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ANALYTICAL RELATION FOR WAKE MOMENTUM THICKNESS AND DIFFUSION RATIO FOR LOW-SPEED COMPRESSOR CASCADE BLADES

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SUMMARY

A simple equation relating wake momentum thickness and suction-surface diffusion ratio (ratio of maximum surface velocity to outlet velocity) of conventional low-speed cascade blades is derived from simplified boundary-layer theory in conjunction with several empirical constants. An analytical relation is thus obtained that describes the experimental correlations between momentum thickness and diffusion ratio reported previously.

Inasmuch as the variation of wake momentum thickness with diffusion ratio is shown to depend on the magnitude of the friction coefficient and the type of boundary-layer flow, an insight is gained into the qualitative effects of such factors as blade-chord Reynolds number, surface roughness, transition location, and extent of local laminar separation. Illustrative calculations show that a wide range of values of allowable diffusion ratio (value above which the wake thickness becomes excessive) can be obtained for different boundary-layer histories. These results indicate that blades should be designed for low friction coefficient (high Reynolds number and low relative surface roughness) and minimum laminar flow (high Reynolds number and high free-stream turbulence) if maximum allowable values of suction-surface diffusion ratio are desired.

INTRODUCTION

The aerodynamic performance of compressor cascade blades is largely determined by the growth and separation of the boundary layers on the blade surfaces. In general, the surface boundary-layer development is a function of many factors such as the surface velocity distribution, the local skin-friction coefficient, and the location and nature of the transition from laminar to turbulent boundary-layer flow. These latter factors are generally related to the design parameters of blade-chord Reynolds number, free-stream turbulence level, and blade surface finish.

In cascade design practice, an accurate prediction of the development of blade surface boundary layers is not currently possible because of the many factors involved in exact boundary-layer theory. However, it has been possible through analysis of experimental data to gain an insight into the gross or first-order behavior of turbulent blade boundary layers insofar as the principal influencing parameters are concerned (refs. 1 to 4). Reference 4, in particular, presents an experimental correlation between the momentum thickness of the blade wake (as formed from the surface boundary layers) and the ratio of maximum suction-surface velocity to outlet velocity for a wide range of conventional two-dimensional-cascade blade configurations at a Reynolds number of about 2×10^5. It was thus established for these cases that the principal determinant of the blade wake and therefore of the cascade total-pressure loss (ref. 5) is the over-all suction-surface velocity ratio $V_{max,s}/V_2$.

The experimental correlations of reference 4 undoubtedly indicate some basic relation between wake thickness and surface velocity ratio (called the diffusion ratio) that can presumably be explained from a consideration of boundary-layer theory. It seemed desirable, therefore, to determine whether the form of the empirical correlations could be reasonably deduced and, furthermore, whether the correlations can be utilized to establish more general analytical relations for blade-wake thickness that would contain some of the other factors affecting the boundary layer. Thus, a means would be obtained for extrapolating the experimental results to show gross effects and trends of variations of other pertinent parameters influencing the blade boundary-layer growth.

The present report presents the derivation of a simple equation for blade-wake momentum thickness as a function of suction-surface diffusion ratio based on the one-dimensional boundary-layer momentum equation in conjunction with simplifying approximations. With the use of empirically derived constants, the equation is shown to satisfactorily describe the previously obtained experimental correlations. Calculations are then made from the derived equation to indicate the comparative effects on the limiting diffusion ratio of such factors as blade-chord Reynolds number, surface roughness, and location of transition from laminar to turbulent flow. The results are utilized for discussions of the empirical diffusion correlations of reference 4 and general blade design considerations.

SYMBOLS

C_D drag coefficient

C_f total skin-friction coefficient

C_f' total skin-friction coefficient for flat-plate flow (based on chord-length Reynolds number)

C_f^* total skin-friction coefficient for flat-plate flow (based on transition-distance Reynolds number)

c chord length

c_f local skin-friction coefficient

c_{l_o} design lift coefficient used in designation of NACA 65-series blades

f_θ averaging factor for momentum-thickness variation in momentum-thickness equation

H form factor of wake or boundary layer (δ^*/θ)

j factor in momentum-thickness equation

k factor in momentum-thickness equation

l length of flow surface

P total pressure

p static pressure

Re_c blade-chord Reynolds number

s blade spacing

V free-stream velocity

v local velocity in wake

x distance along outlet-flow direction or along flow surface

α angle of attack, angle between inlet air direction and blade chord

β air angle, angle between airflow and axial direction

Δ change in quantity

$\Delta\beta$ air turning angle $(\beta_1 - \beta_2)$

δ thickness of wake or boundary layer

δ^* displacement thickness of wake or boundary layer

ϵ factor in momentum-thickness equation

θ momentum thickness of wake or boundary layer

σ solidity (chord/spacing)

Subscripts:

lm laminar

ls laminar separation 4456

max maximum

n normal to outlet direction

p pressure surface

s suction surface

tb turbulent

te trailing edge

tr transition

x outlet flow direction or direction along flow surface

y normal to axial direction

1 cascade-inlet station

2 cascade-outlet measuring station

Superscript:

— average value of quantity

ANALYSIS

The objective of the analysis is to establish an analytical relation between the wake momentum thickness of a cascade, expressed as the ratio $(\theta/c)_2$, and the blade suction-surface diffusion ratio defined as the ratio of maximum suction-surface free-stream velocity to outlet free-stream velocity $V_{max,s}/V_2$. According to the experimental correlations of reference 4 (as shown in fig. 1[1]), the wake momentum thickness should

[1]The diffusion ratio $V_{max,s}/V_2$ in figure 1 is based on a more exact value of V_2 than in reference 4, as discussed on page 10.

increase in an essentially exponential manner with increasing diffusion ratio for the condition of minimum-loss angle of attack.

A relation between $(\theta/c)_2$ and $V_{max,s}/V_2$ is derived from simplifying approximations in the integration of the boundary-layer momentum equation. The development assumes incompressible two-dimensional-cascade flow and considers that the blade peak suction-surface velocity occurs close to the leading edge and that the boundary layer of the suction surface contributes the major portion of the wake. These latter conditions
4456 are generally observed for conventional blade sections in the range of operation from minimum loss to positive (high angle of attack) stall (ref. 4). As in reference 4, the analysis applies only to blade performance in this range from minimum loss to positive angle stall. Sketches of the wake development, velocity variations, and definitions of wake properties are given in figures 2 and 3.

Development of Equations

Basic relations. - In the plane of the blade trailing edge, the momentum thickness of the wake can be expressed as the sum of the boundary-layer momentum thicknesses of the pressure and suction surfaces at the trailing edge.[2] Theoretically, the growth of the boundary-layer momentum thickness θ on a blade surface can be described by the von Kármán momentum equation as a function of the local skin-friction coefficient c_f and the velocity gradient along the surface dV/dx as

$$\frac{d\theta}{dx} = \frac{c_f}{2} - (H + 2) \frac{\theta}{V} \frac{dV}{dx} \tag{1}$$

where V is the local free-stream velocity outside the boundary layer and H is the boundary-layer form factor. Thus, the momentum thickness of a surface boundary layer at the trailing edge is obtained from integration of equation (1) to give

$$\theta_{te} = \int_0^{x_{te}} \frac{c_f}{2}\, dx - \int_0^{x_{te}} (H + 2) \frac{\theta}{V} \frac{dV}{dx}\, dx \tag{2}$$

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[2] If it is assumed that a "dead air" region is formed immediately behind the trailing-edge thickness (as assumed in refs. 5 and 6), then the trailing-edge thickness can have no effect on the wake momentum thickness at the trailing edge since no mass flow is carried in this region.

The integration in equation (2) can be conducted by assuming for simplicity that the maximum free-stream surface velocity V_{max} occurs at the leading edge and that a mean value of $(H_s + 2)(\theta/c)_s$ can be taken so that

$$\left(\frac{\theta}{c}\right)_{te} = \frac{C_f}{2} + j \log_e \left(\frac{V_{max}}{V_{te}}\right) \tag{3}$$

where

$$j = \overline{(H + 2)(\theta/c)} \tag{4}$$

C_f is the conventional total skin-friction coefficient,[3] and V_{te} is the free-stream surface velocity at the trailing edge. In the ensuing analysis, however, since there is generally little change in wake momentum thickness and free-stream velocity between blade trailing-edge and measuring-station location (ref. 7), trailing-edge values will be taken to be those at the measuring station. For the complete wake (both surfaces), the momentum thickness can then be expressed as

$$\left(\frac{\theta}{c}\right)_2 = \frac{C_{f,p}}{2} + \frac{C_{f,s}}{2} + j_p \log_p \left(\frac{V_{max,p}}{V_2}\right) + j_s \log_e \left(\frac{V_{max,s}}{V_2}\right) \tag{5}$$

Examination of surface velocity distributions of conventional cascade blades (as illustrated in fig. 4) indicates that, as blade camber or blade angle of attack is increased, large changes in velocity gradient occur on the blade suction surface, but comparatively small changes occur on the pressure surface. Furthermore, changes in surface friction coefficients will be small compared with the changes in the suction-surface diffusion term. Thus, changes in total wake momentum thickness will result primarily from the diffusion contribution of the suction-surface boundary layer. Accordingly, equation (5) will be written as

$$\left(\frac{\theta}{c}\right)_2 = \epsilon + j_s \log_e \left(\frac{V_{max,s}}{V_2}\right) \tag{6}$$

where

$$\epsilon = \frac{1}{2} (C_{f,p} + C_{f,s}) + j_p \log_e \left(\frac{V_{max,p}}{V_2}\right) \tag{7}$$

[3]It is assumed for simplicity in the integration of the local friction term in equation (2) that the blade surface length is effectively equal to the chord length.

It can be shown from calculations of several common averaging processes that, for the range of values of H and θ/c normally encountered for conventional blades, it is sufficiently accurate to express j in equation (4) in terms of the individual chordwise average values of H and θ/c such that

$$j_s = \overline{(H_s + 2)(\theta/c)}_s \approx (\overline{H}_s + 2)\overline{(\theta/c)}_s \tag{8}$$

Now the average momentum thickness in equation (8) can be expressed as some factor times the momentum thickness at the measuring station, such that

$$\left(\frac{\overline{\theta}}{c}\right)_s = f_\theta\left(\frac{\theta}{c}\right)_{2,s} = f_\theta\left(\frac{\theta_s}{\theta}\right)_2\left(\frac{\theta}{c}\right)_2 \tag{9}$$

where f_θ is some averaging factor that satisfies the equality. Substitution for $\overline{(\theta/c)}_s$ according to equation (9) into equations (8) and (6) then gives for the wake momentum thickness

$$\left(\frac{\theta}{c}\right)_2 = \frac{\epsilon}{1 - k_s \log_e\left(\frac{V_{max,s}}{V_2}\right)} \tag{10}$$

where

$$k_s = (2 + \overline{H}_s)f_\theta\left(\frac{\theta_s}{\theta}\right)_2 \tag{11}$$

An analytical equation for the variation of wake momentum thickness with suction-surface diffusion ratio is thus obtained in terms of a factor ϵ, which is a function primarily of the skin-friction coefficients, and a factor k_s, which involves some mean values of H and θ/c along the surface. It now remains to analyze the ϵ and k_s factors over the range of diffusion encountered in cascades in order to obtain an indication of their magnitude and possible relation to $V_{max,s}/V_2$.

