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HEAT TRANSFER AND THERMAL STRESSES IN SANDWICH PANELS

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SUMMARY

Calculated maximum temperature differences between faces and calculated thermal stresses are presented for sandwich panels with a prescribed linear rate of temperature rise at one face and with the other face insulated. Effects of conduction and radiation are included. Maximum temperature differences between top and bottom faces are considerably less when both radiation and conduction are considered than when radiation is neglected. From the calculated data an equation is derived that relates the maximum temperature difference to the maximum temperature difference when conduction only is considered. An approximate method for including effects of radiation in calculations of temperature difference is presented.

INTRODUCTION

One of the structural configurations being used for supersonic aircraft utilizes a sandwich panel for the load-carrying skin. The state of stress in such panels can be appreciably altered by thermal stresses resulting from aerodynamic heating. In order to calculate thermal stresses, the temperature distribution through the panel must be known.

For solid cores, only heat transfer by conduction need be considered and the temperature distribution can be readily calculated by an appropriate idealization of the sandwich panel into a one-dimensional heat-conduction problem. If, however, air spaces are present in the core, an appreciable amount of heat can be transferred by radiation, with the result that the temperature gradients and the resulting thermal stresses are reduced. The heat transferred by radiation can be included in the one-dimensional heat-balance equation, but because heat transfer by radiation is proportional to the fourth power of the absolute temperature, the result is a nonlinear differential equation which is difficult to solve.

In the present paper equations are derived for calculating the temperature distribution in any sandwich panel in which conduction is predominant. Results of an investigation to determine the effect of including heat transfer by radiation in an analysis of the temperature distribution in honeycomb sandwich panels are also presented. The effect of radiation is evaluated by comparing the maximum temperature difference obtained between the panel faces when radiation is considered with that obtained when radiation is neglected, all other factors being unchanged. These maximum temperature differences are then empirically related.

Most of the calculated results were obtained with a Reeves Electronic Analog Computer. However, an approximate analytical method for calculating temperature when both radiation and conduction must be considered is presented in an appendix. Limited experimental evidence of the validity of the theory is given.

SYMBOLS

A	area
ΔA	solidity of core
B	temperature-rise rate of core
C	arbitrary constant
c	heat capacity of material
D	arbitrary constant
E	modulus of elasticity
$F_{n,m}$	overall configuration factor
$F'_{n,m}$	configuration factor
$G(\tau)$	arbitrary function of time
$g(s)$	Laplace transform of $G(\tau)$
h	core height
K	diffusivity, $k/c\rho$
k	thermal conductivity
L	Laplace transform
L^{-1}	inverse Laplace transform

P	perimeter of cells of core
Q	rate of radiant heat exchange between one point and all other points
q	rate of heat exchange between two points
S	width of square cells in honeycomb panels
T	absolute temperature
T'	absolute temperature less initial temperature
$\overline{T}$	nondimensional temperature
ΔT	difference between the temperatures of the faces of the panel
t	thickness
w	core density
x	distance from unheated face of panel
α	coefficient of thermal expansion
β	roots of the equation $\beta \tan \beta = \frac{h\ \Delta A}{t_F}$
γ	constant
ϵ	emissivity; strain
θ	Laplace transform of T'
λ	dummy variable of integration
ξ	dummy variable of integration
ρ	density of material
σ	Stefan-Boltzmann constant; stress
τ	time
$\overline{\tau}$	nondimensional time, $9K\tau/h^2$

φ	function for configuration factors

Subscripts:

1,2,3,4,5	stations as identified in figure 1
a,b,c,d,e	temperatures defined in appendix B
C	conduction only
eff	effective
F	unheated face
i	positive integer
m,n	integers between 1 and 5
max	maximum
o	initial condition

THEORY

The problem to be considered is the transfer of heat through a sandwich panel, one face of which experiences a prescribed temperature history which is a function of time only. For solid-core sandwich panels, the only mode of heat transfer which must be considered is conduction. If the panel is suitably idealized and suitable boundary conditions are selected, an exact solution can be obtained by employing the well-known one-dimensional partial differential equation of heat balance which governs the transfer of heat by conduction. When air spaces exist in the core, significant amounts of heat may be transferred by radiation from the hot face to the cooler face and to the core elements. When the heat transferred by radiation is incorporated into the differential equation for heat balance, the result is a nonlinear partial differential equation with variable coefficients, the exact solution of which is difficult, if not impossible. The problem can be solved by resorting to an approximate analysis which results in a set of nonlinear ordinary differential equations suitable for solution on an analog computer.

The following assumptions were made in this analysis:

1. Thermal properties are independent of temperature.

2. Convective heat transfer is negligible.

3. The temperature of any plane parallel to the faces is uniform.

In order to obtain a solution which does not involve unnecessary complications, the actual core of the sandwich is idealized as follows. It is assumed that the temperatures of the faces of the sandwich are respectively uniform; therefore, there will be no transfer of heat by conduction in these faces. Because of this, the face material can be considered as concentrated heat capacitors located at the ends of the core elements. It is also assumed that there is no joint resistance between the faces and the core elements, and the thickness of the material used to attach the faces to the core is converted on the basis of heat capacitance into an equivalent thickness of face material which is then included in the concentrated heat capacitors.

The core is considered as a slab with effective thermal properties based on the solidity of the core:

$$k_{eff} = k\,\Delta A$$

$$(c\rho)_{eff} = c\rho\,\Delta A$$

$$K_{eff} = \frac{k}{c\rho}$$

where

k thermal conductivity of core

ΔA solidity of core

$c\rho$ product of heat capacity and density

K diffusivity

Conduction

For honeycomb panels having internal surfaces of low emissivity, or for panels with solid cores (e.g., foam core or laminated plastic core), effects of internal radiation may be negligible. The problem of heat transfer in these panels reduces to one-dimensional heat conduction in the core, governed by the differential equation

$$\frac{1}{K}\frac{\partial T(x,\tau)}{\partial \tau} = \frac{\partial^2 T(x,\tau)}{\partial x^2} \tag{1}$$

The initial temperature of the sandwich is assumed to be uniform:

$$T(x,0) = T_o \tag{2a}$$

The assumed boundary conditions are

$$T(h,\tau) = G(\tau) \tag{2b}$$

$$\frac{\partial T(0,\tau)}{\partial x} = \frac{t_F}{K\,\Delta A}\frac{\partial T(0,\tau)}{\partial \tau} \tag{2c}$$

where

T absolute temperature

T_o initial absolute temperature

t_F idealized thickness of the unheated face

τ time

x distance from the unheated face

h core height

$G(\tau)$ prescribed function of time

Equation (2b) follows from the assumption that there is no thermal resistance between the core and the face, and therefore the end of the core is at the prescribed temperature of the face.

In order to simplify this investigation, the unheated face was assumed to be insulated against heat loss. Equation (2c) is derived in appendix A from this assumption, plus the conditions that the face temperature is uniform and that there is no joint resistance between the core and the face. It is also assumed that the face and core are made of the same material. In order to extend results based on this equation to sandwich panels in which faces and core are made of different materials, an effective face thickness should be used. This effective face thickness is equal to the idealized face thickness multiplied by the ratio of $c\rho$ for the face material to $c\rho$ for the core material.

The solution of these equations for the temperature at any point is derived in appendix A by means of the Laplace transformation and is

$$T(x,\tau) = T_o + \frac{2K}{h^2} \sum_{i=1}^{\infty} \frac{\beta \sin \beta(1 - x)}{\frac{\sin 2\beta}{2\beta} + 1} \int_0^{\tau} G(\lambda) e^{-\frac{K\beta_i^2}{h^2}(\tau-\lambda)} d\lambda \tag{3}$$

where β_i are the roots of the equation

$$\beta_i \tan \beta_i = \frac{h \, \Delta A}{t_F} \tag{4}$$

A partial listing of the roots of this equation is given in table I. A more extensive table is presented in reference 1.

For the temperature of the unheated face $x = 0$, equation (3) reduces to

$$T(0,\tau) = T_o + \frac{2K \, \Delta A}{h t_F} \sum_{i=1}^{\infty} \frac{\cos \beta}{\frac{\sin 2\beta}{2\beta} + 1} \int_0^{\tau} G(\lambda) e^{-\frac{K\beta_i^2}{h^2}(\tau-\lambda)} d\lambda \tag{5}$$

If $G(\tau)$ is a linear function of time, $B\tau$, the following expression is obtained by substituting the transform of $B\tau$ into the transformed equation (A8) before taking the inverse transformation:

$$T(0,\tau) = T_o + B \left\{ \tau - \frac{h^2}{K} \left[\frac{t_F}{h \, \Delta A} + \frac{1}{2} - \frac{2h \, \Delta A}{t_F} \sum_{i=1}^{\infty} \frac{\cos \beta}{\beta_i^4 \left(\frac{\sin 2\beta}{2\beta} + 1 \right)} e^{-\frac{K\beta_i^2}{h^2}\tau} \right] \right\} \tag{6}$$

The series in this equation is more rapidly convergent than the one that would be obtained by substituting $G(\tau) = B\tau$ in equation (5).

