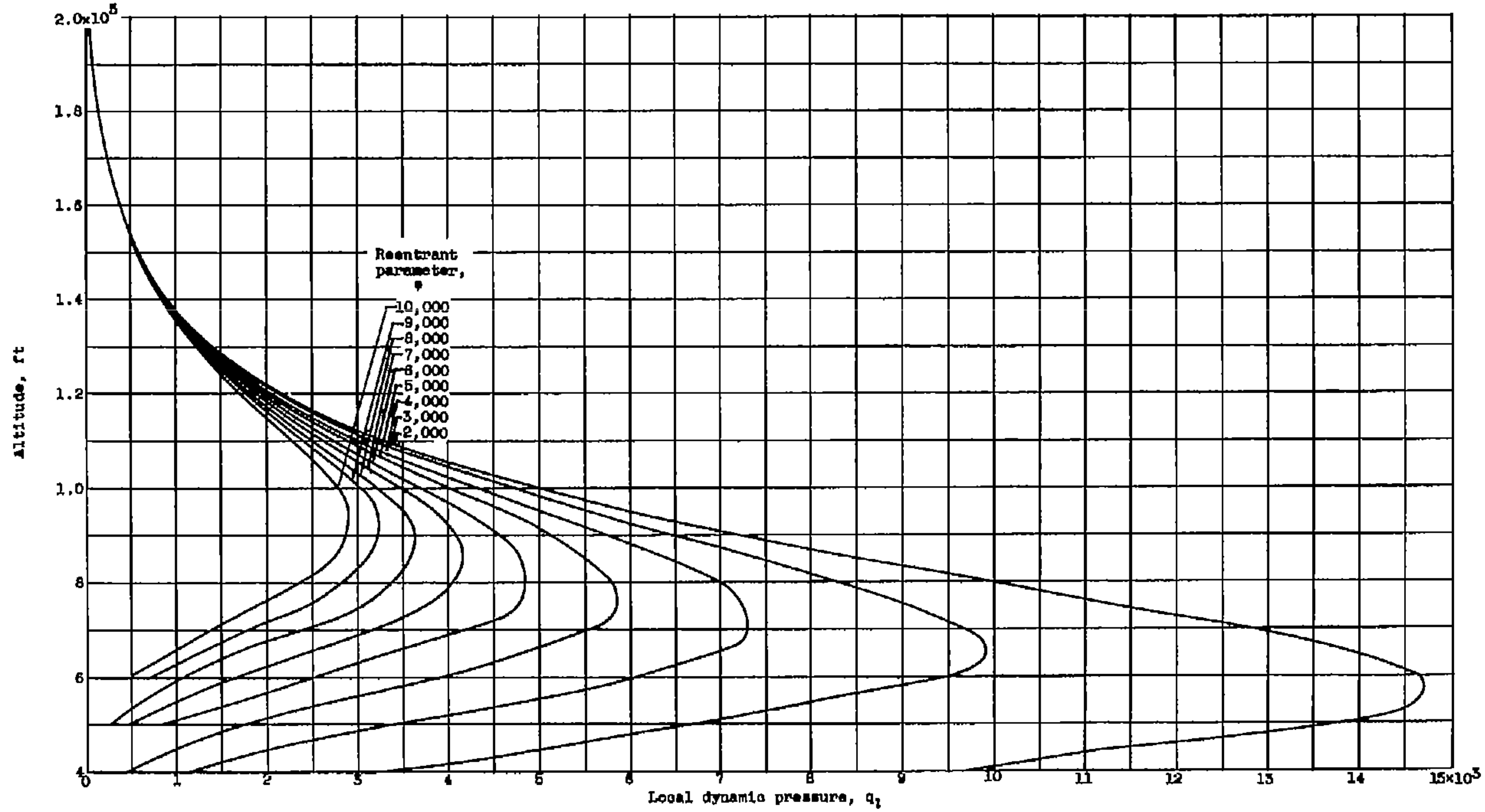

Figure 5. - Local dynamic pressure during reentry as a function of altitude with an initial reentrant velocity of 20,000 feet per second.