Evaluation of terms. - A direct evaluation of the various terms in equations (7) and (11) cannot be made because of the general absence of experimental data for the development of the boundary layer along the surfaces of compressor cascade blades. However, a preliminary indication of the approximate magnitude of the ϵ and k_s terms can be obtained from an examination of available boundary-layer data from similar flow surfaces.

Theoretically, the case of zero diffusion ($V_{max}/V_2 = 1$) is represented by a constant velocity on the blade surfaces, as would be obtained on very thin uncambered blades set at zero angle of attack. The boundary-layer development will therefore be similar to that of the flat plate, so that

$$\epsilon = \frac{1}{2} C'_{f,p} + \frac{1}{2} C'_{f,s} = C'_f \tag{12}$$

where C'_f is the conventional total skin-friction coefficient for the flat plate. A zero diffusion on both blade surfaces, however, cannot be attained with conventional blades of thicknesses of the order of 10 percent, since some variation in velocity will always occur on one or both of the surfaces. The lowest suction-surface diffusion ratio attained for the blades of reference 4 at minimum-loss angle of attack, for example, was about 1.18 (fig. 1). At low diffusion ratios, therefore, since the maximum surface velocities are greater than the inlet velocity (effective Reynolds number is consequently higher) and since negative velocity gradients are present, the total skin-friction coefficient for the blade is expected to be somewhat lower than the conventional flat-plate value based on inlet-velocity Reynolds number C'_f (refs. 8 and 9).

As suction-surface diffusion is increased, the cascade data show that the negative velocity gradients on the suction surface increase markedly, while the negative velocity gradients on the pressure surface generally decrease by a comparatively small amount. A further net reduction in total friction coefficient is therefore likely as diffusion ratio is increased to the separation value.

The third term in the ϵ factor involving the pressure-surface diffusion ratio (eq. (8)) will also generally tend to decrease slightly with increasing suction-surface diffusion ratio. The resulting change in the value of ϵ, however, is expected to be quite small, since the values of pressure-surface diffusion ratio involved are close to unity. Thus, on the basis of these considerations, the value of ϵ is expected to be generally somewhat smaller than the total flat-plate friction coefficient and to tend to decrease slightly with increasing suction-surface diffusion ratio.

The majority of the experimental compressor-blade cascade data considered in the diffusion correlations of reference 4 was obtained for blade-chord Reynolds numbers between 2.0×10^5 and 2.5×10^5. For this range of Re_c, the flat-plate friction coefficient C'_f is about 0.0060 for turbulent flow and about 0.0028 for laminar flow (fig. 5). The factor ϵ in the momentum-thickness equation should therefore have a value somewhere between these limits.

In evaluating the magnitude of the k_s term in equation (11), recourse was made to data from isolated airfoil sections. Experimental

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variations of boundary-layer momentum thickness and form factor in predominantly turbulent boundary layers are shown in figure 6 for the flat plate (constant free-stream velocity) and for the upper surface of several isolated airfoils where flow separation has occurred near the trailing edge. The flat-plate data represent the lower limiting case of cascade flow with zero surface diffusion ($V_{max}/V_2 = 1$), and the isolated airfoil data are representative of the case of a general diffusion ($V_{max}/V_2 > 1$) commencing near the leading edge and resulting in boundary-layer separation. The length l for the isolated airfoils is the distance along the chord to the indicated point of separation.

Although the specific averaging processes required to make the values of f_θ and $\overline{H}_s$ in equation (11) satisfy equations (8) and (9) are not known, it will be assumed for simplicity that the magnitude of these terms can be obtained from the experimental data by considering the mean values of f_θ and $\overline{H}$ as

$$\overline{H} = \int_0^1 H \, d(x/l) \tag{13}$$

and

$$f_\theta = \int_0^1 \frac{\theta}{\theta_{(x/l)=1}} \, d(x/l) \tag{14}$$

Values of $\overline{H}$ and f_θ given by equations (13) and (14) were computed for each flow surface for the data of figure 6. For zero diffusion, from the flat-plate data of figure 6, $\overline{H} = 1.33$ and $f_\theta = 0.57$. (For the classical flat-plate boundary-layer theory with $\theta \propto x^{4/5}$, $f_\theta = 0.556$.) Thus, with essentially symmetrical boundary layers on both surfaces, $(\theta_s/\theta)_2 \approx 0.5$, and the factor k_s (from eq. (11)) will be about 0.95 for the case of zero diffusion.

For the case of diffusion resulting in separation, an essentially constant value of $f_\theta = 0.33$ was computed for all airfoils, while an average value of $\overline{H}_s = 1.60$ was obtained. If similar values of $\overline{H}_s$ and f_θ occur for cascade blades (as is most likely, since the suction-surface velocity distributions of isolated and cascade airfoils are generally similar), then, since $(\theta_s/\theta)_2$ is approximately 0.90 to 0.95 for the stalled condition, k_s values of approximately 1.07 to 1.13 may be obtained. Thus, it appears reasonable to conjecture that k_s might tend to increase somewhat with $V_{max,s}/V_2$ or possibly be essentially constant at a value of the order of unity for conventional cascade blades with turbulent boundary layers.

Comparison with Experiment

The experimental variation of wake momentum thickness $(\theta/c)_2$ with measured suction-surface diffusion ratio $V_{max,s}/V_2$ for a conventional cascade blade over wide ranges of camber, solidity, and air inlet angle 4456 (determined from data of ref. 4) is repeated in figure 7 for the condition of minimum-loss angle of attack (as defined in ref. 4). The diffusion ratio $V_{max,s}/V_2$ in figure 7 (and in fig. 1), however, is based on a more exact value of V_2 than in reference 4 and is given by (from eqs. (1) and (2) in ref. 4)

$$\frac{V_{max,s}}{V_2} = \frac{V_{max,s}}{V_1}\frac{V_1}{V_2} = \frac{V_{max,s}}{V_1}\frac{\cos\beta_2}{\cos\beta_1}\left[1 - \left(\frac{\theta}{c}\right)_2 \frac{\sigma}{\cos\beta_2}\right] \tag{15}$$

where $V_{max,s}/V_1$, β_1, β_2, and $(\theta/c)_2$ are obtained from the measured blade performance and surface velocity distributions. As in reference 4, the values of $(\theta/c)_2$ in figure 7 are taken from faired curves of $(\theta/c)_2$ against angle of attack in an attempt to eliminate some of the obvious increases in momentum thickness due to local laminar separations. The data thus tend to approach the variation for a predominantly turbulent flow without laminar separation bubbles.

As shown by the dashed curve in figure 7, an analytical curve of $(\theta/c)_2$ against $V_{max,s}/V_2$ given by equation (10) can be obtained to produce a reasonable representation of the experimental data on the basis of constant values of $\epsilon = 0.004$ and $k_s = 1.17$. Apparently, for these data, the speculated reduction in ϵ with $V_{max,s}/V_2$ is not significant, or perhaps there is a compensating effect between a decreasing ϵ and an increasing k_s.

Since the boundary-layer flow for the blades of figure 7 is predominantly turbulent, the flat-plate friction coefficient at the Reynolds number of the data should be somewhat less than 0.006. The experimentally determined value of $\epsilon = 0.004$ therefore appears reasonable since, according to the previous considerations, it should be smaller than the flat-plate friction coefficient. It is also noted that the value of $k_s = 1.17$ obtained from the data of figure 7 is somewhat larger than the values deduced previously for the predominantly turbulent boundary layer.

However, the higher value of k_s for the cascade, compared with the previous isolated airfoil case, may reasonably be due to a higher average value of H_s because of the lower Reynolds numbers and existence of local laminar separations in the cascades.

On the basis of these results, it is believed that the use of the flat-plate friction coefficient in the evaluation of the ϵ term and the use of mean chordwise values of H and θ/c in the evaluation of the k_s term are sufficiently valid for the application of equation (11). The derived momentum-thickness equation will therefore be considered as a satisfactory approximate representation of the wake thickness of conventional cascade blades for purposes of comparative analysis.

Theoretical Variations

The approximate momentum-thickness equation derived herein can be utilized to obtain an insight into the qualitative effects of secondary factors such as Reynolds number, surface roughness, transition location, and local laminar separation on the variation of wake momentum thickness with diffusion ratio. This is accomplished by investigating the effect of these factors on the k_s and ϵ terms in equation (10) for turbulent and laminar flow.

The boundary-layer characteristics that influence the magnitude of the k_s term in equation (10) are the chordwise mean values of form factor and momentum thickness, which in turn depend on the type of boundary layer present. As found previously herein, k_s may be about 0.95 to 1.13 for isolated airfoils with turbulent boundary-layer flow at high Reynolds number (order of 10^6 or greater), and about 1.17 for the conventional cascade blades of reference 10 at a Reynolds number of about 2.5×10^5 (predominantly turbulent flow).

Representative values of k_s for the laminar boundary layer can be obtained from the limited available theoretical variations of H and θ shown in figure 8 for the flat plate and for a linearly decreasing velocity with separation at the trailing edge. For zero diffusion (flat plate), $\overline{H} = 2.61$ and $f_\theta = 0.66$ to give $k_s = 1.52$. For the case of diffusion at separation (linearly decreasing velocity), $\overline{H} = 2.88$ and $f_\theta = 0.61$, which gives, with $\theta_s/\theta_2 = 0.90$ to 0.95, a k_s value of about 2.75. The use of a constant value of k_s in equation (10) over the diffusion range therefore may not be representative of the real momentum-thickness development in laminar flow. However, as expected, values of k_s for laminar flow are substantially larger than those for turbulent flow.