Conduction and Radiation

If radiant heat transmission is to be considered, the heat balance must include net radiant heat exchange as well as conduction. Net radiant heat exchange between two isothermal surfaces A_1 and A_2 (as viewed from A_1) can be computed from the expression

$$q_{1,2} = -\sigma\epsilon A_1 F'_{1,2}\left(T_1{}^4 - T_2{}^4\right) \tag{7}$$

where:

σ Stefan-Boltzmann constant

ϵ emissivity

The factor $F'_{1,2}$ is a function of the geometric relation between surfaces A_1 and A_2. This geometric relation is referred to as the configuration factor and is defined as the fraction of the total radiant flux leaving A_1 that is incident on A_2. (See ref. 2.) For the core, the area in the above relation can be expressed as $P\ dx$, where P is the perimeter of the core at a cross section parallel to the faces; therefore equations (1) and (7) can be combined in the following form to include the effects of radiation:

$$\frac{1}{K}\frac{\partial T(x,\tau)}{\partial \tau} = \frac{\partial^2 T(x,\tau)}{\partial x^2} - \frac{\sigma\epsilon P}{k\ \Delta A}\left[T^4(x,\tau) - \int_0^h \varphi_1(x,\xi)\ T^4(\xi,\tau)\ d\xi - \varphi_2(x)\ T^4(h,\tau) - \varphi_3(x)\ T^4(0,\tau)\right] \tag{8}$$

The symbols φ_i denote configuration factors. The terms in brackets represent the net radiant heat exchange at any point; the first term gives the heat emitted from the point, the second term is the heat flux from all other points of the core to the point in question, and the last two terms are the heat flux from the faces to the point.

The initial temperature (eq. (2a)) and the boundary condition at the heated face (eq. (2b)) are unchanged, but the boundary condition at the unheated face becomes:

$$\frac{1}{K}\frac{\partial T(0,\tau)}{\partial \tau} = \frac{\Delta A}{t_F}\frac{\partial T(0,\tau)}{\partial x} - \frac{\sigma\epsilon}{kt_F}\left[T^4(0,\tau) - \int_0^h \varphi_4(\xi)\, T^4(\xi,\tau)\, d\xi - \varphi_5\, T^4(h,\tau)\right] \tag{9}$$

where φ_i again denotes the configuration factors and the terms in brackets represent the net radiant heat flux at the unheated face.

Since no exact methods of solution for equations of this type are known, a finite-difference procedure will be used. This is done by taking three stations in the core and one station at each face. (See fig. 1.) Now equation (8) and its boundary conditions can be expanded by finite differences to a system of four ordinary differential equations. In order to expand the radiation terms of equation (8) into finite-difference form, let $F_{n,m}$ be the fraction of radiant flux leaving all faces at station n which is incident on all faces at station m, multiplied by the area of all the faces at station n, based on unit area of the face plates. Expansion of the radiation terms in equations (8) and (9) into finite differences then gives for each element n a term

$$Q_n = -\sigma\epsilon \sum_{m=1}^{5} F_{n,m}\left(T_n{}^4 - T_m{}^4\right) \tag{10}$$

Therefore, in terms of finite differences, equation (8) and its boundary conditions become

$$T_1 = T_0 + G\left(\frac{h^2\tau}{9K}\right) \tag{11a}$$

$$\frac{dT_2}{d\bar{\tau}} = 2T_1 - 3T_2 + T_3 - \frac{\sigma\epsilon h}{3k\,\Delta A}\sum_{m=1}^{5} F_{2,m}\left(T_2{}^4 - T_m{}^4\right) \tag{11b}$$

$$\frac{dT_3}{d\bar{\tau}} = T_2 - 2T_3 + T_4 - \frac{\sigma\epsilon h}{3k\,\Delta A}\sum_{m=1}^{5} F_{3,m}\left(T_3{}^4 - T_m{}^4\right) \tag{11c}$$

$$\frac{dT_4}{d\bar{\tau}} = T_3 - 3T_4 + 2T_5 - \frac{\sigma\epsilon h}{3k\ \Delta A}\sum_{m=1}^{5} F_{4,m}\left(T_4{}^4 - T_m{}^4\right) \tag{11d}$$

$$\frac{dT_5}{d\bar{\tau}} = \frac{h\ \Delta A}{3t_F}\left(2T_4 - 2T_5\right) - \frac{h\ \Delta A}{3t_F}\frac{\sigma\epsilon h}{3k\ \Delta A}\sum_{m=1}^{4} F_{5,m}\left(T_5{}^4 - T_m{}^4\right) \tag{11e}$$

where

$$\bar{\tau} = \frac{9K\tau}{h^2}$$

and the initial conditions are

$$\bar{\tau} = 0 \qquad T_n = T_o$$

Equations of this type are readily solved on an analog computer equipped to generate fourth powers of the dependent variable.

The solution of equations (11) can be made considerably more general by using a nondimensional temperature parameter $\bar{T}_n$ which is defined by the following relation:

$$T_n = \gamma\left(\frac{3k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}\bar{T}_n \tag{12}$$

where γ is a constant which may be selected so that the problem falls in the desired computer range of temperature.

Substituting this expression for T_n into equations (11) and dividing by $\gamma\left(\frac{3k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}$ gives

$$\bar{T}_1 = \bar{T}_o + \frac{G\left(\frac{h^2\bar{\tau}}{9K}\right)}{\gamma\left(\frac{3k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}} \tag{13a}$$

$$\frac{d\bar{T}_2}{d\bar{\tau}} = 2\bar{T}_1 - 3\bar{T}_2 + \bar{T}_3 - \gamma^3 \sum_{m=1}^{5} F_{2,m}\left(\bar{T}_2{}^4 - \bar{T}_m{}^4\right) \tag{13b}$$

$$\frac{d\bar{T}_3}{d\bar{\tau}} = \bar{T}_2 - 2\bar{T}_3 + \bar{T}_4 - \gamma^3 \sum_{m=1}^{5} F_{3,m}\left(\bar{T}_3{}^4 - \bar{T}_m{}^4\right) \tag{13c}$$

$$\frac{d\bar{T}_4}{d\bar{\tau}} = \bar{T}_3 - 3\bar{T}_4 + 2\bar{T}_5 - \gamma^3 \sum_{m=1}^{5} F_{4,m}\left(\bar{T}_4{}^4 - \bar{T}_m{}^4\right) \tag{13d}$$

$$\frac{d\bar{T}_5}{d\bar{\tau}} = \frac{h\ \Delta A}{3t_F}\left(2\bar{T}_4 - 2\bar{T}_5\right) - \frac{h\ \Delta A}{3t_F}\gamma^3 \sum_{m=1}^{4} F_{5,m}\left(\bar{T}_5{}^4 - \bar{T}_m{}^4\right) \tag{13e}$$

with the initial condition

$$(\bar{T}_n)_{t=0} = \bar{T}_o = \frac{T_o}{\gamma\left(\frac{3k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}}$$

An approximate procedure for calculating the temperature of the unheated face is presented in appendix B.

Thermal Stresses

Temperature differences between faces of sandwich panels cause unequal expansion of the faces and result in thermal stresses. An indication of the magnitude of these stresses can readily be obtained if the following assumptions are made:

(1) Material properties are independent of temperature.

(2) The core is rigid in shear.

(3) The faces remain plane and all edges are free to expand.

(4) The core takes no load.

The stress-strain relations are

$$\begin{aligned} \epsilon &= \frac{\sigma_1}{E_1} + \alpha T_1 \\ &= \frac{\sigma_F}{E_F} + \alpha T_F \end{aligned} \tag{14}$$

where

ϵ strain

σ_1 stress in heated face

σ_F stress in unheated face

E modulus of elasticity

α coefficient of thermal expansion

T_F temperature of the unheated face, T_5 or $T(0,\tau)$

For equilibrium,

$$t_1\sigma_1 + t_F\sigma_F = 0 \tag{15}$$

Therefore, the stresses in the faces are

$$\left.\begin{aligned} \sigma_1 &= -\frac{E\alpha\,\Delta T}{1 + \frac{t_1}{t_F}} \\ \sigma_F &= \frac{E\alpha\,\Delta T}{1 + \frac{t_F}{t_1}} \end{aligned}\right\} \tag{16}$$

where:

$$\Delta T = T_1 - T_F$$

RESULTS AND DISCUSSION

Solutions were obtained for all combinations of the following parameters:

h, in.	0.2, 0.3, 0.4, 0.5
ΔA	0.02, 0.025, 0.03, 0.04
t_F, in.	0.01, 0.02, 0.03, 0.05
B, $^{\circ}R$/sec	5, 10, 20, 50
$c\rho$, Btu/(ft^3)($^{\circ}$R)	50
T_0, $^{\circ}$R	500
k, (Btu)(ft)/(hr)($^{\circ}$R)(ft^2)	12.5

In all cases the face thicknesses listed here were modified in the calculations to include 0.002 inch of face material as the thermal equivalent of the braze or other material used to fasten the core to the face. For cases in which radiation is considered, the core is assumed to consist of 1/4-inch-square cells with emissivity equal to 0.8. Configuration factors used in the calculations are given in table II.

Typical results are shown in figure 2. The curve identified as T_1 is the prescribed temperature of the heated face, which in this case experiences a temperature rise B of 10° R per second. The curve marked $T_{F,C}$ is the temperature response of the unheated face calculated from equation (5), which applies when radiation is neglected. A comparison of this curve with results calculated numerically and by analog computation from equations (13) without the radiation terms revealed differences too small to be shown on this plot. The curve identified as T_F is the temperature of the unheated face calculated from equations (13).

As would be expected, the temperature difference between the heated face and the unheated face is more when radiation is neglected. The temperature difference is essentially constant at its maximum value after 150 seconds when radiation is neglected. (From eq. (6) it is obvious that the maximum occurs as time approaches infinity.)