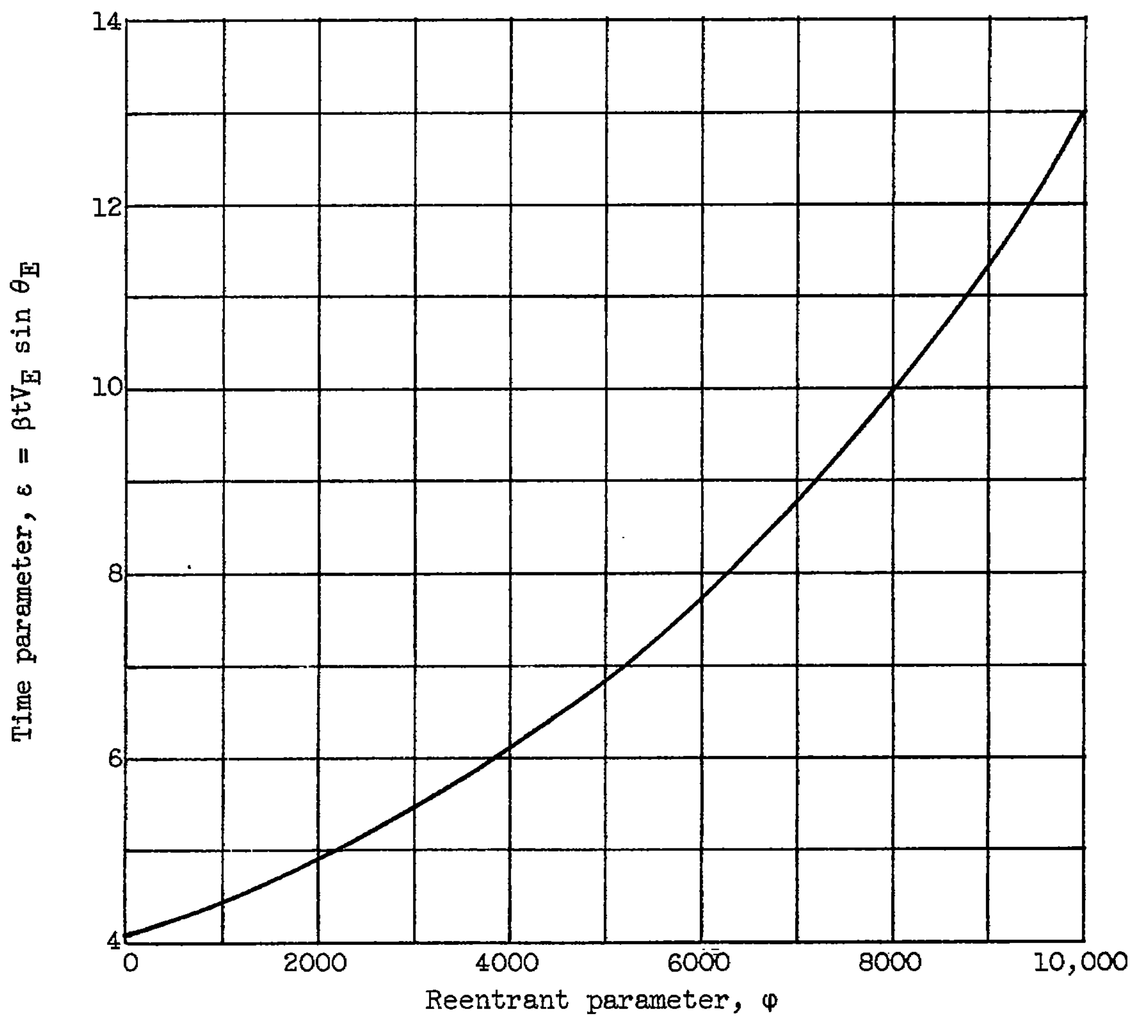

Figure 6. - Time parameter for the missile to fall from 140,000 to 50,000 feet as a function of reentrant parameter.

NACA - Langley Field, Va.