Calculated variations of $(\theta/c)_2$ with $V_{max,s}/V_2$ for several values of k_s representative of conventional cascade flow with predominantly turbulent boundary layers ($\epsilon = 0.004$) and with laminar boundary layers ($\epsilon = 0.0027$, corresponding to C_f' at $Re_c = 2.5\times10^5$) are shown in figure 9. According to equation (10), a limiting value of $V_{max,s}/V_2$ is reached for each value of k_s when $(\theta/c)_2$ approaches infinity. The prime importance of the nature of the surface boundary layer in determining the allowable diffusion ratio is thus clearly indicated.

As indicated by the derived equation (eq. (10)), any increase in the value of ϵ will be directly reflected as an increase in the magnitude of the momentum thickness. This is illustrated by the calculated variation of $(\theta/c)_2$ with diffusion ratio for several values of ϵ shown in figure 10 for a value of k_s representative of predominantly turbulent flow. According to the formulation of equation (11), the variation of ϵ at constant k_s affects only the rate at which the limiting diffusion ratio (value at which $(\theta/c)_2 \to \infty$) is approached. In a real flow, however, it is likely that k_s may also vary as ϵ is varied, since the factors that determine the magnitude of ϵ may also have an effect on the chordwise H and θ variations.

Since the value of ϵ is approximately equal to or at least proportional to the flat-plate friction coefficient C_f', ϵ will be a direct function of the blade-chord Reynolds number and the surface roughness of the blade, values of which can be obtained, for example, from figure 5 and from reference 11 (ch. XXI). For the case of partly laminar and partly turbulent flow, the friction coefficient can be calculated as (ref. 11, p. 434)

$$C_f' = C_{f,tb}' - \left(\frac{x}{c}\right)_{tr} (C_{f,tb}^* - C_{f,lm}^*) \tag{16}$$

where $(x/c)_{tr}$ is the extent of the laminar region, $C_{f,tb}'$ is the turbulent total friction coefficient based on the chord-length Reynolds number, and $C_{f,lm}^*$ and $C_{f,tb}^*$ are the laminar and turbulent total friction coefficients, respectively, based on the Reynolds number at $(x/c)_{tr}$.

In order to obtain an equation in which all friction coefficients are based on blade-chord Reynolds number, use is made of the general relations

$$C_{f,lm,x}' = \frac{1.328}{\sqrt{Re,x}} \tag{17}$$

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and, for simplicity (ref. 11),

$$C'_{f,tb,x} = \frac{0.074}{\sqrt[5]{Re,x}} \tag{18}$$

to give

$$C^*_{f,lm} = \frac{C'_{f,lm}}{\sqrt{\left(\frac{x}{c}\right)_{tr}}} \tag{19}$$

and

$$C^*_{f,tb} = \frac{C'_{f,tb}}{\sqrt[5]{\left(\frac{x}{c}\right)_{tr}}} \tag{20}$$

Substitution in equation (16) then yields, for partly laminar and partly turbulent flow,

$$C'_f = C'_{f,tb}\left[1 - \left(\frac{x}{c}\right)_{tr}^{4/5}\right] + C'_{f,lm}\left(\frac{x}{c}\right)_{tr}^{1/2} \tag{21}$$

where $C'_{f,tb}$ and $C'_{f,lm}$ are obtained from figure 5 for the blade-chord Reynolds number in question. Although the proper relation between ϵ and C'_f cannot be accurately determined from the correlation of figure 7, for practical purposes, values between $\epsilon = 0.8\ C'_f$ and $\epsilon = C'_f$ appear to be satisfactory.

In view of these considerations, it is apparent that the location of the transition from laminar to turbulent boundary-layer flow, and in particular the relation between transition and laminar-separation location, may exert a considerable influence on the terminal value of the momentum thickness of conventional cascade blades. An indication of the possible variation in k_s with ϵ with varying transition location was obtained from consideration of hypothetical form-factor profiles representative of flow with local laminar separation (transition occurs after separation of the laminar boundary layer). Figure 11 shows highly idealized variation of H_s with chord-length distance along the surface for various locations of laminar separation and turbulent reattachment. In the figure, laminar separation is shown to occur at the 0, 0.2, and 0.4 chord points followed by a turbulent reattachment either immediately ($(\Delta x/c)_{ls} = 0$, solid line) or after 0.1 chord length ($(\Delta x/c)_{ls} = 0.1$, dashed line). Turbulent separation is assumed to occur at the trailing edge, as is likely for a heavily loaded blade.

Average values of form factor $\overline{H}_s$ were determined by integration for each curve in figure 11, and corresponding values of f_θ were determined from the simplifying assumption that

$$f_\theta = \left(\frac{x}{c}\right)_{tr} f_{\theta,lm} + \left[1 - \left(\frac{x}{c}\right)_{tr}\right] f_{\theta,tb} \tag{22}$$

where $f_{\theta,lm}$ is taken as 0.61 and $f_{\theta,tb}$ is 0.33. Values of k_s were then determined for each transition location according to equation (11) by assuming $\theta_s/\theta_2 = 0.90$. Values of flat-plate friction coefficient C_f' for each case were determined from equation (21) for $Re_c = 2.5\times10^5$. The results of the calculations for k_s and C_f' for the illustrative cases of figure 11 are listed in the following table:

$(x/c)_{ls}$	$(\Delta x/c)_{ls}$	$\overline{H}_s$	f_θ	k_s	C_f'
0	0	1.58	0.33	1.06	0.0059
	.1	1.80	.36	1.23	.0058
0.2	0	1.88	0.39	1.36	0.0055
	.1	2.10	.41	1.51	.0051
0.4	0	2.19	0.44	1.66	0.0047
	.1	2.41	.47	1.86	.0044

Calculated variations of $(\theta/c)_2$ with $V_{max,s}/V_2$ for the transition configurations of figure 11, assuming the calculated values of k_s and ϵ to be constant over the entire range of $V_{max,s}/V_2$ (assuming $\epsilon = C_f'$), are shown in figure 12 for illustrative purposes. (In an actual flow, of course, it is recognized that k and ϵ might not be constant, since transition and separation characteristics will vary with the diffusion ratio.) Thus, it is seen that a wide range of values of momentum thickness and limiting diffusion ratio may theoretically be obtained for a given cascade if different transition locations and different behaviors of the surface boundary layer in the transition region occur. These latter factors, in turn, are dependent on the blade-chord Reynolds number, the free-stream turbulence level, and the surface roughness.

DISCUSSION

The preceding results, based on the derived momentum-thickness equation, may find application in the interpretation of experimental results and in the establishment of design considerations for compressor or cascade blades. For example, the experimental variation of $(\theta/c)_2$ with

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$V_{max,s}/V_2$ for angles of attack greater than minimum loss for the various cascade geometries of reference 10, as shown in figure 13, exhibits a wide spread of the data for $V_{max,s}/V_2$ greater than about 1.9. Inasmuch as these cascade data were obtained at values of Reynolds number and free-stream turbulence for which local laminar separations were present (ref. 10), it is reasonable to conjecture that the data spread at high values of diffusion ratio may be due, to some extent, to the effects of variations in the nature and location of these separations (as illustrated in fig. 12). Indicated values of stalling diffusion ratio (value above which a sharp rise in loss is possible) derived from experimental data (as in ref. 4) should, therefore, be carefully qualified for the blade-chord Reynolds number and turbulence levels of the tests. It is quite likely, according to the present results, that different stalling limits may be obtained for different values of Re_c and turbulence. The analysis also indicates that, if consistent cascade loss correlations are desired, tests should be run at about the same values of Reynolds numbers and turbulence levels.

For consideration of cascade design, the derived diffusion analysis indicates that considerable variation can be obtained in the allowable value of suction-surface diffusion ratio (value above which θ/c becomes excessive), which in turn depends on the specific boundary-layer behavior of the cascade geometry. In order to obtain a rough indication of the magnitude of the allowable diffusion ratio for a given design, the values of ϵ and k_s must either be known or estimated. Although the effects of Reynolds number and surface roughness on the friction coefficient (and therefore on ϵ) can be readily determined, it is generally not possible to evaluate the effect of these factors as well as the effect of the turbulence and surface velocity distribution on the transition behavior (and therefore on k_s). Furthermore, in view of the limitations involved in the derivation, it is not known whether the derived equation is capable of giving accurate absolute magnitudes for the limiting diffusion ratio over a wide range of cascade and flow configurations.

It is recognized that the derived equation (eq. (10)) represents an oversimplification of the true boundary-layer development on cascade blade surfaces and contains many approximations. The principal reservation lies in the use of mean values of H and θ in the integration of the momentum equation and in the validity of the von Kármán equation in regions of separation where the normal pressure gradients and turbulent stresses may not be zero. At the moment, therefore, the primary utility of the diffusion analysis for design is in providing qualitative considerations and indications of relative effects.

According to the preceding analysis, for a given application in cascade design, the allowable diffusion ratio can be increased by designing the blade and cascade so as to tend to reduce the total friction coefficient and the extent of laminar flow. These objectives can be accomplished in several ways: by utilizing a large value of blade-chord Reynolds number and high free-stream turbulence level, by maintaining smooth blade surfaces, and by designing for peak surface velocity near the leading edge so that the initial boundary layer at the start of diffusion is small.

In axial-flow-compressor design, the situation is more favorable in some respects than in the cascade. For the same Reynolds numbers, the turbulence levels are generally greater in the compressor because of the wakes and disturbances of preceding blade rows. On the other hand, the use of high peak velocities at the blade leading edge may not be consistent with good high-Mach-number performance. The maintenance of high Reynolds number and low relative surface roughness, however, should likewise be effective in increasing the allowable diffusion ratios of compressor blade sections.

SUMMARY OF RESULTS

A simple equation relating the wake momentum thickness and suction-surface diffusion ratio (ratio of maximum surface velocity to outlet velocity) of conventional low-speed cascade blade sections has been derived from boundary-layer theory in conjunction with several simplifying approximations. An analytical relation is thus obtained which corresponds to the experimental correlations between wake momentum thickness and diffusion ratio reported previously. Comparison with the experimental data showed that reasonable values were obtained for the two empirical factors contained in the equation. The derived momentum-thickness equation was therefore considered to be a satisfactory approximate representation of the wake development of conventional cascade blades for purposes of comparative analysis.

Inasmuch as the two empirical factors derived in the momentum-thickness equation are determined by the blade friction coefficient and the type of boundary-layer flow, it was possible to gain an insight into the qualitative effects on the variation of momentum thickness with diffusion ratio of such factors as blade-chord Reynolds number, surface roughness, transition location, and extent of local laminar separation. Illustrative calculations made for ranges of values of friction coefficient and transition behavior showed that a wide range of curves of momentum thickness against diffusion ratio can be obtained for different boundary-layer histories. These results indicated that, for maximum allowable values of suction-surface diffustion ratio (values above which

$(\theta/c)_2$ becomes excessive), blades should be designed for low friction coefficient (high Reynolds number and low relative surface roughness) and minimum laminar flow (high Reynolds number and high free-stream turbulence).