The maximum temperature difference with conduction only can be calculated by subtracting equation (6) from equation (2b). As time becomes large, this difference becomes

$$\left(\Delta T_{max}\right)_C = \frac{Bh^2}{K}\left(\frac{t_F}{h\,\Delta A} + \frac{1}{2}\right) \qquad (17)$$

The maximum temperature difference with radiation and conduction was calculated on an analog computer from equation (11). When these maximum temperature differences were plotted with $\left(\Delta T_{max}\right)_C$ as abscissa and ΔT_{max} as ordinate for each case listed at the beginning of this section, all the points lay within the envelope shown in figure 3.

Since all the points lie in a narrow band, regardless of the values of the parameters, it appears that the effect of radiation depends chiefly on the magnitude of the temperature difference which would be obtained by considering conduction only.

The dashed line in figure 3 represents the data in the interval

$$120 < \left(\Delta T_{max}\right)_C < 3{,}000$$

with a maximum error of about 10 percent.

An approximate equation for this line is

$$\Delta T_{max} = 21.9\sqrt{\left(\Delta T_{max}\right)_C} - 135 \tag{18}$$

If equation (17) is substituted for $\left(\Delta T_{max}\right)_C$ in equation (18), the following expression is obtained for the maximum temperature difference when both radiation and conduction are considered:

$$\Delta T_{max} = 21.9h\sqrt{\frac{B}{K}\left(\frac{t_F}{h\ \Delta A} + \frac{1}{2}\right)} - 135 \qquad \left(120 < \left(\Delta T_{max}\right)_C < 3{,}000\right) \tag{19}$$

Dimensionless Analysis

Calculated maximum temperature differences are presented in dimensionless form in figure 4. The maximum dimensionless temperature difference is plotted as a function of the ratio $h\ \Delta A/t_F$ for several values of dimensionless temperature-rise rate. Each plot is for constant initial dimensionless temperature, since

$$\overline{T}_o = \frac{T_o}{\gamma}\left(\frac{3k\ \Delta A}{\sigma \epsilon h}\right)^{-1/3} \tag{20}$$

Also, each plot is for a fixed value of the ratio h/S, where S is the width of the square cell.

On the logarithmic scale used, the curve of maximum dimensionless temperature difference varies linearly with the ratio $h \, \Delta A/t_F$, and the slopes generally decrease as the temperature-rise rate increases. However, it was not possible to derive a mathematical relation between the slopes of the temperature-difference curves and the temperature-rise rates which would lead to a simpler empirical approximation to the maximum temperature difference than that given by equation (19). Figure 4 can be used to determine maximum temperature differences with greater accuracy than is possible from equation (19).

Correlation of Theory and Experiment

The theory from which the results given in the present paper were obtained assumes that no heat is lost from the unheated face of the sandwich. There is a scarcity of data on sandwich panels tested under conditions which match the theoretical assumptions. However, in a test made in the Langley Structures research laboratory a light-weight steel honeycomb sandwich panel experienced a temperature-rise rate of 19.2° R per second at the heated face and experienced a radiant heat loss at the unheated face of the sandwich. Heat was transferred from the unheated face of the sandwich by radiation across an air gap to a steel backing plate. The sandwich panel properties were $t_F = 0.005$ inch, $w = 7.2 \text{ lb/ft}^3$, and $h = 0.3$ inch. Test results are shown by the data points in figure 5. The prescribed temperature of the heated face is shown by the curve identified as T_1. In order to compute the temperature histories of the sandwich faces, it was necessary to modify the analysis to account for the radiant heat loss at the unheated face. It was therefore assumed that the unheated face of the sandwich panel radiated to a plate which remained at room temperature. The emissivities of the backing plate and the unheated face of the sandwich were assumed to be 0.6 and the internal emissivity of the sandwich was assumed to be 0.8. The calculated temperatures of the unheated face of the sandwich are shown by the solid line identified as T_F and are in excellent agreement with the test results for the first 50 seconds. After that time, the temperature of the backing plate used in the test began to rise and thus invalidated the theoretical assumption of constant temperature and permitted no further basis for comparison. In order to show the effect of radiation, a curve of calculated temperature is included for the case in which radiation is neglected.

Thermal Stresses

In order to give more meaning to the magnitudes of the temperature differences between the faces of a sandwich panel, a simplified elementary stress analysis was made. If it is assumed that the panel remains flat, that the core takes no load, and that there is no external restraint to expansion, the thermal stresses for equal face thicknesses are, from equations (16),

$$\sigma = \frac{E\alpha \, \Delta T}{2} \tag{21}$$

with the heated face in compression and the unheated face in tension. Of course, the analysis overestimates the thermal stresses which actually exist; nevertheless, it provides a convenient means of evaluating the effect of internal radiation. For steel the product $E\alpha$ is about 300 and thermal stresses are

$$\sigma = \pm 150 \, \Delta T \tag{22}$$

Therefore, a plot with maximum temperature difference as the ordinate can be readily converted into a plot with maximum thermal stress as the ordinate. If the temperature differences in figure 3 are converted into thermal stresses, the lowest stress obtained is 15,000 psi. This result indicates that large thermal stresses are to be expected when sandwich panels are heated. It appears from equation (19) that high core density, thin unheated faces, and thin panels will be necessary in order to prevent these stresses from becoming prohibitive.

However, if the ratio of face thicknesses t_1/t_F is increased, compressive stresses diminish and tensile stresses increase. Compressive stresses can be reduced by any desired amount without increasing tensile stresses by more than a factor of 2. This effect will now be considered for a specific example. Take a steel honeycomb panel with $h = 0.3$ inch, $B = 20^{\circ}$ F/sec, and $w = 20$ lb/ft^3. Assume that the panel is to be used in an application requiring a total face thickness of 0.1 inch; that is, $t_1 + t_F = 0.1$. The temperature information necessary for evaluation of the thermal stresses in this example can be calculated from equation (19). It should be noted that, since the temperature of the heated face is prescribed, its thickness does not affect the temperature difference. The face thicknesses given here are the actual face thicknesses and do not include a modification to account for braze material. The stresses are given by the equations

$$\left.\begin{aligned} \sigma_{1,max} &= -3{,}000 \, \Delta T_{max} \, t_F \\ \sigma_{F,max} &= 3{,}000 \, \Delta T_{max}(0.1 - t_F) \end{aligned}\right\} \tag{23}$$

W

Thermal stresses calculated from these equations are plotted against unheated-face thickness in figure 6. If the two faces have the same thickness, the thermal stress in each face is 75,000 psi. Decreasing the unheated-face thickness from 0.05 inch to 0.036 inch and increasing the heated-face thickness to 0.064 inch reduces the stresses in the heated face from 75,000 psi to 46,000 psi. Stresses in the unheated face increase from 75,000 to 80,000 psi. Further reduction of the unheated-face thickness will reduce thermal stresses in both faces.

CONCLUDING REMARKS

Calculations of temperature distributions in sandwich panels, even when internal radiation is present, are not difficult if analog computing equipment is available. If internal radiation is present, the accuracy of an analysis of maximum temperature difference between the top and bottom faces of the panel, based on conduction only, decreases as the temperature difference increases.

If the temperature-rise rate of the heated face of the sandwich is linear, a simple empirical relation exists between the maximum temperature difference between the top and bottom faces of the panel and the maximum temperature difference calculated when only conduction is considered. Approximate temperature differences can be calculated by considering the separate effects of radiation and conduction.

Even with relatively moderate heating, large temperature gradients exist in sandwich panels and cause large thermal stresses. For a given total face thickness, however, the magnitude of thermal stresses can be materially reduced by decreasing the unheated-face thickness while increasing the heated-face thickness by an equal amount.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
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APPENDIX A

CONDUCTION OF HEAT IN SANDWICH PANELS

The differential equation for the linear conduction of heat is

$$\frac{1}{K} \frac{\partial T(x,\tau)}{\partial \tau} = \frac{\partial^2 T(x,\tau)}{\partial x^2} \tag{A1}$$

The initial condition is $T(x,0) = T_0$. The temperature of the heated face, $x = h$, is prescribed as:

$$T(h,\tau) = G(\tau) \tag{A2}$$

The rate at which heat leaves the core and enters the unheated face is

$$q = k \; \Delta A \frac{\partial T(0,\tau)}{\partial x} \tag{A3}$$

Inasmuch as it is assumed that the external surface of the unheated face is insulated, the temperature-rise rate of this face is proportional to the rate at which heat leaves the core:

$$q = c\rho t_F \frac{\partial T(0,\tau)}{\partial \tau} \tag{A4}$$

Equating these expressions for q gives

$$\frac{\partial T(0,\tau)}{\partial x} = \frac{t_F}{K \; \Delta A} \frac{\partial T(0,\tau)}{\partial \tau} \tag{A5}$$

These equations are readily solved by means of the Laplace transformation. Let

$$T'(x,\tau) = T(x,\tau) - T_0$$

The transformed equations are

$$\frac{d^2\theta(x,s)}{dx^2} - \frac{s}{K}\,\theta(x,s) = 0 \tag{A6a}$$

$$\theta(h,s) = g(s) \tag{A6b}$$

$$\frac{\partial\theta(0,s)}{\partial x} = \frac{st_F}{K\,\Delta A}\,\theta(0,s) \tag{A6c}$$

where

$$\theta(x,s) = L\left[T'(x,\tau)\right]$$

$$g(s) = L\left[G(\tau)\right]$$

The solution of equation (A6a) is

$$\theta(x,s) = C \sinh\left(\sqrt{\frac{s}{K}}\,x\right) + D \cosh\left(\sqrt{\frac{s}{K}}\,x\right) \tag{A7}$$

where C and D are constants to be determined from equations (A6b) and (A6c). When C and D are evaluated, the transformed equation is

$$\theta(x;s) = g(s)\,\frac{\frac{t_F}{\Delta A}\sqrt{\frac{s}{K}}\sinh\left(\sqrt{\frac{s}{K}}\,x\right) + \cosh\left(\sqrt{\frac{s}{K}}\,x\right)}{\frac{t_F}{\Delta A}\sqrt{\frac{s}{K}}\sinh\left(\sqrt{\frac{s}{K}}\,h\right) + \cosh\left(\sqrt{\frac{s}{K}}\,h\right)} \tag{A8}$$

An inverse transform of the equation can be expressed immediately as a convolution of the arbitrary heating function with the other terms of the equation:

$$T'(x,\tau) = G(\tau) * L^{-1}\left[\frac{\frac{t_F}{\Delta A}\sqrt{\frac{s}{K}}\sinh\left(\sqrt{\frac{s}{K}}\,x\right) + \cosh\left(\sqrt{\frac{s}{K}}\,x\right)}{\frac{t_F}{\Delta A}\sqrt{\frac{s}{K}}\sinh\left(\sqrt{\frac{s}{K}}\,h\right) + \cosh\left(\sqrt{\frac{s}{K}}\,h\right)}\right] \qquad (x < h) \qquad \text{(A9)}$$

where L^{-1} indicates the inverse transform. The expression within the brackets is single valued with simple poles, and by a formal application of the inversion integral it is found to be

$$\frac{2K}{h^2}\sum_{i=1}^{\infty}\frac{\beta \sin \beta(1 - x)}{\frac{\sin 2\beta}{2\beta} + 1}e^{-\frac{K\beta_i^{2}\tau}{h^2}} \qquad \text{(A10)}$$

where

$$\beta_i \tan \beta_i = \frac{h\,\Delta A}{t_F}$$

Therefore,

$$T(x,\tau) = T_o + \frac{2K}{h^2}\sum_{i=1}^{\infty}\frac{\beta \sin \beta(1 - x)}{\frac{\sin 2\beta}{2\beta} + 1}\int_0^{\tau} G(\lambda)e^{-\frac{K\beta_i^{2}}{h^2}(\tau-\lambda)}\,d\lambda \qquad (x < h) \qquad \text{(A11)}$$

The present paper is concerned with a linear temperature-rise rate at the heated face and solutions can be obtained by substituting $G(\tau) = B\tau$ into equation (A11). However, by substituting the linear temperature-rise rate into equation (A8) and again finding an inverse transform by a formal application of the inversion integral, the following equation is obtained:

$$T(x,\tau) = T_o + B\left[\tau + K\frac{t_F}{\Delta A}(x - h) + \frac{K}{2}\left(x^2 - h^2\right) + 2h^2K\sum_{i=1}^{\infty}\frac{\beta \sin \beta(1 - x)}{\beta_i^{4}\left(\frac{\sin 2\beta}{2\beta} + 1\right)}e^{-\frac{K\beta_i^{2}\tau}{h^2}}\right] \qquad \text{(A12)}$$

The series in this equation is rapidly convergent because of the appearance of β_n^{4} in the denominator.

APPENDIX B

ANALYTICAL APPROXIMATION WITH RADIATION AND CONDUCTION

A simple procedure is presented in this appendix for analytically approximating the temperature of the unheated face of a sandwich panel in which internal radiation is present. The following steps are involved:

(a) Calculate the temperature distribution, considering conduction only.

(b) Calculate the radiant heat transmission to the unheated face, assuming that the temperature distribution is that calculated in step (a). Convert this heat input into a temperature rise.

(c) Correct the temperature of the unheated face for the radiant heat input.

(d) Assume that the actual conduction to the unheated face is proportional to the difference between the temperature of the heated face and the temperature of the unheated face as calculated in step (c). Calculate the temperature rise of the unheated face caused by this heat input.

(e) Add the temperature rises calculated in steps (b) and (d) to obtain the final approximation to the temperature of the unheated face of the panel.

Corresponding to each of these steps are temperatures which will be designated as T_a, T_b, T_c, T_d, and T_e, respectively. The temperatures T_a are those calculated by considering conduction only (eq. (A11)). The radiant heat input to the unheated face (step (b)) can be calculated by integrating equation (10), using the temperatures T_a. This heat input can be converted to a temperature rise by dividing the integral of heat input by the heat capacity of the unheated face. This temperature rise is T_b.

The temperature of the unheated face is now (step (c)):

$$T_c = T_a + T_b$$

From step (d), the heat conducted to the unheated face is

$$q = \frac{k}{h}\left[(T_1)_a - (T_F)_a - (T_F)_b\right]$$

and the temperature rise caused by conduction satisfies the relation

$$q = c\rho t_F \frac{d(T_F)_d}{d\tau}$$

or

$$(T_F)_d = \frac{k}{hc\rho t_F} \int_0^\tau \left[(T_l)_a - (T_F)_c\right] d\lambda$$

The temperature of the unheated face is the sum of the temperature rises due to conduction and radiation:

$$T_e = T_b + T_d$$

Figure 7 compares results calculated on the analog computer from equation (13) with results calculated by the above procedure. The dashed line is the result obtained on the analog computer. The dash-dot line is $(T_F)_c$, which has a correction for the effect of radiation but does not consider any interaction between radiation and conduction. The solid line is $(T_F)_e$. The close agreement between the solid and dashed lines indicate that the approximate analysis is adequate up to the time at which the maximum temperature difference is reached.

REFERENCES

1. Carslaw, H. S., and Jaeger, J. C.: Conduction of Heat in Solids. The Clarendon Press (Oxford), 1947.

2. Hamilton, D. C., and Morgan, W. R.: Radiant-Interchange Configuration Factors. NACA TN 2836, 1952.

TABLE I.- THE FIRST FIVE ROOTS OF THE EQUATION $\beta_i \tan \beta_i = \frac{h\,\Delta A}{t_F}$

$\frac{h\,\Delta A}{t_F}$	β_1	β_2	β_3	β_4	β_5
0.01	0.0998	3.1448	6.2848	9.4258	12.5672
.06	.2425	3.1606	6.2927	9.4311	12.5711
.1	.3111	3.1731	6.2991	9.4354	12.5743
.2	.4328	3.2039	6.3148	9.4459	12.5823
.3	.5218	3.2341	6.3305	9.4565	12.5902
.4	.5932	3.2636	6.3461	9.4670	12.5981
.5	.6533	3.2923	6.3616	9.4775	12.6060
1.0	.8603	3.4256	6.4373	9.5293	12.6453

TABLE II.- OVERALL CONFIGURATION FACTORS

Factor (a)	$\frac{h}{S} = 0.8$	$\frac{h}{S} = 1.2$	$\frac{h}{S} = 1.6$	$\frac{h}{S} = 2.0$
$F_{2,1}$	0.407	0.527	0.620	0.676
$F_{2,3}$	.164	.300	.400	.495
$F_{2,4}$	.092	.133	.145	.125
$F_{2,5}$	.128	.100	.080	.090
$F_{3,1}$	.211	.220	.200	.177
$F_{5,1}$	.252	.152	.100	.040

[a]Equivalent factors are:

$$F_{2,1} = F_{5,4} = F_{4,5}$$
$$F_{2,3} = F_{3,2} = F_{3,4} = F_{4,3}$$
$$F_{2,4} = F_{4,2}$$
$$F_{2,5} = F_{5,2} = F_{4,1}$$
$$F_{3,1} = F_{3,5} = F_{5,3}$$