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, May 26, 1958

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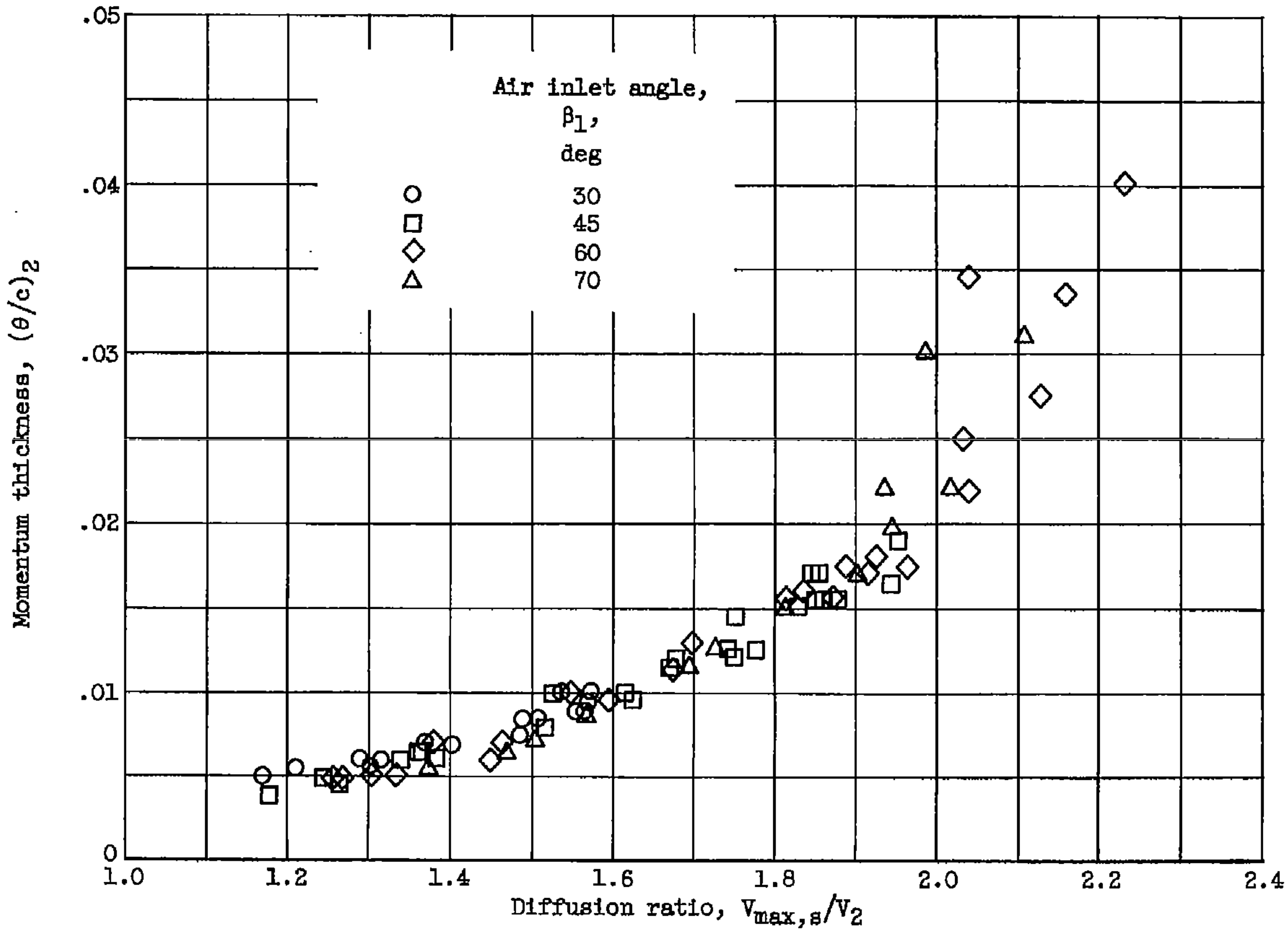

Figure 1. - Experimental variation of wake momentum thickness with suction-surface diffusion ratio at minimum-loss angle of attack for NACA 65-($c_{l_o}A_{10}$)10 blades of reference 10 ($2.0\times10^5 < Re_c < 2.45\times10^5$).

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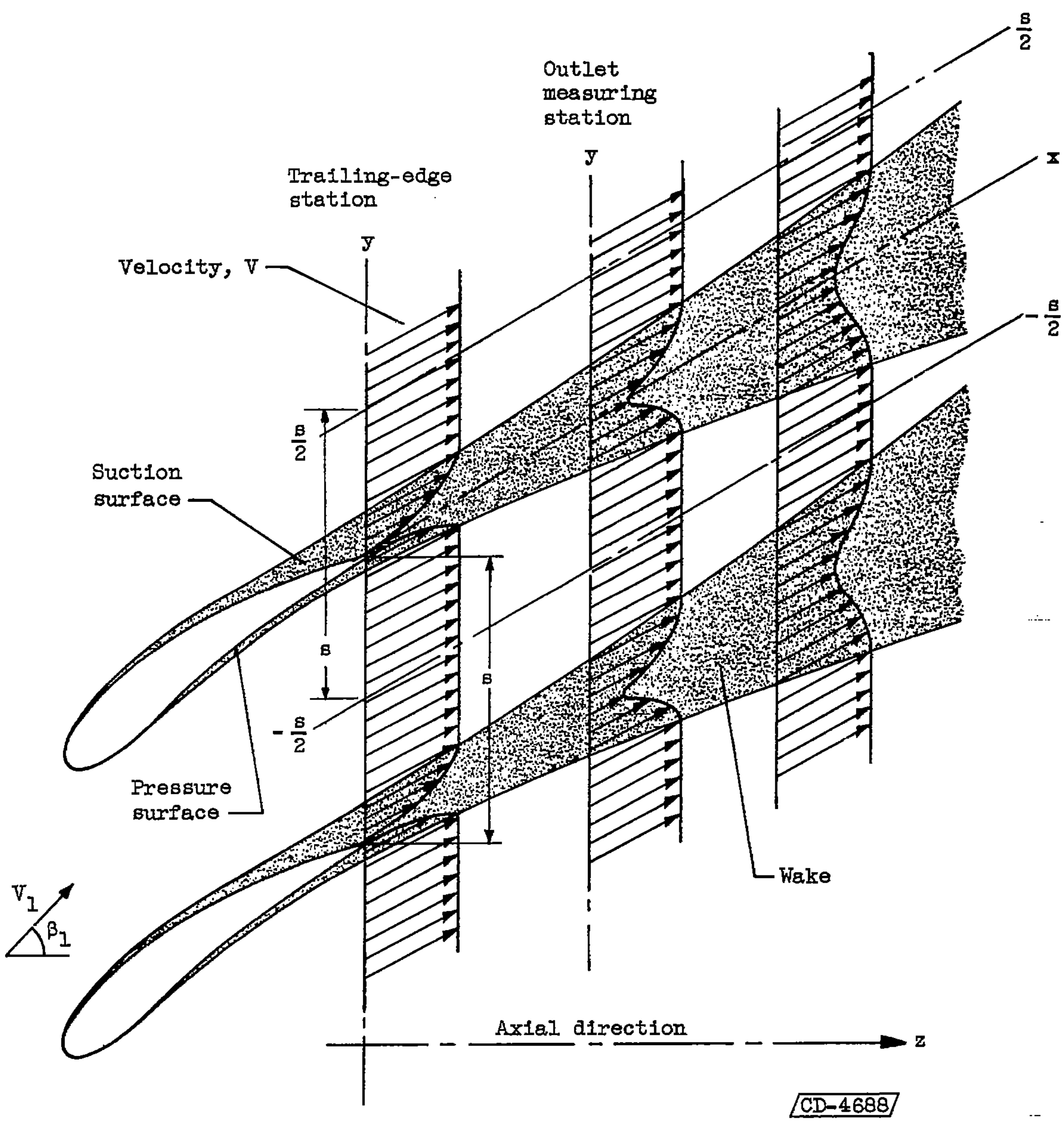

Figure 2. - Schematic representation of wake development in flow about cascade blade sections.

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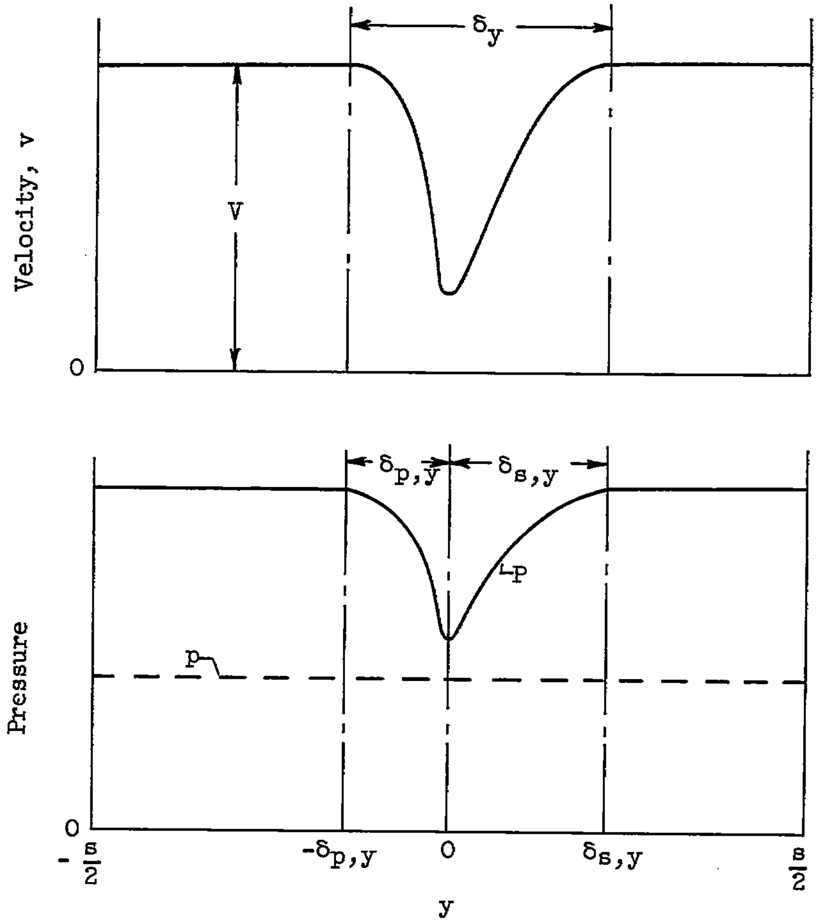