W

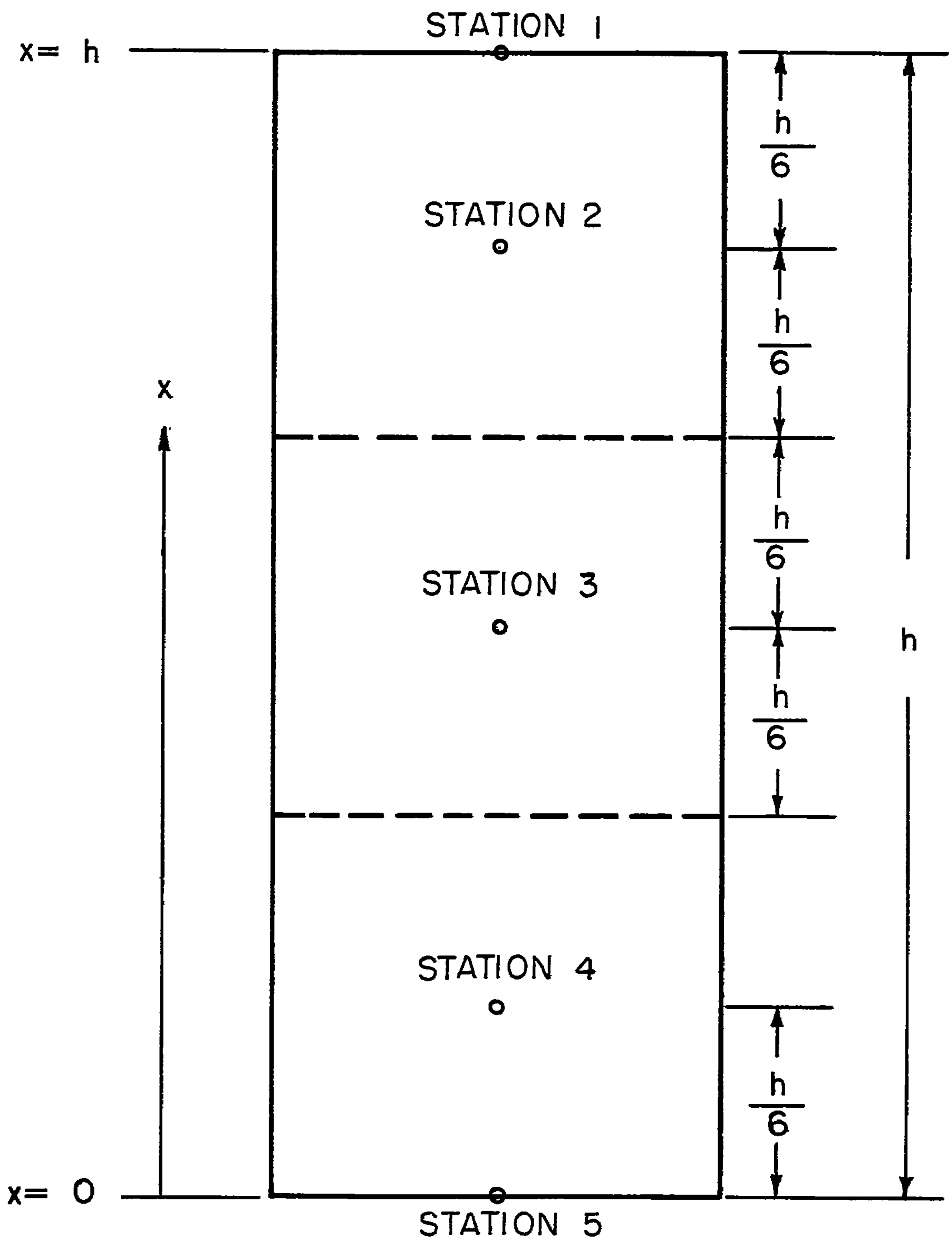


Figure 1.- Location of stations used in finite-difference analysis.

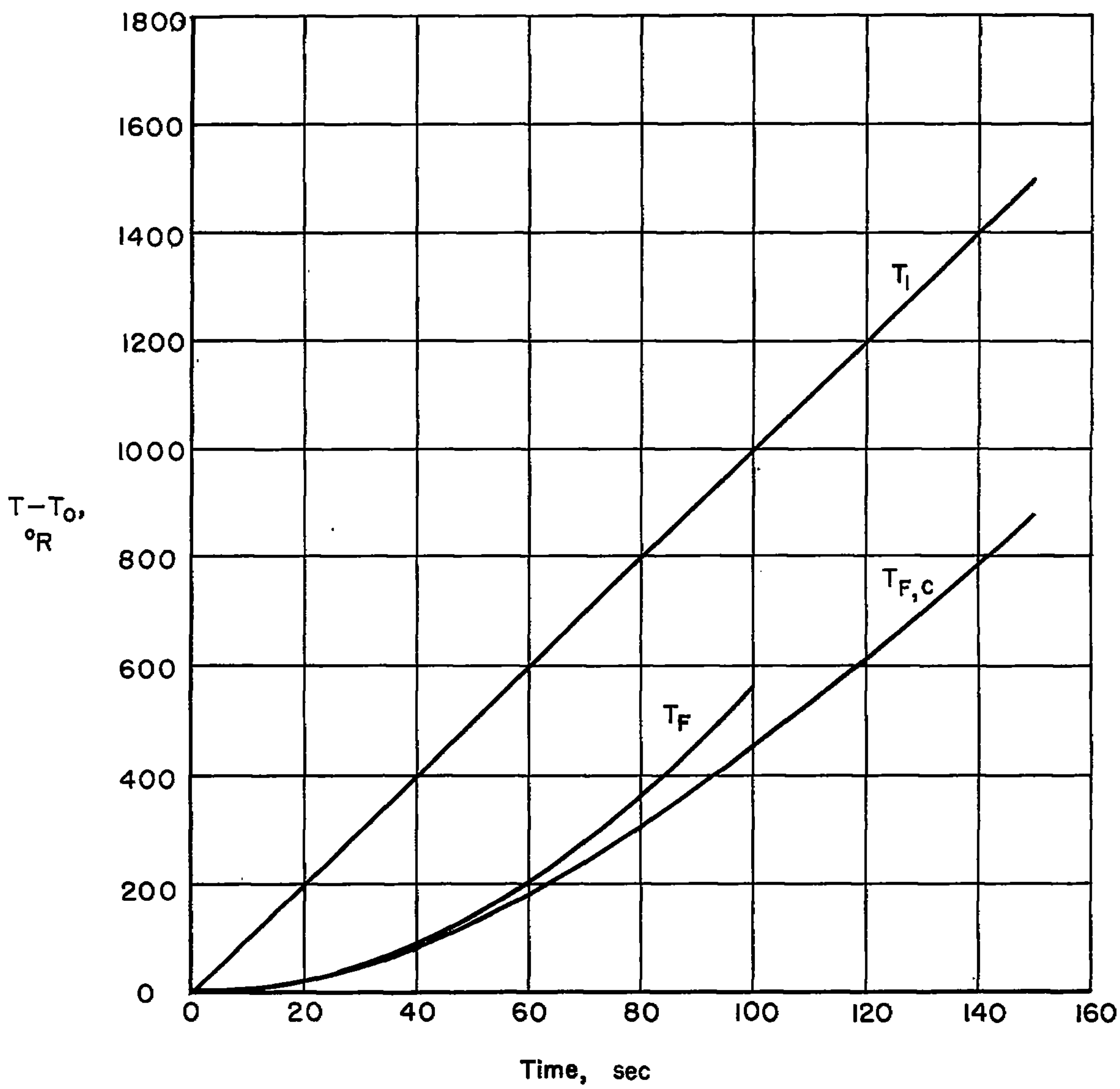


Figure 2.- Typical calculated results. $h = 0.5$ inch; $t_F = 0.03$ inch; $w = 15$ lb/ft^3; $B = 10^\circ$ R per second.