Displacement thickness: $$\delta_y^* = \delta_{s,y}^* + \delta_{p,y}^* = \int_{-\delta_{p,y}}^{\delta_{s,y}} \left(1 - \frac{v}{V}\right) dy$$

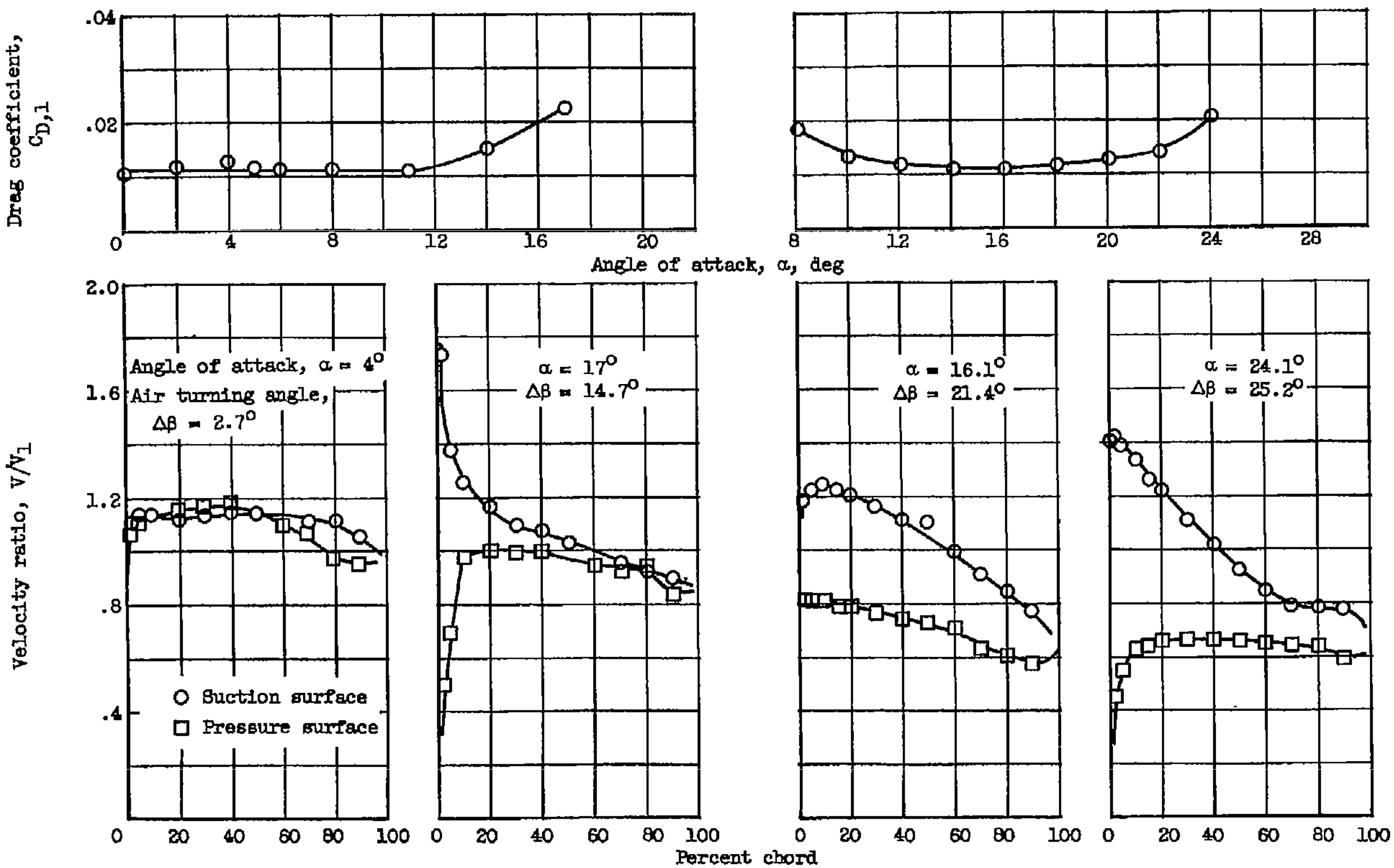
$$\delta^* = \delta_s^* + \delta_p^* = \delta_y^* \cos \beta$$

Momentum thickness: $$\theta_y = \theta_{s,y} + \theta_{p,y} = \int_{-\delta_{p,y}}^{\delta_{s,y}} \left(1 - \frac{v}{V}\right) \frac{v}{V}\, dy$$

$$\theta = \theta_s + \theta_p = \theta_y \cos \beta$$

$$H = \frac{\delta^*}{\theta} = \frac{\delta_y^*}{\theta_y}; \quad \delta = \delta_s + \delta_p = \delta_y \cos \beta$$

Figure 3. - Model variation of velocity and pressure in plane normal to axial direction, and definitions of wake properties. (Subscript y refers to properties in plane normal to axial direction; no subscript indicates properties in plane normal to outlet flow.)



(a) Blade, NACA 65-010; air inlet angle, 45°; solidity, 1.5 (ref. 10).

(b) Blade, NACA 65-($12A_{10}$)10; air inlet angle, 60°; solidity, 1.25 (ref. 10).

Figure 4. - Illustrative surface-velocity distributions for conventional cascade blade.

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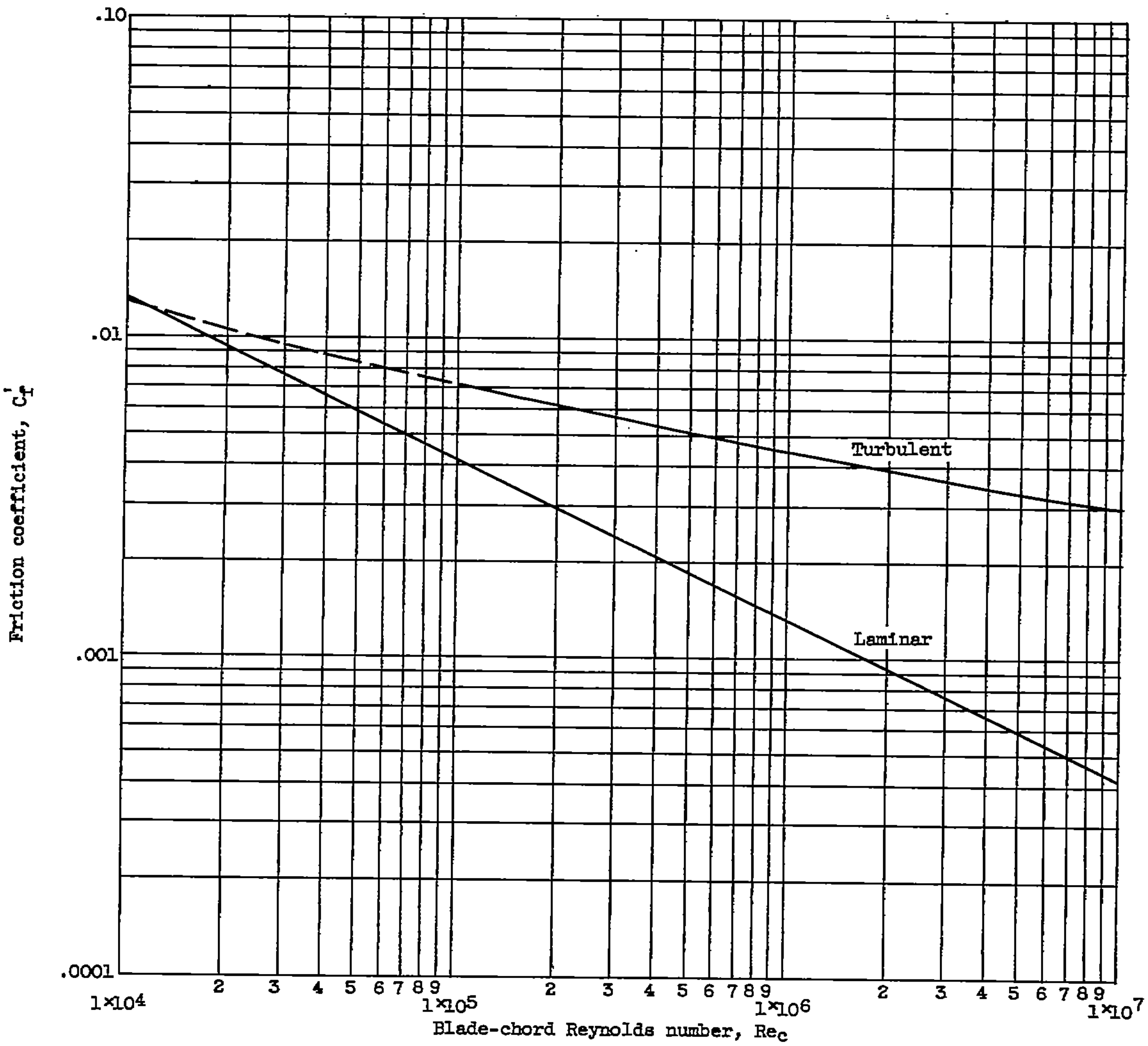
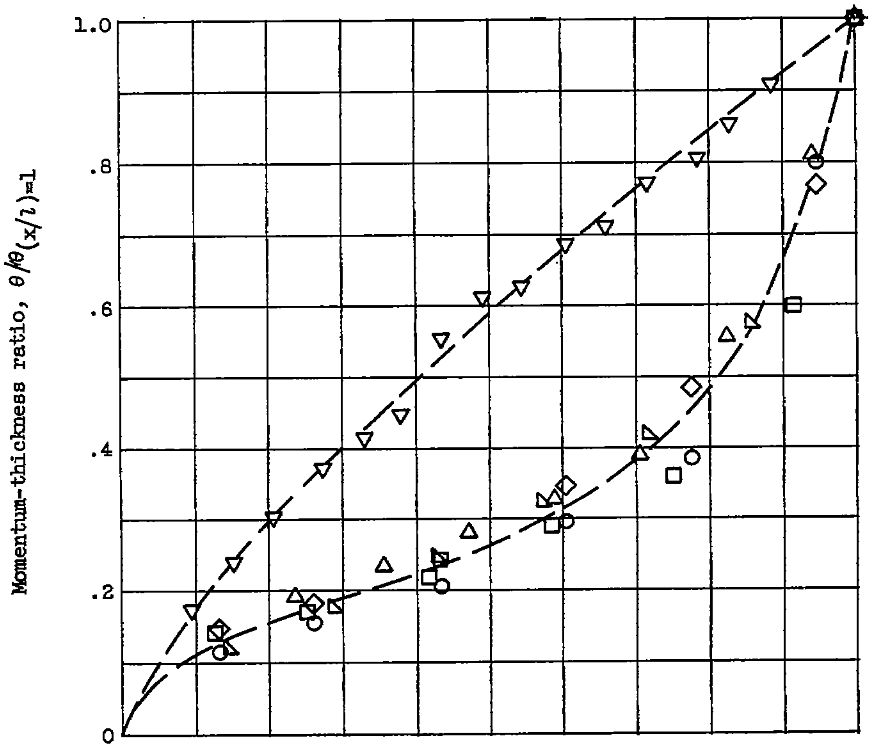
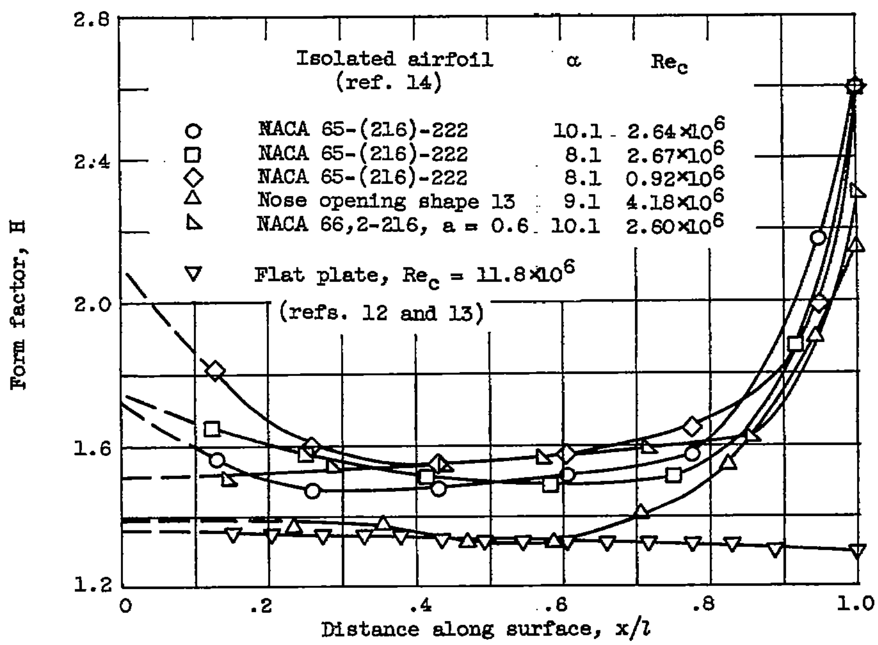


Figure 5. - Total skin-friction coefficient for smooth flat plates (from ref. 11, p. 439). $C'_{f,lm} = \frac{1.328}{\sqrt{Re_c}}$; $C'_{f,tb} = \frac{0.455}{(\log_{10}Re_c)^{2.58}}$.



(a) Momentum thickness.



(b) Form factor.

Figure 6. - Experimental variation of momentum thickness and form factor along several flow surfaces for turbulent boundary-layer flow. Separation at $x/l = 1$ for isolated airfoils.

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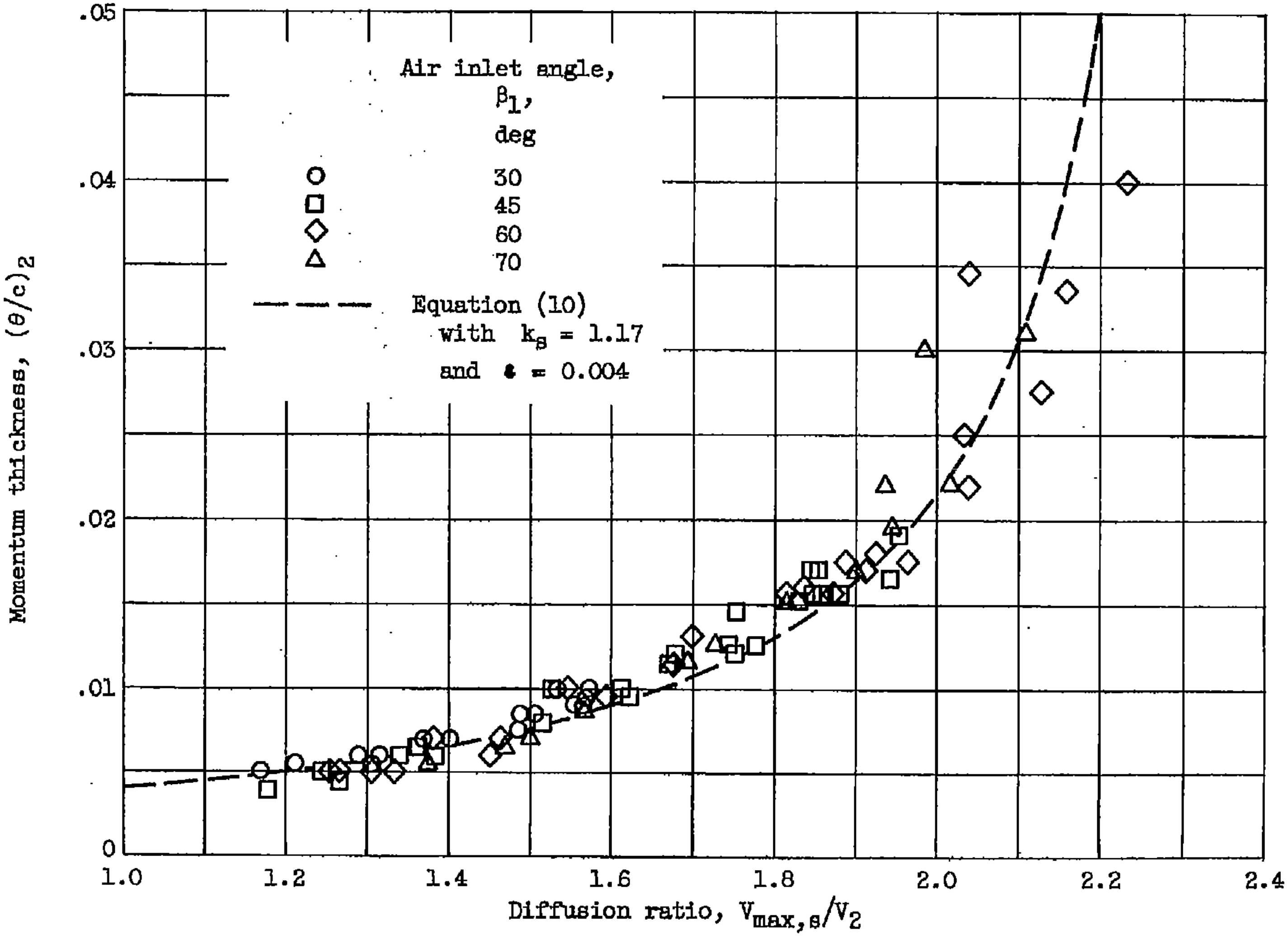
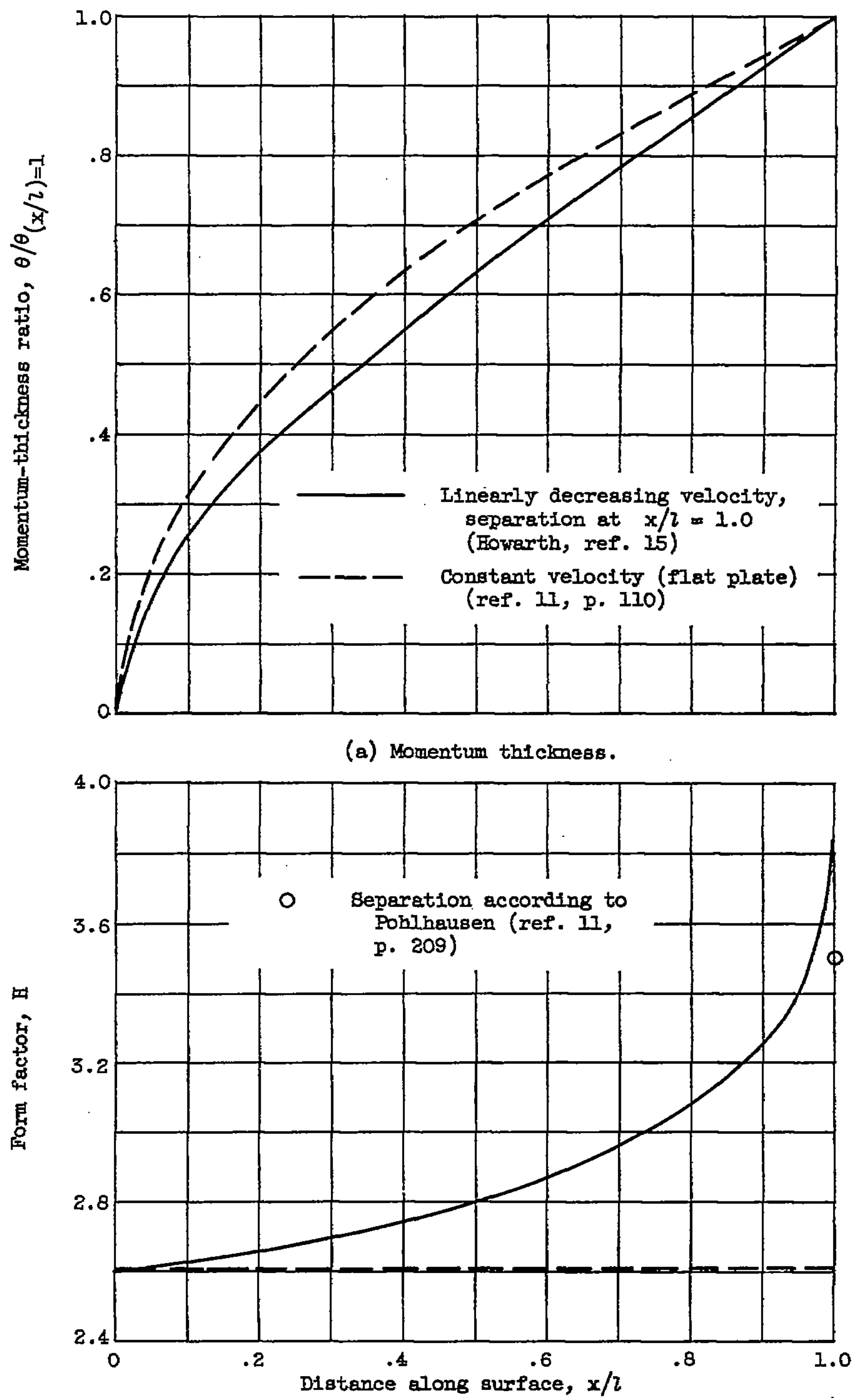


Figure 7. - Comparison between experimental and analytical variation of wake momentum thickness with suction-surface diffusion ratio at minimum-loss angle of attack for NACA 65-($c_{l_o}A_{10}$)10 blades of reference 10 ($2.0\times10^5 < Re_c < 2.45\times10^5$).