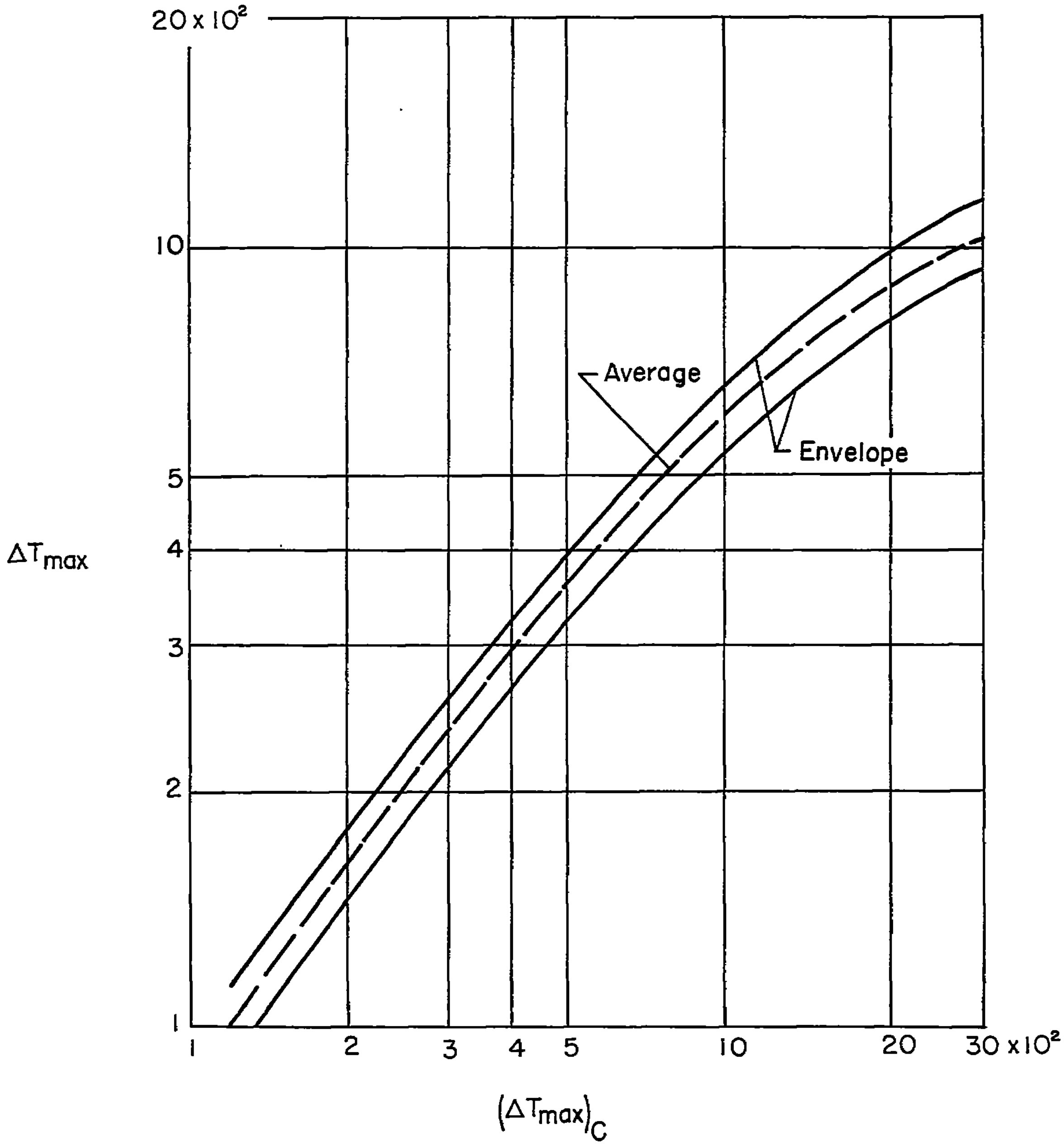
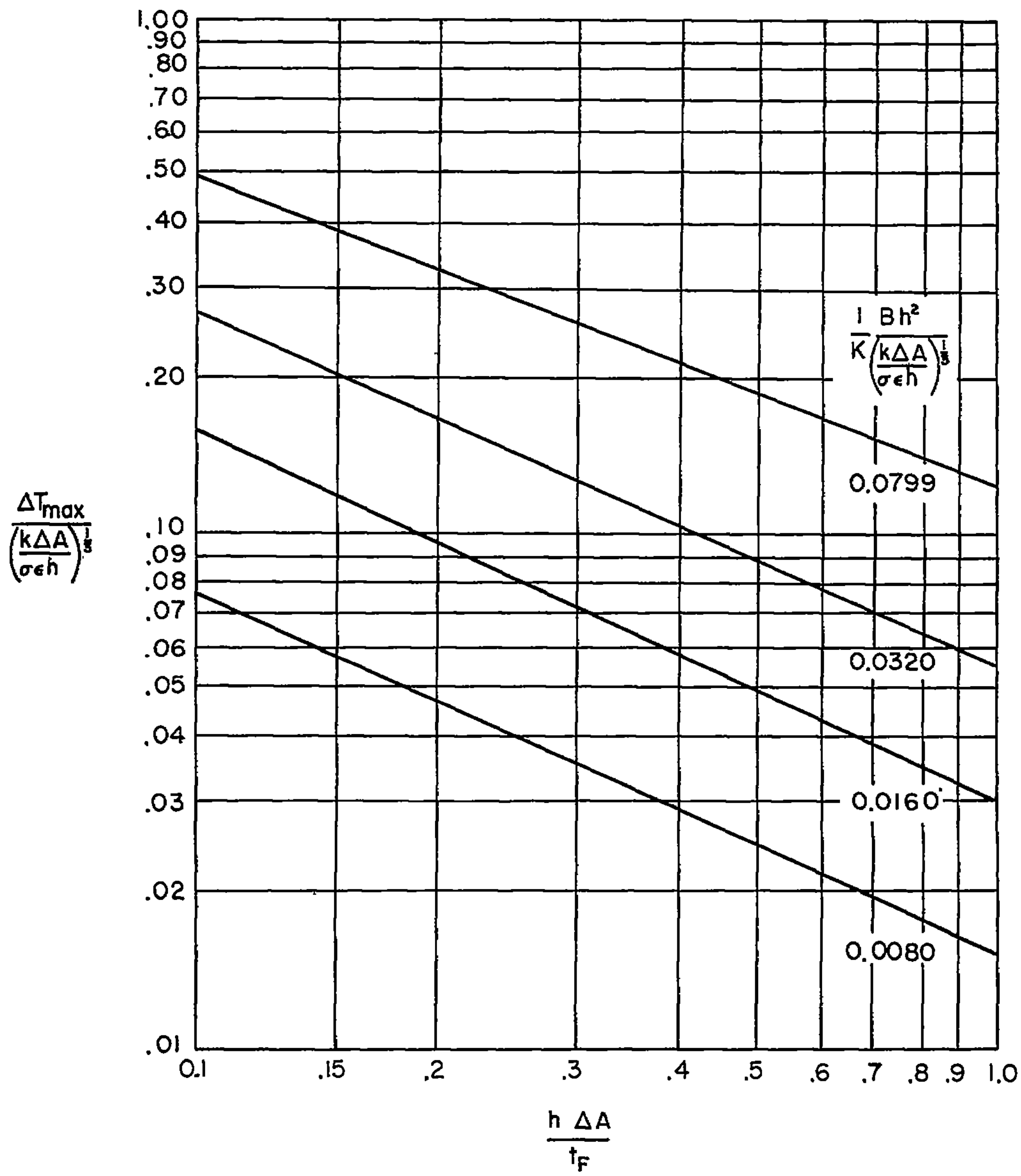
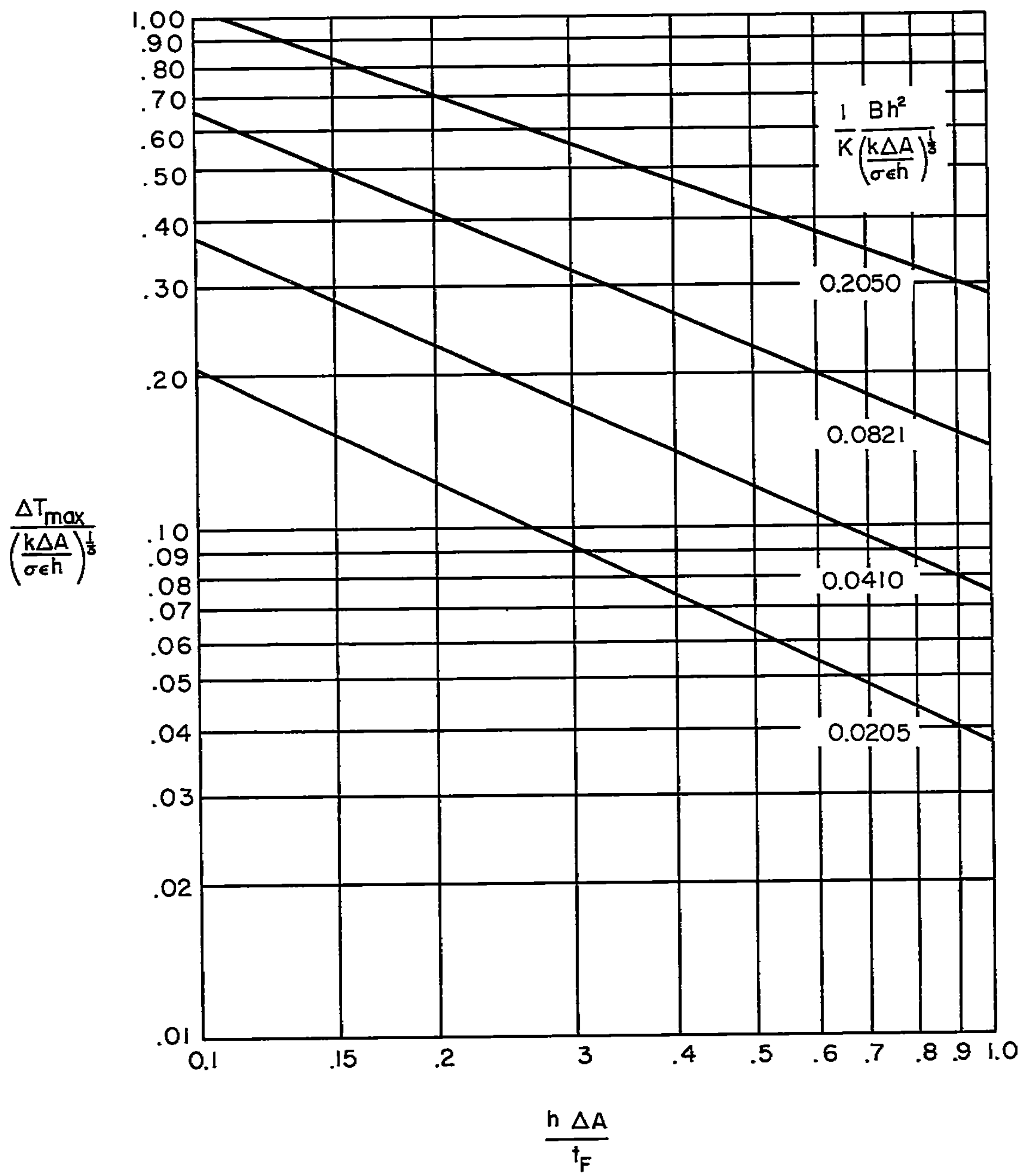


Figure 3.- Maximum temperature difference with radiation and conduction as a function of maximum temperature difference with conduction only.