(a) Momentum thickness.

(b) Form factor.

Figure 8. - Theoretical variation of momentum thickness and form factor along surface for laminar boundary layer.

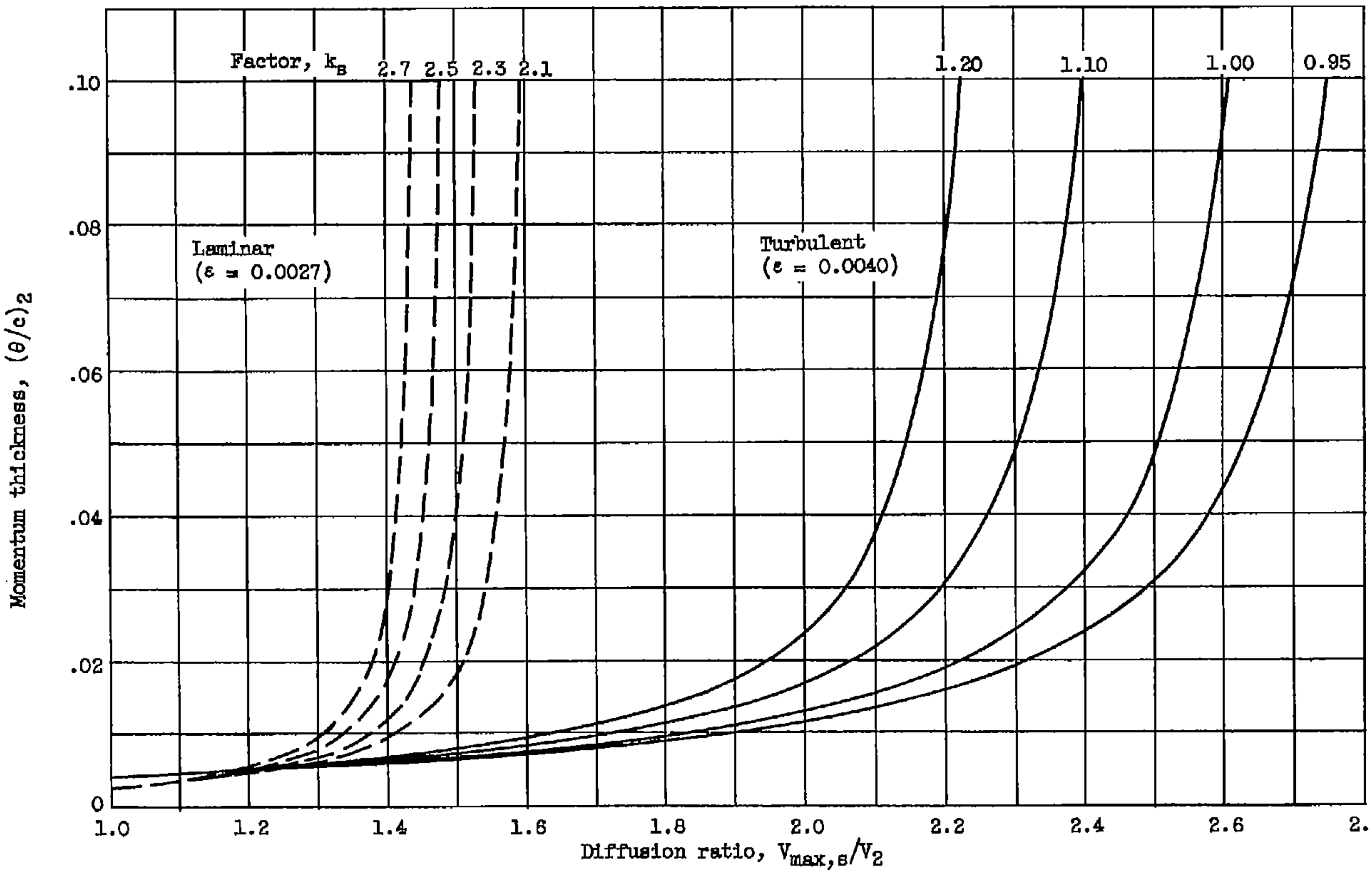


Figure 9. - Calculated variation of wake momentum thickness with suction-surface diffusion ratio for several values of the factor k_s in derived momentum-thickness equation ($Re_c = 2.5 \times 10^5$).

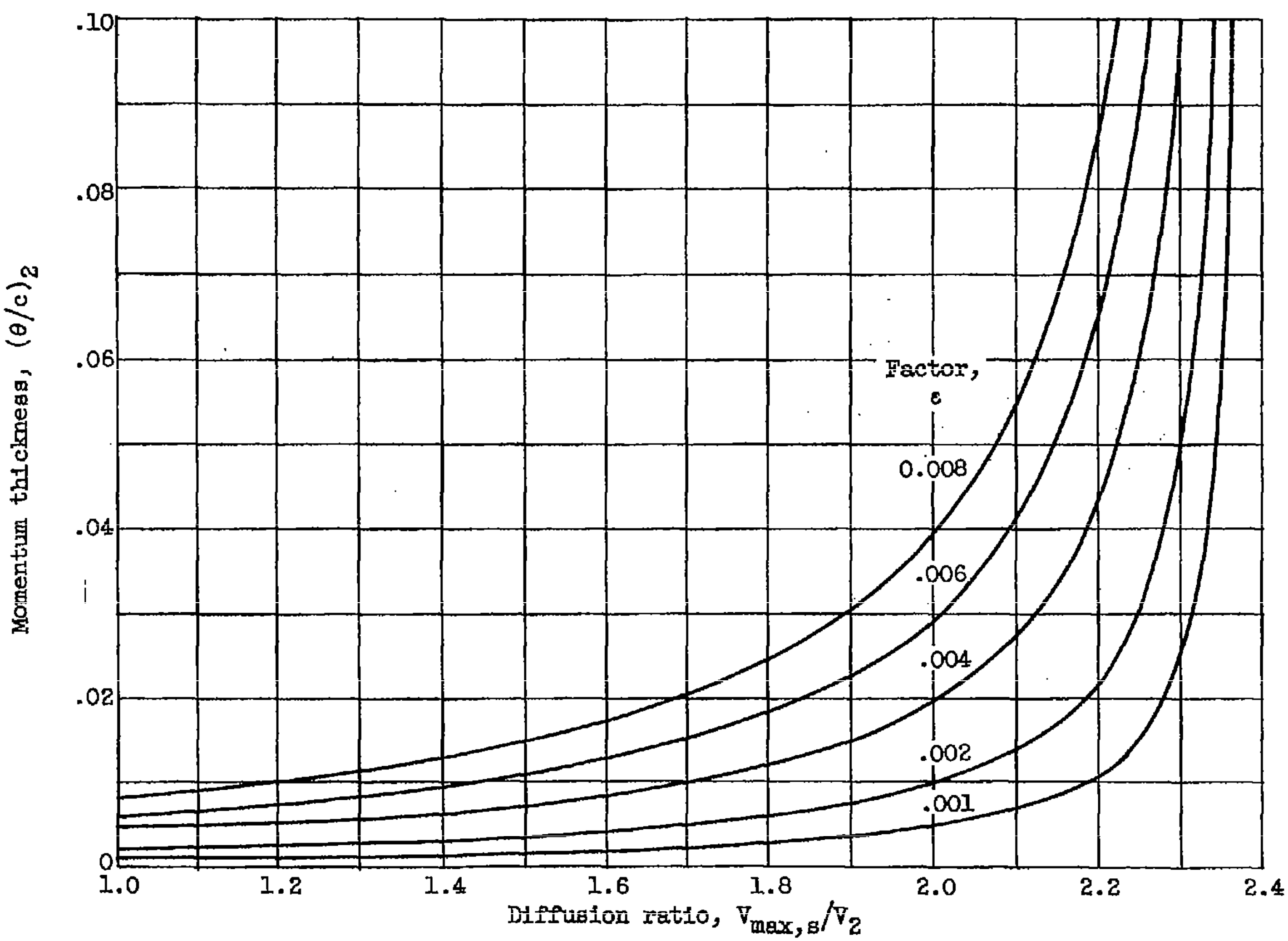


Figure 10. - Calculated variation of wake momentum thickness with suction-surface diffusion ratio for several values of the factor ε in derived momentum-thickness equation for predominantly turbulent flow; factor $k_s = 1.15$.

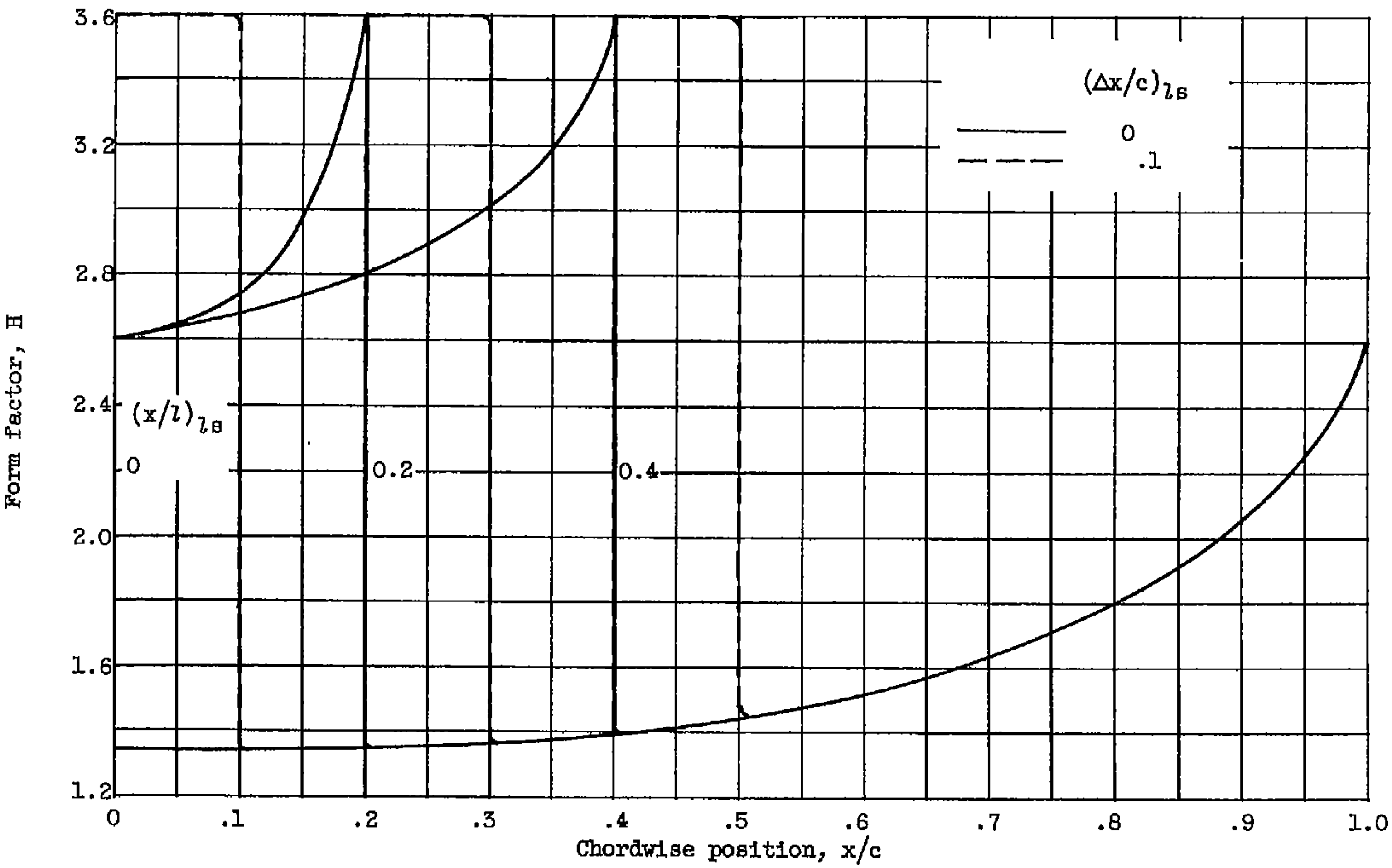


Figure 11. - Idealized variations of boundary-layer form factor along blade surface for various degrees of local laminar flow. Transition and turbulent reattachment after laminar separation and turbulent separation at trailing edge.

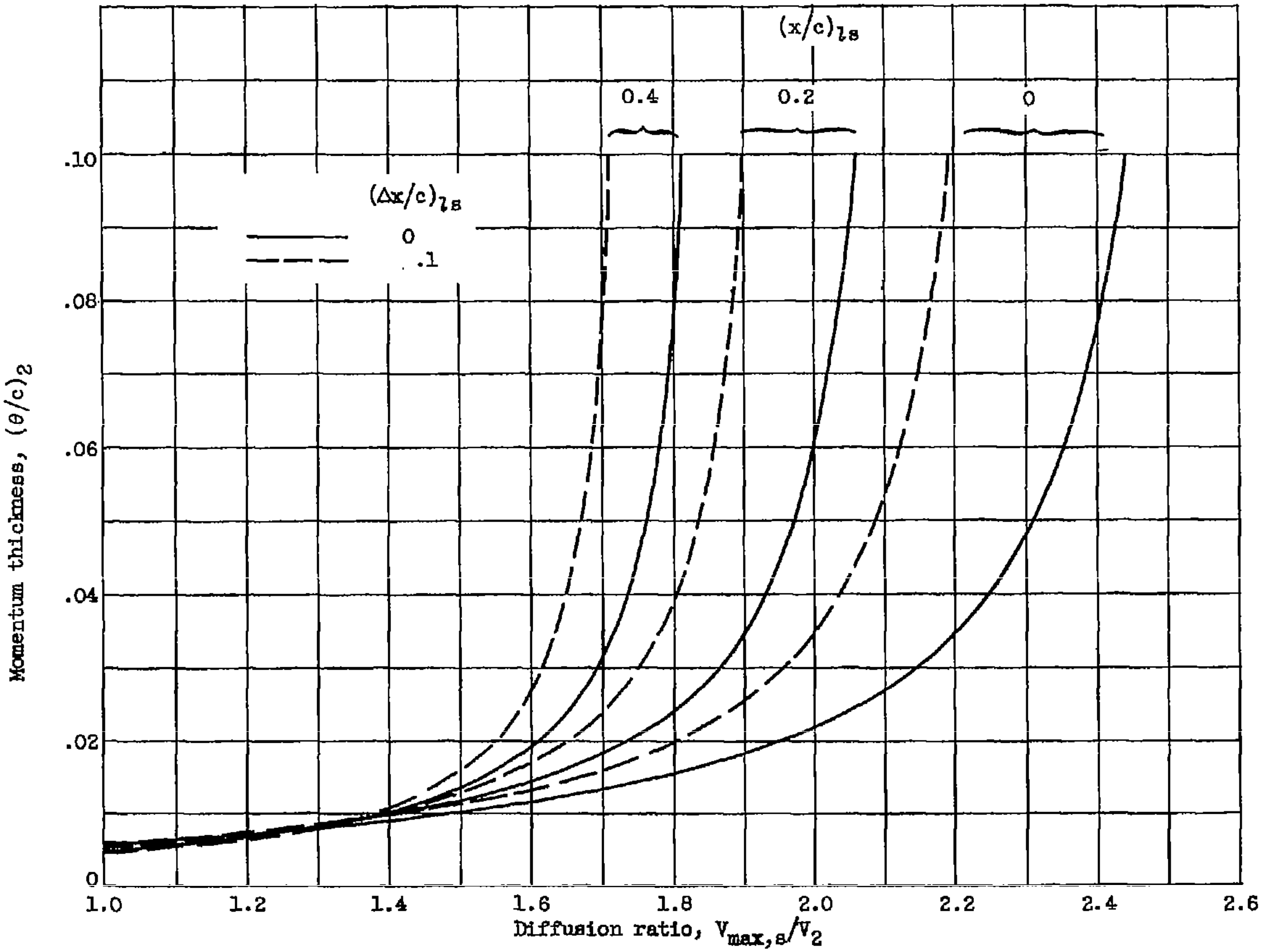


Figure 12. - Calculated variation of wake momentum thickness with suction-surface diffusion ratio for case of transition after laminar separation with turbulent reattachment, as indicated in figure 11 ($Re_c = 2.5 \times 10^5$).

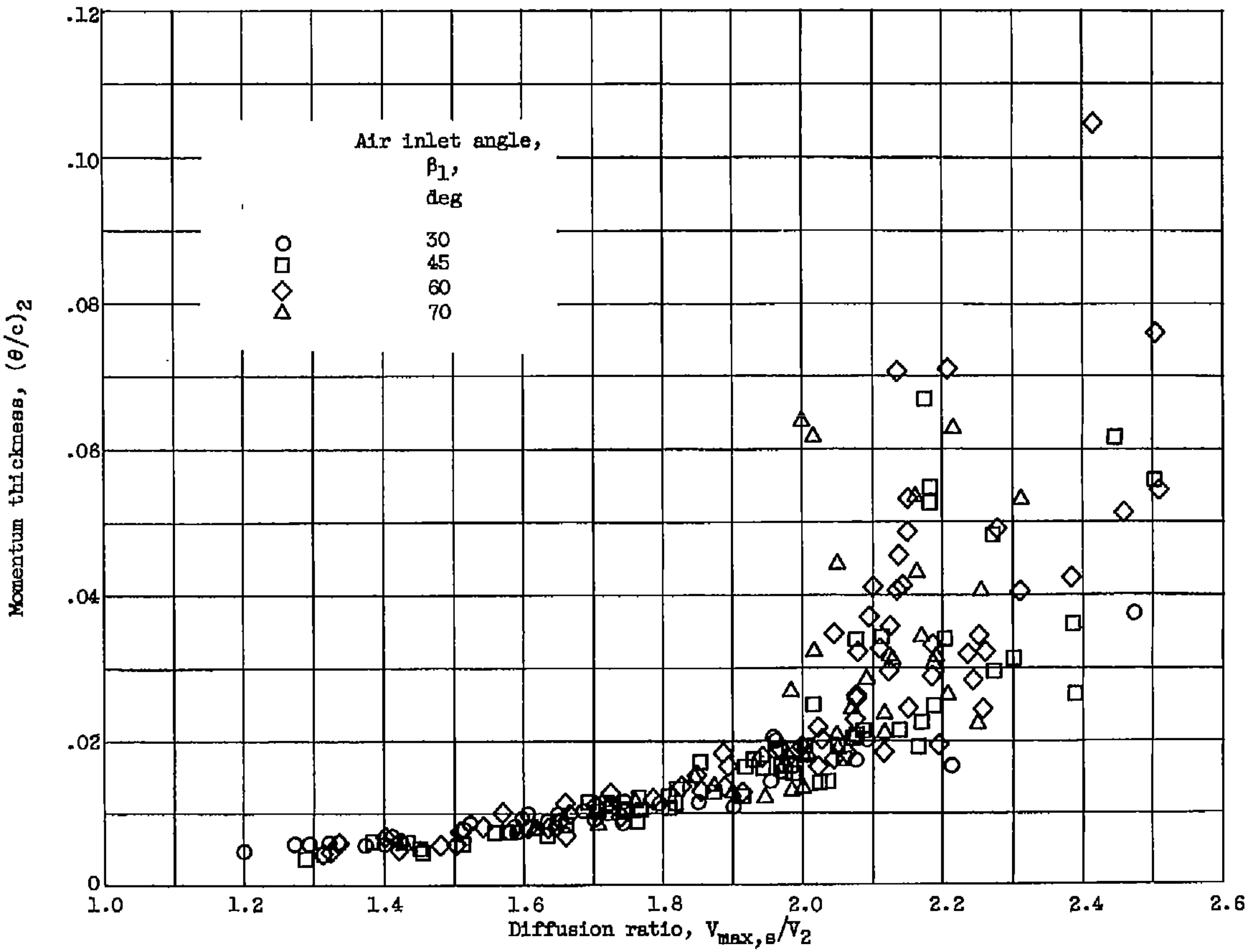


Figure 13. - Experimental variation of wake momentum thickness with suction-surface diffusion ratio at angles of attack greater than minimum loss for NACA 65-$(c_{l_o}A_{10})$10 blades of reference 10 ($2.0\times10^5 < Re_c < 2.45\times10^5$).

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