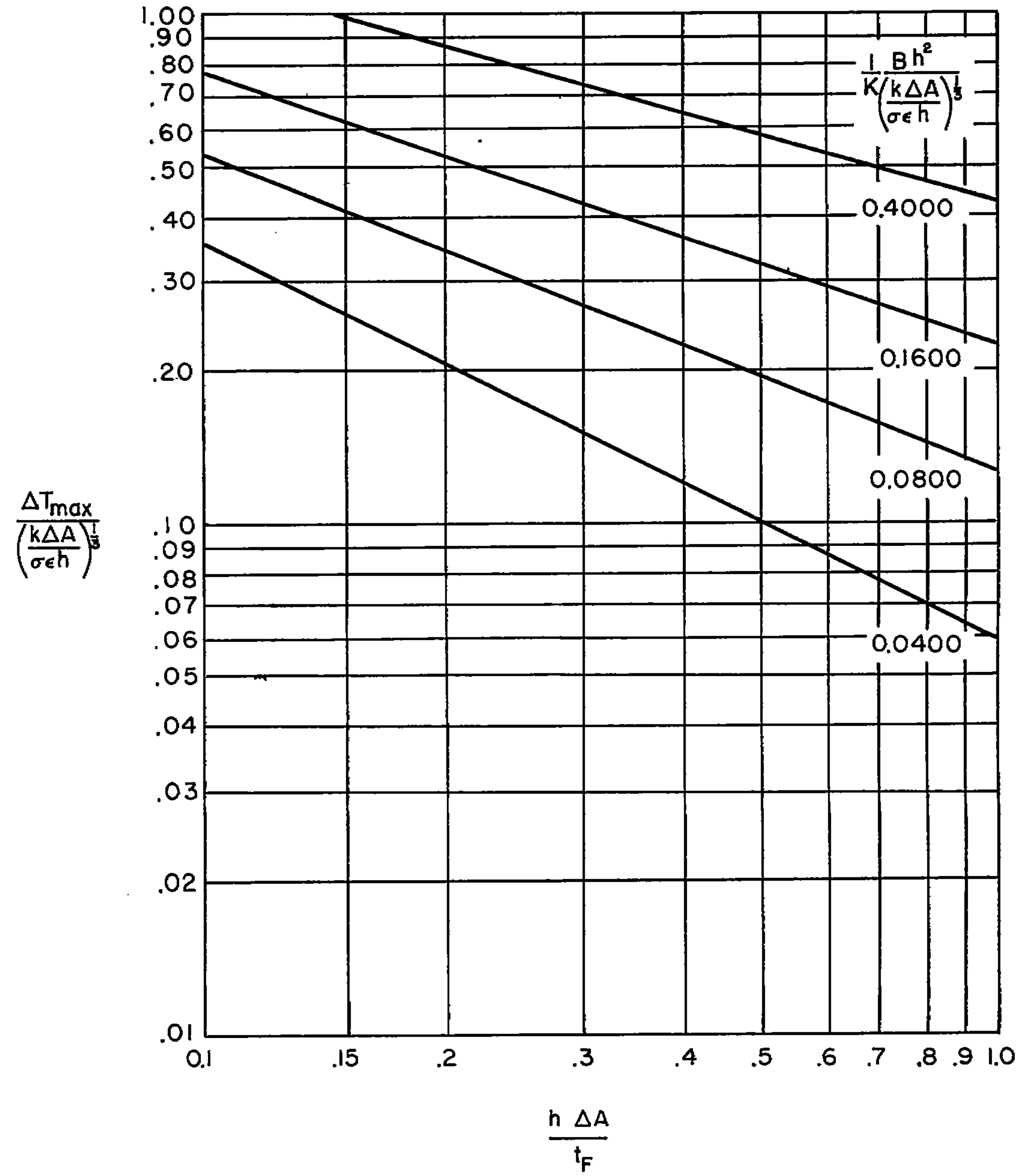
(a) $\frac{h}{S} = 0.8$; $\frac{T_o}{\left(\frac{k\ \Delta A}{\sigma \epsilon h}\right)^{1/3}} = 0.215$.

Figure 4.- Maximum dimensionless temperature difference as a function of $\frac{h\ \Delta A}{t_F}$.



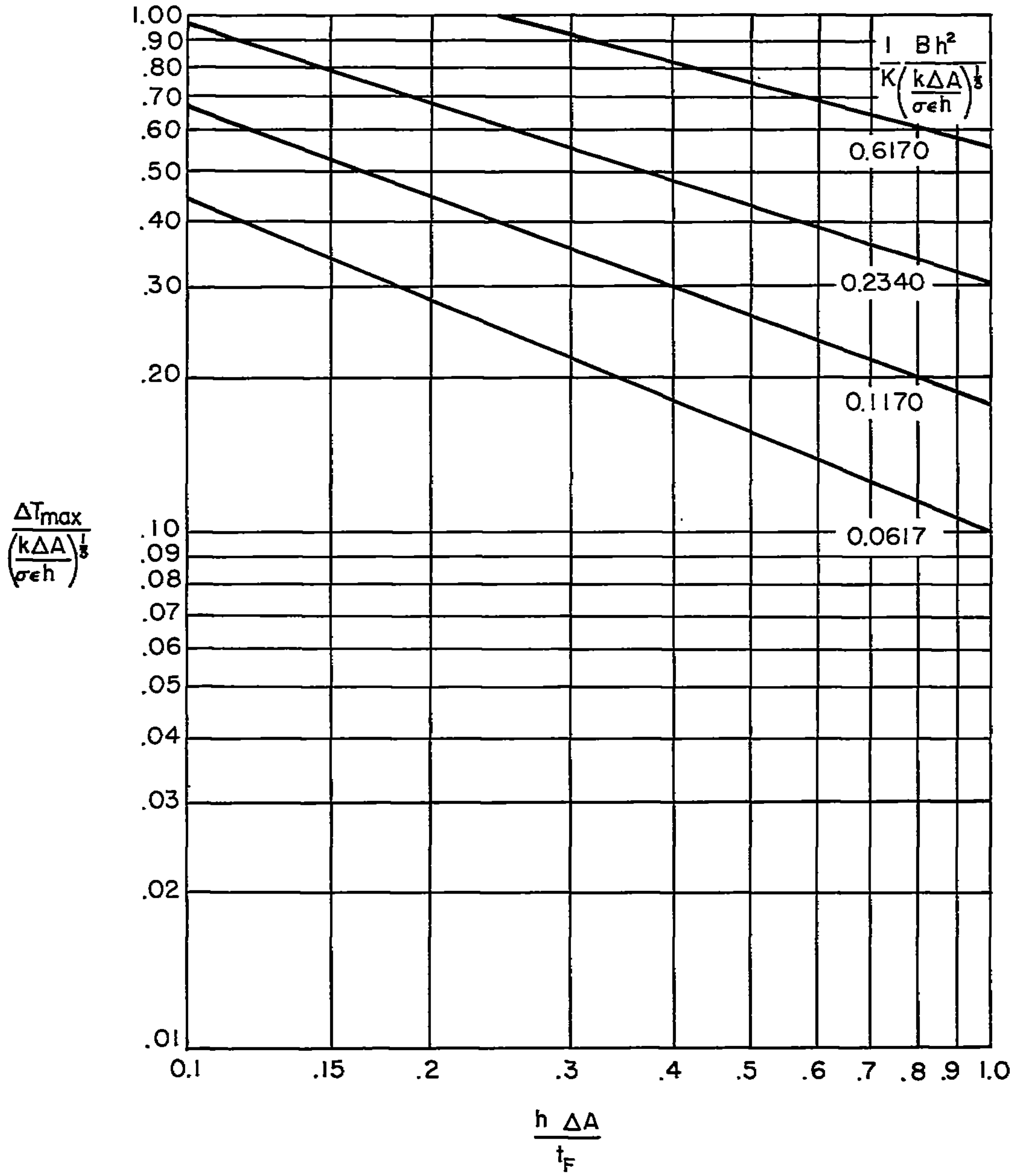
(b) $\frac{h}{S} = 1.2$; $\frac{T_o}{\left(\frac{k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}} = 0.247.$

Figure 4.- Continued.



(c) $\frac{h}{S} = 1.6;\ \frac{T_0}{\left(\frac{k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}} = 0.272.$

Figure 4.- Continued.



(d) $\frac{h}{S} = 2.0$; $\frac{T_o}{\left(\frac{k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}} = 0.292.$

Figure 4.- Concluded.

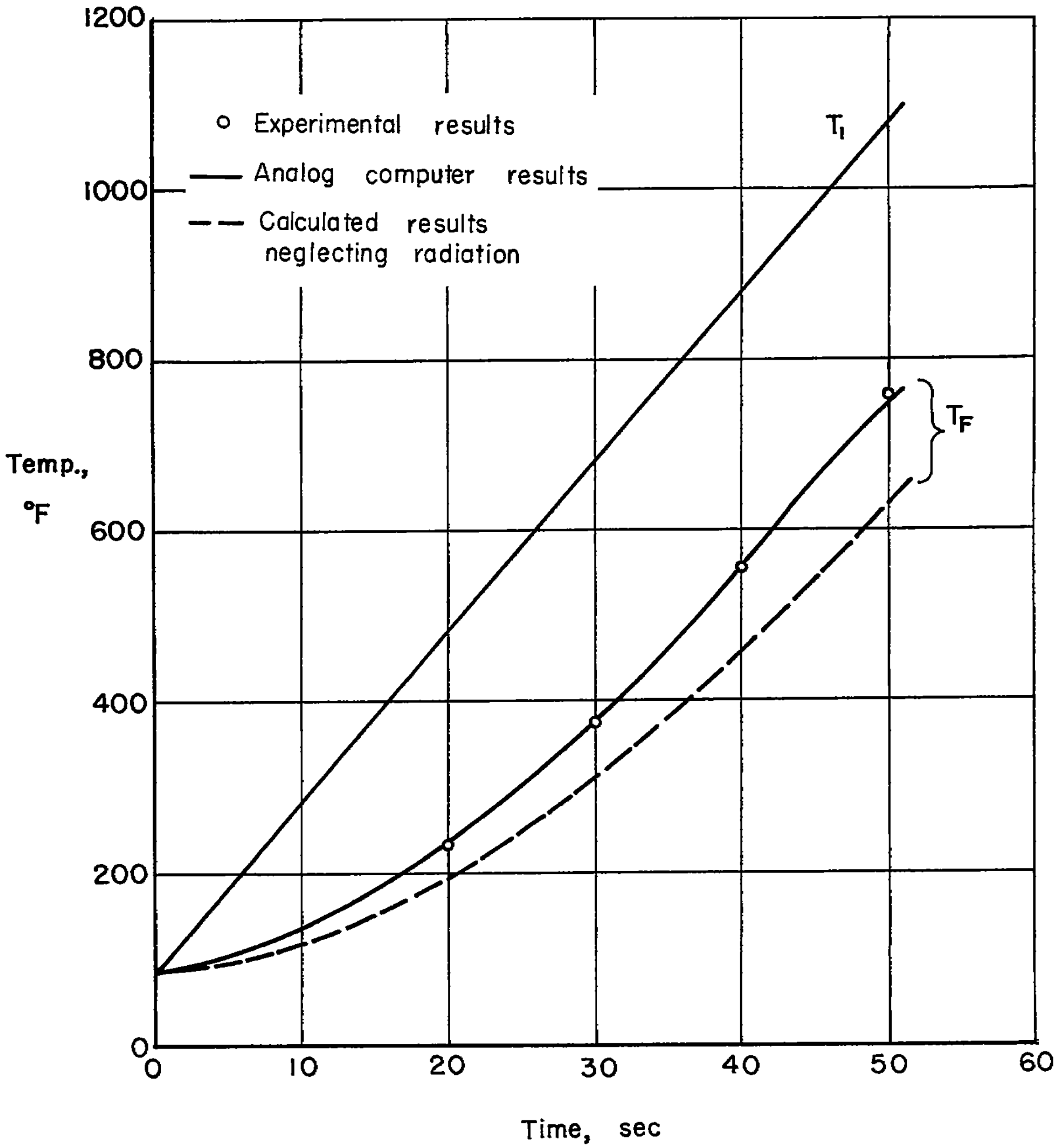


Figure 5.- Comparison of experimental and calculated results.

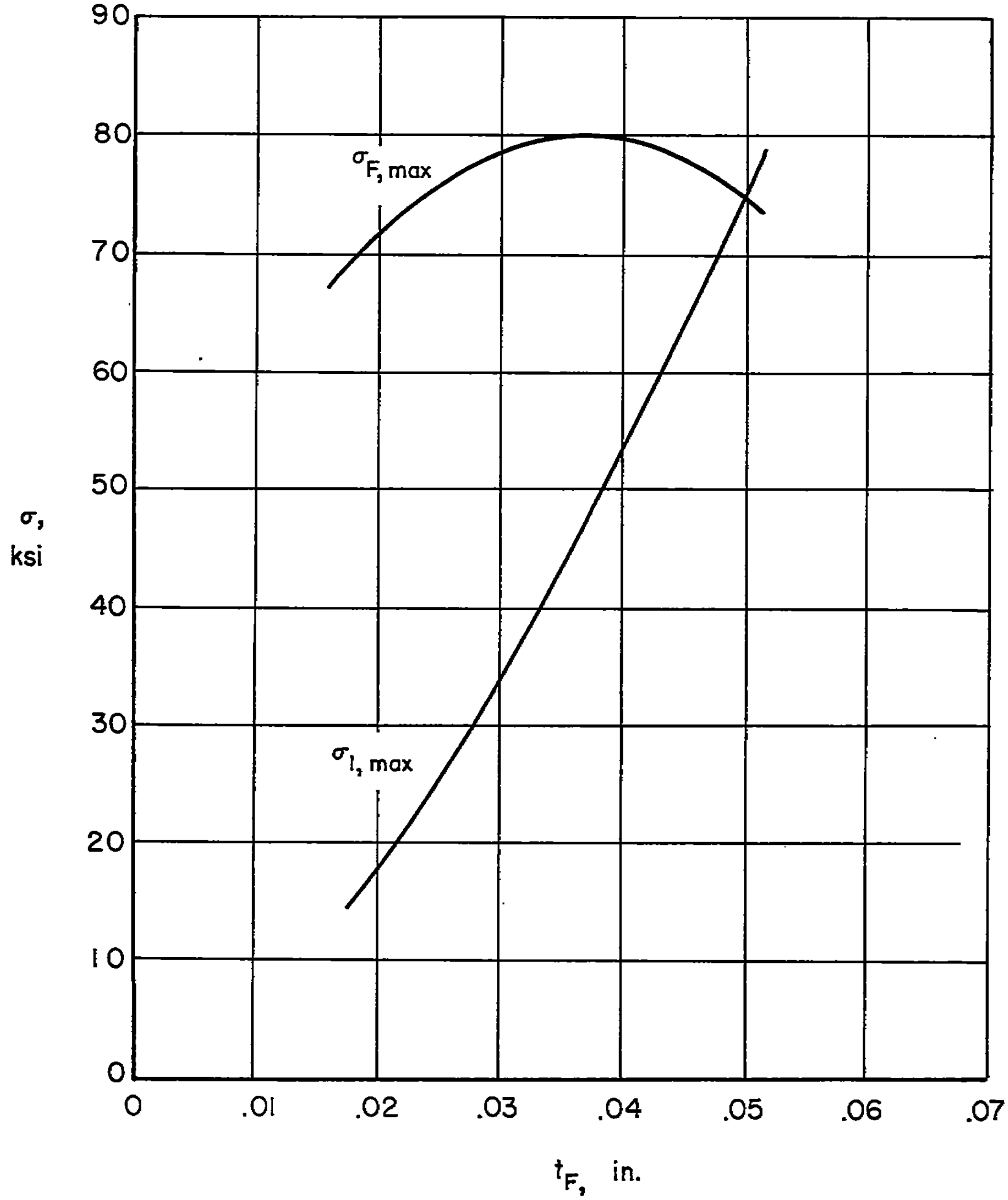


Figure 6.- Maximum thermal stress as a function of unheated-face thickness for constant total face thickness. $t_1 + t_F = 0.1$ inch; $h = 0.3$ inch; $w = 20$ lb/ft^3; $B = 20^\circ$ R per second.

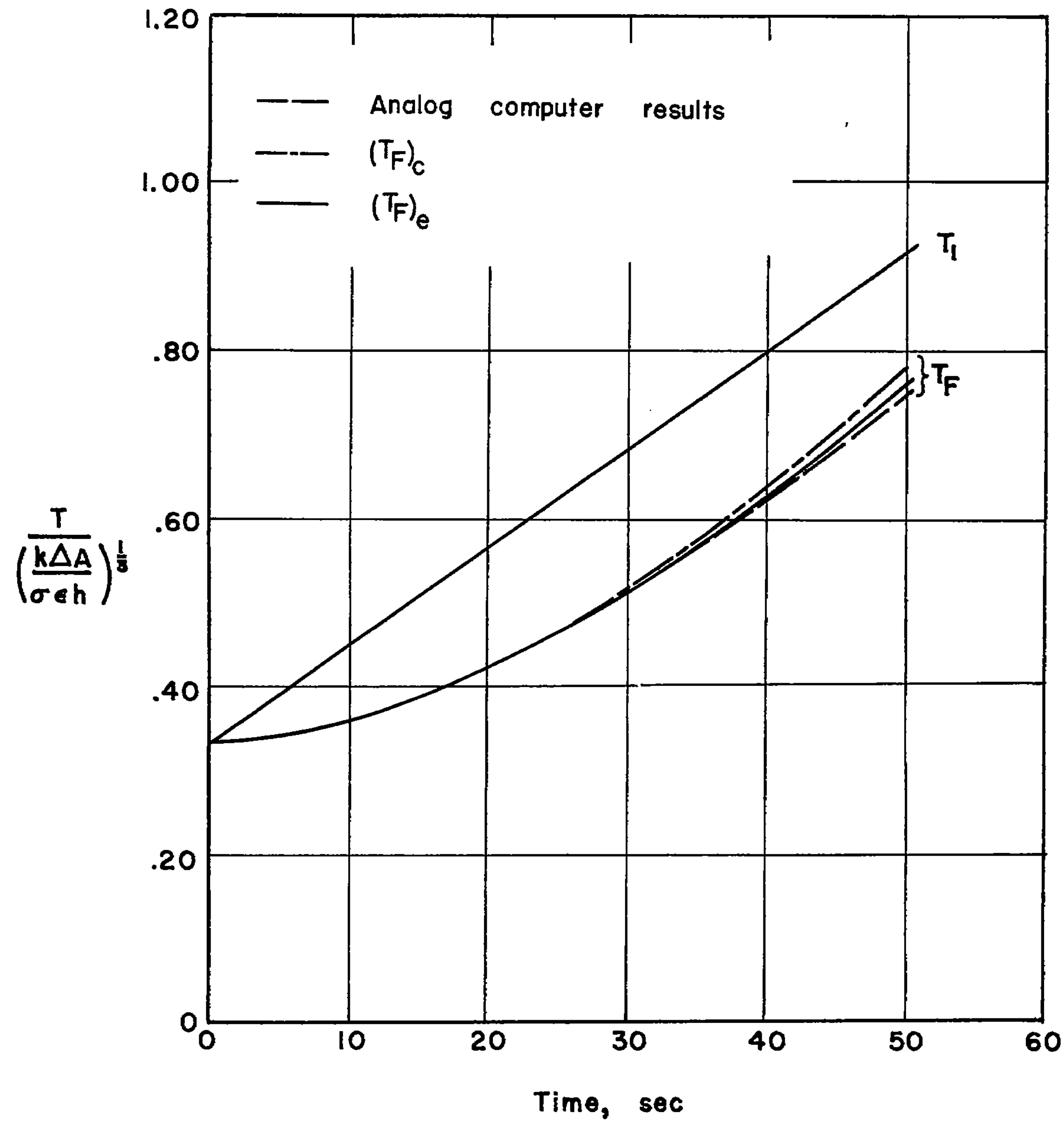


Figure 7.- Results of approximate analytical procedure.

$$\frac{1}{K}\frac{Bh^2}{\left(\frac{k\ \Delta A}{\sigma\epsilon h}\right)^{1/3}} = 0.2050;\ \frac{h\ \Delta A}{t_F} = 0.12;\ \frac{h}{S} = 1.2.$$